HANDBOOK *of* THE HISTORY OF LOGIC

VOLUME 4

BRITISH LOGIC IN THE NINETEENTH CENTURY

Edited by
Dov M. Gabbay
John Woods

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British Logic
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Edited by

Dov M. Gabbay
Department of Computer Science
King's College London
Strand, London, WC2R 2LS, UK

and

John Woods
Philosophy Department
University of British Columbia
Vancouver, BC Canada, V6T 1Z1
and
Department of Computer Science
King's College London
Strand, London, WC2R 2LS, UK
and
Department of Philosophy
University of Lethbridge
Lethbridge, Alberta, Canada, T1K 3M4

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PREFACE

The nineteenth century is widely and rightly held to be the century in which the mathematical revolution in logic achieved its breakthrough. W.V. Quine once remarked that logic is an ancient discipline, but since 1879 it has been a great one. Of course, 1879 marks the publication of Gottlob Frege's *Begriffsschrift*, and 1870 and 1883 the appearance of Charles Peirce's "Description of a Notation for the Logic of Relatives" and "Note B: The Logic of Relatives". Frege and Peirce are the independent co-founders of modern quantification theory. Frege (1848–1925) was a German and Peirce (1839–1914) an American (their contributions are chronicled in volume three of this *Handbook, The Rise of Mathematical Logic: Leibniz to Frege*). Although Frege's work was little recognized and little appreciated by British logicians of the period — Russell was a late exception — important steps toward the mathematicization of logic were taken in Britain. Augustus De Morgan (1806–1871) made significant contributions to the logic of relatives, of which Peirce took respectful heed, and also to probability theory, an interest in which he did much to revive. Until 1847, De Morgan was virtually the lone force in the algebraicization of logic. Then George Boole (1815–1864) published *The Mathematical Analysis of Logic* which appeared on the same day as De Morgan's *Formal Logic* and in which he gives to logic a somewhat different algebraic twist.Boole also did valuable work on probability theory. Even the lesser figure, William Hamilton (1788–1856), had pertinent things to say about predicate quantifiers. But the fact remains that the revolution in logic was not put "over the top" by British logicians of the nineteenth century. Why, then, does the *Handbook of the History of Logic* make a place for an entire volume on this subject? The answer is that there is no better place than Britain to witness the demise of the old logic and the beginnings of the new.

Aristotle's logic — the old logic — was an immense achievement. Aristotle originated the logic of syllogisms, he gave some expression to a separate logic of immediate inference, and he made a number of attempts to extend the syllogistic to modal contexts. The syllogistic was Aristotle's most complete logic. Indeed the perfectability proof of the *Prior Analytics* is an almost sound demonstration of something like the completeness (in the modern sense) of syllogistic logic. Over the centuries, the syllogistic has had its critics. The Megarians and Stoics expressed reservations, as did legions of smart mediaeval logicians. The Renaissance was not a good time for logic, and Aristotle's supremacy was tested (or at least questioned) by the likes of Descartes and the Port Royal logicians, by Locke and Leibniz, by Kant and Hegel. All the same, the theory of syllogisms was logic's paradigm for over two millennia. It provided the essential framework which others would attempt to reinterpret or supplement with additional insights.

All this ended in the 19^{th} century. This was the century in which the hegemony of the syllogistic fell apart like a collapsing empire. It was not, however, a clean death. It was a lingering demise. Some of the century's more original logicians still paid their obeisances to the syllogistic even while they were in fact giving it no role, or no central role, in their own work. A case in point is J.S. Mill (1806–1873), who plights his great admiration for the syllogistic in his *Autobiography*, and who in *A System of Logic,* pretty much concedes deductive logic to Richard Whately (1887–1963), who in turn concedes it to Aristotle. But everything else in that significant work was directed to issues in which it can only have been obvious that the syllogistic would have no place. What makes the 19^{th} century interesting is that while the syllogism is losing its paramouncy, logic launches itself in genuinely new ways, not all of which flow directly to the waters that create the Peircean-Fregean tsunami of mathematical logic. An important additional development is the logic of science.

Squarely in the idealist tradition was the great Romantic poet, Samuel Taylor Coleridge (1772–1834). Hegel had taught that the absolute could be fully articulated, that everything worth knowing is knowable within a dialectical system in which truth is immanent. Coleridge developed a logic which deviated from Hegel's in two principal ways. His logic is foundationalist rather than coherentist; and the knowing subject is beyond conceptualization, and hence ineffable. Lest it be thought that Coleridge's logic is largely an historical curiosity, to say nothing of its being an eccentricity on its author's part, idealism was philosophically dominant in 19^{th} century Britain, and idealism retains a broadly Hegelian orientation not only in philosophy, but in logic as well.

George Bentham (1800–1884), a nephew of Jeremy, was a botanist of note, who took an interest in jurisprudence and logic. His *Outline of a New System of Logic* (1827) is a work of considerable importance. It expressly formulated for the first time the idea of predicate-quantification, the priority of which over Hamilton would in due course be established by Herbert Spencer, and is described by W.S. Jevons (1835–1882) somewhat breathlessly as the most important discovery in formal logic since Aristotle.

Whately's *Elements of Logic* (1826) is a solid and formally correct re-telling of deductive logic in the Aristotelian tradition to which Bentham's book was intended to be a critique, and which Whately ignored in subsequent editions. Whately played a large role in restoring logic to Oxford's curriculum after a period of shameful neglect. In his review of Whately, Mill defended the syllogistic against what he considered encroachments of Scottish philosophers who had proposed its displacement by inductive logic. Mill took this position notwithstanding that *A System of Logic* (1843) itself made a substantial contribution to inductive logic, and was the originator of the deductive-nomological model of explanation. We see in this a blend of something old and something new. In matters deductive, Mill sought no quarrel with the syllogistic. In matters non-deductive, he was in the descendent class of Francis Bacon, as was Mill's contemporary William Whewell (1794–1866), and *A System of Logic* brims with efforts to get at the logic of those inductions that underpin the experimental sciences. Despite his satisfaction with Whately's treatment of the old logic of deduction, Mill was in process of empiricizing it. He sought for deduction the only certitude and the only objectivity that a serious and deep commitment to empiricism could consistently allow. And, in the spirit of the Stoic skeptics, Mill would allow to syllogisms no non-circular place in human cogitation. Deductive logic would impose consistency constraints on consequence-drawing, but it would not give us positive principles of reasoning.

Whewell's approach to the logic of science differed significantly from Mill's. *Philosophy of the Inductive Sciences, founded upon Their History* (1840) expressly rejects the hypothetico-deductive claim that scientific hypotheses are discovered by mere guesswork, and argues that hypothetical entities lie properly in the ambit of induction.

The new work in deductive logic — the work of Boole, De Morgan, and Hugh MacColl (1837–1909) — facilitated the drift away from the syllogistic towards mathematical treatments the newly emerging symbolic logic. We see in this the convergence of three factors: the erosion of the syllogistic paradigm, the mathematicization of deduction, and the rise of inductive logic, not excluding the theory of probability.

Arguably Britain's leading idealist, F.H. Bradley's *The Principles of Logic* appeared in the same year (1883) as Frege's *Grundlagen* and Peirce's "Note B: The Logic of Relatives". One might be forgiven for thinking that, next to Frege and Peirce, Bradley is a backwards looking museum-piece. Bradley, the idealist, like Mill, the empiricist, has a principled aversion to the formalization of reasoning, but he also vigorously attacks the traditional syllogistic's analysis of propositions as a necessary composition of three elements or ideas — the subject-idea, the attribute, and the joining of these two ideas. In so doing, he anticipates difficulties Russell was to have in analyzing propositions as unities of mutually independent constituents, difficulties which crop up in Frege's attempt to preserve a sharp distinction between concepts and objects. Bradley also anticipates Russell's theory of descriptions, and Quine's extension of it to names, arguing that logically proper names are disguised general terms. Russell credits Bradley with the idea of analyzing general propositions as conditionals. Bradley himself thought a further reason to reject the traditional syllogistic was its failure to accommodate the logic of relations. It is therefore simply a misconception to think of Bradley's logic as hide-bound to the old ways.

Hugh MacColl was born in 1837, the first year of Victoria's long reign. In a number of papers he fostered developments that would take hold in the century to come, in Boolean algebra, modal, free and paraconsistent logic, as well as dialogue logic. Notwithstanding that these papers were vigorously — some would say unfairly — condemned by Russell, it is clear that MacColl anticipates the analysis of strict implication, whose precedence C.I. Lewis had the characteristic grace to acknowledge. More generally, it is now clear that MacColl's modal logic is equivalent to the system T of Robert Feys (1937) and the equivalent system of Georg von Wright (1951).

W.S. Jevons (1835–1882), best known as a pioneer of mathematical economics, published in 1874 a substantial and important monograph on logic under the title *The Principles of Science.* Its main achievement was the extension and clarification of the theory of induction developed by Whewell (and criticized by Mill). It knitted together a general theory of probability with the analysis of induction, and is arguably the single most important contribution to logical theory in 19^{th} century Britain.

Virtually every student of logic is familiar with Venn diagrams. They have an even wider currency than the De Morgan equivalences of propositional logic. In this, John Venn (1835–1882), a student of De Morgan's, was anticipated by the figures of Euler a century earlier and of Leibniz in the century before that, and was rivaled by those of his contemporary Lewis Carroll. Of arguably greater originality is Venn's *The Logic of Chance.* Venn was concerned to find a conception of probability that would leave room for freedom of human action. This was the frequentist conception which, among other things, exercised a benign influence on the development of mathematical statistics. His writings on deductive logic are also important, containing early investigations of mechanized proofs.

Lewis Carroll (1832–1898), author of the Alice stories, was an accomplished puzzler and paradoxer. His Achilles paradox convincingly demonstrates the necessity of not requiring principles of inference to appear as premisses in deductions that they themselves validate. He is also thought by some commentators to have originated the method of semantic tableaux.

Before the passing of the century, Bertrand Russell (1872–1970), whose contributions are described in detail in our companion volume, *Logic From Russell to Church*, was largely innocent of developments in mathematics that would shortly shape his and A.N. Whitehead's (1861–1947) logical programme in the next century. It is true that Russell reviewed for *Mind* Louis Courtarat's *De l'infini mathématique* (1896), and thus became acquainted with set theory. He also knew Whitehead's *Universal Algebra* (1898), which contains a version of Boole's logic. Russell's conversion to the new logic appears to have occurred one August morning in 1900 at the First International Congress of Philosophy. Guiseppe Peano (1858–1932) read a paper on forms of definition in mathematics, and bested Ernst Schröder in subsequent discussion. For the remainder of 1900, Russell developed the ideas that would animate *Principles of Mathematics* (1903).

It hardly wants saying that the three themes of British logic in the 19^{th} century – the abandonment of the syllogistic paradigm, the mathematization of deduction, and the advancement of inductive logic – are not all expressly present in the works of most of the logicians of that country and time, although the first two are clearly in Boole. These broad thematic developments were the products of a collective efforts not always graced by individual clarity. But in their net affect these were efforts that made the logic of the period something to sit up and take notice of, both then and now.

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Dov M. Gabbay
King's College London

John Woods
University of British Columbia
and
King's College London
and
University of Lethbridge

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CONTRIBUTORS

Dov M. Gabbay
King's College London, UK.
dov.gabbay@kcl.ac.uk

Michael E. Hobart
Western Washington University, USA.
hobartme@earthlink.net

Dale Jacquette
The Pennsylvania State University, USA.
dlj4@psu.edu

Ralph Jessop
University of Glasgow, Scotland, UK.
r.jessop@crichton.gla.ac.uk

Gordon R. McOuat
University of King's College, Halifax, Canada.
gmcouat@dal.ca

William J. Mander
Oxford University, UK.
william.mander@hmc.ox.ac.uk

Tim Milnes
University of Edinburgh, Scotland, UK.
tmilnes@staffmail.ed.ac.uk

Amirouche Moktefi
IRIST (Strasbourg) and Archives Poincaré (Nancy), France.
amirouche.moktefi@gersulp.u-strasbg.fr

Bert Mosselmans
Roosevelt Academy, The Netherlands.
b.mosselmans@roac.nl

Maria Panteki
Thessaloniki, Greece.
nioka@math.auth.gr

Shahid Rahman
University of Lille, France.
shahid.rahman@univ-lille3.fr

Juan Redmond
Univesity of Lille, France.
juanredmond@yahoo.fr

Joan L. Richards
Brown University, USA.
Joan_Richards@brown.edu

Laura J. Snyder
St. John's University, USA.
snyderl@stjohns.edu

David Sullivan
The Metropolitan State College of Denver, USA.
sullivad@mscd.edu

James van Evra
University of Waterloo, Cananda.
vanevra@watarts.uwaterloo.ca

Ard van Moer
Vrije Universiteit Brussel, Belgium.
Ard.Van.Moer@vub.ac.be

Charissa S. Varma
University of Toronto, Canada.
charissa.varma@utoronto.ca

Fred Wilson
University of Totonto, Cananda.
fwilson@chass.utoronto.ca

John Woods
University of British Columbia, Cananda.
jhwoods@interchange.ubc.ca

BENTHAM'S LOGIC

Gordon R. McOuat and Charissa S. Varma

At the grand age of 71, celebrated English botanist George Bentham (1800–1884) found himself in an uncomfortable position during what should have been his restful retirement years. George Bentham's successful botanical career, exemplified by the canonical *Handbook of the British Flora* (1858) and with J. D. Hooker, the *Genera Plantarum* (1862–1883), was grounded in pre-evolutionary taxonomic styles and practices. These styles and practices valued careful descriptions and definitions of taxonomic groups based on personal observations, and the careful assessment the relevant taxonomic relationships that brought into sharp relief species and genera boundaries. The source of George Bentham's intellectual anxiety in early 1870s was the popular new theory of evolution by natural selection, proposed by Charles Darwin (1809–1882) in his *Origin of Species* (1859) — a theory that seemed to be settling into the taxonomic thinking of British naturalists, and politely ushering out the pre-evolutionary taxonomic styles and practices that underpinned George's work. Darwin's theory replaced relations of similarity with *descent with modification*, challenging the idea that species groups are fixed and unchanging and threatening to undermine centuries of taxonomic tradition. In his 1871 presidential address to the Linnean Society, George Bentham voiced his concerns with Dawin's new theory and its relationship to natural history: "... systematic biology has to a certain degree been cast into the background by the great impulse to the more speculative branches of the science by the promulgation of the Darwinian theory".[1] There was a revolution afoot in natural history, and George Bentham soon realised he was on the losing side.

These pre-evolutionary taxonomic styles and practices formed George Bentham's "distinctive attitude" regarding the kind of data a botanist could use in establishing valid taxonomic groups (such as choice of external morphological features versus internal features of cell-level anatomy), as well as broader issues on the nature of groups and the epistemological status of the types of relationships between taxonomic groups.[2] Throughout his botanical career, George Bentham believed in sharply circumscribed and sufficiently recognisable natural groups.[3] In June of 1827, he drew a tentative connection between species and genera boundaries and genealogical relationships, and by 1856 he was prepared to maintain that

[1] George Bentham, "Presidential Address" *Proceedings of the Linnean Society of London 1870–1871* (1817): xxxv.

[2] P. F. Stevens, *The Development of Biological Classification: Antoine-Laurent de Jussieu, Nature, and the Natural System*. (New York: Columbia University Press, 1994): 105.

[3] P. F. Stevens, *The Development of Biological Classification: Antoine-Laurent de Jussieu, Nature, and the Natural System*. (New York: Columbia University Press, 1994): 121.

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Dov M. Gabbay and John Woods (Editors)

because genealogical relationships could be demonstrated for only a few generations, any emphasis on ancestral relationships in discussions of species and genera boundaries were necessarily speculative.[4] The theory of evolution by natural selection, it seemed, challenged George Bentham's view on the distinctiveness of groups (and consequently the way taxonomists define taxonomic groups), and its emphasis on descent with modification challenged the traditional taxonomically relevant relationships.

But this was not the only revolution in which George Bentham found himself. Shortly after his first botanical publication — *Catalogue des Plantes Indigenes des Pyrenees et du Bas Languedoc* (1825) — George Bentham wrote his *Outline of a New System of Logic* (1827). Written during an 1826–1827 stay in London, the *Outline* hit the press during a transitional period in a tumultuous chapter of British logic, between the last years of Scholastic syllogistic logic and the early years of algebraic logic. On the surface, the *Outline* appeared to be little more than a punchy chapter-by-chapter critique of Richard Whately's popular and polemical defence of Scholastic logic, *Elements of Logic* (1826). However, but beneath the surface lay George Bentham's preliminary work on definition, division, and relations, work that reflected the caution and concern George had consistently exercised in discussing all manner of theoretical or hypothetical matters.[5]

The fact that issues involving definition, division, and relations figured in many of the botanical debates around the time George Bentham was writing the *Outline* certainly played a role in determining which logical issues he would address. However, it was the two subversive reform movements that employed logic in their plan — the role of logic educational reform and the role of logic in the legal reform — that shaped his radical response. Surprisingly, it was not the connection to these reform movements that propelled George Bentham's contributions to the history of logic into the spotlight. His logical contributions gained recognition somewhat circuitously and many years after the *Outline's* publication, largely as a consequence of a priority dispute between Scottish philosopher William Hamilton (1788–1856) and British mathematician Augustus De Morgan (1806–1871) concerning the quantification of the Scholastic predicate.

This priority dispute began in the late 1840s with the announcement that the origins and innovations of algebraic logic were found in De Morgan's lecture "On the Structure of the Syllogism", given 9 November 1846 to the Cambridge Philosophical Society. In this lecture, De Morgan proposed a structure for a new form of logic that included the quantification of the Scholastic predicate. Shortly thereafter, Hamilton and his supporters challenged De Morgan's priority, claiming that Hamilton had been teaching this new form of logic, including the quantification of the predicate, in his Edinburgh classes as early as 1845. As the priority battle raged on (mostly from Hamilton's pen), English philosopher and political theorist Herbert Spencer (1820–1903) and English economist and logician William Stanley

[4]See Bentham Papers, University College London Library ci. 83–87(14–15 June 1827).

[5]P. F. Stevens, "Bentham, George (1800–1884)," in *Oxford Dictionary of National Biography*, ed. H. C. G. Matthew and Brian Harrison (Oxford: OUP, 2004).

Jevons (1835–1882) complicated matters further by not only announcing in 1873 in the *Contemporary Review* that this new system of logic had been spelled out some time earlier by a young botanist by the name of George Bentham, but that Hamilton had reviewed George Bentham's *Outline* in the *Edinburgh Review* in 1833.[6] Our retiring old botanist, it seemed, found himself an unwitting and early participant in a second revolution, this time in logic.

George Bentham's presentation of the quantification of the predicate years before Hamilton, may have given him his ticket into the history of logic. However, because histories of logic covering this period tend to structure their narratives on the development of algebraic logic and the relationship between mathematics and logic, a quick review of George Bentham's life makes it is easy to appreciate why even though he had a ticket, he took a back row seat.[7]

Unlike the usual suspects cited in this logical revolution, George Bentham did not run in mathematical circles and George's lack of a strong connection to mathematical or logical circles makes it hard to establish him as a precursor to the innovations that could characterise algebraic logic. His interest in logic was sparked when he was a teenager, at the industrious hands of his utilitarian uncle, the formidable Jeremy Bentham (1748-1832). George Bentham's affair with logic was decidedly brief, and more often than not, logic played the handmaiden to George's botanical thought. Moreover, George Bentham's first (and last) original logical publication, *Outline of a New System of Logic* (1827), did little to establish his reputation as a logician in logic and mathematical circles. The *Outline* sold a mere sixty copies before the press went bankrupt and the remaining copies destroyed.[8] For the most part, the *Outline* was not widely read, and its reviews were at best lukewarm. Perhaps more importantly, George Bentham did not see his major logical innovations as stemming from mathematical problems or puzzles. Instead, he followed in the footsteps of the educational, legal, and botanical reformers, seeing his innovations in logic as helping to solve more general problems in classification. George Bentham believed his new logic would be more consonant with what people — be they educators, lawyers, or naturalists — needed to do in the business of organising knowledge.

This English botanist, and for a time promising logician, spent a lifetime holding fast to ideals of clarity and precision in language and cautioning against speculation in science. It was these ideals and cautions that placed him on wining and losing

[6] See T. S. Baynes, "Mr. Herbert on Sir Wm. Hamilton and the quantification of the predicate" *Contemporary Review* 21 (1873): 796-798; W. S. Jevons, "Who discovered the quantification of the predicate?" *Contemporary Review* 21 (1873): 821-824.; and H. Spencer, "The study of sociology IX — the bias of patriotism" *Contemporary Review* 21 (1873): 475-502. For the debate see G. McOuat "The Logical Systematist: George Bentham and His Outline of a New System of Logic." *Archives of Natural History* 30 (2003): 206, M. Filipiuk, ed. *George Bentham, Autobiography 1830-1834*. (Toronto: University of Toronto Press, 1997), 484–485.

[7] For example, Kneale and Kneale do not mention George, Styazhkin and Bochenski mention only his quantification of the predicate.

[8] The press went bankrupt and Bentham was not inclined to rescue the remaining copies. M. Filipiuk, ed. *George Bentham, Autobiography 1830–1834*. (Toronto: University of Toronto Press, 1997), 271.

sides of two revolutions. The tale of his life, his influences, and his innovations may perhaps shed some light on the connections between logic and natural history, and logic in legal and educational reform in Britain during the first half of the nineteenth century.

Born 22 September 1800 in Stoke, Plymouth, to Samuel Bentham (1757–1831) and Mary Sophia Fordyce (1765–1858), George was second son, and third of five children. George spent the first years of his life in England, and the Benthams moved frequently between 1805 and 1814, eventually settling in France in 1814, citing reasons of health, finance, and their children's education.[9] Although none of the Bentham children had formal schooling, their parents took their education very seriously. His mother Mary assumed the bulk of this responsibility and completely supervised their education.[10] Both parents encouraged the study of mathematics, in addition to Latin and Greek.

George's most intense period of mathematical study was likely during the first few years in France. George recalled that his father's "first care" upon arriving in France was to find George and Samuel junior a tutor to continue their education. Samuel secured the services of a Monsieur Chiron "one of the professors at the college of Saumur, who was at once a good mathematician, and Latin and to some extent Greek scholar".[11] George recalled happily working through Laplace's arithmetic and algebra with his older brother during the winter of 1814, and fancied himself "pretty well advanced in mathematics, having gone through Euclid and plane trigonometry, and simple and quadratic equations in Algebra, . . ., spherical trigonometry, conic sections and fluxions."[12]

After only two years in France, tragedy struck the Bentham household. Samuel junior died just after his seventeenth birthday, as a result of blood clot triggered by a fall from a swing in their garden. Though separated by about two years, the Bentham boys had been inseparable in almost everything else. George recollected later in his life that "until [Samuel junior's] illness, I had always slept in the same room, taken all my lessons with him, gone through the same exercises with the same books".[13] Not surprisingly, after his brother's death, George lost the desire to engage in those activities they once shared, studies in classics and mathematics were the hardest affected.[14]

[9] See Catherine Pease-Watkin, "Bentham, Samuel (1757–1831)," in *Oxford Dictionary of National Biography*, ed. H. C. G. Matthew and Brian Harrison (Oxford: OUP, 2004)

[10] See Catherine Pease-Watkin, "The Influence of Mary Bentham on John Stuart Mill," *Journal of Bentham Studies* 8 (2006).

[11] M. Filipiuk, ed. *George Bentham, Autobiography 1830–1834*. (Toronto: University of Toronto Press, 1997), 17.

[12] M. Filipiuk, ed. *George Bentham, Autobiography 1830-1834*. (Toronto: University of Toronto Press, 1997), 18.

[13] M. Filipiuk, ed. *George Bentham, Autobiography 1830–1834*. (Toronto: University of Toronto Press, 1997), 29.

[14] Pease-Watkin notes that "The Benthams suspected this outcome for some months, but choice to hide their suspicions from the children. Samuel wrote to Jeremy stating that 'till all was over we concealed all apprehensions from the other children as well as from himself"' Catherine Pease-Watkin, "Jeremy and Samuel Bentham — The Private and the Public," *Journal of Bentham Studies* 5 (2002).

Rather than force George to pursue classics and mathematics during this difficult period, his parents encouraged him to turn attention to other subjects that had once brought him pleasure, specifically history and geography.[15] George immersed himself in history and geography, using of his "natural taste for method and arrangement" to organise geographical and statistical information into tables.[16] George later credited Jeremy with encouraging and developing his interest in method and arrangement. And their shared interest in tabulations during this period did not pass unrecognised. Prussian naturalist, romantic philosopher, and friend of Samuel Bentham, Alexander von Humboldt (1769–1859) recalled:

> it was his [George Bentham's] own "natural taste for method and arrangement, stimulated by uncle's example and the perusal of some of his works" that made him enjoy "tabulating the geographical and statistical information ... as to physical geography, mountain elevations, river courses and their basins, etc...[17]

Before long, George's interest in tabulation and classification shifted away from history and geography, and towards botany.

In 1817, the same year Jeremy's work on education reform, the *Chrestomathia* (1817), was published, Mary Bentham purchased the third edition of the popular *Flore Française* (1815) by French naturalist Jean-Baptiste Lamarck (1744–1829) and French botanist Augustus Pyrasmus de Candolle (1778–1841). George's interest was immediately piqued.[18] He later wrote:

> I was struck by the analytical tables for the determination of plants, which fell in with the methodical and tabulate ideas I had derived from the study of some of my uncle's works and from what I had attempted in geography and statistics...[19]

It was not simply the presence of tables in this botanical work that struck him. George noted that the tables were arranged according to a methodology he had

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[15] George wrote:

> Samuel realised the impact that his brother's death would have on George, writing of 'poor George' and 'the loss of a brother to whom the attachment was as strong as can be'. Indeed many years later, in 1827, George was to write to his elder sister Mary Louisa: 'It is a sad thing to think how those whom I have most loved and confided in have been separated from me, my poor brother whom I had never quitted a single day till his last fatal illness'

See also pages 29 and 33 of M. Filipiuk, ed. *George Bentham, Autobiography 1830–1834*. (Toronto: University of Toronto Press, 1997), 262.

[16] M. Filipiuk, ed. *George Bentham, Autobiography 1830–1834*. (Toronto: University of Toronto Press, 1997), 30.

[17] M. Filipiuk, ed. *George Bentham, Autobiography 1830–1834*. (Toronto: University of Toronto Press, 1997), xxiv.

[18] M. Filipiuk, ed. *George Bentham, Autobiography 1830–1834*. (Toronto: University of Toronto Press, 1997), 36.

[19] M. Filipiuk, ed. *George Bentham, Autobiography 1830-1834*. (Toronto: University of Toronto Press, 1997), 36.

come to recognise in his uncle's legal and educational reform writing. What Jeremy was doing with legal terms, George saw Lamarck and de Candolle doing with plants. George would later include this observation in his translation of the appendix to Jeremy's *Chrestomathia*.[20]

George's reading of *Flore Française* led to a small flurry of reading in more philosophical issues in botany, including de Candolle's *Théorie élémentaire de la botanique* (1813) — a work that tackled questions of the theoretical status of classification systems. His reading of *Théorie élémentaire de la botanique* led to his reading of the work of de Candolle's teacher, Pierre Prevost (1751–1839). De Candolle credited his "logical turn of mind" to Prevost and saw this as leading to the success of the *Théorie élémentaire*.[21] In his lectures, Prevost stressed the importance of forming the genera in accordance with the real value of relations, a view Prevost felt could easily apply to the life sciences.[22] Attempts by Prevost and others to put classification on a solid theoretical foundation resonated with what George was reading in the *Chrestomathia*. Specifically, what George saw in Jeremy's work was an attempt to model educational reform on natural history. Jeremy's interest in logical relations, and the relationship Jeremy assumed between classification and knowledge came out most clearly in his appendix on nomenclature and classification.

In 1819, George began what would become an almost four year project — the French translation the appendix in Jeremy Bentham's *Chrestomathia*. For the first few years, George dedicated little time to this project, perhaps because in 1820, the Benthams bought the château de Restinclières, near Montpellier. The plan was to cultivate the land for profit, and Samuel gave George most of the responsibility of the management of the operation. However, in addition to managing the estate, George read Scottish philosopher Dugald Stewart's (1753–1828) scathing attack on Aristotelian logic and French philosopher Jean le Rond d'Alembert's (1717–1783) encyclopaedia, the latter being a work praised by Jeremy, with regard to the classification systems presented.[23] By 1822, George's work on the translation picked up speed and he was able to have it ready in time for his visit to London in 1823. George published his translation of Jeremy's Appendix, titled "Essai sur la nomenclature et classification", en route to London at the Bossanges Press in Paris. Delighted with his nephew's efforts, Jeremy encouraged George to expand on what he felt were substantial and significant additions.

In spite of his uncle's encouragements, George was more eager to pursue botany

[20]George would also include Duméril's *Zoologie philosophique* (1808) in his list of those that use bifurcating divisions.

[21]J. M. Drouin, "Principles and Uses of Taxonomy in the Works of Augustin-Pyramus de Candolle." *Studies in History and Philosophy of Biology and Biomedical Sciences* 32 (2001): 258.

[22]J. M. Drouin, "Principles and Uses of Taxonomy in the Works of Augustin-Pyramus de Candolle." *Studies in History and Philosophy of Biology and Biomedical Sciences* 32 (2001): 258.

[23]M. Filipiuk, ed. *George Bentham, Autobiography 1830–1834*. (Toronto: University of Toronto Press, 1997), 213.

and spent more time meeting up with old friends and botanists than expanding on his uncle's logic during this visit.[24] Upon returning to France, George resumed his duties as estate manager and devoted almost all of his time to botany, and by 1825, he deemed himself "thoroughly botanical."[25] His botanical efforts centred on building up the collection he had begun five years earlier, a collection from which he wrote the book that would establish his reputation as a serious botanist — *Catalogue des plantes indigenes des Pyrenees et du Bas Languedoc.*[26] Unfortunately, bad luck befell the Benthams in 1826, George found himself seeking assistance from his influential uncle, a situation that would direct his attention once more to issues in logic.[27]

In the first week of August 1826, George travelled to London with his sisters, with the hope of securing his uncle's financial support. When it became clear that he was not going to get financial support from Jeremy and would have to earn a living, George decided to enrol in Lincoln's Inn and train as a lawyer — a decision that infuriated Jeremy.[28] Partly to pacify his irate uncle, George agreed to defer his legal studies and become his uncle's amanuensis. And so began George's work on Jeremy's unruly unpublished logic papers.[29]

Perhaps as an incentive and perhaps to incite interest in these papers, Jeremy intimated that both James Mill (1773–1836) and his son John Stuart Mill (1806–1873) had studied these papers and expressed an interest in editing them. George, however, suspected that the real reason his uncle gave him this particular task was as an alternative means of compensating him. George reported to his sister that:

> [M]y uncle has imagined that he makes my fortune in giving me his logical papers to make a book of; ... if I succeed in putting it into intelligible French, Bossanges [the publisher] may give me something for it — but as for a fortune, if I get any from my Uncle it must be in a more direct way than through the medium of his manuscripts. Besides, he wishes me to do them in English.[30]

However, it is just as likely that George's "Essai" proved to Jeremy that his nephew's training as a botanist coupled with his appreciation of the theoretical

[24]See M. Filipiuk, ed. *George Bentham, Autobiography 1830–1834.* (Toronto: University of Toronto Press, 1997), 98–153.

[25]M. Filipiuk, ed. *George Bentham, Autobiography 1830–1834.* (Toronto : University of Toronto Press, 1997), 97 and 455.

[26]M. Filipiuk, ed. *George Bentham, Autobiography 1830–1834.* (Toronto: University of Toronto Press, 1997), xxx.

[27]The cultivation of Restinclières was reasonably successful for a time, but in the end for various reasons, the family returned to England. One factor was the threat of a lawsuit from neighbouring residents, who objected to Samuel's irrigation system, which, they claimed, was diverting the local water supply. See M. Filipiuk, ed. *George Bentham, Autobiography 1830–1834.* (Toronto: University of Toronto Press, 1997), 234.

[28]For George's account of Jeremy's anger, see M. Filipiuk, ed. *George Bentham, Autobiography 1830-1834.* (Toronto: University of Toronto Press, 1997), 246–7.

[29]In addition, George worked on his uncle's papers on Codification and other legal subjects.

[30]M. Filipiuk, ed. *George Bentham, Autobiography 1830-1834.* (Toronto: University of Toronto Press, 1997), xxxi.

problems would make George an ideal candidate to champion a Benthamite logic. And Jeremy Bentham was right.

While struggling to find some order in Jeremy's logic papers, helping his own father, and pursuing a legal career, George began the *Outline* — a brief and biting Benthamite critique of Richard Whately's *Elements of Logic* (1826). By March 1827, George Bentham had completed his *Outline*, and his proud uncle assumed the printing costs.[31]

The object of George's criticism — Whately's *Elements* — was the first significant logic textbook to grace the British University system since Aldrich's 1691 *Logicae Artis Compendium*. Up until Whately's *Elements*, the two main textbooks used at Oxford and Cambridge were Sanderson's 1615 *Logicae Artis Compendium* and Aldrich's 1691 *Logicae Artis Compendium*. As it turned out, the *Elements* was far more than a textbook. Whately was clear about the polemical nature of his book right from the start. Whately had a religious agenda to push — wanting to encourage and promote sound reasoning in religious men to counteract religious scepticism. Whately wrote: "The adversaries of our Faith would, I am convinced, have been ... more satisfactorily answered ... had a thorough acquaintance with logic been more common than it is".[32] Whately also had an educational reform agenda to push. Whately saw logic as an agent of university reform and argued that logic should be compulsory for candidates at Oxford for academic honours.

In her biography of her father, Whately's daughter Jane reflected on the role of the *Elements* in his educational reform. She writes:

> The task undertaken by [Whately] was one of no ordinary difficulty; it was not the originating of a new science, but the resuscitation of an old and half-defunct one. The study of logic, formerly pursued with great and credible devotion, had, in later years, fallen into disrepute among the more intellectual class in the University. It was pursued in schools in Oxford merely by committing to heart the technical rules of the compendium of Dr. Aldrich. These were by no means without their utility as a tough mental exercise, and many an Oxonian might remember with gratitude the edge which it gave to his powers of reasoning, particularly if unacquainted with the more valuable discipline of mathematics. It was Whately's great and eminently successful effort to raise the study from this inferior condition to something approaching a scientific character.[33]

The impact of Whately's *Elements* was immediate, and its effects were enduring. In his *Historical Sketch of Logic* (1851), published less than twenty-five years after the *Elements*, Blakey recounted the influence of Whately's book:

[31] M. Filipiuk, ed. *George Bentham, Autobiography 1830-1834*. (Toronto: University of Toronto Press, 1997), xxi.

[32] R. Whately, *Elements of Logic, Comprising the Substance of the Article in the Encyclopaedia Metropolitana; with Additions, &c*, (London, B. Fellowes 1831), xxviii.

[33] E. J. Whately, *Life and correspondence of Richard Whately, D.D.*, 2 vols. (1866), 49.

> Archbishop Whately's *Elements of Logic*, is one of the most important and influential logical publications of modern times. It is an able and popular exposition of the scholastic logic; and has, in fact, been the main instrument in producing the revival of the syllogistic system in Great Britain. The work has gone through many editions, and is used, more or less, in several seats of learning, as an ordinary text-book for logical students.[34]

In many respects, the *Elements* owed its existence to the encouragement of Whately's friend, teacher, and mentor Edward Copleston, on whose advice Whately launched his first spirited defence of traditional syllogistic logic in an 1823 article in the *Encyclopaedia Metropolitana*. This article was later expanded considerably and republished in 1826 as *Elements of Logic*.

The *Elements* was regarded as the last successful attempt to breathe life into a dying discipline. The area of Scholastic logic that benefited the most from Whately's pen was the syllogism. Until Whately, the syllogism had not been faring well. Since the seventeenth-century, students educated in the British system were taught that the syllogism was a better instrument of enquiry and proof than induction (the syllogism to remain safely on the bookshelves of Oxford and Cambridge), by the close of the eighteenth century, it was becoming clear syllogistic logic was not flourishing — it was festering.

Some claim the decline of logic during this period reflects the poor quality of the logic compendiums in England used in the universities.[35] At least one recent explanation for the poor quality compendiums during this period argued that it languished because it was not yet a "science", not yet having a clear theoretical framework. Van Evra wrote in his "The Development of Logic as Reflected in the Fate of the Syllogism 1600-1900." (2000):

> In the early portion of the seventeenth century, logic displayed features commonly found in disciplines prior to the emergence of a dominant theoretical framework. At the time, the common logic had neither a secure theoretical structure externally imposed by pure tradition, nor the stabilizing influence of a strong internal theory. In such a vacuum, all features of the subject become, potentially at least, equally relevant.[36]

Other histories emphasise the "silencing" of professors and replacing them with college fellows as being responsible for logic's decline, claiming that although tutors became more popular, they lacked the expertise or skill of logic professors and did

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[34] R. Blakey, *Historical Sketch of Logic: From the Earliest Times to the Present Day* (Baillière, 1851), 454.

[35] See E. J. Ashworth, "Some Notes on Syllogistic in the Sixteenth and Seventeenth Centuries", *Notre Dame Journal of Formal Logic*, XI, no. 1 (1970) 17-33.

[36] Van-Evra James, "The Development of Logic as Reflected in the Fate of the Syllogism 1600-1900." *History and Philosophy of Logic* 21 (2000): 118.

not produce original works during this period, thus causing of the decline of logic.[37]

During the seventeenth and eighteenth centuries, logic also came under attack. Many Renaissance and humanist thinkers of the seventeenth century began to raise concerns about Scholastic logic's status as an art and its monopoly on truth. Promoters of the empirical science of the Scientific Revolution, such as Bacon and Locke, questioned the application of syllogistic logic to the sciences. One of their chief concerns was the ability of logical demonstrations to enable discoveries. Backing the concerns raised by Bacon and Locke, were the "Common Sense" School of Philosophers in Edinburgh that rose to prominence in the eighteenth century. These philosophers began to call for educational reform and logic was one of the subjects on the chopping block. Dugald Stewart in particular, called for the abolishment of syllogistic logic in the curriculum.

For Whately, the first step in restoring the logic to its proper place involved responding to the objections raised from the seventeenth century onwards concerning logic's purpose. One of his goals involved debunking the Scholastic assumption that logic was the art of thinking. Whately's argument that logic was not simply an art, but also a science, served two purposes. First, it provided Whately with a new and powerful response to the seventeenth-century objections to the usefulness of logic. For Whately, the objections of Bacon, Locke, the Scottish Common Sense School, and Watts have at least one thing in common — they targeted not the tools of logic, but their use. Logic was not used to investigate nature (speaking to Bacon and Locke), it was not the instrument of truth (the Scholastic version), and it was not the art of rightly employing the rational faculties. Second, the claim that logic is also a science marks the beginning of a new approach to logic.[38] Although Whately was not the first to claim logic as science, his was the first influential British logic textbook to defend this claim explicitly.[39]

Whately's claim that logic's goal as a science involved providing "the generalised and abstract representation of all demonstration", was also the first step in Whately's new account of the syllogism. The Scholastics understood the syllogism as kind of argument. On Whately's account, the syllogism was a purely formal device. Once axiomatized, Whately believed that syllogisms would serve as a canonical test of the validity of actual arguments, making the syllogism the theoretical core of the science of logic.[40] From this perspective, Whately's redefinition of logic and his re-evaluation of the role of the syllogism have been credited as

[37]Blakey credits this history to Hamilton in his 1833 *Edinburgh Review.* R. Blakey, *Historical Sketch of Logic: From the Earliest Times to the Present Day* (Baillière, 1851), 424.

[38]See J. Van Evra, "Richard Whately and the rise of Modern Logic", *History and Philosophy of Logic* 5, (1984), 1-18; and C. Jongsma, *Richard Whately and the Revival of Syllogistic Logic in Great Britain in the early Nineteenth Century* (unpublished dissertation, Toronto 1982).

[39]Blakey notes that Kirwan, in his 1807 "Logic" volume 1 page 1 makes this same claim, see R. Blakey, *Historical Sketch of Logic : From the Earliest Times to the Present Day* (Baillière, 1851), 449. Also see J. Van Evra, "Richard Whately and the rise of Modern Logic", *History and Philosophy of Logic* 5, (1984) 1-18.

[40]See J. Van Evra, "Richard Whately and the rise of Modern Logic", *History and Philosophy of Logic* 5, (1984) 1–18.

inspiring the direction of the pioneers of algebraic logic.[41] The syllogism, thanks to Whately, was given a whole new lease on life.

On the new definition of logic, George agreed wholeheartedly with Whately. In fact, George began his *Outline* by claiming:

> In reading the elegant exposition of his views, which Dr. Whately has prefixed to his Elements of Logic, I felt that I generally concurred in his observations on the utility of Logic, in his refutation of the arguments of its detractors — of those who set up Common Sense in opposition to Logic, — and in his remarks on the erroneous system proceeded upon with regard to this subject in our University Education. The absurdity of comprehending, within the province of Logic, every branch of art or science to which it may be applicable, will readily be admitted by any reader.[42]

George continued this discussion of the definition of logic as both an art and science in the beginning of chapter two, where he noted that his uncle had presented a similar definition of logic in his *Chrestomathia* a few years before Whately. On Jeremy's view:

> [e]very art had a correspondent science: it was a mistake to think that the field of thought and action could be divided into a series of distinct compartments, some containing an art, some a science, and some containing neither the one nor the other. The fact was that, 'Whatsoever spot is occupied by either, is occupied by both: it is occupied by them in *joint-tenancy*.' The distinction was founded on the distinction between practice and knowledge: '*Practice*, in proportion as *attention* and *exertion* are regarded as necessary to due *performance*, is termed *art: knowledge*, in proportion as *attention* and *exertion* are regarded as necessary to *attainment*, is termed *science*'. There was no 'determinate line of distinction' between an art and its correspondent science, but where 'that which is seen to be *done*' was regarded as being more prominent than 'that which is seen or supposed to be *known*', the more likely it was that it would be considered an art, and in the opposite case a science.[43]

The accolades for this new definition of logic in the *Outline*, did not last long. By annexing logic to "every branch of human knowledge" George felt Jeremy provided too broad definition, and by restricting the science aspect of logic to "mere correct reasoning", George felt Whately provided too narrow a definition. By recasting

[41] See J. Van Evra, "Richard Whately and the rise of Modern Logic", *History and Philosophy of Logic* 5, (1984) 1-18.

[42] G. Bentham, *Outline of a New System of Logic, With a Critical Examination of Dr Whately's "Elements of Logic"* (London: Hunt & Clarke 1827), 1.

[43] J. Bentham, *Chrestomathia*. ed M. J. Smith and W. H. Burston. (Oxford: Clarendon Press 1983), 59–60.

the definition of logic as "the branch of art-and-science which has for its object the advantageous application of the human mind to the study of any other branch of art-and-science" George felt his attempt to make logic entirely general, like a universal grammar, would be just right.[44]

By page five of his introductory remarks, George identified another serious flaw in Whately's logic, and his reaction provides the first real taste of a Benthamite logic. George charged Whately with the failure to distinguish between different types of entities. Though he raised the concern on page five, George developed this criticism in Chapter Three, titled "Analytical Outline". George claimed:

> The remainder of this Analytical Outline is devoted to the definition of the processes of abstraction and generalization, which are here very aptly distinguished, and to the very useful exposure of the common error of ascribing reality to generic terms. This should have been carried still farther; he should have exhibited the pernicious effects resulting from the *realization* of those subject matters which D'Alembert first called *êtres fictifs*, and what Mr. Bentham has described under the name of *fictitious entities*. He should have pointed out the constant but unavoidable *fiction* which must enter into the composition of any discourse, and should have been given some indications by which error, in this respect, may be guarded against.[45]

A significant part of this chapter (fifteen of the twenty-one pages) was dedicated to presenting a Benthamite classification of entities. George's account of entities, as well as his critique of Whately's discussion of exposition in chapter six, came straight from Jeremy's reform work and his unpublished logic papers.

George was quick to recognise that Whately had fallen into the same trap that Jeremy claimed snared other significant thinkers. George followed Jeremy in his claim that explorations into the processes of "abstraction" and "generalization" were useful in uncovering the error of ascribing reality to generic terms, but such investigations should have been pushed further to include a distinction between real and fictitious entities.[46]

Jeremy's distinction between real and fictitious entities marked his first decisive break from Scholastic logic, one that came early in his career and underpinned some of his objections in his legal reform work. One of Jeremy's first targets during his campaign for legal reform was Oxford's first Vinerian Professor of Common Law and Jeremy's teacher, Sir William Blackstone (1723–1780) and part of Jeremy's attack made use of this distinction between real and fictitious entities.[47]In his

[44]G. Bentham, *Outline of a New System of Logic, With a Critical Examination of Dr Whately's "Elements of Logic"* (London: Hunt & Clarke 1827), 14.

[45]G. Bentham, *Outline of a New System of Logic, With a Critical Examination of Dr Whately's "Elements of Logic"* (London: Hunt & Clarke 1827), 29.

[46]G. Bentham, *Outline of a New System of Logic, With a Critical Examination of Dr Whately's "Elements of Logic"* (London: Hunt & Clarke 1827), 29.

[47]Blackstone is remembered for, among other things, establishing English law as an academic discipline. He presented course of private lectures on law at Oxford, which Jeremy attended,

Comment on the Commentaries and A Fragment on Government (1776), Jeremy fixed his radical eye on Blackstone's *Commentaries on the Laws of England* (1765-9), aiming to expose what he saw as a two-fold problem: Blackstone's application of the Scholastic method of definition to the terms of jurisprudence, which prompted Jeremy to present his new classification of entities and his new modes of exposition; and Blackstone's conflation of the role of censor and expositor in discussions of law, which prompted Jeremy to present his new account of methodization.[48]

Jeremy identified the first mistake in Blackstone's discussion of terms of jurisprudence, specifically his concept of a "natural right", was to define the terms of jurisprudence using the Scholastic method of definition — *per genus et differentiam*. Definition, on this account, involved first identifying the group to which the entity in question belongs (or determining the entity's logical genus), and then distinguishing the entity under investigation from the others in the group by specifying the property that makes it what it is and not something else in the group (or identifying the object's logical differentia). Blackstone followed the Scholastic belief that by assigning the logical genus and differentia, a definite meaning is conveyed, giving "a clear idea of the thing it signifies."[49] The Scholastic method of definition may be fine for terms like "turnip" and "table", but for terms such as "obligation", Jeremy saw a problem.

Following Locke, Jeremy claimed that to obtain a "clear idea," involved either having direct sense experience of the entity (a Lockean "simple idea" of a substance) or by constructing "artificial groupings of sensory ideas" (a Lockean "complex idea" of a substance). The problem is that even though terms such as "obligation", hold the subject position in a proposition, they do not refer to an entity accessible by direct sense experience, or for which an artificial grouping could be constructed. Language, for Jeremy, obscured a fundamental distinction between entities that are real and entities that are fictitious.[50]

To help with the exposition of tricky terms, like those of jurisprudence, Jeremy believed we first needed to recognise that they are fictitious entities and that fictitious entities cannot be exposited by the method *per genus et differentiam.* He states this clearly in the *Fragment on Government*:

> The common method of defining — the method *per genus et differentiam*, as logicians call it, will, in many cases, not at all answer the purpose. Among abstract terms we soon come to such as have no *superior genus.* A definition, *per genus et differentiam*, when applied to

and these lectures were later published as *Commentaries on the Laws of England* (1765–1769). Blackstone would later to be a justice of the Court of Common Pleas. See Wilfrid Prest, "Blackstone, Sir William (1723–1780)," in *Oxford Dictionary of National Biography*, ed. H. C. G. Matthew and Brian Harrison (Oxford: OUP, 2004).

[48]Blackstone's confusion of the roles of expositor and censor, see P. Schofield "Jeremy Bentham, The Principle of Utility, and Legal Positivism," *Current Legal Problems* 56 (2003): 1–39.

[49]J. Bentham, 'A Fragment on Government', in *A Comment on the Commentaries and A Fragment on Government*, ed. J.H. Burns and H.L.A. Hart (London, 1977), 587.

[50]See P. Schofield "Jeremy Bentham, The Principle of Utility, and Legal Positivism," *Current Legal Problems* 56 (2003): 14.

> these, it is manifest, can make no advance: it must either stop short, or turn back, as it were, upon itself, in a *circulate* or a *repetend*.[51]

Because the names of real entities refer to entities accessible by sense perception and can be arranged hierarchically (that is, they can be arranged in descending orders of generality according to a principle of division of aggregate masses), they can be defined according to the Scholastic method of definition. In contrast, because the names of fictitious entities, such as "obligation", do not refer to entities accessible by sense perception, and have no superior genus (they cannot be organised hierarchically according to a principle of division of aggregate masses), they demand a different method.

To help appreciate Jeremy's argument, consider applying the method *per genus et differentiam* to the term "obligation". The first step would involve determining the superior genus of "obligation". The only superior genus, according to Jeremy, for a term like "obligation", is the universal genus "fictitious entity". Because we have identified a superior genus, it appears that we can proceed using the method *per genus et differentiam*. However, Jeremy argues that because the species of the genus "fictitious entities" are so many and so comprehensive, any attempt to provide a character by which "obligation" can be distinguished from all others, would lead to an enumeration of properties that may never reach completion.

Later, in the *Chrestomathia,* Jeremy presented his new classification of entities, dividing them into five types: Real entities (perceptible by the senses), Inferential entities (we believe to have real existence, but imperceptible to our senses, like God), Fabulous entities (believed by others, but to the existence of which we can attach no belief, heathen gods), Collective entities (the result of operations of abstraction and generalisation, forming a class), and Fictitious entities (neither have, nor is supposed to have any real existence, but which is grammatically spoken of as real, for example, obligation). George presented this classification of entities in Chapter Three of the *Outline*.

In light of the problem faced when expositing fictitious entities, Jeremy proposed a new method of exposition called "paraphrases". Jeremy's first step in paraphrases rested on a controversial and decidedly unLockean assumption. The logic of Aristotle and the Scholastics, as well as Locke, was a logic of *terms*. On the Scholastic and Lockean account, propositions are the result of combining terms — terms are fundamental. Jeremy turned this assumption on its head, believing that propositions came first, and it is by methods of abstraction and analysis, that we arrive at terms.[52] Jeremy's method of paraphrases brings this controversial as-

[51] J. Bentham, *"A Fragment on Government"* in *A Comment on the Commentaries and A Fragment on Government,* ed. J.H. Burns and H.L.A. Hart (London, 1977), 181n.

[52] Recent scholars have commented on this aspect of Jeremy's work. See Ogden, in his "Theory of Fictions"; H.L.A. Hart *Essays on Bentham* (Oxford: Clarendon Press 1982), 43, and especially W. O. Quine in his *Theories and Things* (Cambridge, Belknap Press 1981), "Five Milestones of Empiricism" in *From a Logical Point of View* (1961) p. 67-72, and "Russell's Ontological Development" *The Journal of Philosophy* 63, No. 21 (1966), 657-667. Quine, for example, saw this focus on propositions in Jeremy's theory of fictitious entities as foreshadowing an innovation that would gain popularity near the end of the nineteenth century and the beginning of the

sumption into the spotlight. For fictitious terms to serve any instructive purpose, they need to be resolved into Lockean simple ideas, that is, they need raise images either of perceived substances or emotions. Rather than simply trying to reduce a fictitious entity to a real entity by a chain of synonyms, paraphrases begins with a proposition that contained the name of the fictitious entity and translates that proposition to an equivalent one that uses the names of only real entities. For Jeremy, if a fictitious entity can be replaced in a proposition by a real entity, without any loss of meaning, then the fictitious entity is meaningful.

Take, for example, the application of paraphrases on the term "obligation". We begin by combining the name of the fictitious entity "obligation" with other words to create a proposition, such as "A man is under an obligation to do X". Then, we find another proposition equivalent to "A man is under an obligation to do X", such as "A man is liable to punishment if he does not do X". The sentence "A man is liable to punishment if he does not do X" is equivalent to "A man is liable to pain if he doesn't do X", provided we adopt the Jeremy's notion of punishment, namely that punishment is "*pain* annexed to an act, and accruing on a certain *account*, and from a certain *source*".[53] The idea associated with the word "pain" is, in Lockean terms, a "simple" idea, and the method of paraphrases has resolved the sentence containing the term "obligation" into a sentence containing the simple idea, namely "pain". So, in the case of obligation, unless an obligation can be enforced with sanctions and pain, an obligation can be dismissed as "absurd in logic".

Getting back to Blackstone and natural rights, for Jeremy, to see why a "natural right" is a nonsense term, we must compare the analysis a "natural right" with a "legal right". Applying paraphrases, both can be analysed in terms of corresponding duties, but only a legal duty can be analysed further into a simple idea. Propositions that include legal duty can be translated to propositions that laws that include the notion of punishment. Legal rights are real rights because they are produced by existing legal systems. Because there is no corresponding law with respect to natural duties, claimed Jeremy, natural rights are just imaginary rights.

In George's eyes, Whatley and Sanderson were behaving much like Blackstone, in placing an undue emphasis in discussions of exposition on one particular mode,

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twentieth century with Russell. Quine, however, qualified the historical significance of Jeremy's idea:

> Bentham was perhaps the first to see the sentence thus as the primary vehicle of meaning. Frege took up the tale. But Russell, in his theory of singular description, was the first to put this insight to precise and effective use. Frege and Peano had allowed singular description the status of a primitive notation; only with Russell did it become an "incomplete symbol defined in use." What suggested the expedient to Russell was not in fact Bentham's work, it seems, but a use of operators in the differential Calculus."

[53] J. Bentham, 'A Fragment on Government', in *A Comment on the Commentaries and A Fragment on Government*, ed. J.H. Burns and H.L.A. Hart (London, 1977), 495 in 95n.

namely definition.[54] Chapter six of the *Outline*, titled "Exposition", George presented the issues with exposition by framing it in terms of teaching and learning concepts (an approach Jeremy took in his discussion of exposition in the *Chrestomathia*) and outlined Jeremy's method of paraphrases, in addition to eleven other Benthamite forms of exposition.

Like Jeremy, George did not see the problems with the tactical aspect of logic ending with new classifications of entities and new modes exposition, and he moved quickly to what he took to be one of the most important subjects in the reform of the tactical aspect of logic — methodization.

George began chapter seven, titled "Methodization" with a discussion of the two operations of methodization — collocation and distribution — but spent the bulk of this chapter on distribution. According to George, distributive methodization is performed by three operations:

1. Dividing an entity into parts: in the case of individuals, this is *analysis*, and in the case of collective entities, this is *logical division.*

2. Uniting entities into a whole: In the case of individuals, this is *synthesis*, and in the case of collective entities, this is *generalization.*

3. Distribution: "for the performance of this operation, a number of *wholes,* as well as *parts*, are supposed to be already given; but, as in the case of real entities, the exhibiting of wholes and parts might appear to constitute the while of this operation, the figure is now changed, the aggregate ideas as *receptacles into* which the several given partial ideas are supposed to be *placed* or *distributed.*"[55]

What George referred to as "distribution" is similar to what Jeremy referred to as "arrangement" in his critique of Blackstone. For Jeremy, the Benthamite expositor needed to do more than define entities properly:

> The function of the Expositor may be conceived to divide itself into two branches: that of *history,* and that of simple *demonstration.* The business of history is to represent the Law in the state it *has* been in, in past periods of its existence: the business of simple demonstration in the sense in which I will take leave to use the word, is to represent the Law in the state it *is* in for the time being.[56]

It is important to note that Jeremy did not borrow the term "demonstration" from logicians and mathematicians, but from naturalists. Jeremy explained his use of the word "demonstration" in the footnote to this passage:

[54] George organised Whately's five kinds of definition on a table on pg 95 of G. Bentham, *Outline of a New System of Logic, With a Critical Examination of Dr Whately's "Elements of Logic"* (London: Hunt & Clarke 1827).

[55] G. Bentham, *Outline of a New System of Logic, With a Critical Examination of Dr Whately's "Elements of Logic"* (London: Hunt & Clarke 1827), 101-2.

[56] J. Bentham, 'A Fragment on Government', in *A Comment on the Commentaries and A Fragment on Government*, ed. J.H. Burns and H.L.A. Hart (London, 1977), p. 414.

> The word *demonstration* may here seem, at first sight, to be out of place. It will be easily perceived that the sense here put upon it is not the same with that in which it is employed by Logicians and Mathematicians. In our own language, indeed, it is not very familiar in any other sense than theirs: but on the Continent it is currently employed in many other sciences. The French, for example, have their *demonstrateurs de botanique, d'anatomie, de physique experimentale, &c.* I use it out of necessity; not knowing of any other that will suit the purpose.[57]

What Jeremy referred to as "demonstration", George called "methodization". For Jeremy, there are three kinds of demonstration: arrangement, narration and conjecture. Jeremy's notion of "arrangement" seems to correspond roughly with George's notion of "distribution". Of the three kinds of demonstration, Jeremy saw arrangement as the most difficult:

> Among the most difficult and the most important of the functions of the *demonstrator* is the business of *arrangement.* In this our Author has been thought, and not, I conceive, without justice, to excel; at least in comparison of any thing in that way that has hitherto appeared. 'Tis to him we owe such an arrangement of the elements of Jurisprudence, as wants little, perhaps, of being the best that a technical nomenclature will admit of. A technical nomenclature, so long as it is admitted to mark out and denominate the principal heads, stands an invincible obstacle to every other than a technical arrangement. For to *denominate* in general terms, what is it but to arrange? and to arrange under heads, what is it but to *denominate* upon a large scale? A technical arrangement, governed then in this manner, by a technical nomenclature, can never be otherwise than *confused* and *unsatisfactory.* The reason will be sufficiently apparent, when we understand what sort of an arrangement that must be which can be properly termed a *natural* one.[58]

Arrangement consists of three tasks: distributing the entities into different groups, for the purpose of a general survey; determining the order in which those groups will be brought to view; and finding a name for each of them. Jeremy's discussion of arrangement in his legal and his education reform revealed the influence of methodology articulated by Swedish naturalist Carl Linnaeus (1707–1778).

Jeremy worried that the language of jurisprudence was corrupt — different objects grouped under one name, and similar objects under different names. This corruption resulted in poor systems.[59] Concerns about naming and the problem

[57] J. Bentham, 'A Fragment on Government', in *A Comment on the Commentaries and A Fragment on Government*, ed. J. H. Burns and H. L. A. Hart (London, 1977), 414 footnote 21.

[58] J. Bentham, 'A Fragment on Government', in *A Comment on the Commentaries and A Fragment on Government*, ed. J.H. Burns and H.L.A. Hart (London, 1977), 414.

[59] S. Jacobs "Bentham, Science and the Construction of Jurisprudence" *History of European Ideas* 12 (1990): 585.

of synonymy was a not just a problem for jurisprudence. Many eighteenth-century naturalists raised this type of concern, and both Jeremy and George saw in the work of these naturalists, especially the botanists, a solution.[60] For Jeremy, naturalists knew that in order for their objects of investigation to be understood and useful, they must be classified and specifically defined. Natural history provided the model for Jeremy's reform.[61]

Although not the botanist George was, Jeremy was no stranger to natural history and natural history classification systems. Jeremy shared a practical interest in botany with George's mother Mary, who was a rather accomplished botanist herself. Jeremy and Mary exchanged letters and botanical specimens for many years, and Jeremy himself kept a small garden.[62] In addition to botanical classifications, Jeremy Bentham refers in his writings to the Linnaean inspired nosologies of French botanist and Professor of Medicine, François Boissier de Sauvages (1706–1767) and Scottish Professor of Chemistry and Medicine, William Cullen (1710–1790). Jeremy's introduction to nosology was likely due to another member of the Fordyce family — George's maternal grandfather, Scottish physician, George Fordyce (1736–1802). George Fordyce was a student of Cullen's, and lectured in London.[63] Fordyce's lectures drew from the work of Cullen, and Jeremy attended some of those lectures.

Jeremy suggested using Linnaeus's botanical classification system — the sexual system of plants — as a guide to arrangement or classification.[64] As a teenager, Linnaeus was fascinated with the sexuality of plants, and after many years of observing the great diversity of sex organs of plants, he decided to make this the basis of his botanical classification system in *Systema Naturae* (1735).[65] Lin-

[60]See L. Daston, "Type Specimens and Scientific Memory." *Critical Inquiry* 31(2004): 153–182.

[61]The following three papers suggest this reading of Jeremy's legal and educational reform work: W. C. Mitchell "Bentham's Felicific Calculus" *Political Science Quarterly* 33 no.2 (1918): 161–183.; S. Jacobs "Bentham, Science and the Construction of Jurisprudence" *History of European Ideas* 12 (1990): 583–594.; A. Perreau-Saussine, "Bentham and The Boot-Strappers of Jurisprudence: The Moral Commitments of a Rationalist Legal Positivist" *Cambridge Law Journal*, 63 no.2 (2004): 346–383.

[62]M. Filipiuk, ed. *George Bentham, Autobiography 1830–1834*. (Toronto: University of Toronto Press, 1997), xx.

[63]N. G. Coley, "Fordyce, George (1736–1802)," in *Oxford Dictionary of National Biography*, ed. H. C. G. Matthew and Brian Harrison (Oxford: Oxford University Press, 2004).

[64]Jeremy mentions Linnaeus in many of his writings. Jacobs notes, for example Jeremy Bentham, *The Correspondence of Jeremy Bentham,* 1, 1752 – 76. ed.T. L. S. Sprigge (London: Athlone Press, 1968), p. 105; also *A Comment on the Commentaries and A Fragment on Government*, ed. J. H. Burns and H. L. A. Hart (London, 1977), 415, 416, 418–419; and in *Principles of Morals,* p. 273 note yl. There are also many references to Linnaeus in the *Chrestomathia*.

[65]For more on Linnaeus, see W. Blunt *The Compleat Naturalist ; A Life of Linnaeus* (New York, Viking Press 1971); G. Eriksson, "Linnaeus the Botanist." in *Linnaeus: The Man and his Work,* ed. F. Frängsmyr. (Berkley, University of California Press, 1985); S. Lindroth, "The Two Faces of Linnaeus" in *Linnaeus: The Man and his Work,* ed. F. Frängsmyr (Berkley, University of California Press 1985); J. Larson *Reason and Experience: The Representation of Natural Order in the Work of Carl von Linné*. (Berkeley, University of California Press 1977); E. Mayr, *The Growth of Biological Thought: Diversity, Evolution, and Inheritance* (Cambridge, Belknap

naeus divided all flowering plants into twenty-three classes (the twenty-fourth being *Cryptogamia* that included flowerless plants like mosses) according to the number, relative length, arrangement, etc., of the male organ or stamen. These classes were then divided into orders based on the female parts, or pistils. That Jeremy would have been familiar with Linnaeus's work comes as no surprise. Linnaeus's system was widely regarded as the most practical system available from 1737-1810, and was the most widely used.[66]

Like Linnaeus, Jeremy saw the practical benefit of singling out a particular character to use as a division principle in an artificial system. Jeremy believed that the consistent use of an established principle of division would result in meaningful groups organised in a coherent way. Borrowing a page from Linnaeus, Jeremy organised all offences hierarchically according to his principle of utility.

Jeremy also saw the merit of arranging entities hierarchically. Linnaeus was not the only example to which Jeremy appealed on the topic of hierarchical arrangements. Jeremy drew attention to d'Alembert, who famously organising his *êtres fictifs* hierarchically in his "tree of knowledge".[67] Jeremy maintained that the hierarchical tree structure effectively organised the terms according to three relations: logical identity, logical diversity, and practical dependence. This arrangement provides the reader, or in the case of the organisation of the terms of jurisprudence, the Legislator, with "an insight — the more clear, correct, and extensive the better, - into the matter of every [...] branch of art and science."[68]

A closer inspection of Jeremy Bentham's classification systems in his legal work, however, demonstrates a far more consistent application of a division principle based on a single character than that found in Linnaeus's systems. But this is not the only difference between Linnaeus and Jeremy. Jeremy seemed to posit a stronger relationship between the division principle employed and the essence of the group when he claimed that a strict adherence to a fundamental division principle would provide a "natural" arrangement for his classification systems:

> That arrangement of the materials of any science may, I take it, be termed a *natural* one, which takes such properties to characterize them by, as men in general are, by the common constitution of man's *nature,* disposed to attend to: such, in other words, as *naturally,* that is readily, engage, and firmly fix the attention of any one to whom they are pointed out. The materials, or elements here in question, are such actions as are the objects of what we call Laws or Institutions.[69]

Press of Harvard University Press 1982).

[66]At least part of the reason for its popularity was because "the information necessary for the construction of more natural systems had not yet been assembled for synthesis" W. Blunt *The Compleat Naturalist; A Life of Linnaeus* (New York, Viking Press 1971), 244.

[67]See J. Bentham, *Chrestomathia.* ed M. J. Smith and W. H. Burston. (Oxford: Clarendon Press1983), 257.

[68]See J. Bentham, *Chrestomathia.* ed M. J. Smith and W. H. Burston. (Oxford: Clarendon Press1983), 218–220.

[69]J. Bentham, 'A Fragment on Government', in *A Comment on the Commentaries and A Fragment on Government*, ed. J. H. Burns and H. L. A. Hart (London, 1977), p. 415.

It is possible that Jeremy's understanding of a natural system is more a vestige of Locke than a modification of Linnaeus. Locke agreed with Aristotle and the Scholastics that essences serve two distinct purposes: a classificatory purpose, that is, the essences of entities should help classifying entities into kinds; and an explanatory purpose, that is, the essence of entities should help explain the properties and behaviours of the entities we observe in the world. However, Locke maintained that each purpose is served by a distinct type of essence, the explanatory purpose by *real essences*[70] and the classificatory purpose by *nominal essences*.[71] For Locke, nominal essences are nothing more than the criteria that we create and use to mark off the members of a group. Nominal essences are the boundaries set by us, not by reality. Reality can supply phenomenal resemblances, but for Locke, resemblances do not constitute natural, real boundaries. At best, phenomenal resemblances indicate underlying structural resemblances.[72] Jeremy's principle of utility provided a "natural" classification of offences, and this principle alone can "render satisfactory and clear any arrangement." In contrast, Linnaeus, like many naturalists during this period, called arrangements based on a single character, "artificial" systems. Artificial systems stood in contrast to "natural" systems, systems that group organisms according to many characters.

A further point George stressed in his discussion of methodization was that both distribution (arrangement) and generalisation are needed for the construction of a

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[70]Locke defines real essences in the following way in *An Essay Concerning Human Understanding*:

> Real essences ... may be taken for the very being of anything, whereby it is what it is. And thus the real internal, but generally (in substances) unknown constitution of things, whereon their discoverable qualities depend, may be called their essence. This is the proper original signification of the word, as is evident from the formation of it; *essentia*, in its primary notation, signifying properly, being. And in this sense it is still used, when we speak of the essence of particular things, without giving them any name. [3:3:15]

In other words, something is a real essence just in case it provides the sufficient condition(s) for the *explanation* of the properties of entities, whether the entities in question are substances or not, so defined.

[71]Locke defines nominal essences in the following way *An Essay Concerning Human Understanding*:

> Nominal essences ... [have] been almost wholly applied to the artificial constitution of genus and species. It is true, there is ordinarily supposed a real constitution of the sorts of things; and it is past doubt there must be some real constitution, on which any collection of simple ideas co-existing must depend. But, it being evident that things are ranked under names into sorts or species, only as they agree to certain abstract ideas, to which we have annexed those names, the essence of each genus, or sort, comes to be nothing but that abstract idea which the general, or sortal (if I may have leave so to call it from sort, as I do general from genus), name stands for. And this we shall find to be that which the word essence imports in its most familiar use. [3:3:15]

In other words, something is a nominal essence just in case it sets a boundary to the class in such a way as to justify our application of a name.

[72]For a more detailed discussion of this claim see Mackie, J., L., *Problems From Locke*, (Oxford University Press, London 1976), 134–6.

successful system. He provided the following example to help appreciate how these operations work together, as well as to show his distinction between division and distribution. George's example began with the naturalist's use of generalization:

> A botanist visits a country with whose productions he is as yet unacquainted; he sees a number of plants which resemble one another very strongly, and which differ considerably from any other plants which he has seen or heard of; he discovers successively several of these *sets* of plants, and by *generalization* he forms as many new *species*, characterized by the properties he has *observed* in these several individual plants.[73]

Then George moved on to a naturalist's use of distribution:

> On referring to his books, he compares the several properties there given as characteristics of general classes, which those which are possessed by his several new species, and thus decides to which of these general classes the species in question belong. This many be termed distributing those species under their superordinate genera.
>
> Suppose that by the addition of these new species, some one class (or logical genus) may now consist of so many, as to render it difficult to compare them or to retain their distinctive properties without the help of some intermediate classes; these classes may be formed either by *dividing* the genus into subclasses, or by *distributing* the species into groups. In this case division and distribution appear so closely allied, as scarcely to be distinguishable otherwise than by the form of expression.[74]

By emphasising the necessity of both generalization and distribution, this passage reflects the impact George's training as a botanist had on his logical thinking. That he would have emphasised both operations and used such an example is not surprising, given his experience as a botanist. Historian of biology Müller-Wille argued that Linnaeus discussed a division of labour in natural history, between the " 'collectors (*collectores*)' who 'primarily cared for the number of species' by collecting, describing and drawing plant specimens, and 'taxonomists (*methodici*)', who 'primarily cared for the classification and denomination of plants'."[75]

For George, a great deal of confusion surrounded the tools used in constructing a classification system. For Jeremy, the principal tool for arrangement or distribution of *real* entities was bifurcation (sometimes referred to as "bipartition" or

[73] G. Bentham, *Outline of a New System of Logic, With a Critical Examination of Dr Whately's "Elements of Logic"* (London: Hunt & Clarke 1827), 102–3.

[74] G. Bentham, *Outline of a New System of Logic, With a Critical Examination of Dr Whately's "Elements of Logic"* (London: Hunt & Clarke 1827), 103.

[75] S. Müller-Wille, "Gardens of paradise" *Endeavour* 25 No. 2 (2001), 49. See also S. Lindroth, "The Two Faces of Linnaeus" in *Linnaeus: The Man and his Work,* ed. F. Frängsmyr (Berkley, University of California Press 1985) and J. Larson *Reason and Experience: The Representation of Natural Order in the Work of Carl von Linné.* (Berkeley, University of California Press 1977).

"exhaustive division"). Jeremy traced this tool back to Porphyry's commentary on Aristotle's logic, but also saw this tool wielded by some late eighteenth-century naturalists. George too found instances of bifurcate divisions in natural history. As mentioned earlier, George saw the dichotomous key typified in the *Flore Française* of Lamarck and de Candolle as an instance of distribution using bifurcate divisions. In the chapter titled "Méthode Analytique" of the *Flore Française*, Lamarck began with the following division: Fleurs dont les étamines & pistils peuvent aisément se distinguer (flowers whose stamen and pistils are distinguishable) or Fleurs dont les étamines & pistils sont nuls, ou ne peuvent se distinguer (flowers whose stamens and pistils are absent or indistinguishable). Amateur naturalists (as George was when he first read the *Flore*) were quick to praise dichotomous identification keys, as they had the virtue of allowing *anyone* to identify plants with relative ease, as compared to the artificial system of Linnaeus[76] (a system that organised plants based on their sexual characters) or the natural systems of Michel Adanson, Bernard de Jussieu, and Antoine Laurent de Jussieu.[77] This is because with bifurcation, large groups are eliminated at each stage by using mutually exclusive characteristics, making it easy to find the name of a plant, provided the reader can identify the requisite parts. This method stands in contrast to the methods of many of the natural systems during the eighteenth century, systems that required the reader to have a great deal of prior botanical knowledge under their belt.

Both Jeremy and George understood bifurcation as taking a class and subjecting it to progressive dichotomous divisions. In other words, every member of class A has the character X in common, and can be divided into two subclasses, B and C, depending on if the member of the class A has the character Y or not (so all the members of B have Y in common and all the members of C do not), then every member of class B has the character Y, and can be divided into two subclasses, D and E, depending on if the member of the class B has the character Z or not (so all the members of D have Z in common and all the members of E do not), and so on. The primary virtue of this method is that results in a classification that is both exclusive and exhaustive. Jeremy continued by claiming that if, when analysing classes and assigning them to divisions to which names are then applied, bifurcation will complement definition by giving a precise definition of a name in relation to the properties that distinguish the class it designates. Jeremy discussed the virtues of bifurcation in *Introduction to the Principles of Morals and Legislation* (1781):

> When a number of objects, composing a logical whole, are to be considered together, all of these possessing with respect to one another a certain congruency or agreement denoted by a certain name, there is but one way of giving a perfect knowledge of their nature; and that

[76]Although Lamarck didn't adopt Linnaeus's artificial system, his *Flore* was one of the first French works to include the Linnaean nomenclature.

[77]J. M. Drouin, "Principles and Uses of Taxonomy in the Works of Augustin-Pyramus de Candolle." *Studies in History and Philosophy of Biology and Biomedical Sciences* 32 (2001): 264-5.

> is, by distributing them into a system of parcels, each of them a part, either of some parcel, or, at any rate, of the common whole. This can only be done in the way of *bipartition*...[78]

While Jeremy encouraged the use of bifurcation whenever possible in arrangement (since classification was designed to assist the logical operation of exposition), George was very careful to outline the limits of bifurcation in his chapter on methodization.[79]

George's careful discussion of the restricted use of bifurcation began by drawing a connection between Whately's and Sanderson's emphasis on definition and Whately's and Sanderson's general rules for division in his discussions of methodization. The problem, as George saw it, was that Whately's and Sanderson's sets of rules apply only to bifurcate divisions, and many things that cannot be effectively organised by bifurcation. Bifurcation, for example, cannot be applied to fictitious entities for the same reason the method of definition cannot be applied fictitious entities — fictitious entities lack a common genus and so cannot be organized according to "any exhaustive plan of arrangement, but must be picked up here and there as they happen to occur".[80] George also mentioned two further restrictions on bifurcation. First, George reminds us that a second purpose of classification was to aid retention, often times this purpose is best served by multifurcate divisions rather than bifurcate divisions.[81] George assumed Sanderson was also aware of this latter restriction on the application of bifurcation:

> But upon reflection, he [Sanderson] was probably led to consider, that, if the successive bifurcate divisions of logical aggregates were carried on to the lowest stage necessary for classification, and if, at the same time, names were given to every one of the intermediate aggregates so formed, — this operation would, on many occasions, be attended with *more labour than advantage* [our italics], and would require too great a strain on the memory. He therefore added the restriction that dichotomy was only to be made use of when it could be *easily performed* [our italics]; and that it should not be sought out on all occasions with superstitious strictness with which the Ramaeans followed it.[82]

Both the Benthams note the impracticality of applying this method to botanical classification. Jeremy wrote:

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[78] J. Bentham, *An Introduction to the Principles of Morals and Legislation*, ed. J. H. Burns and H. L. A. Hart (London, 1970), 187.

[79] George also talks about division in his "Essai" see G. Bentham, "Essai sur la nomenclature et la classification des principales branches d'art-et-science" (Paris, Bossange Freres 1823). 70–72.

[80] J. Bentham, *An Introduction to the Principles of Morals and Legislation*, ed. J. H. Burns and H. L. A. Hart (London, 1970), 53 note c.

[81] J. Bentham, *Chrestomathia* ed M. J. Smith and W. H. Burston. (Oxford, Clarendon Press 1983), 252.

[82] G. Bentham, *Outline of a New System of Logic, With a Critical Examination of Dr Whately's "Elements of Logic"* (London: Hunt & Clarke 1827), 110–1.

> The quantity of surface necessary to the exhibition of such a diagram, presents another circumstance, by which, long enough before the number of the extreme branches had reached to any such number as forty thousand, as above, not to say the tenth or the hundredth part of it, the bar of *impracticality* would be opposed.[83]

Further, George was writing the *Outline* at a time when botanists were racing to organize and classify the ever increasing numbers of known plants. With thousands of plants species already described and thousands more estimated to exist, memorization of species names was simply out of the question, so designs for efficient storage and retrieval systems were the top priority. While dichotomous keys initially held some promise, especially considering how helpful they were for the purpose of identification, George was keenly aware of the many serious flaws resulting from the use of a bifurcating system as a classification system for tens of thousands of plants. The number of dichotomous divisions would be enormous, and revising the system in light of new discoveries of species would be tremendously difficult. It is not surprising for George, then, to find that in natural history classification systems, natural systems and even certain artificial systems (like Linnaeus's sexual system of plants) adopted multifurcate divisions rather than bifurcating divisions in their classification systems. Further, in some instances, bifurcation loses its virtue of being exhaustive. For example, natural history classification systems need to be able to accommodate the new species are always being discovered.[84] For example, in the "Essai" George looks at the effect that the discovery of intermediate groups has on the kind of key he constructed, arguing "our ignorance of so many beings, by implication as yet undiscovered, would be even more damaging to finer subdivisions of the key".[85] Jeremy was also aware of the problem of the number of species not being fixed had on bifurcation:

> Take, for example, *Natural History*, and therein *Botany*. Forty thousand was, some years ago, stated as the number of supposed different species of plants (exclusive of *varieties*) at that time more or less known to the botanic world. But, at that time, the utmost knowledge obtained of them by any person was not, to any such degree clear, correct, or complete, as to enable him, in this way, to show, of every one of them, in any such concise mode, its points of agreement and disagreement with reference to every other. And even if, in and for any one year, the distinctive properties of the whole multitude of individuals contained in the whole multitude of species then known, could have been exhibited in this systematic form, the sketch given them, if with regard to the whole number of species of plants then existing it professed to be,

[83] J. Bentham, *Chrestomathia*. ed. M. J. Smith and W. H. Burston. (Oxford, Clarendon Press 1983), 252.

[84] J. Bentham, *Chrestomathia*. ed. M. J. Smith and W. H. Burston. (Oxford, Clarendon Press 1983), 252.

[85] P. F. Stevens, *The Development of Biological Classification: Antoine-Laurent de Jussieu, Nature, and the Natural System*. (New York: Columbia University Press, 1994): 103.

> and even if it really were, an exhaustive one, would, in and for the next year, no longer posses that quality.

These concerns about bifurcation led George to suggested five considerations for discussions of divisions:

1. The particular end in view, or object of the division. Knowing this will help distinguish between practical or logical division.

2. The nature of its subject.

3. The source or principle of the division.

4. The mode of division, whether it is a contradictory bifurcate division or a loose, irregular multifurcate division.

5. The extent to which the division should be carried out.[86]

George did suggest that, in some cases, a classification system with multifurcate divisions may turn out to be as complete and distinct as a bifurcate system, and we can test its completeness by applying bifurcation to the same subject matter.[87] George used the class of animals *Vertebrata* to illustrate his point. If the goal is simply to give a general idea of the kind of animals that fall in the class *Vertebrata* (in other words, it is not necessary to define exactly the nature of the several subclasses or species), the class can be multifurcately divided into four classes: *Mammiferae* are those who suckle their young; *Birds* are animals which have wings and feathers; *Fish* are those who have fins and live in the water; *Lizards* are those little animals with four legs and scaly skin.[88] For the naturalist, who aims for the all-comprehensiveness of her classes and sub-classes, and requires all classes and subclasses to be distinctly characterized, divisions must be performed using bifurcation. Using the above example, this can be done in three successive bifurcate divisions: every member of class *Vertebrata* are either endowed with lungs (*Mammifera*, *Birds, Reptiles*) or not (*Fish*). Every member of class of animals not endowed with lungs is either endowed with mammae (*Mammifera*) or not (*Birds, Reptiles*). Every member of class of animals endowed with mammae is either endowed with wings (*Birds*) or not (*Reptiles*). This division is complete and its logical species are distinct.[89]

It is in George's chapter eight, titled "Propositions", where we find his famous quantification of the predicate. George prepared us for the quantification of the

[86] G. Bentham, *Outline of a New System of Logic, With a Critical Examination of Dr Whately's "Elements of Logic"* (London: Hunt & Clarke 1827), 104-6.
[87] G. Bentham, *Outline of a New System of Logic, With a Critical Examination of Dr Whately's "Elements of Logic"* (London: Hunt & Clarke 1827), 111.
[88] G. Bentham, *Outline of a New System of Logic, With a Critical Examination of Dr Whately's "Elements of Logic"* (London: Hunt & Clarke 1827), 111–2.
[89] G. Bentham, *Outline of a New System of Logic, With a Critical Examination of Dr Whately's "Elements of Logic"* (London: Hunt & Clarke 1827), 113-4.

predicate early on by introducing us to the confusion that can arise from not being clear about relations. In his Chapter Five, titled "Terms", George drew attention to one kind of confusion that can arise with respect to relations, a confusion that arises when we trade terms between disciplines, specifically between Whately's logical notion of genera and species versus a naturalist's notion:

> As to predicating the "whole essence" of a subject, in one term, that is impossible, unless that term be a strict synonym. Dr. Whately does not appear to have been aware of this; he implies, that, if we predicate the genus, we predicate part of the essence; and if the species, we predicate the whole essence; considering *species* in the sense in which naturalists employ the word, in which case it is, in fact, a logical genus with reference to individuals.[90]

In Scholastic logic and in Whately's logic, wrote George:

> [l]ogical species and genus are only relative, not absolute terms, and the use of them always implies *subalternation*; that is, relation, either to a superordinate genus, or to a subordinate species or individual... As to the particular sense in which naturalists make use of the word species, it is very different from the logical sense of the word, the only one in which it should be made use of on the present occasion.[91]

The naturalist's notion of genera and species, in contrast, is absolute, that is, genus and species have a fixed place in a fixed hierarchy of nested sets. George was also quick to notice that Jeremy's emphasis hierarchical tree structures bring into the spotlight the confusions surrounding the logical relations of logical identity, logical diversity, and practical dependence. The goal for George would be to translate the Aristotelian forms of propositions that contain inclusive relations into equivalent propositions that contained relations of identity and diversity.[92] In this respect, we can begin to see why George would consider the quantification of the predicate.

George gave a fuller discussion of his ideas concerning relations in his chapter on propositions. The idea that propositions should be divided into two terms with the relations of logical identity, logical diversity, or subalternation is first introduced in a footnote on page 123, but explored in the body of the text a few pages later:

> [e]very simple proposition may be reduced to the expression between two ideas: the two ideas are represented by the two *terms* (subject and predicate); the relation itself by the copula. The species of relation here referred to are those of identity and diversity [note: The diversity expressed in logical propositions is always *absolute*, and not *contingent*.

[90] G. Bentham, *Outline of a New System of Logic, With a Critical Examination of Dr Whately's "Elements of Logic"* (London: Hunt & Clarke 1827), 69.

[91] G. Bentham, *Outline of a New System of Logic, With a Critical Examination of Dr Whately's "Elements of Logic"* (London: Hunt & Clarke 1827), 69–71.

[92] N. I. Styazhkin, *History of Mathematical Logic from Leibniz to Peano.* (Cambridge: M.I.T. Press, 1969), 148.

> *Resemblance* and *difference*, and other modifications of diversity mentioned in the classification of entities, are not taken into consideration in the syllogistic process. *Diversity* here signifies *non-identity*.] and if logical subalternation.[93]

For George, the only possible relations within the terms of a proposition are identity, diversity, and logical subalternation, where identity is expressed by the copula "is", diversity by the copula "is not". However, for the copula verb to be an identity between classes, the subject and predicate must be of the same quantifiable form — hence the quantification of both the subject and the predicate.

George began the task of reworking the classification of the forms of propositions by expressing the two terms of a judgment by X (subject) and Y (predicate), their identity by the mathematical sign "=", diversity by the sign "$\|$", universality by the words "in toto", and partiality by the words "ex parte" (for brevity he prefixed the letters t and p as signs of universality and partiality). What resulted were not the four familiar Aristotelian forms of judgment, but eight possibilities dependent on the way in which the quantity and quality of the subject will be combined with the predicate:

1. X in toto $= Y$ ex parte or $tX = pY$
2. X in toto $\|Y$ ex parte or $tX\|pY$
3. X in toto $= Y$ ex toto or $tX = tY$
4. X in toto $\|Y$ ex toto or $tX\|tY$
5. X in parte $= Y$ ex parte or $pX = pY$
6. X in parte $\|Y$ ex parte or $pX\|pY$
7. X in parte $= Y$ ex toto or $pX = tY$
8. X in parte $\|Y$ ex toto or $pX\|tY$

George then took those eight forms and reduced them to five, rather than the four traditional Scholastic forms AEIO.[94] The difference between George's classification of propositions and the Scholastic classification of propositions is more than simple a difference in number of forms of proposition. George was thinking extensionally, in terms of nested sets of classes and reducing propositions and syllogisms to an

[93] G. Bentham, *Outline of a New System of Logic, With a Critical Examination of Dr Whately's "Elements of Logic"* (London: Hunt & Clarke 1827), 127-8.

[94] G. Bentham, G. Bentham, *Outline of a New System of Logic, With a Critical Examination of Dr Whately's "Elements of Logic"* (London: Hunt & Clarke 1827), 133-4. George realised that 1 is equivalent with 7, and 2 with 8, because it is immaterial in his logic which is mentioned first, x or y. Similarly, he thinks we can eliminate 2 as 4 expresses the same meaning in a manner more suitable to the process of deduction. Therefore, thought Bentham, we are left with only five types of judgment.

identity between members of classes. One consequence of this move was that the familiar notions of "resemblance" and "essence" no longer belonged to logical classification.[95]

George was aware that earlier logicians had acknowledged the possibility of quantifying the predicate term, but chose not to. George claimed that such an omission led to fallacies. George wrote:

> Logicians make no mention of the first form, which they consider as useless, and they say that the predicate (or the second term of the proposition) is *never distributed* (note: Elements p, 42) (that is, *universal*). I should think however, that this assertion can scarcely be logical. Many fallacies arise from the considering of terms as synonymous which are not so in reality; and it may be found as advantageous to reduce perfect identity to a logical form; as partial identity, or perfect or partial diversity.[96]

This innovation, the quantification of the predicate, also made George re-think the axiomatization of syllogisms Whately presented in the *Elements*. One of the virtues of Whately's logic is that he provided an axiomatic presentation of the rules for syllogisms — a virtue George happily acknowledged and adopted in his *Outline*. George wrote:

> But, if every legitimate syllogism must, by definition of the word, be self-evident without the help of these rules, might we not suppress altogether a system which requires so much labour to understand it, so much strain on the memory to keep it in mind, and which, after all, rather takes away from, than adds to, this self-evidence? Might not we substitute a few plain and simple axioms, the truth of which cannot be denied, and which may be found to contain, in general terms, every principle upon which a syllogistic conclusion can be founded?[97]

Traditionally, the students of Scholastic logic had to memorise mnemonic devices in order to remember the valid forms of syllogisms. Whately replaced the cumbersome memory tricks with a small set of axioms, an innovation that not only rendered distinguishing syllogisms into moods and figures useless, but spoke to Whately's attempt to see logic as a science with syllogisms as its theoretical core.

In his Chapter Nine, titled "Deduction", George discussed this axiomatization. Like Whately, George set out to axiomatize syllogistic logic, and even credited the first two of his own four axioms to Whately. George cited the two relevant axioms of Whately as: if two terms agree with one and the same third, they

[95]See G. McOuat "The Logical Systematist: George Bentham and His Outline of a New System of Logic." *Archives of Natural History* 30 (2003).

[96]G. Bentham, *Outline of a New System of Logic, With a Critical Examination of Dr Whately's "Elements of Logic"* (London: Hunt & Clarke 1827), 135.

[97]G. Bentham, *Outline of a New System of Logic, With a Critical Examination of Dr Whately's "Elements of Logic"* (London: Hunt & Clarke 1827), 155.

agree with each other; if one term agrees, and another disagrees with one and the same third, these two disagree with each other.[98] George then revised these two axioms in accordance with his new analysis of propositions, that is, he replaced the relationship of agreement in Whately's axioms with the relation of equality. Consequently, George's first two first two axioms read as: things which are equal to the same, are equal to one another; and when two things, one only is equal to a third, and the other is not equal to that third, these two things are not equal to each other.

George's new analysis of propositions also prompted his addition of two more axioms.[99] For George, Whately's axioms are sufficient for a logic that does not quantify the predicate, but if we do quantify the predicate, then we become aware of that rules are necessary for those syllogisms where, for example, the middle term is universal in both premises. George claimed that "for it is then only that the two extremes are precisely equal to the same *mean*."[100] George two new axioms are as follows: parts of a part are parts of the *whole* of that part (that is, of the whole of which that part is a part); and when the whole of a class is said to be equal to, or different from, the whole or any part of another class, it is meant that *every* individual referred to by the first class, is the same as, or different from, any individual referred to by such whole or part of such other class.[101]

George's innovations in this chapter regarding the analysis of propositions marked a shift away from relations of "resemblance" and "essence", and a move towards a logic of classes — an extensional logic with relations of identity and diversity — that would be taken up by the pioneers of algebraic logic.

To conclude, during the 1870s, our retired botanist found himself in a strange position in two revolutions. In spite of the fact that he was President of the Linnean Society of London, George was seen as being mired in the pre-evolutionary styles and practices of a by-gone era during the revolution in natural history. In spite of the fact that his only significant logic publication — the *Outline of a New System of Logic* — was not widely read, George was seen as going well beyond his masters Richard Whately and Jeremy Bentham during the revolution in logic. What remained constant in George were his ideals of clarity and precision in language, and cautioning against speculation in science. The tale of George

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[98] G. Bentham, *Outline of a New System of Logic, With a Critical Examination of Dr Whately's "Elements of Logic"* (London: Hunt & Clarke 1827).

[99] George wrote:

> In order to obviate all these difficulties of expressions with regard to the concordance and the disagreement of collective entities, that I have thought it necessary to modify the expression of Dr. Whately's two axioms, and to add two others, the last of which may perhaps be considered rather as an explanation of the relations of collective entities to one another, than strictly speaking, as an axiom.

G. Bentham, *Outline of a New System of Logic, With a Critical Examination of Dr Whately's "Elements of Logic"* (London: Hunt & Clarke 1827), 158.

[100] G. Bentham, *Outline of a New System of Logic, With a Critical Examination of Dr Whately's "Elements of Logic"* (London: Hunt & Clarke 1827), 156.

[101] G. Bentham, *Outline of a New System of Logic, With a Critical Examination of Dr Whately's "Elements of Logic"* (London: Hunt & Clarke 1827), 155-6.

Bentham's life, his influences, and his innovations show that logic and natural history were intimately connected during the nineteenth century, but not in ways normally discussed in the histories of either discipline.

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COLERIDGE'S LOGIC

Tim Milnes

1 WHY READ COLERIDGE'S *LOGIC?*

In 1834, Thomas De Quincey wrote of a fellow essayist, philosopher and opium addict that 'logic the most severe was as inalienable from his modes of thinking as grammar from his language.'[1] This assessment of Samuel Taylor Coleridge reveals more, perhaps, than De Quincey intended. It not only indicates the importance of logic to Coleridge's thought, but also the unconventional use he made of it. One of Coleridge's main intellectual ambitions, indeed, was to unite the disciplines of grammar and logic. This task, daunting enough in itself, was however, only part of a much broader undertaking. The reconciliation of grammar, or language, with logic could only be effected through a universal organon that encompassed subjects as diverse as poetry, epistemology, natural philosophy, hermeneutics, metaphysics and theology: in short, as he saw it, all aspects of human life. Even in the work which he devoted to the subject, the *Logic* manuscript, which, like so many of the tantalising, unfinished or fragmentary works that littered his career, remained unpublished at his death, Coleridge showed little inclination to separate logical questions from ethical and existential concerns. The reader who approaches Coleridge for the first time with the assumption, as Quine puts it, that 'logic in the strictest sense is quantification theory, and a logical deduction in the strictest sense consists in establishing a quantificational implication,' is likely to find Coleridge's treatment of logic exotic, often baffling, sometimes even frustrating.[2]

Why then, as James McKusick asks, 'should we care about Coleridge's *Logic, a* book that was unpublished in his lifetime and had no contemporary influence?'[3] Most of its readers today are professional academics attached to English Literature departments whose interest in Coleridge is primarily literary, historical, or a mixture of both. Such commentators have on the whole, long since discarded what Owen Barfield disparagingly refers to as 'biographical/comparative' and 'biographical/psychological' approaches to Coleridge's thought that characterised

[1] Thomas De Quincey, 'Samuel Taylor Coleridge,' *The Collected Writings of Thomas de Quincey,* rev. ed. David Masson, vol. 2 (Edinburgh: Adam and Charles Black, 1889) 153.

[2] Willard Van Orman Quine, *Elementary Logic,* rev. ed. (Cambridge, MA: Harvard University Press, 1980) 116.

[3] James C. McKusick, 'Coleridge's "Logic": A Systematic Theory of Language,' *Papers in the History of Linguistics,* eds. Hans Aarsleff, Louis G. Kelly and Hans-Joseph Niederehe (Amsterdam: John Benjamins Publishing Company, 1987)479-480.

Handbook of the History of Logic. Volume 4: British Logic in the Ninteenth Century.
Dov M. Gabbay and John Woods (Editors)

much scholarship in the early twentieth century, preferring instead to concentrate on the materiality of either Coleridge's language or the historical conditions through which it was determined.[4] This adjustment in focus has been linked to developments in critical theory that make any attempt to assess what Coleridge *thought* appear hazardous, to say the least.[5] For his part, McKusick responds to his own challenge by arguing that the *Logic* is one of Coleridge's 'most coherent and intellectually sophisticated works,' and 'arguably the most important theory of general linguistics produced in England during the early nineteenth century.'[6] Whether such claims stand up to scrutiny remains to be seen. Nonetheless, what can be averred at this stage, albeit tentatively, is that it is the very same qualities that make Coleridge's logic appear so exotic and alien to a modern reader that make it worthy of attention, and then not merely as some historical curio, but as highlighting a significant part of the genealogy of modern European thought, one that is too easily forgotten.

This claim will come as no surprise to Coleridge's commentators, who have long since established that Coleridge's reputation rests upon more than a slim collection of poems and an influential if gnomic apology for the creative imagination. His wide- ranging writings, collected in the Bollingen edition — which, after half a century's work, has recently been completed — reflect the ebbing of the eighteenth-century faith in empirical science and reason and the nineteenth century's dawning interest in religion, history and idealism. Coleridge's world is that of post-revolutionary optimism swiftly followed by war, paranoia and the search for a new resolution. For Coleridge, one of the lessons of the age is the inevitable failure of any attempt to reform society upon purely abstract principles. Instead, modernity must be allowed to evolve as a higher synthesis of the new energies of reason and science with the best traditions present in the collective and often unconscious experience of the people.

In this new synthesis, logic plays an important but ambivalent role. On one hand it is the levelling instrument of the revolution, a principle of abstract understanding, which, overplayed by the Enlightenment *philosophes,* is all too easily appropriated by the discourse of calculated utility, by a system of capital that hi-jacks the language of Hume and Bentham. The logic of Enlightenment, as Wordsworth expresses it in his poem 'The Tables Turned,' represents the 'meddling intellect' by which 'We murder to dissect.' On the other hand, and against Wordsworth, Coleridge argued that logic could be rehabilitated once it had been reconnected with an older wisdom, with the higher logic he found in scripture, ancient philosophy, and above all in the new German idealism.

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[4]Owen Barfield, *What Coleridge Thought* (Connecticut: Wesleyan University Press, 1971) 3. Nigel Leask's *The Politics of Imagination in Coleridge's Critical Thought* (Basingstoke: Macmillan, 1988) remains one of the best studies of the historical and political context of Coleridge's philosophy.

[5]Thus, a decade after Barfield's work, Jerome Christensen sought in *Coleridge's Blessed Machine of Language* (Ithaca: Cornell University Press, 1981) 'not [...] to reproduce what Coleridge thought,' but 'to produce the way Coleridge writes' (15-16).

[6]McKusick, 'Coleridge's "Logic"' 480.

How this rehabilitation was to be effected was already the subject of debate. By the end of the eighteenth century there was a widespread feeling that logic was in need of a serious overhaul. Traditional logic was struggling to adapt to the epistemological implications of an accelerating culture of scientific discovery. In particular, before Mill developed an inductive logic, there seemed to be a disjuncture between empirical psychology and the canons of logical deduction. Coleridge registers this friction, noting with regret that Aristotle's 'Organum,' which had survived largely intact for centuries, had recently been deposed in France and 'made to give way to the Logic of Condillac,' although it would 'be more accurate perhaps to say that the Study of Logic altogether is exploded in France, for Condillac's Book is rather psychological than logical.'[7] Written in 1803, these remarks form part of the preparatory notes Coleridge made outlining a plan for his own 'History of Logic.' It is a full twenty years, however, before he writes to John Taylor Coleridge, announcing the readiness of the *Logic,* which at this stage still bears the working title, ' "The Elements of Discourse, with the Criteria of true and false Reasoning, as the ground-work and preparation for Public Speaking and Debate — addressed to the Students and Candidates for the Pulpit, the Bar, or Senate".' By this time, Coleridge has apparently made up his mind about the subordinate status of logic within the hierarchy of intellectual disciplines. The 'Elements,' he claims, is 'a Work of Logic for [...] *forensic* purposes, denying its applicability, as a positive Organ, to all subjects [...] in which the absolute Truth is sought for,' and of use 'in all subjects of discussion or inquiry, in which the Truth relatively to the Sense and Understanding of man in all his social and civil Concerns and Functions is alone required or of pertinence.'[8] Nonetheless, and despite Coleridge's careful demarcations, it was to prove difficult to confine logic to the purely *'forensic* purposes' of intellect, a problem that was to have significant ramifications for his conception of the relationship between logic, truth, and 'man in all his social and civil Concerns.'

2 THE POLITICS OF LOGIC

The civic function of logic is of vital importance to Coleridge. As Heather Jackson has noted, after the French Revolution the concept of logic becomes highly politicised, one of the 'red-flag terms,' like method, theory or even philosophy, which could be relied upon to send sections of British society into paroxysms of anxiety.[9] Increasingly disaffected with the 'logic' of French rationalism, but equally contemptuous of the British tradition of 'common sense,' Coleridge comes to the

[7]Samuel Taylor Coleridge, *Shorter Works and Fragments,* eds. H. J. Jackson and J. R. de J. Jackson, vol. 1 (Princeton University Press, 1995) 128.

[8]Samuel Taylor Coleridge, 'To John Taylor Coleridge,' 5 June 1823, letter 1335 of *Collected Letters of Samuel Taylor Coleridge,* ed. Earl Leslie Griggs, vol. 5 (Oxford: Clarendon Press, 1956-71) 275.

[9]See Heather J. Jackson, 'Coleridge's Lessons in Transition: The "Logic" of the "Wildest Odes",' *Lessons of Romanticism: A Critical Companion,* eds. Thomas Pfau and Robert F. Gleckner (Durham: Duke University Press, 1998) 221-222.

conclusion that what both lack is *logic,* in its strictest sense. Around the time he writes his 'Outlines of the History of Logic,' he makes a notebook entry that reflects upon the decline of the discipline in both Britain and France:

> Of Logic & its neglect, & the consequent strange Illogicality of many even of our principal writers — hence our Crumbly friable Stile/each Author a mere Hour-Glass/ — & if we go on in this way, we shall soon have undone all that Aristotle did for the human Race, & come back to Proverbs & Apologues — /The multitude of Maxims, Aphorisms, & Sentences & their popularity among the French, the beginners of this Style, is it some proof & omen of this?[10]

Coleridge's particular concern with the 'Crumbly friable Stile' of the modern writer in this entry is telling. Deprived of a unifying logic, language becomes atomised, contingent, 'each Author a mere Hour-Glass' of running particulars. In this way, Coleridge hopes to show that 'The multitude of Maxims, Aphorisms, & Sentences' to be found in the French materialist writers who supported the revolution, so far from being the *neplus ultra* of logic, was thoroughly illogical. Once again, he found the neglect of ancient thought, and the declining reputation of Aristotle in particular, to be symptomatic of a deep intellectual malaise. The virtue of Aristotle's logic, in contrast to the psychologising of the French followers of Locke, was its understanding of the deep and noncontingent connection between thought and language, a theme he examines at length in his 1818-1819 lectures on the history of philosophy:

> So Aristotle first of all determined what were the laws common to all coherent thinking, and therein he founded not only the science of logic, but with it he made general throughout all the civilised world the terms of the connection: "we", "me", "our", ["us"], our "ands", and our "thes", and our "therefores", and so forth.

In contrast, he continues, like 'oriental writing' in which 'thought is put on thought with little other connective than "ad[d]" for "and",' the 'new French writings' are 'aimed at destroying all the connections of thought' as well as 'all the connections of society and domestic life.'[11] At this point, Coleridge's concern for the links between language, logic and civic harmony breaks the surface of his writing. Above all, ancient Greek thought demonstrated the compatibility between principles of democracy, social cohesion and logic in a way that did not depend upon the reductive, aggregative kind of philosophy that had, as he saw it, reduced France and much of Europe to dust. Indeed, as he argues in the 'Outlines of the History of Logic,' 'Ethics & argumentative Metaphysics were the Offspring of Democracies whom superior Courage & superior Intellect had rendered victorious

[10]Samuel Taylor Coleridge, *The Notebooks of Samuel Taylor Coleridge,* ed. Kathleen Coburn et al., vol. 1 (London: Routledge, 2002) 1759 (text).

[11]Samuel Taylor Coleridge, *Lectures 1818-1819 On the History of Philosophy,* ed. J.R. de J. Jackson, vol. 1 (Princeton University Press, 2000) 234.

over Despots.'[12] In this way, having 'scramble[d] the familiar codes,' as David Simpson puts it,[13] disassociating method, theory and logic from Jacobinism, Coleridge felt able to have his radical cake and eat it, arguing that 'the origin of Logic' was in 'a democracy when the Varieties of Character & moral habit find sufficient Space & free playroom.'[14] The challenge facing modernity then, was one of recovering that democratic intellect in an age when the increasing dominance of a mechanised and levelling understanding had rendered democracy itself dangerous. To this end, Coleridge believed, philosophy must be enlisted in the process of reform, binding particular, empirical or abstract truths to spiritual, practical, organic Truth. It is on this basis that *The Friend* mounts its politically conservative defence of 'theory':

> The Friend, however, acts and will continue to act under the belief, that the whole truth is the best antidote to falsehoods which are dangerous chiefly because they are half-truths: and that an erroneous system is best confuted, not by an abuse of Theory in general, nor by an absurd opposition of Theory to Practice, but by a detection of the errors in the particular Theory. For the meanest of men has his Theory: and to think at all is to theorize.[15]

The faintly unpatriotic air of the final remark is neutralised by Coleridge's insistence that the ultimate destiny of all theory is the ideal. The past excesses of theory, philosophy, method, and logic will be overcome not by rejecting theory as such, but by pursuing theory to its limit as an ideal of reason: in other words, to the point where it merges with being. This notion of atonement through higher synthesis, through the reconciliation of positions apparently opposed or contradictory, is crucial to Coleridge's changing view of logic, as the radical philosophies of the young Unitarian poet and lecturer give way to the more harmonious visions of the German idealist. For the latter figure, increasingly reliant upon opium, upon his doctor and landlord in Highgate for managing that addiction, and upon friends, students and the flattering attention of curious visitors for his sense of self-worth, feelings of guilt combine with a sense of lost opportunities and wasted strength to colour his view of human intellect. In the struggle to comprehend the failures of revolution and his own personal inadequacies, ideas of expiation and the search for a higher unity become the dominant figures of Coleridge's later thought: in particular, they shape his belief that the quantificational logic of an overweening, 'mechanical' understanding must be redeemed by the higher, qualitative logic of a divinely sanctioned *summum bonum.*

Indeed, one of the few constant elements throughout Coleridge's career is his Christian faith, and his refusal to treat questions of faith and reason separately.

[12] Coleridge, *Shorter Works* 1 127.

[13] David Simpson, *Romanticism, Nationalism, and the Revolt Against Theory* (University of Chicago Press, 1993) 60.

[14] Coleridge, *Shorter Works* 1 128.

[15] Samuel Taylor Coleridge, *The Friend,* ed. Barbara E. Rooke, vol. 1 (Princeton University Press, 1969) 189.

Logic, like any dimension of human intelligence, can only ultimately be comprehended in terms of Christianity. Christianity, however, is not a 'subject' for philosophy; indeed, it is questionable whether it is susceptible of 'comprehension' at all. As he puts it in *Aids to Reflection,* 'Christianity is not a Theory, or a Speculation; but a *Life.* Not a *Philosophy* of Life, but a Life and a living Process.'[16] The notion that logic has an integral role to play within a Christian theosophy, and that only as part of that broader theosophical order can logic itself be demonstrated to its fullest extent, is one of the larger gulfs that lie between the modern reader and Coleridge's work. One useful way of approaching this problem is to remain alert to how elastic the concept of 'logic' remains in this period. Reflecting on his own early education in *Biographia Literaria*, Coleridge reveals a climate in which, as the boundaries between disciplines are being contested and reformed, definitions are up for grabs. Recalling his old schoolmaster at Christ's Hospital, James Bowyer, he writes:

> I learnt from him, that Poetry, even that of the loftiest, and, seemingly, that of the wildest odes, had a logic of its own, as severe as that of science; and more difficult, because more subtle, more complex, and dependent on more, and more fugitive causes.[17]

Comments such as this hint at Coleridge's ambition to forge a link between the logical and the existential, between ratiocination and 'life.' This aspiration, however, wrestled with a more positivist impulse to regulate any new illumination under the auspices of a master discipline that embodied the codes of traditional philosophy. Turning to his early schooling in the rules of poetic diction, Coleridge recounts his dissatisfaction with conventional authorities on prosody:

> But as it was my constant reply to authorities brought against me from later poets of great name, that no authority could avail in opposition to Truth, Nature, Logic, and the Laws of Universal Grammar; actuated too by my former passion for metaphysical investigations; I laboured at a solid foundation, on which permanently to ground my opinions, in the component faculties of the human mind itself, and their comparative dignity and importance.[18]

These attempts on one hand to poeticise logic and, on the other, to underpin poetry *with* logic might appear, at least potentially, to conflict. For Coleridge, they are two sides of the same coin, as the common principles of both are to be found 'in the component faculties of the human mind itself.' And yet, even though Coleridge writes *Biographia* in 1816 with the confidence of someone who, having studied long and hard the lessons of German idealism, is able to flourish

[16]Samuel Taylor Coleridge, *Aids to Reflection,* ed. John Beer (Princeton University Press, 1995) 202.

[17]Samuel Taylor Coleridge, *Biographia Literaria,* eds. James Engell and Walter Jackson Bate, vol. 1 (Princeton University Press, 1995) 9.

[18]Coleridge, *Biographia* 1 22

as his trump card Kant's account of the 'comparative dignity and importance' of the faculties of understanding and reason (of which more later), his concern with the relationship between logic and the language of poetry indicates the continuing influence of a different tradition. As Paul Hamilton argues, the imported German philosophy of self-consciousness to which Coleridge was first introduced around 1800 had to find an accommodation in his thought with a concern with linguistic propriety — specifically, with the normativity of 'ordinary' language — that he had inherited from late eighteenth-century British philosophies of language and literary theory.[19] It is within this tradition that Coleridge's original logical speculations had originally taken shape.

3 ETYMOLOGIC

The question of the relationship between logic and language is one to which Coleridge returns throughout his life, with varying results. Here, as in so many areas, his views change over time. An 1800 letter to William Godwin, however, reveals one preoccupation that was to remain central to his thinking: the idea that language is not merely a system of conventions, but a manifestation of divine providence:

> I wish you to write a book on the power of words, and the processes by which human feelings form affinities with them — in short, I wish you to *philosophize* Horn Tooke's System, and to solve the great Questions [...]. "Is Logic the Essence of Thinking?" in other words — Is *thinking* impossible without arbitrary signs? & — how far is the word "arbitrary" a misnomer? Are not words &c parts & germinations of the Plant? And what is the Law of their Growth? — In something of this order I would endeavour to destroy the old antithesis of *Words & Things,* elevating, as it were, words into Things, & living Things too.[20]

Coleridge's initial willingness to identify logic with 'arbitrary signs' reveals the increasing influence of John Home Tooke's work, particularly *The Diversions of Purley,* upon his thought at this point. And yet, his desire *'to philosophize* Horn Tooke's System' shows that he is already trying to link Tooke's etymological theories to a metaphysical order based on organic principles, according to which words are 'parts & germinations of the Plant.' The idea that logic and language grow from the same seed underscores Coleridge's conviction that the linguistic sign is not arbitrary in its reference, and thus that etymology and logic are fundamentally engaged in the same enterprise: that of understanding their common 'germination.' Following Stephen Prickett, we can roughly divide Coleridge's thinking about language into three phases: (1) an early stage, up to around 1800, when his theories are still dominated by the ahistorical, materialistic associationism of

[19] Paul Hamilton, *Coleridge's Poetics* (Basil Blackwell, 1983) 3.
[20] Coleridge, 'To William Godwin,' 22 September 1800, letter 352 *of Letters* 1 625-626.

David Hartley; (2) a relatively brief period in the first decade of the nineteenth century, during which the influence of Tooke made him increasingly 'aware of the illogical complexities of language,' and a final phase (3) in which, under the sway of German idealism, he developed a metaphysics of language as constantly evolving, 'with words related not so much to things as changes in human consciousness itself.'[21]

(1) Seen in this frame, Coleridge's comments to Godwin are pivotal, in that they attest to his gathering sense that Hartley's claim that words are simply impressions that *'excite Ideas in us by Association, and [...] by no other means'* was damagingly reductive, a poor explanation of 'the power of words, and the processes by which human feelings form affinities with them.'[22] Similarly, Hartley's vision of the growth of language as fundamentally metaphorical, producing meaning as new relations of 'likeness' replaced older, 'literal expressions,' sat uneasily with Coleridge's belief that the principle of words and the 'Law of their Growth' must be rooted in a power more dignified than that of contingent association.[23] In this light, Hartley's assumption that the bond between language and the world was a secure but inscrutable part of God's design is cold comfort. In response, Coleridge's plan 'to destroy the old antithesis of Words & Things, elevating, as it were, words into Things, & living Things too,' should be read as beginning a move away from a correspondence theory of language, towards a view of words as in some way constitutive of thought. Words, as he writes in a notebook entry of 1810, 'are not mere symbols of things & thoughts, but themselves things.'[24]

Coleridge's reaction against Hartley thus helps to form his own contribution to the ongoing search for a 'natural' language of humanity. By rejecting Hartley's conventionalist linguistics of association, he aligns himself with eighteenth-century nonconventionalists like Thomas Reid and Lord Monboddo, who argued that the origins of language were to be found in the nascent concepts of inarticulate cries in primitive societies.[25] Indeed, Coleridge's involvement in the debate over natural language feeds directly into his famous argument with Wordsworth on poetic diction. Like Wordsworth and Shelley, Coleridge was given to drawing parallels between the inchoate lyricism of early societies and the embryonic linguistic skills of children. However, he drew the line at making a fetish of everyday language, or prizing childish utterance as worthy of poetic emulation. Natural language was emphatically *not* 'ordinary' language. Accordingly, in the second volume *of Biographia Literaria,* he denies that everyday language of rural folk should in be taken any way as normative:

[21] Stephen Prickett, *Words and The Word* (Cambridge University Press, 1986) 147.
[22] David Hartley, *Observations on Man,* vol. 1 (1791, Poole: Woodstock Books, 1998) 268.
[23] Hartley 1 292.
[24] Coleridge, *Notebooks* 3 3762.
[25] Monboddo, in turn, aligns himself with Rousseau when he claims 'that, before men used language, they conversed together by signs and inarticulate cries [...]. And that the first languages had a greater greater deal of prosody, or musical tones [...]' *(On the Origin and Progress of Language,* 2nd ed., vol. 1 [Edinburgh, 1774] x).

> The best part of human language, properly so called, is derived from reflection on the acts of the mind itself. It is formed by a voluntary appropriation of fixed symbols to internal acts, to processes and results of imagination, the greater part of which have no place in the consciousness of uneducated man [. . .].[26]

More revealing still is Coleridge's own advice to the poet:

> But if it be asked, by what principles the poet is to regulate his own style, if he do not adhere closely to the sort and order of words which he hears in the market, wake, high-road, or plough-field? I reply; by principles, the ignorance or neglect of which would convict him of being no *poet,* but a silly or presumptuous usurper of the name! By the principles of grammar, logic, psychology![27]

The origins of language are 'primitive,' then, in a metaphysical, not a social, sense. Just as the principles of poetic language are bound up with logic, grammar and psychology, the origins and fates of words are bound up with the universal Logos. The appearance of 'grammar' in Coleridge's list of poetic desiderata is significant, notwithstanding its philosophical subordination, for it betrays the extent to which Tooke's influence on Coleridge persisted, despite the immersion in transcendentalism that helped to produce *Biographia.*

(2) For Coleridge, Tooke's work had two major implications, one of which was deplorable, the other welcome. The first — the hub of Tooke's argument — was the linguistic deflation of ideas. As he argues in *Diversions,* any truly empirical 'consideration of *Ideas* [...] will lead us no farther than to *Nouns:* i.e. the signs of these impressions, or names of ideas.'[28] Coleridge found this reduction unacceptable, an error driven, as he saw it, by a regrettable hangover from empiricism, namely the assumption that 'Words are the *signs of things.*'[29] Against Tooke's linguistic atomism, Coleridge proposes a dynamic model of language based upon the power of the verb, an agency he links to the act of consciousness and the Logos. As Mary Anne Perkins argues, the Logos, presenting the underlying spiritual unity of truth, language and being, is 'the "key" to understanding every area of his thought after 1805.'[30] Vital to this idea is the conception of being as an act, not a thing. Reality and thought are formed by verb substantive, the 'I am,' or living word of God. The most basic presupposition of thought then, is unity, not difference. Coleridge makes this point unambiguously in one of his 1819 lectures on philosophy, when he tackles the question: what is thought?

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[26] Coleridge, *Biographia* 2 54.
[27] Coleridge, *Biographia* 2 281.
[28] John Home Tooke, *Epea Pteroenta or the Diversions of Purley,* rev. ed. by Richard Taylor, vol. 1 (London, 1829) 49.
[29] Tooke, *Diversions* 1 18.
[30] Mary Anne Perkins, *Coleridge's Philosophy: The Logos as Unifying Principle* (Oxford: Clarendon Press, 1994) 3.

> We are [so] apt to use words derived from external objects, that I verily believe that many a man before he asked himself the question would imagine his mind made of thoughts, as a wall is of bricks, till he asks whether he ever separated a thought from a thought, whether there was a meaning in the word "thought," except as the mind thinking in such a situation; whether a notion of any plurality, or anything we can call construction, takes place in our experience. We are aware of no such thing [...].[31]

Such speculations, however, inevitably raise the question: is logic subordinate to grammar? Despite the claims of his apologists, this is a conundrum that Coleridge never quite resolves. McKusick, for instance, defends the coherence of Coleridge's belief that the apparent dependence of logical canons upon rules of grammar merely indicates the existence of a higher logic in the form of the Logos, in which truth and meaning have not yet been alienated. Language is prior to syllogistic logic, in other words, but the Logos is the spirit of a higher reason in words. Accordingly, language *speaks us,* but only because the spirit of God, and thus humanity, already inhabits language.[32] Further, McKusick agrees with Heather Jackson that it is in this way, by drawing the deflationary sting from Tooke's theory of language, that Coleridge is able to welcome the second major implication of Tooke's work: the constitutive role of language in thought and, correspondingly, the centrality of etymological inquiry to any philosophical investigation. As Coleridge himself wrote in literary correspondence published in 1821:

> Etymology [...] is little else than indispensable to an insight into the true force, and, as it were, freshness of the words in question, especially of those that heve passed from the schools into the marketplace, from the medals and tokens [...] of the philosopher's guild or company into the current coin of the land.[33]

Coleridge accepts Tooke's etymology as a method of inquiry, but sides with Tooke's opponents like Monboddo in arguing that, far from being accidental, the structures of language are determined by 'logical categories which are themselves intrinsic to thought.'[34] Indeed, Coleridge's disagreement with Tooke even finds its way into his lectures on Shakespeare, as J. Tomalin's contemporary notes testify:

> Home Tooke had called his book Epea Pteroenta, winged words. In Coleridge's judgement it might have been much more fitly called Verba Viventia, or "living words" for words are the living products of the living mind & could not be a due medium between the thing and the

[31] Coleridge, *Lectures 1818-1819* 2 577.

[32] James C. McKusick, *Coleridge's Philosophy of Language* (New Haven: Yale University Press, 1986) 50.

[33] Coleridge, *Shorter Works* 2 927-928.

[34] McKusick, *Coleridge's Philosophy of Language* 42.

> mind unless they partook of both. The word was not to convey merely what a certain thing is, but the very passion & all the circumstances which were conceived as constituting the perception of the thing by the person who used the word.[35]

Etymology then, as Jackson puts it, becomes a 'tool won from the enemy': effectively, 'etymologic.'[36] This in turn enables Coleridge to revive the Enlightenment quest for a theory of universal language based on the metaphysical primacy of the Absolute *act* of self-creation made incarnate in the divine Logos. By the time he writes *Aids to Reflection* in the late 1820s, Coleridge has even abandoned earlier talk of words as 'living things,' denying that words are 'things' at all: 'For if words are not things, they are living powers, by which the things of most importance to mankind are actuated, combined, and humanized.'[37]

It is worth pausing for a moment to consider further the significance of this turn in Coleridge's theosophy of language. So long as he had sought to 'philosophize' Tooke's system by rendering it more dynamic, by drawing truth and meaning closer together, Coleridge's conception of 'etymologic' had had the potential to lead to conclusions similar to those of Tooke. Increasingly linked to meaning and to the everyday activity of interpretation, Coleridge's concept of truth, for a moment, begins to appear more holistic and linguistic than metaphysical or absolute. In a notebook entry of December 1804, for instance, he once again attempts to improve on Tooke by demonstrating how both 'Word' and 'Truth' are etymologically related to terms of action:

> Word, Werden — that which is — <Worth, wirthy — Rede, Redlich> — Truth is implied in Words among the first men. Tale = tale. Word, wahr, wehr — truth, troweth, throweth i.e. hitteth = itteth = it is it. The aspirate expresses the exclamatia of action. Through, & Truth — Etymol.[38]

In his haste to demonstrate that truth and language originate in a primordial 'action,' however, Coleridge chooses not to explore the further implications of his remark that 'Truth is implied in Words among the first men.' The possibility that

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[35]Samuel Taylor Coleridge, *Lectures 1808-1819 On Literature,* ed. R.A. Foakes, vol. 1 (New Jersey: Princeton University Press, 1987) 272-273.

[36]See Heather J. Jackson, 'Coleridge, Etymology and Etymologic,' *Journal of the History of Ideas* 44.1 (1983) 81. Similarly, James McKusick argues in *Coleridge's Philosophy of Language* that 'Home Tooke was a seminal influence throughout most of Coleridge's intellectual career' (39).

[37]Coleridge, *Aids* 10.

[38]Coleridge, *Notebooks 2* Text 2354. In her note to this passage, Kathleen Coburn argues that Coleridge was misled by Adelung's dictionary, which 'suggests, wrongly, that wahr is identical with the past tense, war, of the verb "to be".' As for 'truth, troweth, throweth': 'The first two words are obviously connected, but neither of them with the third.' Coleridge's speculations on the origins of 'hitteth,' meanwhile, 'seem devoid of any philological basis, nor is there any etymological connexion between "through" and "truth".'

truth might simply be the presupposition of communication, the basic precondition for language to have meaning at all, is passed over. In Coleridge, linguistic explanations must ultimately rest upon ontological or metaphysical theories. Not for the last time, the potential of 'etymologic' is sacrificed to Coleridge's quest for ultimate etymologoi.

(3) This brings us to Prickett's 'third' stage in Coleridge's changing views on language. Parallel to his etymological speculations, Coleridge developed a metaphysical picture of language evolving through a spiritual process of dialectic. Etymologic was certainly one way of comprehending this evolution. But while etymologic helped one to understand the outward signs of this progression — by, as it were, putting the process into reverse — it could provide no insight into its basic principles. Consequently, when Coleridge writes *Logic* (ostensibly a handbook on logic but, as McKusick notes, equally a theory of language) and *Aids to Reflection* in the 1820s, the role of etymologic is reduced to that of under-labourer, a mere supplement of theosophy. Thus, when Coleridge declares in the Preface that one of the main aims of *Aids to Reflection* is to highlight the advantages of knowing words in 'their primary, derivative, and metaphorical senses,' these varieties of meaning are seen as determined by a prior metaphysical order of unity, antithesis and synthesis.[39] As Michael Kent Havens notes, the 'three types of meaning correspond respectively to a primal unity of language, the ongoing process of distinction or desynonymization, and the reuniting of distinguished meanings in a newly holistic language.'[40]

Desynonymization is a key element in Coleridge's theory of language.[41] Coleridge sees the development of language as an organic process whereby equivalences between terms are broken down, just as branches and leaves emerge from an originally undifferentiated trunk. As Coleridge explains in one of his philosophical lectures of 1819, one must be conscious

> that the whole process of human intellect is gradually to desynonymise terms, that words, the instruments of communication, are the only signs that a finite being can have of its own thoughts, that in proportion as what was conceived as one and identical becomes several, there will necessarily arise a term striving to represent that distinction.[42]

This emergence of difference, however — a process that is both conceptual and historical — is merely the furtherance of a richer unity already inherent in the totality of the organism, or language. For Coleridge then, the fate of formal logic is bound up with the progress of language as it strives throughout history to

[39] Coleridge, *Aids* 1.

[40] Michael Kent Havens, 'Coleridge on the Evolution of Language,' *Studies in Romanticism* 20 (1981): 167.

[41] Paul Hamilton argues that Coleridge repressed his argument concerning desynonymy in Biographia Literaria because of his concerns about how its potentially radical political implications might be interpreted. See *Coleridge's Poetics* 72.

[42] Coleridge, *Lectures 1818-1819* 1 212.

become more fully itself, growing through stages of spiritual improvement towards the vanishing point of ultimate reunification with the Logos in higher synthesis. Consequently, the condition of a language at any given time is an expression of the stage of a culture's spiritual and intellectual development. In an 1818 lecture to the London Philosophical Society, Coleridge contrasts the primitive unity of ancient Greek with the atomistic, aggregative logic of northern European, or 'Gothic' languages. In this way, Greek societies, leaning 'to the manifold and popular, the unity in them being purely ideal, namely of all as an identification of the whole' differed from the 'northern or Gothic nations,' in which 'the individual interest was sacred':

> In Greek the sentences are long, and structure architectural, so that each part or clause is insignificant when compared with the whole. But in the Gothic [...] the structure is short, simple, and complete in each part, and the connexion of the parts with the sum total of the discourse is maintained by the sequency of the logic, or the community of feelings excited between the writer and his readers.[43]

In this light, it is the privilege of modernity to complete the dialectic by which these traditions are united. Only in the greatest writers, however, does Coleridge find clear evidence of the growth of the Logos through language, and only in Shakespeare does the fallen status of human language as a system of arbitrary signs achieve partial redemption. As he writes in his lecture notes, Shakespeare's language is neither the original 'Language of Nature' that 'was with the Thing, ⟨it⟩ represented, & it was the Thing represented,' nor the artificial language of the civilized intellect, but

> a something intermediate, or rather it is the former blended with the latter, the arbitrary not merely recalling the cold notion of the Thing but expressing the reality of it, & as arbitrary Language is an Heir loom of ⟨the⟩ Human Race, being itself a part of that which it manifests.[44]

For Coleridge then, as he puts it in *Biographia,* 'besides the language of words, there is a language of spirits,' and 'the former is only the vehicle of the latter.'[45] The position of logic in this arrangement, however, remains ambiguous: formally, a mere stepping-stone of abstraction in language's return to the Logos, it is also destined, in its higher, metaphysical incarnation, to form the Logos itself.

4 THE IDEA OF *LOGIC*

Coleridge's views on Shakespearean language in 1818 show how his reading in German philosophy is leading him to think of language and logic in more frankly

[43] Coleridge, *Lectures 1808-1819* 2 231.
[44] Coleridge, *Lectures 1808-1819* 1 429.
[45] Coleridge, *Biographia* 1 290.

idealistic terms. And it is principally the influence of German thought, particularly that of Kant, which leads to Coleridge's hesitation over whether to confine 'logic' to the abstract understanding or renew it as a higher discipline of knowledge. Not for the first time, the dilemma facing Coleridge was between revolution and reform. Having already resolved to recover reason from the Jacobins, rehabilitating it as spiritual insight, could he do the same for logic? In both cases, it was German idealism that suggested the path to redemption. In *Biographia,* Coleridge recounts how his early reading of Kant introduced him to a new way of thinking about the nature of experience:

> I began then to ask myself, what proof I had of the outward *existence* of any thing? Of this sheet of paper for instance, as a thing in itself, separate from the phenomenon or image in my perception. I saw, that in the nature of things such proof is impossible; and that of all modes of being, that are not objects of the senses, the existence is *assumed* by a logical necessity arising from the constitution of the mind itself, by the absence of all motive to doubt it, not from any absolute contradiction in the supposition of the contrary.[46]

The use of the term 'pheænomenon' here — together with the way in which he distinguishes the type of thought process he describes from empirical 'perception' on one hand, and, on the other, formal questions relating to 'absolute contradiction' — reveals the influence of Kant's method upon Coleridge's changing conception of 'logical necessity.' The kind of argument that Coleridge identifies with the new 'logic' is nowadays, following Kant, widely classed as a transcendental argument, whereby the truth of a statement in dispute is shown to be necessarily implied by the same conditions that constitute the possibility of an indubitable statement. What varies between different transcendental arguments, however, is the interpretation of 'necessarily.' Kant's arguments have long been criticized for their tendency to blur the distinction between conceptual necessity and mere psychological incorrigibility. If anything, however, Coleridge is even more inclined than Kant to fall into what Frege would later deplore as the conflation of formal and psychological arguments, equating 'logical necessity' with the limits on thought dictated by 'the constitution of the mind itself.' Moreover, as demonstrated by a schema of the human faculties sketched by Coleridge around the same time, he saw this new kind of knowledge not (as Kant did) as the foundation of knowledge in general, but as a 'logical' intermediary connecting empirical perception and spiritual truths accessed through intuition:

> We may usefully distribute all our knowleges into four sorts [? or] Orders. First, those derived from the senses by the aid of the Understanding.
>
> Second, those derived from the Understanding by reflection on its own acts and processes.

[46]Coleridge, *Biographia* 1 200-201.

> Third: those derived from the pure Reason.
>
> Fourth: those derived from the Conscience.[47]

The new logic is covered by the 'second' order of knowledge, which is the province of the understanding. Coleridge's scheme, it should be noted, is hierarchical: the knowledge of understanding is subordinate to that of reason, and that, in turn, to Conscience (which he often refers to as practical reason). This architectonic lies behind Coleridge's ambition to complete his great work on the intellect, the work that in a contemporaneous letter to John May he describes as the 'Logosophia: or on the Logos, divine and human, in six Treatises.' Logic, accordingly, takes its place in volume two:

> The first, or preliminary treatise contains a philosophical History of Philosophy [...]. The second Treatise is [...] on the science of connected reasoning, containing a system of Logic purified from all pedantry & sophistication, & applied practically to the purpose of ordinary life, the Senate, Pulpit, Bar, &c. [...] The III. (Logos Architectonicus) on the Dynamic or Constructive Philosophy — preparatory to the IV. or a detailed Commentary on the Gospel of St John [...]. The Vth. [...] on the Panthesists and Mystics [...] The Vlth on the Causes & Consequences of modern Unitarianism.[48]

And yet, the master method of this system remains the 'logic' of transcendental argument — only, in Coleridge's hands, Kant's transcendental method is extended from the theoretical or purely cognitive into the realm of the practical. As he explains in *Biographia,* the use of spatial intuitions as postulates in geometry supplies philosophy with an example of the foundation 'from which every science that lays claim to *evidence* must take its commencement. The mathematician does not begin with a demonstrable proposition, but with an intuition, a practical idea.'[49] Conceived as inherently moral, the foundational intuition reveals the original unity of knowing and being. It is in this context that Coleridge urges his favourite Delphic injunction upon his reader as the 'first postulate' of philosophy:

> The postulate of philosophy and at the same time the test of philosophic capacity, is no other than the heaven-descended know thyself! [...]. And this at once practically and speculatively. For as philosophy is neither a science of the reason or understanding only, nor merely a science of morals, but the science of being altogether, its primary ground can be neither merely speculative or merely practical, but both in one.[50]

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[47]Coleridge, *Shorter Works* 1 412.

[48]Coleridge, To John May,' 27 September 1815, letter 976 *of Letters* 4 589-590.

[49]Coleridge, *Biographia* 1 250. See also *Aids* 136: 'we proceed, like the Geometricians, with stating our postulates; the difference being, that the Postulates of Geometry no man can deny, those of Moral Science are such as no good man will deny.'

[50]Coleridge, *Biographia* 1 252. On similar grounds, Coleridge later criticises Aristotle for

As a result, Coleridge's treatment of logic bears the strain of two competing imperatives: first, that philosophy should be unified and complete, a seamless panoply of thought and faith, and second, that 'logic' as such falls within the remit of the understanding alone. In particular, it is when he tries to define a metaphysical logic on the basis of the model of transcendental argument inherited from Kant that Coleridge runs into difficulties. It proved impossible to dovetail the relatively modest, conceptual aims of transcendental argument with the higher dialectic of knowing and being planned for the *Logosophia,* involving as the latter did questions of will, faith, and the overcoming of logical categories and conceptual clarity associated with the mere understanding.

This tension is one reason why *Logic,* putatively Coleridge's study of 'the science of connected reasoning' in the understanding and the propaedeutic to the 'Dynamic or Constructive Philosophy,' remains incomplete. Coleridge divides his subject into the 'organon or *logice organica, heuristica* [...] the criterion or *logice dialectica,* and the canon or *logice simplex et syllogistica*.'[51] In other words, logic subdivides into the metaphysical logic of discovery, the transcendental logic of psychology, and the purely formal logic of the syllogism. However, only the sections on canonic and dialectic logic were completed. On the face of it, this absence is surprising, given the fact that *Logic* is full of apologetic digressions where Coleridge teasingly anticipates the transition between conceptual understanding and spiritual insight. Indeed, at times it seems as if he cannot wait to cut through the dry formalities in order to penetrate the metaphysical order behind reason.[52] In this respect, *Logic* bears an uncanny resemblance to that earlier and more notorious case of deductive interruption, *Biographia,* in which Coleridge left incomplete his metaphysical argument designed to underpin a poetics grounded in the principles of imagination. The reasons behind Coleridge's omission of organic logic, together with his decision not to include 'polar logic' in the rubric of the work, will be considered later.

5 THE CANON, OR LOGIC AS SYLLOGISM

Coleridge's tendency in the *Logic* to digress from pure or formal logic into metaphysics is one symptom of his impatience to broaden the subject and branch out into what he sees as deeper and more urgent questions. Another is his painstaking articulation of the place of logic within the hierarchical structure of knowledge. Noting that the ancient Greeks considered 'the mind in the threefold relation,'

having 'confounded Science with Philosophy,' when in truth, 'Philosophy is the middle state between Science or Knowledge and Wisdom or Sophia' (Samuel Taylor Coleridge, *Table Talk,* ed. Carl Woodring, vol. 1 [Princeton University Press, 1990] 173-174).

[51] Samuel Taylor Coleridge, *Logic,* ed. J.R. de J. Jackson (Princeton University Press, 1981) 52.

[52] See, for example, *Logic* 87: while ostensibly still treating of 'common or syllogistic logic,' Coleridge admits to drifting into territory concerned with 'the last and concluding stage of our present subject, viz. the transition of the dialectic into the organic [...].'

that is, relative to the evidence of reason, understanding and the senses, he ranks the metaphysical disciplines accordingly:

A — Noetics = the evidence of reason

B — Logic = the evidence of the understanding

C — Mathematics = the evidence of sense

Under the heading 'physics,' he lists:

D — Empiric = evidence of the senses

Scholium. The senses = sense+sensation+impressions.[53]

In this Kantian division of labour, 'Logic' is clearly defined as the province of the understanding, mediating between Mathematics and Noetics:

> Thus by the mathematic we have the immediate truth in all things numerable and mensurable; or the permanent relations of space and time. In the noetic, we have the immediate truth in all objects or subjects that are above space and time; and, by the logic, we determine the mediate truths by conception and conclusion, and by the application of all the world to the senses, we form facts and maxims of experience which is one of the two provinces in and on which the formal sciences are to be employed and realised.[54]

The distinctly un-Kantian definition of 'Noetics' as concerned with 'the immediate truth in all objects or subjects that are above space and time' betrays, once again, Coleridge's impatience with the Critical Philosophy and his tendency to equate Kant's work with logic as the science of understanding. On this picture, Logic is limited to only one facet of human knowledge because it expresses only one side of the power of self-consciousness. As he expresses it in marginal comments to Kant, 'in Logic the mind itself being the Agent throughout does not take itself into question in any one part. It is a Teller which does not count itself; but considers all alike as Objective, because all alike is in fact subjective.'[55]

This lack of self-consciousness betrays a naivete inherent in formal logic, one which Coleridge attempts to explain in the introductory chapters of *Logic* in both conceptual and historical terms. Here, the first emergence of logic is figured as an adolescent stage through which the fallen or finite human mind must develop in its journey towards a reunion with its origin in the plenitude of the Logos, the unity of being and act. 'In the infancy and childhood of individuals (and something analogous may be traced in the history of communities),' he claims, this logical phase is preceded by a kind of 'happy delirium, the healthful fever, of the physical, moral, and intellectual being, Nature's kind and providential gift to childhood,' in

[53] Coleridge, *Logic* 43-44.

[54] Coleridge, *Logic* 36.

[55] Samuel Taylor Coleridge, *Marginalia,* eds. H. J. Jackson, George Whalley, et al, vol. 3 (Princeton University Press, 1980-2001) 275.

which 'the first knowledges are acquired promiscuously.'[56] As it grows, the infant mind / society develops the ability to trace similarities between things, and thus form abstractions. The 'earliest products of the abstracting power' are words, which also 'become the first subject matter of abstraction; and consequently the commencement of human education.'[57] As language becomes increasingly abstract and complex, it nonetheless retains an organic relation to its origin. As Coleridge explains, the community of parts of speech 'is a necessary consequence of their common derivation or rather production from the verb substantive.' For 'the verb substantive ("am" [...]) expresses the identity or coinherence of being and act. It is the act of being. All other words therefore may be considered as tending from this point [...].'[58]

This organic solution to the relationship between logic and language can be seen as Coleridge's contribution to a debate about the possibility of a 'universal grammar.' Inherited from Port Royal Logic, the notion of a purely rational language had been undermined in the late eighteenth century by the nominalism of Tooke and, more damagingly, by Kant's insistence that logic was the domain of understanding rather than reason. As we have seen, however, Coleridge, ever the philosophical magpie, manages to borrow what suits him from both these thinkers. He audaciously combines an adapted form of Kantian logic with a metaphysical rendering of Tooke's etymology to argue for the *ideal* unity of grammar and logic. This line of argument is pursued further in the *Magnum Opus:*

> in fact, the science of grammar is but logic in its first exemplification, or rather in its first product, *Λογος*, discursus, discourse, meaning [...] either, i.e. thoughts in connexion, or ⟨connected⟩ language, and the ⟨primary⟩ distinctions of identity and alterity, ⟨of essence and form⟩, of act and of being, constituting the groundwork and, as it were, the metaphysical contents ⟨and preconditions⟩ of logic.[59]

The claim that 'the science of grammar is but logic in its first exemplification' is also Coleridge's main premise in *Logic,* where he argues that one cannot understand logic without understanding language, and that one cannot understand language without understanding the psychological and metaphysical *act* that underlies being. Tellingly, the relatively short chapter on 'Pure Logic or the Canon' that opens the main body of *Logic* (53-59) is swiftly followed by a more substantial chapter 'On the Logical Acts' (60-103), in which the reader is urged not 'to think too meanly of the process' of reasoning by syllogism: 'If $A - C$, and B be contained in A, then B is contained in C. This [...] is a truth of mind: a somewhat that in the mind actually exists, as any object of the senses exists without us.'[60]

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[56] Coleridge, *Logic* 7-8.
[57] Coleridge, *Logic* 15
[58] Coleridge, *Logic* 16-17.
[59] Samuel Taylor Coleridge, *Opus Maximum,* eds. Thomas McFarland and Nicholas Halmi, (Princeton University Press, 2002) 208.
[60] Coleridge, *Logic* 65.

Armed with his idealistic form of etymology, Coleridge outlines the metaphysical and grammatical foundations of canonic or syllogistic logic in a logical primer inscribed for his son, Derwent, in a copy of the 1818 edition of *The Friend:*

> All Logic, as far as it is pure Science and of course purely formal, is reducible to three Acts of the Intellect — i.e. to the first Figure of Syllogism. These are: 1. Seclusion. 2. Inclusion. 3. Conclusion — and of these the first is the whole, as far as any effort of Judgement is concerned. The 2^{nd} and 3^{rd} are self-evident. To think absolutely or indefinitely is impossible, for a finite mind at least. To think (Ding, denken; res, reor) is to thingify. Thing = The Ing [...] is a somewhat set apart — thus Ingle = the Hearth. Conceive the indistinguishable all of our Perceptions, Conceptions and Notions as a vast Common — In or from this I seclude a determinate portion [...] by assigning it's Terminus = Terminus Major sive communis. 1. All men (= the portion) are mortal = Term. Major. Within this common ring-fence I include with a fence of it's own Socrates = Terminus Minor.[61]

Having set out to demonstrate the basis of syllogistic logic in 'three Acts of the Intellect,' Coleridge suddenly sweeps his reader away on an etymological excursion whereby, first, the origin of thinking is linked to 'thingifying,' and secondly, the process of thingifying is itself traced, via the suffix 'ing,' to the way in which we 'ring-fence' the vast, indistinguishable 'Common' of our perceptions in logical reasoning. What seems at first a whimsical flight of speculation is entirely characteristic of Coleridge's thinking on this subject, and consistent with his other writings. The same premise that logic, grammar and mind emerge from the primordial unity of an *act* forms the basic assumption *of Logic.* By 1818, Coleridge is confident that he has answered the questions he posed to Godwin fifteen years earlier.

Canonic logic, however, constitutes only one part of Coleridge's tripartite conception of logic proper. A full account of logic, he maintains, must include an account of the relationship between logic and truth, and for the logician, 'all truth, and consequently all true knowledge, rests on the coincidence of the object with the subject.'[62]

6 THE CRITERION, OR LOGIC AS DIALECTIC

This brings us to the form of logic Coleridge terms the 'criterion' or 'dialectic.' The appearance of the latter term, together with Coleridge's identification of the central concern of dialectical logic as the problem of the 'coincidence of subject and object,' attests to Kant's influence on this part *of Logic.* Indeed, it has long

[61] Coleridge, 'To Derwent Coleridge,' November 1818, letter 1152 *of Letters* 4 885. Coleridge presents the same etymological account of 'thing' in *Logic* 114-115: 'The "ing" [...] is the universal exponent of whatever is inclosed, bounded.'

[62] Coleridge, *Logic* 37.

been accepted that how one reads the *Logic* as a whole will be in no small part determined by how one views Coleridge's relation to Kant. Controversy over the exact use Coleridge makes of Kant's philosophy dates back to contemporaries such as Henry Crabb Robinson, who wrote the following, not uncritical, assessment:

> To Kant his obligations are infinite, not so much from what Kant has taught him in the form of doctrine as from the discipline Kant has taught him to go through. Coleridge is indignant at the low estimation in which the Post Kantians affect to treat their master. At the same time Coleridge himself adds Kant's writings are not metaphysics, only a propaedutic. Were Coleridge in Germany he would not be suffered to hold this language; he would be forced to make his election between the critical and the absolute philosophy, or he would be equally proscribed by both.[63]

Robinson's parting shot — the claim that, were he writing in Germany, Coleridge would not be allowed to have it both ways, borrowing from the Critical Philosophy where it suited him while simultaneously adopting the more full-blooded idealism of Kant's successors, Fichte and Schelling — prefigures the kind of debates over Coleridge's debts to Kant that have occupied scholars and commentators for at least the past century. The fact that *Logic* is widely seen as Coleridge's most explicitly 'Kantian' work means that much of this controversy has centred on the manuscript originally intended as primer for the *'forensic* purposes' of discourse. Ranged on one side are those who argue that Coleridge's handling of logic contains little new of significance, either (as Rene Wellek argues) because Coleridge simply misunderstood Kant's logic or (as J.R. de J. Jackson maintains) because an exposition of Kant was the limit of his aims in this particular work, or indeed (as G.N.G. Orsini allows) for both these reasons.[64] On the other side of the argument, however, is a tradition of criticism that has defended Coleridge as an innovator in logic who improves on Kant, whether (as John Muirhead claims) because of the way his use of triadic conception of reason logic trumps the dichotomic logic of the first *Critique,* or (as James McKusick, Tim Fulford, Gerald McNeice, and Thomas McFarland have all proposed) because, by playing 'Tooke,' as it were, to Kant's 'Locke,' Coleridge successfully introduces a 'linguistic turn' into transcendental idealism.[65]

[63] Coleridge, *Table Talk* 2 485.

[64] In *Immanuel Kant in England 1793-1838* (Princeton University Press, 1931), Rene Wellek claims that Coleridge failed to see 'that nothing of the Kantian epistemology can be preserved in a new system' (80). However, in his Editor's Introduction to the standard edition, J.R. de J. Jackson counters that Coleridge 'does not offer any new arguments' in the *Logic* because it is 'essentially a popularisation of the *Critique of Pure Reason*' (lxii). G.N.G. Orsini combines both points of view in *Coleridge and German Idealism* (Southern Illinois University Press, 1969), arguing that while *Logic* 'is professedly an exposition of Kant' (115), Coleridge was hampered by the fact that he 'did not fully grasp, or perhaps did not fully accept' (256) Kant's argument for the transcendental unity of apperception in the second edition of the *Critique of Pure Reason.*

[65] As long ago as 1930, in *Coleridge as Philosopher* (George Allen & Unwin Ltd., 1930), John H. Muirhead praised Coleridge's proto-Hegelian 'triadic logic' as 'an attempt to carry the dialectic

Whatever one's position on this perennial topic, the imprint of Kant's influence on *Logic* is undeniable. Having covered 'canonic' or syllogistic logic in Part One in a tidy fifty pages, and with no third section on 'organic' logic ever written, Part Two of *Logic,* on 'The Criterion or Dialectic,' occupies the remaining one hundred and sixty four pages of the standard edition, thus making up the bulk of the work as a whole. With chapter titles such as 'Judicial Logic, Including the Pure Aesthetic' and 'On Synthesis *a priori,'* The 'Dialectic' makes little attempt to disguise its reliance upon the *Critique of Pure Reason.*

The concern of dialectic, as Coleridge makes clear, is with truth as the 'coincidence' of subject and object. However, he notes, this is an ambiguous question. The question, 'what is truth?' could mean three things, quite apart from the verbal definition of truth, whereby ' "If you ask for the meaning of the word [...] I reply, 'the coincidence of the word with the thought and the thought with the thing'".' First, the question could mean, what is truth, relatively to God? In this case, the question has either no meaning or admits of but one reply, viz. "God himself." God is the truth, the identity of thing and thought [...]' But this cannot be a 'criterion' of truth, since a criterion implies an abstraction, which God is not. 'Truth, therefore, is its own criterion [...].'[66] Secondly, the question may refer to the 'coincidence between thought and the word' most often implied in the 'communication' of truth. While acknowledging that 'the criteria must vary with the occasion and the purpose,' Coleridge links this kind of question with the search for what 'metaphysicians have named the thing-in-itself,' independent of all human conception, a search that he claims permits of 'no answer at all.'[67] Thirdly, however, '[w]e may mean subjective truth, in which truth may be defined [as] the coincidence between the thought and the thinker, the forms, I mean, of the intellect.' And this is the very office of logic as dialectic, which 'as far as it concerns the knowledge of the *mere form* abstractedly from the *matter,'* presents 'the criterion of truth, that is, of formal truth [...].'[68]

Significantly, this has the consequence that, strictly speaking, logic is 'not philosophical; for logic [...] consists in the abstraction from all objects. It is wholly and purely subjective,' and 'has no respect to any reality independent of the mind.'[69] The dialectic of logic is necessary, however, as a propaedeutic and a guard against attempts to identify the 'thing-in-itself,' to rush into the ontology of truth without having first established one's grounds in the psychology of truth. For

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of Kant's thought a step farther and turn criticism against the critic' (89). More recently, Tim Fulford in *Coleridge's Figurative Language* (MacMillan, 1991) (106), Gerald McNiece in *The Knowledge that Endures: Coleridge, German Philosophy and the Logic of Romantic Thought* (MacMillan, 1992) (21), and Thomas McFarland — in his editorial 'Prolegomena' to *Opus Maximum* — have agreed with McKusick's assessment in *Coleridge's Philosophy of Language* that, in McFarland's words, '[m]uch more clearly than Kant, Coleridge sees that epistemological questions cannot be resolved without recourse to a prior analysis of the linguistic structures that constitute the means of intellectual inquiry' (ccxxix).

[66] Coleridge, *Logic* 111.

[67] Coleridge, *Logic* 112-114.

[68] Coleridge, *Logic* 112.

[69] Coleridge, *Logic* 127.

Coleridge, the worst offenders in this respect are materialist philosophers such as Hartley and Godwin, who through their neglect of logic unwittingly substituted philosophical Hypopoeisis for philosophical 'Hypothesis.' As he argues in an 1809 notebook entry, 'Hypothesis' consists in 'the placing of one known fact under others as their ground or foundation. Not the fact itself but only its position in a [...] certain relation is imagined.' Where both the position and the fact are imagined, it is Hypopoeisis not Hypothesis, 'subfiction not supposition.'[70] Increasingly influenced by Kant, Coleridge came to see logic as the art of supposition, preliminary to philosophy. Thus, the architectonic of Kant's Transcendental Aesthetic and deduction of the categories of understanding became exercises in a preparatory discipline of logical hypothesis whereby the mind shed itself of the naive objectivism that would only produce subfiction or Hypopoeisis. Later, in *Aids to Reflection,* he associates this discipline with an awareness of forethought, writing that it 'is at once the disgrace and the misery of men, that they live without forethought. Suppose yourself fronting a mirror. Now what the objects behind you are to their images at the same apparent distance before you, such is Reflection to Fore-thought.'[71]

A crucial element in the hypothesis of dialectic is transcendental method. Already in *Biographia Literaria,* Coleridge shows how Kant's demonstration of the necessary conditions of experience has affected his insight into the Hypopoeisis of the materialists. The errors of Hartley and associationist philosophy, he argues,

> may be all reduced to one sophism as their common genus; the mistaking of the *conditions* of a thing for its *causes* and *essence;* and the process by which we- arrive at the knowledge of a faculty, for the faculty itself. The air I breathe, is the *condition* of my life, not its cause. We could never have learnt that we had eyes but by the process of seeing; yet having seen we know that the eyes must have pre-existed in order to render the process of sight possible.[72]

In other words, knowing that eyes are the precondition of sight requires more than sight itself, it requires a kind of reasoning that goes beyond the mere 'conditions of a thing,' to the 'cause' that makes it possible. Later in *Biographia*, Coleridge gives the same point a more recognizably Kantian formulation: 'We learn all things indeed by occasion of experience; but the very facts so learnt force us inward on the antecedents, that must be pre-supposed in order to render experience itself possible.'[73] In *Logic,* following Kant still more closely, he terms this method of reasoning 'transcendental,' noting that 'the term "transcendental" means the same as "sciental," but with an additional significance. All knowledge is excited or occasioned by experience, but all knowledge is not derived from experience, such, for instance, is the knowledge of the conditions that render experience itself

[70] Coleridge, *Notebooks* 3 text 3587.
[71] Coleridge, *Aids* 12.
[72] Coleridge, *Biographia* 1 123.
[73] Coleridge, *Biographia* 1 142.

possible [...].'[74]

Thus far, Coleridge's logic of dialectic looks similar to Kant's transcendental method. This is why much of the later sections *of Logic* in particular read as free translations and adaptations from the *Critique of Pure Reason.* It is when the 'Criterion' is placed within Coleridge's broader conception of philosophy, however, that serious differences emerge. Coleridge learns from Kant the power of transcendental argument, or thinking by way of presuppositions, but he has a very different conception of the status and nature of those presuppositions. 'Fore-thought' for Coleridge represents not just the conceptual or even psychological conditions that make thought or experience possible, but the half-hidden presence of an Idea that bears life, encompassing the very dimensions of will, love, and intuition that Kant had taken such care in the first *Critique* to isolate from transcendental deduction. Thus, like Kant, Coleridge is apt to hypostatize the preconditions of thought, assuming that the thing presupposed must itself have foundational status. But while for Kant this foundation is the 'I think' or transcendental unity of apperception, for Coleridge it is the 'I am,' the self-inaugurating word, or Logos.

Unsurprisingly, Coleridge finds it difficult to connect Kant's transcendental method to the principles of his theosophy. The closest he comes to success is in the 'Essays on the Principles of Method,' written for the 1818 edition of *The Friend* (first published in 1809). 'Method,' he notes, 'implies *a progressive transition'* in reasoning. 'But as, without continuous transition, there can be no Method, so without a pre-conception there can be no transition with continuity. The term, Method, cannot therefore, otherwise than by abuse, be applied to a mere dead arrangement, containing in itself no principle of progression.'[75] Thus, the presupposition or 'pre-conception' underlying Method proper cannot simply be a truth of formal logic, much less an empirical fact. It must be an Idea with a 'life' of its own; it must, in short, be an organism, capable of growing and seeding further thought. Coleridge calls this *'leading Thought'* in Method the 'Initiative.'[76] Towards the end of his life, in conversation with Henry Nelson Coleridge, he refers to this beefed-up transcendental method as 'structive' or 'Synthetic':

> There are three ways of treating any subject. 1. Analytically. 2. Historically. 3. structively or Synthetically. Of these the only one complete and unerring is the last. [...] You must begin with the philosophic Idea of the Thing, the true nature of which you wish to find out and manifest. You must carry your rule ready made if you wish to measure aright. If you ask me how I can know that this idea — my own invention — is the Truth, by which the phenomena of History are to be explained, I answer, in the same way exactly that you know that your eyes are made to see with — and that is — because you *do* see with them.[77]

[74] Coleridge, *Logic* 146.
[75] Coleridge, *Friend* 1 457.
[76] Coleridge, *Friend* 1 455.
[77] Coleridge, *Table Talk* 1 364-366.

The deployment once again of the 'sight' analogy indicates Coleridge's continuing belief in the link between Kant's transcendental argument and the seminative and self-authorizing power of the Ideal, 'the philosophic Idea of the Thing.' And yet, an 'Idea' is more than just a logical construct, it is a postulate with an existential component. Every 'initiative' of reasoning involves the input of the volitional, emotional and spiritual life of an individual. Thus, Coleridge opens the *Magnum Opus* with the declaration that '[i]n every science something is assumed, the proof of which is prior to the science itself,' adding that 'the one assumption, the one postulate, in which all the rest may assume a scientific form [...] is the Existence of the *Will* [...].' Linking logic to the Will would be the task of the polar logic of the Divine Tetractys: in turn, bridging the logic of understanding (the canon and the criterion) and this higher logic of reason was to be the task of the 'organon or logice organica.'[78]

7 THE ORGANON, OR LOGIC AS DISCOVERY

This brings us to the missing logic *of Logic.* In long footnote to the 'Aphorisms on Spiritual Religion' *in Aids to Reflection* in which he presents his conception of polar logic as the schema of a 'Noetic Pentad' (to which I shall return later),[79] Coleridge directs his more logical-minded reader to the final section of his *Logic:*

> In the third and last Section of my 'Elements of Discourse'; in which (after having in the two former sections treated of the Common or Syllogistic Logic, the science of legitimate *Conclusions;* and the Critical Logic, or the Criteria of Truth and Falsehood in all *Premisses)* I have given at full my Scheme of Constructive Reasoning, or 'Logic as the Organ of Philosophy,' in the same sense as the Mathematics are the Organ of Science [...].

Not for the first time, Coleridge's redirected reader is destined to be disappointed. Yet later in the same footnote, Coleridge hints at the reasons behind the absence of the 'organon':

> As this third Section does not pretend to the forensic and comparatively popular character and utility of the parts preceding, one of the Objects of the present Note is to obtain the opinions of judicious friends respecting the expedience of publishing it, in the same form, indeed, and as an Annexment to the 'Elements of Discourse,' yet so as that each may be purchased separately.[80]

As the appeal to 'judicious friends' (his usual coded alert for a project that has stalled) shows, Coleridge was already thinking of the 'organon' as sufficiently different to the 'canon' and 'criterion' to warrant publication in a separate volume.

[78] Coleridge, *Opus Maximum* 5, 11, 33.
[79] Coleridge, *Aids* 180.
[80] Coleridge, *Aids* 183.

The ostensible reason for this is clear enough: the first and second sections *of Logic* are 'forensic' and 'comparatively popular,' insofar as they are intended to be formal guides to the principles of reasoning for young professionals. They do not (explicitly at least) presuppose any theological doctrine. However, logic conceived as 'Constructive Reasoning,' as the 'Organ of Philosophy,' or 'instrument of Discovery and universal Method in Physics, Physiology, and Statistics' was, as Coleridge came to see, a method that required *content.*[81] At this point, we realise the extent of Coleridge's ambitions in logic: in addition to what he saw as the older logic of syllogism and exposition, he wanted to create a new logic that was not only fit for modern science, but managed to anchor the growth of knowledge in an ancient tradition of Christian metaphysics. It was to be a synthetic logic of discovery, explaining and regulating the expansion of human consciousness. Such was the goal of polar logic. The job of organic logic was to connect this higher scientific/metaphysical logic to the formal/transcendental logic of the canon and the criterion. Coleridge thought of this task as the attempt to build a bridge between reason and understanding. Ultimately, it seems, the organon proved to be a bridge too far.

As with transcendental method, Coleridge adapted the distinction between reason and understanding from Kant. For Coleridge, Kant's act of desynonymisation could not have been more timely. He saw the confusion of the human mind with the merely formal and mechanical understanding as the great *mal de siecle,* a problem that loomed with the same menace as it had when Kant published the first *Critique.* Indeed, if anything, the situation was worse in 1830, as Coleridge complains in *On the Constitution of the Church and State,* declaring that thanks to industry and utilitarianism, 'we live [...] under the dynasty of the understanding: and this is its golden age.'[82] Coleridge's later works signal a return to Kant after the ill-fated flirtation with the idealism of Schelling and Fichte in *Biographia Literaria.* As Gian Orsini notes, one result of this is that after 1815 Coleridge places less emphasis on the restorative power of the imagination. Instead, 'the proper use of Reason serves to correct the errors of the Understanding.'[83] Indeed, Thomas McFarland argues that the distinction between understanding and reason 'was the keystone in the arch of Coleridge's thought; it was the ultimate, the *ne plus ultra,* of all his mentation.'[84] While this might be a slight exaggeration, it is certainly true that Coleridge hoped, 'by converting the reason of Hume and Diderot into understanding, and marching to relieve the beleaguered garrison of Christianity under the generalship of a new conception of reason [...] to wrest "reason" from the hands of the Antichrist [...].'[85] As early as 1812 Coleridge is writing in the

[81] Coleridge, *Aids* 183.

[82] Samuel Taylor Coleridge, *On the Constitution of the Church and State,* ed. John Colmer (Princeton University Press, 1976) 59.

[83] Orsini 140.

[84] Thomas McFarland, 'Aspects of Coleridge's Distinction Between Reason and Understanding,' *Coleridge's Visionary Languages: Essays in Honour ofJ.B. Beer,* eds. Tim Fulford and Morton D. Paley (Cambridge: D.S. Brewer, 1993) 168.

[85] McFarland, 'Aspects' 171.

margins of his copy of Moses Mendelssohn of

> [t]he unspeakable importance of the Distinction between the Reason, and the Human Understanding, as the only Ground of the Cogency of the Proof a posteriori of the Existence of a God from the order of the known Universe — . Remove or deny this distinction, and Hume's argument from the Spider's proof that Houses &c were spun by Men out of their Bodies becomes valid.[86]

This innovation also had profound implications for Coleridge's conception of logic. Unlike Kant, Coleridge conceives of pure reason as constitutive and cognitive *as well as* regulative and moral. Consequently, he believes that the canons of logic must be demonstrably based upon higher principles of reason. This is because reason as a faculty is not just superior to understanding, but a superintending presence for all the faculties. As he argues in the *Statesman's Manual,* 'the Reason without being either the Sense, Understanding or the Imagination contains all three within itself, even as the mind contains its thoughts, and is present in and through them all.'[87] As Coleridge explains in *The Friend,* reason differs from understanding in two key respects. First, it is immutable: 'the understanding may be *deranged, weakened, or perverted,'* he maintains, 'but the reason is either *lost* or *not* lost, that is, wholly present or wholly absent.'[88] Second, reason is consubstantial with its objects: 'Thus, God, the Soul, eternal Truth, &c. are the objects of Reason; but they are themselves *reason* [...]. Whatever is conscious *Self-knowledge* is Reason; and in this sense it may be safely defined the organ of the Super-sensuous [...].' Understanding, on the other hand, 'supposes something that is *understood*'; it is the organ of the sensuous, depending upon a combination of logic and the objects of experience.[89]

The logic of the understanding then, derives its validity from principles or ideas of reason, which, being 'Super-sensuous' and verbal in nature, supersede logical canons. There are few areas where the subordination of formal logic to Coleridge's notion of rational principle is more glaring than in his treatment of predication and the law of contradiction. Predication for Coleridge is the product of reflection, which is itself a lower power of intuition. Since logic depends upon the pre-reflective unity in judgement, an *act* of mind, 'neither the subject nor its predicate,' taken separately or together contain the principle of 'their reality or objective being.' This principle is only realised 'when the mind bears witness to its own unity in the subject represented to it, and this act with this consciousness of the same is conveyed or expressed in the connective "is".'[90] At the level of logic or understanding then, it is through the connective or copula 'is' that the mind affirms not only its own reality and unity, but its power to *create* that reality. As

[86]Coleridge, *Marginalia* 3 848.

[87]Samuel Taylor Coleridge, *Lay Sermons,* ed. R. J. White (Princeton University Press, 1972) 69-70.

[88]Coleridge, *Friend* 1 153.

[89]Coleridge, *Friend* 1 156.

[90]Coleridge, *Logic* 79.

Coleridge puts it, 'the mind is, and it is a form, and it is formative,' and predication is a reflection of this formative act of consciousness.[91] Barfield explains this idea further in *What Coleridge Thought:*

> The moment of predication is the moment in which the presence of reason in the understanding is manifested in its effect, but only as effect. To meditate faithfully on the principle of contradiction, upon which predication and syllogism are based, is to have one's attention drawn [...] to the effective reality of Reason as a universal and constitutive principle.[92]

However, as we saw with Coleridge's etymological treatment of the syllogism, the act of consciousness, indeed the act of being itself, is formed through language. 'If we ask ourselves how we know anything,' he notes in *Logic,* 'that rose, for example, or the nightingale hidden in yonder tree [...] what are these but the goings from the subject, its words, its verb? The rose blushes, the nightingale sings.'[93] Thus, just as the identity of subject and object is found in self-consciousness, 'the first rule of logic is in accordance with the first rule of grammatical syntax — that the case which follows the verb substantive, and as such stands for the objective, is the same with the case that precedes, that is, the subjective case.'[94] The principles of reason that govern the logic of understanding are both ideal and linguistic.

Coleridge's insistence on the metaphysical and linguistic roots of logic leads him to distinguish between 'natural' and 'verbal' applications of logic. The former, in which there is 'an actual inclusion' of the predicate by the subject, is 'almost automatic and spontaneous' to the understanding. It is 'a science of nature's rather than of our conscious self.' Verbal logic however, is an echo of the divine Logos, 'partaking of this communicative intelligence' through 'ideas or communicable forms.'[95] It is within the latter concept of logic, rooted in the primordial verbal performance, the 'I am' of the communicative Logos, that Coleridge believes the canons of predication and contradiction must ultimately be framed. Thus, if the principle of contradiction, which Coleridge gives as *'impossibile est idem simul esse [ac non]* esse' (it is impossible for the same thing at the same time to be and not be), is to be *a principle,* 'there ought to be something that should follow':

> [B]ut what is to follow in the present instance? How can we arrive from this negative to a positive? Through another negative? As whatever is not *ens* and *non ens* [being and not-being] at the same time is possible[?] But the rule that two negatives make a positive is grammatical, not logical.[96]

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[91] Coleridge, *Logic* 80.
[92] Barfield 190.
[93] Coleridge, *Logic* 82.
[94] Coleridge, *Logic* 80.
[95] Coleridge, *Logic* 95-96.
[96] Coleridge, *Logic* 89.

At the same time, the 'rival position,' the principle of identity, or "'whatever is, is",' is 'merely a repetition during the act of reflection of the term "is". I mean that it expresses no more than my consciousness that I am reflecting, that is, consciously reflecting the truth of being.'[97] Thus, '[i]nstead of truth, principle, or axiom, it is in reality a mere narration of a fact: I reflect on being.' What troubles Coleridge, like Hume and Kant before him, about the so-called principles of contradiction and identity is that they tell us nothing *new* about the world, and actually 'encourage the error of presenting as particular truths the mere exponents of our universal or essential consciousness.' Ultimately, these axioms are analytic, but the copula 'is' is itself 'grounded, not in our reflection, or the analytic unity, but in the synthetic.'[98] Their validity derives from a unity whose origins lie in the emergence of the universal consciousness into language and being through the communicative Logos, in the form of the verb substantive, the 'I am.' The only human faculty that can even approach this power is the intuitive reason.

We can now see that the missing logic in Coleridge's philosophy, the organon or logic of discovery, was to be a logic that enabled the mind to make the transition from understanding to reason, from 'natural' to 'verbal' logic, and from predication and contradiction to the more fundamental act of the divine consciousness, which, echoing in the human mind, subtended the division between analysis and synthesis. We can also now appreciate why Coleridge runs into trouble. In effecting the shift from 'dialectic' or transcendental argument to a metaphysical logic worthy of theosophy and the ideas of reason, one might expect Coleridge to show how argument by way of presupposition (dialectic) cannot account for the totality of its own presuppositions. Thus, by effectively presupposing its own supersedence, it would pave the way for a higher 'polar logic' of alterity. Indeed, it is for just this proto-Hegelian move, as he saw it, that Muirhead applauds Coleridge, claiming that the latter succeeds in turning 'criticism against the critic.'[99]

However, while this may be fine in theory, in Coleridge things are not so straightforward. First, Coleridge does not confine the criterion or transcendental logic to questions of understanding, that is, to questions regarding the way in which concepts are applied to experience, but frequently deploys the method in the realm of 'Noetics,' or intuitive reason. Indeed, Coleridge routinely uses transcendental argument in support of ideas, not concepts (what he calls 'conceptions'). Thus, in the *Magnum Opus,* the existence of ideas of reason, which are 'contradistinguished alike from the forms of the sense, the conceptions of the understanding, and the principles of the speculative reason by containing its reality as well as the peculiar form of the truth expressed therein,' is established by transcendental argument, since '[t]hat without which we cannot reason *must be presumed* [...] as the ground

[97] Coleridge, *Logic* 89.

[98] Coleridge, *Logic* 89.

[99] Muirhead 89. See also Kathleen Wheeler, 'Coleridge's Theory of Imagination: a Hegelian Solution to Kant?' *The Interpretation of Belief: Coleridge, Schleiermacher and Romanticism,* ed. David Jasper (London, 1986) 22: 'Coleridge's conclusion is one with Hegel's: "The Subjectivity of Reason is the great error of the Kantean system".'

of the reasoning.'[100]

Second, there is nothing in Coleridge to compare with Hegel's argument in the *Phenomenology of Spirit* that any 'dialectic' based on transcendental method inevitably gives way to 'dialectic' based upon negation.[101] As I have shown elsewhere, this is because the conception of alterity that Coleridge locates at the heart of his higher logic of reason is not driven by the power of negation, but by the altogether obscurer agency of the Pythagorean Tetractys, which (among other things) involves a metaphysics of Will.[102] As he indicates in the passage from *Magnum Opus* mentioned above, the 'source' of ideas is 'neither in the reason without the Will nor in the Will without the reason.' What this means is that Coleridge cannot, like Hegel, *demonstrate* how the Kantian dialectic itself presupposes negativity. Instead, he relies upon para-philosophical means such as illustrations, flashes of insight, aids to reflection and above all the willingness of his reader to be *guided,* in order to lead his audience towards an illumination that cannot be attained without an element of volition, or faith. In this respect, as Katherine Miles Wallace points out, Coleridge saw his relation to the reader more as a guide, albeit a 'Chamois-hunter,' than as a preceptor.[103] As he argues in his Appendix to the 1831 edition *of Aids to Reflection,* within Noetics, practical and theoretic reason must work together:

> The Practical Reason alone is Reason in the full and substantive sense. It is reason in its own Sphere of perfect freedom; as the source of ideas, which Ideas, in their conversion to the responsible Will, become Ultimate Ends. On the other hand, Theoretic Reason, as the ground of the Universal and Absolute in all Logical Conclusion, is rather the Light of Reason in the Understanding [. . .].[104]

As Coleridge found in the case of organic logic, one can gesture towards the light of reason in the understanding as the Absolute of logical conclusion, but one cannot demonstrate it. In the end, Coleridge's failure to develop an organon betrays the extent to which his interest in 'dialectical' logic or transcendental argument is motivated by his unwarranted assumption that this form of reasoning was merely a preparatory exercise for metaphysical illumination, as if Kant's method was merely the stepladder needed to reach the first step on the marble staircase of 'polar logic.'

[100] Coleridge, *Opus Maximum* 270-271. Emphasis added.

[101] See G.W.F. Hegel, *Phenomenology of Spirit,* trans. A.V. Millar (Oxford University Press, 1977) 29.

[102] Tim Milnes, 'Through the Looking-Glass: Coleridge and Post-Kantian Philosophy,' *Comparative Literature* 51.4 (1999): 309-323.

[103] Catherine Miles Wallace, *The Design of the Biographia Literaria* (London: George Allen & Unwin, 1983) chapter 1: 'The Chamois Hunter.' Coleridge uses the metaphor in the 1818 *Friend* 1 55: 'Alas! legitimate reasoning is impossible without severe thinking, and thinking is neither an easy nor an amusing employment. The reader, who would follow a close reasoner to the summit and absolute principle of any one important subject, has chosen a Chamois-hunter for his guide.'

[104] Coleridge, *Aids* 413.

8 POLAR LOGIC AND THE NOETIC PENTAD

This brings us to the logic that transcended the science of understanding covered by *Logic:* the logic of reason. Indeed, with polar logic we are immediately confronted by the question of what logic *is,* if it is not to be the articulation of demonstrable principles of reasoning. As we have seen, this is more than just a problem of nomenclature, in that its answer is bound up with Coleridge's estimation of the legitimate reach of philosophy within human life. Ever conscious of what he saw as the hollow rationalism that lay behind the French Revolution, Coleridge is vigilant about the dangers of an unregulated understanding. Consequently, logic plays a systematically ambiguous role in his thought, potentially both a menace and a comfort. On one hand, as the instrument of understanding, it threatens to break out of its allocated canonical and dialectical functions. Unfettered, formal logic has dangerous tendency to become reified, collapsing back into a damaging irrationalism. Thus, in *Aids to Reflection,* Coleridge complains of 'Ideas or Theories of pure Speculation, that bear the same name with the Objects of Religious Faith' being taken for those objects themselves thanks to the natural tendency of mind to form

> certain Essences, to which for its own purposes it gives a sort of notional *Subsistence.* Hence they are called *Entia rationalia:* the conversion of which into *Entia realia,* or real Objects, by aid of the Imagination, has in all times been the fruitful Stock of empty Theories, and mischievous Superstitions.[105]

For this reason, he maintains, following Kant, the purely spatio-temporal categories of understanding should not be extended 'beyond the sphere of possible Experience. Wherever the forms of Reasoning appropriate only to the *natural* world are applied to *spiritual* realities, it may be truly said, that the more strictly logical the Reasoning is in all *its parts,* the more irrational it is as a *whole.*'[106]

Upon this reasoning, Coleridge bases his argument that the irrationalism of his age is merely the reflex of an overextended faculty of understanding. One way in which this pyrrhic triumph of logic is expressed is pantheism, a heresy with which, as Thomas McFarland has shown, Coleridge struggles throughout his career.[107] Dictating the fragments that were to form the uncompleted manuscripts for *Logosophia* or *Magnum Opus,* Coleridge claims to have demonstrated in the *Logic* 'that Dichotomy, or the primary Division of the Ground into Contraries,' though 'the necessary form of reasoning as long as and wherever the intelligential faculty of Man [weens] to possess within itself the center of its own System,' can easily excite a 'delusive conceit of Self-sufficiency,' the 'inevitable result' of which, as with 'all consequent Reasoning, in which the Speculative intellect refuses to acknowledge a higher or deeper ground than it can itself supply, is [...] Pantheism.'[108]

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[105] Coleridge, *Aids* 167.
[106] Coleridge, *Aids* 254.
[107] Thomas McFarland, *Coleridge and the Pantheist Tradition* (Oxford University Press, 1969).
[108] Coleridge, *Opus Maximum* 104-106.

On the other hand, Coleridge's careful restriction of his critique to the dangers threatened by ratiocination dominated by 'Dichotomy,' reminds us of his attraction to logic, and his desire to free it from the confines of the understanding. Thus, while in *Logic* he urges his reader not to 'think too meanly'[109] of formal logic, elsewhere he criticises Robert Leighton for assuming that an argument 'in point of Logic, legitimately concluded' is the same as a truth of reason. On the contrary, 'the truths in question are transcendent, and have their evidence, if any, in the Ideas themselves, and for the Reason; and do not and cannot derive it from the conceptions of the understanding [...].'[110] Similarly, in his marginalia to Johann Christian Heinroth' s *Lehrbuch der Anthropologie,* he complains of the 'Followers of Fichte, Baader, Schelling and Steffens' that 'they neglect Logic — or rather do not understand what Logic is. Thus, what Kant asserted as an assumption for the purposes of a formal Science, Heinroth asserts as a matter of fact. It is a necessary fiction of pure Logic [. . .].' [111]

Where all these thinkers fail, according to Coleridge, is in their inability to recognise the point where dichotomic logic (the logic of understanding) must give way to polar or trichotomic logic (the logic of reason). The same distinction informs the transition that Coleridge envisages in his 1815 letter to John May between the preparatory discipline of the *Logic* or 'Elements of Discourse' and that of the 'Dynamic or Constructive Philosophy' or 'Logos Architectonicus.' The exposition and articulation of the latter was to be one of the main tasks of the *Magnum Opus.* The limitations of the former, he claims, are evident in Emanuel Swedenborg's *Prodromus Philosophiae:*

> The Reasoning in these pages might be cited as an apt example of the inconvenience of the Dichotomic Logic: which acts in a contrary direction to the prime end and object of all reasoning, the reduction of the Many to One [...]. Two terms in manifest correspondence to each <other> are yet opposed as contraries, without any middle term: the consequence of which is, that one [...] of the [...] two becomes a mere negation of the other [...].[112]

For Coleridge, duality, opposition, difference, contradiction, all presuppose a more fundamental unity. As he continues to explain, the negation in dichotomic logic is 'a mere act of the mind, arising from a defect of perception.' It is therefore necessary to establish a logic that is able to express the way in which contraries are always mediated by a 'middle term,' yoking dualities within a dynamic unity. The only way this can be done, he maintains, is with a logic of 'Trichotomy,' similar to that involved in 'Pythagorean Tetractys.'[113] Kant had introduced trichotomic logic to eighteenth-century though by arguing that all logical oppositions presuppose a fundamental unity of consciousness in apperception, but as usual this did

[109] Coleridge, *Logic* 65.
[110] Coleridge, *Marginalia* 3 516.
[111] Coleridge, *Marginalia* 2 1002.
[112] Coleridge, *Marginalia* 5 459.
[113] Coleridge, *Marginalia* 5 459.

not go far enough for Coleridge. In his marginalia to Richard Baxter, Coleridge complains that Kant takes trichotomy 'only as a Fact of Reflection — [...] in which he seems to anticipate or suspect some yet deeper Truth latent & hereafter to be discovered.' Baxter, on the other hand, is commended not just for prefiguring Kant's arguments a century earlier, but for grounding trichotomy 'on an absolute Idea presupposed in all intelligential acts.'[114] Coleridge makes the same point in a different way when, in *The Statesman's Manual,* he distinguishes three kinds of necessity. There is, as he puts it, 'a threefold Necessity':

> There is a logical, and there is a mathematical, necessity; but the latter is always hypothetical, and both subsist *formally* only, not in any real object. Only by the intuition and immediate spiritual consciousness of the idea of God, as the One and Absolute, at once the Ground and the Cause, who alone containeth in himself the ground of his own nature, and therein of *all* natures, do we arrive at the third, which alone is a real *objective,* necessity. Here the immediate consciousness decides: the idea is its own evidence, and is insusceptible of all other. It is necessarily groundless and indemonstrable; because it is itself the ground of all possible demonstration. The Reason hath faith in itself, in its own revelations.[115]

In this passage we can see how the principal forces in Coleridge's thought conspire to ensure the subordination of 'formal' (both logical and mathematical) necessity: the prioritisation of intuitive reason and 'immediate spiritual consciousness'; the insistence on unity; the idea of the self-grounding and therefore fundamental agency of consciousness in its act of 'faith,' and above all the primacy of a revelatory God 'as the One and Absolute.' For Coleridge, these principles confirm the ascendancy of reason over understanding, and thus of intuitive Ideas over logical rules. The law of contradiction, for example, is seen as secondary to the indemonstrable principle (itself the engine of the growth of language through desynonymy) of difference within unity, or distinction without division. Thus, in a fragment on consciousness from around 1816, he notes that 'all Ideas, when interpreted into Conceptions [...] must appear to involve contra-dictions' to those who never move beyond the understanding: 'in other words, as the Ideal *Power* can only [...] manifest itself for the Senses in opposite Forces, so the Idea [...] relatively to Speculation, can only be conceived by incompatible Conceptions [...].'[116]

Difference, in other words, will always remain secondary to a fundamental unity, which means that while difference might appear as contradiction in the understand-

[114]Coleridge, *Marginalia* 1 347-348. In a further note, Coleridge links 'the principle of Trichotomy' to the 'Polar Logic' of 'Giordiano Bruno's Logica Venatrix Veritatis' and the 'Pythagorean Tetractys'. For a meditation on the significance of Bruno's sudden appearance in this note, see Barfield's Appendix to *What Coleridge Thought.*

[115]Coleridge, *Lay Sermons* 32.

[116]Coleridge, *Shorter Works* 1 430.

ing, it retains a one-ness in the Reason.[117] Thus, dysynonymising terms in *Church and State,* Coleridge remarks on 'the essential difference between *opposite* and *contrary,'* arguing that unlike contraries '[o]pposite powers are always of the same kind, and tend to union, either by equipoise or by a common product.'[118] 'Opposition,' however, is prior to contrariness; just as all contradictions presuppose a unity, all contraries presuppose an opposition. Indeed, without this principle, Coleridge's theory of desynonymy as the means by which thought enriches and renews itself through differentiation in language would lose its foundation: the undifferentiated Logos, from which language descends and to which it seeks to return.

Nor is this the only consequence of the superiority of the 'polar' logic of 'oppositions' over the formal logic of 'contraries.' In *Church and State,* Coleridge's opposite / contrary distinction is finally deployed to explain how the principles of the state, like those of language, can be traced to the two opposite or 'antagonist' powers of 'Permanence and of Progression.'[119] Similarly, in his contributions to J.H. Green's lectures on aesthetics, Coleridge maintains that 'the Beautiful excludes the distinct consciousness [...] of the forms of the Understanding — for these are determined by a logical necessity' and do not consider 'an ultimate *end'* but merely the means or 'the mode of the conspiration of the Manifold to the One.' However, he continues, 'the direct contrary is the character of the Beautiful. The manifold must be melted into the *one,'* just as a 'beautiful Piece of Reasoning' is 'not beautiful because it is [...] understood as truth; but because it is felt, as a truth of Reason, i.e. *immediate,* and with [...] a facility analogous to Life.'[120]

Above all, however, the impetus behind polar logic was theological. As McFarland observes, having already established through trichotomic logic that no two positions in a dyad are equal, and that 'trinal conceptions arise inevitably from dyadic conceptions,' it seems obvious to Coleridge that the next step is to replace the 'I-Thou' dyad implicit in most unreflective conceptions of the relationship between the self and God, with an account that would vindicate the 'reasonableness of the Trinity.'[121] This brings us back to the 'Noetic Pentad,' which Coleridge models on the Pythagorean Tetractys. Included in a footnote to the 'Aphorisms on Spiritual Religion' in *Aids to Reflection,* this schematism incorporates the dynamics of polar logic in a relational model of reality that Coleridge hopes will give a philosophical underpinning to Trinitarianism. He first proposes adopting the terms *'objective and subjective reality,* &c. as substitutes for *real* and *notional,* and to the exclusion of the false antithesis between *real* and *ideal*,' the relationship between the two being that of 'Thesis' and 'Antithesis.'[122] Once again, however, the relation between these points is dictated not by negation, as in Hegel, but by

[117]Indeed, Perkins claims that the principle of difference-in-unity 'is the very foundation of Coleridge's system' (39).
[118]Coleridge, *Church and State* 24.
[119]Coleridge, *Church and State* 24.
[120]Coleridge, *Shorter Works* 2 1313.
[121]McFarland, 'Prolegomena,' *Opus Maximum* cxlii.
[122]Coleridge, *Aids* 178.

a dynamic produced by Coleridge's conflicting needs. On one hand, he wishes to install the identity of mind and world as the foundation of knowing and being; on the other, he seeks to secure a dynamic of mediation between the two that will obviate any suggestion of a return to pantheism. As a result, the Pentad is organised according to Coleridge's metaphysics of language, centring on the Prothesis as 'Verb Substantive,' the Absolute copula or 'I am,' which expresses 'the *identity* or co-inherence of Act and Being':[123]

	1. Prothesis [Verb Substantive: 'I am']	
2. Thesis [Substantive: 'thing']	4. Mesothesis [Infinitive: 'to act']	3. Antithesis [Verb: 'I act, undergo']
	5. Synthesis [Participle: 'acting']	

It is here that the significance of Coleridge's desynonymisation of 'opposite' and 'contrary' becomes clear, since it is only through the oppositions of polar logic, and not the contraries of the logic of equation and contradiction, that the dynamic relationship between world and self, thesis and antithesis, can come into being. Thus, the subordination of the laws of formal logic to the principle of difference-in-unity allows Coleridge, not for the first time, to have it both ways. Accordingly, while the Prothesis is installed as a noumenous foundational identity, the 'Punctum invisibile, et presuppositum [the invisible and presupposed point],' by which the Pythagoreans 'guarded against the error of Pantheism,' a mediating term is introduced in the form of the 'Mesothesis,' expressing the *'Indifference'* — but not the identity — of subject and object.[124] Inserting the mesothetical point as 'the Indifference of the two poles or correlative opposites,'[125] as Perkins points out, enables Coleridge finally to sever his links with Schelling by showing that the Absolute foundation is the identity of 'unity and distinction' and not (as Schelling claimed in his Identity Philosophy) the identity of 'identity and distinction.'[126]

9 BEYOND LOGIC

Ultimately, Coleridge's philosophy aspires to move beyond argument, beyond even polarity, into a horizon where knowledge merges with power. As the 1816 fragment on the 'four sorts of knowledge' makes clear, only the 'fourth' source of knowledge, the conscience, enables the reflective intellect to recover its original unity with the transcendent agency of Will.[127] Consequently, we run the risk of misrepresenting Coleridge's logic if we ignore the role it plays within a broader logosophic system

[123]Coleridge, *Aids* 180.
[124]Coleridge, *Aids* 180-181.
[125]Coleridge, *Aids* 179.
[126]Perkins 65.
[127]Coleridge, *Shorter Works* 1 412.

that incorporates conscience, imagination, and faith. 'We (that is, the human race) live by faith,' he maintains in *The Statesman's Manual*; faith 'is scarcely less than identical with its own being,' thus 'it is the Copula — it contains the possibility of every position, to which there exists any correspondence in reality. It is itself, therefore, the realising principle, the spiritual substratum of the whole complex body of truths.'[128]

Statements such as this, as we have seen, are typical of Coleridge's curious willingness to continue to deploy transcendental arguments (concerning 'the possibility of every position') in areas where argumentation itself might appear to be ceding ground to volition. Nonetheless, for much of his career, as Coleridge struggled to reconcile the claims of philosophy with those of his religion, the balance of his thinking shifted increasingly towards the noncognitive. 'For a very long time indeed I could not reconcile personality with infinity,' he recounts in *Biographia,* 'and my head was with Spinoza, though my whole heart remained with Paul and John.'[129] It was to be the power of personality that finally triumphed in this contest, as the teachings of Paul and John overcame the limitless philosophical planes of Spinoza. Will, rather than reason, becomes the keystone of Coleridge's thought as he endeavours to vindicate his belief that '[t]he Ground of Man's nature is the Will in a form of Reason.'[130]

Indeed, for Coleridge, even the alterity that determines Noetics, or the science of reason, is itself the product of Absolute Will. In an unpublished fragment dating from around 1818-1819, Coleridge maintains of the Will that 'being causative of alterity it is a fortiori causative of itself[,] and conversely the being causative of itself it must be causative of alterity [...]. Consequently the Will is neither abstracted from intelligence nor can Intelligence be conceived of as not grounded and involved in the Will [...].'[131] The trichotomic logic upon which all logic depends thus ultimately rests upon an alterity grounded in the Will. One consequence of this is that Coleridge conceives of the most fundamental relations governing reality as personal *relationships* rather than logical relations.

This is most powerfully expressed in the familial model deployed to explain the emergence of consciousness (both human and Absolute) in *Magnum Opus.* Here, Coleridge distinguishes three relationships — mother/child, father/son, I/thou — whose interdependence forms the condition of possibility for the communicative Logos. Without these relationships, indeed, it is impossible to explain how difference emerges from identity. 'The whole problem of existence,' he argues, 'is present as a sum total in the mother: the mother exists as a One and indivisible something.' Alterity, and with it language, is only made possible by the intervention of the father, introducing the difference-in-unity expressed in the Logos: 'The father and the heavenly father, the form in the shape and the form affirmed for itself are blended in one, and yet convey the earliest lesson of distinction and alterity. There

[128] Coleridge, *Lay Sermons* 18.
[129] Coleridge, *Biographia* 1 201.
[130] Coleridge, *Shorter Works* 2 1368.
[131] Coleridge, *Shorter Works* 1 779.

was another beside the mother, and the child beholds it and repeats [...].' Finally, having left the maternal knee, 'the child now learns its own alterity'.[132] Viewed this way, personality is no less than the key to the relation between unity and difference. Only when one thinks of the most fundamental relations as *personal* relationships, Coleridge argues, can one understand how alterity itself is possible. Only through the metaphysics of personality is it possible, by acknowledging that the claims of otherness touch us at the deepest level of our being, to resist the lure of hyperrationalism or Spinozism.

This is not to say that our personal relationship with God is identical to our relationships with other people. Being perfect, God's personhood is prothetical, it is 'personeity, differing from personality only as rejecting all commixture of imperfection associated with the latter.' This installation of God as 'at once the absolute person and the ground of all personality,'[133] represents Coleridge's most mature attempt to bridge religion and philosophy by reconciling 'personality with infinity.' By making the alterity outlined in his 'higher' polar logic or noetic dependent upon the *willing* relationships that sustain the relations between persons, Coleridge avoids collapsing these relations into an undifferentiated foundation that could once again be made the exclusive property of philosophy. For the later Coleridge, personhood is prior to being, just as faith is prior to knowing.

Thus, in the unpublished 'Essay on Faith,' written in 1820, Coleridge defines 'Faith' as *'Fidelity* to our own Being as far as such Being is not and cannot become an object of the sense,'[134] arguing that 'even the very first step [...] the becoming conscious of a Conscience, partakes of the nature of an *Act* [...] by which we take upon ourselves [...] the obligation *of Fealty*.'[135] As Steven Cole points out, in 'The Essay on Faith,' 'Coleridge offers his fullest, and most compelling, explanation of how the idea of personhood is contextually enacted,' based on how "'fidelity to our own being" establishes a relation our being has to the being of others.'[136] But Coleridge goes further even than this. He intimates that the obligation of *acknowledgement* is the most fundamental precondition of *all* recognition; whether perceiving nature or other persons, we are bound to enter into a moral relationship that presupposes an element of will, and thus faith. Consequently, 'Conscience is the root of all Consciousness, and a fortiori the precondition of all Experience'; it 'is a witnessing respecting the unity of the Will and the Reason effected by the Self-subordination of the Will, as = Self, to the Reason, as = the Will of God.'[137]

In essence, as Anthony John Harding indicates, what Coleridge attempts in the 'Essay on Faith' and the *Magnum Opus* is to reverse Kant's proof of the exis-

[132] Coleridge, *Opus Maximum* 131-132. As McFarland notes, this account of the Logos or 'I am' was crucial in freeing Coleridge from Schelling's dyadic, 'either/or' conception of Absolute Identity (cxxxi).
[133] Coleridge, *Opus Maximum* 176-177.
[134] Coleridge, *Shorter Works* 2 834.
[135] Coleridge, *Shorter Works* 2 836.
[136] Steven E. Cole, The Logic of Personhood: Coleridge and the Social Production of Agency,' *Studies in Romanticism* 30 (1991) 103-105.
[137] Coleridge, *Shorter Works* 2 837-838.

tence of free will from the moral law, basing the moral law on the existence of free will. In this, he was influenced by recent German criticism of Kant's system as crypto-Spinozism, and in particular Jacobi's argument in *Concerning the Doctrine of Spinoza in Letters to Herr Moses Mendelssohn* (1785) that '[f]aith is the element of all human cognition and activity.'[138] It was Jacobi's work that encouraged Coleridge in his conviction that '[t]he conscience [...] is not a mere mode of our consciousness, but presupposed therein.'[139] And yet, Coleridge was not prepared to swallow Jacobi's antidote to philosophy, what the latter termed in his 1815 Preface to *David Hume on Faith* as a 'knowing not-knowing.'[140] Instead, as Harding indicates, by establishing conscience as the means by which 'the self becomes aware of its own existence,' Coleridge does 'what Kant did not do for himself, that is, establish *a priori* the possibility of recognising other human beings as themselves possessed of conscience and selfhood.'[141] Ultimately, Jacobi's insistence on the priority of personhood and faith in human knowledge is transformed by Coleridge into a metaphysics of personality designed to prevent the higher logic of Noetics from folding into pantheism. Against Kant's stricture that there could never be a theology of reason, Coleridge envisages religion and philosophy in perfect equipoise, the logic of understanding blending with a logic of reason that is itself part logic, part revelation. Such is the place of logic in a theosophy according to which reason must always incorporate the illumination of faith, as Coleridge's Appendix to *On the Constitution of the Church and State* makes clear:

> Finally, what is Reason? You have often asked me; and this is my answer;
>
> Whene'er the mist, that stands 'twixt God and thee
>
> Defecates to a pure transparency,
>
> That intercepts no light and adds no strain —
>
> There Reason is, and there begins her rein![142]

10 COLERIDGE'S LOGIC TODAY

The story of Coleridge's influence on modern logic is not one likely to detain the historian of ideas for very long. This is almost entirely due to the fact that the *Logic* manuscripts lay almost unnoticed for much of the nineteenth century, and were first published, in excerpted form, in Alice Snyder's 1929 *Coleridge on Logic*

[138] Friedrich Heinrich Jacobi, *The Main Philosophical Writings and the Novel* Allwill, trans. George di Giovanni (McGill-Queen's University Press, 1994) 234.
[139] Coleridge, *Opus Maximum* 73.
[140] Jacobi 545.
[141] Anthony John Harding, *Coleridge and the Idea of Love: Aspects of Relationship in Coleridge's Thought and Writing* (Cambridge University Press, 1974) 189-91.
[142] Coleridge, *Church and State* 184.

and Learning.[143] Even then, a further 62 years elapsed before J.R. de J. Jackson produced a text for the Bollingen *Collected Coleridge* that accurately reflected the content of the *Logic* manuscripts. Rather like Bentham's contemporary work on logic and language, Coleridge's logic suffered from the chaotic state of the writer's corpus at the time of his death. While in the case of Bentham this was largely due to carelessness about publication, with Coleridge the causes were excessive caution and procrastination. This, combined with the fact that Coleridge tends to avoid the technicalities of logic in his published works — typically deferring full exposition with an apology and a promissory note for the 'Elements of Discourse' — means that little of Coleridge's logical theory was available in the decades following his death. His influence in the spheres of ethics and the theory of government (famously impressing John Stuart Mill), theology (encouraging Newman and the Oxford movement with his defence of Trinitarianism), philosophy (transmitting his ideas via Carlyle to Emerson and the American Transcendentalists), and aesthetics (single-handedly inventing the concept of practical criticism that would later be developed by LA. Richards) is immense and well documented.[144] In logic, however, the dissemination of Coleridge's thought, at least until recently, has largely been limited to footnotes and the inferences of his more attentive readers.

That said, since the end of the nineteenth century a succession of critics has deplored the neglect of Coleridge's logic. Among these commentators there is near unanimity in the view that Coleridge's single greatest achievements in this field stem from his exploration of the interconnectedness of logic, language, and the noncognitive matrices of faith and personhood. Snyder set the tone by arguing that, more than his efforts 'to schematise the dialectic movement of the reason,' Coleridge's 'negative criticisms of the lower faculty, the understanding [...] threw out informal suggestions for which thinkers today are still expressing their gratitude.'[145] Less patronisingly, Muirhead embraced Coleridge's rejection of conventional logic, praising 'a method which proceeds, as he expresses it, by "enlargement" instead of by "exclusion," and by inner development instead of by mere external synthesis,'[146] as well as Coleridge's anticipation of the principle — of which later idealists such as Bradley, Green and Bosanquet made so much — that

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[143] Alice D. Snyder, *Coleridge on Logic and Learning* (New Haven: Yale University Press, 1929).

[144] Coleridge's impact on these areas is too vast, and the literature devoted to the subject too extensive to document here. However, for a comprehensive study of Coleridge's influence on Mill and the thought of nineteenth-century Britain, see Ben Knights, *The Idea of the Clerisy in the Nineteenth Century* (Cambridge University Press, 1978). For his influence on Cardinal Newman, see Philip C. Rule, 'Coleridge and Newman: The Centrality of Conscience,' *The Fountain Light: Studies in Romanticism and Religion in Honor of John L. Mahoney,* ed. Robert J. Barth (NY: Florida University Press, 2002). Basil Willey's 'I.A. Richards and Coleridge,' in *LA. Richards: Essays in his Honor,* eds. Rueben Brower, et al. (Oxford University Press, 1973) assesses the role Coleridge played in Richards' conception of the function of criticism, while the poet's relation to American transcendentalism is examined by Kenneth Marc Harris in 'Reason and Understanding Reconsidered: Coleridge, Carlyle and Emerson,' *Essays in Literature* 13.2 (1986). The wider impact of Coleridge's thought is mapped in *Coleridge's Afterlives, 1834-1934,* eds. Jane Wright and James Vigus (Palgrave, forthcoming).

[145] Snyder 13.

[146] Muirhead 86-87.

'what is necessary and at the same time possible must be real.'[147] More recently, as well as McKusick's championing *of Logic* as a landmark of nineteenth-century linguistics, Nicholas Reid has published a series of articles in which he argues that by indexing meaning to an imaging *process* rather than to fictional entities called images, Coleridge evades the Fregean-Wittgensteinian attack on psychologism, presenting a view of logic and language, which, 'admittedly shorn of its idealist and theistic roots, is ripe for revival.'[148]

Apologias such as the above notwithstanding, it is difficult to see how, for the foreseeable future at least, Coleridge's position in the history of logic, sandwiched between Kant and Mill, can avoid appearing to many as an embarrassing example of romantic overreaching. This is partly because of the way in which the status of logic as a 'discipline' is contested in his writings, but mainly because of how he challenges philosophy to justify itself at the bar of 'life.' Yet it is for precisely this reason, I would argue, that we should applaud Coleridge and continue to read him. Driven by an urgent desire to define a new index of rationality for a post-revolutionary age, the sheer range and audacity of Coleridge's theosophical resuscitation of logic in an era dominated by the spectre of irrationalism can reacquaint a modern reader with the historical conditions of what Habermas calls the plurality of the voices of reason.[149] In particular, his endeavours remind us how, before Frege, logic briefly assumed an existential form, which, however outlandish it might appear today, rightly refused the Humean severance of thought that analytic philosophy was later to inflict on human intellect.

Admittedly, one result of this is that navigating what McFarland aptly describes as 'the reticulation of Coleridge's thought' is never without its difficulties.[150] In particular, as his later work strives to adjudicate between competing conceptions of reason (instrumental, dialogic, intuitive, incarnate, practical), Coleridge's increasingly elaborate attempts to square the circle between faith, logic and communication acquire a dogmatic character. Suspicious of foundationalism, Coleridge originally grounds thought and being in a linguistic act. Yet, rather than leave reason to the pragmatics of communication, to the everyday function of 'etymologic,' he hypostasizes this act in the form of a metaphysical principle. Instead of dispensing with philosophical foundations altogether, Coleridge ultimately installs a

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[147]Muirhead 110.

[148]Nicholas Reid, 'Coleridge, Language, and the Imagination,' *Romanticism on the Net,22,* May 2001, 31 March 2006 `www.erudit.0rg/revue/ron/2001/v/n22/005977ar.html`. See also Nicholas Reid, 'Form in Coleridge, and in Perception and Art More Generally,' *Romanticism on the Net* 26, May 2002, 31 March 2006 `www.erudit.0rg/revue/ron/2002/v/n26/005699ar.html`, and ' "That Eternal Language," or Why Coleridge was Right about Imaging and Meaning,' *Romanticism on the Net* 28, November 2002, 31 March 2006 `www.erudit.org/revue/ron/2002/v/n28/007208ar.html`.

[149]See Jürgen Habermas, 'The Unity of Reason in the Diversity of its Voices,' *Postmetaphysical Thinking: Philosophical Essays,* trans. William Mark Hohengarten (Cambridge: Polity Press, 1992): '[T]he unity of reason only remains perceptible in the plurality of its voices — as the possibility in principle of passing from one language to another — a passage that, no matter how occasional, is still comprehensible' (117).

[150]McFarland, *Pantheist Tradition* xl.

super-foundationalism. Thus, the 'copula' that the verb substantive, the Prothesis or 'I am' presents in thought becomes a ground of grounds, a logical, grammatical, psychological *and* metaphysical principle underwriting logic and theory of method alike. But is this principle itself logical? The closing words of Barfield's study will serve just as well here, in that they elegantly capture the question that Coleridge bequeaths to modern logic. What finally matters when considering Coleridge's logic, Barfield concludes, is 'whether there is indeed a sense in which it is proper to characterise as a nuclear — or polar — logic the exactness of the understanding, not blurred or cancelled, but pierced to its empty heart at each moment, by the energy of imagination as the bearer of related qualities.'[151]

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RICHARD WHATELY AND LOGICAL THEORY

James Van Evra

1 INTRODUCTION

Richard Whately (1787–1863) is better known for his influential defense of logic than for adding anything to the formal core of the subject. That defense, set out in his *Elements of Logic* [EL] of 1826,[1] appeared at the beginning of a period of significant change in the subject. The popular logic[2] that Whately inherited had been the subject of unremitting criticism at the hands of major thinkers for more than two centuries. As a result, the received logic, which consisted largely of texts that were formally defective and contained little evidence of theoretical insight, was in disarray. Soon after the publication of *Elements*, by contrast, the first wave of great 19^{th} century works in logic appeared, works by George Boole and Augustus De Morgan in Britain, Bernard Bolzano on the continent, and Charles Sanders Peirce in the United States. Many factors contributed to this reversal of fortune, including, perhaps most importantly, the growing recognition of interesting ties between logic and mathematics. Although Whately's defense of the subject preceded the mid century shift, it was nonetheless more than a defense of the existing tradition. Rather, Whately's contribution to the process was a reconception of the broad theoretical context within which logic occurs. In effect, his theory cast logic in a form more congenial to the formal developments soon to come. In addition to its impact on the core conception of the subject, his reconception had the further effect of clarifying the boundary conditions on logic, something sorely needed in a period in which logic had often been uncritically run together with epistemology or rhetoric. Settling logic's boundary conditions, in turn, served to better define the scope and limits of the neighboring disciplines (in particular rhetoric, the topic of another of Whately's works, *Elements of Rhetoric* [1828]).

[1]The date of the first monograph edition; the work first appeared in 1823 in several parts in a popular cyclopedia. See Jongsma 1983 for a full account of its provenance.

[2]While they contain no innovations, popular texts are nonetheless a good indicator of the state of logic at that time. E. J. Ashworth puts the point well: "... the textbook-writers and schoolteachers of a period may be as important as the leading intellectuals, for it is by these minor figures that all innovations are accepted, altered, and made into the new commonplace. To concentrate solely on the great thinkers is to obscure the reality of the university and school, of the mainstream orthodoxy which lies behind these thinkers and which feeds them." [Ashworth, 1985: LIV]

Handbook of the History of Logic. Volume 4: British Logic in the Nineteenth Century.
Dov M. Gabbay and John Woods (Editors)

2 BIOGRAPHY

2.1 Life and Works

Richard Whately was born in London on February 1, 1787, to an established family whose members held positions of responsibility in government service, banking, law and the clergy. Throughout his life, he is described as being a brilliant, independent thinker who bore little routine allegiance to established tradition. The youngest (by six years) of nine children, his independence was at first a matter of necessity; often isolated and left to his own interests, he learned to read and write at an early age, studied nature on solitary walks, and displayed prodigious ability at arithmetic calculation. Later in life his independence took the form of advocacy of less than uniformly popular causes, such as ending legal disadvantages borne by English Catholics, and, while serving as president of a royal commission on the Irish poor (1835/6), arguing for making improvements to agriculture rather than forcing the impoverished into workhouses.

After early schooling near Bristol, Whately entered Oriel College in 1805. His life in Oxford was a series of successes, beginning with a double second BA (in Classics and Mathematics) in 1808. The degree was one of the first double degrees awarded there, and one of its components, mathematics, was just beginning to gain acceptance in Oxford as an independent field of study. He was elected fellow of Oriel in 1811, awarded an MA in 1812, and became a private tutor. In 1822, he left Oxford to become a parish priest, but returned in 1825 as principal of St. Alban Hall. He became professor of Political Economy in 1829, and in 1831 left Oxford permanently to become Archbishop of Dublin, a post he occupied until his death on October 8, 1863.[3]

Throughout his working life, Whately was known more for his talent as a teacher ("no don," it was said at the time, "was ever less donnish")[4] than for his published scholarship. However, while not extensive in quantity, and while his two primary works were texts, his writings were nonetheless influential; they are cited, for instance, 237 times in the Oxford English Dictionary. Among them, three are particularly noteworthy: the two *Elements* already mentioned, and his early *Historic Doubts Relative to Napoleon Buonaparte* [1819], an ironic critique of David Hume's argument on miracles based on the claim that the same argument could be used to cast doubt on the existence of historical figures such as Napoleon.

Logic played a relatively minor role in the full scope Whately's working life; he was primarily a theologian and philosopher. However, the fact that he was not a logician is unremarkable, for at that time, no one was a logician.[5] What interest he had in logic had two origins, one theological, the other pedagogical. Regarding the former, while at Oxford, Whately came under the influence of Edward Copleston,

[3]A full account of Whately's life can be found in E. Jane Whately 1875.

[4]Cf. Brent 2004/5. Mary Prior [1967] likens him to an early 19th century P. T. Geach.

[5]None of the text writers in the tradition were primarily logicians. Much later, Peirce (correctly) claimed that he was the first since the middle ages to completely devote his life to logic. Cf. [Fisch, 1985, xviii].

the founder of a group (the Noetics) devoted to defending Christian doctrine on the ground of its presumed reasonableness, and to producing clergy capable of providing a reasoned defense of the teachings of the Church of England. Whately considered logic to be an important part of the university curriculum insofar as it strengthened rationality to such ends:

> The cause of Truth universally, and not least, of religious Truth, is benefited by everything that tends to promote sound reasoning and facilitate the detection of fallacy. The adversaries of our Faith would, I am convinced, have been on many occasions more satisfactorily answered ... had a thorough acquaintance with Logic been a more common qualification than it is. [1826, xxvii][6]

The other source of his interest in logic stems from changes then taking place in Oxford. At the time, logic was being forced to prominence as a result of curricular reform, one effect of which was that more students were required to study it. At the same time, the university came under attack from external critics for promoting the teaching of what they took to be a worthless subject. With logic so much a matter of contention, there was obvious need for a clear account of the nature of the subject and a rationale for studying it. Whately provided both.[7]

2.2 Reception

Elements was one of the most influential logic texts of the 19^{th} century. When referring to the background against which his own work was set, for instance, George Boole immediately recognized *Elements* as the standard text in logic.[8] In reaction to *Elements*, De Morgan called Whately the "restorer of logical study in England",[9] and Peirce's interest in the subject was initially spurred by a reading of the work at age 11.[10] Beyond such direct influence, *Elements* was broadly popular, appearing in nine editions during Whately's lifetime, and in many reprint editions

[6] This was a common sentiment at the time. Others (e.g. Wesley quoted below) held the same view. Unlike them, however, Whately sharply distinguished the science itself from its presumed benefits.

[7] A more complete account of the reform process and reactions to it can be found in my [1984, sec. 2.3].

[8] In *Laws of Thought*, Boole says "that portion of this work which relates to logic presupposes in its reader a knowledge of the most important terms of the science, as usually treated, and of its general object. On these points, there is no better guide than Archbishop Whately's *Elements of logic* [Boole, 1854, pref.].

[9] [De Morgan, 1860, 247]

[10] Peirce's first encounter with logic, as Max Fisch [1982, xviii] describes it, occurred when Charles found a copy of *Elements* in his brother's room, and asking what logic was, got a simple answer, stretched himself on the carpet with the book open before him, and over a period of several days absorbed its contents. Since that time, he often said in later life, it had never been possible for him to think of anything, including even chemistry, except as an exercise in logic. And so far as he knew, he was the only man since the middle ages who had completely devoted his life to logic.

throughout the nineteenth and early twentieth centuries.[11] It was so popular in the United States that, as Whately (correctly) observed, "... it is in use, I believe, in every one of their Colleges" [9^{th} ed., xviii] The reason for its popularity is clear: *Elements* contains a well reasoned and engaging defense of logic generally, an accurate presentation of what was then its formal core, and a coherent theory of the subject.

3 WHATELY'S CONCEPTION OF LOGIC

3.1 Logic as Whately Found it

Whately wrote *Elements* in part as a critique of a long standing tradition in common logic on the one hand, and in reaction to the pointed criticisms of logic which arose in reaction to that tradition on the other. His point throughout was that logic had been misrepresented by the authors of the texts and misunderstood by the critics.

The tradition in question began as a revival of Aristotelianism in the late sixteenth century. By that time, the sophistication of Medieval logic had been lost to critics (e.g. the "Humanists") who deplored the imposition of what they considered barbarous technicalities on the language, and yearned instead for a return to the elegant Latin of Cicero and Seneca. By the mid-sixteenth century, popular works such as the *Dialectic* of Peter Ramus [1543] appeared which played to literary fashion by freely drawing illustrations from favored Latin authors. Rather than conceiving of logic as having to do with semantic problems, forms of argument, or paradoxes as it had been during the medieval period, logic was now described as "an art which teaches one to dispute well". As a result, such

> Lack of interest in, and open hostility to, the older logical methods were widely accompanied by an increased concern with rhetoric, so that logic, for long the 'art of arts', was now required to be patterned on logically untutored thought and speech in a way as free as possible from subtle technicalities and rigour. [Thomas, 1964, 300]

Later works in the tradition identified the syllogism as the central formal feature in logic, but often gave faulty accounts of such basic topics as rules for identifying middle terms, and for distinguishing major and minor premisses. Clearly, the atmosphere of continuing hostility to the older logic had taken its toll:

[11]So popular was *Elements* that it was featured in an 1867 poem by W. S, Gilbert: ("Sir Macklin". Cf [Gilbert, 1968, 96]):

> Then I shall demonstrate to you,
> According to the rules of Whately
> That what is true of all, is true
> Of each, considered separately.

> Having discarded the whole corpus of late medieval novelties and discoveries, it is clear that [the authors] left themselves little room for manoeuvre. In strictly formal logic, about all that they set themselves to do was to give an account of the syllogistic. Their normal ill success is a measure of the general loss of logical nerve at this time. [*Ibid.*, 302]

Prominent works in the tradition include those by Robert Sanderson [1615], Richard Crakanthorpe [1622], The mathematician (and teacher of John Locke) John Wallis [1687], and Henry Aldrich [1691]. Of these, versions of the first and especially the last were still in prominent use in Britain in the early 19^{th} century. Indeed, Whately acknowledged Aldrich's ("concise, but generally accurate") *Compendium* as the source of the formal core of *Elements*. When attention turned to theory, however, any allegiance Whately felt to his predecessor vanished.

To give some indication of what texts of the period were like, what follows is a brief characterization of those by Sanderson and Aldrich. Both texts begin with essentially the same description of logic. Here, in translation, is Aldrich's version:

> Logic, which by synecdoche (or the figure which takes the part for the whole) is denominated Dialectics, has been called the art of reasoning; or, an instrumental art, directing the mind into the knowledge of all intellectual things. For this reason it ought to be the first of all disciplines, as being necessary to the acquirement of the rest. [Jackson, 1836, 1n][12]

As we will see, such an optimistic estimate of logic's role is an example of what for critics was a sure sign of its abject failure.

Sanderson's *Logicae Artis Compendium* [1615] was popular throughout its history and still in use in the late 18^{th} and early 19^{th} centuries.[13] Little of it is devoted to what would now be considered logic. An account of the figures and rules of the syllogism, for instance, occupies just three (of 357) pages. Further, the syllogism is interpreted, following Aristotle, as a kind of discourse, but not one based on a distinct conception of logical form (there is, for instance, no formal representation of the syllogism in the text). Also, Sanderson recognizes only three syllogistic figures on the Philoponian arrangement of syllogistic terms. The remainder of the work is devoted to topics which, while related to logic (e.g. speech, or the mind), are no longer considered proper parts of the subject. Sanderson's conception of logic is broad enough, in fact, to include everything to which logic might *apply*, including all of (material) existence (and non-existence!) The syllogism appears only derivatively, i.e. as a sub-class of second intentions (used to group things taken materially. (The other sub class being demonstration.))

[12]Sanderson, more than a century earlier, put it as follows: "Logica, quae & Synecdichice Dialectica, est ars instrumentalis, dirigens mentam nostrum in cognitionem omnium intelligibilum". (1) Obviously, little had changed in the intervening period.

[13]Cf. [Ashworth, 1974]. Jeremy Bentham studied Sanderson as an undergraduate, and used it in designing a new curriculum for logic in the schools. See also [Bentham, 1827, 13, 22].

Befitting its status as a compendium, is Aldrich's *Artis Logicae Compendium* [1691] is spare (in most versions, approx. 50 pages in length), and contains little more than the basic components of an Aristotelian text, i.e. a classification of terms and propositions, an account of syllogistic structure, a few pages on method, and an appendix on fallacies. While slight, Aldrich's text contains evidence that by the latter stages of the 17^{th} century, at least some formal progress had been made:

> [Aldrich] defines the terms in the Philoponian way and works out the consequences accurately and completely, arriving at twenty-four moods in four figures, tables them in full, and effects a deductive rejection of the 232 invalid moods. To come on these few pages after the logical rags and tatters of the previous two centuries is to be presented with a creation of haute couture. [Thomas, 1964, 310–311][14]

Though not part of the same tradition, other 17^{th} century works displayed much the same character. Arnauld and Nicole's Port Royal logic (La *Logique, ou l'art de Penser* (1662)) was, like Aldrich compendium, formally accurate in its (brief) presentation of the syllogism, but was otherwise in the thrall of Cartesian epistemology, and promoted Descartes' critical attitude toward logic as traditionally conceived.[15]

The same extravagant claims made for logic in the 17^{th} century can be found in the 18^{th} century as well. In his "Address to the Clergy", for instance, John Wesley (a translator of Aldrich in addition to being a noted theologian) characterized logic in this way:

> Some knowledge of the sciences also, is, to say the least ... expedient. Nay, may we not say, that the knowledge of one, (whether art or science), although now quite unfashionable, is even necessary next, and in order to, the knowledge of the Scripture itself? I mean logic. For what is this, if rightly understood, but the art of good sense? of apprehending things clearly, judging truly, and reasoning conclusively? What is it, viewed in another light, but the art of learning and teaching; whether by convincing or persuading? What is there, then, in the whole compass of science, to be desired in comparison of it? [Wesley's, *Works*, 1756. Jackson ed. 1872, Vol. 10. 483]

3.2 The Critics

As Whately received it, logic was caught in the rational equivalent of a perfect storm, i.e. a set of circumstances that converged to make progress in the subject highly unlikely. During the same period in which logic was being described

[14]Whately incorporated this feature of the *Compendium*, with some modifications, in *Elements*. It is one of the few features of the work that he used without extensive alteration.

[15]See [Descartes, quote below, 82].

as playing a role in gaining knowledge, (while at the same time being based on modes of thought fixed by tradition), major thinkers in the great outwash from the Renaissance were busy promoting their own new methods for gaining knowledge (including the fundamentals of the scientific method as we now know it) and making major additions to the theory of knowledge itself. To them, not only did the logic of terms, propositions and syllogisms play no role in uncovering new truths, it was an example of what they were intent on leaving behind, i.e. a device based on discredited tradition that did nothing but merely reshuffle knowledge already in hand. Here is Whately's description of the situation:

> By representing logic as furnishing the sole instrument for the discovery of truth in all subjects, and as teaching the use of intellectual faculties in general [members of the preceding tradition] raised expectations which could not be realized, and which naturally led to a reaction. [1826, vii]

The reaction is well known. Some critics focused on the general Aristotelian context. Thus Francis Bacon:

> For as water ascends no higher than the level of the first spring, so knowledge derived from Aristotle will at most arise no higher again than the knowledge of Aristotle. And therefore, though a scholar must have faith in his master, yet a man well instructed must judge for himself. [Bacon, 1605, 20]

And in the 18^{th} century, Thomas Reid:

> [Aristotle's]'s works carry too evident marks of pride, vanity and envy which have often sullied the character of the learned. He determines boldly things above all human knowledge.
>
> He delivers his decisions oracularly, and without any fear of mistake. Rather than confess his ignorance he hides under hard words and ambiguous expressions, of which his interpreters can make what they please. [Reid, 1843, 553][16]

Others focused on what they took to be the nature of logic. Thus Locke's famous remark that

> If the syllogism be taken as the only proper instrument of reason and means of knowledge, it will follow that before Aristotle, there was not one man that did or could know anything by reason, and that since the invention of the syllogism there is not one in ten thousand that doth.
>
> But God has not been so sparing to men to make them barely two-legged creatures and left it to Aristotle to make them rational. [Locke, 1706, 264]

[16]In his review of *Elements*, John Stuart Mill [1828, 138] indicated that even before Whately, such extreme views had already lost favor.

The main criticism directed at logic, however, and one repeated frequently, was that it could not do what had been claimed for it. Again in Bacon's words,

> The syllogism consists of propositions, propositions of words are the signs of notions. If, therefore, the notions (which form the basis of the whole) be confused and carelessly abstracted from things, there is no solidarity in the superstructure. Our only hope, then, is induction. [Bacon, 1620, 316]

And in a similar vein, Descartes says that

> I noticed that as far as logic was concerned, its syllogism and most of its other methods serve rather to explain to another what one already knows, or even, as in the case of Lully, to speak freely and without judgement of what one does not know, than to learn new things. [Descartes, 1637, 14] [17]

Comments such as these are only a small sampling of the widespread contempt for logic. In a period of intense activity in epistemology, the logicians' claims seemed both antiquated and false. The point constantly made was that there are better ways to gain knowledge than those offered in logic, methods such as Baconian induction, or the new methods of Descartes or Locke.

Such criticism gave rise, in turn, to a variety of works, identified as logics, that were designed substantially to modify, or completely replace, syllogistic texts. The common rationale was that if the syllogism could not lead to knowledge, then syllogistic texts should be replaced by accounts of what *could.* Thus, for instance, as Rolf George points out,

> Denis Diderot's article on logic in the *Encyclopedie*, the most widely consulted work of the 18^{th} century, has nothing whatever to say about logic. It is claimed here, simply, that reasoning is a *natural* ability, and that to conduct logical inquiries is like "setting oneself the task of dissecting the human leg in order to learn how to walk" (*Encyclopedie, Logique*; [George, 2002, 36]).

Other works, including Arnauld's [1662] work already mentioned, Issac Watts' *Logick, or the right use of reason* [1725], and *Elements of logick* [1748], by William Duncan, relegate the syllogism to a minor role, or discard it altogether, and replace it with the fruits of 17th century Epistemology.

While such alternatives to the syllogistic manuals were popular, they did not entirely displace the 17th century texts. Support for the standard texts, for instance, remained particularly strong in Oxford. That support, in turn, led inevitably to renewed criticism, and the debate over logic that resulted was an important factor in the genesis of *Elements.*

[17]'Lully' is Ramon Lull (1232? –1315), best known as the author of an *Ars Magna* [1274].

3.3 Whately's alternative

Whately made two initial assumptions that set his view of logic apart from those of his forbears. The first is that logic is a science on a par with other prominent sciences (e.g. chemistry, physics or algebra). While logic had been considered to be equally an art as well as a science, that is, Whately considered it to be primarily a science, and an art only derivatively and in application (see [1826, 1, 127–28]). In calling logic a science, Whately meant that it is a discipline based on clear theoretical principles.[18]

For Whately, it was the longstanding lack of a strong theoretical foundation, by contrast, that helped to explain why logic had progressed so little in preceding centuries, for, as he says, no science progresses unless it is founded on such principles [1826, 2]. And, while centuries of stagnation suggested to its critics that logic had become moribund, Whately remained optimistic about its prospects. One had to recall, he suggested, that sciences such as physics and chemistry had been dormant for long periods before receiving the theoretical support needed for scientific progress [1826, 11].

The same lack of 'right principles' also explained, according to Whately, why logic had been the subject of so much criticism, for without a proper foundation, it had been subjected to the claims and demands of common opinion, but not science:

> The vanity ... by which all men are prompted unduly to magnify their own pursuits, has led unphilosophical minds ... to extend the boundaries of their respective sciences, not by the patient development and just application of the principles of those sciences, but by wandering into irrelevant subjects. ... none is more striking than the misapplication of logic, by those who treated it as the 'art of rightly employing the rational faculties', or who have intruded it into the province of natural philosophy, and regarded the syllogism as an engine for the investigation of nature. [1826, 6–7].

The second assumption Whately made is that logic is immediately concerned with language. Earlier logicians had held that language serves logic only as a representative of thought, which in turn is the primary subject.[19] Whately, on the other hand, took language itself to be the primary subject:

> Logic is entirely conversant about language: a truth which most writers on the subject, if indeed they were fully aware of it themselves, have

[18] Sir William Hamilton criticized Whately for suggesting that the definition of logic as a science introduced something new. He pointed out that it had often been so described in the past [Hamilton, 1833, 131 ff.]. Hamilton was right; there had been considerable debate in the 16th and 17th centuries concerning whether logic was an art or a science. As then understood, however, 'science' was understood in the broad Aristotelian conception of that term (i.e. one that encompassed any classificatory scheme). Whately's conception of science, by contrast, is of the modern theoretical variety, and not the Aristotelian.

[19] See [Aldrich,, Jackson, 1836, 3]

> certainly not taken care to impress on their readers. Aldrich's definition of logic, for instance, does not give any hint of this. [1826, 56n]

And,

> . . . since logic is wholly concerned with the use of language, it follows that a Syllogism (which is an argument stated in regular logical form) must be "an argument so expressed, that the conclusiveness of it is manifest from *the mere force of the expression*", i.e. without considering the meaning of the terms . . . [1826, 88][20]

While placing language in such a central role is reminiscent of earlier Hobbesian nominalism, the view is also a precursor of the view that it is possible to deal with a variety of subject areas as they are represented in language.

Within the scope of these two broad assumptions, the heart of Whately's view is that logic is a 'generalized and abstract statement of all demonstration whatever' [1826, 34]. In terms of the sort Boole would later use, it is 'a method of analyzing that mental process which must *invariably* take place in all correct reasoning' [1826, 11]. What makes it abstract is that, unlike those who treat the syllogism as one kind of argument among many,[21] Whately separates logic from its intended field of application. Thus thinking of the syllogism as a particular sort of argument is, Whately says (in obvious reply to Locke), a "mistake no less gross than if any one should regard Grammar as a peculiar Language, and should contend against its utility, on the ground that many speak correctly who never studied the principles of grammar" [1826, 11, 13/14]. In his conception, by contrast, using logic as an abstract analytical device 'is like using chemical analysis to examine the elements of which any compound body is composed' [1826, 11/12].

Perhaps the clearest example of Whately's conception of the abstract nature of logic can be seen in the analogy he draws between logic and mathematics:

> All numbers must be numbers *of some things*. But to introduce into the science any notice of the things respecting which calculations are made, would be evidently irrelevant, and would destroy its scientific character: we proceed therefore with arbitrary signs representing numbers in the abstract. So does logic pronounce on the validity of a regularly constructed argument, equally well, though arbitrary symbols have been substituted for the terms: and, consequently, without any regard to the things signified by those terms. [1826, 13–14]

And,

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[20]The word 'force' in the passage is likely a misprint of the word 'form.' Cf. [1826, 37] for a similar passage using the latter term.

[21]Whately cites as an instance the rhetorician George Campbell, who juxtaposed syllogistic and moral reasoning.

> In order to trace more distinctly the different steps in the abstracting process by which any particular argument may be brought into the most general form, we may first take the syllogism stated accurately ... and then somewhat generalize the expression by substituting (as in algebra) arbitrary unmeaning symbols for the significant terms that we originally used. [1826, 35]

Within this context, Whately considered the syllogism to be a purely formal device for the expression of any argument, as opposed to being an argument of a particular kind. In Whately's words, an argument, when cast into the proper form

> is called a syllogism; which is ... not a particular kind of argument, but only a peculiar form of expression, in which every argument may be stated. [1826, 24]

Again, logic,

> which is, as it were, the grammar of reasoning, does not bring forward the regular syllogism as a distinct mode of argumentation, designed to be substituted for any other mode; but as the form to which all correct reasoning may be ultimately reduced; and which, consequently, serves the purpose ... of a test to try the validity of any argument. [1826, 11–12; See also 124n]

Unlike earlier theories, that is, Whately held the syllogism to be an abstract canonical form, into which (perhaps elliptically stated) arguments may be formulated for validity testing. So regarded, the syllogism has more in common with algebraic expressions containing variables, than with arguments as they appear in ordinary language. Whately later strengthened this conception of the syllogism as a normal form by insisting on a completely tenseless interpretation of the copula (see [1826, 57]).

Defining the syllogism as abstract in relation to arguments permitted Whately to adopt a somewhat more liberal approach to the relation between the syllogism and various sorts of common arguments than that found in the 17^{th} century texts. He allowed, for instance, (where the earlier logicians had not) the possibility that other sorts of arguments might have their own rules:

> ... rules have been devised for ascertaining the validity of [hypothetical syllogisms] at once, without bringing them into categorical form. [1826, 108]

And, although he held that the categorical syllogism remains the ultimate ground for the validity of arguments, no longer were various kinds of arguments treated as *de facto* incomplete syllogisms.

Whately's formal development of the syllogism (in a section occupying less than one fifth of the whole work) bears an external resemblance to earlier logics. On closer inspection, however, the effects of his theoretical stance are immediately apparent. Thus he initially restricts the scope of the discussion to reasoning and argumentation, and then only as they occur in language. In addition, while he begins in the traditional manner with a classification of terms, his classification differs fundamentally from any that could be regarded as Aristotelian. He counts, for example, the traditional distinctions (e.g. between univocal and equivocal terms, or terms of first and second intention) not as classifications of *terms*, but of ways in which they are used. Whately's classification is confined to just one major division (singular and common terms), all others being relegated to derivative status.

One consequence of the abstract character of logic, according to Whately, was that it has value even when it has no direct application [1826, ix, 20]. Judging logic by its direct effects (as those who held that it was an art had done), he says, reflects a confusion about the nature of theories. Thus finding fault with logic for not making people think better 'is as if one should object to the science of optics for not giving sight to the blind' [1826, 12]. Obviously, Whately's association of logic with pure science was based on the recognition of similarities between the structural properties of theories in areas such as optics with those in logic.

Whately's identification of argument forms containing variables as the proper focus for logic now seems to be nothing more than a mundane recognition of the obvious. In the early 19^{th} century, it was nothing of the kind. While there had been earlier logicians whose work included abstract components, Whately's theory explicitly recognized this as a defining feature of logic.

3.4 Other aspects of Whately's approach to logic

Beyond Whately's general conception of logic as a formal science with similarities to algebra, etc., there are two more specific features found in *Elements* that are worthy of mention. The first concerns his identification of logical individuals as things which are incapable of *logical* division, which replaces the traditional notion of individuals as essentially simple things [1826, 68]. This interpretation is later prominent in the works of both Boole and De Morgan). Second, he includes (for the first time since the Port-Royal logic), the principle of conditionalization:

> A conditional proposition ... may be considered as an assertion of the *validity* of a certain argument; since to assert that an argument is *valid*, is to assert that the conclusion necessarily results from the premisses, whether those premisses be *true* or not. [1826, 110]

3.5 Whately on method

Separate from the core of *Elements*, but appended to it, Whately includes a lengthy section (50 pp.) entitles 'Dissertation on the province of reasoning', which concerns such traditional topics as induction, the discovery of truth, and inference and proof.

Unlike earlier treatments of the same topics, in which they are run together with logic proper, Whately explicitly separates the two:

> Logic being concerned with the theory of reasoning, it is evidently necessary, in order to take a correct view of this science, that all misapprehensions should be removed relative to the occasions on which the Reasoning-process is employed — the purposes it has in view — and the limits within which it is confined. [1826, 205]

In this way he separates logic from the substantive theory of reasoning (a point which Mill later failed to appreciate), the former being concerned with the latter as theory to model, i.e. in this case to typical ways in which reasoning is used and justified. Logic itself remained for Whately the formal theory of the theory of reasoning.

Nowhere is this separation more evident than in Whately's treatment of induction. Attempting to establish a relationship of precedence or dependence between logic and induction (a continual arguing point in the 17^{th} and 18^{th} century), is, says Whately, a mistake:

> Logic takes no cognizance of induction ... as a distinct form of argument. ... The essence of an inductive argument (and so of the other kinds which are distinguished for it) consists not in the *form of the Argument*, but in the relation which the *subject-matter* of the Premises bears to that of the Conclusion. [1826, 124]

And, as Whately notes on several occasions, the mistake of treating the syllogism as a kind of argument rather than as a common logical form has a common origin with the mistake of treating induction as a form of argument.

4 CRITICAL REACTION TO WHATELY'S THEORY

Reactions to *Elements* began to appear shortly after its first monograph publication. While there was general agreement that Whately had mounted an admirable defense of logic, and that he was responsible for stimulating new interest in the subject, specific reaction to his revision of the context within which logic is done fell into two groups. Those still committed to the Aristotelian style of logic with all its accoutrements interpreted Whately as being one of them due to his retention of the syllogism, but found his conception of logic as a pure formal science too spare. Those who had moved beyond Aristotelian logic, by contrast, tacitly adopted his formal style, but (again owing to his retention of the centrality of the syllogism), regarded him as still a member of the older tradition.

The first substantial review of *Elements* was written by Jeremy Bentham's nephew, George Bentham (1800–1884). His book, entitled *Outline of a new system of logic, with a critical examination of Dr. Whately's "Elements of logic"*, [1827], is an account of his own ideas on logic (which include the first use of equations and symbolic quantifiers in the formalization of syllogistic premisses), combined with a running commentary on *Elements*. To Bentham, *Elements* was simply the 'last and most improved edition of the Aristotelian system' [Bentham,, 1827, vii], rather than something fundamentally new. While Bentham lauds Whately for having done much to 'divest the science of that useless jargon, of those unmeaning puerilities, with which it had been loaded by the schoolmen', [Bentham, 1827, v] he nonetheless finds Whately's conception of logic too confining. Reflecting older views, Bentham advocated a return to the earlier view that logic should be concerned with the acquisition of knowledge as well as its formal representation. Logic, he says, is the 'branch of art-and-science which has for its object the advantageous application of the human mind to the study of any other branch of art-and-science' [1827, 14]. As such, logic should be concerned with both deduction and induction. Being confined only to deductive reasoning, Bentham held, Whately's theory is too restricted to be of use.

In the following year, the young John Stuart Mill's review of *Elements* appeared in the *Westminster review*. Like Bentham, Mill complimented Whately for having written a 'clear exposition of the principles of syllogistic logic, and vindicating it against the contemptuous sarcasms of some modern metaphysicians.[22] Then, prior to expressing reservations similar to those of Bentham, Mill makes a comment that would be repeated in the 20th century: '[Whately] has written rather excellently concerning logic, than expounded in the best manner the science itself" [1828, 138]. Mill's criticism of Whately, repeated later in his *System of logic* [1843], again concerns the abstract nature of Whately's conception of logic. In opposition to Whately, for instance, Mill says that logic *can* remove ambiguity, for the 'analysis, to which it subjects any process of reasoning, affords the readiest and most certain means by which a latent ambiguity in any one of the term employed, or the tacit assumption of any false or doubtful propositions, can be detected' [1828, 144].

Mill's main complaint, however, was that while Whately was on solid ground when he dealt with terms and propositions as they are used in *syllogisms*, 'Aristotelians did not stop here, nor confine within these narrow bounds the dominion of their science' [1828, 154]. Rather, he says, they included also instruction for the right employment of words as an instrument for the investigation of truth. As an alternative, Mill offers the prospect of the development of a logic of induction. Once again, Whately was being judged by the standards of the older tradition.

Even Sir William Hamilton, no friend of Oxford logic generally, grudgingly admitted Whately's salutary influence on the study of logic by saying that just when logic seemed dead,

> . . . a new life was suddenly communicated to the expiring study, and hope at least allowed for its ultimate convalescence under a reformed system. [Mill, 1828, 199]

After ascribing the revitalization to the publication of *Elements*, however, Hamilton nonetheless dismisses Whately (and others) by saying that they relied exclu-

[22] [Mill, 1828, 137]

sively on Aldrich, who in turn knew little of the Aristotelian heritage of the subject. The new works, he says, show ignorance of the Greek commentaries on the *Organon*, of the scholastic, Ramistic, Cartesian or Wolfian approaches, or the Kantian dialectic.[23] In addition, Hamilton accused Whately of contradicting himself by saying that logic was about the regularities which underlie reasoning on the one hand, and that logic is about language on the other.[24] Once again, Whately's theory was judged to be too narrow.

One further 19^{th} century commentator deserves mention. Robert Blakey, the author of the first modern work entirely devoted to the history of logic, characterized Whately's legacy in the following manner:

> A great change has been effected in Oxford of late years, and almost solely through the labours of Dr. Whately. Since the publication of his *Elements*, many excellent works have made their appearance from that venerable seat of learning. . . . [Blakey, 1851, 427]

By the 20^{th}century, the changes Whately advocated had become so routinely assumed that he was no longer associated with them. When he is mentioned at all, [25] it is as a syllogistic logician who defended the subject but added nothing to it.[26] Arthur Prior's assessment, for instance, is that 'in the early 19th century the common logic was rescued from oblivion by Richard Whately but was not enlarged by him.'[27] Mary Prior is equally explicit: 'Whately's achievement is not so much in logic as in moral metalogic; he explained what logicians should have been doing.' [M. Prior, 1967, 287]

The conclusion that Whately is to be counted among the "old" logicians also gains support from the view that he had little impact on subsequent events in logic. Mary Prior, for instance, says that

> Between 1826, the year Whately's *Elements of Logic* was published, and 1860, George Boole, De Morgan, and John Stuart Mill were writing. It is therefore natural to expect to find adumbrations of their work in Whately, but in his systematic and formal treatment of logic there are remarkably few. [M. Prior, 1967, 287]

I suggest, on the contrary, that there *are* adumbrations of the work of later logicians in Whately, but they are in the context in which he locates the subject rather than in his 'formal treatment of logic'.

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[23] [Hamilton, 1833, 127]

[24] See [Hamilton, 1833, 135].

[25] He is mentioned, for instance, neither in the Kneale's *Development of logic* [1961], nor in Bochenski's *History of formal logic* [1961]. Also interesting is the fact that while Louis Liard [1878], and C. I. Lewis [1918], mention Bentham's commentary on *Elements*, neither mentions *Elements* itself.

[26] A conclusion drawn also in two theses which deal with Whately. See [Jongsma, 1983], and [Brody, 1967].

[27] [A. Prior, 1967, 541]

5 WHATELY'S ROLE IN THE DEVELOPMENT OF LOGIC

There are those (most noticeably Thomas Kuhn in the mid 20^{th} century) who think that change in a science is exclusively the result of transitions between its major theories. Richard Whately, among others, stands as a counterexample to such a view. He is one of those individuals who have an affect on a science without making a direct addition to the theories at its core. In this sense, his contribution is more like Francis Bacon's in the 17^{th} century, or Whewell's or Mill's in the 19^{th}, than that of Newton or Frege. Influence of Whately's sort on a science is often all but invisible; By our standards, his conception of the broader theory of logic is so much like our own that his role in its inception is easily forgotten. Indeed, the extent of his contribution is obvious only in comparison with earlier conceptions of the subject. When that comparison is made, however, Whately stands out as having had prescient insight into what logic would eventually become.

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THE LOGIC OF SIR WILLIAM HAMILTON: TUNNELLING THROUGH SAND TO PLACE THE KEYSTONE IN THE ARISTOTELIC ARCH

Ralph Jessop

1 BACKGROUND AND INTRODUCTION

> Every *learner* in science, is now familiar with more truths than Aristotle or Plato ever dreamt of knowing; yet, compared with the Stagirite or the Athenian, how few, even of our *masters* of modern science, rank higher than intellectual barbarians! (Hamilton, 'Philosophy of Perception', p. 40).[1]

The Enlightenment initiated the modern world as the product of a hitherto unsurpassed devotion to reason and scepticism. The scientific successes, ideals of progress, material advancement, hopes of social amelioration and freedom of the Enlightenment, were accompanied by catastrophic failures, conspicuous atrocities perpetrated in the name of reason and authority, and increasing fears of a dreadful new age of barbarism. Several Western countries incurred massive rifts, upheavals, wars, and profound societal changes that impinged upon or were feared to be the results of Enlightened thought. Following the shock-waves of the American and French revolutions, as some of the first effects of the industrial revolution were beginning to be felt, divisions at the heart of the Enlightenment between reason and scepticism resurfaced in varying guises in England, France, Germany, and Scotland. During the 18^{th} century Scotland had undergone major economic and political changes that both weakened the country's autonomy and yet liberalised its intelligentsia in ways that helped foster that great flourishing of intellectual talent we now call the Scottish Enlightenment. This intellectual movement laid the groundwork for so many succeeding cultural and material changes across the world. A number of its leading lights were members of the University of Glasgow.

[1] Sir Willam Hamilton, *Works of Sir William Hamilton*, with an introduction by Savina Tropea, 7 vols (Bristol: Thoemmes Press, 2001), vol. 1, *Discussions*. All references to Hamilton's articles are to this reprint edition and in this form. Page numbers correspond to, *Discussions on Philosophy and Literature, Education and University Reform*, 2nd edn (London: Brown, Green and Longmans; Edinburgh: MacLachlan and Stewart, 1853).

Handbook of the History of Logic. Volume 4: British Logic in the Nineteenth Century.
Dov M. Gabbay and John Woods (Editors)

It was here, in one of the homes in the Professors' Court of this University that William Stirling Hamilton was born on 8 March 1788.

The cultural antecedents within Hamilton's family background are interesting. In the late 17^{th} century, two ancestors were leading Covenanters; in the 18^{th}, several became somewhat distinguished academics. One of Hamilton's namesakes became the Professor of Divinity and later Principal of the University of Edinburgh — intriguingly he 'acquired a high reputation [...] for theological erudition' (Veitch, p. 5).[2] But it was in medicine that Hamilton's direct male ancestors excelled. His grandfather, Thomas Hamilton, a professor of medicine at the University of Glasgow, was fairly close to some of the more eminent medics at the University, such as William Cullen, Joseph Black, and William and John Hunter. However, Thomas was also frequently in the company of Adam Smith and James Watt, since not only were they connected through their respective roles within the University, but they were also members of the literary Anderston Club, presided over by the classical scholar and, to some extent still renowned, Professor of Mathematics, Robert Simson. Hamilton's father followed in his own father's academic footsteps but, having been Professor of Anatomy from his early 20s, he died young, aged just 31.

William Hamilton's academic lineage, the mainly Glasgow-based Enlightenment figures of his father's and grandfather's acquaintance, and the general educational ethos contributed to by a good number of the University's alumni and professors during the century of Hamilton's birth, probably played important roles in helping to mould the academic he would later become. Certainly it does seem as though Hamilton looked back into his past and may have found there sources of inspiration with regard to his somewhat pugilistic critical approach to philosophy, his legendarily extensive erudition, distinctive and in many ways exemplary pedagogical style, understanding of the nature of philosophy, and (in the works of Thomas Reid and Dugald Stewart) subject matter of extensive later study. Particularly with regard to Reid (who was the Professor of Moral Philosophy at Glasgow and is generally recognised as the founding father of the Common-Sense school), Hamilton's own development of Reidian philosophy is a significant factor that I shall briefly return to later. However, although Hamilton's intellectual inheritance from mainly Glasgow-based Enlightenment scholars must have helped shape his intellect, the educative role of his mother, Elizabeth Hamilton, should not be forgotten.

Hamilton was just 2 years old when his father died. His mother played a crucial role in his educational development. Elizabeth probably imbued in him a great keenness to excel, while balancing against this her various attempts to ensure that he did not develop too fast by, for example, returning him to school education in England following a period at Glasgow University and affording him ample leisure time during vacations to enjoy various physical pursuits and the companionship of other boys and his younger brother Thomas (who later gained some fame as a writer and the author of the novelistic account of pre-industrial Glas-

[2] John Veitch, *Memoir of Sir William Hamilton* (Edinburgh and London: Blackwood, 1869).

gow, *Cyril Thornton* (1827)).[3] Hamilton's early childhood and overall educational experiences under the supervision of a strong mother were therefore, as it were, the complete obverse of the deeply unpleasant regime so notoriously inflicted by James Mill and Jeremy Bentham on John Stuart Mill, the man who would later become Hamilton's greatest antagonist (notably some nine years after his death). Mill's one-time famous attack on Hamilton in his longest philosophical work, the *Examination of the Philosophy of Sir William Hamilton*, still stands far in excess of virtually any other attempt to disparage his standing as a philosopher.[4] Since Mill, most commentators who in one way or another berate Hamilton, either merely add minor footnotes to Mill's *Examination* or uncritically accept his authority. Since Mill's *Examination* — peppered with numerous misreadings of Hamilton — is in so many ways misleading, it deserves a thorough critical reassessment which I have not judged to be appropriate or even possible within the scope of this chapter.[5]

Hamilton's principal biographer, John Veitch, claims that 'no son could cherish greater regard or a more loyal affection for a mother than he did' (Veitch, p. 12). However, Hamilton's early letters to his mother often suggest a surprisingly direct and at times high-handed manner towards her (see Veitch, pp. 25-6). Fiery, imperious, peremptory, Hamilton's style of writing in these letters possibly indicates a certain fierceness of temperament tolerated or even enjoyed by his mother. Several of his somewhat more mature letters suggest increasing tenderness towards her (see letter to his mother, dated 27 November 1807, Veitch, p. 31). By the time of her death in 1827, which profoundly affected him, Hamilton had lived with her for almost his whole life and there can be little doubt that he was deeply attached to her (Veitch, pp. 134-5). Veitch claims that Hamilton wrote to his mother 'with the familiarity of an equal in point of years, without reserve, and often strongly' but he credits her with having been conscious of 'those qualities of mind which became afterwards so remarkable' and he accords to Elizabeth the praise of resolving 'to give him every advantage of education which lay in her power' (Veitch, p. 27).

Hamilton's early school education was mainly at the Glasgow Grammar School, followed by a brief spell in the Latin and Greek classes at the University of Glasgow in 1800. He was at this time just twelve years old. Though it was not uncommon for boys to attend Scottish universities at such a young age, Hamilton was certainly

[3] Thomas Hamilton, *The Youth and Manhood of Cyril Thornton*, ed. by Maurice Lindsay (Aberdeen: The Association for Scottish Literary Studies, 1990).

[4] John Stuart Mill, *An Examination of Sir William Hamilton's Philosophy and of the Principal Philosophical Questions Discussed in his Writings* (London: Longman, Green, Longman, Roberts & Green, 1865).

[5] For one attempt to reconcile the logics of Mill and Hamilton prior to Mill's *Examination*, see, [Alexander Campbell Fraser], 'Province of logic and Recent British Logicians', *North British Review*, 33 (Nov. 1860), 401-427. There were also several attempts to defend Hamilton following Mill's attack — for example, see, [Alexander Campbell Fraser], 'Mill's Examination of Sir William Hamilton's Philosophy', *North British Review*, 43 (Sept 1865), 1-58. For a brave attempt to defend Hamilton against Mill, though not specifically addressing his logic, see Dallas Victor Lie Ouren, 'HaMILLton: Mill on Hamilton: A Re-examination of Sir William Hamilton's Philosophy' (unpublished doctoral thesis, University of Minesota, 1973).

on the younger side of the norm. But Elizabeth Hamilton decided against William continuing his studies at Glasgow in the following academic session and he and Thomas were therefore moved in 1801 to schools in England. After returning to Scotland both boys entered the University of Glasgow in 1803, where William seems to have performed well in Latin, though 'In the classes of Logic and Moral Philosophy Hamilton was greatly distinguished, having in each carried off the highest honour of the year, which was then [...] awarded by the votes of the members of the class' (Veitch, p. 21).

Hamilton's most notable teacher from this time was Professor George Jardine (1742-1827) in the Logic class. Jardine's teaching made a lasting impression on him (Veitch, p. 21). As his studies continued, he studied medicine from 1804, studying botany and anatomy from 1805. His medical studies continued at Glasgow throughout 1806 and, during the winter of 1806-7, at Edinburgh. However, his book purchases from around this time included a fairly broad array of philosophical, medical, and historical works (see Veitch, p. 24). Though greatly impressive by today's academic standards for undergraduates, the breadth of his reading was very much in line with the generalist nature of Scottish educational practice and was not particularly atypical of other students who would later become eminent scholars and writers. By the time of Hamilton's death he had amassed some ten thousand volumes, around eight thousand of which were purchased by the University of Glasgow where they are currently held in a special collection. Within this collection there are about one hundred and forty editions of Aristotle's works and a good number of the texts he reviewed, some of which display neat manuscript marginalia at times evincing a peculiar degree of care in, for example, comparing earlier and later editions of works by Archbishop Richard Whately — one of the Oxford logicians whom Hamilton repeatedly criticised.

If by this time Hamilton was beginning to distinguish himself as a student of marked ability at the University of Glasgow, probably the most conspicuous educational advantage Elizabeth bestowed upon her son, was her determination that he should complete his university education at Oxford. In 1807 he secured a Snell Exhibition and entered Balliol College where he continued his studies until taking his Bachelor of Arts in 1810 — he of course obtained a First. His Oxford days seem to have been highly stimulating — certainly he made many acquaintances during this period and he read voraciously. The vast extent of Hamilton's learning became somewhat legendary from around the time of his final examination at Oxford. According to one account, 'He allowed himself to be examined in more than four times the number of philosophical and didactic books ever wont to be taken up even for the highest honours [...]. Since that time [...] there has been no examination in this University which can be compared with his in respect to philosophy' (Veitch, p. 60). However, his first career was not in philosophy but instead in law.

He became a member of the Bar in 1813 and having returned to Edinburgh he lived with his mother and his cousin, whom he later married in 1829. His wife was a devoted companion and without her hard work as an amanuensis, perhaps little of Hamilton's lectures would have survived. Though as yet we know too little about her, Lady Hamilton must have been or become through her marriage to William, one of those many women during the 19^{th} century whose knowledge of literature and philosophy far excelled the attainments accredited to them by posterity. Now that Hamilton was an advocate attempting to make a living at Edinburgh, with rather too few cases to attend to, he began to investigate his family history, though not as a light hobby but with a real purpose. Family tradition had it that William was an indirect descendant of Sir Robert Hamilton of Preston, a staunchly fierce Covenanter who died in 1701, after which time the baronetcy was not assumed by the heir and from thenceforth had lapsed into a mere family memory. After three years of research Hamilton finally presented a case to the Edinburgh Sheriff which proved that he was the heir-male to his Covenanting ancestor. Henceforth, William Stirling Hamilton became Sir William Hamilton, Baronet of Preston and Fingalton (Veitch, p. 69). This may seem a curious moment in Hamilton's personal history but no doubt he was motivated by several practical considerations, not least of which must have had to do with social and career advancement. From what I can gather from the occasionally sketchy accounts of his life by Veitch and Monck, Hamilton had sufficient employment as a lawyer but was only moderately successful: law was 'but a secondary pursuit for him' and instead he haunted the Advocates library with the bibliophilic zeal of an antiquarian (Veitch, p. 75).[6]

If he was less suited to the law than he might have been, his politics were also an obstruction to great material success since he was a Whig, the ruling party of the day Tory. As Veitch assesses Hamilton's politics, he was 'a man of progress' and liberal principles, though little if at all involved in party politics of any kind (Veitch, p. 78). Of course he knew and socialised with many of the leading Scots of the day, Sir Walter Scott, Thomas de Quincey, Francis Jeffrey, J.G. Lockhart, Macvey Napier, and many others, but he also had a fair number or European friends from Russia, France, and Germany. He visited Leipzig in 1817 and again travelled to Germany in 1820, visiting libraries in Berlin and Dresden. He was largely instrumental in the Advocates Library's purchase of an extensive collection of valuable German works. His interest in German literature and philosophy, which would later grow to unrivalled proportions among his contemporaries, dates from around this time.

Also in 1820 Hamilton applied for the Moral Philosophy professorship at Edinburgh University. Although he had strong support for this chair, not least of all from the elderly Dugald Stewart, John Wilson (better known as the famous 'Christopher North' of *Blackwood's Magazine*) secured the post due to the Town Council's patronage, though he was by no means a suitable candidate. In many ways this was quite scandalous and seems to have been entirely due to Hamilton's Whig politics (Veitch, pp. 96-103). Hamilton had to settle for a poorly paid Professorship of Civil History to which he was appointed in the following year

[6] W.H.S. Monck, *Sir William Hamilton* (London: Sampson Low, Marston, Searle & Rivington, 1881).

and he had to wait for a further 15 years before he secured a post in Philosophy. Finally, in 1836, though with only a small majority over the other candidate, he was elected to the Chair of Logic and Metaphysics at the University of Edinburgh. Veitch gives a fairly thorough account of Hamilton's appointment and the strong testimonials that supported him, but the narrowness of his majority and his failure to secure the earlier appointment of the Moral Philosophy chair indicate that University appointments in Scotland were handled in an altogether shameful manner — Hamilton was without doubt one of the most philosophically erudite and talented men in Scotland at that time but party politics, personal preferences, and unfounded doubts about his religious beliefs were allowed to prevail. In some ways little had changed since the more understandable yet equally non-academic rejection of David Hume by the University of Glasgow in the previous century, nor since the huge debacle that erupted in 1805 when John Leslie was accused of being an infidel and as a result nearly failed to secure the Mathematics Chair at Edinburgh in 1805 because he had endorsed Hume's theory of causality (see Veitch pp. 183-210).[7]

Though there is much to relate about Hamilton's life from this time on, like many scholars and dedicated teachers he led an industrious and comparatively uneventful life, though not unmarred by damaging vicissitudes, such as the deaths of his son in 1836, his brother Thomas in 1842, and a daughter in the winter of 1844-5. From the time of his appointment to the Logic and Metaphysics Chair in 1836 until around the mid 40s, Hamilton was clearly working far too hard and under a great deal of personal strain. Then, in July 1844, aged 56, he suffered a physically debilitating stroke that partially paralysed him for the rest of his life. Two years later he became embroiled in a controversy with Augustus de Morgan concerning their respective quantification systems. Academically, this is undoubtedly the most troublesome and embarrassing moment in Hamilton's career and several have agreed that he behaved rather foolishly. According to William Kneale, Hamilton was 'a pedantic Scottish baronet' who was 'properly ridiculed by De Morgan'.[8] General opinion about the affair has been that de Morgan came out on top. [9] Possibly the intensely desperate times of the mid 40s in Britain generally, a prevailing sense of crisis in Scotland following the massive upheaval due to the disruption of the Scottish Kirk in 1843, the various bereavements Hamilton had suffered, and his loss of physical vigour may be factors that ought to be considered. Hamilton may have acted in an imperious but also a somewhat desperate manner towards de Morgan but one cannot help but wonder whether his judgment merely faltered amidst a context of great personal and social difficulties.

The details of the de Morgan controversy are tediously complex, the complexity exacerbated by Hamilton's forensic analyses of the events and his sustained suspi-

[7]On Leslie see, Ralph Jessop, *Carlyle and Scottish Thought* (Basingstoke: Macmillan, 1997), pp. 36-9.

[8]William Kneale, 'Boole and the Revival of Logic', *Mind*, 57 (Apr. 1948), 149-75 (p. 152n).

[9]For example, see, William Kneale and Martha Kneale, *The Development of Logic* (Oxford: Clarendon, 1962), pp. 353-4.

cions over some years that de Morgan had plagiarised or at least was in some way trying to steal all credit from him. This issue lasted for many years well beyond Hamilton's death and although Hamilton backed down considerably from his first accusations of plagiarism in 1847, neither men really gave up. But, as de Morgan's fame rather prospered as a result of the conflict, Hamilton's prestige diminished. Peter Heath has analysed and recounted this controversy in admirable detail and therefore I shall defer to his general understanding of the whole affair, if not to his tendency to agree that de Morgan's assessment of Hamilton's quantification is right.[10] Some, though perhaps not all, of the relevant letters and other documents were collected together by Hamilton.[11] In very general terms, it would seem that both de Morgan and Hamilton did not fully understand each other's respective positions and arguably 'the two systems are not only distinct from, but opposed to each other.'[12] As I shall argue later in agreement with Robert Fogelin, de Morgan was mistaken concerning a fundamental point and Hamilton's system can thus be shown to be consistent and much more robust than many who subscribed to de Morgan's standpoint have assumed it to be. One of the most fruitful and important outcomes of the controversy was the effect it had on George Boole whose interest in and subsequent mathematization of logic was in no small part inspired by the rather public disagreement between de Morgan and Hamilton.[13] Heath insightfully remarks of the de Morgan controversy, that de Morgan's notes 'contain the following, which might well serve (and was perhaps so intended) as an epigraph for the whole encounter: "Two French squadrons at B — cannonaded each other — why? Because each took the other for *Russians*. 'Then why did they fight?' Said a little girl'.[14]

Although Hamilton was not affected mentally by the stroke he suffered in 1844 and managed to continue in his Chair at Edinburgh for a good many years after, it is fair to say that he was greatly impeded by this disablement. Indeed, it is arguable that his standing as a philosopher may have suffered more from this than from Mill's *Examination*, since one of the greatest problems in studying Hamilton's works has always been simply this: he produced no *magnum opus* either in metaphysics or logic. It is reasonable, if yet somewhat whimsical, to say that, had he not been struck down in 1844 he would have at some stage during his remaining years brought his philosophical endeavours together in at least one definitive and fully mature volume. But this did not happen and after his paralysis he produced relatively little until his death at Edinburgh on 6 May 1856, aged 68.

Although Hamilton did not produce a fully definitive work on metaphysics or on logic, he nevertheless did write rather extensively, producing material which, in the

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[10] Peter Heath, 'Introduction' in, Augustus de Morgan, *On the Syllogism and Other Logical Writings* (London: Routledge & Kegan Paul, 1966), vii-xxxi (pp. xii-xx).

[11] *Miscellaneous Writings and Correspondence*, *Works*, vol. 7.

[12] [H.L. Mansel], 'Recent Extensions of Formal Logic', *North British Review*, 15 (May 1851), 90-121 (p. 95n).

[13] See, Luis M. Laita, 'Influences on Boole's Logic: The Controversy between William Hamilton and Augustus De Morgan', *Annals of Science*, 36 (1979), 45-65 (p. 61; p. 65).

[14] Heath, p. xvi.

recently reprinted edition of his works by Thoemmes Press, fills seven volumes. In addition to this, he also produced an extraordinarily, many would say excessively footnoted edition of the *Works of Thomas Reid* in which he first published his fragment on Logic, the 'New Analytic of Logical Forms' in 1846.[15] His footnotes in Reid's Works are at times quirky and needlessly pedantic. However, by sharp contrast with this, towards the end of his life he produced a much less cumbersome and indeed rather elegant edition of the *Works* of Dugald Stewart.[16] However, long before these scholarly works Hamilton contributed a series of substantial articles for the *Edinburgh Review* all of which, despite his failing health, he managed to publish in a single volume entitled *Discussions on Philosophy and Literature, Education and University Reform* in 1852. The first of these essays, the 'Philosophy of the Unconditioned' (1829) immediately struck readers as largely unintelligible or overly philosophically sophisticated, mystical even in its complexity[17] — but it was this article that properly launched Hamilton's career and first made him famous as the first truly eminent Scottish philosopher since Dugald Stewart who, after many years of ill health, had died in the previous year.

In 'Philosophy of the Unconditioned' Hamilton reviews the work of Victor Cousin and his attempt to establish an eclectic philosophy of the Infinito-Absolute. The main thing to note here is that Hamilton constructs his Law of the Conditioned, a law prescribing the domain of positive knowledge or the realm of what may be said to be knowable — in some ways it might be regarded as a forerunner of Ayer's logical positivism, though Hamiltonian positivism is a far cry from rejecting either metaphysics or theology. A full account of this article is not appropriate here — suffice to say that Hamilton argues that all knowledge lies in a mean between two extremes of unknowables or inconditionates, which is to say all things that may be described as absolute or as infinite comprise the boundaries of knowledge and are thus strictly incomprehensible to us. The absolute and the infinite are posited as contradictories, neither of which can be positively construed to the mind but one of which, on the basis of the laws of excluded middle and non-contradiction, must obtain — though which of the two obtains is incognisable. Hence, Hamilton inaugurates what I call his doctrine of *nescience*, or *learned ignorance as* the 'consummation' of knowledge ('Philosophy of the Unconditioned', p. 38). As all thought, and hence all knowledge, is conditional, of the plural, phenomenal, limited, Hamilton declares: '*To think* is *to condition*; and conditional limitation is the fundamental law of the possibility of thought' ('Philosophy of the Unconditioned', p. 14).

Educated in philosophical discourses of the much more sedate and even-tempered stateliness that typified so much earlier 18^{th} century and contemporary philosoph-

[15] Thomas Reid, *The Works of Thomas Reid*, preface, notes, and supplementary dissertations by Sir William Hamilton (Edinburgh: MacLachlan, Stewart; London: Longman, Brown, Green and Longmans, 1846).

[16] *The Collected Works of Dugald Stewart*, ed. by Sir William Hamilton, 11 vols (Edinburgh: Thomas Constable, 1854-60).

[17] 'Francis Jeffrey to Macvey Napier, 23 November, 1829', in Macvey Napier [Jnr.], ed., *Selections from the Correspondence of the Late Macvey Napier* (London: Harrison, 1877), p. 68.

ical prose from Francis Hutchison, to Hume, Smith, Reid and Dugald Stewart, few at the time may have been able to understand what Hamilton wrote. His highly concentrated style, densely philosophical critical argumentation, frequent references to various German philosophers including Kant, Fichte, and Hegel — though all this must have dazzled and excluded many, few can have failed to catch the sense of excitement that pervaded 'Philosophy of the Unconditioned'. But this must have been more the case with his second major article 'Philosophy of Perception' (1830) which was written in an even more vigorous and racy style than his first. He now turned his attention to scepticism, principally to the scepticism of Hume, though one could be forgiven for missing this since more prominently 'Philosophy of Perception' launches an extraordinarily vitriolic attack on the man who had gained such a high reputation as a philosopher and who for many years continued to be admired by established English philosophers and writers such as John Stuart Mill and Leslie Stephen and whose job, following his death, Hamilton failed to get in 1820 — Thomas Brown. Hamilton savaged Brown for leading philosophy back into the morass of Humean scepticism and for fundamentally failing to grasp the true import of Thomas Reid's critical philosophy of Common Sense, which, according to Hamilton, had given a successful, if nonetheless relatively unsophisticated answer to Hume.

So much needs to be said about this article but I shall confine myself to just a few points: in 'Philosophy of Perception' Hamilton develops his own version of Common Sense philosophy in the form of a doctrine of perception which holds that in the *act* of perception the self and the not-self were instantaneously revealed in one indivisible moment of cognition — he calls this theory of perception *natural dualism* or *natural realism* and maintains that, contradistinguished from all *representationist* theories of perception that in one way or another tend towards scepticism, *natural dualism* is a theory of immediate or *presentative* perception. Leaving aside all consideration of just how *natural dualism* was proffered by Hamilton as the best and most successful counter-argument to what some may think of as a straw man scepticism of purely theoretic indeterminacy — the absolute scepticism of Hume — I want to draw attention to Hamilton's non-Kantian notion of the relativity of knowledge by means of just one quotation:[18]

> Relatives are known only together: the science of opposites is one. Subject and object, mind and matter, are known only in correlation and contrast [. . .]. Every conception of self, necessarily involves a conception of not-self: every perception of what is different from me, implies a recognition of the percipient subject in contradistinction from

[18]Hamilton has often been mistakenly thought of as borrowing heavily from Kant. However, in several places he is critical of Kant and his relativity of knowledge needs to be distinguished from Kant's. On this see, Manfred Kuehn, 'Hamilton's Reading of Kant: A Chapter in the Early Scottish Reception of Kant's Thought', in George MacDonald Ross and Tony McWalter, eds, *Kant and His Influence* (Bristol: Thoemmes Antiquarian Books, 1990), 315–347 (pp. 333–45). It should also be noted that Hamilton's notion of the relativity of knowledge is more subtle and more complex than Mill represents it as being. On this see, John Veitch, *Hamilton* (Edinburgh And London: Blackwood, 1882), pp. 201–222.

> the object perceived. [...]. In Perception, as in the other faculties, the same indivisible consciousness is conversant about both terms of the relation of knowledge. ('Philosophy of Perception', pp. 50–51).

Although the details of Hamilton's theory of the relativity of knowledge deserve separate examination, in the above we can see indications of a notion that is of critical importance to understanding Hamilton's logic, namely, the notion that subject and object exist only in correlation with one another, such that opposites, or the distinguishable terms of subject and predicate in a given proposition, may be thought of as being held together in a relationship of equation or non-equation — in this lies the germ of Hamilton's emphasis on the relativity of concepts and his quantification of the predicate.

The influence that Hamilton's two articles on the unconditioned and perception had on Victorian thought is also a subject that deserves separate study. Noah Porter, professor of moral philosophy and metaphysics (1846), and later president of Yale, wrote effusively and at some length in Veitch's biography of the considerable extent of Hamilton's influence on American students (Veitch, pp. 421-8). However, although Porter thought that Hamilton had been a positive religious force in American thought, there is another side to this story. Emphasising the vastness of our ignorance in 'Philosophy of the Unconditioned', while regarding this as a prompt for our wonderment and faith, the most significant direct and lasting effect of the 'Philosophy of the Unconditioned' is perhaps best assessed in terms of the influence it had upon Henry L. Mansel, a prominent follower of Hamilton who wrote several articles defending Hamilton's logic, developed his own version of it in his *Prolegomena Logica* where he specifically acknowledges his debt to Hamilton, and, more popularly, in his Bampton Lectures, gave rise to a doctrine of Christian Agnosticism.[19] But as the agnostic movement developed during the 19^{th} century, Hamilton's 'Philosophy of the Unconditioned' in comparison with its transmutation into Mansel's Christian Agnosticism can also be seen as having a profound effect on anti-Christian agnostics such as Thomas Huxley (Darwin's bulldog). As Sheridan Gilley and Ann Loades nicely put it: 'Huxley saw in Mansel the suicidally honest theologian, sitting on an inn sign and sawing it off.'[20] Hamilton's importance to the growth of agnosticism, although not widely known, has certainly been established not only by more recent scholarship but also in some of the earlier responses to Hamilton.[21] Hence, though firstly, inspiring a new religious piety and apparent salvation from scepticism, but secondly, becoming infused into

[19]For example, see Henry Longueville Mansel, *Prolegomena Logica: An Enquiry into the Psychological Character of Logical Processes* (Oxford, 1851) pp. xi–xii; 'The Philosophy of the Conditioned: Sir William Hamilton and John Stuart Mill', *Contemporary Review*, 1 (1866), 31-49; 185-219; [Bampton Lectures], *The Limits of Religious Thought* (London : John Murray, 1858).

[20]Sheridan Gilley and Ann Loades, 'Thomas Henry Huxley: The War between Science and Religion', *The Journal of Religion*, 61 (1981), 285-308 (p. 297).

[21]Bernard Lightman, *The Origins of Agnosticism: Victorian Unbelief and the Limits of Knowledge* (Baltimore and London: John Hopkins University Press, 1987), p. 16; Robert Flint, *Agnosticism* (Edinburgh and London: Wm Blackwood, 1903).

succeeding waves of religious doubt and the growth of agnostic principles more powerfully damaging to orthodox belief than even Hume's or Voltaire's more full-frontal atheistic attacks on religious belief, the full significance that Hamilton's 'Philosophy of the Unconditioned' would come to have on Victorian society, philosophy, literature, and culture, was nothing short of immense.[22]

But Hamilton's influence can also be seen through certain of his friends or acquaintances and, of course, his students. One of his former students, who was profoundly influenced by him, became the Professor of Moral Philosophy at the University of St. Andrews, James Frederick Ferrier, a philosopher whose personal and philosophical connections with Hamilton run deep and whose work is now just as undeservedly but even less well known than that of Hamilton. In literature, Hamilton was a major influence on E. S. Dallas whose two volume work of literary theory, *The Gay Science,* positively teems with Hamiltonian philosophy — but Dallas has also now shrunk into the shadows and is barely known.[23] George Davie claims that Hamilton inspired a number of brilliant scholars including Ferrier and the physicist so greatly admired by Einstein, James Clerk Maxwell.[24] The Maxwell connection is particularly interesting since a recent work of literary criticism has opened up a whole new field of study — the Victorian relativity movement. Christopher Herbert argues in *Victorian Relativity* that the cultural and philosophical antecedents of Einstein's special theory of relativity are to be found in a succession of Victorian philosophers and thinkers, the writings of whom have been largely neglected for over a century. Interestingly, just as scholars who have written on agnosticism have traced its origins in the 19th century back to Hamilton, Herbert also finds in Hamilton's theory of the relativity of knowledge a significant starting point for his study, seeing Hamilton at the beginning of a major Victorian re-invention of the fundamentally Protagorean relativist rejection of absolutism.[25] First suggested in 'Philosophy of the Unconditioned', developed in 'Philosophy of Perception', and, as we shall see, a key component of Hamilton's logic, Hamiltonian relativism intriguingly re-positions Hamilton as one of the 19th-century's key *avante garde* thinkers.

It is too easy to see Hamilton in caricature and as of marginal importance, since this is where the later Victorians placed him and where scholars of the 20th century left him. However, for all that he was hampered by a corrupt system of university patronage and indeed may not have been the most self-promoting of figures, within the context of British philosophy and literature of the 1830s,

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[22]See, Ralph Jessop, 'Carlyle's Agnosticism: An Altar to the Unknown and Unknowable God', *Literature and Belief,* 25 (1&2) (2005), 381-433 (pp. 395-404).

[23]E.S. Dallas, *The Gay Science* (London: Chapman and Hall, 1866).

[24]See, George Elder Davie, *The Democratic Intellect: Scotland and Her Universities in the Nineteenth Century*, Edinburgh University Publications, History Philosophy & Economics: 12 (Edinburgh: Edinburgh University Press, 1961; repr. 1982), p. 261; John Hendry, *James Clerk Maxwell and the Theory of the Electromagnetic Field* (Bristol and Boston: Adam Hilger, 1986), p. 112.

[25]Christopher Herbert, *Victorian Relativity: Radical thought and Scientific Discovery* (Chicago and London: The University of Chicago Press, 2001), pp. 35-7.

Hamilton was a leading and highly dynamic writer who played an important part in re-invigorating Reidian philosophy while also turning British attention arguably in a different but by no means unrelated direction towards German philosophy. It must be noted that in early 19^{th}-century Britain very few people could read German and Hamilton was one of almost a handful of intellectuals during the 1820s and 30s who had any direct knowledge of the works of Kant and other German philosophers such as Hegel. Hamilton was one of a tiny number of Germanizers who promoted the study of German philosophy and, given the rise of neo-Kantian and neo-Hegelian philosophy in Britain amongst philosophers who owed some debt to Hamilton, in this alone his influence was probably much more profound than has generally been appreciated.

Another major Germanizer was Hamilton's one-time friend, the literary man of letters, Thomas Carlyle, who knew Hamilton from at least the late 1820s. Carlyle, though a literary giant of the Victorian period and well beyond, of proportions it would be difficult to exaggerate, is yet another figure who is now increasingly little known and poorly understood. Not much is positively known about their friendship but at the time of its publication in 1829 Carlyle read Hamilton's 'Philosophy of the Unconditioned' with admiration and in later life he wrote a highly reverent Reminiscence of Hamilton which is published in Veitch's biography, followed by a fairly intimate letter from Carlyle written in 1834 in which he says, 'Think kindly of me; there are few in Scotland I wish it more from' (Veitch, pp. 121–7). There are undoubtedly many interesting parallels between Carlyle and Hamilton, some of which were detected in Carlyle's own lifetime by his close friend David Masson.[26] I have discussed some of the interconnections between Carlyle and Hamilton elsewhere.[27]

Hamilton produced a good number of other articles for the *Edinburgh Review*, including the following selection: 'On the Revolutions of Medicine in Reference to Cullen' (1832); 'On the Study of Mathematics as an Exercise of the Mind' (1836) — controversially, though rather in keeping with his Scottish predecessors attitudes about mathematics, Hamilton did not think that mathematics was a good exercise of the mind as part of a liberal education and instead advocated philosophy while in several ways indicting the emphasis on mathematics at Cambridge;[28] 'On the Patronage and Superintendence of Universities' (1834); 'On the State of the English Universities with More Especial Reference to Oxford' (1831). Several of his articles were pointedly critical of Oxford and Cambridge and it must be said that Hamilton undoubtedly set himself up for retributive attacks from some scholars in response to his various denunciations of the established ancient universities of

[26]David Masson, *Recent British Philosophy: A Review with Criticisms including some Comments on Mr Mill's Answer to Sir William Hamilton*, 3rd edn (London: Macmillan, 1877),p. 69.

[27]For example see, Jessop, *Carlyle and Scottish Thought*, pp. 28-31. Also see Alex Benchimol, 'William Hamilton' in, *The Carlyle Encyclopedia*, edited by Mark Cumming (Cranbury, NJ:Farleigh Dickinson University Press, 2004), 207-9.

[28]For example compare, Richard Olson, 'Scottish Philosophy and Mathematics 1750-1830', *Journal of the History of Ideas*, 32 (1971), 29-44 (pp. 41-4).

England. More directly germane to our subject matter and also involving his first of many salvos against English logicians, is his 'Logic: The Recent English Treatises on that Science' (1833).

In this article he gave yet another virtuoso performance that at once established his reputation as one of the foremost logicians of his day. But, as with some of his other slashing remarks on contemporary scholars and, as he saw it, the present philistinism of learning in Britain, in 'Logic' he was unsparing in his treatment of several Oxonian logicians. Supposedly reviewing eight recent publications on logic, he focuses almost exclusively on Whately's *Elements of Logic*, making only brief reference to George Bentham's *Outline of a New System of Logic* (1827). As some commentators have noted, the cursory treatment of Bentham is odd since a system for quantifying the predicate is given in the work of this nephew of the more famous Jeremy Bentham.[29] Hamilton's paternity of this doctrine has often been called in doubt but since most of the specific characteristics of his quantification are largely, if not exclusively, peculiar to him, and since his own theory is so thoroughly grounded in a painstaking explication of the grounds for quantifying the predicate, I shall not engage with the tortuous historical complexities and shall instead attempt in the following sections to outline how Hamilton arrives at his quantification system and what that system itself is. Furthermore, it is likely that Hamilton was so dismissive of Bentham's *Outline* that he simply did not read all of the text and cast it aside in order that he might focus on the main logician, his principal target, Richard Whately.

Though Hamilton intimates a certain degree of respect for the natural abilities of the authors under review, he denounces their lack of genuine originality — the source of Hamilton's ire is clear and is a much repeated complaint elsewhere in his work, namely, the inadequacy of the authors' learning:

> None of them possess — not to say a superfluous erudition on their subject — even the necessary complement of information. Not one seems to have studied the logical treatises of Aristotle; all are ignorant of the Greek Commentators on the Organon, of the Scholastic, Ramist, Cartesian, Wolfian, and Kantian dialectic. ('Logic', p. 129).

And so he continues to cut and hew his way through the inadequate learning of contemporary Oxonians. But, as Whately's *Elements* stood pre-eminent, it is this work that Hamilton takes to task and he proceeds to make point after forensic point against Whately, all the time demonstrating his own vastly superior knowledge of swathes of the literature of logic and outlining several features of his own system of logic.

As part of his sustained attempt to claim priority in discovering his system of quantification, in a much later footnote in *Discussions* Hamilton asserts that on the basis of a rather vague authority — 'the tenor of the text'– the 'Logic' article shows that he 'had become aware of the error in the doctrine of Aristotle and the logicians, which maintains that *the predicate in affirmative propositions could only*

[29] Heath, p. xvi.

be formally quantified as particular' ('Logic', p. 162n). More strongly, and with much better justification, he writes in an appendix in *Discussions*, 'Touching the principle of an explicitly *Quantified Predicate*, I had by 1833 become convinced of the necessity to extend and correct the logical doctrine upon this point. In the article on Logic [...] the theory of Induction there maintained proceeds on a thoroughgoing quantification of the predicate, in affirmative propositions' (*Discussions*, p. 650). Certainly the 1833 'Logic' article does suggest this but it does not display the full quantification system (see 'Logic', p. 163). As I shall indicate later, although the quantification system was not fully articulated until the publication of his 'New Analytic' in 1846, a good deal of Hamilton's treatment of logic in his lectures does proceed painstakingly towards the quantification. That he was teaching this system during the late 1830s, several years ahead of de Morgan's quite different quantification, seems to have been generally agreed and is patently evident in the wonderfully clear account given by one of his students, Thomas Spencer Baynes, whose winning essay written for a competition set by Hamilton in 1846 was later published as *An Essay on the New Analytic of Logical Forms* in 1850.[30]

In 'Logic' Hamilton very much lays out his stall. He begins with a brief definition of logic that he later elaborates:

> Nothing, we think, affords a more decisive proof of the oblique and partial spirit in which philosophy has been cultivated in Britain, for the last century and a half, than the combined perversion and neglect, which Logic — the science of the formal laws of thought — has experienced during that period. ('Logic', p. 119).

Just a few years after writing this, Hamilton commenced his Lectures on Logic as the Professor of Logic and Metaphysics at Edinburgh. These were posthumously published in four volumes, two on Metaphysics in 1859 and two on Logic in 1860. In the *Lectures on Logic*, Hamilton spends a lot more time carefully explaining his above definition of logic as 'the science of the formal laws of thought'. Though his definition of logic in the lectures is interesting and deserves discussion, I shall merely summarise some main points here.

Hamilton basically holds that Logic proper or Formal Logic is Abstract logic, dealing only with necessary inference and devoid of all adventitious or extra-logical matter or contingent considerations. However, the nature of Logic as a pure science had been greatly misunderstood in Britain: 'Bacon wholly misconceived its character in certain respects; but his errors are insignificant, when compared with the total misapprehension of its nature by Locke' (LL.I.29).[31] The British had

[30]Thomas Spencer Baynes, *An Essay on the New Analytic of Logical Forms, Being that which Gained the Prize Proposed by Sir William Hamilton, in the Year 1846, for the Best Exposition of the New Doctrine Propounded in his Lectures*; with an Historical Appendix (Edinburgh: Sutherland and Knox; London: Simpkin, Marshall, and Co., 1850).

[31]All references in this form are to the *Lectures on Logic*, ed. by H.L. Mansel and John Veitch (Edinburgh and London: Blackwood, 1860), vols I or II; vols 5 or 6 in the Thoemmes reprint edition of Hamilton's *Works*.

mistakenly departed from a certain general agreement on the formal nature of Logic among ancient and more recent German logicians. They had ignorantly or perversely deviated from centuries of collective wisdom and while German logicians from the time of Leibniz had probably done more than most to further the science, Hamilton seems to have regarded the more recent and more serious perversions and confusions in the work of recent English authors, in particular Whately, as a sort of further entrenchment of the very kind of misconstrual most likely to hinder the science (LL.I.40). It was also reprehensible that such recent British scholars should be ignorant of the German logicians: 'Great Britain is, I believe, the only country of Europe in which books are written by respectable authors upon sciences, of the progress of which, for above a century, they have never taken the trouble to inform themselves' (LL.I.33).

Distinguishing between Special or Concrete Logic (logic in its particular applications or instantiations in the several arts and sciences) and Pure or General or Abstract Logic, for Hamilton, Pure Logic has to be contradistinguished from any particular subject or discipline, the object-matter of which must necessarily be contingent due to the nature of the topics it addresses or to which Logic is being in some sense applied or put into practice (LL.I.56). However, while Special Logic is dismissed this is not to say that practical matters to do with Logic must be altogether excluded from consideration (see LL.I.61-2). He coins the term 'Modified Logic' to describe what he argues had been improperly called Applied Logic by Kant and some other German philosophers, defining Modified Logic as 'a science, which considers thought not merely as determined by its necessary and universal laws, but as contingently affected by the empirical conditions under which thought is actually exerted' (LL.I.60). Although Hamilton's treatment of Modified Logic in the second volume of his *Lectures on Logic* as the correlative second main branch of Abstract Logic, is certainly interesting, I have chosen not to discuss it but instead focus on what for Hamilton was clearly of much greater immediate importance, namely, Formal or Pure Logic. He insists that Pure Logic really comprises the whole of Abstract Logic — Modified Logic is 'a mere mixture of Logic and Psychology'; 'There is in truth only one Logic, that is, Pure or Abstract Logic', 'Modified Logic being only a scientific accident, ambiguously belonging either to Logic or to Psychology' (LL.I.63).

With such points in mind, he provisionally defines Logic as 'the Science of the Laws of Thought as Thought' (LL.I.4). Extruding the contingent, inasmuch as this is possible, Hamilton claims that Logic is only concerned with those phenomena of formal (or subjective) thought that are necessary or 'such as cannot but appear' as opposed to the contingent phenomena of thought, or 'such as may or may not appear' (LL.I.24). Hence, through his final introduction of the notion of necessary laws being the sole province of Logic, Hamilton asserts that 'Logic, therefore, is at last fully and finally defined as the science of the necessary forms of thought' (LL.I.24). Though this is Hamilton's final definition of Logic, he takes care to explain the sense or 'quality' of 'necessary' by extensively quoting Wilhelm

Esser's *System der Logik*.[32] Hence, to summarize Esser's arguments as translated by Hamilton: the necessity of a form of thought is contradistinguished from contingency by being *subjective*, which is to say that a necessary form of thought 'must be determined or necessitated by the nature of the thinking subject itself' as opposed to being determined objectively; since we are incapable of conceiving the possibility of its non-existence, such incapacity warrants the notion that the form of thought in question is *original* to, or a constitutive feature of, the thinking subject or human mind, and thus the subjective necessity of a form of thought 'must be original and not acquired'; as necessary, subjective, and original in the sense explained, the form of thought that comprises the object-matter of Logic cannot necessitate on some occasions and not on others, and thus it must also be *universal*; and finally, further enriching the sense or quality of 'necessity', according to Esser, 'if a form of thought be necessary and universal, it must be a law; for a law is that which applies to all cases without exception, and from which a deviation is ever, and everywhere, impossible, or, at least, unallowed' (LL.I.24-5). With the sense of 'necessity' within the phrase 'necessary forms of thought' hereby explained in terms of something subjectively determined by the human mind as subject and not objectively determined or extraneous to the mind, original and not acquired, universal, and a law, Hamilton gives what he regards as his 'most explicit enunciation of the object-matter of Logic' as: 'Logic is the science of the Laws of Thought as Thought, or the science of the Formal Laws of Thought, or the science of the Laws of the Form of thought; for all these are merely various expressions of the same thing.' (LL.I.25-6).

Many more recent formal logicians would generally agree that formal Logic, as contradistinguished from informal Logic and the subject matter or topics of Rhetoric, is entirely or principally concerned with deductive arguments and thus with valid inference and the necessary laws that pertain to or determine validity. But we need to take notice of the state of logic in 1830s Britain as Hamilton understood it: his contemporaries appeared to him to be woefully misguided, unscholarly in their reading of the Aristotlean tradition, and blissfully ignorant of the more recent German tradition in logic since the time of Leibniz. Hamilton's peers were, as far as he could tell, largely unaware of how the recent German writers including and after Kant had far excelled the Oxford logicians in their knowledge and understanding of the subject. Furthermore, it seems clear that part of Hamilton's crusade was against the prevailing tendency of philosophy in Britain towards the increasingly prevalent mediocrity and barbarity of his times that almost inevitably was following in the wake of and was implicitly collusive with an era of rapidly advancing materialism. Pervaded from its outset by reason and scepticism, during the early decades of the 19^{th} century the Enlightenment was evolving into new forms of a more socially pervasive utilitarian rationality and sceptical subversions of faith. Acutely conscious of such trends in philosophical

[32] Hamilton translates for his students many passages from Esser. His inclusion of these numerous quotations implies the commencement of a substantial realignment in British philosophy with German logicians.

discourse Hamilton took considerable pains to establish his standpoint as one much more closely in line with the German approach that regarded Logic as a pure science.

2 DOCTRINE OF CONCEPTS

> Logic [...] is exclusively conversant about thought,–about thought considered strictly as the operation of Comparison or the faculty of Relations; and thought, in this restricted signification, is the cognition of any mental object by another in which it is considered as included,– in other words, thought is the knowledge of things under conceptions. (LL.I.40).

Shortly after the above quotation Hamilton distinguishes between the act of conceiving (conception) and the thing conceived (concept) and briefly suggests a similar distinction with regard to perception, somewhat tentatively coining the term 'percept' some 40 years before its first recorded use in the *Oxford English Dictionary.* He takes particular care to point out that Logic is concerned with 'thought considered as a product; that is, as a concept, a judgment, a reasoning' (LL.I.74). The operation of Comparison, or as he later calls it, the Faculty of Comparison, produces Concepts, Judgments, and Reasonings. However, Hamilton argues that Concepts and Reasonings are modifications of Judgments, 'for the act of judging, that is, the act of affirming or denying one thing of another in thought, is that in which the Understanding or Faculty of Comparison is essentially expressed' (LL.I.117). This means that for Hamilton, 'A concept is a judgment' and as such it collects together or is 'the result of a foregone judgment, or series of judgments, fixed and recorded in a word,–a sign' which may be supplemented or extended by additional attributes, themselves judgments (LL.I.117). Thus, as a concept in a sense fixes a judgment or series of judgments, collecting together various attributes within a single term, so also can it be analysed into these components or amplified by the annexation of further attributes.

An important point to note here that will later be of fundamental significance to Hamilton's quantification of the predicate, is that for Hamilton an oft-ignored and oft-violated postulate or principle of Logic is that the import of the terms used in a judgment or reasoning should be fully understood or made explicit, which is to say that 'Logic postulates to be allowed to state explicitly in language all that is implicitly contained in thought', that Logic demands licence to make explicit the full import of any particular concept or term, much as it attempts to do in making overt all of the steps and relations involved in a given process of reasoning or argument. Though this fundamental postulate is simple in its statement, Hamilton clearly regarded its significance as central to his project to evolve or develop Logic as a Pure science: 'This postulate [...], though a fundamental condition of Logic, has not been consistently acted on by logicians in their development of the science; and from this omission have arisen much confusion and deficiency and error in our present system of Logic' (LL.I.114). Interestingly, in one place he makes the postulate more precise by replacing the term 'implicit' with 'efficient' (NA.LL.II.270).[33] The analysis of concepts into their often implicitly-held component attributes as also their amplification by means of adding new attributes, effectually constitutes an adherence to this postulate. Indeed, arguably, Hamilton's emphasis on the importance of being rigorously explicit elevates this postulate to the status of what one might call a meta-theoretical principle of explicitness.

With reference to the Latin '*concipere'* he explains that traditionally conception indicated 'the process of embracing or comprehending the many into the one', 'the act of comprehending or grasping up into unity the various qualities by which an object is characterised' (LL.I.120). Within our consciousness this process is typified by two cognitions, one immediate and 'only of the individual or singular', the other 'a knowledge of the common, general, or universal' (LL.I.122). Thus 'a Concept is the cognition or idea of the general character or characters, point or points, in which a plurality of objects coincide' (LL.I.122, and see, 122-3). Mirroring his notion of the formation of language, which he outlines in his *Lectures on Metaphysics*,[34] as a synthesising process that moves from chaos to the construction of general and universal terms which enable a complex and iterative relationship with the individuals or particulars into which such concepts may be analysed — a process of composition and decomposition that he occasionally describes as organic — Hamilton regards the formation of concepts as a complex process involving human agency or the *exertion* of an 'act of Comparison' upon an otherwise chaotic or confused array of presentations. Reminiscent of his highly significant relativist maxim, '*To think* is *to condition*', Hamilton is thus acutely aware of the role of human agency or volition in the process or mental *activity* involved in the formation of concepts as unities consisting of various points drawn together by an act of comparison and the implicit intentionality within this action 'of discovering their similarities and differences' (LL.I.123). This awareness of the part played by human agency or volition, and the artificiality and partiality of concepts, becomes clearer when he invokes 'the act called *Attention*', by means of which certain objects and qualities forming any given concept become strongly highlighted (LL.I.123). As Hamilton explains the mental operations of attention and abstraction, these two processes involved in the formation of concepts 'are, as it were, the positive and negative poles of the same act', by means of which as some objects and qualities become highlighted, others 'are thrown into obscurity' (LL.I.124, 123). He is claiming that the thinking subject's point of view or perspective, or conditioning role is crucial to the formation of concepts, and as we shall see later, point of view also plays an important role with regard to Hamilton's treatment of propositions.

What Hamilton is describing here, and in what follows this observation about the act of abstraction and attention, is the *relativity of concepts*, and thereby he

[33] All references in this form are as above to the *Lectures on Logic* with 'NA' prefixed to indicate the 'New Analytic of Logical Forms' as given in vol.II.249-317.

[34] *Lectures on Metaphysics*, II.327-332.

begins to point up certain implicit features of concepts that emphasise their fluidity or their adaptability, as well as their inherent inadequacy or insufficiently explicit nature. For Hamilton the act of comparison combined with that of abstraction and attention *reduces* in consciousness the multiple (or really differing objects of consciousness) into the unity that is a concept (see LL.I.124). By throwing out of view the non-resembling marks or characters or points that in reality individuate one individual from another, and by attending to the resembling points alone, we treat these *resembling* points as though they were identical and thereby synthesise them into a unity, a concept through which we think of the individuals or to which we relate these individuals. Implicitly, in opposition to mechanistically grounded (or mechanically modelled) theories of the mind, for Hamilton the formation of concepts in the mental act of conception is a single or indivisible, quasi-organic process of thought that may be analysed into components for the purpose of speaking about and better comprehending concepts (see LL.I.133). No doubt integral to or consistent with Hamilton's *natural dualism*, he regards our knowledge of what is presented and represented in consciousness — the phenomena of consciousness — as 'a direct, immediate, irrespective, determinate, individual, and adequate cognition' (LL.I.131). Such cognitions of the phenomena of consciousness, though virtually self-sufficient as cognitions, do not, for Hamilton, constitute *thought,* or so it would seem, since repeatedly he construes thought in terms of a relational or relative process in which the thinking subject is more distinctively active or operates as an agent conditioning the objects of thought. Hence, by contrast with the mere phenomena of consciousness as self-sufficient cognitions, by means of which the human mind may be thought of as, if not merely sentient, then active but unthinkingly or non-rationally so, 'A concept, on the contrary, is an indirect, mediate, relative, indeterminate, and partial cognition of any one of a number of objects, but not an actual representation either of them all, or of the whole attributes of any one object' (LL.I.131). This means that concepts are non-absolute or 'not capable of representation as absolute attributes' (LL.I.137). Earlier, in attempting to explain the inadequacy of concepts and their relativity, Hamilton pointed up the partiality of concepts, their dependence upon selective 'representation of a part only of the various attributes or characters of which an individual object is the sum' — for example, as we think *Socrates* through a particularly small range of attributes, say, *man, biped, animal*, our representation of him will be proportionately less adequate than when we think him through a greater range of attributes. Hence, a concept, construed as a collection of attributes to which the concept refers, are always one-sided or partial and to a greater or lesser extent inadequate by contrast with the plenitude of attributes that constitute any given individual.

However, this partiality or incompleteness of concepts, arising due to the combined act of abstraction and attention that conceiving necessarily involves, while it suggests the relativism of the thinking subject's conditioning influence on the objects of thought, does not seem to be the relativity that Hamilton particularly wishes to emphasise. Instead, the relativity of concepts to which he draws our attention inheres in the relational nature that a concept does not overtly make explicit but rather, in a sense, disguises by means of the fiction and even illusion of unitariness we tend to impose upon the collection of more or less resembling but nonetheless differing individual attributes that comprise any given concept. According to Hamilton, 'A concept or notion, as the result of a comparison, necessarily expresses a relation. It is, therefore, not cognisable in itself, that is, it affords no absolute or irrespective object of knowledge, but can only be realised in consciousness by applying it, as a term of relation, to one or more of the objects, which agree in the point or points of resemblance which it expresses' (LL.I.128). Although Hamilton's account of the relativity of concepts is far from being unproblematic, he asserts that the passage just quoted resolves 'the whole mystery of Generalisation and General Terms' and thus that his notion (that a concept expresses a relation, and is not therefore an 'absolute or irrespective object of knowledge') resolves the disputes between Conceptualists and Nominalists. That concepts constitute absolutes or objects of knowledge is an 'illusion' that fundamentally arises due to our conversion of similarity into identity, and thereby, according to Hamilton, 'the real plurality of resembling qualities in nature is factitiously reduced to a unity in thought; and this unity obtains a name in which its relativity, not being expressed, is still further removed from observation' (LL.I.128).

This may sound as though Hamilton is saying that concepts are fundamentally fallacious; that they are fallaciously unities, actually pluralities. However, it needs to be noted that while he may be saying that the view that concepts are absolute objects of knowledge, or that they express unities, arises due to our conversion of similarity into identity, the illusion of concepts as objective absolutes thus generated is not due to some inherent fallaciousness within our mental faculties. Rather, this illusion is due to an insufficiently rigorous attention to the internal components, structure, and relational processes involved in any given instance of conceiving as deeply integrant to the characteristic features and nature of the product of such thought, namely, a concept. Furthermore, as he later argues, concepts, formed by comparison and hence relative or expressive of a relation, 'can only be thought of in relation to some one of the individual objects they classify', but as such 'they fall back into mere special determinations of the individual object in which they are represented. Thus it is, that the generality or universality of concepts is potential, not actual' (LL.I.134). Though Hamilton does not go on to explain that this implies that concepts have to be regarded always as in some sense provisional and thereby capable of modification, supplementation, or some other adjustment, and available for examination from differing points of view, this relativist attack on the implicit absolutism of concepts construed as non-relativistic objects of knowledge, is further pursued by Hamilton's excursus into the role of language in a way that further brings to the fore the sense in which concepts are provisional artefacts, the establishment and permanence of which is dependent on the human subject — that concepts are necessarily subjective.

He apologizes for his digression into the extra-logical domain of metaphysics, a metaphysics which he admittedly here only sketches (LL.I.131). However, it would seem that, for Hamilton, in order to analyse the products of thought (concepts) as principal components within demonstrative arguments, we need to understand the relativity of concepts by making this explicit. To do so may involve some digression into metaphysical discourse, and even the enunciation of a metaphysical standpoint consistent with or flowing out of Hamilton's *natural dualist* position, a position more or less contentious for some readers but of substantive interest in relation to the Scottish Common Sense tradition in philosophy out of which Hamilton developed the term 'natural dualism'. Clearly concerned that his constriction of Logic to the necessary laws of thought is being violated by metaphysical considerations, he also apologizes for digressing onto a similarly extra-logical consideration of language. However, just as metaphysical discourse is at the interface between Logic and the nature of the object-matter of Logic, an important part of Hamilton's description of the generation of concepts as illusorily absolute unities or objects of knowledge, though actually relative, is his eloquent and fairly extensive treatment of language. He attempts to explain how a name or term or linguistic sign somehow fixes a concept within our consciousness as though it expresses an absolute object of knowledge and not a process or relational bundle of attributes brought together by some contributing act or relationship of agency by means of which the thinking subject participates with or conditions the phenomena of consciousness to make its otherwise chaotic or confused and unfixed plenitude meaningful.

His digression onto the relationship between language and thought further deepens the relativism of his whole approach as he brings to the fore the reciprocal nature of language and thought: 'Considered in general, thought and language are reciprocally dependent; each bears all the imperfections and perfections of the other; but without language there could be no knowledge realised of the essential properties of things, and of the connection of their accidental states' (LL.I.137). He prioritises thought over speech or language but does so in such a way as to suggest that this priority really pertains to the origination of the phenomenon of language rather than being a necessary condition of all speech (see LL.I.138). Be that as it may, Hamilton's more important concern has to do with describing the reciprocal relationship between thought and language, the process of conception and the claim that, were it not for our ability to fix 'and ratify in a verbal sign' all of the constituents of a concept, it would otherwise 'fall back into the confusion and infinitude from which it has been called out' (LL.I.137). He illustrates this for his students with some nice metaphors, such as the following:

> You have all heard of the process of tunnelling, of tunnelling through a sand bank. In this operation it is impossible to succeed, unless every foot, nay almost every inch in our progress, be secured by an arch of masonry, before we attempt the excavation of another. Now, language is to the mind precisely what the arch is to the tunnel. The power of thinking and the power of excavation are not dependent on the word in the one case, on the mason-work in the other; but without these subsidiaries, neither process could be carried on beyond its rudimentary commencement (LL.I.139).

Hamilton elaborates upon the metaphor of tunnelling through a sandbank, but while his simple claim may be altogether unexceptionable (that without language thought would at best remain in the most elementary and fragile state of almost total impermanence), it is interesting to note that what his tunnelling metaphor most strongly indicates is the relationship that the thinking subject is caught within, a relationship between the ever-shifting sand of a multitudinous plenum and the crafting of signs that, unlike the relative nature of concepts with regard to their constituent parts, may seem to render in some sense or to some degree permanent for consciousness the collected attributes of any given concept, but which still leaves such concepts relative and *factitiously* unitary. But as the thinking subject's struggle to secure or make permanent through language the confusion and infinitude of the phenomena of consciousness is here highlighted, Hamilton's awareness of Logic's contest with a disintegrative atomism or absolute relativism, at once suggests his consciousness of the volitional nature of reason and the abyss of a more thoroughgoing relativism that threatens to undermine the warrantability of his project entirely. But though this exaggerates the danger of Hamilton's relativism with regard to his own project, it draws to our attention the acute sense that Hamilton has of logic's relationship through language to the surrounding and teeming chaos of the universe within which our intellects struggle to achieve order — logic, like language, is for Hamilton a process of inching forward, of a tunnelling through sand only made possible by constructing arches to hold back and make orderly the chaos that perpetually threatens to engulf us. Any elementary concepts we might have the capacity to form without the assistance of language, which Hamilton admits may be a possibility, would be 'but sparks which would twinkle only to expire', implying that without language, and we must add, without logic, whatever thought might be possible is barely worth considering (LL.I.139). It is therefore of the greatest moment for Hamilton that the logic we construct should be robust and built upon proper foundations established through the most exacting scrutiny of the work of others — this is not just a task for those who *profess* to be logicians; it is also a task involving sound architectural skills, considerable scholarship, the craftsmanship of the master builder, and a critical engagement with and demolition of the crumbling and imperfect buildings of the past.

But it all must be carried out with an acute consciousness of the materials one has to work with and the instability of the substance that only logic can hold in place. Hamilton has, more or less wittingly, but nonetheless in a most profound way, highlighted the imperfect, inchoate, factitious, anthropocentric, volitional, indeterminate, and inherently relative nature of concepts. However, in doing so he has overburdened the tenability of the fixative term or word or sign to such an extent that the permanence or ratification he seems to claim we are capable of establishing or ascribing to concepts — their 'acquired permanence' as he later describes it (LL.I.225) — begins to look questionable. That this should be the case — that Hamilton's conceptualism is in fact a dismantling of Logic's object-matter into a purely formal kind of object that describes a process only artificially or analogically rendered as a material object consisting of identifiable components —

should perhaps not greatly surprise us, given that this idealism or immaterialism is so clearly congruent with his doctrine of *nescience*, and several other aspects of his philosophical position related to this doctrine.

But this is not to say that Hamilton actually undercuts the whole point of examining concepts and attempting to evolve a more complete science of the necessary laws of thought. Rather, it is to his credit that, as he persists with his enquiry, though the relativism and in general the perilously fragile condition of concepts (as factitiously wholes, and indeed thereby factitiously permanent fixtures in consciousness), is brought to his students' attention, he effectually engages in a quasi-Kantian critique of reason that at once points up the queerness, or idealism, of Logic's object-matter, while yet exploring what can be made determinate and placed under the regulation of irrefragable laws with regard to thought, an entity that he brings before us with great authenticity not as something already fixed or identical or closely analogous to material entities and physical nature (or to conceptions of physical nature that construe the material mechanistically), but rather as an object-matter evincing an indeterminacy with which it behoves exacting logical scrutiny to engage.

Having pointed up the indeterminate and relative nature of concepts he goes on to discuss the three main relations of concepts, namely, the relation they hold to their objects, to their subject, and to each other. The first relation, to their objects, is of course encapsulated by the term 'quantity' since all concepts are said to consist of a greater or lesser number of attributes (the objects of a concept). However, as is now well known but, according to Hamilton, had been largely overlooked by many contemporaneous and earlier logicians, the quantity of a concept can be distinguished into two different kinds, denominated by the terms 'extension' and 'intension' (or Hamilton's more frequently used term 'comprehension'). Although the terms 'extension' and 'intension' are well known to present-day logicians, that Hamilton regarded his contemporaries as being largely ignorant of these terms and that their importance to his own attempts to improve traditional logic is so great, provides at least two good reasons for elucidating his treatment of these terms here.

He claims that the distinction between extension and intension 'forms the very cardinal point on which the whole theory of Logic turns' (LL.I.119). He buttresses this claim by repeatedly returning to the significance of his distinction between and treatment of extension and intension in several later lectures, for example, when he argues that propositions can be distinguished as intensive or extensive depending on whether the subject or the predicate is respectively the containing whole (see, LL.I.231-3). However, as we shall see, the way in which he handles this distinction further deepens his underlying notion concerning the relativity of concepts to show, by demonstrating how extension and intension are correlatives of one another, that a relativistic analysis of concepts and arguments is possible and indeed further evolves the science of Pure Logic. However, importantly, the relativistic analysis that Hamilton's coordination of extension and intension enables, is one that is nonetheless anchored in the fundamental rule of containment, namely, the axiom which 'constitutes the one principle of all Deductive reasoning', 'that the part of a part is a part of the whole' (LL.I.119, 145, 144). It is perhaps worth pointing out that this axiom is also given in two Latin phrases by Hamilton, one of which is: '*Prœdicatum prœdicati est prœdicatum subjecti'*. The Editor of his lectures points out in a footnote that this is 'A translation of Aristotle's first antipredicamental rule as given in the *Categories* (see LL.I.144).

Hamilton's thorough and fairly extended explication of extension and intension in his Lecture VIII may be summarised as follows: a concept is a thought that *embraces* or, in the sense explained earlier, *brings into unity* an indefinite plurality of characters and it is also *applicable* to an indefinite plurality of objects about which it may be said, or *through* which these objects may be thought. As such, *a concept is a quantity of two different and opposed kinds*, denoted by the terms 'intension' and 'extension' (LL.I.140-52).

'Extension' refers to the *external* quantity of a concept, being determined by the number of objects — concepts or realities — to which the concept may be applied or which it classifies and hence *under* which these entities are said to be contained (a concept's extensive quantity is comprised of the number of objects that can be thought mediately through the concept). The extensive quantity of a concept is also referred to as its *sphere* or *breadth* and 'the parts which the total concept contains, are said to be contained *under* it, because, holding the relation to it of the particular to the general, they are subordinated or ranged under it. For example, the concepts *man, horse, dog*, &c., are contained under the more general concept *animal'* (LL.I.145). When these parts of a concept's extension are exposed, this is called *Division*.

'Intension' refers to the *internal* quantity of a concept, being determined by the number of objects — concepts or realities — that constitute the concept and hence *in* which these entities are said to be contained (a concept's intensive quantity is the conceived sum of the attributes that constitute it, formed into a whole or unity in thought). The intensive quantity of a concept is also referred to as its *comprehension* or *depth* and 'the parts [...] which go to constitute the total concept, are said to be contained *in* it. For example, the concept *man* is composed of two constituent parts or attributes, that is, of two partial concepts,–*rational* and *animal*; for the characters *rational* and *animal* are only an analytical expression of the synthetic unity of the concept *man*' (LL.I.143-4). When these parts or characters of a concept's intension are exposed, this is called *Definition*.

According to Hamilton, logicians 'have exclusively developed' the extensive quantity of concepts. However, he asserts that the extensive and intensive quantities comprise 'the two great branches of reasoning' and that intension 'is at least of equal importance' in comparison with extension (LL.I.144-5). This claim is significant in that, placing intension and extension on an equal footing, as two main branches of reasoning, immediately brings the analysis of concepts under, as it were, dual aspects or two main *perspectives* from which concepts may be viewed, analysed, and through which Hamilton can further elaborate his thesis concerning the relativity of concepts by bringing any given concept's extensive and intensive quantities into relation with one another — which is precisely what he does at this stage in his Lectures as also in *Discussions* and his 'New Analytic of Logical Forms'.

Hamilton seems to suggest at this stage what he will later make much more explicit, that the axiom or fundamental and sole principle of all Deductive reasoning, that a part of a part is a part of the whole, originates with regard to the intensive quantity of concepts — he certainly introduces this axiom of containment by illustrating how the intensive quantity of a given concept, such as is signified in the term 'man', may be analysed into ever diminishing parts contained *in*, or implicit within, the concept 'man', 'till we reach attributes which, as simple, stand as a primary or ultimate element, into which the series can be resolved' (LL.I.144). If he is suggesting that the axiom of containment has been, as it were, translated for application to the extensive quantity by the notion expressed as 'whatever is contained under the partial or more particular concept is contained under the total or more general concept', it is unclear whether he is at all troubled by the idea that the necessity of relation implicit in the axiom of containment, particularly as expressed with regard to a concept's intension, is grounded on an analogy with physical containment that he elsewhere rejects as unwarranted — that is, with regard to the assumption that mind and body are analogically related (LL.I.145).[35] However, he has effectually described concepts as necessarily factitious or artificial constructions, given a kind of sufficient or provisionally adequate permanence by means of language. He has also described concepts as relative continua in thought, the *analysis* of which is itself, purposive or tendentiously conducted in order that we can, as I indicated earlier, both speak about and thereby better comprehend concepts (see LL.I.133). Thus, Hamilton can perhaps claim some licence in deploying terms such as 'contain' without issuing caveats to guard against some of the assumptions antithetical to his general philosophical standpoint of *natural dualism* that, as a term analogically related to physical containment, 'contain' itself may be said to contain or imply. However, Hamilton is relying upon a traditional technical language of containment to elucidate the intensive and extensive quantities which he will later claim is better replaced by the more accurate 'substantive verb, (*is, is not*)' to express the equation or affirmation or negation of identity between a given concept and the objects constitutive of it or to which it may be related (or through which other concepts or particulars may be thought) (LL.I.154). As we shall see later, he does regard the axiom or sole principle of all Deductive reasoning (that a part of a part is a part of the whole), as rather crucially originating in a thought unshakeably natural to us, namely, our knowledge of the quantity of intension and the ways in which intensive containment provides as it were a natural grounding for all purely logical inference — Hamilton, strenuously striving to craft Logic into a pure science, abstracted from all extra-logical matters, cannot resist bringing the laws of thought into an intimate relation with the natural, the human as irrevocably an interrelated whole consisting of the self and the not-self.

But this aside, he applies the axiom of containment to both the intensive and

[35] *Discussions*, pp. 61-2.

the extensive quantities of concepts. Both may be said to *contain* attributes, but they crucially differ in that, while the intensive quantity is said to be contained *in* a concept, the extensive is said to be contained *under* it. Thus, while both of the quantities of extension and intension *contain* in differing senses, and while both are determined by wholes that may be identified and thereby quantified, 'These two quantities are not convertible. On the contrary, they are in the inverse ratio of each other; the greater the depth of comprehension [intension] of a notion the less its breadth or extension, and *vice versa*' (LL.I.142). This inverse ratio relationship between extension and intension is well illustrated by Hamilton as follows:

> When I take out of a concept, that is, abstract from one or more of its attributes, I diminish its comprehension [intension]. Thus, when from the concept *man*, equivalent to *rational animal*, I abstract from the attribute or determination *rational*, I lessen its internal quantity. But by this diminution of its comprehension I give it a wider extension, for what remains is the concept *animal*, and the concept *animal* embraces under it a far greater number of objects than the concept *man* (LL.I.147).

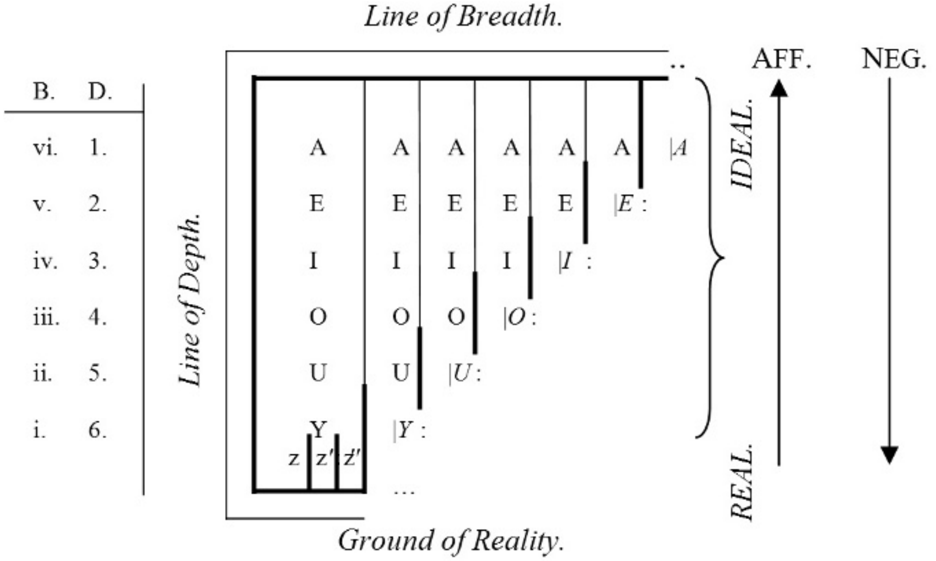
Hamilton explains the way in which this inverse ratio operates in fuller detail, pointing up as he does so that as we continue the analytic process outlined in the above quotation, the diminishment of a concept's intensive quantity, with the correspondent amplification or expansion of its extension, must finally result in 'that concept which all comprehension and all extension must equally contain, but in which comprehension is at its minimum, extension at its maximum,–I mean the concept of *Being* or *Existence*' (LL.I.149). And by contrast with this, when a concept's intension is at its maximum, its extension being then at its minimum, the concept we must end up with is that of an individual, 'the concept being a complement of the whole attributes of an individual object, which, by these attributes, it thinks and discriminates from every other' (LL.I.148).

His notion that, when extension is at a maximum and thereby intension at a minimum we must end with existence, raises the interesting topic of the existential import of concepts and propositions. However, leaving this aside, it should now be clear that, having described concepts as generally relative, Hamilton is laying bare the sort of things that this relativity enables with regard to how we may view and analyse concepts. In an ingenious table in his *Discussions*, which the editors of the Lectures appended to the end of Lecture VIII, Hamilton further exposes how their relational or relative nature enables us to view any given part of a concept under different relational aspects, for example: as species of a genus from one end, the genus of a species from its opposite end; how the expansion of a concept's extension implies a coordinate reduction in its intension; the sense in which we may say that each part 'in opposite respects, *contains* and is *contained'* (LL.I.153); and how 'the *real* identity and *rational* differences of Breadth and Depth' become exposed, such that it becomes more apparent that extension and intension 'though denominated *quantities*, are, in reality, one and the same quantity, viewed in counter relations and from opposite ends. Nothing is the one, which is not, *pro tanto*, the other' (LL.I.153).

When explicating these points to do with a concept's extension and intension in his Lectures Hamilton used 'a circular machine' which he had devised. Similar to a now-familiar child's toy consisting of a circular base with a wooden rod at its centre onto which are placed differently coloured circular discs of various circumferences, the discs of Hamilton's machine represented concepts that could be placed on the central rod representing the individual to which these concepts might refer. Each disc was placed on the rod in the correct order to illustrate several things to do with the relation of extension to intension, the completed device standing as an inverted cone. This 'machine' is represented and discussed by Veitch (see Veitch, pp. 250-2). As though trying to reinforce Hamilton's claim that his doctrine of the quantification of the predicate had been taught in his Lectures prior to de Morgan's quantification, the Editors' insertion of the table and the explanatory pages from his *Discussions* at the end of Lecture VIII reveal how his distinction between extension (Breadth) and intension (Depth), and the way in which he brings these two quantities into relation with one another, form the basis of his quantification of the predicate. That Hamilton's editors thought it necessary to do this, strongly suggests their awareness of how easy it might be not to grasp from his lectures, the extent to which Hamilton's treatment of Concepts and the two principal quantities of extension and intension lead to and are indeed an integral part of Hamilton's quantification of the predicate. However, the editors' insertion of this table is perhaps best understood as an attempt to bring to the fore the very sort of thing that it may be assumed Hamilton explained at this stage in his Lecture VIII by use of his 'circular machine'. I have reproduced Hamilton's table below.

The table displays the lines of Breadth and Depth, the conceptual range as Ideal, the objective or particular range of individuals or singulars as Real, and gives lines of direction to indicate affirmation and negation (which I shall not attempt to explain). The table distinguishes between individuals or singulars (z, z', z'') and classes (A, E, I, O, U). The highest genus or widest attribute is given as A, A, A, etc., the subaltern genera and species as E, I, O, U, and the lowest species or narrowest attribute as Y. Each class is represented as 'a series of resemblances thought as one', symbolised by the same letter to denote that they are *thought* of as one, though really distinct or differing from one another in some respect(s) (this being intimated by the vertical lines separating each letter). This is in line with Hamilton's notion that a concept is relative in the sense that it brings into a unity in thought what are really discrete though resembling characters. The narrowest attribute (Y) is shown as a simple term constituted by the individuals z, z', z''. Though simple or singular in the sense that it has no extension or has a minimal extension, it is dichotomised by using a thick line '|' to denote 'not' — that is to say, if 'Y' is thinkable, as a strict logical necessity one must also be able to think 'not Y', but the narrowest attribute must otherwise be a singular attribute and not a class (as in, say, U). This is not to say that some given attribute represented

Table 1. Schemes of Two Quantitites



(LL.I.152; for further detail see, pp. 152-6, and *Discussions*, pp. 699-701).

by 'Y' could not be translated into 'A' in another table since any 'Y' is only the least or narrowest attribute in *relation* to U and the other letters above this, and the least must be in thought, if not in fact, a unity or attribute directly relatable to the individual constituents as shown beneath it. In other words, while the table posits a narrowest attribute Y, as something not consisting of any other resembling Y, closer examination of Y's intension could involve translating it into another table of the same form in which it might be translated into A. As the thick line '|' denotes 'not' (e.g. to contradistinguish between say, 'animal' and 'not animal', the thinner lines 'merely discriminate one animal (A) , from another (A)' (LL.I.156). Hamilton's elucidation of the table more clearly begins to indicate just how his understanding of the relativity of concepts, and what a concept consists in with regard to the two quantities of extension and intension, combine to illustrate the necessity of quantifying the predicate: 'A is only A, not A, A, A, &c.; some Animal is not some Animal; one class of Animals is not all, every, or any other; this Animal is not that; Socrates is not Plato; z is not z'. On the other hand, E is EA; and Y is $YUOIEA$; every lower and higher letter in the series coalescing uninterruptedly into a series of reciprocal subjects and predicates, as shown by the absence of all discriminating lines. Thus Socrates (z'), is Athenian (Y), Greek (U), European (O), Man (I), Mammal (E), Animal (A)' (LL.I.155).

This needs to be further elucidated as follows: reading from left to right, the first A contains under it the concepts $EIOUY$, and the full range of all individual and hence actual Athenians, represented by 'z, z', z''.' However, the second A contains under it $EIOU$, and *not* Y ('$|Y$'), and hence neither z nor z' nor z''. The first A is a term identical *qua* term to all of the other As in the row of As, and it *resembles* (according to the thinking subject) the other As to such an extent as to be unified in thought and fixed as a unity by means of the single classificatory or general term 'A'. However, just as the first A and the second A are in all respects identical, *except* that the first A contains under it Y (and hence, in this case, z') whereas the second A does not, similarly each of the other As may be differentiated from one another and from the first A by means of reference to their respective depths or intensive quantities. This implies that each of the terms A is, as Hamilton puts it, 'only A, not A, A, A, &c. [...] one class of Animals is not all' (LL.I.155). This suggests the basis of quantification of the subject term in a simple proposition such as 'Some Animals are Mammals'. But, as we shall see shortly, it also suggests the basis of quantifying the predicate term.

Suppose that: A represents the term 'Animal'; E represents the subordinate or species 'Mammal' to A as the genus Animal; I represents the subordinate or species 'Man' to E as the genus Mammal. For ease of explanation it is necessary to follow Hamilton and ignore a reasoning that starts by quantifying the class with the universal term 'All', for it can easily be seen that were we to commence, in Breadth, with 'All A' ('all Animal') then, contained under this class are 'All E and some not E' ('all Mammal and some not Mammal'), which unnecessarily complicates explication of how Hamilton's table demonstrates the need to quantify the predicate. Now, if we only attend to the range encompassed by the first five As, then this implies we must only be looking at *some* A and not all (in this case the first five). What is contained under these five As is the range of all Es (i.e. 'not E' ('$|E$') has been excluded by abstracting or ignoring the sixth A). Translating this selection of the first five As and the range of Es listed under them, we have 'some Animals are all Mammals.' This process can of course be continued by repeating the process of removing one attribute or class of E, so that the next step in the reasoning process that examines what is contained under E, involves a further abstraction or removal of, in this case, the 4^{th} E, resulting in: 'Some E (Mammals) are all I (Man)', and so on.

If this begins to illustrate how reducing the extension of a concept deepens or expands its Depth, and *vice versa* — the inverse ratio principle Hamilton claims to exist between extension and intension — the table also illustrates that, in effect, the same reasoning can be applied whether we are arguing in Depth (i.e. with regard to the intensive quantity) or in Breadth (i.e. with regard to the extensive quantity). This, according to Hamilton, is because, 'Though different in the order of thought, (*ratione*), the two quantities are identical in the nature of things, (*re*). Each supposes the other; and Breadth is not more to be distinguished from Depth, than the relations of the sides, from the relations of the angles, of a triangle' (LL.I.154). The table illustrates how the same reasoning can be applied to both the extension and intension of a concept, so long as both the subject term and the predicate term in all of the resulting propositions are quantified consistently in accordance with the correct understanding of what the table indicates at each step in the reasoning with regard to containment (in or under). Hamilton's own illustration of this is as follows:

> In effect it is precisely the same reasoning, whether we argue in Depth — "z' is, (i.e. as subject, contains *in* it the inherent attribute), some Y; all Y is some U; all U is some O; all O is some I; all I is some E; all E is some A — therefore, z' is some A:" or whether we argue in Breadth — "Some A is, (i.e. as class, contains *under* it the subject part), all E; some E is all I; some I is all O; some O is all U; some U is all Y; some Y is z' — therefore, some A is z'." The two reasonings, internally identical, are externally the converse of each other; the premise and term, which in Breadth is major, in Depth is minor. (LL.I.154)

But, as this displays how the inverse ratio principle operates, and how so relating extension and intension reveals that it is only *externally* that the two quantities determine a difference in reasoning whereas, with each term consistently quantified using 'some' or 'all' as appropriate, subjects and predicates can swap position to yield formally identical conclusions ('z' is some A' and 'some A is z''), Hamilton is quick to point out in this more developed treatment of the topic explicating his 'Schemes of the Two Quantities' table that: 'In syllogisms also, where the contrast of the two quantities [extension and intension] is abolished, there, with the difference of figure, the differences of major and minor premise and term fall likewise.' (LL.I.154). With these few words Hamilton is touching on just how, through examining concepts extensively and intensively to show the inverse ratio relationship between these two quantities, the resultant quantification of all terms in a reasoning that hinges on the identification of the subject and predicate terms of a proposition, enables a significant simplification of traditional Formal Logic. However, he is also touching on a claim, the significance of which he brings to the fore much later in his Lectures, namely, that 'In fact, the *two quantities* and the *two quantifications* have by logicians been neglected *together*' (LL.I.155). As attending to the quantities of extension and intension reveals their '*real* identity and *rational* differences', such that it becomes more apparent that they are 'in reality, one and the same quantity', Hamilton will later show, through careful stages of his teaching of logic in the Lectures, that this identification of the two quantities is highly significant with regard to both propositions and syllogisms (LL.I.153; compare *Discussions*, pp. 701–2). However, before the full relevance of Hamilton's inverse ratio principle with regard to extension and intension can be illuminated with regard to propositions and syllogisms, it is necessary to outline some of the other points he makes in his Lectures concerning concepts.

If by the end of Lecture VIII it is beginning to emerge, particularly with the editors' helpful insertion of Hamilton's 'Schemes of the Two Quantities' table given above, that Hamilton is carefully working towards his quantification of the predicate, in some of the Lectures that follow, he still has much to say that will further reinforce his notion that concepts are relative, not simply in the sense that they are relational (or wholes constituted by and referring to a plurality of resembling entities, or relative with regard to their objects), but also in the sense that our construction and grasp of the depth (intension) and breadth (extension) of any given concept is dependent upon a subjective interrelation of clearness/obscurity and distinctness/indistinctness. This introduces the notion of the *quality* of a concept (see LL.I.157).

Thus, having elucidated the inverse ratio relation between the extension and intension of a concept, he goes on to consider the *subjective* relation of concepts (i.e. the relation to the subject that thinks a concept) in relation to their clearness and distinctness as similarly relative terms determining the quality of a given concept. He explains the terms 'clearness' and 'distinctness' at some length, acknowledging as he does so a considerable debt to Leibniz (see LL.I.159–65).

Hamilton points out that while intensive distinctness is at a maximum when we reach simple notions that are thereby indefinable, and while extensive distinctness is at a maximum when, to quote his translation of Esser, 'we touch on notions which, as individual, admit of no ulterior division', such distinctness is only ideal and that in fact this ideal distinctness is something we are always approaching but never in reality attaining (LL.I.170). As this ideal distinctness is regarded by Esser as an incentive to re-analyse the intension and extension of concepts, it is clear that Hamilton is suggesting to his students that this relativity and incompleteness or non-absolute condition of concepts is an important aspect of his overall approach to the study of Logic, a crucial incorporation of an awareness of the ultimate indefinability and non-absolute dimension of Logic's object matter. And in this Hamilton is arguably being entirely consistent with other aspects of his metaphysics and overall philosophical position.

In Lecture X he has more to say about the imperfection of concepts, this time returning to the problem of language. Concepts have, as it were, a propensity to be obscure and indistinct and these vices are due, partly to their very nature as wholes that bind together 'a multiplicity in unity', and partly from their dependence upon language as that which fixes concepts in consciousness (LL.I.172). He explains the problem of language by means of an illustrative analogy with methods of exchange in countries lacking an established currency. Thus, language operates much like the handing over of unquantified bags of precious metals which may or may not be closely scrutinised to see if they yield the value they purportedly signify — on most occasions the language user takes on trust that a particular term binds together what it seems to claim for itself, namely, that it does in fact represent a multitude of entities collectively amounting to a certain sum or value; but at other times, this will not be the case. This analogy of course teems with significance but there seem to be two main points that Hamilton is attempting to emphasise: firstly, 'that notions or concepts are peculiarly liable to great vagueness and ambiguity, and that their symbols are liable to be passed about without the proper kind, or the adequate amount, of thought' (LL.I.173-4); and secondly, that an important distinction, originated by Leibniz, can be made with regard to our knowledge that divides cognition into the *blind* or *symbolical* and *intuitive* (see LL.I.180-86). In short, Leibniz's notion of symbolical knowledge refers to concepts as terms taken to signify entities obscurely and imperfectly presented to the mind but which may potentially be exposed or made explicit, though we cannot think all of the ingredients that comprise the symbol or term used to refer to them. By contrast with this, intuitive knowledge is of these ingredients themselves inasmuch as this is possible. Hamilton fails to explain the significance of this distinction except to claim that thereby Leibniz and his followers in Germany superseded or overcame 'the whole controversy of Nominalism and Conceptualism,–which, in consequence of the non-establishment of this distinction, and the relative imperfection of our philosophical language, has idly agitated the Psychology of this country [Britain] and of France' (LL.I.179). However, it should be fairly obvious that by means of this distinction between symbolical and intuitive knowledge, Hamilton is providing a warrant for treating concepts in a purely formal manner in order to examine more closely the relations between concepts considered blindly or symbolically, while at the same time drawing attention to the interface between what the symbols represent with regard to the imperfect but real condition of our intuitive knowledge of the particulars to which concepts relate, and through which we both think these particulars and constitute our concepts. Once again, the relativism, imperfection, inchoateness, and mutability of concepts and the phenomenal and plural nature of our knowledge as relative, relational, provisional, and so on, is being emphasised by Hamilton, while at the same time he attempts to establish a domain or object matter of Logic at once stable, constricted, and discriminated from the material or actual, though both domains of the intuitive and symbolical are held in relation to one another as mutually informative and only theoretically discrete.

Though Hamilton is clearly aware that his various points concerning the relativity of concepts is foregrounding matter that might easily be thought of as extra-logical, having distinguished between symbolical and intuitive knowledge, in Lecture XI he appropriately turns to what he regards as 'The Relation proper' of concepts, namely, *their relation to each other* — something that can be represented symbolically and diagrammatically and which thereby establishes a set of relationships familiar to logicians. Again he discusses this in relation to the two principal quantities of extension and intension. Taking extension first, he outlines five principal relations: Exclusion, Coextension, Subordination, Co-ordination, and Intersection. All of these he illustrates with simple circle diagrams, a practice used by some earlier philosophers, such as, according to Hamilton, the late 16^{th} century Christian Wiese, and of course developed later by John Venn. It is needless to reproduce Hamilton's diagrams (see LL.I.189; 256), but it is important to note that of all of these relations between concepts considered with regard to their extension, 'those of Subordination and Co-ordination are of principal importance, as on them reposes the whole system of classification' (LL.I.189-90). He elucidates Subordination and Co-ordination with crystal clarity. However, as yet further evidence of his interest in and even fascination with relativism, he continues to draw into consideration: that there is no absolute exclusion in the relation between concepts known as Exclusion (LL.I.188); the ways in which our *perspective* on concepts results in re-describing a genus as a species and a species as a genus; how speculatively, if not practically, we may always divide a concept *ad infinitum* (LL.I.192-3); how different abstractions result in different relations of genus and species in both subordination and co-ordination; and how, admitting that there can be both a highest genus and a lowest species, neither of which are convertible into a species nor a genus respectively, within Subordination there are gradations of genus and species known as subalterns or intermediates such that, the genus lower than the highest genus, can become a species, whereas the species higher than the lowest species, can become a genus. This, as Hamilton points out all comes from Porphyry's Introduction to Aristotle's *Categories*, but it is nonetheless notable that Hamilton should emphasise the non-absolute aspects of the relations between concepts and the dependence of these various reversals of species and genus on how we regard them (LL.I.196). Point of view is indeed something he is at pains to emphasise as important to several matters, not least of which is the relation of whole to part and the distinction, which he regards as erroneous, between Logical and Metaphysical wholes, these being, contra to previous logicians, 'equally logical' (see LL.I.201-2).

Hamilton is evidently working towards some important claims to do with the whole-part relationship that will give priority to the intensive quantity of concepts which in part relies upon his notion of *Involution* (see LL.I.202-3). Skipping over the various kinds of whole that Hamilton elucidates, the notion that a genus contains its species either potentially or actually (see LL.I.205-6), along with various other points of interest, it is in Hamilton's treatment of Comprehension or the quantity of intension in Lecture XII that he first makes fully explicit one of his major disagreements with traditional logic, a disagreement nevertheless that he has been hinting at in one way or another from a fairly early stage. He claims that the relations of Involution and Co-ordination have been:

> altogether neglected by logicians: and, in consequence of this, they have necessarily overlooked one of the two great divisions of all reasoning [...]. In each quantity there is a deductive, and in each quantity there is an inductive, inference; and if the reasoning under either of these two quantities were to be omitted, it ought, perhaps, to have been the one which the logicians have exclusively cultivated [i.e. the deductive reasoning in extension]. For the quantity of extension is a creation of the mind itself, and only created through, as abstracted from, the quantity of comprehension [intension]; whereas the quantity of comprehension is at once given in the very nature of things. The former quantity is thus secondary and factitious, the latter primary and natural. (LL.I.217-8).

Now, it must be noted that by the term 'inductive' Hamilton departs from what we might call the standard treatment of or distinction between induction and deduction. Thus, he does not mean that kind of inference in which the conclusion may be said to exceed or amplify the content of the premises. He is not talking here about probable reasoning or judgements that go beyond what is delivered in the premises — his restriction of induction to a form of necessary reasoning is therefore substantially different from the empirical reasoning or induction of the physical sciences. Instead, as he makes clear in a later lecture, he is treating induction in a formal sense as differentiated from an informal or material sense, to mean a process of reasoning from *all* of the parts to the whole that these parts *entirely* constitute (see LL.I.319-26). This aside, that he is now distinguishing as he does in the above quotation between extension and intension may seem to be at odds with the claim he made earlier about concepts being factitiously unities, but really pluralities — now it would seem they are only partially factitious, projected, mind-dependent and mind-generated, but partially 'primary and natural'. Also, this prioritisation of intension seems to sit uncomfortably with his attempt to hold, by means of his inverse ratio relation between the two quantities, that extension and intension are non-convertible but equivalent with regard to the reasoning that we apply to both quantities. Leaving aside this weaker objection, that he has now introduced a distinction between extension and intension that prioritises intension as the real and primary, creates problems for Hamilton that he does not satisfactorily resolve. For example, is not the intensive quantity also a binding together of resembling but nonetheless discrete entities held in a relationship of containing and contained, and thus is not intension every bit as factitious as extension? Furthermore, how can an *abstraction* from the intensive quantity be satisfactorily described as factitious, given that it is just that, an abstraction from what is not factitious but rather 'in the very nature of things'? Perhaps, by 'factitious' Hamilton is merely asserting that, in relation to the thinking subject, extension is a product of the mental action of abstracting from the real, intensive quantity, and in this sense the product that is a concept's extensive quantity may be more or less arbitrarily or selectively constructed, that extension is both factitiously a unity, really a plurality of resembling attributes, and that its factiousness in relation to the reality of a given concept's intension is a matter of degree. That is to say, both with regard to the degree of resemblance between a concept's attributes in extension, and with regard to how close its extension is to its definition, its distinctness, or the thinking subject's grasp of its intensive quantity, the extensive quantity may be *more or less* factitious.

Be that as it may, Hamilton does seem to be making an important distinction and claim here that may be more sympathetically understood as an attempt to ground in reality the formation of our concepts as emanating from or evolving out of a deeper or more intimately connected relationship of resemblance between the parts of a whole than is tenable of the resembling attributes in extension. He does this by pointing up the difference between the senses of 'contain': in extension a concept contains *under* it, its various attributes in subordination; whereas in intension, a concept contains *in* it, its various attributes as constitutive of and as the definitional properties that make the concept what it is, just as the multitude of parts go together to constitute any given individual. To distinguish between these two different senses of 'contain', Hamilton introduces the notion of Involution mentioned above, which he explains as follows:

> In the quantity of comprehension [intension], one notion is involved in another, when it forms a part of the sum total of characters, which together constitute the comprehension of that other; and two notions are in this quantity co-ordinated, when, whilst neither comprehends the other, both are immediately comprehended in the same lower concept. (LL.I.220).

He gives two illustrations of the Involution relationship between a concept's attributes, the second of these pointing up the inter-relatedness of concepts as involving and involved, such that we may be said to think a certain concept only in and through that of another concept:

> In this quantity [of intension] the involving notion or whole is the more complex notion; the involved notion or part is the more simple. Thus *pigeon* as comprehending *bird*, *bird* as comprehending *feathered*, *feathered* as comprehending *warm-blooded*, *warm-blooded* as comprehending *heart with four cavities*, *heart with four cavities* as comprehending *breathing with lungs*, are severally to each other as notions involving and involved. (LL.I.223).

Suggesting as this does, a relative overlapping and even partial integration of one concept with another, such that they mutually imply and yet collectively constitute the individual or unity they define, he immediately follows this somewhat sketchy account of Involution, by differentiating between this relativist aspect (containing and contained) and the non-relative (since not *necessarily* containing and contained) co-ordination:

> Again, notions, in the whole of comprehension, are co-ordinated, when they stand together as constituting parts of the notion in which they are both immediately comprehended. Thus the characters *oviparous* [egg producing] and *warm-blooded*, *heart with four cavities*, and *breathing with lungs*, as all immediately contributing to make up the comprehension of the notion *bird*, are, in this respect, severally considered as its co-ordinate parts. These characters are not relative and correlative,–not containing and contained. For we have oviparous animals which are not warm-blooded, and warm-blooded animals which are not oviparous. Again, it is true, I believe, that all warm-blooded animals have hearts with four cavities [...], and that all animals with such hearts breathe by lungs and not by gills, But then, in this case, we have no right to suppose that the first of these characters comprehends the second, and that the second comprehends the third. For we should be equally entitled to assert, that all animals breathing by lungs possessed hearts of four cavities, and that all animals with such hearts are warm-blooded. They are thus thought as mutually the conditions of each

> other; and whilst we may not know their reciprocal dependence, they are, however, conceived by us, as on an equal footing of co-ordination. (LL.I.223)

Hamilton hints that the significance of Involution and Co-ordination will come to the fore when he later moves on to tackle the syllogism and, seemingly regarding this as virtually a digression, his explanation of Involution in Lecture XII is unfortunately rather brief and might have been more fully elucidated, particularly given that it may be much more crucial than perhaps Hamilton realised, to how we might best understand at least one major aspect to do with the co-identity relationship within universal propositions which I shall discuss in Section 4. However, he says just about enough to suggest what one might call a grounding relativism in which his few words on this, though offered to explain the wholeness of a concept's intensive quantity, verge on further pointing up the artificial nature of the fundamental laws of logic of identity, excluded middle, and non-contradiction. If this is less evident in the above quotation, it is more so in how he relates the involution within a concept's intensive quantity to his non-linear notion of the formation or *evolution* of general and particular terms:

> Our notions are originally evolved out of the more complex into the more simple, and [...] the progress of science is nothing more than a progressive unfolding into distinct consciousness of the various elements comprehended in the characters, originally known to us in their vague or confused totality [the condition in which they may be said to be maximally complex].
>
> It is a famous question among philosophers,–Whether our knowledge commences with the general or with the individual,–whether children first employ common, or first employ proper, names. In this controversy, the reasoners have severally proved the opposite opinion to be untenable; but the question is at once solved, by showing that a third opinion is the true,–viz. that our knowledge commences with the confused and complex, which, as regarded in one point of view or in another, may easily be mistaken either for the individual, or for the general. [...]. It is sufficient to say in general, that all objects are presented to us in complexity; that we are at first more struck with the points of resemblance than with the points of contrast; that the earliest notions, and consequently, the earliest terms, are those that correspond to this synthesis, while the notions and the terms arising from an analysis of this synthesis into its parts, are of a subsequent formation. But though it be foreign to the province of Logic to develop the history of this procedure; yet, as this procedure is natural to the human mind, Logic must contain the form by which it is regulated. It must not only enable us to reason from the simple and general to the complex and individual; it must likewise, enable us to reverse the process, and to reason from the complex and individual to the simple

> and the general. And this it does by that relation of notions as containing and contained, given in the quantity of comprehension [intension]. (LL.I.221-2).

In short, the notion of Involution takes us once again into the relativism of Hamilton's metaphysical epistemology and theory of language which he outlines more extensively in his *Lectures on Metaphysics*.[36] And, if he is, perhaps rather shakily, attempting to give priority to the reality of intension over and against the factitiousness of extension, he at least does so in a way that coheres interestingly with his theory of language with regard to the iterative or relative or non-linear origination of general and particular terms as a natural process of thought and indeed reasoning. It must be noted, however, that Hamilton is here attempting to foreground not the *relativity* of concepts but rather, that concepts considered intensively enable the natural tendencies in reasoning and thought generally 'to reason from the simple and general to the complex and individual [and also] reverse the process, and [...] reason from the complex and individual to the simple and the general' — this, he clearly sees as a particular virtue of the intensive quantity, namely, that, as contradistinguished from the extensive quantity, the reversals in the direction of reasoning that we do in fact or naturally make, can only legitimately be carried out with regard to the intensive quantity.

Although the somewhat fuzzy logic of this notion of Involution within, or characterising, the relation of the internal components of a given concept, or its intension (as contradistinguished from Subordination in extension), is not handled as fully as one might wish, it does seem that Hamilton is attempting to ground in an indeterminate reality of chaos or confusion, concepts as originating in, wrested out of, and partaking in a significant degree of relativism. However, I would claim that, though fluxive, quasi-organic, and arguably suggesting an indivisible and interminably iterative process, the involution relation that Hamilton seems to be suggesting inheres within a concept's intension, warrants, or is itself peculiarly suggestive of, the distinction of constituent parts comprising wholes and the defensibility and inherent reasonableness of regarding such parts as being contained in the whole such that their relations one to the other may be understood as necessary and as such fundamental to Pure Logic as the science of the necessary laws of thought. The naturalness of this reasoning as grounded in the notion that the parts of parts *involve and are involved*, and thereby permit reversing the process of reasoning, is brought out or hinted at by Hamilton in several ways. For example, just what he seems to think of as Involution may be being suggested in what he says about *partes integrantes* with regard to Mathematical, or the Quantitative, or Integrant Whole (see LL.I.204). Perhaps the naturalness and primacy Hamilton is trying to claim of intension, as contrasted to the factitiousness of extension, is being indicated in the natural involutions of physical entities as we encounter them in, say, the human body, but also as may be postulated of all material phenomena as the relative, plural, and confused constituents of consciousness which,

[36] See, *Lectures on Metaphysics*, II.319-27.

conditioned by thought, we naturally or inevitably collect together (or *grasp*) as resembling particulars. Organised into wholes, as part of the natural processes of thought and our conditioning propensity to cognise, these wholes avail, and indeed require, a corresponding analysis into their particulars and a relation of these to the concept they constitute, our *act* of grasping or even perceiving them as collected together into the unity of the concept, being the synthetic reconstruction of the particulars constituting it, albeit to some degree provisionally. For Hamilton, it is as though a power of the mind — the faculty of comparison – is itself reinforced by and originates in the involution relationship as something that at once suggests the greater chaos or confused complexity into which the involutions of the parts seem to be capable of collapsing, while also suggesting the sharper discrimination into non-involved particulars that are thereby not constitutive of but which rather divide a concept into its extensive quantity. If so, Involution at once suggests and is itself the suggestion of what a concept's intension may be evolved into by the native processes of conception and its negative corollary, the reasoning of an analysis only prevented from disintegrating into infinite divisibility by an aptly pragmatic limitation on the extent of this science's explicitness, which Hamilton explicitly imposes (see LL.I.192-3; 210).

But in order to go at least some way towards understanding why Hamilton is now claiming that the reasoning of intension is natural, or has a basis in reality — and indeed in a reality that he regards as otherwise confused or chaotic and theoretically if not practically open to infinite divisibility in its unconditioned state as the raw and unorganised mere phenomena of consciousness — and hence why he regards intension as prior or superior to the quantity of extension, we need to note at least something of the extent to which he regards the history of logic since Aristotle as flawed. We also need to note that Hamilton quite pointedly regards his prioritisation of intension as a significant contribution towards placing the keystone in the arch of the Aristotelian logic. He claims that logicians following Aristotle rather surprisingly neglected the process of reasoning to do with intension, even though they explicitly stated and relied upon the axiom: 'The character of the character is the character of the thing; or, The predicate of the predicate is the predicate of the subject' (LL.I.218). However, according to Hamilton, Aristotle understood the application of this axiom:

> In fact I think it even possible to show in detail, that his whole analysis of the syllogism has reference to both quantities, and that the great abstruseness of his *Prior Analytics*, the treatise in which he develops the general forms of reasoning, arises from this,–that he has endeavoured to rise to formulæ sufficiently general to express at once what was common to both kinds;–an attempt so far beyond the intelligence of subsequent logicians, that they have wholly misunderstood and perverted his doctrine. They understood this doctrine, only as applied to the reasoning in extensive quantity; and in relation to this kind of reasoning, they have certainly made palpable and easy what in Aristotle is abstract and difficult. But then they did not observe that Aristotle's

> doctrine applies to two species, of which they only consider one. [...]. This mistake,–this partial conception of the science,–is common to all logicians, ancient and modern: for in so far as I am aware, no one has observed, that of the quantities of comprehension and extension, each affords a reasoning proper to itself; and no one has noticed that the doctrine of Aristotle has reference indifferently to both. (LL.I.218-9).

3 DOCTRINE OF JUDGMENTS

> In the proposition *Men are animals*, we should be allowed to determine whether the term *men* means *all* or *some men*,–whether the term *animals* means *all* or *some animals*; in short, to quantify both the subject and predicate of the proposition. This postulate ['To state explicitly what is thought implicitly'] applies both to Propositions and to Syllogisms. (NA.LL.II.252).

In elucidating some aspects of Hamilton's 'Schemes of the Two Quantities' table and certain features of his doctrine of Concepts in the previous section, I have so far been indicating something of how his development and understanding of extension and intension underpinned his quantification of the predicate. Before examining the quantification itself, we need to look at a selection of some of the other construction work.

By 'judgment' Hamilton means 'proposition' *or* the forming of a proposition, such as '*water rusts iron',* by means of the process of judging (LL.I.227). What is relative within a judgment or proposition has at least the appearance of being much more stable than the relativity of concepts discussed at various stages above, though the very fact that Hamilton calls attention to propositions as *judgments*, keeps in focus how it is that a proposition may be regarded as at least related to some mental agency, a judgment being the product of the act of judging (akin to a concept being the product of conception). Introducing his students to the standard terms of 'subject', 'predicate', and 'copula', he points out that the subject is the determined or qualified notion, whereas the predicate is the determining or qualifying notion, the relation of determination between them being signified by the copula. He therefore defines a proposition as:

> the product of that act in which we pronounce, that, of two notions thought as subject and as predicate, the one does or does not constitute a part of the other, either in the quantity of extension, or in the quantity of Comprehension. (LL.I.229).

Hamilton is clearly regarding subject and predicate as being held in a relation of quantity in accordance with the axiom that a part of a part is a part of the whole, but he is also, consistent with his treatment of extension and intension, incorporating both of these quantities, and this is how he does so:

> The first great distinction of Judgments is taken from the relation of Subject and Predicate, as reciprocally whole and part. If the Subject or determined notion be viewed as the containing whole, we have an intensive or Comprehensive proposition; if the Predicate or determining notion be viewed as the containing whole, we have an extensive proposition. (LL.I.231-2).

Hence, whether a proposition is extensive or intensive depends on how we view the Subject and the Predicate. Of course Hamilton acknowledges that in single propositions it is rarely clear which way the proposition is to be viewed, since it is generally unclear whether it is the Subject or the Predicate that is being regarded as the containing whole. However, according to Hamilton, 'It is only when propositions are connected together into syllogism, that it becomes evident whether the subject or the predicate be the whole in or under which the other is contained' — he thus regards the distinction of propositions into extensive and intensive as highly general, though of great importance, since 'it is only in subordination to this distinction that the other distinctions [he is about to introduce] are valid' (LL.I.233). The other distinctions that relate to the extensive/intensive distinction with regard to the subject and predicate relation may be summed up as: the distinction between Categorical (or simple propositions, in which, following the practice of Aristotle's followers, the predicate is either simply affirmed or denied of the subject, as in '*A is B*', or '*A is not B*') and Conditional propositions, Conditional propositions being further distinguished into Hypothetical ('*if A then B*'), Disjunctive ('*A or B*', or '*D is either B, or C, or A*'), and Dilemmatic or Hypothetico-disjunctive ('*If X is A, it is either B or C*') (see LL.I.233-242).

Although it is crucial to grasp that Hamilton is showing his students how in principle the subject and predicate terms may be viewed as reciprocally related to one another in any given proposition as expressions solely concerned with a whole-part relationship, either in extension or intension, it is needless to elucidate this any further here. More importantly, is Hamilton's major point of difference with Aristotle and 'The doctrine of Logicians' concerning the division of propositions into four classes or species, since it is in this that Hamilton may be said to be making his most explicit statement within his Lectures so far, concerning how his system and much if not all of his previous discourse on logic has been working towards and in turn will rely upon a thoroughgoing quantification of the predicate, a quantification which notably traditional logic had failed to achieve due, not only to exclusively focusing on extension, but also to the establishment of a class of proposition that admitted vagueness or ambiguity with regard to the subject term's quantification and thereby disabled quantification by failing to make explicit the quantity pertaining to the subject term which, according to Hamilton, is 'involved in every actual thought' though at times not in its linguistic expression (LL.I.244). But, as we shall see, for Hamilton, as this notion that quantity is either explicit or implicit enables the removal of a class of propositions in which the subject term is not quantified in expression, it also underpins the whole notion that the *predicate* term's quantity may also be made explicit.

Following Aristotle, logicians traditionally divided propositions with regard to their extensive quantity by categorising them as: *Universal* or *General*; *Particular*; *Individual* or *Singular*; and, *Indefinite.* According to Hamilton these terms were applied with the following meanings: in Universal propositions 'the subject is taken in its whole extension'; in Particular propositions 'the subject is taken in a part, indefinitely, of its extension'; in Individual propositions 'the subject is at a minimum of extension'; and in Indefinite propositions, 'the subject is not [...] overtly declared to be either universal, particular, or individual' (LL.I.243). With regard to quantification generally, Hamilton claims that *commonly* only the Subject is regulated, whereas the predicate 'Aristotle and the logicians do not allow to be affected by quantity; at least they hold it to be always Particular in an Affirmative, and Universal in a Negative' (LL.I.244). However, he claims that this doctrine is untenable, incomplete, that it resulted in confusion, and that this confusion and incompleteness is partly due to logicians paying insufficient heed to the fundamental principle of explicitness that I explained earlier. At this stage in his lectures, Hamilton might have simply gone straight to quantifying the predicate to show that, by making the quantification of the predicate term explicit, it is no longer requisite that we consider whether the proposition is being considered extensively or intensively. However, he is much more careful to take his students through the evolution of his logic as it involves a critique and a significant modification of traditional logic's assumption that the predicate term in an affirmative proposition is always Particular whereas in a negative proposition it is always Universal.

By contrast with the Aristotelian doctrine of the logicians, which divides propositions into the four classes or species of Universal, Particular, Individual, and Indefinite, Hamilton proposes that they should be differentiated quite differently, and he does this largely by redefining 'indefinite' and thereby eradicating as a distinct class traditional logic's Indefinite propositions, while retaining the notion of indefiniteness as describing, within any given whole, an indeterminate range of quantities sufficiently competent to be classed as the quantifier of particularity we normally express by the term 'some' — in other words Hamilton's 'indefinite' constitutes that species of judgments/propositions the logicians formerly called 'Particular'.

His redefinition of 'indefinite' seems to amount to this: for the logicians 'indefinite' meant little more than that the extensive quantity of the subject (universal, particular, or individual) was unexpressed and thus indefinite in the sense of being unclear or inexplicit; by contrast with this, for Hamilton, 'indefinite' refers more directly to the *quantity* expressed by the terms 'some', 'many', and various other expressions so long as they designate 'some indefinite number less than the whole'. Hence, his definition of 'indefinite' refers to any quantity within a whole, ranging from (possibly) a singular to a number less than the whole to which the indefinite term refers (LL.I.246). It should perhaps be noted here that Hamilton's definition of 'indefinite' may not at first sight seem to differentiate itself sufficiently from the quantity of Individual judgments or propositions, nor from Universal propositions.

As Fogelin claims, Hamilton 'held complicated views on the quantifier *some*', in which, while he sometimes referred to 'some' as meaning 'some but not all', he also used the definition 'some perhaps all', and as Fogelin argues, there seem to be good grounds for saying that Hamilton purposely incorporates both readings of 'some' (Fogelin, 153).[37]

These complicated views on the meaning of Particulars, as that class of propositions which are indefinite, only come to the fore in Hamilton's 'New Analytic' (see NA.LL.II.279-80). However, that by 'some' Hamilton meant 'some but not all', and that he distinguishes between Individual and Particular Propositions does seem to be fairly clear if we confine attention solely to Lecture XIII. Not only does he give little or no indication in Lecture XIII that 'some' might mean 'some perhaps all', but he also makes it very clear that a proposition may be expressed using the indefinite article, and thereby be classed as a Particular proposition, as in one of his examples, '*An Englishman generalised the law of gravitation*'; whereas, when expressed using a proper name, it will be an Individual proposition, as in '*Newton* generalised the law of gravitation' (LL.I.247). This, however, merely illustrates what is required to transform an ordinary language statement from being a Particular proposition to an Individual one and Hamilton's explicitness principle ought to allow that all such indefinite propositions, not involving a plurality, must be expressible as individual or, where the 'some' really means 'all', as universal propositions.

Though Hamilton may not have squarely tackled certain problems to do with his definition of the quantifier 'some', it is at least possible to detect in his virtual eradication of the traditional logic's class of Indefinite propositions, that he is making the treatment of the quantity of propositions more internally consistent by accommodating the indefinite within Particular propositions by removing the obstruction to them being accommodated within Particular propositions, namely, the mistaken condition of their failure to *express* whether they were universal, particular, or individual — a failure that, Hamilton might have pointed out, was entirely due to the logicians not grasping that, in an extensive proposition, where the quantity of the subject is unexpressed it must either be a part (and hence particular) of the predicate as whole; or, in an intensive proposition, its quantity must be capable of being expressed as the containing whole determined by the predicate as part of that whole. To re-state the quotation given earlier: 'If the Subject or determined notion be viewed as the containing whole, we have an intensive or Comprehensive proposition; if the Predicate or determining notion be viewed as the containing whole, we have an extensive proposition' (LL.I.231-2). By itself this will not imply what quantity ought to be appended to the subject of a given proposition where that quantity remains unknown. However, this is

[37] Robert J. Fogelin, 'Hamilton's Quantification of the Predicate' in, *Philosophical Interpretations* (Oxford: Oxford University Press, 1992), first published in *Philosophical Quarterly*, 26 (1976), 149-65. All references to Fogelin are to this edition. Fogelin also supplies in this edition a second article entitled 'Hamilton's Theory of Quantifying the Predicate — A Correction' (166-8), also first published in *Philosophical Quarterly*, 26.

irrelevant as the traditional class of Indefinite propositions regards the proposition in extension alone, and as I have claimed in support of Hamilton, viewing the traditional Indefinite proposition as extensive, determines that the subject term must be particular in relation to the predicate term as the determining whole containing the subject term. What Hamilton has achieved here is the removal of an anomalous class of propositions that is only possible by means of his understanding of the whole-part relationship in propositions viewed extensively and intensively. However, although much more might be said about all this, there is another crucial factor to removing the traditional class of problematic and confusing indefinite or indeterminate propositions, namely, his introduction of the terms 'predesignate' and 'preindesignate'.

It is easy to see how the application of these terms to the traditional class of Indefinite propositions warrants a discrimination between propositions that may thereby be deemed to be either logically adequate or inadequate. When a proposition (presumably considered as an external expression that may thus be more or less precise, as contradistinguished from a judgment, construed here as the mental process) articulates its quantity by prefixing terms such as 'all' and 'some', this is a *predesignate* proposition; whereas when a proposition does *not* articulate its quantity, the proposition is called by Hamilton, *preindesignate*. Hence, the subject and predicate terms of a proposition may also be either predesignate or preindesignate terms. But, for Hamilton, though in a proposition's external expression one or both of its terms may be preindesignate (the quantity often being 'elided in its expression'), the unexpressed quantity is *always* involved in thought (definite or indefinite 'quantity being involved in every actual thought', though not always marked by a quantifier) (NA.LL.II.250; LL.I.244). Hence, adhering to his principle of explicitness, such preindesignate terms may be translated into predesignate ones — which is to say, that unexpressed quantities may always be expressed at least in principle and indeed should be expressed in adherence to the fundamental principle of explicitness. One might therefore say that, with regard to the traditional class of Indefinite propositions, if in fact it is impossible to determine the predesignate term(s) (as quantified in thought) of a given statement purporting to be a proposition but which is somehow quite indeterminate in its meaning by means of its use of a preindesignate term(s) (hence unquantified in expression), then such a statement must be deemed to be non-propositional and inadequate for consideration within a reasoning or argument.

The differences between propositions, with regard to quantity, according to Hamilton, arise, on the one hand, 'from the necessary condition of the Internal Thought' (when we consider them specifically as Judgments), and on the other hand, 'merely from the accidental circumstances of [a proposition's] External Expression' (when we consider them as propositions) (LL.I.243). Thus, he characterises three classes of proposition as properly adequate for logical consideration: *Universal judgments or propositions*, 'in which the whole number of objects within a sphere or class are judged of,–as *All men are mortal*, or *Every man is mortal'*; *Individual judgments or propositions*, in which 'the whole of a certain sphere is judged of, but in which sphere there is found only a single object, or collection of single objects,–as *Catiline is ambitious,–The twelve apostles were inspired',* the individual(s) in question here constituting what Hamilton describes, with possible oblique reference to his notion of involution, 'determinate wholeness or totality in the form of oneness [or] indivisible unity'; and, *Particular judgments* or *propositions*, 'in which, among the objects within a certain sphere or class, we judge concerning some indefinite number less than the whole,–as *Some men are virtuous–Many boys are courageous–Most women are compassionate.* The indefinite plurality, within the totality, being here denoted by the words *some, many, most*' (LL.I.245-6). As he explains by means of reference to the example cited above concerning the conversion of a Particular proposition ('*An Englishman* ...') into an Individual proposition ('*Newton* ...'), although the logicians are right to treat Universals and Individuals as convertible, their correspondence one with the other is not merely due to 'the oneness of their subject', but rather: 'The whole distinction consists in this,–that, in Universals and in Individual Judgments, the number of the objects judged of is thought by us as definite; whereas, in Particular Judgments, the number of such objects is thought by us as indefinite' (LL.I.246). Hence, the major distinction between propositions in terms of quantity is between the definite (universal or individual) and the indefinite (particular). This distinction between definite and indefinite quantity, Hamilton declares most forcefully in his 'New Analytic': '*definite* and *indefinite* are the only quantities of which we ought to hear in Logic; for it is only as indefinite that particular, it is only as definite that individual and general, quantities have any (and the same) logical avail' (NA.LL.II.250).

Thus, bearing in mind that, since Universals and Individuals with regard to quantity, constitute one class of *definite* propositions, whereas Particulars constitute the only alternative quantity and thus class of *indefinite* propositions, all that needs to be added to this twofold *definite* (Universal and Individual) and *indefinite* (Particular) distinction between propositional forms to yield the traditional (yet Hamiltonized) fourfold distinction, is the standard distinction between affirmation and negation, known as the *quality* of the proposition. Hamilton briefly outlines the notion of quality, which he regards as an unfortunately ambiguous and yet generally accepted term to denote affirmation and negation (LL.I.250). Important to his quantification of the predicate, he sensibly argues against some contemporary and earlier logicians who held that affirmation and negation properly belong to the copula and not to the subject nor to the predicate terms. Drawing attention to the non-literal or non-grammatical sense in which the copula should be regarded as expressing the form of the relation between subject and predicate, he argues against certain previous and some modern logicians, that negation does not belong to the predicate term but rather to the copula, allowing him to treat subject and predicate as being held together in a reciprocal relation of whole to part, such that in a negative judgment a part is taken out of a whole, whereas in an affirmative judgment a part is put into a whole (see LL.I.251-4). All that thus belongs to the subject and to the predicate, for Hamilton, is their respective definite (Universal or

Individual) or indefinite (Particular) quantities; but as this enables a distinction between propositional forms according to their quantity (definite or indefinite), traditionally given as Universal or Particular, when these are both distinguished according to their quality (affirmative or negative), this results in the traditional fourfold A, E, I, O distinction of propositional forms: A (universal affirmative); E (universal negative); I (particular affirmative); O (particular negative).

He represents the forms A, E, I, O by using simple circle diagrams (see LL.I.255). I shall not provide these since they merely illustrate the quantity and quality aspects of the four propositional forms and since I shall shortly provide one of Hamilton's later tables from the 'New Analytic' which, by deleting one of the diagrams as redundant and adding one that expresses co-extension, incorporates these circle diagrams into a new set of four figures to which he relates both the traditional forms A, E, I, O, and his additional forms. Although it is highly likely that Hamilton pointed out to his students the quantifications of the predicate and the subject terms implied by the diagrams, for example, that the Universal Affirmative (A) in traditional logic implied an indefinite or Particular predicate, the text does not at this stage make any explicit reference either to the assumptions concerning quantification nor does it advance the thoroughgoing quantification that results in Hamilton doubling the traditional four propositional forms A, E, I, O. It must be noted, however, that much later in the Lectures he does assert that 'The nineteen useful [syllogistic] moods admitted by logicians, may [...] by the quantification of the predicate, be still further simplified, by superseding the significance of Figure' (LL.I.402). Although his quantification of the predicate does not seem to have been made fully explicit, he was clearly at the very least intimating aspects of it to his students some years before his controversy with de Morgan.

Lecture XIV is the last lecture on Hamilton's doctrine of Judgments. Before he terminates the lecture, he makes at least one significant claim worth mentioning, namely, his rejection of Modal propositions as a separate class and his argument that, for example, the modal proposition '*Alexander conquered Darius honourably*' ought to be treated as merely a complex proposition in which the mode is regarded as part of the predicate. As Hamilton points out, the predicate can be more or less complex and there is no need for the Aristotelian logic's modal propositions as 'modified by the four attributions of Necessity, Impossibility, Contingence, and Possibility. [...] in regard to these, the case is precisely the same; the mode is merely a part of the predicate, and if so, nothing can be more unwarranted than on this accidental, on this extra-logical, circumstance to establish a great division of logical propositions' (LL.I.257). Once again Whately comes under fire concerning this, as also when Hamilton moves on to discuss and outline some basic points concerning the subject of the conversion of judgments or propositions, such as when the subject and predicate are transposed in a categorical proposition (see LL.I.258–9; 262–3). I shall come back to the subject of conversion briefly later, but for the time being I shall leave Hamilton's lectures and the commencement of his introduction to reasoning and the syllogism, and instead leap forward to his later work in the fragmentary but nonetheless insightful and more mature work, his

'New Analytic of Logical Forms'. For, within Lecture XIV Hamilton has arrived at a significant stage in laying the groundwork for his quantification of the predicate. Although by this stage he has yet much more to construct, it would seem that he has reached a critical point that has by now established a warrant for providing the thoroughgoing quantification that I shall illustrate in the following section.

4 QUANTIFICATION OF THE PREDICATE

> THIS NEW Analytic is intended to complete and simplify the old — to place the keystone in the Aristotelic Arch. (NA.LL.II.249)

It is abundantly clear in the 'New Analytic' that Hamilton's principle of explicitness, or rather the thoroughness of his *adherence* to this principle, is a major driving force behind his quantification of the predicate. He cites the postulate or principle as something insisted upon by Logic, but insufficiently adhered to by previous logicians, claiming on the basis of this principle, 'that, logically, we ought to take into account the *quantity*, always understood in thought, but usually, and for manifest reasons, elided in its expression, not only of the *subject*, but also of the *predicate*, of a judgment' (NA.LL.II.250; and see p. 252). Making explicit the quantities of both the subject and the predicate in a judgment or proposition facilitates regarding the subject and predicate terms as comprising 'an equation'. It is important to understand how it can be said that the whole-part relationship in extension or intension may be thought of as constituting either an equation or non-equation. Though in some places Hamilton does not seem to be as clear as he might have been on this point, at other places his explanation is substantially more helpful than the often rather submerged hints in the Lectures. For example, in the 'New Analytic' Hamilton's treatment of the subject of the Conversion of Categorical propositions clarifies what he thinks erroneous in traditional logic's Conversion of propositions with regard to quantity.

He regards the doctrine of Conversion as 'beset with errors' but that these errors are generated from two principal ones — Hamilton is worth quoting at length here:

> The First cardinal error is,–That the quantities are not converted with the quantified terms. For the real terms compared in the Convertend [the original proposition], and which, of course, ought to reappear without change, except of place, in the Converse [the proposition converted], are not the naked, but the quantified terms. This is evident from the following considerations:
>
> 1^o, The Terms of a Proposition are only terms as they are terms of relation; and the relation here is the relation of comparison.
>
> 2^o, As the Propositional Terms are terms of comparison, so they are only compared as Quantities,–quantities relative to each other. An Affirmative Proposition is simply the declaration of an equation, a Negative Proposition is simply the declaration of a non-equation, of its

terms. To change, therefore, the quantity of either, or of both Subject and Predicate, is to change their correlation,–the point of comparison; and to exchange their quantities, if different, would be to invert the terminal interdependence, that is, to make the less the greater, and the greater the less.

3^o, The Quantity of the Proposition in Conversion remains always the same; that is, the absolute quantity of the Converse must be exactly equal to that of the Convertend. It was only from overlooking the quantity of the predicate [. . .] that two propositions, exactly equal in quantity, in fact the same proposition, perhaps, transposed, were called the one universal, the other particular, by exclusive reference to the quantity of the subject.

4^o, Yet was it of no consequence, in a logical point of view, which of the notions collated were Subject or Predicate; and their comparison, with the consequent declaration of their mutual inclusion or exclusion, that is, of affirmation or negation, of no more real difference than the assertions,–*London is four hundred miles distant from Edinburgh,–Edinburgh is four hundred miles distant from London.* In fact, though logicians have been in use to place the subject first, the predicate last, in their examples of propositions, this is by no means the case in ordinary language, where, indeed, it is frequently even difficult to ascertain which is the determining and which the determined notion. [. . . .]

The Second cardinal error of the logicians is, the not considering that the Predicate has always a quantity in thought, as much as the Subject; although this quantity be frequently not explicitly enounced, as unnecessary in the common employment of language; for the determining notion or predicate being always thought as at least adequate to, or co-extensive with, the subject or determined notion, it is seldom necessary to express this, and language tends ever to elide what may safely be omitted. But this necessity recurs, the moment that, by conversion, the predicate becomes the subject of the proposition; and to omit its formal statement is to degrade Logic from the science of the necessities of thought, to an idle subsidiary of the ambiguities of speech. [. . . .]

1^o, That the predicate is as extensive as the subject is easily shown. Take the proposition,–*All animal is man*, or, *All animals are men.* This we are conscious is absurd, though we make the notion man or men as wide as possible; for it does not mend the matter to say,–*All animal is all man*, or, *All animals are all men.* We feel it to be equally absurd as if we said,–*All man is all animal*, or, *All men are all animals.* Here we are aware that the subject and predicate cannot be made coextensive. If we would get rid of the absurdity, we must bring the two notions into coextension, by restricting the wider. If we say,–*Man is animal*,

> (*Homo est animal*), we think, though we do not overtly enounce it, *All man is animal.* And what do we mean here by *animal?* We do not think,–*All*, but *Some, animal.* And then we can make this indifferently either subject or predicate. We can think,–we can say, *Some animal is man*, that is, *Some or All Man*; and, *e converso*,–*Man (some* or *all) is animal*, viz. *some animal.*
>
> It thus appears that there is a necessity in all cases for thinking the predicate, at least, as extensive as the subject. Whether it be absolutely, that is, out of relation, more extensive, is generally of no consequence; and hence the common reticence of common language, which never expresses more than can be understood — which always, in fact, for the sake of brevity, strains at ellipsis. (NA.LL.II.257-9)

This lengthy quotation brings several things to our attention: that Hamilton regards the quantifying terms as integrant components of the 'naked' terms or concepts or judgments used in any given proposition — and hence whenever the 'naked' terms are transposed in some conversion, their clothing (the quantifier) goes with them; the terms of a proposition only exist *qua* terms as related to one another by a comparison of their respective quantities, and that their quantities will be identical or equated in an affirmative proposition, non-identical or not equated, but rather related by some confliction to do with their quantities, in a negative proposition — violation of this relation of comparison or equation/non-equation is to change without warrant the relation being asserted in their very predication; that it is mistaken to deem a proposition to be Universal or Particular solely by attending to the status of the quantification of its subject; and, Hamilton's reliance on his principle or postulate of explicitness as the fundamental principle of Logic is invoked as warranting exposure of what *must* (on pain of otherwise thinking an absurdity) be thought with regard to the predicate's quantity and thereby its relation of equation or non-equation with the quantified subject.

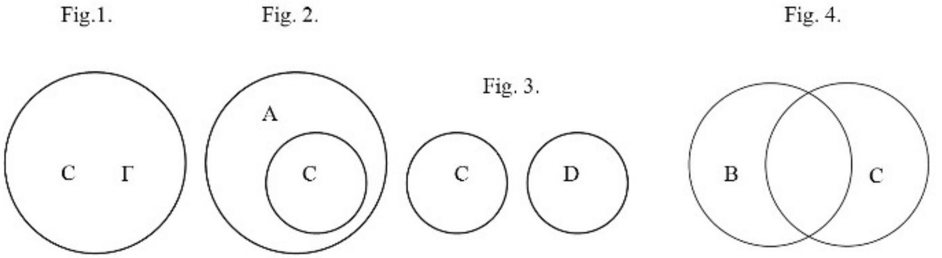
This last point concerning how in thought if not in linguistic expression (due to our propensity in linguistic expression to elide quantities into the 'naked' terms) strongly suggests a mental and in this sense private language of subject-predicate equation/non-equation, a mental relation of the respective quantities of both subject and predicate, that is often but not always behind the scenes and tantamount to the fundamental necessary laws of logic themselves. However, Hamilton seems to be at pains to describe these laws as in some sense natural, informing actual discourse, and in turn operating as the standard against which rhetorical utterance may be tested and to which rhetoric may be reduced without attempting to modify logic's laws to suit grammar: 'We should not do as the logicians have been wont,–introduce and deal with ['the rhetorical enouncements of common speech'] in their grammatical integrity; for this would be to swell out and deform our science with mere grammatical accidents; and to such fortuitous accrescences the formidable volume, especially of the older Logics, is mainly owing. In fact, a large proportion of the scholastic system is merely grammatical' (NA.LL.II.262). The tendency in Hamilton's logic to correct and simplify traditional logic, thus marks a major departure from regarding logic as an attempt to capture the vagaries and complexities of grammatical rules, real-life argumentation, and rhetoric in order to keep logic focused on the laws of necessary inference. However, although this approach warrants the unnatural sounding expressions that result from quantifying the predicate, as in 'All men are some mortal', Hamilton is not such a purist as to eschew utterly the notion of Pure Logic's relevance to and capacity to translate 'the rhetorical enouncements of common speech', nor is his quantification of the predicate justified solely by means of reference to a merely dogmatic assertion that quantity is 'always understood in thought' though often elided in its linguistic expression, since his arguments in support of quantifying the predicate incorporate appeals to common instances in which the quantification is made explicit: 'in fact, ordinary language quantifies the Predicate so often as this determination becomes of the smallest import' (NA.LL.II.259).

Be all that as it may, it becomes clear in the 'New Analytic' that one of Hamilton's major achievements with regard to the complex and, as he often asserts, confusing doctrines, rules, and practices of the logicians has to do with how his system effectually sweeps away various different types of conversion. With the establishment of his quantification of the predicate the only defensible type of conversion is the simple conversion he advocates. His simple conversion relies wholly upon the predicate's quantification being made as explicit as the quantification of the original subject term. Hence, simple conversion merely involves whatever transposition of terms is possible so long as the respective quantifiers of the original subject and predicate terms remain attached to these terms in order to retain in conversion any given proposition's meaning as an equation or non-equation of the quantities of the two terms in the original proposition. Over and again Hamilton emphasises the erroneousness of earlier species of conversion, both with regard to affirmative and negative propositions (see NA.LL.II.256-76). The logicians had missed 'the one straight road' of conversion, simple conversion, and Hamilton makes the ambitious but clearly defensible claim that if, by means of his quantification of the predicate, he is right in having reduced all species of conversion to the simple conversion he advocates, then 'the whole doctrine of logical Conversion is superseded as operose and imperfect, as useless and erroneous. The systems, new and old, must stand or fall with their doctrines of the Conversion of propositions' (NA.LL.II.276).

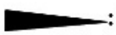
Though some elements of Hamilton's construction of his system of quantifying the predicate have been overlooked above, I think I have given an ample outline of his notions concerning the quantification and how this radically supersedes much of the traditional logic while yet incorporating and building upon at least some part of it. However, we now need to look at Hamilton's quantification procedure in some more detail and the best place to commence this is by examining the table supplied at Appendix V section (d) in the second volume of his *Lectures on Logic*, which, with only a few minor adjustments, I have replicated below as Table 2.

The only significant modification I have made to Hamilton's table, is to adopt

Table 2. Application of Doctrine of Quantified Predicate to Propositions
New Propositional Forms — Notation



Affirmative.

(1)	[A f A]	C :  : Γ	All Triangle is all Trilateral [fig. 1].
(ii)	[A f I]	C :  , A	All Triangle is some Figure (A) [fig. 2].
(3)	[I f A]	A ,  : C	Some Figure is all Triangle [fig. 2].
(iv)	[I f I]	C ,  , B	Some Triangle is some Equilateral (I) [fig. 4].

Negative.

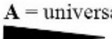
(v)	[A n A]	C :  : D	Any Triangle is not any Square (E) [fig. 3].
(6)	[A n I]	C :  , B	Any Triangle is not some Equilateral [fig. 4].
(vii)	[I n A]	B ,  : C	Some Equilateral is not any Triangle (O) [fig. 4].
(8)	[I n I]	C ,  , B	Some Triangle is not some Equilateral [fig. 4].

Key:

1, 3, 6, 8 – Hamilton's forms

ii, iv, v, vii – Aristotelic or traditional forms. Hence, using the scholastic letters A, E, I, O: A (Universal affirmative) – ii; E (Universal negative) – v; I (Particular affirmative) – iv; O (Particular negative) = vii.

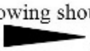
A = universal; **I** = particular; **f** = affirmation; **n** = negation; **,** = some; **:** = all.

 = the affirmative copula (*is*)

 = the negative copula (*is not*)

The two arrow lines above indicate which of the terms is subject and which predicate thus: where the proposition is being read in Extension, the thick end of the arrow line denotes the subject, the thin end the predicate; where the proposition is being read in Intension, subject and predicate are reversed and hence the thick end of the arrow line denotes the predicate, the thin end the subject.

To illustrate how to read Hamilton's symbolic representation of a proposition in Table 2, note that the following should be read thus:

C :  , A

Extensively: *All* C is *some* A – i.e. All C *is contained under* some A

Intensively: *Some* A is *all* C – i.e. Some A *contains in it* all C.

his later method of symbolising the Universal and Particular terms using the letters A for Universal, and I for Particular (enclosed in square brackets). This alteration should also be of some help to readers who may wish to make comparisons between Table 2 and Hamilton's more detailed 'Table of the Mutual Relations of the Eight Propositional Forms on Either System of Particularity' to which I shall refer later (see NA.LL.II.284).

Table 2 is interesting in several ways. Firstly, the various configurations of A (universal/definite) and I (particular/indefinite) under Affirmative and Negative regularly display all of the possible permutations of a thorough quantification of both subject and predicate as neatly displayed in the symbols given in square brackets such as $[A\ f\ A]$. Secondly, the arrow lines indicate which of the terms is the subject and which the predicate, depending on whether the proposition is to be read as extensive or intensive — hence, the terms may be easily transposed according to Hamilton's method of simple conversion without any alteration in the proposition's meaning using the same symbolic notation for both an extensive and an intensive reading (though it must be said that by this stage in Hamilton's logic, having made so much of extension and intension to establish the dual perspectives from which propositions may be viewed, how thus the subject and predicate terms may be transposed, and how the number of syllogistic forms can thereby be amplified, the extension-intension distinction seems to fall out of account as superseded by the full quantification itself). Thirdly, using Hamilton's own terminology and categorisation, the four circle diagrams need to be thought of as expressing not four but three possible principal relations of: *Toto-total Coinclusion* (fig. 1); *Toto-total Coexclusion* (fig. 3); and, brought together under one class of counter-related relations, *Incomplete Coinclusion* and *Coexclusion* (fig. 2 and fig. 4). Fourthly, Hamilton's example of proposition (1) uses the symbol 'Γ', but since it only appears in this proposition, it is not immediately clear which of the other propositions may be said to be its contradictory or negation, though with a simple change of terms proposition (v), describing the relation of Toto-total Coexclusion (fig. 3), seems to be the most obvious contradictory of proposition (1) as describing Toto-total Coinclusion (fig. 1). These last two points require further explanation and as we shall see this will involve some discussion of a major source of difficulty and controversy concerning Hamilton's quantification.

Firstly, my third point above: in Hamilton's system the relationship between subject and predicate in each proposition needs to be thought of as a relationship of *mutuality.* This becomes much more clear when we take note of Hamilton's 'Observations on the Mutual Relation of Syllogistic Terms in Quantity and Quality' at Appendix V (e) in the *Lectures on Logic* (see NA.LL.II.285). With reference to the circle diagrams in Table 2 above, these relations can be given as follows:

1. *Toto-total Coinclusion* (fig. 1) — the relation of 'coidentity, absolute convertibility or reciprocation'.
2. *Toto-total Coexclusion* (fig. 3) — the relation of 'non-identity, absolute inconvertibility or non-reciprocation.

3. *Incomplete Coinclusion* and its counter-relation *Incomplete Coexclusion* (fig. 2 and fig. 4) — the relations of 'partial identity and non-identity, relative convertibility and non-convertibility, reciprocation and non-reciprocation'. Under this counter-related pair of Incomplete Coinclusion and Coexclusion, Hamilton details all of the propositional forms he regards as intermediaries between the extreme opposites of proposition (1) Toto-total Coinclusion, and proposition (v) Toto-total Coexclusion as:

   (ii) Toto-partial coinclusion (fig. 2)
   (3) Parti-total coinclusion (fig. 2)
   (iv) Parti-partial coinclusion (fig. 4)
   (6) Toto-partial coexclusion (fig. 4)
   (vii) Parti-total coexclusion (fig. 4)
   (8) Parti-partial coexclusion (fig. 4)

   (see NA.LL.II.285)

As Hamilton argues in his Lectures, in keeping with traditional logic a universal (A or E) may be treated as an individual or as a universal proposition — hence, the *Toto-total Coinclusion* and *Coexclusion* relations represented in Table 2's proposition (1) [fig. 1], and proposition (v) [fig. 3] respectively, may be translated in two ways as referring either to: two single/individual entities (A and B) that are (1) identical or (v) non-identical; or, two groups (collections) of things, all of which in each collection (A and B) are *sufficiently resembling to be asserted as* (1) identical or (v) non-identical — hence Fogelin seems to be right to claim that 'Hamilton develops his theory of universal propositions on an existential (rather than a Boolean) interpretation)', though arguably, as seems to be the case with his specific illustrations of propositions (1) and (v), he does also accommodate *a priori* truths constituted by wholes that may be thought of as having no existential import as physical realities, the whole and thus individual that is 'All Triangle' existing in thought, though its ontological status, as a purely ideal whole, is by no means unrelated to the whole that might be any more or less complete collection of real triangles as instantiations of the unity of thought that is 'All Triangle' (see Fogelin, p. 152). Incidentally, the complex and interesting topic of Hamilton's understanding of the existential import of propositions, which I have merely touched on here, is not perhaps as one at might first think, since Hamilton is conscious that 'the Logician has a right to suppose any material impossibility, any material falsity; he takes no account of what is objectively impossible or false, and has a right to assume what premises he please, provided they do not involve a contradiction in terms' (LL.I.322; also see, p. 338; p. 360).

Though in his Lectures Hamilton seems to keep to just one sense of the quantifier 'some' it becomes clear in the 'New Analytic' that particulars, though in each and every case an *indefinite* quantity, may be indefinite in two different senses, either: as *Indefinite Definitude*, in the sense expressed by '*some, at least*' (which is to say, 'at least one, possibly more, but not all'); or, as *Definite Indefinitude*, in the sense of '*some, at most*' (which is to say, 'some, perhaps all, but not less than one') — hence, the *Incomplete Coinclusion* and *Coexclusion* relations represented in Table 2's fig. 2 and fig. 4, may also be translated in two different ways to accommodate this difference in the possible senses of the Particular quantifier 'some'. The significance of these points should become clear later, but in the meantime, I want to claim that while Hamilton made a great deal of the importance of Logic as a Pure science that must not become intermixed with any grammatical, linguistic, rhetorical, or other material concern (inasmuch as this is practically possible), as Fogelin rightly points out, 'Hamilton acknowledges both interpretations of the quantifier *some*, but only insists that each interpretation must be examined in order to capture all the everyday inference patterns a logician should study' (see Fogelin, p. 153). It does seem as though Hamilton is attempting to make his system sufficiently accommodating, such that it can encompass different readings of 'some' (or can embrace different degrees of indefiniteness). His system can also be applied, not simply to both individual and universal quantities (as the only definite quantities), but also, on the one hand: to *a priori* truths or universals, the wholeness of which is as an unanalysable individual/singular, their ontological status being ideal and thus *potentially* existing only in thought, though *actually* never out of relation to, real entities; and, on the other hand and more conspicuously, his system can be applied to those universals that we might better describe as *general* terms, the *a posteriori* nature of which implies that at best they are only approximate or provisional universals which may admit of some exceptions without nevertheless losing their applicability in a syllogistic reasoning as universal terms.

Now, to come to the second main observation I want to make about Table 2: Hamilton's proposition (1) is diagrammatically represented by figure 1 in Table 2 and figures 2, 3, and 4 all look as though they illustrate relations that must stand counter to proposition (1), but since the example Hamilton gives uses a term 'Γ' that he does not replicate elsewhere in Table 2, it is not immediately clear which of the other propositions may be said to be its contradictory or negation. Figure 1 absolutely equates C with Γ and thus describes an absolute identity or Toto-total coinclusion relation between C and Γ. This is also illustrated by Hamilton's example of a possible proposition that might express this relation between the judgments or concepts C and Γ: 'All Triangle is all Trilateral'. However, surely both C and Γ collect together respectively the entire class of C (all possible shapes and sizes of triangles) and Γ (all possible Trilateral figures) and, linked by the copula, the proposition 'All C is all Γ' brings these two quantities into a relation of comparison? If so, does this mean that de Morgan's interpretation of proposition (1) is right, namely, that it expresses a complex proposition constructed by compounding 'every C is Γ' and 'every Γ is C' (de Morgan, p. 257)?

In Fogelin's first attempt to express the meaning of Hamilton's proposition (1), he accepts de Morgan's interpretation and (though shown using *A* and *B* as terms) Fogelin thus translates proposition (1) as ‘All C is Γ and all Γ is C’ (Fogelin, p. 151). On this reading of proposition (1), according to de Morgan, it is contradicted by one of either proposition (6) or (vii), which we may here give, not as in the illustrative propositions given in Hamilton’s Table 2 but, to conform to the letter symbols used in proposition (1), as follows: (6), ‘some Γs are not Cs’; (vii), ‘some Cs are not Γs’ — these are, for de Morgan, the contradictories of proposition (1), ‘All C is all Γ’ (de Morgan, p. 257). To be sure, as is clearly seen simply by comparing figure 1 (illustrating proposition (1)) and figure 4 (illustrating propositions (6) and (vii)), both (6) and (vii) must stand in a negative relation to proposition (1). In a helpful but somewhat complex and possibly inaccurate table, Hamilton asserts that (6) and (vii) stand as what he calls *unilateral contraries* of (1) — this seems to suggest that he would accept that (6) and (vii) do severally contradict (1). However, as *contraries* neither (6) nor (vii) properly constitute a contradiction of (1) (see, NA.LL.II.284). Now, it might at first sight seem odd that neither Fogelin nor (in his original critique of Hamilton) de Morgan, mention proposition (v) as the contradictory of (1), since, translating this into symbols consistent with (1) and in line with de Morgan’s reading, proposition (v) should be read as stating ‘No C is Γ’ and indeed, to be consistent with de Morgan’s translation of proposition (1), he ought to have translated this as ‘No C is Γ and No Γ is C’. However, de Morgan does mention proposition (v) in a later footnote as being offered as the contradictory of (1) by ‘an eminent defender’ of Hamilton’s system (de Morgan, p. 258n1). But de Morgan brushes this aside as not being ‘*in the system’*. Quite what de Morgan means by this is rather unclear, especially since as Fogelin points out, ‘de Morgan is just wrong in suggesting that the system of propositions do not pair up into proper contradictories’, and he goes on to list, (1) and (8); (ii) and (vii); (3) and (6); (iv) and (v) as contradictory pairs. But still, it may seem puzzling why (1) Toto-total coinclusion and (v) Toto-total coexclusion should not be thought of as contradictories. Proposition (v)’s relation to (1), is displayed in one place by Hamilton in such a way as to suggest that (v) and (1) mutually contradict one another (see NA.LL.II.286). However, in another place Hamilton gives their relation as one of *bilateral contraries* — this is to say that both the Toto-total coinclusion of the terms in (1) as shown in figure 1 of Table 2, and the Toto-total coexclusion of the terms in (v) as shown in figure 3 of Table 2, stand *not* as contradictories of one another but both potentially false; which is of course to say, that if (1) is true though (v) must be false and *vice versa,* since (1) and (v) may both be false they are not strictly contradictories (see NA.LL.II.284).

Be all that as it may, as Fogelin rightly points out, the trouble lies with proposition (8) which de Morgan rejects as having no contradictory within the system. To establish against de Morgan’s rejection of proposition (8) that Hamilton’s system is comprehensive and not inconsistent, we need to be able to answer the question: what is the true contradictory of (8) *in the system*? This is an important question since if there is no contradictory of (8) within the system, de Morgan is right to assert that it has not been generated from any necessary laws of thought but rather, as de Morgan so derisorily claims, on the basis of ‘an arbitrary extension of the application of language' (de Morgan, p. 258n1). However, as intimated above, Fogelin ably answers de Morgan's rejection of (8) by arguing that its true contradictory is (1). Incidentally, although de Morgan can be fulsome in his praise of Hamilton — but then so can Mill — de Morgan's treatment of Hamilton's quantification scheme is at times rather scurrilously worded. It would seem that both de Morgan and Mill were, rather excessively, much given to resort to more or less veiled abusive *ad hominem* attacks on Hamilton, and I cannot help but comment here that while Hamilton's frequent denunciations of others, including de Morgan, must have to some extent provoked such responses, his achievements and reputation as a logician have most certainly suffered unduly from the cheap rhetorical tricks of opponents whose conduct ought to have been exemplarily fair, not solely as a mark of respect for Hamilton's considerable endeavours but also as a generally more virtuous way of conducting their discourse — paradoxically, it would seem that rather too often winning the argument is much more important to those who should be most concerned with striving to resolve it satisfactorily.

Fogelin's defence of Hamilton is an incisive attempt to redress the balance and although I shall not rehearse the full extent of his critical examination of Hamilton's system, to date it stands as one of the strongest defences of Hamilton's quantification of the predicate. Fogelin's defence is carried out in part by translating the controversial proposition (8), which de Morgan also claims with complete disdain was erroneously, and, so he implies, foolishly offered by one of Hamilton's defenders as the true contradictory of (1) (de Morgan, p. 259n3). As Fogelin rightly says about proposition (8), 'it is this proposition that has been the constant source of confusion' (Fogelin, 151). However, as Fogelin rightly attempts to show in his first article on Hamilton's quantification, proposition (8) is indeed the contradictory of proposition (1). But, Fogelin's second article on Hamilton makes an important correction to his first attempt to establish that proposition (8) genuinely contradicts proposition (1), and it is therefore to this second article that I shall now refer.

According to Fogelin's reading of proposition (1) this should be interpreted as: 'Anything that is an A is identical with anything that is a B' — which 'means that there is but one thing that is an A, one thing that is a B, and these things are identical' (Fogelin, p. 167). This is to say, as Fogelin argues, that de Morgan's interpretation of proposition (1) — 'All A is B and all B is A' — is wrong. However, de Morgan was right in his interpretation of proposition (8) and hence (8) can be stated as 'Some A is not some B', which may then be translated as the contradictory of (1) as: 'Something that is an A is not identical with something that is a B'. Fogelin goes on to demonstrate just how this interpretation renders a certain syllogism (which on his previous interpretation of proposition (8) is invalid), can be shown to be valid using his second (and de Morgan's original) reading, thereby proving that Hamilton's system is 'saved from inconsistency' (Fogelin, p. 168). Although the relation of Hamilton's quantification system to syllogisms is of course important, instead of looking at this I want to enrich Fogelin's argument somewhat by considering his interpretation of proposition (1) as meaning 'that there is but one thing that is an A, one thing that is a B, and these things are identical'.

To return to Hamilton's own example, 'All Triangle is all Trilateral': as expressing the relation of Toto-total coinclusion, this expresses the nearest thing (along with Toto-total coexclusion) Hamilton would call an absolute, which is to say that the subject and predicate terms are co-inclusively related as maximally similar such that they may be said to be absolutely convertible one with the other — they are thus absolutely *reciprocal*. Whatever, if anything, *might* be said to differentiate 'all Triangle' from 'all Trilateral', such that they can be clearly if not distinctly separated into two entities (and this may merely be that the terms themselves are different signs both of which signify the same single entity), as two terms brought into a mutual relation of coidentity, this coinclusion is nonetheless one in which all material difference has been abstracted from thought, such that the co-identities are indeed absolute and the proposition that articulates this is thus effectually an implicit denial of their *non*-identity. Hence, such propositions asserting Toto-total coinclusion that involve merely a *terminal* differential assert a co-identity between two terms, and as such these propositions can be most directly contradicted by the assertion of the most minimal difference in their quantity, since with regard to their quantity alone their co-identification implies a unity or singularity — hence the assertion of parti-partial coexclusion in proposition (8) is this most direct contradiction of the unity expressed by proposition (1) in terms of two unities being co-identical, since the very ground of (1) being the expression of absolute identity is at once negated by proposition (8)'s assertion that the co-identities are co-excluding.

But, we need to keep in mind that, for Hamilton, there is no absolute *exclusion* in that relation between concepts known as Exclusion. Since he rejects absolute exclusion, but also since Hamilton so permeates his system with correlations of one sort or another, it seems fair to regard his system as ultimately one in which, as there can be no absolute exclusion, there can also be no absolute *inclusion* or perfect identity/unity. Rather, the universals of, say, the Toto-total Coinclusion proposition (1), as relating at least two terms together, are assertions of either: an *approximate* (adequate) but non-maximal co-identification; or, an *absolute* co-identification that *is* total, but *only to the extent* that the subject and predicate being equated in the proposition rely upon some merely nominal/terminal differential. For Hamilton it would seem that *some* differential is the minimum requirement for any concept, judgment, or reasoning to be *possible*, whether this differential is actual (thus rendering the universal approximate), or is so crucially dependent upon the merely terminal as to render all other distinction between them impossible. Even when proposition (1) may seem to be an affirmation or assertion of perfect identity, for (1) to exist as a proposition or material expression of the unity of thought in which A is identified with B, it must consist of at least two entities. As such, 'All A is all B' is most directly contradicted by the assertion of the sole proposition that most adequately breaches or contradicts the *relation* of coinclusion affirmed by 'All A is all B'. For (1) and (8) to be contradictories, the 'all' in (1) quantifies both subject and predicate as individuals absolutely co-identified (though the 'absolute' here, must involve *some* differential), while the 'some' in (8) must quantify the subject and predicate of (8) to be at least the individuals referred to in (1) also, which (8) must then be asserting are coexclusive to an extent beyond the necessary differential making (1) possible, such that these individuals in (8), contradicting (1), may be said to be co-exclusive. The contradictory of proposition (1), as before, is hence the assertion of Parti-partial Coexclusion in proposition (8), 'Some A is not some B', or as Fogelin interprets this, 'Something that is an A is not identical with something that is a B'.

Now, the unity of A and B's coinclusion or co-identification in thought, being expressed as a coinclusion relationship between the two terms 'A' and 'B', is the expression of a bringing into unity concepts which, as Hamilton took much trouble to explain in his Lectures, are themselves relative, since a concept consists of disparate entities thought as one by means of the degree of resemblance between their several attributes — this is why in the bulk of actual cases of proposition (1) expressing Toto-total coinclusion we need to think of their co-identity as approximate. However, the *best examples* of proposition (1) may be thought of as propositions in which the subject and predicate terms are concepts or singularities/individuals in which the constituents that respectively define them, which is to say their intensive quantities, are not mere bundles of pluralities possibly thought erroneously or merely approximately as constituting two unities. Instead, in a best case example of proposition (1) the subject and predicate terms will be constituted by attributes that so overlap or co-inform one another in meaning that they comprise what one might just as well call a true whole, a whole the parts of which are as notions involving and involved. Thus, the intensive quantities of both the subject term and the predicate term in a best case example of proposition (1) will be involuted wholes, which are in turn related to one another by Involution to form what we might call a true whole. Such true wholes are best exemplified by *a priori* truths in which each of the terms is involved and involving in each other. Now, while this may not be fully satisfactory, some such extrapolation from at least Hamilton's notion of involution and some other elements of his work on Logic, seem to go a long way to justify Fogelin's reading of the real meaning of proposition (1), namely, that it is the assertion that there is but *one* thing that is an A, and *one* thing that is a B, *and* these things are identical. The unities co-identified in a best case example of proposition (1), as involuted wholes, themselves both involved and involving one another, bespeak the nearest true whole or unity that Hamilton can admit into his system. Hence, the only possible and most efficient and immediate contradiction of the coidentity of A and B, yet again, must be the assertion of Parti-partial coexclusion between 'A' and 'B' as expressed in proposition (8), for this is *not* to find a mere single *exception* within A or B that is not a co-identical attribute of both, but rather this is to declare the *non*-identity or co-exclusion of the two things that are A and B, and thus (8) is the contradiction of the unity that proposition (1) asserts.

However, this story, complicated enough, is not yet fully resolved. For Hamilton lists propositions (1) and (8) as *Compossible*, which is to say, that they are mutually but not existentially contradictory of one another (i.e. though contradictory they can coexist). Intriguingly this Leibnizian notion of compossibility is expressed by means of empty spaces in the relevant columns of Hamilton's 'Table of the Mutual Relations of the Eight Propositional Forms' (see NA.LL.II.284). But if (1) and (8) are compossible does this not run contra to Fogelin's strong defence of Hamilton's system? As I understand it, on Fogelin's reading, the contradictoriness of (1) and (8) is existential. So, how can Hamilton be right to class (1) and (8) as compossible if they must be regarded as existentially contradictories?

What we are looking for is an interpretation of (1) and (8) that shows that they mutually contradict one another and yet may be said to be coexistent. One way in which (1) and (8) may be said to be compossible has to do with the sense in which (1) expresses an absolute unity, and the sense in which (8) expresses indefiniteness. For Hamilton, (8) expresses either one of two different species of indefinitude. Thus, all that (8) *may* be asserting against (1) is the very differential implicit within (1)'s terminal difference between its subject and predicate. For, while (8) may operate in some instances and be used within a syllogism as if it *is* the existential contradictory of (1), as soon as it in fact loses its Particular/indefinite status and becomes either an individual or a universal in both quantities — as soon as we *know* that its terms have to be quantified as universal/individual (hence definite) — it must immediately be transformed into proposition (v), 'Any A is not any B'/'No A is any B'. Of course, were we able to introduce, as a known, *partial* definitude, (8) would similarly metamorphose into either (6) or (vii). Thus, while (1) and (8) do mutually contradict one another as conflicting assertions, the indefiniteness of (8) and the strictly non-absolute nature of even a best case example of (1) does not warrant *knowledge* of *existential* contradiction (mutual annihilation), though such coexistence may be for us unimaginable or inconceivable.

But this does not resolve the problem since it seems to amount to saying that we can *regard* (8) as *functionally* the existential contradiction of (1) though possibly not its true contradictory but rather the *condition* of (1)'s existence and vice versa, and hence that (1) and (8) are propositions the compossibility of which is dependent on our epistemological limitation. Now, granted that (1) is the assertion of unity, (8) the assertion of divisibility, it might be the case that Hamilton's classification of (1) and (8) as compossible simply *serves to remind us* of the very epistemological limitation — our *nescience* — with regard to anything absolute and anything infinite similar to what he enounced in 'Philosophy of the Unconditioned' by means of his Law of the Conditioned. But, the problem with this is that if Hamilton's classification of (1) and (8) as compossible relies solely on the *possibility* that their mutual contradiction is already inherent within both of these propositional forms, then the attribution of compossibility is pointless since it merely reiterates what we might regard as an implicitly admissible distinctness between the subject and predicate terms in (1), and were it the case that (8) only functioned to perform this reiteration, as Hamilton himself suggests, (8) itself would be useless (see *Discussions*, p. 695).

But these points notwithstanding, the compossibility of (1) and (8) may reside quite simply in Hamilton's principle of explicitness, as follows: in logic we must be allowed to express what we must be allowed to think and therefore we can think and express the contradiction even of that which seems to disallow contradiction (for example, 'All Triangle is all Trilateral') — and in this sense (8) is the expression of what contradicts (1), even though its contradiction ('Some Triangle is not some Trilateral') may be or seem to us to be an expression lacking all existential import and merely derived as the incomprehensible negative of an assertion of a necessary truth and absolute unity. If this is so, then the compossibility of (1) and (8) is simply the coexistence of what *formally* contradicts but does not thereby *entail* existential contradiction. This does not rule out that in fact a given particular instance of (1) and (8) may existentially contradict one another, but since the indefinite status of (8) cannot provide positive knowledge, excepting that it asserts co-exclusion where (1) asserts co-identity, (1) and (8) have to be regarded as *formally* compossible.

For all its seeming weirdness, baulked at and ridiculed by de Morgan, and for all the difficulty of attempting to resolve this problem of compossibility, I think Hamilton is right to defend proposition (8) as importantly the *declaration* that a whole of any kind is divisible, or that the assertion of an indivisible unit may be negated by the assertion of divisible plurality (see *Discussions* pp. 695-6). As such (8) does contradict or is the negation of proposition (1) as the declaration or 'judgment of indivisibility, of unity, of simplicity' (see *Discussions*, p. 695). However, retaining (1) and (8) as formally compossible draws attention to a certain freedom of thought within Hamilton's system that underscores the importance of being able to think and thus express our ability at least to *entertain* the negation of any assertion of an absolute or indivisible whole. This liberal approach is one that endorses the enormous importance of the dialectical nature of logic and argumentation as it recalls from obscurity the seemingly innocent, impotent, and merely theoretic, but sophisticated, potent, and practically useful parti-partial negative of otherwise unquestionable absolutes. If I am right so to interpret at least part of the *significance* of Hamilton's defence of proposition (8), and thus of the complete range of his thoroughgoing quantification system, what we seem to have between (1) and (8) is something akin to the contradictory relationship between absolute and infinite that, as it first brought Hamilton fame with his controversial and yet highly potent and profoundly influential Law of the Conditioned, placed the conditioned nature of legitimate thought within a frame or field of reference at once vertiginously complex and yet competent to enable a rational negotiation with and within an awareness of this complexity.

5 THE SYLLOGISM: SOME IMPLICATIONS

> The science now shines out in the true character of beauty,–*as One at once and Various.* Logic thus accomplishes its final destination; for as 'Thrice-greatest Hermes,' speaking in the mind of Plato, has expressed it — '*The end of Philosophy is the intuition of Unity.*' (NA.LL.II.252)

Hamilton's treatment of the syllogism deserves a whole chapter in its own right — I shall only be able to deal with it rather briefly in this section. In the Lectures Hamilton takes a considerable amount of time to explain many detailed aspects of the syllogism to his students. He distinguishes and displays four different classes of syllogism — the Categorical, Disjunctive, Hypothetical, and the Hypothetico-disjunctive (see LL.I.291-2). The categorical syllogism is also displayed in both extensive and intensive forms, something he will later capture in his symbolic notion as indicted in Table 2 earlier and as fully detailed in the final table of the second volume of Lectures (LL.I.295-300).[38] Importantly, in drawing attention to the different reasonings between extensive and intensive syllogisms, where the copula signifies respectively 'contains under' and 'contains in', Hamilton makes the point that from what can be observed of the inverse ratio relation between extension and intension with regard to syllogisms, 'it is not to the mere external arrangement of the terms, but to the nature of their relation, that we must look in determining the character of the syllogism' (LL.I.300; and compare, p. 348). This is important to how he will proceed to regard the syllogism within the lectures but it also bears within it the necessity of quantifying the predicate, even though at this stage he does not seem to have produced the full system of quantification as given in Table 2 earlier. With extensive and intensive syllogisms differentiated, Hamilton constructs three rules, in place of Whately's six, three for extensive, and three for intensive syllogisms, the intensive merely inverting the extensive rules (see LL.I.305-6; 315).

In all this we can see Hamilton working, as it were, from the ground up — as we shall see shortly, his ultimate position will be even more simple or general as, on the basis of his thoroughgoing quantification system, he develops a single general rule or Canon governing all valid syllogisms in both affirmative and negative moods. With regard to what is happening in the Lectures it is therefore important to remember both their instructional function and that some aspects in the Lectures are later superseded, such as, for example, his later rejection of the Rule of Reason and Consequent in favour of just three main Rules of Identity, non-Contradiction, and Excluded middle (see LL.I.290n).

In his treatment of the syllogism in the Lectures, Hamilton introduces his students to the usual suspects: the four figures, moods, the ingenious mnemonics of, for example, Barbara, Celarent, Darii, the formal fallacies, and so on, a great deal

[38] Compare William Thomson, *An Outline of the Necesary Laws of Thought: A Treatise on Pure and Applied Logic*, 3rd edn. (London: Longman, Brown, Green and Longmans, 1854), pp. 245–9. Thomson fully endorses and offers some explanation of this table which he also reproduces.

of which he lays out with painstaking detail (see LL.I.394-468). He also discusses the various forms of conversion, but since I have already touched on this subject in the previous section and how Hamilton's quantification effectually displaces other kinds of conversion with his simple conversion, it is needless to say anything more about his treatment of it in the Lectures (see LL.I.262-5; NA.LL.II.264-76). Using simple circle diagrams to illustrate the relations between the extremes or subject and predicate terms of the conclusion and the middle term, his explanations of syllogisms must have given his students an excellent grounding in the differing figures and moods. However, as so often occurs in Hamilton's expositions of the traditional logic, he marks some significant differences between his treatment and that of both his predecessors and contemporaries. One major example of this is his rejection of the fourth figure which is, of course, simply shown, in extension, as follows:

$$\frac{\begin{array}{c} P \textit{ is } M \\ M \textit{ is } S \end{array}}{S \textit{ is } P}$$

He argues that, though the fourth figure can be shown to be valid, 'the logicians, in consequence of their exclusive recognition of the reasoning in extension, were not in possession of the means of showing that this figure is a monster undeserving of toleration, far less of countenance and favour' (LL.I.424). I shall not rehearse Hamilton's arguments against the fourth figure, except to note that he shows that in this figure there is an unwarranted switch from reasoning in extension to intension or *vice versa* and thus it performs 'a feat about as reasonable and useful in Logic, as the jumping from one horse to another would be reasonable and useful in the race-course. Both are achievements possible; but, because possible, neither is, therefore, a legitimate exercise of skill' (LL.I.427). But, Hamilton's principal reason for rejecting the fourth figure is that it involves a mental process 'which is not overtly expressed' — in other words, when we adhere rigidly to the principle of explicitness, the fourth figure's reliance upon an intermediary conclusion becomes evident (see LL.I.427-8).

There are many other interesting features in the lectures worth mentioning, such as his treatment of the difference between Induction and Deduction, which I briefly touched on in Section 2 above. He discards what is now much more typically classed as Inductive argument, instead regarding Deduction and Induction both as formal forms of demonstrative reasoning — logicians had 'corrupted and confounded' logical deduction 'governed by the necessary laws of thought' with contingent matter and probability (LL.I.325; and see 319–26). This formal approach, according to Hamilton, is more in keeping with Aristotle's understanding of induction (see LL.I.325–6). It also has important implications for how Hamilton regards both the Sorites and Enthymeme, both of which he discusses in terms of their formal characters as syllogisms. He does of course explain that the Sorites became associated with that sophism or informal fallacy commonly referred to or illustrated by means of the examples of piling up grains of sand until what was once maintained to be a small quantity becomes large (a Progressive Sorites), and also the famous bald man example (Regressive Sorites) (see LL.I.376–8; 464–6). According to Hamilton, the Sorites only became associated with such sophistic or fallacious reasoning some time in the 15^{th} century and the failure of logicians to incorporate the Sorites as a legitimate chain-syllogism was all down to their exclusive concentration on extension and not keeping in mind that, for Aristotle 'all our general knowledge is only an induction from an observation of particulars' (LL.I.377; 380; and see, 366–85).

His treatment of the Enthymeme is similarly interesting and informative, arguing that it is only the external form of the enthymeme that may be said to be imperfect or incomplete. As Hamilton rightly shows, an enthymeme is not merely an argument in which one of the premises is missing or suppressed; it may also be that the *conclusion* has been suppressed/omitted. But, whether the major or minor premise or the conclusion is not made explicit, this does not, for Hamilton, warrant calling an enthymeme a special or defective syllogism — it 'constitutes no special form of reasoning', nor did Aristotle maintain that it did (LL.I.387). The enthymeme illustrates an important principle that pervades so much of Hamilton's approach to logic, namely, that the mere verbal accident of elision (or in the case of the enthymeme we might say more or less deliberate omission, often serving purposes of persuasion which Hamilton somewhat oddly does not address directly) is something that the logician, in staunchly adhering to the principle of explicitness, can make explicit as something that is in thought, though not in expression (for Hamilton's treatment of the enthymeme, see LL.I.386-94). He provides some nice examples of enthymemes but I shall only quote one of these since it involves a nice quip against Hegel that suggests something of Hamilton's capacity for at least occasional touches of humour: 'There is recorded [...] a dying deliverance of the philosopher Hegel, the wit of which depends upon [its] ambiguous reasoning. 'Of all my disciples,' he said, 'one only understands my philosophy; and he does not.' But we may take this for an admission by the philosopher himself, that the doctrine of the Absolute transcends human comprehension' (LL.I.398). There is in this also more than a hint of not only Hamilton's opposition to Hegel but also to absolutism more generally.

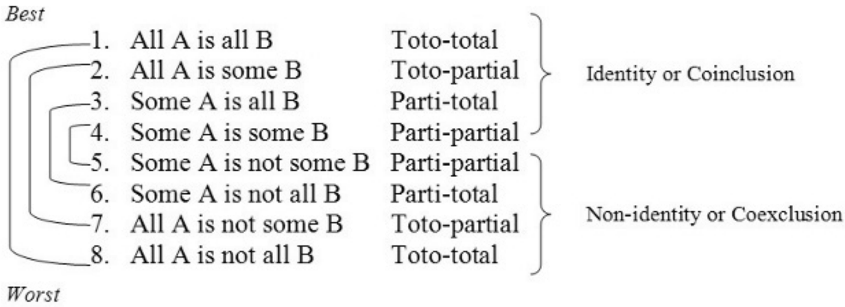
To his credit Hamilton reminds his students of certain points he made much earlier in the lectures to do with the status of propositions with regard to their discrete components, pointing out that a syllogism is an integrated mental act and that it ought therefore to be thought of not in a merely mechanical manner:

> It is [...] altogether erroneous to maintain, as is commonly done, that a reasoning or syllogism is a mere decompound whole, made up of judgments; as a judgment is a compound whole, made up of concepts. This is a mere mechanical mode of cleaving the mental phenomena into parts; and holds the same relation to a genuine analysis of mind which the act of the butcher does to that of the anatomist. It is true, indeed, that a syllogism can be separated into three parts or propositions; and that these propositions have a certain meaning,

> when considered apart, and out of relation to each other. But when thus considered, they lose the whole significance which they had when united in a reasoning; for their whole significance consisted in their reciprocal relation,–in the light which they mutually reflected on each other. We can certainly hew down an animal body into parts, and consider its members apart; but these, though not absolutely void of all meaning, when viewed singly and out of relation to their whole, have lost the principal and peculiar significance which they possessed as the coefficients of a one organic and indivisible whole. It is the same with a syllogism. The parts which, in their organic union, possessed life and importance, when separated from each other, remain only enunciations of vague generalities, or of futile identities. Though, when expressed in language, it be necessary to analyse a reasoning into parts, and to state these parts one after another, it is not to be supposed that in thought one notion, one proposition, is known before or after another; for, in consciousness, the three notions and their reciprocal relations constitute only one identical and simultaneous cognition. (LL.I.275-6).

The notion of interrelation and simultaneity in the above is important to how Hamilton will go on to view the syllogism's structure as, for example, something in which it is only mere convention that always places the conclusion last, and that the relative positions of major and minor premise themselves can easily be switched around without any loss of meaning. However, I have quoted the above passage at length since it eloquently apprises us of an important general dimension of Hamilton's whole approach to Logic — for all that he is striving for a rigorously pure science free from the extra-logical and although he will later in the 'New Analytic' speak of a 'Symbolic Notation [that will display] the propositional and syllogistic forms, even with a mechanical simplicity', Hamilton constantly opposes the mechanical or rigidly structural in favour of considering the syllogism less like a dismembered material body or constructed building and more like a process — multiplex, organic, interrelated, and even fluid — and his logic needs to be seen as an attempt to capture, not only the dual perspectives afforded by extension and intension, but also a greater sense of the richness and complexity of formal reasoning (NA.LL.II.251).

In the previous section I referred to Hamilton's 'Observations on the Mutual Relation of Syllogistic Terms in Quantity and Quality' at Appendix V (e) in the *Lectures on Logic* (see NA.LL.II.285). In this 'Observations' section, he provides the relations between any given proposition's subject and predicate terms and, by inverting the order of the negative propositional forms given in the previous section in Table 2, he displays the best-worst quantification relationships between each of the four affirmative propositions and their corresponding negatives (see NA.LL.II.286). I have reconfigured Hamilton's presentation of these relations as follows:



The significance of this schema becomes clear when we note how Hamilton's system of quantification brings into consideration certain aspects of syllogistic reasoning formerly ignored by Aristotelian logic. According to Hamilton:

> Former logicians knew only of two worse relations,–a particular, worse than a universal, affirmative, and a negative worse than an affirmative. As to a better and worse in negatives, they knew nothing; for as two negative premises were inadmissible, they had no occasion to determine which of two negatives was the worse or better. But in quantifying the predicate, in connecting positive and negative moods, and in generalising a one supreme canon of syllogism, we are compelled to look further, to consider the inverse procedures of affirmation and negation, and to show [. . .] how the latter, by reversing the former, and turning the best quantity of affirmation into the worst of negation, annuls all restriction, and thus apparently varies the quantity of the conclusion. (NA.LL.II.285-6).

I shall not attempt to explain this in detail. Suffice to say that the above schema of best-worst quantification relationships is used by Hamilton to construct what he calls his 'General Canon' to determine the relationship between the subject (S) and predicate (P) of the conclusion in a syllogism, on the basis of a best-worst comparison between the relationships between the subject term and middle term (M), and the middle term and predicate term of the syllogism. Hence, depending on the relationship between the subject and predicate terms of a syllogism, the relationship between S and P in the conclusion will be, for example, *totally coexclusive*, or *partially coexclusive*, or *toto-partially coexclusive*, and so on. Hamilton's 'one supreme canon of syllogism' is given as follows:

> General Canon. What worst relation of subject and predicate, subsists between either of two terms and a common third term, with which one, at least, is positively related; that relation subsists between the two terms themselves. (NA.LL.II.285)

He translates this 'General Canon' into twelve clear rules governing 36 syllogistic moods in the first three figures (the fourth figure having been rejected as pointed out above), plus 24 negative moods — these forms are detailed in the final table given at the end of the *Lectures on Logic* (see LL.II.475). However, it might be better to think of these not as rules but rather as instantiations of his General Canon since, once one has grasped how Hamilton's system operates, each of them can be translated from this Canon with relative ease (see NA.LL.II.285-9). I shall not discuss these instantiations in any detail but will instead use just one of them to illustrate how Hamilton's General Canon operates, using an example which Fogelin uses to demonstrate the correctness of his reading of propositions (1) and (8) as discussed in the previous section.

Fogelin gives the following syllogism as an example that he claims Hamilton's system allows as valid, yet which can be shown, using a Venn diagram, to be invalid if translated using his earlier translation of proposition (8), though valid using de Morgan's interpretation of this parti-partial coexclusion proposition:

Some P is not any M
Some S is some M
$\therefore$ Some S is not some P

Having already accepted Fogelin's treatment of the above syllogism which shows how it can be read as valid (and hence what proposition (8) expressing Parti-partial coexclusion means, or can be read as meaning), all I want to do here is merely illustrate how Hamilton's General Canon can be invoked to produce from the two premises the conclusion as given. I think that all this will demonstrate is that Fogelin is right that the above syllogism would be declared valid in Hamilton's system as he asserts. However, I am not entirely sure that Hamilton's table and list of interpretations of his General Canon are completely free from anomalies, and it must be said that to check this thoroughly, given the complexities of co-ordinating various mixtures of extensive and intensive readings with Hamilton's own symbolic representation of the negative moods, is beyond the scope of this chapter. Hence, I shall simply give Fogelin's example as unequivocally exemplifying adherence to Hamilton's General Canon, even though by treating this so simplistically I may be overlooking certain niceties and complexities concerning how it ought to be used:

Fogelin's Example:	***Relation:***	***Hamilton's rule from General Canon:***
some P is not any M	Parti-total coexclusion	VII.a: *A term parti-totally coexclusive, and a term partially coinclusive, of a third [M], are partially coexclusive of each other.* (NA.LL.II.288)
some S is some M	Parti-partial conclusion	
some S is not some P	Parti-partial coexclusion	

From what I have been so far able to ascertain, though not articulate in full, certain possible anomalies (or doubts that I still have) notwithstanding, Hamilton's General Canon does seem to work.

If, in agreement with Fogelin, I am right about this, Hamilton's system marks a most significant advancement in the treatment of the syllogism that almost entirely replaces the previous systems which Hamilton so often decried as being imperfect, flawed, confused, and misleading. No wonder that Fogelin, though acknowledging that Hamilton's achievement may not seem important 'from a modern point of view', regards his quantification system as 'a radical departure from traditional theory' (Fogelin, pp. 163–4). If Hamilton's ultimate regulation or General Canon of syllogistic forms is fully adequate, comprehensive, sufficiently general, and flexible, his treatment of the syllogism arguably marks the considerable improvement on and replacement of traditional logic's treatment of the syllogism that he himself believed it did. In place of numerous logical rules and botched attempts to bring the Aristotelic system out of the chaos in which Hamilton found it, by judiciously interpreting but not slavishly falling under the spell of Aristotle's enormous authority, Hamilton may well have developed, albeit within certain limitations, a system of formal logic that, as it culminated in a simple mechanism for making all quantities in both affirmative and negative propositions explicit, only required the capstone of his single or supreme canon of the syllogism, to warrant his grand claim that he had produced a system whose beauty resided in the very naturalness of being '*as One at once and Various.*'

6 CONCLUSION

> A mere knowledge of the rules of Rhetoric can no more enable us to compose well, than a mere knowledge of the rules of Logic can enable us to think well. (LL.I.48-9)

Augustus de Morgan misunderstood the complexities that Hamilton's system could either accommodate or was pointing towards. As Fogelin claims, though de Morgan clearly did try to understand Hamilton's system 'he failed to do so and commentators since have hardly done better' (Fogelin, p. 162). According to Fogelin this has much to do with the strangeness of Hamilton's language, such as we find in his non-standard exchanges of 'all' for 'any'. True, these things do create problems, as also do Hamilton's now strange symbolism, dense tables, and the fragmentary nature of much of his work on Logic, excepting the Lectures which are abundantly clear, always informative, and even occasionally somewhat entertaining. However, I suspect that the linguistic difficulties have more to do with Hamilton's enormous ability to compress the language of his texts into an often overly taut, quasi-litigious style that is at first off-putting and certainly at times not for the faint-hearted. However, there are some more substantial reasons why de Morgan and others have floundered and in the end did Hamilton a great disservice.

In order to gain more than a foothold in Hamilton's logic it would seem that one also has to have a foothold in and possibly be to some extent persuaded by his metaphysical standpoint of *natural dualism*, for in this doctrine's opposition to absolutism — not least of all in its opposition to absolute scepticism — inheres

Hamilton's relativity and the germ of what becomes a much more pervasive *correlativism* throughout so much of his writing. Hamilton's relativity places subject and predicate into simple relations of equation and non-equation, distinguishes indefinite quantities into two kinds or degrees of indefiniteness, and is founded on recognising the significance of perspective with regard to a given concept's dual quantification dimensions of extension and intension. All this seems to be an attempt to frame subject and predicate as the unity of thought from which they originate. However, the frame is really much larger, and herein lies a great part of the problem and yet potential of Hamilton's system. A part of Hamilton's relativism also brings into play not just perspective (and thereby different possible readings of propositions), but both the relations between a concept's attributes and the conditioning nature of the human subject's agency. The relations between logic (the laws that constitute the conditions of thought) and language (the terms without which we would otherwise be incapable of participating in any productive thought, argument, discourse, analysis, or articulation of logic's laws, propositional forms, and so on), and the surrounding chaos or ever-shifting sands against, and yet in relation to which, logic and language comprise our attempts to make whole, divide, and recombine this otherwise unintelligible plenum of indeterminate entities under which we are continually at risk of being submerged — as these relations form the frame or field within which Hamilton's logic is conducted, he was constructing a logical system both highly suggestive of a deeper relativism he eschewed, and yet which he virtually postulated was the condition within which logic had to operate as the very function of logic and language had to do with stabilising the multiple within unity. This, perhaps more than anything else, makes his writing complex. However, much more simply, Hamilton's enormous erudition — also a factor in the diversity he attempts to bring into order — remains an intimidating force that few, excepting some of his more devoted students and followers, have found a congenial companion.

Hamilton regarded philosophy — particularly logic and metaphysics — as the greatest gymnastic of the mind and with such a conception of his subject matter providing a large part of philosophy's *raison d'être*, he was attempting to set his students on a course of study that would challenge their intellectual abilities to their utmost through his attempts to make the study of Logic measure up to and (it must have been his hope) compete with the higher educational, intellectual, and scholarly standards that had been demanded on the Continent. The German logicians in particular, made even the Professors of Britain's one-time most prestigious university, Oxford, appear to Hamilton's eyes as little better than intellectual philistines or dilettantes. In his definition of Logic as a pure science of the necessary laws of thought, single-handedly Hamilton was laying the groundwork for improving upon and in many ways replacing the traditional Aristotelian logic that he regarded as having been sunk into a mass of confusion by centuries of almost slavish or insufficiently critical adherence to the Stagirite, coupled with numerous misinterpretations of the true nature of their subject.

Within the boldness of Hamilton's emphatic assertions and robust denunciations of the errors and confusions that he argues arose due to 'the passive sequacity of the logicians [in following] obediently in the footsteps of their great master', Aristotle, there is a genuine sense of his excitement concerning this claim that his system supersedes all previous systems new and old (NA.LL.II.262). By this stage in his work on logic it must have seemed to Hamilton as though, after many long years of arduous industry during which he had been diligently examining, summarising, and critically assessing the logics of others written mainly in Greek, Latin, English, and German, at last the numerous errors and confusions riddling centuries of Aristotelian logic could be removed. Through the earlier works of ancient and scholastic logicians, these errors and confusions had continued to sift themselves, but they also permeated the work of his Oxonian contemporaries. But now, after years of clearing away the detritus of former ages and chastising the dilettantism of more recent logicians, he could with one final push radically brush aside those aspects of the traditional logic that had significantly impeded its development or evolution into a system at once more complex and yet more orderly (see NA.LL.II.252). At last he had constructed a robust system that he could proudly advocate as the 'keystone in the Aristotelic Arch', and while this keystone had doubled the number of propositional forms, these forms constituted a structural whole based on the logic of Aristotle and thus at once the traditional logic was brought one significant step closer to the beauty and perfection which Aristotle's work seemed to promise but had not realised (see NA.LL.II.249). However, Fogelin rightly counters Hamilton's claim that his 'New Analytic' completes traditional Aristotelian logic — for, instead of merely placing the keystone in the Aristotelic arch, 'By introducing an entirely different system of classifying propositions in virtue of their potential roles in syllogisms, Hamilton made a radical departure from traditional theory' (Fogelin, p. 163). The keystone that is Hamilton's quantification of the predicate one might thus say, is far from being a pretentious claim to glory undeserved — rather, Hamilton's 'keystone' is an overly modest description for a much more radical, yet substantial and more stable construction that involved the destruction and removal of logical ruins from bygone ages, though it is plain to see that at least some of the foundations were incorporated to support Hamilton's arch. But with a final and fitting twist of irony, all too quickly the course of logic would develop during the 19^{th} century in ways that left so much of Hamilton's endeavour far behind as a curiously flawed relic.

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"THE WHOLE BOX OF TOOLS": WILLIAM WHEWELL AND THE LOGIC OF INDUCTION

Laura J. Snyder

William Whewell (1794–1866), well-known polymath, has been criticized for his view of inductive logic ever since publishing it. His friend, the logician Augustus DeMorgan, may or may not have intended a cruel reference to Whewell's lowly social origins as the son of a master carpenter when he complained, in his 1847 textbook on logic, about certain writers incorrectly using the term "induction" as including "the whole box of [logical] tools." Other writers on logic, from John Stuart Mill on, have similarly claimed that Whewell did not present a valid inductive logic. Mill accused Whewell of allowing "no other logical operation [in scientific discovery] but guessing until a guess is found that tallies with the facts". In the twentieth century, followers of Karl Popper made that criticism a compliment, claiming that Whewell had "anticipated" Popper's anti-inductive methodology of conjectures and refutations. Whewell's response to later criticisms of DeMorgan is, I believe, as apt as it was then: "I do not wonder at your denying [my induction] a place in Logic; and you will think me heretical and profane, if I say, *so much the worse for logic*".[1] As I will show in this chapter, Whewell's method of discoverers' induction is, as he always claimed, an inductive logic, one that was strongly influenced by Whewell's reading of Francis Bacon. Though Whewell's inductive logic does include the "whole box of tools", this is a benefit, rather than a liability, of his view.

1 WHEWELL'S EDUCATION AND CAREER

Whewell was born in 1794 in Lancaster, England, the eldest child of a master carpenter. His prodigious abilities were recognized early. As a child, the master of the local grammar school saw his potential and offered to teach him for free at his facility. Soon after, Whewell was examined by a tutor from Trinity College, Cambridge, who predicted that the young boy would one day place among the top six wranglers (the holders of first-class degrees). Whewell was then sent to the Heversham Grammar School in Westmorland, some twelve miles to the north, where he would be able to qualify for a closed exhibition to Trinity. In the nineteenth century and earlier, these "closed exhibitions" or scholarships were set aside for the children of working class parents. Whewell studied at Heversham

[1] William Whewell to Augustus DeMorgan, January 18, 1859, in [Todhunter, 1876, 2: 417].

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for two years, and received private coaching in mathematics from John Gough, the blind mathematician to whom reference is made in Wordworth's "Excursion". Although Whewell did win the exhibition it did not provide full resources for a boy of his family's means to attend Cambridge; money had to be raised in a public subscription to supplement the scholarship money. He thus came up to Trinity in 1812 as a "sub-sizar" (scholarship student).[2]

When he arrived at Trinity, Whewell became involved with the Analytical Society, formed the year before by Charles Babbage, John Herschel and others. The Analytical Society had been founded for the purpose of reforming the teaching and practice of mathematics at Cambridge. The members of the society wished to introduce continental methods of mathematics to a university which had excluded progress since the time of Newton (specifically, they hoped to introduce Lagrange's algebraic and formalistic version of the calculus, which included replacing Newton's fluxion dot notation with Leibniz's "d" notation) (see [Fisch, 1994] and [Becher, 1980].) In addition to his involvement with the Analytical Society, Whewell's years as a Cambridge undergraduate were enriched by his friendships with Richard Jones (the future political economist) and Herschel, each of whom had great effect on his future thinking about science. He also formed friendships with Hugh James Rose, Julius Charles Hare, and Connop Thirlwall, men greatly interested in and influenced by the German Romantic movement. Whewell finished his studies in 1816 as Second Wrangler and Second Smith's Prizeman. After a short post-graduate period as a private tutor, Whewell won a college fellowship in 1817.

After winning his fellowship, Whewell was assistant tutor from 1818 until 1823, and tutor from 1823 until 1838. He was elected to the Royal Society in 1820, and ordained a priest — as required for Trinity Fellows to maintain their position after an initial seven year period — in 1825. He took up John Henslow's Chair in Mineralogy in 1828 (when Henslow vacated it for the Botany Professorship), and resigned it in 1832. During the early 1830s Whewell was instrumental in the formation of the British Association for the Advancement of Science (BAAS), the Statistical Section of the BAAS, and the Statistical Society of London. In 1833, at a meeting of the BAAS, he invented the word "scientist", an achievement emblematic of his broader influence upon science and culture. In 1838 Whewell became Professor of Moral Philosophy. Shortly after his marriage to Cordelia Marshall on 12 October 1841, he was named Master of Trinity College, having been recommended to Queen Victoria by Prime Minister Robert Peel. He was Vice-Chancellor of the University in 1842 and again in 1855. In 1848 he played a large role in establishing the Natural and Moral Sciences Triposes at the University. Whewell engaged in scientific research, winning a medal from the Royal Society for his work on the science of the tides. He corresponded with the most eminent scientists of his day. He published his own translations of German novellas and poetry, and, with his popular translations of Plato's dialogues, was partly responsible for a "Platonic revival" in Britain in the 1850s (see [Turner, 1981, 371–2]). Towards the end of

[2]Whewell's status as a sub-sizar meant that he had to serve the other boys at dinner. See [Garland, 1980, 11], and [Rothblatt, 1981].

his life Whewell was asked to tutor the young Prince of Wales (the future Edward VII) in political economy.

As this brief summary of his life indicates, Whewell's rise from what we would probably call the middle (or lower-middle) class to the intellectual and social elite was spectacular. If this rise tended to make him a bit arrogant, and even imperious, in his later life, it also I think imbued him with a sense of mission. Whewell very much saw himself as having a "calling" to reform philosophy; he refers to this as his "vocation".[3] As we will see in the next sections, Whewell's project, throughout his career, was to reform inductive philosophy, and to do so by revising Francis Bacon's inductive logic.

2 REFORMING INDUCTION

From his days as an undergraduate at Trinity College, to his final years spent as Master of that institution, Whewell considered his project to be the reform of the inductive philosophy, a reform that was intended to apply to all areas of knowledge. Whewell intended that his reformed inductive philosophy would provide the groundwork for the reshaping of more than natural science: morality, politics, and economics would also be transformed. In this chapter, however, I will focus only on Whewell's renovation of induction (details about how he intended this reform to be applied to the other areas can be found in my [Snyder, 2006]).

Whewell and his friend Richard Jones together planned to reform induction, and saw their task as consisting in two parts. The first was *defining* a "true idea of induction". In an early notebook entry, Whewell lamented "that the true idea of induction has not been generally fixed and agreed upon must I think be very obvious".[4] The second task was "to get *the people* into a right way of thinking about induction";[5] that is, to publicize the nature and value of induction in all areas of thought. In one of his many letters to Jones, Whewell refers to induction as the "true faith", wondering how it can "best be propagated".[6]

Whewell and Jones framed their reforming mission as a battle against those they referred to as the "downwards mad", that is, those who preferred a deductive approach to the logic of the natural and moral sciences.[7] One of these "deductive savages" was Richard Whately, fellow of Oriel College, Oxford, who became archbishop of Dublin in 1831.[8] Early that year Jones wrote to Whewell after seeing

[3]In his 1831 review of Herschel's *Preliminary Discourse*, Whewell referred to "the history and philosophy of physical science" as his "vocation" (see [Whewell, 1831, 374]). Later, he wrote to Herschel that "the reform of our philosophy is the work to which I have the strongest vocation, and which I cannot give up if I were to try" (letter to Herschel, 9 April 1836, in [Todhunter, 1876, 2: 234]).

[4]See notebook dated 28 June 1830, WP R.18.17 f. 12, pp. v–ix.

[5]William Whewell to Richard Jones, 25 February 1831, WP Add.Ms.c.51 f. 99.

[6]William Whewell to Richard Jones, 25 February 1831, WP Add.Ms.c.51 f. 99.

[7]See William Whewell to Richard Jones, 20 January 1833, WP Add.Ms.c.51 f. 149; and 22 July 1831, Add.Ms.c.51 f. 110.

[8]William Whewell to Richard Jones, 19 February 1832, WP Add.Ms.c.51 f.129.

the third edition of Whately's *Elements of Logic*, objecting to Whately's "strange notion" that induction was a type of deductive reasoning.[9] Indeed, Whately had claimed that *all* forms of reasoning could be assimilated to the syllogism [Whately, 1827, 207]. Induction, for example, was said to be reasoning in which the major premise, which is generally suppressed, can be expressed as "what belongs to the individual or individuals we have examined, belongs to the whole class under which they come" [Whately, 1827, 209]. Thus, to take Whately's example, if we find, from an examination of the history of several tyrannies, that each of them lasted a short time, we conclude that "all tyrannies are likely to be of short duration". In coming to this conclusion we make use of a suppressed major premise, namely, "what belongs to the tyrannies in question is likely to belong to all" [Whately, 1827, 209]. Whately admitted that some would complain that his notion of induction was too narrow, in that it did not account for how the *minor* premises are obtained; i.e., for how it was ascertained that each of the examined tyrannies were "short-lived". But he distinguished between "logical discoveries", which occur when syllogistic reasoning alone is used to deduce a conclusion from known premises, and "Physical Discoveries", which involve more than syllogistic reasoning because various methods are used for ascertaining the premises — including observation, experiment, the selection and combination of facts, abstraction of principles, and others [Whately, 1827, 234–6]. On Whately's view logic was concerned only with reasoning from premises, not with ascertaining the premises. Thus inductive logic was also only concerned with reasoning from premises, or what Whately called logical, as opposed to physical, discovery.[10]

Whewell and Jones saw Whately's characterization of induction as more than just a point of logic. Rather, it presented a potential obstacle for the reform of philosophy they sought. They realized that many people associated induction with scientific method, even if they were unclear about the precise meaning of the term. If people accepted Whately's definition of induction, they might be led to the erroneous conclusion that science is essentially deductive, concerned only with deducing conclusions from assumed "first principles". Whewell and Jones linked this deductive view of science with that of the scholastic Aristotelians; indeed, Whately had explicitly framed his work as a defense of Aristotelian logic against the "confused" views of induction that resulted from its connection with Bacon.[11] But Jones and Whewell were eager to endorse a renovated version of

[9]Richard Jones to William Whewell, 24 February 1831, WP Add.Ms.c.52 f.20.

[10]As Pietro Corsi notes, Whately thus distinguished between two senses of induction: one having to do with the collection of facts, the other being a form of inference. The first was not of concern to Whately. Rather, his aim was to demonstrate that the second was "under the dominion of logic" [Corsi, 1988, 151]. This aim arose out of the context of attacks on the Oxford curriculum from those, such as John Playfair, who argued that progress in the mathematical sciences had been impeded by the reliance on Aristotelian logic. Whately and his teacher, Edward Copleston, defended the curriculum and the prominence of Aristotelian logic within it (see also [McKerrow, 1987]).

[11]See [Whately, 1827, ix and 9] and [McKerrow, 1987, 172]. In a letter to Jones, Whewell claimed that Whately and his followers were even worse than Aristotle, "far more immersed in verbal trifling and useless subtilty" (7 April 1843, WP Add.Ms.c.51 f. 227). Pietro Corsi has

Baconian inductivism. From their undergraduate days, they referred to Bacon as the "Master", and framed their task in terms of revising and popularizing his inductive logic. Although his view was not without flaws, it was the closest thing to a true inductive logic. Certainly, it was a vast improvement over the logic of the scholastic Aristotelians.

Because of their emphasis on the syllogism, Whewell complained, the scholastics "talked of experiment" but "showed little disposition to discover the truths of nature by observation of facts" [Whewell, 1860, 48]. In a notebook from 1830, Whewell wrote of the Aristotelian method that such a method "could lead to no such truths, and in the development of physical science especially was entirely barren. ... The business of speculative men became, not *discovery*, but *argumentation*".[12] Later, in his *History of the Inductive Sciences*, Whewell would refer to the Middle Ages as a "stationary period" in science. Whewell and Jones believed that the correct, Baconian view of induction needed to be brought before the public in order to prevail against sterile deductive approaches to scientific knowledge. Whewell expressed his "confidence" to Jones that "by and by the whole world will think [the deductive definition of science] as nonsensical as we do". But before this could happen, he and Jones would need to spread the "true faith".[13]

3 WHEWELL'S ANTITHETICAL EPISTEMOLOGY

The early influence of Bacon strongly inclined Whewell toward an inductive, empirical view of epistemology. At the same time, however, Whewell had a deep appreciation for a priori, deductive forms of reasoning, due to his interest in mathematical sciences. His early work in mechanics exposed him to a physical science which seemed to incorporate both empirical and a priori elements.[14] But it was his experience studying mineralogy abroad that struck Whewell with the need to combine empirical and a priori elements in an epistemology and scientific methodology. In 1825, when Henslow vacated the chair of Mineralogy at Cambridge to take up the Botany Professorship, Whewell announced himself a candidate for the position. Although he had published mathematical papers on crystallography, Whewell did not have much empirical knowledge of mineralogy; thus he went abroad to study with experts such as Friedrich Mohs. (This was not unusual at the time. When Adam Sedgwick was elected to the Woodwardian professorship of geology in 1818, he knew little of the discipline and needed to learn it quickly) (see

noted that Whately and the "Oxford Noetics" used Stewart's writings in defense of Oxford's educational system, with its emphasis on Aristotelian logic, a position opposed to Stewart's own [Corsi, 1987].

[12]William Whewell, Notebook, WP R.18.17 f. 12, p. 94. See also a notebook dating from 1831-2 (WP R.18.17 f. 15, p. 24). Thirty years later, in the final edition of the *Philosophy of the Inductive Sciences*, Whewell criticized Descartes on similar grounds. See [Whewell, 1860, 163].

[13]William Whewell to Richard Jones, 25 February 1831, WP Add.Ms.c.51 f. 99.

[14]Whewell's first book was a textbook on dynamics, published in 1819. He went on to produce four more textbooks on mechanics, a book on analytical statics, and a text on the differential calculus.

[Garland, 1980, 96]). Whewell was strongly impressed with the "German school" in mineralogy, especially with the elegant mathematical treatment given by Mohs of a science which Whewell had previously considered to be purely empirical. He wrote to his friend Hugh James Rose that "I am afraid ... that I may not bring back my faith as untainted as you have done: for I find my mineralogical supernaturalismus giving way in some respects. It may be possible to bring about a union between the two creeds [i.e., the a priori and the empirical], which I hope will not be such a thing in science as you hold it to be in faith".[15]

Whewell eventually reconciled the empirical and a priori elements of science in an epistemology that is "antithetical", in that it expressed what Whewell called the "Fundamental Antithesis", or dual nature, of knowledge.[16] According to Whewell's mature epistemology, all knowledge involves both an ideal, or subjective, element, as well as an empirical, or objective, element. Although his experience with mineralogy had sparked Whewell's desire to "bring about a union between the two creeds" of empiricism and a priorism, Whewell's notebook writings on induction prior to 1831 present induction as a purely empirical process, consisting in enumerative induction of observed instances.[17] He had not, up to this point, found a way to synthesize the a priori elements of scientific knowledge with the empirical epistemology he wished to follow Bacon in endorsing. By February of 1831, while working on his review of John Herschel's *Preliminary Discourse on the Study of Natural Philosophy*, Whewell for the first time used a metaphor indicating his initial attempt to combine these empirical and a priori or purely rational elements of science. Whewell explained that

> Induction agrees with mere Observation in accumulating facts, and with Pure Reason in stating general propositions; but she does *more* than Observation, inasmuch as she not only collects facts, but catches some connexion or relation among them; and *less* than pure Reason ... because she only declares that there *are* connecting properties, without asserting that they *must* exist of necessity and in all cases. If we consider the facts of external nature to lie before us like a heap of pearls of various forms and sizes, mere Observation takes up an indiscriminate handful of them; Induction seizes some thread on which a portion of the heap are strung, and binds such threads together. [Whewell, 1831, 379]

He asked Jones to look over his draft.[18] Jones's assessment proved to be valuable for Whewell:

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[15]William Whewell to H.J. Rose, 15 August 1825, in [Todhunter, 1876, 2: 60]. See also [Fisch, 1991, 67].

[16]Whewell's epistemology is referred to as "antithetical" in [Fisch, 1985] and [Fisch, 1991].

[17]See especially the earliest draft of *Philosophy*, dated 28 June 1830, where his view of induction sounds much like the view expressed later by Mill in his *System of Logic*, a view that Whewell strongly criticized (WP R.18.17 f. 12, 98, 104).

[18]William Whewell to Richard Jones, 11 February 1831, WP Add.Ms.c.51 f. 98.

> I go along with you in your use of the word induction only I fear to a certain extent — I do not myself like to oppose it to or contrast it with either observation or pure reason — Induction according to me and Aristotle (admire my modesty) *is the whole process by which the intellect gets a general principle from observing particulars or individuals and in that process both observation and pure reason have a part* — when observation has collected the facts abstraction. ... seizes on the law or principle and then the inductive process is compleat in all its parts.[19]

In this passage Jones suggested to Whewell a way to synthesize the empirical and the ideal in his epistemology: namely, by seeing induction as an act that includes *both* observation *and* pure reason. Whewell seems to have been inspired by this characterization of induction. He soon began to describe induction as a *process* involving observation and reason. The rational element of induction was provided by certain "conceptions of the mind". In a notebook dated 1831-2, Whewell for the first time characterized induction as involving such conceptions: "induction supposes a power of clearly representing phenomena by the mind as subordinate to the conceptions of space, number, etc."[20] In another notebook of this period Whewell described the importance of distinctly conceiving conceptions in order to make proper inductions,[21] and added what appears to be his first list of conceptions regulating different sciences.[22] By 1833 Whewell had come to believe that, as he explained in a notebook entry, "knowledge implies passive sense [i.e., observation] and active thought [i.e., reason]".[23] In an address to the British Association for the Advancement of Science in that same year, Whewell claimed that "a combination of theory with facts ... is requisite" in order to discover new truths" [Whewell, 1834a, xx]. And in a letter to Jones in 1834 Whewell described his "Philosophy of Induction", claiming that "you will see that a main feature is the assertion of ideas and facts as equally and conjointly necessary to science".[24]

By 1837, Whewell was ready to formulate more systematically this position, and to express it publicly. He did so in his "Remarks on the Logic of Induction", which was appended to his textbook *Mechanical Euclid*. In this essay Whewell explained that induction requires both an idea provided by the mind and facts provided by the world [Whewell, 1838, 181]. A "general idea", which is not given by the phenomena but "by the mind", is "*superinduced* upon the observed facts" [Whewell, 1838, 178]. In his *History of the Inductive Sciences*, published that same year, Whewell similarly noted that both facts and ideas are requisite for the "formation of science". "Real speculative knowledge", Whewell claimed, "demands the combination of the two ingredients" [Whewell, 1857a 1: 5–6]. By the time

[19]Richard Jones to William Whewell, 7 March 1831, WP Add.Ms.c.52 f. 26, emphasis added.
[20]William Whewell, Notebook, WP R.17.18 f. 13, p. 54.
[21]William Whewell, Notebook, WP R.17.18 f. 15, p. 2.
[22]William Whewell, Notebook, WP R.17.18.f. 15, p. 47, in an entry dated 2 July 1831.
[23]William Whewell, Notebook, WP R.18.17 f. 8, facing p. 20.
[24]William Whewell to Richard Jones, 5 August 1834, WP Add.Ms.c.51 f. 174.

he published the first edition of the *Philosophy of the Inductive Sciences* in 1840, he had worked out more details of the position, and developed an argument situating his epistemological view as a "middle way" between stark empiricism and full-blown rationalism.

Because of the dual or antithetical nature of knowledge, Whewell claimed, gaining knowledge requires attention to both Ideas and Sensations. Both are required for knowledge: "without our ideas, our sensations could have no connexion; without external impressions, our ideas would have no reality; and thus both ingredients of our knowledge must exist" [Whewell, 1858a, 1: 58]. An exclusive focus on one or the other side of the antithesis is to be avoided. Whewell criticized both Kant and the German Idealists, for their exclusive focus on the ideal or subjective element, and Locke and his followers of the "Sensationalist School", for their exclusive focus on the empirical, objective element.

Whewell generally referred to these ideas, which comprise the ideal or subjective element in his antithetical epistemology, as "Fundamental Ideas". Whewell explained that "I call them *Ideas*, as being something not derived from sensation, but governing sensation, and consequently giving form to our experience; — *Fundamental*, as being the foundation of knowledge, or at least of Science" [Whewell, 1860, 336]. They are supplied by our minds in the course of our experience of the external world; they are not simply received from our observation of the world [Whewell, 1858a, 1: 91]. This is why Whewell claimed that the mind is an active participant in our attempts to gain knowledge of the world, not merely a passive recipient [Whewell, 1860, 218].

Although these Ideas are supplied by our minds, they are such that they enable us to have real knowledge of the empirical world. They do so by connecting the facts of our experience; this occurs because the Ideas provide the general relations that really exist in the world between objects and events. These relations include Space, Time, Causation and Resemblance, among numerous others. By enabling us to connect the facts under these relations, the Ideas provide a structure or form for the multitude of sensations we experience [Whewell, 1847, 1: 25]. Thus, for example, the Idea of Space allows us to apprehend objects as existing in space, in spatial relations to each other, and at a particular distance from us. Indeed, we need these Ideas in order to be able to make sense of our sensations: "our sensations, of themselves, without some act of the mind, such as involves what we have termed an Idea, have not form. We cannot see one object without the Idea of Space; we cannot see two without the idea of resemblance or difference; and space and difference are not sensations" [Whewell, 1858a, 1: 40]. Every science, Whewell believed, has one or more Fundamental Ideas particular to it, which provide the structure for all the facts with which that science is concerned ([Whewell, 1858b, 137] and [Whewell, 1858a, 1: 3]). The Idea of Causation is especially associated with the science of Mechanics, while the Idea of Space is the Fundamental Idea of Geometry. Whewell explained further that each Fundamental Idea has certain "conceptions" included within it; these conceptions are "special modifications" of the Idea applied to particular types of circumstances.[25] For example, the conception of force is a modification of the Idea of Cause, applied to the particular case of motion (see [Whewell, 1858a, 1: 184–5, 236]).

Thus far, this discussion of the Fundamental Ideas may suggest that they are similar to Kant's forms of intuition and categories, and there are indeed some similarities. Because of this, numerous commentators argue that Whewell's epistemology was derived from his reading of Kant, or perhaps a view of German philosophy refracted through the writings of Coleridge and the Coleridgian circle at Cambridge, which included Whewell's close friends Rose and Julius C. Hare.[26] However, although there are some similarities between his view of the Fundamental Antithesis and certain notions of Kant's and Coleridge's, there are important differences, which are frequently overlooked by commentators on Whewell.

On the one hand, Whewell did read and appreciate Kant, in a time when German philosophy was not terribly popular in England (see [Wellek, 1931]). He knew German well enough to publish translations of German prose and poetry,[27] and, if a story related by his student Isaac Todhunter is true, well enough to pass in Germany as a native-speaker.[28] A notebook from 1825 contains reading notes of the *Critique of Pure Reason*.[29] Like Kant, Whewell was interested in how it is possible for us to have knowledge which has a universality and necessity that experience alone can not give it. And his answer, like Kant's, involves certain conceptions and ideas which are in some sense a priori, because they are not derived from experience. As he wrote to Herschel, "My argument is all in a single sentence. You *must* adopt such a view of the nature of scientific truth as makes universal and necessary propositions possible; for it appears that there are such, not only in arithmetic and geometry, but in mechanics, physics and other things. I know no solution of this difficulty except by assuming *a priori* grounds".[30] Moreover, Whewell rather modestly admitted that his discussions of the Fundamental Ideas of Space and Time are mere "paraphrases" of Kant's discussion of the forms of intuition in the first *Critique* [Whewell, 1860, 335]. A contemporary translator of Kant claimed, in fact, that in his *Philosophy* Whewell "more elegantly expressed" certain doctrines of Kant.[31] These most basic Ideas of Space and Time (in some

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[25] [Whewell, 1858b, 187]. Whewell was not always consistent in maintaining the distinction between Ideas and conceptions. For instance, in this work he referred both to the "Idea of Number" (p. 54) as well as the "conception of number" (pp. 50 and 56).

[26] On readings of Whewell as a "British Kantian", see [Butts, 1965a], [Buchdahl, 1991], [Ruse, 1979], and [Marcucci, 1963]. Claims that Whewell's view is closely linked to that of Coleridge can be found in [Preyer, 1985], [Cantor, 1991], [Cannon, 1976], and [Sloan, 2003].

[27] In 1847, Whewell edited *English Hexameter Translations from Schiller, Goethe, Homer, Callinus, and Meleager*, which contained Whewell's translation of Goethe's *Herman and Dorothea.*

[28] Isaac Todhunter quotes a letter from the scientist Alexander von Humboldt, who bemoaned the fact that he had "lost the pleasure" of seeing Whewell in Potsdam because he had told his servant to admit an English gentlemen, but Whewell was turned away because "you have spoken German like an inhabitant of the country!" (see [Todhunter, 1876, 1: 411].)

[29] See William Whewell, Notebook, WP R.18.9 f. 13, p. 19.

[30] William Whewell to John Herschel, 22 April 1841, in [Todhunter, 1876, 2: 298].

[31] In Whewell's library there is a translation of the *Kritik der reinen Vernunft* by Francis

works, Whewell includes the Idea of Cause among these) do function in Whewell's epistemology as similar to Kant's forms of intuition in being conditions of our having any knowledge of the world; indeed, Whewell referred to them as "conditions of experience" [Whewell, 1858a, 1: 268] and "necessary conditions of knowledge" [Whewell, 1841, 530]. For instance, on his view, experiencing objects as having form, position and magnitude requires the Idea of Space (see [Whewell, 1844b, 487]). Because of these similarities, Whewell was accused by his contemporaries (not all of whom had themselves studied Kant) of trying to import Kant into British philosophy; in his review of the *Philosophy* DeMorgan expressed surprise that "the doctrines of Kant and Transcendental Philosophy are now promulgated in the university which educated Locke" [DeMorgan, 1840, 707]. The logician H. L. Mansel — who was the closest thing to a British Kantian in those days — thought Whewell had not gone far enough, criticizing him for his "stumble on the threshold of Critical Philosophy" [Mansel, 1860, 258].

However, this was a threshold that Whewell did not intend to cross. There are important differences between Kant's transcendental philosophy and Whewell's antithetical epistemology. Whewell did not follow Kant in distinguishing between the a priori components of knowledge provided by intuition (*Sinnlichkeit*), the Understanding (*Verstand*) and the faculty of Reason (*Vernunft*). Thus Whewell drew no distinction between "precepts", or forms of intuition, such as Space and Time, the categories, or forms of thought, in which Kant included the concepts of Cause and Substance, and the transcendental ideas of reason. Further, Whewell included as "Fundamental Ideas" many which function not as conditions of experience but as conditions for having knowledge within their respective sciences: although it is certainly possible to have experience of the world without having a distinct Idea of, say, Chemical Affinity, we could not have any knowledge of certain chemical processes without it ([Whewell, 1858a, 2: 2-3] and [Whewell, 1860, 349]). Unlike Kant, Whewell did not attempt to give an exhaustive list of these Fundamental Ideas; rather, he believed that there are others which will emerge in the course of the development of science. Moreover, and perhaps most importantly for his philosophy of science, Whewell rejected Kant's claim that we can only have knowledge of our "categorized experience". The Fundamental Ideas, on Whewell's view, accurately represent objective features of the world, independent of the processes of the mind, and we can use these Ideas in order to have knowledge of these objective features.[32] Indeed, Whewell criticized Kant for viewing external reality as

Hayward (1848) inscribed "The Rev. Dr. Whewell/With the Translator's respects". In the text, the translator added a note to Kant's Preface to the 2^{nd} edition: Kant wrote "If the intuition must regulate itself according to the property of the objects, I do not see how one can know anything with regard to it, *a priori*, but if the object regulates itself, (as objects of the senses,) according to the property of our faculty of intuition, I can very well represent to myself this possibility" (pp. xxv-xxvi). The added note reads "See Whewell's *Philosophy of the Sciences,* vol. ii, p. 479, where this is more freely translated and more elegantly expressed" (p. xxvi).

[32]In 1865 Charles Peirce expressed a similar view of Whewell's divergence from Kant's view: "Dr. Whewell has usually been considered a Kantian. Up to a certain point this is true. He accepts Kant's division of the matter and form of our knowledge and also his theory of space and time but he seems to have cast away from the doctrine of the limits of our knowledge which

a "dim and unknown region" [Whewell, 1860, 312]. Whewell's justification for the presence of these concepts in our minds takes a very different form than Kant's transcendental argument. For Kant, the categories are justified because they make experience possible. For Whewell, though the categories *do* make experience (of certain kinds) possible, the Ideas are justified by their origin in the mind of a divine creator. And finally, the type of necessity that Whewell claimed is derived from the ideas is very different from Kant's notion of the synthetic a priori (we will return to this final point in section VIII).

Thus we should take seriously Whewell's frequent denials that his epistemology is identical or even particularly similar to Kant's. In his response to DeMorgan's review, a privately-published pamphlet, Whewell noted that his critic had gone too far in associating him with the Kantian philosophy: "It might have occurred to him, ... that by the very circumstance of classing many other ideas with those of space and time, I entirely removed myself from the Kantian point of view" [Whewell, 1840b, 4]. In his reply to Mansel, later published as part of the *Philosophy of Discovery*, Whewell clearly noted that he never intended to follow Kant's view, and pointed out ways in which his philosophy differs from Kant's [Whewell, 1860, 335–46]. What he particularly admired of Kant's work was his having shown the untenable nature of the account of knowledge given by the Lockean school [Whewell, 1860, 308]. Whewell's dislike of Locke began early, even before his 1825 reading of the first *Critique*: in 1814 he wrote to his former headmaster, the Reverend G. Morland, that while reading Locke he "grew out of humour with [him]".[33] In his chapter on "The Influence of German Philosophy in Britain" in the final edition of the *Philosophy*, Whewell set up a tripartite division showing himself to be in between the idealism of the Germans and the strict empiricism of Mill, standing in here for the Lockean school. Speaking of himself in the third person, Whewell wrote: "Kant considers that Space and Time are conditions of perception, and hence sources of necessary and universal truth. Dr. Whewell agrees with Kant [only] in placing in the mind certain sources of necessary truth; he calls these Fundamental Ideas, and reckons, besides Space and Time, others, as Cause, Likeness, Substance, and several more. Mr. Mill, the most recent and able expounder of the opposite doctrine, derives all truths from Observation, and denies that there is such a separate source of truth as Ideas".[34] Whewell was, as he insisted, seeking a "middle way" between the excesses of Locke and Kant.

It is noteworthy that, while working out his theory, there are no references to Kant or uses of technical Kantian terminology such as "forms of intuition", "categories", "analytic and synthetic truths" in his letters to Jones, his notebooks,

is the essence of the critical philosophy" [Peirce, 1982, 205].

[33] William Whewell to George Morland, n.d., April 1814, in [Todhunter, 1876, 2: 2].

[34] [Whewell, 1860, 336]. Buchdahl claims in his [1991] that Whewell shares Kant's "idealist" or "subjectivist" approach to the Ideas, and therefore also to the knowledge we gain using them. Butts [1994, 278] maintains that Whewell "owes his theory of science to Kant". Fisch has a more nuanced view, claiming that there are "Kantian undertones" in Whewell's epistemology, but less than generally supposed (see his [1991, 105]).

or his early drafts of the *Philosophy.*[35] This suggests that Kant was not explicitly on Whewell's mind as he strove to develop an epistemology that could incorporate both empirical and ideal elements of knowledge. While there are, as we have seen, some aspects of Whewell's view that are similar to Kant's epistemology, it is not necessarily the case that Whewell derived these directly from his reading of Kant in 1825. Whewell certainly appreciated Kant's critique of the Sensationalist school, and took note of his ideal solution. But his reading of Dugald Stewart, Thomas Reid and other Scottish philosophers similarly impressed Whewell, and may well have first exposed him to the type of ideal solution given by Kant.[36] Whewell expressed an appreciation of the work of the Scottish philosophers as early as 1814, when he sent a letter full of their praise to Morland.[37] Years later, when he published the *Philosophy,* Whewell applauded the "intelligent metaphysicians in Scotland", including "Reid, Beattie, Dugald Stewart, and Thomas Brown", for arguing against the "sensationalism" of Hume and Locke (see [Whewell, 1860, 214] and [Whewell, 1847, 2: 309]. He noted that their shared view, "according to which the Reason or Understanding is the source of certain simple ideas, such as Identity, Causation, Equality, which ideas are necessarily involved in the intuitive judgments which we form, when we recognize fundamental truths of science, approaches very near in effect to [my] doctrine ... of fundamental ideas ..." [Whewell, 1860, 215] (see also [Whewell, 1847, 2: 310–11]). Stewart had argued, for example, that every act of observation involves an "interpretation of nature". In his *Elements of the Philosophy of the Human Mind*, he explained that "without theory or, in other words, without general principles inferred from a sagacious comparison of a variety of phenomena, experience is a blind and useless guide" (cited in [Corsi, 1987, 97]). It seems likely that Stewart was at least as much an influence upon the development of Whewell's epistemology as Kant.

Similarly, Coleridge's writings did not inspire Whewell's antithetical epistemology. Commentators have made much of the network of friendships at Trinity including classical and German scholars influenced by Coleridge and German philosophy such as Hare, Rose, Thirwall, F.D. Maurice and John Sterling, and men of science such as George Peacock, G.B. Airy, Sedgwick, Herschel, and Whewell (see [Preyer, 1985] and [Cantor, 1991, especially p. 77]. However, friendship, in these cases, did not imply philosophical agreement. Whewell was especially close to Hare and Rose, but he did not join them in their enthusiasm for the Romantic movement in general or its specific views of education, history, poetry or philosophy. Whewell particularly objected to the way the Romantics set up an opposition between reason and feeling or "enthusiasm", elevating the latter over the former. He argued this point with Rose. "Why will you not see that in speculative matters, though Reason may go wrong if not guided by our better affections, you cannot

[35]This point is also noted by [Fisch, 1991, 105].

[36]Pietro Corsi has suggested, for example, that Whewell's earliest views of space and time as conditions of experience may have been sparked initially by his reading of Dugald Stewart's *Philosophical Essays,* prior to his reading of Kant (see [Corsi, 1988, 155]).

[37]William Whewell to George Morland, 15 June 1814, in [Todhunter, 1876, 2: 6].

do without her? All your efforts not to reason at all will only end in your reasoning very ill ... Finding that Reason alone cannot invent a satisfactory system of morals or politics, are you not quarrelling with her altogether, and adopting opinions *because* they are irrational?"[38] In a notebook entry from 1816-17, containing reading notes on Coleridge's *On the Constitution of the Church and State According to the Idea of Each, Lay Sermons,* Whewell accused Coleridge of trying to "make reason commit suicide" by ceding ground to enthusiasm.[39]

At first glance, however, there are similarities between Whewell's epistemology and the view expressed by Coleridge in his *Preliminary Treatise on Method* (1817), which served as the general introduction of the *Encyclopedia Metropolitana.* Unlike some Cambridge admirers of Coleridge, such as Whewell's friend Rose, Coleridge himself did not disparage natural science, and intended his treatise on method to illustrate the application of his philosophy to the scientific study of nature.[40] Coleridge noted that discoveries of truth are not made by accident, but by the distinct presentation of an *Idea.* He claimed that the science of Electricity had progressed more rapidly than that of Magnetism because the former contained a clear Idea of Polarity, while the latter had no clear regulative Idea [Coleridge, 1849, 17–21]. Coleridge described the "perfect" scientific method as involving the placing of particulars under a general conception, which becomes their "connective and bond of unity" [Coleridge, 1849, 54]. So far, this is not opposed to Whewell's view of the role of fundamental ideas in our knowledge. Yet, as Whewell perceived, Coleridge was attempting to "separate the poles of the Fundamental Antithesis", by asserting an absolute division between the ideal and the empirical parts of knowledge [Whewell, 1860, 424–5]. Coleridge drew a distinction between "Metaphysical" and "Physical" Ideas, explaining that "Metaphysical Ideas, or those which relate to the essence of things as possible, are of the highest class ... Physical Ideas are those which we mean to express, when we speak of the *nature* of a thing actually existing and cognizable by our faculties" [Coleridge, 1849, 20]. Metaphysical Ideas, then, have nothing to do with empirical experience, while Physical Ideas exclusively concern empirical experience. In his *Aids to Reflection* and *The Friend*, Coleridge asserted further that there are two distinct faculties, the Understanding and the Reason, claiming that these different faculties are responsible for our having knowledge of different types [Coleridge, 1965-80, 9: 252n and 4: 158]. The Understanding, according to Coleridge, is the "conception of the Sensuous, or the faculty by which we generalize and arrange the phenomena of perception"; it is the faculty that deals with our perceptions of material objects, which we gain through the senses [Coleridge, 1965-80, 4: 156]. The faculty of Reason is "the organ of the

[38]William Whewell to H.J Rose, 29 December 1823, in [Stair Douglas, 1882, 95].

[39]See a notebook entry from 1816-17, containing reading notes on Coleridge's *On the Constitution of the Church and State According to the Idea of Each, Lay Sermons.* Quoted in [Todhunter, 1876, 1: 349].

[40]See [Coburn, 1975]. Although, to my knowledge, Whewell never referred explicitly to Coleridge's *Preliminary Treatise* in his letters to Rose or in his notebooks, he surely must have read it, given his interest in scientific method and given that Rose was one of the editors of the *Encyclopedia Metropolitana.*

Super-sensuous", in that it does not depend on the senses. Rather, Reason is the source of necessary and universal principles of mathematics and science, as well as the laws of thought. Understanding is the faculty that leads us to Theories, which, according to Coleridge, describe relations that are only contingent, being "the result of observation". Reason is the faculty that leads us to Laws, which he claims describe necessary relations between things.[41] Thus Coleridge maintained that the Ptolemean system and the Newtonian system were developed by different faculties: the Ptolemean system by the Understanding, the Newtonian system by Reason [Coleridge, 1965-80, 9: 252–3].

Whewell pointedly criticized Coleridge's view of the discoveries of Ptolemy and Newton, calling it "altogether false and baseless". He noted that "the Ptolemaic and the Newtonian system do not proceed from different faculties of the mind, but from the same power, exercised more and more completely" [Whewell, 1862, 122]. As we have seen, Whewell believed that *all* knowledge requires the use of both sensations and conceptions; he explained that "there is in science no faculty which judges according to sense without doing more; and no creative or suggestive faculty which must not submit to have its creations and suggestions tested by the phenomena" [Whewell, 1862, 123]. Further, Whewell objected to Coleridge's denigration of the understanding by associating it with the instinct of animals (in *Aids to Reflection* Coleridge compares this faculty to the instinct of bees and ants). In this way Coleridge made even more explicit his disdain for the faculty of perception and the empirical facts with which it deals in the attainment of human knowledge.[42] Like Plato, whom Whewell also criticized, Coleridge was guilty of "disparaging or neglecting facts" [Whewell, 1860, 11]. After writing one of his papers on Plato, Whewell sent it to DeMorgan, writing "I hope you will think that in the paper I send you I have demolished the Coleridgean account of *Reason* and *Understanding*".[43]

4 WHEWELL'S DISCOVERERS' INDUCTION

Once he had developed his antithetical epistemology, Whewell was able to construct an inductive methodology that accounted for the discovery of laws while incorporating both empirical and a priori elements. Whewell's first explicit, lengthy discussion of this inductive method — which he called (in an 1859 letter to DeMorgan) "Discoverers' Induction" — appeared in the 1840 first edition of the *Philosophy of the Inductive Sciences, Founded Upon Their History*.[44] His view of inductive method remained essentially unaltered through his publication of the

[41][Coleridge, 1849, 21]. As we will see below, on Whewell's view this distinction between laws and theories is invalid, because there is no fundamental division between necessary truth and empirical truth

[42]See [Coleridge, 1965-80, 9: 243–5] and [Whewell, 1862, 123–6]. See also Whewell's notes on Coleridge's *Aids to Reflection* (WP R.G.13 f. 39, p. 21, quoted in [Levere, 1996, 1686–89]).

[43]William Whewell to Augustus DeMorgan, 2 February 1858, WP O.15.47 f. 23.

[44]William Whewell to Augustus DeMorgan, January 18, 1859, in [Todhunter, 1876, 2: 417].

second edition of the Philosophy in 1847, and the third and final edition, which appeared as three separate works between 1858 and 1860 (*The History of Scientific Ideas*, in two volumes [1858], the *Novum Organon Renovatum* [1858] and the *Philosophy of Discovery* [1860]). In describing Discoverers' Induction, Whewell began by noting that the standard view of induction holds that it is "the process by which we collect a General Proposition from a number of Particular Cases" [Whewell, 1847, 2: 48]. However, Whewell rejected this overly narrow notion of induction, casting induction in the light of the Fundamental Antithesis by arguing that, in scientific discovery, it is not the case that "the general proposition results from a mere juxta-position [sic] of the cases" (that is, from simple enumeration of instances) [Whewell, 1847, 2: 48]. Rather, Whewell explained that "there is a New Element added to the combination [of instances] by the very act of thought by which they were combined" [Whewell, 1847, 2: 48]. As Jones had suggested years before, induction was described by Whewell in the various editions of the *Philosophy* as an "act of the intellect" which includes both observation and reasoning.[45] In the *Philosophy*, Whewell coined the term "colligation" to describe this "act of thought".

Colligation, Whewell explained, is the mental operation of bringing together a number of empirical facts by "superinducing" upon them a fundamental conception that unites the facts and renders them capable of being expressed by a general law. The conception provides the "true bond of Unity" tying together the phenomena, by providing a property shared by the known members of a class (note that, in the case of causal laws, this can be the property of sharing the same cause) [Whewell, 1847, 2: 46]. We have already seen that, in his 1831 review of Herschel's *Preliminary Discourse*, Whewell used a metaphor suggesting that the facts of nature are like pearls, and the conception is the string upon which the pearls can be threaded. Without the string we have nothing but an "indiscriminate" heap of pearls, while with the string we have something ordered and beautiful. In all editions of the *Philosophy*, Whewell continued to believe that laws are formed by connecting facts with a uniting conception.

In order to colligate facts with a conception, we must first have suitable facts. Whewell noted that it is necessary to have "already obtained a supply of definite and certain Facts, free from obscurity and doubt" [Whewell, 1847, 2: 26]. It is useful at this point to consider what Whewell precisely meant by a "fact". In some places Whewell oversimplified matters, as when he claimed that the colligation of facts involves the establishing of a connection "among the phenomena which are presented to our senses".[46] This suggests that "facts" are simply observed phenomena. However, the situation is complicated for Whewell by his antithetical epistemology. As we have seen, his view of the nature of knowledge entails that all observation is mediated by our Ideas and conceptions; thus there can be no

[45] See [Whewell, 1858b, 36], and compare to letter from Richard Jones, 7 March 1831, WP Add.Ms.c.52 f. 26.

[46] [Whewell, 1847, 2: 36]. He was being sloppy here, as the passage follows the pages in which he described the need to decompose facts.

conception-free observation [Whewell, 1858a, 1: 32]. One consequence of this, Whewell explained, is that we cannot easily separate in our perceptions of things the element that our mind contributes and the element that comes from outside our mind [Whewell, 1847, 2: 27]. It is, indeed, impossible to do so [Whewell, 1847, 2: 30]. Nevertheless, we can attempt to make explicit the Ideas and conceptions that are involved in our perception of specific phenomena. Once we do so, we can use those facts that have reference to the more "exact Ideas" of Space, Time, and Cause, or the conceptions associated with these Ideas, such as Position, Weight, Number, as the "foundation of Science" [Whewell, 1847, 2: 30–2]. This is useful because these Ideas and conceptions are particularly definite and precise [Whewell, 1847, 2: 38]. The operation by which we separate complex facts into more simple facts exhibiting the relations of Space, Time and Cause, is called the *Decomposition of Facts* [Whewell, 1847, 2: 33]. Whewell described it as a method of "render[ing] observation certain and exact" [Whewell, 1847, 2: 35]. He outlined a number of "Methods of Observation" which can be used in this process.[47]

Once we have decomposed facts, Whewell explained, we can examine them with regard to other Ideas and conceptions. We can bind together the facts by applying to them a "clear and appropriate" conception; although such a conception may not be as exact and precise as the conceptions associated with Space, Time and Cause, it must still be clear enough to be capable of giving "distinct and definite results" [Whewell, 1847, II: 39]. It is necessary to "explicate conceptions", that is, clarify them and render them as precise as possible. Conceptions used in colligation must be not only clear, but also "appropriate" to the facts involved. As Whewell pointed out, attention is generally not given to this aspect of discovery. He noted that "the defect which prevents discoveries may be the want of suitable ideas, and not the want of observed facts" [Whewell, 1840b, 7]. An "appropriate" or "suitable" conception is one which expresses a property or cause shared by the facts which it is used to unify. As Whewell put it, conceptions must be "modifications of that Fundamental Idea, by which the phenomena can really be interpreted" [Whewell, 1858b, 30]. Often scientists apply an inappropriate conception to a set of facts, as when astronomers prior to Kepler applied the conception of epicycles to colligate planetary motions. This was not the appropriate conception, because it does not describe the way that planets do, in fact, move. Scientific discoveries are made not merely when accurate observations are obtained, as was the case after Tycho Brahe's observations of the orbit of Mars, but when in addition to accurate observations the appropriate conception is used, as when Kepler applied the conception of an ellipse rather than that of the epicycle. Whewell observed that finding this correct conception is often the most difficult part of discovery, the part that gives it its "scientific value". As Whewell noted, Tycho Brahe already

[47] [Whewell, 1847, 2: 337–58]. Whewell's interest in methods of observation was not merely theoretical, but also practical. In the mid-1830s, Whewell organized a vast international project of tidal observations for the Royal Society, which awarded him a gold medal in recognition of this work in 1837. As part of this enterprise Whewell wrote a *Memorandum and Directions for Tide Observations.* For more on Whewell's work on tidal science, see [Snyder, 2002; Cartwright, 1999].

had access to the "facts", but only Kepler was able to determine that these facts could be correctly colligated by the conception of an ellipse. Another example used by Whewell is Aristotle's inability to account for the mechanical forces in the lever by attempting to use the conception of a circle to colligate the known facts regarding the proportion of the weights that balance on a lever. Archimedes later showed that this was an inappropriate conception, and instead used the idea of pressure to colligate these facts [Whewell, 1847, 1: 71–2]. The problem with other notions of induction — most notably Whately's — is that they omit precisely this crucial and difficult step of finding the appropriate conception (or, in Whately's terms, the minor premise).

How does a scientist discover the appropriate conception with which to colligate a group of facts? Whewell believed that this was an essentially rational process, but one that was not susceptible of being put in algorithmic form. What is most important to this process is that the discoverer's mind contains a clarified or "explicated" form of the appropriate conception. The explication of conceptions is necessary because, although Whewell claimed that the Ideas and their conceptions are provided by our minds, they cannot be used in their innate form. Indeed, in an 1841 paper read before the Cambridge Philosophical Society, "Demonstration that all Matter is Heavy", Whewell denied that his Fundamental Ideas were innate ideas in the typical sense of the term. Unlike innate ideas, the Fundamental Ideas are not "self-evident at our first contemplation of them" [Whewell, 1841, 530]. Later, Whewell introduced the term "germs" to describe the original form of the conceptions in our minds. In the third edition of the *Philosophy*, Whewell explained that "the Ideas, the germ of them at least, were in the human mind before [experience]; but by the progress of scientific thought they are unfolded into clearness and distinctness" [Whewell, 1860, 373] (see also [Whewell, 1858b, 30–49] and [Snyder 2006, 55–60]).

Whewell is a realist about our knowledge of the world, as he reassured Herschel (who had questioned his commitment to realism in his review of the *Philosophy* and the *History* [Herschel, 1841]).[48] Our Idea of Space, Whewell explained, is what enables us to conceive things as existing in space with spatial characteristics; but the reason we conceive things as so existing in space is that "they do so exist" [Whewell, 1844b, 488]. Objects in the world really do exist in spatial relations to each other, and with spatial characteristics of size and shape. By the 1850s, Whewell came to argue that our Fundamental Ideas correspond to the world because both Ideas and world have a common origin in a divine creator. Our minds contain the germs of these ideas because God "implanted" these germs within our minds. Eventually, then, Whewell placed his epistemology on a theological foundation.

This theological foundation for Whewell's epistemology was inspired by the work

[48]When a person has knowledge about something, Whewell elsewhere claimed, "we conceive that he knows it because it is true, not that it is true because he knows it" [1844a, 479]. I thus disagree with Buchdahl's reading of Whewell in his [1991].

of his childhood friend, the comparative anatomist Richard Owen.[49] In the early to mid-40s, Owen developed his theory of the archetypal vertebrate.[50] Owen first published his theory in 1846 in his work on the *Anatomy of Fishes* and developed it further in *On the Archetype and Homologies of the Vertebrate Skeleton* (1848) and *On the Nature of Limbs* (1849). Owen argued that individual vertebrate animals could be seen as modified instantiations of patterns or archetype forms that existed in the Divine Mind. This "unity of plan" view allowed Owen to explain homologies, that is, similar structures that have quite different purposes, such as the wing of a bird and the forelimb of a quadruped. According to Owen, the similarity of these structures that were used in such different ways was a consequence of their being variations on the vertebrate archetype. Similarly, structures without apparent purpose, such as male nipples, did not contradict the claim that all of creation was designed; rather they were the result of the application of general archetypes (for more on Owen, see [Rupke, 1994]).

Whewell applied Owen's archetype theory to his epistemology in five chapters originally written for the *Plurality,* which Whewell deleted at the last moment before publication.[51] (Whewell took this drastic step on the advice of his friend Sir James Stephen, who had been Regius professor of history at Cambridge since 1849, and who was concerned that these chapters were too "metaphysical" for the general reader.) Whewell added a discussion of this point in the last volume of the final edition of the *Philosophy of the Inductive Sciences,* the *Philosophy of Discovery*, published in 1860; presumably, the intended audience for this book could be expected to follow metaphysical expositions (see [Whewell, 1860, chapters 30 and 31]). Whewell argued that there were many archetypical Ideas in the Divine Mind, in accordance with which the universe was created. As he put it, God "exemplified" in his creation certain Ideas existing in His mind, by creating the universe in accordance with these Ideas.[52] For example, God exemplified an Idea of Space in his universe by creating all physical objects as having spatial characteristics, and as existing in spatial relations to each other. The Idea of Space, then, became on Whewell's view an archetypical Divine Idea similar to the Idea of the vertebrate skeleton: it was no less embodied in the physical world. In creating us in his own image, God implanted us with the germs of the Divine Ideas. Whewell claimed that "our Ideas are given to us by the same power which made the world". We can know the world because God has created us with the Ideas needed to know it. More precisely, our minds are created with the "germs" from which the Ideas can develop [Whewell, 1860, 373]. God, then, has implanted within

[49]In his [2003], Sloan argues that Whewell's philosophy was an influence upon Owen's development of his archetype theory, specifically Owen's concept of the archetype as both empirical and transcendental (see pp. 58–9).

[50]For more details regarding the development of Owen's theory and its appearances in his lectures in the early 40s, before its publication, see [Sloan, 2003, 55–7].

[51]See William Whewell's letter to Richard Jones, 30 December 1853, WP Add.Ms.c.51 f. 278.

[52][Whewell, 1860, 371–379], and *Of the Plurality of Worlds*, printer's proofs including five chapters not included in the published version, WP Adv.c.16 f.27, p. 277. These chapters are included in the edition of the *Plurality of Worlds* edited by Michael Ruse [Whewell, 2001].

us the germs that need to be unfolded into Ideas representing those archetypical Divine Ideas upon which the universe was created. Gaining a clear view of these Ideas enables us to have knowledge of both the natural world and its Creator.[53]

Because the Fundamental Ideas exist innately within us only in the form of germs which must be unfolded, the explication of Ideas and conceptions is a necessary part of science. Explication of conceptions is a necessarily social process, proceeding by "discussion" and "debate" among scientists; explication is not a process that can take place solely within the mind of an individual genius. Whewell noted that disputes concerning different kinds and measures of Force were important in the progress of mechanics, and the conception of the Atomic Constitution of bodies was currently being debated by chemists [Whewell, 1847, 2: 6–7]. Whewell explained that by arguing in favor of a particular meaning of a conception, scientists are forced to clarify and make more explicit what they really mean. This is beneficial, whether or not the original expression prevails. If it does not, then it is replaced by a more accurate or clearer expression of the conception. If it does, the original expression has been improved by the scientists' efforts. Thus, like Bacon, Whewell argued that "the tendency of all such controversy is to diffuse truth and to dispel error. Truth is consistent, and can bear the tug of war; Error is incoherent, and falls to pieces in the struggle" [Whewell, 1847, 2: 7]. Whewell claimed that the explication of conceptions is a "necessary part of the inductive movement" [Whewell, 1858b, vii]. Indeed, a large part of the history of science is the "history of scientific ideas"; i.e., the history of their explication and subsequent use as colligating concepts [Whewell, 1858a, 1: 16]. This is why, even though Whewell worried about his philosophy being considered too metaphysical by Jones and Herschel, ultimately he believed that metaphysics is a necessary part of science at all stages; he explained that "the explication or ... the clarification of men's ideas ... [is] the metaphysical aspect of each of the physical sciences" [Whewell, 1858b, vii]. Disagreeing with Auguste Comte, who had claimed that at its highest level of advancement the intellect becomes "scientific" by purging itself of metaphysics, Whewell argued that successful discoverers differ from "barren speculators" not by rejecting metaphysics, but by having "*good* metaphysics" rather than bad [Whewell, 1858b, vii].

Although the explication of conceptions is a process that occurs by discussion and debate among groups of scientists, scientific discoveries are, Whewell noted, generally made by individuals. Why is it that particular individuals are able to discover the appropriate conception to apply to a set of facts? Whewell pointed to a facility for "invention", the quality of "genius"; yet he strongly denied that there is anything "accidental" about scientific discoveries. Whewell explicitly opposed the view popularized by David Brewster in his *Life of Newton* (1831) that the discovery of the law of universal gravitation was a "happy accident". Whewell stressed that discovery is always preceded by much intellectual and scientific preparation.

[53]Just as Owen's archetype theory was supplanted by Darwin's theory of evolution, so too could Whewell's theological foundation for his epistemology be replaced by an evolutionary epistemology. For an early attempt, see [Snyder, 1996].

Thus, Whewell noted that Kepler was able to discover the elliptical orbit of Mars because his mind contained the clear and precise Idea of Space, and the conception of an ellipse derived from this Idea. Whewell explained that "to supply this conception, required a special preparation, and a special activity in the mind of the discoverer ... To discover such a connection, the mind must be conversant with certain relations of space, and with certain kinds of figures" [Whewell, 1849, 28–9]. The fact that Kepler's mind was more "conversant" with the long-explicated conceptions derived from the Idea of Space, such as that of the ellipse, explains why he was able to recognize that the ellipse was the appropriate conception with which to colligate the observed points of the Martian orbit, while Tycho Brahe (who had made the most precise observations) was not.[54]

Another useful quality for the discoverer is a certain facility in generating a number of possible options for the appropriate conception. Often, before the appropriate conception is applied to the facts, the discoverer must call up in his mind a number of possibilities. Whewell sometimes used the term "guesses" to describe this stage of the discovery. Because of this, twentieth-century commentators, for the most part, have incorrectly viewed Whewell's methodology as similar to the "method of hypothesis" (or, as it is now known, the "hypothetico-deductive" method). (For examples of this interpretation, see [Achinstein, 1992], [Buchdahl, 1991], [Butts, 1987], [Hanson, 1958], [Hempel, 1966], [Laudan, 1971], [Laudan, 1980], [Niiniluoto, 1977], [Ruse, 1975], [Wettersten, 1994], and [Yeo, 2004].) On that view, there is no rational inference to a hypothesis; its formation is generally described as a "guess". However, Whewell claimed that the *application* or the *selection* of the appropriate conception, in Kepler's case and in all cases of discovery, is not a matter of guesswork. Whewell described this process as being one in which "trains of hypotheses are called up and pass rapidly in review; and the *judgment* makes its choice from the varied group" [Whewell, 1847, 2: 42, emphasis added]. Thus, even though at a certain point in his investigation (i.e., once he had inferred that the Martian orbit was some type of oval) Kepler called up in his mind "nineteen hypotheses" of possible ovals, his choice of the appropriate ellipse conception was based on "calculations", and hence on a rational process (and, certainly, his conclusion that the orbit was oval involved much rational inference). Nor is the selection of an appropriate conception a matter of mere observation. Whewell claimed that to choose the appropriate conception requires more than this: "there is a special process in the mind, in addition to the mere observation of facts, which is necessary" [Whewell, 1849, 40]. (See also [Whewell, 1831, 379] and [Whewell, 1860, 256–7].) This "special process of the mind" is inference. In order to colligate facts with the appropriate conception, "we infer more than we see", Whewell explained [Whewell, 1858a, 1: 46]. Whewell claimed that, in the case of choosing the conception of force to colligate the observed motions of a needle towards a magnet, an inference, or an "interpretative act" of the mind, is required (see [Whewell, 1858a 1: 31 and 45]). Inference is required before we see

[54]For a more detailed discussion of how Kepler's discovery conformed to Whewell's methodology, see [Snyder, 1997].

that "force" is the appropriate conception with which to colligate magnetic phenomena. Since the selection of the appropriate conception with which to colligate the facts involves inference, Whewell noted that discoveries are made "*not* by any capricious conjecture of arbitrary selection" [Whewell, 1858a, 1: 29], that is *not* by guesswork.

Indeed, selecting the appropriate conception typically requires not just one inference, but a series of inferences. This is why Whewell claimed that discoverers' induction is a process, involving a "train of researches" [1857a 1: 326]. (See also [Lugg, 1989].) Whewell explicitly opposed limiting inductive discovery to enumerative inference, writing that "induction by mere enumeration can hardly be called induction" [1850, 451]. Indeed, enumerative inference could not, in most cases, account for the discovery of the appropriate conception with which to colligate the data. Rather, Whewell allowed that any form of valid inference could be used. He especially stressed the power of analogical reasoning. He extolled the importance of analogical inference in two reviews from the early thirties, his review of Herschel's *Preliminary Discourse* and his discussion of the second volume of Lyell's *Principles of Geology* (see [Whewell, 1831, 385] and [Whewell, 1832, 110]. See also [Whewell, 1837, 2: 391]). Decades later, in his *Plurality of Worlds*, Whewell more systematically discussed the importance of defining the varying degrees of precision and relevance to different kinds of analogies [Whewell, 1855]. Not everyone was pleased with this liberal notion of inductive reasoning. As we have seen, DeMorgan complained about certain writers using the term "induction" as including "the use of the whole box of tools" [DeMorgan, 1847, 216].

Although inductive discovery is not a matter of accident, according to Whewell, there is no "logic of discovery" in the sense of an algorithmic procedure. According to Whewell, "no maxims can be given which inevitably lead to discovery. No precepts will elevate a man of ordinary endowments to the level of a man of genius" [Whewell, 1858b, 94]. As we have seen, Whewell's antithetical epistemology entails the need for the existence of clear conceptions in the mind of the discoverer; and, while there are methods to aid in the clarification and selection of the appropriate conception, there is no algorithm or mechanical method for this process. This is why invention requires "genius". However, the fact that Whewell denied the possibility of a mechanical method of discovery does not entail his rejection of a rational discovery-method. His discoverers' induction is a rational method for the discovery of laws, even though it is not purely mechanical. His method does not contain rules that are universally applicable but does offer rational "instruments" for aiding in discovery. Thus, in a letter to DeMorgan, Whewell noted that it is possible to have an "*Art* of Discovery"; he quipped "if I had £20,000 a year which might be devoted to the making of discoveries, I am sure that some might be made".[55]

[55] William Whewell to Augustus DeMorgan, 14 February 1859, in [Todhunter, 1876, 2: 416].

5 CONFIRMATION OF THEORIES

Once Discoverers' Induction results in a new hypothesis, Whewell claimed that there were three ways to confirm it. These confirmation criteria are first, that "our hypotheses ought to *fortel* phenomena which have not yet been observed; at least phenomena of the *same kind*" as those which the hypothesis originally colligated [Whewell, 1858b, 86]; second, that they should "explain and determine cases of a *kind different* from those which were contemplated in the formation" of the hypotheses [1858b, 88]; and third that hypotheses should "become more coherent" over time [1858b, 91]. These verification criteria are known respectively as prediction, consilience, and coherence.

On Whewell's view, prediction includes a temporal element: a "successful prediction" occurs when some fact unknown at the time that the theory was discovered is predicted by the theory, and is afterwards found to be true. Such predictions, when they turn out to be successful, are evidence in favor of the hypothesis. (However, contrary to Mill's assertion in his [1963-91, 7: 501], Whewell did not argue that successful predictions were *conclusive* evidence for a theory; indeed, as examples of hypotheses that led to successful predictions, Whewell included several that were already known to be false, such as the epicycle theory.) Moreover, Whewell claimed even more strongly that successful predictions were better evidence than mere explanations of known facts. He maintained that "to predict unknown facts found afterwards to be true is ... a confirmation of a theory which in impressiveness and value goes beyond any explanation of known facts" [Whewell, 1857a 2: 464].

Whewell argued for his two claims about predictive success by invoking an analogy between constructing a true theory and breaking a code. "If I copy a long series of letters of which the last half-dozen are concealed, and if I guess these aright, as is found to be the case when they are afterwards uncovered, this must be because I have made out the import of the inscription" [Whewell, 1860, 274]. Successful prediction of formerly unknown facts, more so than the colligation or explanation of known facts, Whewell thus argued, is proof that we have broken the code of Nature, that we have "detected Nature's secret" [Whewell, 1858b, 87]. Additionally, he compared our ability to colligate facts into hypotheses by his methodology to knowing the alphabet of the language of Nature. As in learning a new language, however, this is not enough; we must also know how to construct intelligible words and sentences using the alphabet, and thus learn how to use "the legislative phrases of nature" [Whewell, 1858b, 87]. Whewell noted that it is easier to learn the alphabet than to use these legislative phrases; thus successful predictions of unknown facts serve as *better* evidence than colligations of known facts. Our prediction of some new fact is analogous to the attempt of a language student to form a proper sentence in her new language. When our prediction is confirmed, as when the attempted sentence elicits an appropriate response from a native speaker, it is a sign that we have spoken correctly: Nature is "respond[ing] plainly and precisely to that which we utter, [and] we cannot but suppose that we have in great measure made ourselves masters of the meaning and structure of her language" [Whewell, 1858b, 87].

Whewell's view of the power of prediction seemed to be borne out in 1846, when a successful prediction appeared to provide stunning evidence for the truth of Newton's theory. In that year, the planet Neptune was discovered after its existence, position and even its mass had been predicted mathematically. Perturbations in the orbit of Uranus expected on Newtonian theory had led some to conclude that there must be an unobserved planetary body external to Uranus' orbit exerting an additional gravitational force on the planet. Using Newton's theory, the French mathematician U.J.J. Le Verrier calculated mathematically the mass and orbit of this postulated planet.[56] Acting upon Le Verrier's calculations, astronomers at the Berlin Observatory found the planet less than one degree from its expected location. From Newton's theory, then, it was possible to predict successfully the existence, position and mass of a previously unexpected planet. This success was considered further and quite strong evidence for Newton's theory of Universal Gravitation. Whewell praised the discovery as a triumph of astronomy, which he termed the "Queen of the Sciences". He argued that predictive success is strong confirmation of a theory, because the agreement of the prediction with what is found to be true is "nothing strange, if the theory be true, but quite unaccountable, if it be not" [Whewell, 1860, 273–4]. If Newtonian theory were not true, Whewell was suggesting, the fact that from the theory we could correctly predict the existence, location and mass of Neptune would be bewildering, and indeed miraculous, equivalent to the feat of a non-speaker of Russian forming an intelligible and meaningful question in that language which elicited a proper response from a native speaker.

Whewell did not offer an argument for his claim about the superiority of "new evidence" over "old evidence", that is, of successful predictions over explanations of known facts. He merely stated that "If we can predict new facts which we have not seen, as well as explain those which we have seen, it must be because our explanation is not a mere formula of observed facts, but a truth of a deeper kind" [Whewell, 1849, 60]. The intuition behind this claim, which has been made by numerous modern commentators, is that if a theorist knows what fact must be explained by his theory, it is simple for him to "cook" the theory in order to somehow explain the known fact; whereas, on the other hand, if a theory predicts a novel (unknown) fact it is not due to the ingenuity of the theorist, but to the truth of the theory. However, this intuition has been contested in recent years (see [Brush, 1989; Snyder, 1998]). Interestingly, the discovery of Neptune, Whewell's paradigmatic case of a novel prediction, may be considered not only as a prediction of a new fact, but equally well as an explanation of a known fact, namely the perturbations of the planet Uranus. The problem for Newton's theory was described in this way by Airy in his report to the BAAS in 1832: "I need

[56]At roughly the same time an Englishman, James Couch Adams, also made these calculations, which very nearly caused an international priority war between the British and the French after the planet was discovered (see [Smith, 1989]).

not mention that there are other subjects (the theory of Uranus, for instance) in which the existence of difficulties is known, but in which we have no clue to their explanation" [Airy, 1833, 189]. Thus whether the postulation of an unseen planet is considered an explanation of a known fact or the prediction of a new fact depends upon the way it is described, not on any objective features of the evidence itself or of its relation to the theory. So it would be odd, to say the least, if the way of *describing* the evidence made a difference to its evidential value. It should be noted that Whewell only supported the thesis about the superiority of new evidence in very strong cases of predictive success, such as the discovery of Neptune; he explained of such success that "It is a confirmation which has only occurred a few times in the history of science, and in the case only of the most refined and complete theories, such as those of Astronomy and Optics" [Whewell, 1857a 2: 464].

Whewell's next, and most interesting, confirmation test is consilience. Whewell explained that "the evidence in favour of our induction is of a much higher and more forcible character when it enables us to explain and determine cases of a *kind different* from those which were contemplated in the formation of our hypothesis. The instances in which this have occurred, indeed, impress us with a conviction that the truth of our hypothesis is certain" [Whewell, 1858b, 87–8]. Whewell called this type of evidence a "jumping together" or "consilience" of inductions. To understand what Whewell means by this, it may be helpful to schematize the "jumping together" that occurred in the case of Newton's law of universal gravitation, Whewell's exemplary case of consilience. In book III of the *Principia,* Newton listed a number of "propositions". These propositions are empirical laws that were inferred from certain "phenomena" (which are described in the preceding section of book III). The first such proposition, inferred from phenomena of "satellite motion", is that "the forces by which the circumjovial planets are continually drawn off from rectilinear motions, and retained in their proper orbits, tend to Jupiter's centre; and are inversely as the squares of the distances of the places of those planets from that centre". The result of another, separate induction from the phenomena of "planetary motion" is that "the forces by which the primary planets are continually drawn off from rectilinear motions, and retained in their proper orbits, tend to the sun; and are inversely as the squares of the distances of the places of those planets from the sun's centre". Newton saw that these laws, as well as other results of a number of different inductions, coincided in postulating the existence of an inverse-square attractive force as the cause of various phenomena.[57] According to Whewell, Newton saw that these inductions "leap to the same point"; i.e., to the same law [Whewell, 1858b, 88]. Newton was then able to bring together inductively (or "colligate") these laws, and facts of

[57]See [Newton, 1968]. As Ernan McMullin notes, the notion of the "attractive force" was problematic for Newton; he was never satisfied in his search for an "agent cause" of gravitational behavior, a "cause of gravity", as Newton put it. But Newton did succeed in causally unifying disparate phenomena by the concept of this attractive force (see [2001, 296]). It is this aspect of Newton's theory that Whewell pointed to as the exemplar of consilience.

other kinds (for example, the event kind "falling bodies"), into a new, more general law, namely the universal gravitation law: "All bodies attract each other with a force of gravity which is inverse as the squares of the distances". By seeing that an inverse-square attractive force provided a cause for different kinds of events — for satellite motion, planetary motion, and falling bodies — Newton was able to perform a more general induction, to his universal law.

As I have argued elsewhere [Snyder, 2005; 2006], understanding Whewell's notion of natural kinds is useful for comprehending his notion of consilience. Whewell believed that there are natural kinds of objects, such as minerals and species, but also natural kinds of events, such as diffraction and planetary motion. Consilience occurs when a theory brings together members of different kinds, showing that they belong to a more general classification. In the case of event kinds, individual types of events are members of the same kind when they share the same cause.[58] Newton discovered that what makes "the orbit of Mars" a member of the class "planetary motion" is that it is caused to have the properties it does by an inverse-square attractive force of gravity between Mars and the other bodies in the universe. He also found that other event kinds share this kind essence. What Newton did, in effect, was to subsume these individual event kinds into a more general kind comprised of sub-kinds sharing a kind essence, namely being caused by an inverse-square attractive force. Consilience of event kinds results in *causal unification*. More specifically, it results in unification of natural kind categories based on a shared cause. Phenomena that constitute different event kinds, such as "planetary motion", "satellite motion", and "falling bodies", were found by Newton to be members of a unified, more general kind, "phenomena caused to occur by an inverse-square attractive force of gravity" (or, "gravitational phenomena"). In such cases, according to Whewell, we learn that we have found a "vera causa", or a "true cause", i.e. a cause that really exists in nature, and whose effects are members of the same natural kind.[59] Moreover, by finding a cause shared by phenomena in different sub-kinds, we are able to colligate all the facts about these kinds into a more general causal law. Whewell claimed that "when the theory, by the concurrences of two indications ... has included a new range of phenomena, we have, in fact, a new induction of a more general kind, to which the inductions formerly obtained are subordinate, as particular cases to a general population" [Whewell, 1858b, 96]. He noted that consilience is the means by which we effect the successive generalization that constitutes the advancement of science (see [Whewell, 1847 2: 74]).

Note that consilience is importantly different from the type of reasoning known as "inference to the best explanation", even when the "best explanation" is defined as a causal hypothesis. The causal law expressing the essence of different event kinds is not one postulated merely because it explains or accounts for these

[58]For more details of Whewell's notion of natural classification, see [Snyder, 2006, chapter three].

[59]See [Whewell, 1860, 191]. Ruse [1976, 232] agrees that Whewell's notion of the vera causa lies at the center of his criterion of consilience. See also [Butts, 1977, 81] and [Ruse, 1975, 161–2].

different classes of facts. Rather, in the case of each class, the law has emerged from a process of inductive reasoning following Whewell's methodology. Thus the causal law is not imposed from above as a means of tying together different event kinds. Instead, it wells up from beneath in each separate case. What is important in the case of consilience, then, is not merely that a single causal mechanism or law can explain or account for different event kinds (as the inference to the best explanation allows), but rather that separate lines of induction lead from each event kind to the same causal mechanism or law, that there is a convergence of distinct lines of argument. At each step, there is inductive warrant for the causal law besides its explanatory utility.

Whewell discussed a further, related test of a theory's truth: "coherence" [Whewell, 1858b, 91]. Coherence occurs when we are able to extend our hypothesis to colligate a different event kind without ad hoc modification of the hypothesis; that is, without suppositions which are added merely for the purpose of saving the phenomena, for which there is no independent evidence. When Newton extended his theory regarding an inverse-square attractive force, which colligated facts of planetary motion and lunar motion, to the natural class "tidal activity", he did not need to add any new suppositions to the theory in order to colligate correctly the facts of this event kind (facts about particular tides).[60] The situation was rather different for phlogiston theory (which explained combustion, before the discovery of oxygen, by the presence of an "essence" in the burning body called "phlogiston"). According to Whewell, phlogiston theory colligated facts about "chemical combination". But when the theory was extended to colligate facts about the weight of bodies, it was unable to do so without an ad hoc and implausible modification (namely, the assumption that phlogiston has "negative weight") [Whewell, 1858b, 92–3]. The emission or particle theory of light was claimed by its adherents to be capable of colligating the facts of different kinds (indeed, the same facts that the wave theorists claimed they could colligate). However, according to Whewell and other proponents of the wave theory, the particle theory could do so only by the admission of suppositions that were ad hoc or implausible. For example, in order to account for the facts of the velocity of light (in particular, the uniformity of velocity), the particle theorists needed to suppose that light particles are emitted with different velocities based on the mass of the luminous body, but that, as Humphrey Lloyd acerbically put it, "among these velocities is but one which is adapted to our organs of vision" [Lloyd, 1835, 300–1. See also 296]. (Of course, the wave theorists were themselves accused of postulating an implausible or ad hoc "luminiferous ether"; it was argued by Whewell and others that there was independent evidence for the ether's existence, and hence that it was not an ad hoc supposition.) In non-coherent cases such as these, the modifications have no independent evidence; they are added to the theory solely because they are needed

[60]Facts about the tides were not known systematically, until Whewell engaged in his tidology research program. Part of the importance of his project, Whewell believed, was to strengthen the claim that Newton's law could colligate these facts without modification (see [Whewell, 1834b, 15–17).

to colligate a new or problematic set of facts. As this discussion suggests, coherence is closely related to consilience. The distinction between them is as follows. In cases of consilience, two independent colligations of different event kinds yield the same conclusion. In cases of coherence, the result of one or more colligations of one or more event kinds is extended, without ad hoc modification, to another event kind. This latter process generally occurs over time as theoretical systems are developed.[61]

Consilience, then, tells us that we have found a law that colligates members of a natural kind — either members of a natural object kind or a natural event kind.[62] In the case of object kinds, consilience is, Whewell suggested, evidence for the truth of a non-causal empirical law such as "all metals are crystalline when solid", and thus is evidence that we have grouped certain individuals into a kind ("metals") that is natural and hence supportive of the law. In the case of event kinds, consilience is evidence for having found a causal law, such as the inverse-square law discovered by Newton. This is at the same time evidence for the naturalness of the kind "gravitational phenomena".

We are now in a position to see why a common argument against Whewell's criterion of consilience is flawed. A number of commentators have proposed that consilience is fundamentally a relativistic or contextual criterion. According to this view, whether some theory is consilient depends upon our knowledge-state with respect to the classes involved; specifically, whether we know that certain classes of facts are members of the same general type of thing (see [Laudan, 1971]; [Forster, 1988]; [Harper, 1989] and [Morrison, 1990]). One critic notes that for Descartes, planetary and terrestrial motions were phenomena of the same type, because both were the result of vortices; thus, relative to the Cartesian system, Newton's theory was *not* consilient [Laudan, 1971, 374] (see also [Butts, 1977, 74–5; Fisch, 1985; Morrison, 1990]). However, as we have seen, the criterion for consilience is that a theory connects two (or more) different natural kinds into a more general natural kind *in virtue of inductively inferring* the same cause for each natural sub-kind. The issue, then, is not the relativistic one of whether or not we previously *knew* or *suspected* that these sub-kinds all belong to a more general kind, but rather with whether they do all fit into the same more general kind in virtue of sharing the same cause, and whether we have reached this conclusion by the proper sort of inductive reasoning (thus conferring on the conclusion inductive warrant), and not merely by postulating a common cause hypothetically. Whewell famously believed that Descartes did not reach his conclusion by proper reasoning. On the other hand, Newton's theory was consilient because it satisfied both conditions.

This erroneous criticism of Whewell's criterion of consilience may arise from a confusion of Whewell's view with that of Herschel. Herschel's brief discussion of

[61][Whewell, 1858b, 90–1]. The claim that consilience *results in* coherence can be found in [Forster, 1988] and [Harper, 1989]. However, it is more accurate to say that consilience and coherence are similar means for obtaining the same result, namely simplification in the sense of (causal) unification.

[62]Herschel recognized this in his review of Whewell's *History* and *Philosophy* [Herschel, 1841, 227]. See also [Ruse, 1976, 240].

consilience in the *Preliminary Discourse* focuses on the subjective, psychological aspects of consilience, and does not explicitly include the requirement of causal unification under more general natural kinds (see [Herschel, 1830, 170]). Herschel explained that "the surest and best characteristic of a well-founded and extensive induction ... is when verifications of it spring up, as it were, spontaneously, into notice, from quarters where they might be least expected, or even among instances of that very kind which were at first considered hostile to them" [1830, 170]. Herschel's view of consilience is prone to the relativistic criticism. Like Herschel, Whewell did at times stress the psychological aspect of consilience, noting the impact of "the unexpected coincidence of results" [Whewell, 1847, 2: 67]. Nevertheless, he emphasized the logical aspect of consilience, which does not depend upon the psychological. Whewell's consilience not only concerns the psychological element of surprise, but also the logical element of causal unification of different event or process kinds into more general kinds, in virtue of sharing a common cause.

Whewell justified this criterion by claiming that consilience is "a criterion of reality, which has never yet been produced in favour of falsehood". Elsewhere he noted that "there are no instances, in which a doctrine recommended in this manner has afterwards been discovered to be false" (see [Whewell, 1858b, 90] and [Whewell, 1860, 192]. He can thus be seen as making an inductive argument in favor of the confirmation value of consilience: namely, the argument that, since consilient theories in the past have all turned out to be true, we can infer that (probably) all consilient theories are true, and thus that our current consilient theories are (probably) true.[63] Whewell has been much ridiculed for this inductive argument by Mill and his twentieth-century followers (see, for example, [Van Fraassen, 1985, 267]). As Whewell's detractors today like to point out, the one positive instance strongly adduced by Whewell for his inductive argument has been shown to be a counter-instance: Newton's theory, which seemed irrefutable in Whewell's time, has since been shown to be false. Newton's theory employs conceptions that, it turns out, are not true of the world, such as absolute space and absolute time. The cause postulated by the theory, the inverse-square attractive force of gravity propagated through space and time, is also non-existent (it is not a *vera causa*). Thus, although Newton's theory was highly consilient, more so than any other theory of Whewell's time — it brought together different event kinds, by having inductive warrant for the inference that they shared a common cause — it turned out to be false. Whewell's inductive argument does seem to fail in establishing any confirmatory value for consilience.

However, even though Newton's theory turned out to be false, there is something about the theory which has not been disconfirmed. Newton's theory still seems to have been correct in bringing together event kinds that belong to a more general

[63]Whewell also made an additional argument for consilience's value: he claimed that consilience is like the coinciding testimony of two witnesses. In my [2005], I show how this argument can be understood as a form of "common-cause argument". But I here focus on the argument Whewell most relied upon.

kind, showing something true about the natural kind structure of the physical world, although it was wrong about what the shared kind essence of the natural kind was. Newton showed that the phenomena of free fall, pendulum motion, lunar acceleration, satellite motion and planetary motion share a cause, and in this way constitute a general event kind. This insight has not been proved false, although the particular shared cause he postulated has been rejected. Einstein's general theory of relativity proposed instead that the cause of these phenomena (and others) is the curved structure of space-time. It may well be that this proposed cause will be replaced by another; but so far it seems unlikely that a later theory will separate these different phenomena and attribute to each of them different causal structures or mechanisms. These event kinds do seem to be sub-kinds of a more general kind that is connected in nature. Thus the one truly consilient theory endorsed by Whewell is still considered to have gotten things right, at least in terms of the natural kind structure of the physical world.[64]

Of course, the fact that Newton's theory was wrong about the particular cause he postulated does show that Whewell went too far in claiming that consilience is conclusive evidence for a theory, in the sense of proving that a theory will *never* be shown to be false. And it is certainly possible that even the natural kind structure Newton's theory proposed might turn out to be wrong. It would have been more accurate — as well as more consistent with his general view of the progress of science — for Whewell to have claimed that hypotheses satisfying the condition of consilience are our best theories, but are still subject to correction. He suggested this kind of revision when he asserted, referring to Newton's Rules of Reasoning in Book Three of the *Principia*, that "the really valuable part of the Fourth Rule is that which implies that a *constant verification*, and, if necessary, rectification, of truths discovered by induction, should go on in the scientific world. Even when the law is, or appears to be, most certainly exact and universal, it should be constantly exhibited to us afresh in the form of experience and observation" [Whewell, 1860, 196, emphasis added].

6 CONSILIENCE AND DARWIN'S *ORIGIN OF SPECIES*

The power of Whewell's criterion of consilience was appreciated by Charles Darwin, who appealed to consilience in his *Origin of Species*, though not by name. Darwin was acquainted with both Whewell and his writings; he made extensive notes in his copy of Whewell's Bridgewater Treatise and the *History of the Inductive Sciences*, and probably knew details of Whewell's *Philosophy* by reading Herschel's review of it for the *Quarterly Review*. While writing the *Origin of Species*, Darwin often expressed his concern that his theory would be criticized for not being sufficiently "inductive".[65] It is therefore not surprising he would present his theory as conforming to the method of justifying inductive theories proposed

[64]See my [2005] where I show how Whewell's criterion of consilience can be used to argue for a natural-kind realism.

[65]See, for instance, his letter to Asa Gray, 29 November 1857, in [Darwin, 1983-2004, 6: 492].

by one of the acknowledged scientific experts of the day. In successive editions of the *Origin* (as his expectations of the criticisms were met and exceeded) Darwin strengthened the appeal to consilience. By the sixth edition of 1872 Darwin explained that: "It can hardly be supposed that a false theory would explain, in so satisfactory a manner as does the theory of natural selection, the several large classes of facts above specified ... It is a method used in judging of the common events of life, and has often been used by the greatest natural philosophers. The undulatory theory of light has thus been arrived at".[66]

Darwin's method of justifying his theory in the *Origin* is similar to Whewell's notion of consilience in several ways. First, Darwin's theory explains not just many facts, but many *kinds* of facts. That is, the theory provides an explanation for facts in the realms of classification, geography, paleontology, embryology, comparative anatomy, and geology. Indeed, before writing the *Origin*, Darwin spent much time observing, experimenting, and collecting data on the different kinds of phenomena and phenomenal laws that he wanted his theory to explain (see [Ospovat, 1981, 170]). In the sixth edition, Darwin added a chapter on "Miscellaneous Objections to the Theory of Natural Selection", in which he outlined more explicitly the classes of facts that can be explained by his theory but not by the theory of special creation. It is clear that Darwin was impressed with the ability of his theory to explain facts of different kinds, rather than just many facts of one class. It is possible to see these "classes of facts" as similar to Whewell's event kinds. For example, the adaptation of organisms to their environment can be seen as a kind, whose members (the polar bear's white fur, the woodpecker's thin beak) share a single cause, which the special creationists claimed was God's intentional design, while Darwin claimed it was evolution by natural selection. Another way in which Darwin's strategy is similar to consilience is that he wanted to include in his theory the laws or theories accepted by the experts in various fields, much as Whewell believed that a consilient theory includes (by causally unifying) the already-known phenomenal laws of various kinds [Ospovat, 1981, 114 and 148–9]. Thus, Darwin stressed that his theory explained phenomenal laws such as the law of embryonic resemblance (see [Ospovat, 1981, 165]).

Moreover, Darwin believed his theory to be superior not only because it provided an explanation for many different kinds of facts, but because it provided a *causal* explanation, one framed in terms of natural causes similar to those appealed to in science. For instance, his theory provided a cause for the observed cases of homological structures. Darwin claimed that neither Lamarckian evolutionary view nor the doctrine of special creation gave a natural cause of this phenomenon. But, as he noted, his theory did provide an explanation: because homologous structures descended from a common ancestor, and because the changes that eventually resulted

[66]Darwin, *Origin of Species*, 6^{th} edition, in [Darwin, 1986-89, 16: 438–9]. In a letter to Asa Gray in 1859, Darwin already expressed his confidence about the theory in these terms: while admitting that there were still some difficulties with the theory, Darwin noted that "I cannot possibly believe that a false theory would explain so many classes of facts, as I think it certainly does explain" (11 November 1859, in [Darwin, 1983-2004, 7: 369).

in the branching off of different species happened gradually, the main pattern of structure remained the same [Darwin, 1859, 434–5]. Darwin's theory provided the causal unification of Whewell's criterion of consilience as applied to classes of phenomena, by showing that all these different kinds of phenomena share a common cause, namely the alteration of original organisms by gradual modification, through the mechanism of natural selection. Darwin is an example of an eminent scientist of Whewell's own time who embraced a key part of his methodology.

However, this is not to say that Whewell believed Darwin had successfully proven his theory. Indeed, Whewell continued to reject evolutionary theory until his death seven years after the publication of the *Origin of Species.* Darwin's nephew Francis spread the story that the Master of Trinity had even kept the book out of the college library, due to his religious prejudices against evolution. However, this is an oversimplified and inaccurate understanding of Whewell's reaction to Darwin's theory as well as of his view of the relation between religion and science. Whewell had always believed that "all truths must be consistent with other truths": that truths of natural science and truths of theology do not conflict, even if we do not have full insight into how they coincide.[67] Moreover, he held that apparent conflict between science and scripture did not necessarily mean that science was to blame. In his *Philosophy*, Whewell had noted that "when a scientific theory, irreconcilable with [the Bible's] ancient interpretation, is clearly proved, we must give up the interpretation, and seek some new mode of understanding the passage in question, by means of which it may be consistent with what we know…". [Whewell, 1840a, 2: 148]. Indeed, in 1864 Whewell refused an invitation to sign a "Declaration of students of the natural and physical sciences", which claimed to support a "harmonious alliance between Physical Science and Revealed Religion", but which was seen by many scientists as attempting to put theological restraints on scientific inquiry. (Of Whewell's old friends and acquaintances, only Sedgwick and Brewster signed; De Morgan and Herschel were quite vocal in their public denunciations of the document) (see [Brock and Macleod, 1976]). For this reason, when he read Darwin's book Whewell wrote to him, "You will easily believe that it has interested me very much, and probably you will not be surprised to be told that I cannot, *yet at least*, become a convert to your doctrines. But there is so much of thought and fact in what you have written that it is not to be contradicted without a careful selection of the ground and manner of the dissent…".[68]

However, Whewell did not believe that Darwin's theory was yet fully consilient, and he had legitimate grounds for this view. Indeed, Whewell had at least four reasons for doubting the consilient status of Darwin's theory. First, while evolution by natural selection did explain different kinds of phenomena, it is not so clear that Darwin reached this theory by numerous inductions from each of these facts.

[67]Whewell expressed this view privately in a letter to H. J. Rose in 1826 (12 December; WP R.2.99 f.27) and publicly in his [1857a 3: 487]; this claim also appeared in the 1837 first edition.
[68]William Whewell to Charles Darwin, 2 January 1860, in [Darwin, 1983-2004, 8: 6, emphasis added].

Recall that, for Whewell, it was the convergence of separate lines of induction that was important: the reading of the cause independently from each class of facts. This was necessary to ensure that, in each case, the assertion of the cause as a cause for a particular event kind has high inductive warrant (is more likely than not). Otherwise, on Whewell's view, we cannot be certain that we have a *vera causa*. Darwin did not emphasize that separate processes of induction led him to his mechanism of natural selection. Indeed, Darwin believed that any hypothesis, no matter how it was first obtained, could be proven to be probably true by explaining numerous facts. In a letter of 1860 he explained that "it seems to me fair in Philosophy to invent *any* hypothesis and if it explains many real phenomena it comes in time to be admitted as real".[69] Thus it may well have seemed to Whewell that Darwin, like Herschel, was using consilience as a means to justify a bold hypothesis, rather than a true induction.

Moreover, although Whewell was a proponent of the wave theory of light, he expressed some ambivalence about the degree to which it was consilient. Though this theory did causally unify numerous optical phenomena, Whewell believed it had not attained the truly consilient status of Newton's law of universal gravitation, in part because it was unable to give a specific causal law which explained this unification. Darwin's theory suffered from the same problem: he was not able to give the causal law by which variations in organisms are produced, nor the causal mechanism by which they are inherited by offspring. Darwin admitted that "Our ignorance of the laws of variation is profound" [Darwin, 1859, 167]. Without the causal laws of genetics, Darwin's theory could have had, at best, a status for Whewell similar to that of the wave theory of light. And, thirdly, Darwin's theory had not yet stood the test of time in order to become coherent, the way the wave theory had done, even without a causal law.

Whewell had a final reason for believing that Darwin's theory was not fully consilient. In his 1838 Presidential address to the Geological Society, Whewell had laid out the challenge for any naturalistic account of man's origin. "Even if we had no Divine record to guide us", Whewell argued, "it would be most unphilosophical to attempt to trace back the history of man without taking into account the most remarkable facts in his nature" [Whewell, 1839, 642]. Whewell claimed that it was not possible to account adequately for the origin of humans without accounting for our nature as intellectual and moral beings. When he referred to Darwin's *Origin pf Species* as a "most unphilosophical book", he was indicating the fact that Darwin had not explained how an evolutionary view could account for the appearance of a rational and moral creature. As he had written in his *Plurality of Worlds* six years before the appearance of the *Origin*, "the introduction of reason and intelligence upon the Earth is no part nor consequence of the series of animal forms. It is a fact of an entirely new kind" [Whewell, 1855, 164]. Whewell believed that Darwin's theory did not colligate the most important fact about man, and so could not be fully consilient.[70]

[69]Darwin to Charles James Fox Bunbury, 9 February 1860, in [Darwin, 1983-2004, 8: 76].

[70]Darwin himself recognized the need to explain how man's reason and morality could arise

7 RENOVATING BACON

Whewell explicitly framed his reformist project in Baconian terms; he wished to "renovate" Bacon's inductive logic (indeed, one volume of the third edition of his *Philosophy* is entitled *Novum Organon Renovatum*). Yet it might seem that Whewell's discoverers' induction, because of the antithetical epistemology underlying it, is inherently opposed to Bacon's inductivism. Herschel, who shared Whewell's desire to renovate Bacon, thought that Whewell's epistemology undercut his Baconianism. Later commentators have read Whewell in a similar way, as having rejected inductivism for some kind of hypothetical methodology by the time he published the *Philosophy* (if not sooner).[71] However, as we have seen, this is not the case. In the *Philosophy,* Whewell clearly endorsed an inductive view of discovery. Moreover, Whewell continued to support an inductive methodology in later works. In *Of Induction* (1849), his work responding to Mill's criticisms of him in *System of Logic*, Whewell noted, in opposition to Mill's characterization of Kepler, that new hypotheses are properly "collected from the facts", and not merely guessed [Whewell, 1849, 17]. In his *Plurality of Worlds*, first published in 1853, Whewell strongly criticized the "bold assumptions", both "arbitrary and fanciful", that led some people to claim we had good reason to believe that there was intelligent life on other worlds. Since we have no inductive evidence for this, Whewell claimed, we should not engage in such "conjectures" and "speculations" [Whewell, 1855, 41, 122, 141, and *passim*]. In his 1857 review article of the Spedding, Ellis and Heath edition of the collected works of Bacon, Whewell continued to praise Bacon for his emphasis on the gradual successive generalization which Whewell believed characterized the historical progress of science. He also stressed this in the third edition of his *History of Inductive Sciences*, published the same year. In his "Additions" to the first volume, Whewell explained that "laborious observation, narrow and modest inference, caution, slow and gradual advance, limited knowledge, are all unwelcome efforts and restraints to the mind of man, when his speculative spirit is once roused: yet they are the necessary conditions of all advance in the Inductive Sciences". He criticized the "bold guesses and fanciful reasonings of man unchecked by doubt or fear of failure" [Whewell, 1857a, 1: 339–40]. And, to take one final example, in his *Philosophy of Discovery,* published in 1860, Whewell referred to the belief that "the discovery of laws and causes of phenomena is a loose hap-hazard sort of guessing", and claimed that this type of view "appears to me to be a misapprehension of the whole nature of science"

by evolution by natural selection; he did so in his *Descent of Man*, published five years after Whewell's death. I am not suggesting that Whewell, had he still been alive then, would have changed his mind about Darwin's theory. In fact, he seems to have prejudged the issue, believing that man's intellectual faculty could not arise from other organic beings (see, e.g., [Whewell, 1845, 43–4]). I merely note that, during his lifetime, Whewell had methodological grounds for questioning the probable truth of Darwin's theory.

[71] Richard Yeo has argued that, by the time of writing his review of Herschel's *Preliminary Discourse*, Whewell had already abandoned an early endorsement of inductivism in favor of a hypothetical method (see [Yeo, 1993, 98–9). Fisch similarly argues in his [1991] that Whewell abandoned his early commitment to Baconianism at an early stage of his career.

[Whewell, 1860, 274].

Moreover — importantly for someone so concerned with the role of language in science — Whewell consistently continued to call himself an inductivist, and his philosophy an inductive one. It seems unlikely that Whewell, who argued that scientific terminology "fixes" discoveries by connoting definitions of explicated conceptions, would continue to characterize his view as "inductive" if he had decided it was really hypothetical or "deductive" instead.

Thus, Whewell never abandoned his commitment to some form of Baconian inductivism, although he certainly believed that his own view had improved upon Bacon's. I will here show that there are several central aspects of Whewell's discoverers' induction that mirror Bacon's inductive methodology. (More details of the comparison between Bacon and Whewell can be found in [Snyder, 1999]). First, and most importantly, Whewell agreed with Bacon in claiming that hypotheses are invented by inference from the data, and in rejecting the method of anticipation, which "leaps" too quickly to wide generalizations. As a means of protecting against such hasty and illicit generalizations, Whewell agreed with Bacon's emphasis upon the "gradual and continuous ascent" to hypotheses (see [Whewell, 1860, 130, 131, 145]. See also [Whewell, 1857a, I: 7–8]. Whewell claimed that Bacon's emphasis upon such a gradual inferential process is where his importance and originality lie, not in the claim that knowledge must be sought in experience, which many others prior to Bacon had professed [Whewell, 1857b, 158]. Whewell's conception of science followed the gradualism of Bacon in two respects. First, the method of inventing hypotheses from data involves a "connected and gradual process" of inference, namely discoverers' induction. The second way in which Whewell followed Bacon's gradualism concerns Whewell's view of the history of science. Whewell claimed that the progress of science over time is slowly cumulative, in the sense that theories of progressively greater generality are derived from less general theories once those are proven to be true. In this way, Whewell noted, Kepler's law of planetary motion, invented by discoverers' induction, was used by Newton to derive a law of greater generality, the universal inverse-square law of gravitation [Whewell, 1860, 182] (see also [Yeo, 1985, 273]. The development of the theory of electromagnetism is another case in the history of science pointed to by Whewell as exemplifying the Baconian process of gradual generalizing [Whewell, 1857b, 159–60]. Whewell's own "Inductive Tables" are meant to illustrate the successive generalization that occurs over the history of science. In his earlier works Whewell even called them "Inductive Pyramids", echoing Bacon's use of the term "Pyramids of Knowledge" to describe the process of successive generalization in the sciences (indeed, Whewell explicitly drew this comparison in his [1860, 132]). Whewell's Inductive Tables can be constructed only for sciences which have achieved a large degree of generality, that is, where phenomenal laws have been subsumed under causal laws of greater generality, and these causal laws are themselves seen as instances of laws of even greater generality. The two sciences able to support the formal structure of the inductive tables, thus far, according to Whewell, are astronomy and (to a certain extent) optics. So central to Whewell discoverers' induction is its gradualism that Whewell claimed these inductive tables present the "Logic of Induction, that is, the formal conditions of the soundness of our reasoning from the facts" [1860, 207]. He suggested by such comments that the soundness of our reasoning from facts to law is the greater the more gradual is the generalization, and that this soundness is therefore exhibited in the formal structure of the tables.[72]

Whewell's discoverers' induction shares another important feature of Bacon's method of interpretation: namely, the claim that the inference from data to hypothesis is not limited to inductive generalization. We have seen that, on Whewell's view, the selection of the appropriate conception with which to colligate the data involves an inferential process, and that this process can involve any type of inference. Bacon too argued that mere enumerative induction was not sufficient; he criticized the "logic of the schoolman" as being "puerile". Nor was it enough to add eliminative induction. In particular, Bacon, like Whewell after him, emphasized analogical inference.[73] Analogical inference plays several important roles in Bacon's method. First, it is a crucial element in the construction of the "Natural History" the philosopher must construct for the property of things under study. A natural history consists of a table of presence, listing instances that share the property in question, a table of absences, which lists instances that are each similar to one of the instances on the table of presences but which differ in lacking the property in question, and a table of variations, in which the investigator describes instances in which the property is varying to greater or lesser degrees. The construction of a table of absences involves both positive and negative analogy. Positive analogies are used in determining the instances that are similar to those on the table of presence, while negative analogies are used in determining which of these instances lack the property in question. So, for example, in the investigation into the property of heat, the table of presence includes rays of the sun, which have this property. Positive analogy is used to suggest that rays of the moon are similar enough to rays of the sun to be included in the table of absence; these different types of rays share the properties of being rays of light and emanating from celestial bodies. Negative analogy suggests that rays of the moon do not share with rays of the sun the quality of heat, because we feel often warm when we are in the sunlight but not when we are in the moonlight. Another role for analogical inference arises in Bacon's discussion of "learned experience" (*experientia literata*). In this type of reasoning, an investigator argues by analogy from a situation in which a particular kind of experiment was fruitful, to another, similar situation in which the same kind of experiment might yield informative results. In the case of the rays of the sun and moon, experientia literata suggests that the known experimental result of passing sun rays through a magnifying lens

[72]See also [Whewell, 1860, 134], [Whewell, 1858b, 115], and Whewell's letter to DeMorgan, 18 January 1859, in [Todhunter, 1876, 2: 416–17]. Indeed, Whewell criticized Bacon for not being sufficiently gradualist in his depiction of science. See [Whewell, 1860, 136–7].

[73]Bacon's use of analogical inference is discussed in more detail in [Snyder, 2006] and [Snyder, 1999].

in order to intensify the heat and thus set combustible material on fire might be applied to the rays of the moon, in order to see if heat is thus increased. Analogical inference is also a crucial part of what Bacon called "prerogative instances" (see, for example, [Bacon, 1877-89, 4: 203]). Although Whewell does not explicitly discuss the use of analogical reasoning by Bacon, it is likely that his view on this topic was influenced by that of his young friend and protégé J. D. Forbes, who wrote an essay on the importance of analogical reasoning in Bacon's inductive philosophy a few years before meeting Whewell in 1831.[74]

Another similarity between the views of Whewell and Bacon is that both insist that hypotheses obtained inductively must be tested by their empirical consequences. And like Bacon, Whewell believed that science can yield knowledge about the unobservable part of the natural world. Bacon claimed that his interpretation of nature could discover the unobservable forms of simple natures. Whewell, as we have seen, intended his method to allow for the discovery of hypotheses referring to unobservable entities and properties. For example, he claimed that Fresnel and the other wave theorists had good inductive grounds for postulating the existence of unobservable light waves in an unobservable ether, and scoffed at Mill's rejection of the wave theory on the basis of its postulation of these unobservable entities.

Although I have shown that Whewell correctly considered his discoverers' induction as following in the tradition of Bacon's inductivism, he certainly also recognized that their views were not identical. From the time of his earliest notebooks on induction, Whewell expressed the need to improve upon Bacon's inductivism. In 1836, he wrote to Herschel that the *Novum Organum* "requires both to be accommodated to the present state of thought and knowledge, and to have its vast vacuities gradually supplied".[75] Indeed, he believed himself to be renovating — i.e., improving upon — Bacon's method. This renovation takes the form of grafting his antithetical epistemology onto Bacon's empirical methodology. However, this does not constitute a contradiction of Bacon's view; for, as we will see, there are seeds of an antithetical epistemology already in Bacon's works, as Whewell often pointed out.

Whewell's epistemology entails that certain ideal conceptions, as well as facts, are necessary materials of knowledge. We have already seen two ways in which conceptions are crucially involved in the discovery of empirical laws according to Whewell. First, conceptions are involved in the very process of perception; Whewell claimed that all perception is "conception-laden". Second, conceptions are necessary to form theories from facts in the process of colligation. The appropriate conception must be superinduced upon, or applied to, the facts in order to bring the facts together under a general law. But there is a third way in which Whewell emphasized the role of conceptions in science. He noted that some

[74] J. D. Forbes, "On the Inductive Philosophy of Bacon, His Genius, and Atchievements [sic]", quoted in [Olson, 1975, 225–26]. A later paper, [Forbes, 1835], which Whewell discussed in letters to Forbes, argued for the importance of analogical reasoning in scientific discovery. See letters from Forbes to Whewell in [Shairp *et al.*, 1873, 115–17], and from Whewell to Forbes in [Todhunter, 1876, 2: 203–4].

[75] William Whewell to John Herschel, 9 April 1836, in [Todhunter, 1876, 2: 234].

conceptual framework is necessary in order to serve as a guide in the collection of empirical data. In his Address to the British Association meeting in 1833, Whewell claimed that "it has of late been common to assert that *facts* alone are valuable in science ... [But] it is only through some view or other of the *connexion* and *relation* of facts, that we know what circumstances we ought to notice and record" [Whewell, 1834a, xx]. That is, we cannot and do not collect facts blindly, without some theory or conception guiding our choices for what to include and exclude from the collection of data.[76]

At times, Whewell argued that Bacon did not adequately take note of the antithetical nature of knowledge; Bacon did not "give due weight or attention to the ideal element in our knowledge" [Whewell, 1860, 135]. Other times, Whewell claimed that Bacon did not ignore the conceptual side of knowledge altogether. The problem is that Bacon never was able to complete his task of reforming philosophy: "if he had completed his scheme, [he] would probably have given due attention to Ideas, no less than to Facts, as an element of our knowledge" [Whewell, 1860, 136]. It has been suggested that Whewell more or less invented this reading of Bacon in order to "detach" the British inductive tradition from French positivism [Yeo, 1993, 247] (on this point see also [Perez-Ramos, 1988, 26]). However, this conceptual element *can* be found in Bacon's writings in various ways, though not as explicitly as it is developed in Whewell's philosophy.

Bacon claimed, for instance, that he "established for ever a true and lasting marriage between the empirical and the rational faculty" [Bacon, 1877-89, 4: 19]. He elaborated on this marriage in his famous aphorism urging the scientist to emulate the bee:

> Those who have handled sciences have been either men of experiment or men of dogmas. The men of experiment are like the ant; they only collect and use: the reasoners resemble spiders, who make cobwebs out of their own substance. But the bee takes the middle course; it gathers its material from the flowers of the garden and of the field, but transforms and digests it by a power of its own. Not unlike this is the true business of philosophy; for it neither relies solely or chiefly on the powers of the mind, nor does it take the matter which it gathers from natural history and mechanical experiments and lay it up in the memory whole, as it finds it; but lays it up in the understanding altered and digested. Therefore from a closer and purer league between these two faculties, the experimental and the rational, (such as has never yet been made) much may be hoped. [Bacon, 1877-89, 4: 92–3]. (See also [Rossi, 1984, 255].)

[76]For example, Whewell explained that "the laws of the tides have been in a great measure determined by observations in all parts of the globe, *because* theory pointed out what was to be observed. In like manner the facts of terrestrial magnetism were ascertained with a tolerable completeness by extended observations, *then*, and then only, when a most recondite and profound branch of mathematics had pointed out what was to be observed, and most ingenious instruments had been devised by men of science for observing" [1860, 155].

Notice that the "blind" gathering of facts that Whewell criticized is characteristic of the "men of experiment", or the "empirics", whose methods Bacon rejected in the above passage.[77] Moreover, Bacon pointed to the importance of the conceptual side of knowledge — in a way similar to Whewell — when he cautioned that forms cannot be discovered until conceptions or "notions" are clarified (see [Bacon, 1877-89, 4: 49–50 and pp. 61–2]). Such comments evoke Whewell's claim that, as we saw above, laws cannot be discovered until conceptions are clarified or explicated. Whewell was aware that Bacon made this point; indeed he chastised Bacon for ignoring his own advice. Whewell complained that, in his investigation into heat, "his collection of instances is very loosely brought together; for he includes in his list the *hot* taste of aromatic plants, the *caustic* effects of acids, and many other facts which cannot be ascribed to heat without a studious laxity in the use of the word" [Whewell, 1860, 139].

There is, however, one real difference in their respective views on the role of conceptions in science, and that concerns Whewell's claim that perception itself is conception-laden. Bacon famously warned against the imposition of our internal concepts upon the external world. He admonished that "... all depends on keeping the eye steadily fixed upon the facts of nature and so receiving their images simply as they are. For God forbid that we should give out a dream of our imagination for a pattern of the world" [Bacon, 1877-89, 4: 32–3]. Bacon claimed that in constructing our natural histories we must record phenomena that correspond as much as possible to pure, non-conception-laden observations; moreover, he believed that the correspondence can be quite high.[78] His "new way", the method of interpretation, is intended to begin "directly from the simple sensuous perception" [Bacon, 1877-89, 4: 40]. Human-derived conceptions cannot, according to Bacon, aid us in understanding a God-made world [Bacon, 1877-89, 4: 110]. In contrast to this view, Whewell believed, as we have seen, that our conceptions aid us in perceiving and understanding the created world precisely because they correspond in some degree to Ideas in the Divine mind. Whewell rejected the straightforwardly empiricist epistemology of Bacon, for the reasons he rejects the views of Locke and the "Sensationalist School". He replaced this purely empiricist epistemology with his antithetical epistemology. But as we have seen, there are elements of Bacon's view that seem to allow for the importance of the conceptual, as well as the empirical, side of knowledge. Whewell had reason to believe that this epistemological alteration was more of an organic extension of Bacon's philosophy, than an outright rejection of it.

[77]See [Bacon, 1877-89, 4: 91; see also pp. 70 and 81], and [Rossi, 1996] and [Rossi, 1984, 250]. In a footnote added to *De Augmentis*, James Spedding rejected the proposal that Bacon intended fact-collecting to occur in the absence of any theory [Bacon, 1877-89, 1: 623n.1]. Spedding suggested that Bacon's notion of *experientia literata* was meant to provide a provisional theory of the collection of facts. [Jardine 1990, 60 and 63n.15] agrees with this point. See also [Sargent, 1995, 34].

[78]See [Jardine, 1974, 135]. There seems, however, to be a conflict between this optimism and his claim that the Idols of the Cave and of the Tribe cause us almost inevitably to "distort and discolor" our sense experience of nature.

Because Bacon did not adequately cultivate the conceptual side of knowledge, Whewell claimed, he was led into another error: namely the false notion that there can be a purely mechanical method of discovery [Whewell, 1860, 138]. Bacon began his work with the claim that what was new about his *Novum Organum* was the realization that "the entire work of the understanding [must] be commenced afresh, and the mind itself be from the very outset not left to its own course, but guided at every step; and the business be done as if by machinery" [Bacon, 1877-89, 4: 40]. He compared his task to that of providing a compass for the purpose of enabling any person to draw a perfect circle [Bacon, 1877-89, 4: 62–3]. Such comments have been taken to suggest that Bacon sought to develop what his follower Robert Hooke later called a "philosophical algebra". Yet, as we have seen, Whewell denied that there could be a mechanical discovery method.

However, it is not clear that Bacon's goal was, indeed, to give a "philosophical algebra". Many of the elements of his method of interpretation cannot be reduced to a mechanical rule. The construction of the tables of presence, absence and variation cannot be; Bacon himself described *experientia literata*, which is a crucial tool in their construction, as "rather a sagacity, and a kind of hunting by sense, than a science" [Bacon, 1877-89, 4: 241]. Moreover, after the more or less mechanical exclusion is performed, it is still not obvious what the true form is. One needs to postulate a provisional form, seemingly by analogical reasoning, for which Bacon gives us no mechanical algorithm. Bacon's comments about creating a discovery "machine" can perhaps be read as expressing the intention to create a machine that we may use to supplement our creative rationality, not to supplant it, just as a compass aids the hand in drawing a circle but does not render the hand itself unnecessary (on this point see also [McMullin, 1990, 83]). Such a view is suggested by the following passage in the Preface to the *Novum Organum*: "Certainly if in things mechanical men had set to work with their naked hands, without help or force of instruments, just as in things intellectual they have set to work with little else than the naked forces of the understanding, very small would the matters have been which. . .they could have attempted or accomplished" [Bacon, 1877-89, 4: 40]. This is not so far from Whewell's own view.

8 NECESSARY TRUTHS IN EMPIRICAL SCIENCE

Whewell not only wished to describe how scientists discover empirical laws. He also sought to account for how scientists come to know necessary truths. This was a problem that concerned Whewell from 1819, while he was completing his first textbook on mechanics. By the time he published the *Philosophy* in 1840, Whewell believed he had solved what he referred to in a letter to Herschel as the "ultimate problem" of philosophy of science, by showing that necessary truths can emerge in the course of empirical science [Whewell, 1844b, 489]. This is a rather striking claim and has not been well-understood by other commentators (see [Butts, 1965a; 1965b; Fisch, 1985; 1991; Morrison, 1997; Walsh 1962]). Comprehending Whewell's solution requires viewing it in the context of his antithetical epistemology (for a more detailed exposition of what follows, see [Snyder, 1994]).

One way in which Whewell described the antithetical nature of knowledge was by claiming that "there is no fixed and permanent line" to be drawn between the empirical and ideal elements of knowledge [Whewell, 1858a, 1: 23]. For example, as we have seen, sensations cannot be entirely differentiated from Ideas. Whewell claimed further that there is no permanent line to be drawn between fact and theory; facts are joined together by the use of an Idea to form a theory, but a true theory is itself a fact, and can be used to form theories of even greater generality.[79] The fact/theory distinction is only relative, then, because where we draw the line between them changes as we discover that our theories are true (and thus that they are "facts"). Whewell implied that the same relation holds for the pair "experiential and necessary truth" [Whewell, 1844a, 465]. Although there is no "fixed and permanent line" between experiential and necessary truth, Whewell did allow that they can be distinguished philosophically, for the purposes of understanding them (as can the pairs "fact and theory" and "sensation and Idea"). Experiential truths are laws of nature that are knowable only empirically [Whewell, 1858a, 1: 26]. Necessary truths, for Whewell, are propositions expressing laws which "can be seen to be true by a pure act of thought" [Whewell, 1858a, 1: 60]. That is, they are knowable a priori, without any experience. Further, experiential truths are recognized by us as being contingent; they are such that "for anything which we can see, might have been otherwise" [Whewell, 1858a, 1: 25]. Necessary truths, by contrast, are those "of which we cannot distinctly conceive the contrary" [Whewell, 1844a, 463]. These distinctions between experiential and necessary truths are epistemic, grounded upon how we come to know a truth, and whether we can conceive of its contrary. But there is also a non-epistemic distinction: Whewell claimed that necessary truths "must be true" — whether we recognize this or not [Whewell, 1858a, 1: 25–6]. Moreover, Whewell suggested that the epistemic criteria are reliable tests for this non-epistemic necessity. If a general proposition satisfies the epistemic criterion for necessary truth, then we can be certain that it must be true. Note that Whewell did not imply the converse. That is, a proposition might be necessary in the non-epistemic sense even if it does not meet the epistemic criteria (only, in that case, we would not know it was a necessary truth).

Like facts and theories, however, experiential and necessary truths "are not marked by separate and prominent features of difference, but only by their present opposition, which is only a transient relation" [Whewell, 1860, 305]. There is, Whewell claimed, merely a temporary division between truths which are experiential and those which are necessary. Whewell believed that science consists in a process called the "idealization of facts", whereby experiential truths are "transferred to the side" of necessary truths [Whewell, 1860, 303]. The same proposition

[79]William Whewell [1844a, 467]; see also [1860, 305]. Whewell characterized the relation between facts and theories in this way as early as his 1830 book on architecture. In this work, Whewell referred to theories as "general facts", and expressed the hope that his architectural descriptions would be intelligible to "those who prefer facts to theories, that is, particular facts to general ones". See the third edition [1842, 40].

moves from one side of the fundamental antithesis to the other — hence the dividing line between them is "transient". Whewell claimed that, by this process, "a posteriori truths become a priori truths" [Whewell, 1860, 357–8]. Truths which are first knowable only empirically become knowable a priori. Self-evident truths, then, *become* self-evident. In order to understand how this can occur, it is important to grasp the relation between necessary truths and the fundamental Ideas.

Whewell believed that necessary truths, or the "axioms" of science, can be known a priori from the fundamental Ideas, because they are "necessary consequences" of these Ideas [Whewell, 1858a, 1: 99]. As we have seen, every science is organized by one or more fundamental Ideas. Each Fundamental Idea has several axioms which follow from it. Axioms are necessary consequences of an Idea in the sense that they express the meaning of the Idea. Whewell explained that the axioms "in expressing the primary developments of a fundamental Idea, do in fact express the Idea, so far as its expression in words forms part of our science" [Whewell, 1858a, 1: 75] (see also [Whewell, 1858a, 1: 58; Whewell, 1858b, 13]). For example, one of the three axioms of the Idea of Cause is "every event must have a cause"; and Whewell noted that "this axiom expresses, to a certain extent [because it is only one of the axioms] our Idea of Cause" [Whewell, 1858a, 1: 185]. The connection between an Idea and its axioms seems to be that the meaning of the axiom is contained in the meaning of the Idea, and expresses nothing but what is already contained in the Idea. The proposition "Causes are such that every event has a cause" is therefore analogous to the proposition "bachelors are never-married men", where the predicate ("never-married men") expresses only what is already contained in the subject ("bachelors"). (Kant called such statements "analytic judgments".) As we have seen, Whewell argued that a crucial part of science is the "explication" of Ideas and their conceptions. By explicating Ideas scientists gain an explicit, clarified view of the meaning of the Idea — it becomes "distinct". Once an Idea is distinct enough that its meaning is understood, the scientist can see that the axioms are necessary consequences of the Idea, in virtue of the fact that they express part of this meaning.[80] That is, when the scientist's mind contains an explicated form of the Idea of Cause, she will know that it must be true that every event has a cause.

The two epistemic criteria Whewell gave for necessary truths follow from this understanding of the relation between the Ideas and the axioms. Once the meaning of the concept "bachelor" is understood, no empirical knowledge is required to know the truth of the proposition "bachelors are never-married men": this proposition follows from the meaning of the concept, and hence can be known a priori. (On the other hand, if the meaning of "bachelor" is not understood, the truth of this proposition will not be knowable a priori.) Further, once the meaning of bachelor is understood, it will not be possible to conceive of someone who is both a bachelor and married. Similarly, Whewell claimed, only someone with a distinct understanding of the Idea of Space can know a priori that two straight lines cannot

[80]This is why the ability to recognize the necessity of axioms is one test for the distinctness of our Ideas. See [Whewell, 1858a, 1: 101], and [Whewell, 1835b].

enclose a space; moreover, a person who does know this a priori would be unable to conceive of two lines that are straight but yet contradict this necessary truth.[81]

Thus, Whewell's notion of necessary truth is opposed to Kant's conception of the synthetic a priori (contra [Butts, 1965a; Fisch, 1991; Metcalfe, 1991; Stoll, 1929]). Whewell disagreed with Kant's claim that necessary truths of mathematics are synthetic a priori. Kant explained of the truth "5+7=12" that "this concept of 12 is by no means already thought in merely thinking this union of 7 and 5; and I may analyze my concept of such a possible sum as long as I please, still I shall never find the 12 in it" (see [Kant, 1929, 53]). That is, for Kant, "5+7=12" is not an analytic judgment such that merely by knowing the meaning of "5+7" we can know the truth and necessity of "5+7=12". Whewell was certainly aware of Kant's view; in one of his notebooks he transcribed this discussion of the synthetical nature of mathematical principles in the first *Critique*.[82] In published work, nevertheless, Whewell presented the opposing position. Using the similar example "7+8=15", Whewell claimed that "we refer to our conceptions of seven, of eight, and of addition, and as soon as we possess the conceptions distinctly, we see that the sum must be 15".[83] Merely by knowing the meanings of "seven", "eight", and "addition", we see that it follows necessarily that "7+8=15". Hence, for Whewell, mathematical truths (like all necessary truths) are analytic and not synthetic in Kant's sense.The Kantian Mansel duly complained that

> Dr. Whewell lays too much stress on *clearness* and *distinctness* of conceptions as the basis of the axiomatic truths of science. But the clearness and distinctness of any conception can only enable us more accurately to unfold the virtual contents of the concept itself; it cannot enable us to add *a priori* any new attribute. In other words, the increased clearness and distinctness of a conception may enable us to multiply to any extent our analytic judgments, but cannot add a single synthetical one. [Mansel, 1860, 258]

Whewell's view of mathematical truth also differs from the position of William Hamilton and Dugald Stewart. They had argued that the axioms of mathematics are conventional or "hypothetical", because axioms are deductive consequences of definitions that are not themselves necessary (see [Whewell, 1838, 151] and [Whewell, 1858a, 1: 107–8]. Mathematical truths such as "two straight lines cannot enclose a space" are necessarily true (within Euclidean geometry) only because of how Euclidean geometry defines "straight line" (i.e., "a straight line is that which lies evenly between its extreme points"). The axiom would not even be true, let alone necessary, if our geometry defined "straight line" as one which

[81]For the contrary claims of other commentators, and my arguments against these claims, see [Snyder, 1994].

[82]William Whewell, Notebook, WP R.18.19 f.13, p. 10. Although the entry is undated, based on the date of an earlier entry it appears to originate from the mid-1820s.

[83][Whewell, 1844a, 471]. Whewell made the same claim for the axioms of geometry: "the *meaning of the terms being understood*, and the proof being gone through, the truth of the proposition must be assented to" [1844a, 462, emphasis added].

lies unevenly between its extreme points. Importantly, their view suggests that we might have chosen to define "straight line" differently. But Whewell rejected this merely hypothetical necessity. On his view, the necessary truths that follow from definitions are necessary because they are deductive consequences of definitions which are themselves necessary. Mathematical definitions are not merely conventional.[84] Rather, they are "descriptive" (on this point see [Richards, 1988, 22–3]). On Whewell's view the definitions of geometry and arithmetic describe the properties of certain mathematical conceptions such as point, line, circle and number. These conceptions are necessary consequences of the Ideas of Space and Time (see [Whewell, 1858b, 30–1; 1858a, 1: 74]). Like the axioms, the conceptions of an Idea are "included" in the meaning of the Idea (hence their definitions express part of its meaning) [Whewell, 1858a, 1: 75]. When we verbalize the definition of "straight line" we are not conventionally *assigning* the properties of straight lines, but are expressing or *describing* what these properties really are.[85] We could not correctly define straight line in any other way. Like most British thinkers prior to the late 1870s, Whewell considered geometrical definitions to be descriptive not only of mathematical conceptions that exist in our minds, but of physical reality as well; geometry was thus held to have an ontological foundation. As we have seen, Whewell's Fundamental Ideas correspond to the structure of the physical world. The Idea of Space which conforms to the geometry of physical space is (according to classical Newtonian physics) *Euclidean*.[86] On Whewell's view, only definitions which follow from this Idea of Space can serve as the source of necessary truths of geometry. This is why Whewell claimed that it is not the case that a necessary truth of mathematics "*merely* expresses what we mean by our words"; rather, it expresses a truth about some fundamental feature of physical reality as well.[87]

[84]See [Whewell, 1858b, 36 and 39; 1858a, 1: 74–5], and Whewell's letter to Frederic Myers, 6 September 1845, in [Stair Douglas, 1882, 327]. For Hamilton's view see his [1836]. Hamilton denied Whewell's claim that mathematics is an important subject of study for university students; Hamilton's view of mathematics entailed that it was too purely formal to be of use in developing basic reasoning skills.

[85]Whewell claimed that "a definition, to be admissible, must necessarily refer to and agree with some conception which we can distinctly frame in our thoughts". See [Whewell, 1838, 153–5]. Like the axioms, these definitions are recognized by us as being correct and necessary when our Ideas are adequately distinct.

[86]It was not until the late 1870s — ten years or so after Whewell's death — that British thinkers began to question the assumption that the nature of geometry was defined by Euclidean geometry (see [Richards, 1988, chapter two]). But we may still speculate upon what Whewell's reaction would have been to non-Euclidean geometries. Since the structure of physical space was still considered to be Euclidean, Whewell would have maintained that only Euclidean geometry is "science", in the sense of containing necessary truths which conform to the nature of physical reality. However, Whewell would have needed to address the question of how geometers, who seem to have a distinct Idea of Euclidean Space, can nevertheless distinctly conceive propositions contrary to the Euclidean axioms.

[87][Whewell, 1858a, 1: 59, emphasis added]. By denying the conventionality of mathematical truths, Whewell did not thereby deny their analyticity, as some commentators have claimed. Analyticity need not be limited to truths of conventional definition, but can apply to definitions that describe the true nature of physical reality. For opposing views see [Butts, 1965a, 167] and [Fisch, 1991, 155–7].

To understand this assertion, we must recall Whewell's theological justification for the claim that the Ideas existing in our minds correspond to the nature of physical reality. As we have seen, this justification is based on the notion that our Ideas and the world share a Divine creator. In his *Philosophy of Discovery*, Whewell explicitly asked how it is possible that propositions we know a priori are informative about, and indeed necessarily true of, the physical world. Answering this question, Whewell claimed, required asking another: "how did things come to be as they are?" [Whewell, 1860, 354–5 and 358]. Whewell answered this with the claim that God created the physical universe in accordance with certain of his "Divine Ideas". For example, God made the world such that it corresponds to the Idea of Cause partially expressed by the axiom "every event has a cause". Hence in the universe every event conforms to this Idea, not only by having a cause but by being such that it could not have occurred without one. Whewell's necessary truths are not logically necessary, in the sense of being true in all possible worlds (here I disagree with [Ruse, 1977, 251–2]). On Whewell's view, God could have chosen to create the world in accordance with different Ideas, in which case different axioms would be necessary truths. Even the axioms of mathematics are not logically necessary: "the propositions of space and number and the like, must be supposed to be what they are by an act of the Divine Mind", i.e., by the act of God choosing one set of Ideas over another.[88] Given the Ideas God did choose, the axioms are necessarily true of the world, because they follow necessarily from the meanings of these Ideas. But this is not a view of hypothetical truth in the sense that Hamilton and Stewart proposed for mathematics. We do not conventionally assign meanings to the Ideas; now that the world has been created, only one set of meanings is possible.

We are at last in a position to understand what Whewell meant by claiming that a proposition that is at first knowable only empirically can become knowable a priori — that is, how it is possible to "idealize the facts". Since necessary truths follow necessarily from the meaning of our Ideas, the a priori intuition of necessary truths is possible only once our Ideas are distinct: when this is the case, we apprehend that an axiom is necessarily true because the meaning of the axiom is, in fact, contained in the meaning of a Fundamental Idea to which the universe necessarily conforms (given God's choice of Ideas to use as archetypes in creating it). But if our Idea is not distinct, we do not — and indeed cannot — apprehend this. We have already discussed the explication of conceptions and Ideas which occurs in the course of empirical science, and by which Ideas are made distinct. Once an Idea is distinct — once we understand its meaning — truths which we may have discovered empirically are seen actually to follow from the meaning of the Idea. That is, the experiential truth becomes knowable a priori from the now-understood meaning of the Idea; the experiential truth has been "idealized" into a necessary truth. The a priori intuition of necessary truths is "progressive", then, because our Ideas must be explicated before it is possible for us to know their axioms a

[88]See printer's proofs of *Plurality of Worlds*, WP Adv.c.16 f. 27, chapter XII, p. 276 and [Whewell, 2001].

priori. This is why Whewell claimed that necessary truths become *knowable* a priori and not merely *known* a priori. Thus, for example, Whewell argued that "though the discovery of the First Law of Motion was made, historically speaking, by means of experiment, we have now attained a point of view in which we see that it might have been certainly known to be true independently of experience" [Whewell, 1847, 2: 221]. The First Law of Motion, then, is a necessary truth that has undergone the process of idealization: though it was first knowable only empirically, it has become knowable a priori.[89]

We have seen that, by the process of idealization of facts, experiential truths come to satisfy the criteria of necessary truths. That is, they become knowable a priori from a distinct Idea, and it becomes impossible (for those who have the Idea in its distinct form) to conceive clearly their contraries. But recall that these epistemic criteria are intended to be reliable tests of a deeper kind of necessity. If a law satisfies these epistemic criteria, then we know that the law "must be true". Yet it is not the case that laws change their status regarding this non-epistemic necessity. These truths which *become* necessary in the epistemic sense, are *always* necessary in the non-epistemic sense. As discussed earlier, the non-epistemic sense in which an axiom must be true is that it follows as a necessary consequence of one of the Divine Ideas used by God in creating the world. Since God created the world to conform to a particular Idea of Cause, the axioms which express the meaning of this Idea, and the necessary truths which are a priori derivable from these axioms, must be true of the objects and events of the world: and this is so even if we have not explicated our Idea of Cause enough to see their necessity.

Thus, through the idealization of facts, truths become necessary truths in the epistemic sense, which were always necessary in the non-epistemic sense. There is a rather interesting and important consequence of this understanding of the idealization of facts. Recall that this discussion began with the fundamental antithesis, according to which no fixed line divides experiential and necessary truths. We now see how it is that the line we draw between them, like that between fact and theory, is a relative one, based upon epistemic distinctions that change as our Ideas become more distinct. As we explicate our Ideas, we recognize empirical truths to be necessary consequences of these Ideas; and the truths are thus transferred from the empirical to the necessary side of the antithesis. But since there is no firm division between these two classes of truths, any experiential truth can, in

[89]Whewell's view of necessary truth altered between the writing of his [1835a] and his [1840]. In the earlier work, he claimed that the laws of motion were experiential truths, while in the later one he characterized the first law of motion as a necessary truth. In [1835, 573] he argued that the reason to be cautious about considering the laws of motion to be necessary truths is that "we know that, historically speaking, men did at first suppose the laws of motion to be different from what they are now proved to be", and this would be "impossible" if the laws were necessary and "self-evident". See also [1833a, 179]. By 1840, however, Whewell had come to hold his mature view that — as he put it later — the intuition of a priori truths require "a certain growth and development of the human mind", and thus that there is no contradiction in supposing a law to be a necessary truth even if previous science did not recognize it as such (see [1860, 347]). Those who have focused only on the earlier essay have consequently misunderstood Whewell's mature position.

principle, become knowable a priori (and therefore it will become impossible to conceive distinctly its contrary). Since satisfying the epistemic criteria is a reliable test for truths which are necessary in the non-epistemic sense, it follows that every experiential truth is, in fact, necessary in this sense. That is, every law of nature is a necessary truth, in virtue of following necessarily from some Idea used by God in creating the universe.

Whewell's view thus destroys the line traditionally drawn between laws of nature and the axiomatic propositions of the pure sciences of mathematics; mathematical truth is granted no special status. Mansel seems to have had this consequence in mind when he argued against Whewell that the difference between a priori principles and empirical laws "is not one of degree, but of kind; and the separation between the two classes is such that no conceivable progress of science can ever convert the one into the other" [Mansel, 1860, 275]. For Whewell, there is no such separation. In virtue of their connection to the Divine Ideas, the laws of nature have the same rigorous necessity as geometrical axioms (see [Whewell, 1835b, 160]). Moreover, the axioms of geometry and arithmetic are themselves laws of nature, "established by the Creator of the Universe".[90] In principle, then, it is possible to idealize all experiential truths into necessary truths knowable a priori. Hence, Whewell claimed it was possible (again, and importantly, only in principle) for all science to become purely deductive, like the mathematical sciences. Once all the axiomatic laws of a science are knowable a priori, the only task left for the scientist would be to deduce further theorems from these laws. Eventually this would mark the end of empirical science. However, there is still much work left for the empirical scientist; Whewell vehemently disagreed with Mill's claim that most remaining scientific work is deductive [Whewell, 1849, 73–6]. Indeed, it is clear that Whewell believed we will never, in fact, idealize all empirical laws. Many such laws will be seen by us only as experiential truths, as being what they are "not by virtue of any internal necessity which we can understand" [Whewell, 1833a, 165].

9 WHEWELL'S DEBATE WITH MILL

In the twentieth-century, Whewell was "rediscovered" as the antagonist of John Stuart Mill in a debate about induction (they also publicly argued about moral philosophy and political economy). Because of the widespread influence of the *System of Logic,* the view of induction presented within its pages became the standard view of induction, remaining so today. It is thus because of Mill that Whewell's inductive logic, which differs from Mill's, is generally interpreted as being non-inductive. Indeed, as we have seen, Mill is the original source for the claim that Whewell's methodology is identical to that later endorsed by twentieth-century hypothetico-deductivists. In *System of Logic*, Mill wrote that "Dr. Whewell ... allows of no logical process in any case of induction other that ... guessing until

[90]William Whewell, *Of the Plurality of Worlds,* WP Adv.c.16 f.27, p. 274. See a;sp [Whewell, 2001].

a guess is found which tallies with the facts, and accordingly ... he rejects all canons of induction, because it is not by means of them that we guess" [Mill, 1963-91, 7: 304].

Mill's characterization of Whewell's view as non-inductive is due to Mill's denial that there is any contribution of the mind in induction; leaving room for any such contribution was thought by Mill to open the door to intuitionism, a philosophical position he associated with conservative political views. Indeed, as he indicated in his *Autobiography*, Mill's motivation for writing *System of Logic* was primarily political. As he explained,

> The notion that truths external to the mind may be known by intuition or consciousness, independently of observations and experience, is, I am persuaded, in these times, the great intellectual support of false doctrines and bad institutions. ... There never was such an instrument devised for consecrating all deep seated prejudices. And the chief strength of this false philosophy in morals, politics, and religion, lies in the appeal which it is accustomed to make to the evidence of mathematics and of the cognate branches of physical science. To expel it from these, is to drive it from its stronghold ....In attempting to clear up the real nature of the evidence of mathematical and physical truths, the "System of Logic" met the intuition philosophers on ground on which they had previously been deemed unassailable. [Mill, 1963-91, 1: 233-5]

Thus, as I discuss elsewhere, Mill's philosophy of science as a whole must be read in the light of his political concerns (this point is a central contention in my discussion of Mill in [Snyder, 2006]).

In attacking Whewell's view of induction, Mill criticized his characterization of Kepler as an inductive discoverer. Whewell often praised Kepler as being an exemplary discoverer.[91] Specifically, he commended Kepler's discovery of the elliptical orbit of Mars, considering it a model use of discoverer's induction. In his *System of Logic,* Mill used the example of Kepler in order to argue that Whewell's discoverers' induction was not, in fact, a form of induction. He explained that induction "is a process of inference from the known to the unknown; and any operation involving no inference, and any process in which what seems the conclusion is no wider than the premises from which it is drawn, does not fall within the meaning of the term".[92] However, Whewell's discoverers' induction, Mill argued, does not satisfy this definition. Mill concentrated his attack on Whewell's notion of colligation. According to Mill, what Whewell called "colligation" is nothing more than a matter of observation and description, and thus it does not go

[91] See, for example, Whewell's earliest Induction notebook (WP R.18.17 f. 10, p. 49), [Whewell, 1858a, 1: 318], and [Whewell, 1860, 121].

[92] [Mill, 1963-91, 7: 288]. Thus Mill, like Jones and Whewell, rejected Whately's claim that inductive inference can be reduced to the syllogism. This rejection reflects a change of position from his 1828 review of Whately's *Logic*, in which Mill endorsed Whately's view.

beyond the premises (i.e., the observations) themselves. In this way he denied Whewell's claim that colligation, as in the example of Kepler's discovery, involves inference. He made a series of arguments against Whewell's position. First, Mill argued that finding a shared property in a set of facts is typically a matter of direct observation. When we assert "all observed crows are black", we are simply summarizing numerous observations of black crows; we are not going beyond these observations as in an ampliative inference. (Of course, Mill would agree that the projecting of this property, to reach the conclusion "all [observed and unobserved] crows are black", is an ampliative inference.) Similarly, to determine the curve defined by the observed positions of Mars, Mill insisted, "there was no other mode than that of direct observation" [Mill, 1963-91, 7: 292–3]. Thus Mill accused Whewell of "confounding a mere description, by general terms, of a set of observed phenomena, with an induction from them" [Mill, 1963-91, 7: 288]. In this way he suggested that Kepler found the property shared by the observed positions by simple curve-fitting; that is, that Kepler determined that the observed points of the orbit share the property of lying on an elliptical curve merely by plotting the observations of Mars and then "connecting the dots", as it were, in order to see what curve included them. Far from being impressed by Kepler's achievement, as Whewell was, Mill sniffed that "the only wonder" was that no one had made this discovery before, once Tycho-Brahe's accurate observations had been recorded [Mill, 1963-91, 8: 652].

However, Mill's claim ignores the obvious fact that our observations are from the vantage point of the Earth, whereas determining the orbital path of Mars requires determining its path around the Sun. Observations made from the Earth do not yield an ellipse. Kepler needed to develop a theory of the Earth's motions before he could infer the true path of Mars from its Earth-observed positions (on this point see [Wilson, 1968, 5–10; Stephenson, 1987, 49–61; Kozhamthadam, 1994, 155–61]). Further, even if Kepler's theory of the Earth plus the observations could have yielded "connectable dots", or points of the orbit around the Sun, this itself would not have enabled Kepler to see the orbital path as being elliptical, since this particular ellipse is very nearly circular. In certain places in *System of Logic,* Mill (inconsistently with his other claim) seemed to acknowledge that the elliptical property of Mars' orbit could not be determined by direct observation, because we are not in a privileged position to see its path around the sun. Mill wrote that, in the case of Kepler's discovery, "the facts [were] out of the reach of being observed, in any such manner as would have enabled the senses to identify directly the path of the planet" [Mill, 1963-91, 7: 296]. Nevertheless Mill continued to reject the view that Kepler's discovery of the ellipse required any inference. He offered a second argument against Whewell's claim. Sometimes, Mill explained, a conception can be obtained from earlier experience and applied to present observations. In Kepler's case, the property of lying on an elliptical curve was "derived from his former experience", presumably from his experience of mathematical curves [Mill, 1963-91, 8: 651]. However, Mill neglected to explain why the discovery that this property is shared by the members of the set of observed points of Mars' orbit does not constitute an inference; obviously, to apply a property not directly observed in the facts to these facts *is* to go beyond premises about what is observed.

Mill's argument here relies on a curious counterfactual. He noted that if we had adequate visual organs, or if the planet left a visible track as it moved through the sky, and if we occupied a privileged position with which to view this path, we could directly observe the planet's orbital path [Mill, 1963-91, 7: 297]. Mill suggested that Kepler's discovery that the observed positions of Mars share a property "derived from his former experience" did not constitute an inference because it was merely an "accident", or a contingent fact, that this property was not directly observed by him. Thus he explained that "if the path [of the planet] was visible, no one I think would dispute that to identify it with an ellipse is to describe it: and I cannot see why any difference should be made by its not being directly an object of sense" [Mill, 1963-91, 7: 296]. Later, in Book VI, Mill made a similar point [1963-91, 8: 651].

This strange argument of Mill's obviously does not defeat Whewell's claim that Kepler's discovery of the ellipse required inference from the known to the unknown. The problem with Mill's argument is that it invalidly narrows the scope of ampliative inference. Whewell claimed that Kepler's hypothesis required an inference from the observed positions of Mars to what was, in fact, unobserved — namely, the shape of an orbital path that included these positions. The shape of such a curve was not directly observed by Kepler. Mill's argument against considering this operation to be a type of inference from the known to the unknown is that this property would be observable *under certain conditions* (if we were at the proper viewing angle, if the planet left a visible trail). But this is surely irrelevant to the question of inference. What matters is what is, in fact, observed. This is so in the case of enumerative induction as well. After all, every individual crow is observable, yet we (and Mill) still allow that the conclusion "all crows are black" can be reached only by ampliative inference if it is the case that every crow has not *been* observed. It is exactly because Kepler did not see the orbit's path directly and at the correct angle that he needed to make an inference to a property shared by the points of the orbit; the fact that this property may be, under some idealized conditions, "observable", is irrelevant.

Mill realized that he needed to explain what type of non-inferential procedure could be used to obtain the true description of the facts when the conception connecting them is not directly observed. He argued that, in such cases, a conception may be applied to a set of facts by non-rational guesswork. Thus he wrote that Kepler's discovery of the ellipse involved nothing but "guessing until a guess is found which tallies with the facts" [Mill, 1963-91, 7: 304]. He claimed that Kepler merely made a series of non-rational guesses, using previously observed conceptions, until he found the conception which best fit the observed positions of Mars. That is, he supplied the ellipse conception "hypothetically ... from among the conceptions he had obtained from other portions of his experience" [Mill, 1963-91, 7: 296]. There are, however, two problems with this claim about Kepler. First, it is clear that Kepler did not merely guess his ellipse hypothesis. As Whewell correctly claimed, Kepler made a series of rational inferences to his discovery; even if one wanted to say, with Newton, that he "guessed" the orbit to be elliptical, clearly Kepler used inference to arrive to the oval, and this was, to a great degree, the more difficult and revolutionary part of his discovery. The second problem with Mill's claim is that it is inconsistent with his own claims about the role of hypotheses in science. Mill frequently argued that a hypothesis merely guessed at can have only a heuristic role in science; it cannot be proven to be true or likely merely by being found to fit the data — even if it leads, in addition, to a successful prediction of an unexpected consequence. Here, however, Mill seems to be claiming that hypotheses may be proven to be true solely by testing whether they conform to the observations.

Mill rather oddly claimed that "Dr. Whewell ... pass[ed] over altogether the question of proof" [Mill, 1963-91, 7: 304]. But Mill's analysis of Whewell's position is grounded in his mistaken belief that Whewell allowed hypotheses with no inductive support to be confirmed by predictive success. Mill himself required successful prediction for theories as part of his so-called "deductive method", where the first step was the induction to a theory, the next was the deduction of consequences, and the third was verifying testing of them. But he emphasized that the second two steps were useless without the first; this was what he disdainfully called the "hypothetical method". Since Whewell's view was seen by Mill as hypothetical, Mill believed that he had no proper method for testing inductive theories.

In 1849 Whewell published a work devoted to responding to Mill: *Of Induction, with especial reference to Mr. J. Stuart Mill's "System of Logic"*. More than one quarter of this work is devoted to a critique of Mill's view of Kepler's discovery. According to Whewell, Mill illegitimately attempted to set up a distinction between description and induction. Whewell characterized Mill as arguing in the following way: "when particular facts are bound together by their relation in *space*, Mr. Mill calls the discovery of this connection *Description*, but when they are connected by other general relations, as time, cause and the like, Mr. Mill terms the discovery of the connection *Induction*". Mill asserted that the discovery of Mars' orbit, in which the particular facts were connected with the spatial property of an elliptical curve, was merely a description, while the discovery that the planetary orbits are connected by the causal property of being acted upon by the Sun's gravitational force, was an induction [Mill, 1963-91, 7: 299]. Whewell claimed that there is no obvious argument to warrant such a distinction. If inference is needed to discover the property connecting particular facts, then it is wrong to call the act a mere (non-inferential) description, whether the property is a spatial one such as an elliptical curve or a causal one such as gravitational force. Whewell noted "that the orbit of Mars is a Fact — a true description of the path — does not make it the less a case of Induction", because inference was needed in order to discover this true description [Whewell, 1849, 23].

As we have seen, Mill's claim about Kepler arose from his desire to reject any internal or subjective element in knowledge. Whewell recognized that the real point of contention between them here had to do with the *source* of a conception or colligating property used in inference. Recall that, in Whewell's antithetical epistemology, the conceptions are modifications of Fundamental Ideas that are supplied by our minds in our contemplation of the world around us. Thus the conceptions used in colligating particular facts have an ideal, subjective source, yet correspond to the relations that exist in the physical world. Mill strongly denied this claim. "Conceptions", he wrote, "do not develop themselves from within, but are impressed from without". That is, "the conception is not furnished *by* the mind till it has been furnished *to* the mind" [Mill, 1963-91, 8: 653 and 655]. This is why he insisted, as we have seen above, that finding a colligating conception is merely a matter of describing what is observed outside the mind, rather than being an inference involving conceptions provided from within. In his response to Mill, Whewell claimed that Mill was ignoring how difficult it often is to find the appropriate conception with which to colligate a set of particular facts; indeed, as we have seen, Mill rather naively believed that Kepler's task in discovering the elliptical orbit was an almost trivially simple one (see [Whewell, 1849, 31]). Mill claimed that Whewell's "Colligation of the Facts by means of appropriate Conceptions, is but the ordinary process of finding by a comparison of phenomena, in what consists their agreement or resemblance" [Mill, 1863-91, 8: 648]. Whewell rather archly responded by noting that "of course" discovering laws involves finding some general point in which all the particular facts agree, "but it appears to me a most scanty, vague, and incomplete account" to suggest that the commonality is found merely by observation, with no inference at all [Whewell, 1849, 41–2].

Moreover, Whewell protested Mill's mischaracterization of his view as being a non-realist one. Mill wrongly accused Whewell of believing that these conceptions, because they are supplied by the mind, are merely ideal, mental constructs that correspond to nothing in the world but simply organize our experience in useful ways. Mill asserted, for instance, that Whewell held that the conception of an ellipse "did not exist in the facts themselves" [Mill, 1963-91, 7: 294]. Whewell had argued against this understanding of his view in his response to Herschel's review of his works. Contrary to the way in which Herschel and Mill interpreted his view, Whewell believed that our conceptions do "exist in the facts", in the sense that they provide shared properties and relations that really exist between objects and events, even though it takes an "act of the intellect" to find them there. Of the conception of the ellipse, Whewell wrote, "Kepler found it in the facts, because it was there" [Whewell, 1849, 23]. Drawing upon an analogy earlier employed by Bacon, Whewell claimed that Nature is a book, and her laws are written within it, but we cannot read these laws without knowing the language in which they are written. The laws exist "in the facts", but our acquisition of knowledge of the laws requires that we develop and use the rules of grammar that exist in our minds [Whewell, 1849, 34]. Mill was therefore wrong to claim that Whewell was a non-realist.

Whewell also objected to the radically empiricist tenor of Mill's philosophy. Whewell recognized that Mill's inductivism rejected inference to theories referring to unobservable entities or properties. Indeed, Whewell complained that "Mr. Mill rejects the hypothesis of a luminiferous ether, 'because it can neither be seen, heard, smelt, tasted, or touched"' [Whewell, 1849, 34]. In two later works, Whewell associated Mill with Comte, whom Whewell criticized for "rejecting the inquiry into causes" (see [Whewell, 1866, 356] and [Whewell, 1860, chapter 21]). Whewell's method, in contrast, did allow for the inquiry into unobservable causes; like Bacon, he believed this to be the ultimate aim of science. Thus Whewell noted that "to exclude such inquiries, would be to secure ourselves from the poison of error by abstaining from the banquet of truth..." [1860, 233]. The history of science shows that it is both possible and important to seek these kinds of causes.

Further, Whewell strongly criticized Mill for the methodology he developed based on this radically empiricist epistemology. Whewell rejected "Methods of Experimental Inquiry" as extreme oversimplifications of scientific discovery [Whewell, 1849, 44]. Mill had noted that the method of difference can only be applied in controlled laboratory settings; and he was correct that the method could be useful in such cases, as it is today, along with the other methods, in devising studies in medical research.[93] However, Mill had also called these methods "the *only* possible modes of experimental inquiry", and suggested that they are not difficult to apply, at least in the physical sciences [Mill, 1963-91, 7: 406]. Indeed, Mill claimed that he wanted to find rules of induction analogous to the rules of the syllogism; that is, something like a "discovery machine", in the very sense that Whewell rejected [Mill, 1963-91, 283].

But the main complaint Whewell had of Mill's methods concerned Mill's means of justifying them. "Who will carry these formulae through the history of the sciences, as they have really grown up; and shew us that these four methods have been operative in their formation...?" [1849, 45]. Mill himself had not justified his methods by showing that they have, in fact, been used to make successful discoveries. Instead of surveying the history of science to find whether scientists have used his methods of experimental inquiry, Mill focused his examples on a narrow range of not-well understood cases. He spent much time on Wells' researches on Dew, which Herschel had discussed in the *Preliminary Discourse*. Whewell blamed Herschel for suggesting that one or two examples are sufficient to understand scientific discovery. As he wrote to Jones, "tell Herschel he has something to answer for in persuading people that they could so completely understand the process of discovery from a single example".[94] Moreover, the Wells example was inappropriate to illustrate inductive methods, according to Whewell, because it was not really an original discovery, but rather a deduction of particular phenomena from already-established principles (see [Whewell, 1849, 50]). Mill's other favorite example concerns the work of Liebig in physiological chemistry, specifically his theories regarding the cause of death. But, as Whewell noted, there are two problems with

[93]Interestingly, though, Mill himself claimed that his methods of experimental inquiry were *not* useful in medical science because of the intermixture of effects (see [1963-91, 7: 451]).

[94]William Whewell to Richard Jones, 7 April 1843, WP Add.Ms.c.51 f. 227.

the example of Liebig. First, Liebig's theories were not yet verified, so Mill could not know whether they were real discoveries [1849, 47]. As Whewell complained to Jones, these theories were so recent that "the most sagacious physiologists and chemists cannot yet tell which of them will stand as real discoveries".[95] Moreover, the science of physiology was a new and not well-established one. Whewell exclaimed, "nor can I think it judicious to take so large a proportion of our examples from a region of science in which, of all parts of our material knowledge, the conceptions both of ordinary persons and men of science themselves, are most loose and obscure, and the genuine principles most contested...".[96] Whewell believed that it was only possible to understand the process of reaching truths by examining fields of knowledge in which truth is uncontested. Thus he held that "the philosophy of science is to be extracted from the portions of science which are universally allowed to be the most certainly established ....The first step towards shewing how truth is to be discovered, is to study some portion of it which is asserted to as beyond controversy". For this reason Whewell also objected to Mill's introduction of moral and political subjects into his discussion of induction in the *Logic* even though Whewell himself was quite concerned with these topics (indeed, his original plan for the *Philosophy* included the "hyperphysical branches" of science, including morality and political economy) [Whewell, 1849, 5–6].

10 SCIENTIFIC METHOD AND THE HISTORY OF SCIENCE

These last criticisms of Mill grew out of Whewell's view of the relation between philosophical discussion of scientific method and the history of science. Whewell claimed to have developed his philosophy of science from his study of the history of science; thus, he wrote his *History of Inductive Sciences* (1837) before the *Philosophy* (1840), claiming that any philosophy of the sciences must be "founded upon their history". As he put it in the preface to the *History*, "it seemed to me that our study of the modes of discovering truth ought to be based upon a survey of the truths which have been discovered" [Whewell, 1857a, 1: viii]. Some commentators have claimed that, on the contrary, Whewell first developed an a priori philosophy of science and then shaped his *History of the Inductive Sciences* to conform to his own view (for example, see [Stoll, 1929] and [Strong, 1955]). To a limited extent this is no doubt true. As we have seen, from his days as an undergraduate at Trinity College Whewell considered his "vocation" to be the advancement of the inductive method in the sciences. So from the very beginning Whewell had an inductive view of scientific method, which influenced his writing of the *History* in one important sense: by leading him to the view that learning about scientific method must be inductive and therefore historical. As he wrote to Jones in 1831, "I do not believe the principles of induction can be either taught

[95] William Whewell to Richard Jones, 7 April 1843, WP Add.Ms.c.51 f. 227.

[96] [Whewell, 1849, 48.] See also letters to Jones, 5 August 1834 [WP Add.Ms.c.51 f. 174] and 21 August 1834 in [Todhunter, 1876, 2: 185–188].

or learned without many examples".[97]

Examples, then, are needed to fill out the details of this broadly inductive view, and they are to come both from knowledge of current science and knowledge of the history of science. That these examples were expected to come from both sources is indicated in numerous letters as well as in Whewell's early induction notebooks. Whewell's earliest attempt at a draft of a work on induction appears in a notebook dated 1830. In this notebook Whewell claimed that in order to judge the methodology of Bacon, it was necessary "to shew how this method has been exhibited and exemplified since it was first delivered". In order to do so it is necessary to discuss the history of science: this may explain why Whewell put aside this draft and began working on his history of science.[98] In this notebook and in several which follow over the next three to four years there are many notes on recent discoveries in science, as well as citations from contemporary scientific works in which scientists express a view of proper scientific method.[99] Yet there are also numerous entries describing the histories of various scientific fields. These entries are interwoven with Whewell's early thoughts on an inductive philosophy of science. In one of these notebooks alone, Whewell took reading notes on Davy's *Elements of Chemical Philosophy*, Gilbert's *De Magnete*, Brewster's book on Newton, as well as works by Cuvier, Copernicus, Galileo, da Vinci, and Harvey; described recent discoveries in Optics by Biot, Young, Fresnel, Arago, and Airy; and discussed historical material, giving details about the work of Aristotle, Euclid, Plato, Alhazen (ibn al-Haytham), Newton, Roger Bacon, Brahe, Kepler Huyghens, and Fraunhofer. Whewell then used this examination of the history of Optics and current research in the field to outline the "Steps of the Induction of the Theory of Light".[100] In a notebook dated 1831-32, a discussion of the use of conceptions in induction includes notes on the scientific work of Archimedes, Pascal, Aristotle, Descartes, Mersenne, Galileo and Torricelli.[101] In another notebook, dated December 1833, Whewell discussed Herschel's *Preliminary Discourse*, pointing out one problem with his friend's work: namely, that "we do not here find the view of physical science to which we hope to be led:–that if its history and past progress be rightly studied we shall acquire confidence in truth of all kind...".[102] Moreover, in several letters to Jones in 1834 Whewell described himself as eager to get to his philosophy of science but determined to finish the history first.[103]

[97]William Whewell to Richard Jones, 25 February 1831, WP Add.Ms.c.51 f. 99.

[98]See William Whewell, Notebook, WP R.18.17. f. 12; for quotation see p. xv.

[99]For example, in an entry dated July 1831 there is a quote from Dalton followed by a comment by Whewell: "[this is] an excellent description of induction and a good proof of the difficulty in presenting things inductively" (WP R.18.17 f. 15, p. 40). In the induction notebook of 1830 there is an extensive discussion of recent discoveries in Geology (WP R.18.17 f. 12; pp. xxiv-xxv and 1ff).

[100]See William Whewell, Notebook, WP R.18.17 f. 13; the notebook is undated but it is headed "Induction IV"; the other induction notebooks are dated from 1826-1832.

[101]William Whewell, Notebook, WP R.18.17. f. 15.

[102]William Whewell, Notebook, WP R.18.17 f. 8.

[103]ee William William Whewell to Richard Jones, 27 July, 5 August and 6 August 1834; WP Add.Ms.c.52 f. 173, f. 174 and f. 175.

It is important to note that, while writing both the *History* and the *Philosophy*, Whewell attempted to ensure he had a real understanding of the scientific work he was describing. Contrary to Robert Brown's sneer about Whewell ("yes, I suppose that he has read the prefaces of very many books"[104]), Whewell did more than simply read: he actively engaged in scientific research in several areas. For example, at the time Whewell announced himself a candidate for the Chair in Mineralogy, he had already published a paper on crystallography, and two on three-dimensional geometry, which later set the foundation for mathematical crystallography (see [Whewell, 1822], [Whewell, 1825], [Whewell, 1827], [Becher, 1986] and [Deas, 1959]). But he did not have much empirical knowledge of mineralogy, so he went to Berlin and Vienna to study with Mohs. While in Germany Whewell was impressed by the natural classification system in mineralogy, and rejected the artificial system then in vogue in England. He published a monograph on mineralogy, and several more papers (see [Whewell, 1828b] and [Becher, 1986]). Whewell also performed experiments with his friend (and future Royal Astronomer) G.B. Airy, in order to determine the mean density of the earth. In June of 1826 Whewell and Airy went to Cornwall, where they spent time in the Dolcoath copper mine comparing the effect of gravity on pendulums at the surface and at a depth of 1200 feet (Bacon had suggested this experiment in his *Novum Organum*) [Whewell, 1828a]. Some years later, Whewell became interested in tidal research, based in part on his friend John Lubbock's work on the topic (it was also an area that had greatly interested Bacon, who wrote an essay "On the Ebb and Flow of the Sea").[105] Whewell helped Lubbock get grants from the newly-formed BAAS for his research, and suggested the term "cotidal lines" to him, to designate lines joining high water times (see [Cartwright, 1999, 111] and [Whewell, 1833b]. Whewell also pushed for a large-scale research project of tidal observations. Aided by Captain (later Admiral) Beaufort, Hydrographer of the Navy, and with the support of the Duke of Wellington, Whewell managed to organize simultaneous observations of tides at 100 British coast guard stations for two weeks in June 1834. In June 1835 he organized three weeks of observations along the entire coast of N.W. Europe and Eastern America, including 101 ports in 7 European countries, 28 in America from the mouth of the Mississippi to Nova Scotia, and 537 in the British Isles, including Ireland.[106] This resulted in over 40,000 readings; Whewell

[104]Reported by Charles Darwin in his [1958, 131]. Darwin related this story in order to illustrate the point that Brown, though possessing some positive qualities, "was rather given to sneering at anyone who wrote about what he did not fully understand: I remember praising Whewell's *History of the Inductive Sciences* to him, and he answered, 'yes, I suppose that he has read the prefaces of very many books'."

[105]In this essay, Bacon had postulated a northward progressing tide over the whole globe (see [Bacon, 1877-89, 5: 443–58]). But he realized that there was not enough empirical data to support this supposition strongly, and he urged mariners to record tidal times along the coast of West Africa, among other places. After Bacon, in the late seventeenth and eighteenth centuries, analysis of the tides became mainly mathematical, utilizing models which were overly-idealized, such as that of Laplace which postulated an ocean covering the whole globe. But by the nineteenth century even the scant empirical data which existed showed the inadequacy of such models.

[106]Whewell noted that the observations were made from June 8-28, and occurred in 28 places

himself reduced all the data [Whewell, 1836, 291]. Eventually Whewell presented 16 papers to the Royal Society and several summary reports to the BAAS on the topic of the tides between 1833 and 1850. In recognition of his work in tidal research, Whewell was awarded a gold medal by the Royal Society in 1837 [Cannon, 1964, 183].

Whewell, then, had first-hand knowledge of the methods of empirical research. But he was also up-to-date on the discoveries of other scientists. Indeed, Whewell consulted scientists about their own discoveries or those of others throughout the history of their respective fields, and sent proof sheets of the *History* to them for their approval. For instance, Whewell asked his friend Airy to look over his section on the history of Astronomy and to send references to French works on the polarization of light.[107] He sent queries on physics to Forbes[108] and on anatomical science to Owen. While Whewell was revising his *History*, he asked Faraday whether there were any errors describing his work in the first edition.[109] (Faraday responded by saying there were no errors that he could see.)

So, while it would be an overstatement to say that Whewell had no ideas about philosophy of science until after he completed all three volumes of the *History*, it would also be a vast understatement to suggest that the *History* was written to conform to a fully fleshed-out a priori methodological position. Knowledge of both current scientific practice and the history of science were important to Whewell in developing his philosophy of science.

On the other hand, Mill's relationship with science was rather less intimate, as he admitted, prompting Whewell's dismissal of Mill's scientific abilities, in a letter to Herschel soon after the publication of *System of Logic*: "Though acute and able". Whewell wrote of Mill, " he is ignorant of science".[110] Yet Whewell did not focus on whether Mill inferred his philosophy of science from his own knowledge of science and its history. Rather, Whewell criticized Mill for the fact that his methods were not applied to a large number of appropriate historical cases. He complained that "if Mr. Mill's four methods had been applied by him ... to a large body of conspicuous and undoubted examples of discovery, well selected and well analysed, extending along the whole history of science, we should have been better able to estimate the value of these methods" [Whewell, 1860, 264–5]. On Whewell's view, even though Mill did not infer his philosophy from science and its history, it should be possible, if his methods are valid, to find examples of their use in actual scientific practice throughout the history of science.

This sheds some light on what Whewell held to be the important relation between science and philosophy of science. What is important according to Whewell

in America, 7 in Spain, 7 in Portugal, 16 in France, 5 in Belgium, 18 in the Netherlands, 24 in Denmark, 24 in Norway, 318 in England and Scotland, and 219 in Ireland [Whewell, 1836]. See also [Cartwright, 1999, 112-14].

[107]See letters from G.B. Airy, 11 October 1856 (WP Add.Ms.a.200 f. 114) and 21 April 1831 (WP Add.Ms.a.200 f. 9).

[108]See, e.g., William Whewell to J.D. Forbes, 19 February 1840, WP O.15.47 f. 51a.

[109]See William Whewell to Michael Faraday, 7 August 1846, WP O.15.49 f. 56.

[110]William William Whewell to John Herschel, 8 April 1843, in [Todhunter, 1876, 2: 315].

is not whether a philosophy of science is, in fact, inferred from knowledge of past and present scientific practice, but rather, whether a philosophy of science is *inferable from* such knowledge. Any well-founded philosophy of science must be shown to be exemplified in actual scientific practice throughout the history of science. Thus, even though Bacon did not infer his philosophy of science from a study of the history of science — and indeed, as Whewell noted, he did not have much history of modern science behind him — his philosophy can still be found to be legitimate if it is shown to be exemplified in the science that has come since him. (To some extent this is the project Whewell himself had taken on.)[111] Even if Whewell did not develop his philosophy of science only after his study of the history and practice of science was completed, he showed us in his works — through numerous apt examples — that his philosophy has been embodied in the practice of science throughout its history. Mill was unable to do so.

11 CONCLUSION: RECEIVED VIEWS OF WHEWELL

As I have already noted, most commentators have claimed, following Mill, that Whewell presented a view similar to what is known today as "hypothetico-deductivism". However, I have shown this claim to be incorrect. Whewell believed that before his "tests of hypotheses" could be applied, the hypothesis must have been invented by some inductive reasoning process. Some commentators have proposed instead that Whewell's methodology resembles the view proposed in the late nineteenth century by Charles Peirce, initially called "abductivism" and later known as "retroductivism" (see, for example, [Fisch, 1991, 168n.13] and [Laudan, 1971, 370]). Peirce himself admired Whewell greatly, and suggested that his own methodology was following in the tradition of Whewell's [Peirce, 1982]. Peirce's view of the impossibility of any pure observation, apart from theory, may also have been informed by his reading of Whewell's work on the Fundamental Antithesis; Peirce came to share Whewell's view that "there is a mark of theory over the whole face of nature" [Whewell, 1858a, 1: 46]. But the methodology Peirce endorsed is not equivalent to Whewell's discoverers' induction. Peirce referred to abduction as a form of reasoning, classifying it, along with induction and deduction, as the three types of logic. Moreover, he maintained that testing the consequences of a hypothesis could only occur after abductive reasoning prior to testing.[112] However, he also claimed that abductive reasoning amounted to a kind of "guessing". In abductive reasoning, a set of surprising facts gives rise to a hypothesis that would, if true, explain these facts. But as Peirce stresses, the abduction itself "has no pro-

[111] Whewell saw this as his project from the beginning; see the notebook entry dated 28 June 1830, WP R.18.17. f. 12, p. xv.

[112] See Peirce, 1960, 7: 124. A very different characterization of "retroduction" is given by Ernan McMullin in his [1992]. McMullin recognizes that his characterization of retroduction is broader than the form of inference called retroduction by Peirce and his twentieth century followers such as N.R. Hanson. In fact, McMullin's version of retroductivism shares important features with Whewell's discoverers' induction.

bative force". It is merely a method of "engendering" new ideas. Any abduction is worthy of inductive and deductive testing, as long as it fulfills certain "economies of research". As we have seen, however, Whewell's discoverers' induction is not a form of guesswork. While it shares certain attributes of Peirce's abduction — especially the introduction of a new conception or idea — it diverges from Peirce's method in requiring the use of different forms of inductive reasoning.[113]

What is the reason for these common mischaracterizations of Whewell's view? Certainly, the fact that Whewell's works are peppered with the terms "guess" or "conjecture" has contributed to the misinterpretations. However, the use of these terms by itself does not entail the hypothetical-deductive methodology, especially if we read them (as we must) in the context of Whewell's complete writings and the common meanings of these terms in his own time. We have already seen that sometimes Whewell uses the term "guesswork" in connection with the generation of numerous possible conceptions, yet he considers the selection of the appropriate one for colligating certain facts to be a matter of inference. Other times, he uses the term "guess" or "conjecture" to refer to a conclusion that is simply not yet confirmed. For instance, in speaking of Kepler's move from his discovery of Mars' elliptical orbit to his first law of planetary motion, Whewell claimed that "When he had established his premise, that 'Mars does describe an Ellipse around the Sun,' he does not hesitate to guess at least that ... 'All the Planets do what Mars does"' [1858b, 75]. But surely this was no mere guess on Kepler's part, but was, rather, a generalization of a property found to exist in one member of the class of planets to all its members. Although such an inference may be a rather weak one, because he was inferring from a single case to a universal generalization, in this instance Kepler surely had additional rational support for inferring that all the planets shared the property of the elliptical orbit from the premise that one of them had this property; there would be strong analogical and causal reasons for thinking that the planetary orbits all lie on the same type of curve (because it was reasonable to suppose that each orbit is caused by the same physical mechanism). Whewell used the term "conjecture" in a similar way. Whewell's employment of these terms is consistent with ways they were used prior to the twentieth century, when "conjecture" was often used to connote not a hypothesis reached by non-rational means, but rather one which is "unverified", or which is "a conclusion as to what is likely or probable" (as opposed to the results of demonstration). Writers whose work was well-known to Whewell, including Bacon, Kepler, Newton and Stewart, utilized the term in this way. This common usage of the term explains why Whewell was not interpreted by nineteenth-century reviewers (besides Mill) as advocating the method of hypothesis. Indeed, his sometime-nemesis David Brewster criticized him for *not* proposing such a view, claiming that, contra Whewell, "It cannot, we think, be questioned that many of the finest discoveries in science have been the result of pure accident" [Brewster, 1837, 121]. And as we have seen, DeMorgan criticized Whewell for including *too many* forms of reasoning in his inductive method.

[113]Further work remains to be done in tracing the influence of Whewell on Peirce.

Another reason for the view that Whewell proposed a non-inductive logic similar to twentieth-century hypothetico-deductivism is that modern commentators tend to read his work through the lens of Mill. One legacy of the influence of Mill's *System of Logic* is the notion that "induction" refers to a narrow logical operation involving only enumerative or eliminative forms of reasoning, as defined by Mill's famous "Methods of Experimental Inquiry". Indeed, in the twentieth century, this legacy led to the delineation of a false dichotomy: between a narrow Millian inductivism and hypothetico-deductivism. It was argued by some proponents of hypothetico-deductivism that scientific discovery could not be a matter of merely calculating enumerations of observed instances, perhaps together with some eliminative process; there was, rather, an element of creativity involved, so that discovery could not proceed by a logical "rule-book". Further, it was often noted that theoretical entities were an important part of modern science, and, since Mill's methods could not reach theoretical entities, these methods could not be the proper path to discovery. In this way it was concluded that scientific discovery was not, and could not be, inductive. Instead, it must consist in a non-inferential process, one that does not involve any "logic". Mill's influence, ironically, resulted in the position popular among a number of philosophers that any methodologist proposing a means for inventing theories about unobservables was proposing a hypothetical method, and Whewell's theory was read in this light. But Mill's definition of "induction" involved a narrowing of the term, whose earlier more Baconian uses allowed a broader scope for various methods of reasoning. On his classification, Whewell proposed a non-inductive method. But why should we apply Mill's definition of induction to Whewell?

Whewell believed himself to be proposing an inductive method and, in the sense that it was a rational method for discovering theories, one that relied heavily on analogical inference, it *was* inductive. Moreover, it is a view that provides an interesting and fruitful way to view scientific reasoning today. Because of its broader notion of inductive logic, Whewell's view of method allows for the rational inference to unobservable entities and properties; thus it defuses one criticism of inductive methods by the proponents of hypothetico-deductivism. Because of Whewell's intensive survey of the history of science, and his own scientific researches, it is a method that draws upon much knowledge of the actual workings of scientists, particularly in the physical sciences. It is not surprising that a number of scientists in Whewell's day — not only Darwin, but also Faraday and Maxwell — thought his view was a valuable one, and consistent with certain aspects of scientific discovery and justification.[114] Indeed Whewell's view of inductive logic remains valuable for those interested in scientific reasoning. If we follow Mill in his reductive view of inductive logic, Whewell's riposte to DeMorgan is still apropos: so much the worse for logic.

[114]For the relation between Whewell and Faraday, see [Snyder, 2002]. For Whewell and Maxwell, see [Fisch, 1988] and [Harman, 1998].

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For more on Whewell and his debate with Mill, see [Snyder, 2006].

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THE LOGIC OF JOHN STUART MILL*

Fred Wilson

John Stuart Mill argued that the highest human aim was the promotion of the general happiness of humankind, and was the greatest defender of social reform in the nineteenth century He argued that the promotion of liberty was essential to human happiness, and that one could promote liberty and the general happiness only through clear thinking and the development of reasonable policies. The person who thought for him- or herself was better for that reason, better for their own self and better for their fellows. We all err, and sometimes error is dangerous: our aim should be to eliminate it. The latter in turn requires critical debate, the tool of which was logic – deductive logic to keep our thought consistent, inductive logic as the basis for the search after truth, and a knowledge of the ways in which fallacies might arise so that they can be uncovered and avoided. He argued, in his *Inaugural Address* as Rector of St. Andrews University, that "in the operations of the intellect it is much easier to go wrong than right," and that

> Logic is the great disperser of hazy and confused thinking; it clears up the fogs which hide from us our own ignorance, and which make us believe that we understand a subject when we do not.[1]

He developed his "system of logic" with these aspirations to guide him.

John Stuart Mill was born in 1806 in Pentonville which was then a suburb of London. He was the eldest child of James Mill, who trained for the ministry in his native Scotland but who had come to London to earn his living as a writer and later in the employ of the East India Company. In London he had come under the influence of the utilitarian philosopher Jeremy Bentham. He became, with Bentham, a leading advocate of political reform. He began the education of his son when the latter was very young – lessons in Greek and arithmetic began when he was three and by the time he was six he was enjoying Hume and Gibbon. The education was based on the principles of associationist psychology defended by Bentham and his father. The younger Mill was raised to become the leader of the reformers. This he did become and his influence in Victorian Britain was enormous. However, in 1826 he went through a state of depression in which he questioned himself and his values. He emerged from the depression (through the reading of Wordsworth) a changed person and a changed philosopher. He never abandoned the basic associationist and utilitarian framework of Bentham and his

*References to John Stuart Mill are to the editions in the *Collected Works*, ed. J. Robson (Toronto: University of Toronto Press, 1963-). Cited as "*CW*".

[1] *Inaugural Address*, *CW*, vol. 21, p. 238.

Handbook of the History of Logic. Volume 4: British Logic in the Nineteenth Century.
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father, but he modified it considerably to accommodate what he came to see were the insights of such thinkers as Coleridge and Carlyle. He published many essays in major reviews, including a serious review (1828) of Richard Whately's *Elements of Logic*,[2] and his first major work was his *System of Logic, Ratiocinative and Inductive* (1843).[3]

In his *Autobiography*, he tells how his father had him read Aristotle's *Organon* when he was 13, followed by several scholastic texts and Hobbes *Computation sive Logica*, and later records[4] how he and a few friends studied logic in the early 1820's. They began with Aldrich's textbook on logic, which had been for a century the standard text used in Oxford. Mill and the others in his group found it seriously deficient as a book in logic (which indeed it was – then [as now] Oxford was seriously out of date in logic). They turned to a better book on scholastic logic, by Phillippe Du Trieu, *Manuductio ad logicam* (which however, while better, was only a little better), then took up Whately (which was much better), and (again) Hobbes' *Computatio sive Logica* (which was also a good logic, and provided as a metaphysical context a nominalism that opposed Whately's Platonism). Mill also spent coonsiderable time and effort in editing Bentham's *Rationale of Judicial Evidence*, which gave him a practical knowledge of the rules of evidence as well as a knowledge of the many fallacies to be found in English law.[5] The issues he discussed and studied with his friends were of course logic in the narrow sense, but also (as Mill says in the *Autobiography*) "metaphysical". Certainly, in the *Logic*, and already in the review of Whately, Mill not only articulates a set of rules for determining validity (logic in the narrow sense) but locates them in a metaphysical context in which they can be defended. We are therefore not surprised to find further comments on the nature of logic and of logical inference in his only purely metaphysical work, *An Examination of Sir William Hamilton's Philosophy* (1865).[6] The *System of Logic* itself is subtitled as "a connected view of the principles of evidence and of the methods of scientific investigation," and consistently with that purpose it contains not what we would refer now to as "logic", that is, deductive logic but also an inductive logic and a well developed "philosophy of science," including the social sciences. Nor is Mill simply content to expound a set of principles, but he undertakes to defend them, against, on the one hand, critics such as the earlier empiricists, Locke for example, or Reid, who denigrated logic as useless, and against, on the other hand, representatives of the rationalist, or what he calls the intuitionist school, who would aggrandize logic at the expense

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[2] "Whately's Elements of Logic," *CW*, vol. 21, pp.3-35. The review is of Richard Whately, *Elements of Logic* (London: Mawman, 1826).

[3] *System of Logic, Ratiocinative and Inductive*, *CW*, vols. 7-8. The text here is the eighth edition of 1872, Mill's last revised version; it contains all variants through the various editions from the first on. The editor of the series provides an informative introduction, describing the history of the composition and the history of the various editions. There is also a philosophical introduction by R. F. McCrae.

[4] *Autobiography*, *CW*, vol. 1, ch. i and iv.

[5] *Autobiography*, *CW*, vol. 1, ch. iv.

[6] *An Examination of Sir William Hamilton's Philosophy*, *CW*, vol. 9.

of empirical research methods, that is, such thinkers as Sir William Hamilton or William Whewell[7] or, for that matter, also Richard Whately, though he does give great (and deserved) credit to the latter for producing a readable and competent text on logic, including what we would now call informal logic. Mill's discussion of formal logic cannot be separated from his account of empirical science and the nature of inductive inference, and also from his view that the principles of reasoning must be a guide to forms of evidence and rational discourse in the pursuit of truth. The latter aims imply, for Mill, that any complete exposition of any system of logic should include an account not only of correct reasoning but also an account of bad reasoning, and those aims therefore also imply that an account of the logical fallacies has (as it had for Whately) an important place in the exposition of any system of logic. There is thus a breadth to the treatment of logic in Mill's *System of Logic* (supplemented by the work in the *Examination*) that is seldom to be found in standard treatments. At the same time, it must be said that Mill was hardly a visionary in the development of formal logic, which, even as he wrote the *Logic*, was beginning to undergo its flourishing growth that led to what we now recognize as (the science of) formal logic – Mill does not mention Boole nor Jevons in any edition, the two references to Venn are to his work in probability, and the references to de Morgan are not to his brilliant contributions to formal logic. But again, a qualification must be made, for Mill's logic and the framework he provided are such that those developments could in them find a natural place and defence. And so the empiricism expounded by Bertrand Russell in second decade of the twentieth century is essentially Mill supplemented by the new formal and mathematical logic.

1 DEFINITION OF LOGIC

Mill argues that "Logic ... is the science of the operations of the understanding which are subservient to the estimation of evidence."[8] It is a science, not an art, and insofar as it can be taken as an art, it is, like all arts, subservient to its background science. As a science it is concerned with inferences, not with those truths of individual fact known by intuition, that is, observation by sense or by inner awareness. Observation knows facts immediately, and not by inference, and there is therefore no logic for this part of our knowledge. These facts are the starting point for knowledge; what we know is known by observation or by inference therefrom. Their study is more properly the domain of metaphysics than it is of logic. But some metaphysics cannot be avoided in developing a *system*

[7]Mill, in the *System of Logic*, refers often and critically to William Whewell, *History of the Inductive Sciences*, 3 vols. (London: Parker, 1837) and *The Philosophy of the Inductive Sciences*, 2 vols. (London: Parker, 1840). Whewell replied to Mill in later editions, and Mill replied to these replies in later editions of the *Logic*. The history of the debate can be followed in the edition of the *Logic* in the *Collected Works*. Whewell was a defender of the *a priori* in science, intuitionism in ethics, and unreformed universities; it has been said that, if Whewell had not existed, Mill would have had to invent him.

[8]*System of Logic*, Intro., § 7, *CW*, vol. 7, p. 12.

of logic, since logic has been rooted in a particular metaphysical tradition. This tradition claimed that there are, besides those truths known by observation, other truths known by non-sensory intuition, synthetic necessary truths about the ontological structure of the universe. Mill aims to free logic from these metaphysical constraints: knowledge, he argues, starts in, and ends in, observable individual facts.

Inferences are made as part of the search after truth. There are two kinds of inference. On the one hand, there are inferences to propositions from those less general. On the other hand, there are inferences from propositions which are equally or more general. The former inferences are inductions, the latter are deductions. What we now generally understand as logic is the latter, but Mill argues that this, what he calls "ratiocination," should be seen as part of a larger logic, the logic of truth.

2 HISTORICAL BACKGROUND: ARISTOTLE'S LOGIC

The logic that came down to the nineteenth century was largely the invention of Aristotle.

This logic dealt with propositions of two terms, referred to as the subject and the predicate terms. Arguments had three terms; there was the subject and predicate of the conclusion and the third term, called the middle, which was shared by the two premises. There were arrangements like this:

$$\frac{\begin{array}{c} M - P \\ S - M \end{array}}{S - P} \tag{1}$$

The three terms were general terms, and the propositions either general ("all S is P") or particular ("some S is P"). An argument like

$$\frac{\begin{array}{c} \text{All } M \text{ is } P \\ \text{All } S \text{ is } M \end{array}}{\text{All } S \text{ is } P} \tag{2}$$

was called a syllogism. Syllogisms with the three terms arranged as in (1) were in the "first figure". So (2) is in the first figure. Other arrangements of the terms yielded other figures (second to fourth). Some syllogisms were valid, in the sense that if the premises were true then the conclusion must be true; in these syllogisms the conclusion is contained implicitly in the premises. (2) is a valid syllogism. But

$$\frac{\begin{array}{c} \text{All } P \text{ is } M \\ \text{Some } S \text{ is } M \end{array}}{\text{No } S \text{ is } P \ (= \text{All } S \text{ is not } P)} \tag{3}$$

(which is in the second figure) is not valid. There were various rules concerning which syllogisms are valid and which are not ((3) violates both the rule that if one premise is particular then the conclusion is particular, and the rule that a negative conclusion requires a negative premise).

Aristotle also showed that syllogisms of the other figures could be proven to be valid by constructing proofs that reduced them to a valid syllogism in the first figure. Thus,

$$\frac{\begin{array}{c}\text{All } S \text{ is } M\\ \text{All } M \text{ is } P\end{array}}{\text{All } S \text{ is } P} \tag{4}$$

which is in the fourth figure can be reduced to (2) in the first figure by simply transposing the premises. Similarly, the syllogism

$$\frac{\begin{array}{c}\text{No } P \text{ is } M\\ \text{All } S \text{ is } M\end{array}}{\text{No } S \text{ is } P}$$

which is in the second figure can be reduced to the first figure syllogism

$$\frac{\begin{array}{c}\text{No } M \text{ is } P\\ \text{All } S \text{ is } M\end{array}}{\text{No } S \text{ is } P}$$

by the simple conversion of the first premise

$$\frac{\text{No } P \text{ is } M}{\text{No } M \text{ is } P}$$

which is a valid immediate (one premise) inference. The second figure syllogism is valid because it reduces by a valid inference to a valid first figure syllogism. Other reductions are more complicated, but the rules for reduction in effect establish that validity in the other figures can be reduced to validity in the first figure, which was therefore known as the "perfect figure".

So far, this is how Aristotle developed the formal logic of syllogistic (in his *Prior Analytics*). Formal logic of this sort could be used by the debater, the lawyer or the politician, to make a point in a conclusive way, provided that the premises were given, perhaps by custom, perhaps by the legal code, perhaps by ordinary experience of the world. (Aristotle so used it himself in his book on the *Topics*.) But Aristotle had a more particular, and deeper, purpose for his syllogistic. He intended his logic to apply to, and display, the ontological structure of the universe, conceived in terms of his metaphysics. (These developments can be found in his *Posterior Analytics*.)

On this metaphysics, the world consists of substances. Substances have properties present in them; these properties are given in sense experience and are predicated of the substance in which they are present: they constitute the being of the substance, they are what it *is*. Simple sentences that describe the world, for example,

$$\text{That apple is green} \tag{5}$$

have a subject term, here 'that apple' and a predicate term, here 'green.' The subject term refers to a substance and the predicate term refers to a property. The 'is' of predication represents that the property is present in the substance.

Each substance has a form — its species. (In (5) the form of the substance is that of an apple.) This species is not given in sense experience, but nonetheless explains, constitutes the reason for, the substance being and becoming what it is. The properties presented to us in sense reflect in their order the inner form: there is as it were the outer form given in sense and the inner form, the deeper reality, that is the cause or reason for that outer form. Now, the properties may come and go, but the substance and its form remains the same and endures through change. The substance is active in determining its own being, this activity is in conformity with its specific form — the form is an active form. Substances have properties; these change in certain ways determined by their form; and these changes are explained by the active form. The form determines the end towards which the substance is, through its activity, striving to become. The explanation is plainly teleological and has human activity as its model.

Substances have a specific form, but this form will have an internal structure. It shares features with other species, and so is falls under a genus. It is distinguished from other members of this genus by its specific difference. For Aristotle, a scientific syllogism has P as a genus, S as a species and M as the specific difference. Sentences like (5) do not appear in scientific syllogisms; the latter contain no terms that refer to individual substances, only terms that refer to forms, which are general and not individual. Thus, as an example of a scientific syllogism, we have

$$\frac{\begin{array}{c}\text{All rational is animal}\\ \text{All human is rational}\end{array}}{\text{All human is animal}} \tag{6}$$

which is an instance of the form (2) in the first figure. This syllogism *displays* the logical and ontological structure of the species human. This form is given equivalently in the *real definition* of the species human:

$$\text{human is rational animal} \tag{7}$$

This real definition is a necessary truth, a timeless truth about the logical and ontological structures of the forms of substances. "Human" is the species, "animal" is the genus, and "rational" is the specific difference — the first of these is the subject of the conclusion of the scientific syllogism, the second is the predicate of the conclusion, and the third is the middle term — the syllogism thus displays the connection between species and genus that is in the real definition of the species. The premises of the scientific syllogism are thus necessary in the sense of constituting the logical structure of the specific form mentioned in the conclusion. A scientific syllogism thus, for Aristotle, shows the necessity of its conclusion by displaying how that structure is necessarily contained in the structures, themselves also necessary, mentioned in the premises.

As for the substantial forms, knowledge of these is presupposed by the use of scientific syllogism. These forms are not given in our sense experience of the world. They are, rather, known by a rational intuition. Some held (following Plato) that such rational intuitions are innate, others (following Aristotle himself) held that they are products of a process through which the mind abstracts the form of a substance from the properties it presents to us in sense experience, but in any case it is an intuition and not an inference and rational because it an intuition of the forms that constitute the ontological reasons for the being of things (substances), that is, for their being and becoming what they are, for their having and coming to have present in them the properties that are predicated of them. In fact, on the traditional view, again coming from Aristotle, the rational intuition of the form consists of that form being itself in the mind of the knower ("like knows like"). In knowing, the knower becomes identical with the known. The form in the mind is the concept — the abstract idea — of the thing known. In thinking the thing that thing is there in the mind of the thinker as the concept of the thing being thought.

3 HISTORICAL BACKGROUND: ARISTOTLE: THE EMPIRICIST CRITIQUE

According to this tradition coming down from Aristotle, in order to explain the world we know in sense it is necessary for the mind to go beyond this world to entities that are outside this world, to the substantial forms of things. This metaphysics of forms and substances was subject to critique in the ancient world, but the sceptical tradition largely disappeared during the long period during the middle ages and beyond when philosophy was subordinated to theology. In the early modern period, the sceptical critique was revived by Montaigne, and developed more systematically by a series of British thinkers, Bacon, Hobbes, Locke, Berkeley and Hume.

The idea that there were things beyond the world of sense was criticized by Locke, Berkeley and Hume: there are neither substances nor substantial forms. Hobbes, Locke, Berkeley and Hume attacked the older doctrine of our concepts of things: all our concepts derive from sense experience, and there are no abstract ideas in the required sense — not only are there no substances and no substantial forms but they cannot even be thought — the whole metaphysics is, in other words, simply non-sense.

It follows that the traditional understanding of the syllogism, or at least of the scientific syllogism, is in fact gravely in error. On the tradition, the terms in the syllogism represent concepts which are the substantial forms of things, and the syllogism itself displays the ontological structure of these forms. The syllogism thus expresses and makes explicit our knowledge of this form. But, the critique argued, there are no forms and therefore none of the relevant concepts and none of the necessary connections supposedly displayed in the "scientific syllogism." Far from representing knowledge, the "scientific syllogism" was empty nonsense.

Syllogisms of course still had their place in debates, but they were certainly incapable of providing the superior sort of knowledge claimed for it by the tradition. Concerning the knowledge of things, syllogisms were simply trivial: they could not yield knowledge. And even their utility in debates could be questioned. Or rather, debate, where they might find a use, was itself a trivial exercise, incapable in itself of yielding knowledge of things: the knowledge of things that yielded understanding could be got only through the methods, the empirical methods, of the new science. Such, at least, was the argument developed by Locke in his *Essay concerning Human Understanding.*

The concept of what it is to understand events in the world in fact undergoes a radical change. In understanding, one wants to know what brings things about: what brings things about are the causes that are the reasons for things, in the sense of providing the reasons why the world goes this way rather than that. In the traditional metaphysics, the causes of things are the substantial forms, where these active forms provide the necessary connections among the events in the world of ordinary sensible experience that explain why this rather than that follows so and so. To understand meant intuiting — rational intuiting — the necessary connections established by the logical structures of the forms. But now there are no forms. All that can be said to bring about something is that it was preceded by another event, where events of the same sort as the first are regularly preceded by events of the second sort. For example, this kettle of water is boiling and being heated is what brought about this event, since water, when heated, boils.

Mill puts it this way:

> An individual fact is said to be explained, by pointing out its cause, that is, by stating the law or laws of causation, of which its production is an instance.[9]

Thus, in general an explanation is of the form

$$\frac{\begin{array}{c}\text{All } F \text{ are } G\\ \text{This is } F\end{array}}{\text{This is } G} \tag{8}$$

the event of this being F explains the event of this being G just in case that the pair can be subsumed under the generality or regularity that All F are G. The point is that, given this regularity, then bringing about an F will produce a G, and the F is the reason for the G. In other words, in the world without substances and forms, reason, the capacity to know the reasons for things, consists in the grasp of regularities. This account of explanation and of the human understanding was developed most thoroughly by the philosopher David Hume. It was adopted and defended by both the Mills. It is a major theme in John Stuart Mill's *System of Logic*.

(Karl Popper, later in the twentieth century, was to claim that he was the first to propose that scientific explanation involves subsumption under a general law or

[9] *System of Logic*, Bk. III, ch. xii, § 1, *CW*, vol. 7, p. 465

regularity. He may have discovered it for himself, but he could have saved himself the trouble by reading Mill!)

For Mill human understanding consists in the discovery of matter-of-empirical-fact regularities, and it such regularities that form the major premise in explanatory arguments of the form (8). In causal explanation, it is a regularity that explains: contrary to the older tradition there is no ontologically necessary tie that links cause and effect and accounts metaphysically for the observed regularity. Already in his review of Whately's *Logic*, Mill had criticized the book for its taking seriously the traditional doctrine of real definition and the traditional doctrine that scientific syllogisms have necessary truths for their premises, and this attack continues in the *System of Logic*. Right from the beginning Mill in various ways criticizes the doctrine of substances and of substantial forms, and the doctrine consequent upon these that there is no causal connection rooted in objectively necessary connections. To the contrary, causal explanations are given by matter-of-empirical-fact regularities, that is, by contingent generalizations. As for substances and forms, these concepts are confusions and the thought that there are such things is a matter of fallacious reasoning. Thus, for example, Mill argues that those philosophers who, like Descartes, or Mill's contemporary William Whewell, think there are objective synthetic necessary connections do so because it is fallaciously supposed that what is inseparable in thought is logically inseparable in reality. Mill's understanding of the syllogism is therefore different from that of the older tradition, and even from that of Whately.

But Mill does not take so critical an attitude towards the utility of formal logic as do other critics of the tradition such as Locke. To be sure, syllogistic is no longer taken as displaying through intuited necessary connections the objective ontological structure of the world. Argumentation is not, however, the trivial matter Locke supposed it to be: the scholasticism Locke encountered in Oxford was still in its bareness there in Mill's time, but that was not all that could be said for debate — critical thought was much needed everywhere. Even in the search after truth, however, logic had a role unacknowledged by Locke.

In the discovery of causes, there is first observation.

$$\begin{array}{l}\text{That } F \text{ is } G\\ \text{That other } F \text{ is } G\\ \text{Yet another } F \text{ is } G\end{array} \tag{9}$$

Then there is inductive inference from observation: essentially we have the inference

$$\frac{\text{All observed } F \text{ are } G}{\text{so, All } F \text{ are } G} \tag{10}$$

But then the generalization that is the conclusion of the inductive inference must be tested: one tests it by using it predictively — if things turn out as it predicts then the generalization is confirmed, if things do not turn out as predicted then the generalization is falsified and rejected — in which case the process of forming hypotheses to try to understand nature begins over with the new data. The point

Mill makes is that prediction involves deduction: predictive arguments are, like explanatory arguments, of the form (8)

$$\frac{\begin{array}{l}\text{All } F \text{ are } G\\ \text{This is } F\end{array}}{\text{This is } G}$$

— the explanatory argument applies to an F already known to be G, the predictive argument applies to an individual which is F but where it is not (yet) known whether it is G or not. So, according to Mill, deduction plays an important role in scientific theorizing — deduction elaborates and makes explicit what is contained in the hypotheses arrived at through inductive inferences, and then tests those hypotheses by testing those consequences against observed facts.

The disappearance of substances leads to a new account of basic statements of individual fact such as (5)

That apple is green

Since there are no substances, there is no reference to them, nor are sensible qualities predicated of them. A sentence like (5) is now understood as saying something simply about how the world appears, and not about what metaphysically transcends this world, not about substances or active forms. Formerly, on the old view, and also on the new, the predicate 'green' refers to a sensible quality that is in the thing of which it is predicated. But where the subject term 'that apple' was formerly taken to refer to a substance in which sensible qualities inhere, including the one that is predicated of it, it is taken, as before, to refer to the thing that remains the same through change, which now, however, can only be the whole itself of all the sensible properties predicated of it. Where formerly the relation of predication represented the tie of inherence, it now represents the relation of a part to a whole. As for the term 'apple', where this formerly was taken to refer to the active substantial form of the substance, it is now taken to describe the *pattern* of appearances characteristic of apples, the pattern that distinguishes those wholes from other wholes such as sticks and rainbows. But this is not the end of the story. As science comes further to investigate the regularities that give the causal structure of the world it locates the sensible appearances of things as rooted in a common source, a "material object", which is unlike the sensible appearances to which it gives rise, unlike them in quality but like them in having a similar structure. These material objects are not traditional substances: the supposition that there are such entities is not a matter of metaphysical speculation but rather is rooted in the inferences from sensible experience of empirical science: the evidence for their existence is empirical, not metaphysical. Mill defends this view in the *System of Logic*, and in still greater detail in the *Examination of Sir William Hamilton's Philosophy.* He sums up his view there in this way:

> When ... I say, The sky is blue, my meaning, and my whole meaning, is that the sky has that particular colour .... I am thinking only of

> the sensation of blue, and am judging that the sky produces this sensation in my sensitive faculty; or (to express the meaning in technical language) that the quality answering to the sensation of blue, or the power of exciting the sensation of blue, is an attribute of the sky.[10]

4 RATIOCINATION: MILL'S DEDUCTIVE LOGIC

As the case of Whately makes clear, one can present a good defence of formal logic without accepting the empiricist critique of the traditional doctrine of substances and forms and objective necessary connections; and as the case of Locke makes clear, one can accept the empiricist critique, and its rejection of the traditional doctrine of the syllogism, without accepting that formal logic has a reasonable place in the structure of knowledge. Mill accepts the empiricist critique and also defends formal logic. But as one might expect, the formal logic that he defends is not quite that of the tradition.

Mill argues that the meaning of terms in a proposition is to be understood as involving *denotation* and *connotation*. Proper names denote individuals. Thus, 'John Stuart Mill', the name, denotes the individual person who was so called. 'This' and 'that' are also pure denoters, but they denote a different individuals on each occasion of their use. Predicates such as 'red' also denote, but besides that they connote: they connote a certain property or quality and denote the individuals that have that property or quality. A basic sentence such as

$$\text{This is red} \tag{11}$$

or

$$\text{This is } F$$

is true just in case that the denotation of the subject term is among the individuals denoted by the predicate, which is to say that it is true just in case the properties or qualities connoted by the predicate are present in the individual denoted by the subject term.

In taking this view, Mill is situating his account of logic in a tradition other than that of the great logician Gottlob Frege. In a proposition like (11), Mill takes it, if it is true, to be true by virtue of a combination of entities, namely the property connoted by the predicate 'red' and the individual denoted by the subject term 'this'. The 'is' of predication represents the tie between the property and the individual which has that property. The complex which makes (11) true, if it is true, consists of an individual, a property and the tie represented by the relation of predication. For Frege, there is no such complex. There is the individual, that is, the individual which the subject term 'this' denotes. But there is no tie of predication. Rather, the predicate as it were carries the tie with it. We should

[10] *Examination of Sir William Hamilton's Philosophy*, *CW*, vol. 10, p. 386

not think of the predicate of (11) as 'red' alone but as 'is red'. This term Frege takes to be, not a property present in the individual, but a *function*, or at least an entity analogous to a function, in the mathematical sense of 'function', where, for example, the function

square of

maps one number onto another, as

the square of 2

maps 2 onto 4. Here the basic sentences are like

the square of 2 = 4

so that 'is red' in (11) maps the individual *this* onto another entity. This entity is, Frege proposes, *the True*. Thus, (11) is really of the form

the redded of this = the True

Needless to say, this entity, *the True* (and its evil twin *the False*), is hardly one given in our ordinary sensible experience of the world, and would for that reason alone be unacceptable to Mill. But Mill is hardly alone on this matter: Bertrand Russell and Ludwig Wittgenstein both rejected *the True* and *the False* on these grounds. To use a phrase of Russell, any philosopher with a robust sense of reality could not accept such an ontological monster.

For Mill, predicates in sentences like (11) represent classes, not functions. In defending this view of such propositions (and the consequent class interpretation of general propositions), Mill is in fact locating himself in what was to become the central tradition in formal logic, the tradition developed by Boole, de Morgan and Peirce, and in effect made final by Russell, the tradition, ubiquitous in logic texts, in which predicate logic is understood as a logic of classes.

In giving his account of propositions like (11), Mill has other opponents in mind besides the defenders of substantial forms. Mill also argues against Hobbes and other nominalists about the correct account of predication. According to the nominalists, the meanings of terms is given by their denotation alone. A predicate is distinguished from a subject term only in this, that it has been decided that it denotes several objects rather than just one. Just which individuals it denotes is a matter of decision; there is no objective ground for the choice. The extension of any predicate is therefore arbitrary, and which propositions are true is a matter of convention. (One is reminded of more recent views of such philosophers as Quine, Sellars, and Davidson.) But, Mill argues, truth is not a matter of convention: we apply a predicate like 'red' to things by virtue of their being red, and not simply because we have decided these rather those other things are red. To account for this we must allow, what the nominalists miss, that terms like 'red' have connotation as well as denotation. The nominalist account works for verbal propositions such as

Cicero is Tully

where truth is a matter of linguistic convention, but not for others like (11), where the truth is not a matter of convention.

But Mill sides with the nominalists against the Aristotelian tradition that general terms refer to substantial forms This leads to a different understanding of the general propositions that appear in syllogisms. For the Aristotelians, the general proposition

S is P

states a relation about substantial forms, and not about individuals. Given the non-existence of substantial forms and the new understanding of predication, the S and P terms denote individual and connote qualities or properties that determine classes. This means that the form

All S is P

is now taken to assert that all members of the class of individuals denoted by 'S' are members of the class of individuals denoted by 'P', or, equivalently, that every individual having the qualities connoted by 'S' also has the qualities connoted by 'P'. Being S is a sign of, or evidence for, being P.

This extensional understanding of the propositions of the syllogism enabled Mill to include among the forms of logic not only syllogisms like

$$\frac{\begin{array}{c}\text{All } M \text{ is } P\\ \text{All } S \text{ is } M\end{array}}{\text{All } S \text{ is } P}$$

and

$$\frac{\begin{array}{c}\text{All } M \text{ is } P\\ \text{Some } S \text{ is } M\end{array}}{\text{Some } S \text{ is } P}$$

but also syllogisms like

$$\frac{\begin{array}{c}\text{All } S \text{ is } P\\ \text{This is } S\end{array}}{\text{This is } P}$$

involving singular propositions where the subject term denotes a single individual. Inclusion of the latter within the scope of logic was in fact a major step in making legitimate forms of reasoning excluded by the metaphysics of substantial forms. But none of this required any serious revision of the traditional rules, e.g., concerning reduction to the first figure.

Others were beginning to propose more major changes. Sir William Hamilton had introduced his doctrine of the "quantification of the predicate." This doctrine refers to traditional propositions like "S is P," understood as Mill understood them, as propositions about classes of individuals. When not construed as about a substantial form, but about a group or class, then the assertion that "All S is P" may be meant to apply to every member of the class or only to some of them; it is, therefore, necessary to indicate this, or to express the quantity of the subject: "All S is P" in contrast to "some S is P". In the traditional forms, the predicate is not similarly quantified. But a quality always determines a class — the class of things which possess that quality. This implies that the predicate can be also be treated as if it were a class and also assigned a quantity. Thus, Hamilton modifies quantitatively the predicate as well as the subject in "All S is P" to produce two forms, "All S is all P" (where the classes determined by S and P have exactly the same members) and "All S is some P" (where the class determined by S is contained in the class determined by P, but that determined by P has more members than that determined by S). This would apply to all the traditional forms, so that we would also have, for example, in place of "Some S is P" the two forms "Some S is all P" and "Some S is some P", and similarly for the other forms. Hamilton thus extended the range of classification of propositions. His "new analytic" depends upon the contention that the quantity thus implied should be always explicitly stated, and consists in following out the changes in formal procedure which seem to him to result from this being done. But Hamilton was not thorough enough in the elaboration of his theory, and was too often less than careful in elaborating the rules for validity for the now enlarged class of syllogistic forms. He did not see that the change from the traditional view to the "class view" of the proposition would lead to a very different classification of propositions from his, and, in general, to a much more radical revision of logical forms than he contemplated. Two mathematicians contemporary with Hamilton (and with Mill) — Augustus de Morgan and George Boole — went further than he did. The latter's treatise on *The Laws of Thought* (1854), in which he accepted Mill's view of logic as dealing with classes determined by qualities connoted by predicates, working on a careful analogy with algebra, uses the symbolical methods of the latter to greatly enlarge our knowledge of the laws of logic beyond those of the traditional syllogistic. Boole here laid the foundations of the modern logic as a symbolic calculus. Boole's work represents the commencement of that flourishing of logic that has taken it far beyond syllogistic which is now but a minor part of the discipline. Mill notes none of these developments in which formal logic became modern symbolic logic: his account comes at the end of the older tradition and not at the beginning of the new.

Hamilton, in contrast, did point the way to these new developments, however imperfectly he worked out his proposals. Mill did see the soundness from a formal point of view of Hamilton's move to quantify the predicate, but like Hamilton did not see the implications that de Morgan and Boole were to develop. Moreover, Mill argued that Hamilton's innovation was of little interest to those who wished to study the judgments and inferences that people actually make. He argued that, while we could quantify the predicate in our ordinary judgments, we in fact do not do this: normally, Mill argued, we do not know whether the class S does or does not coincide in membership with the class P and so use the form "All S is

P" in a way that leaves this open. Mill is no doubt correct, and he is no doubt correct in his allowing that Hamilton's point is formally sound, given how they both understood in an extensional or "class" way the propositions with which syllogistic deals; but his anti-Hamilton polemic, on this point at least, goes on for rather too long.

Apart from allowing syllogisms including singular propositions, Mill does not alter the formal structure of syllogistic as it had come down to him through Whately. Rules for such things as the reduction of syllogisms to the first figure are those of the tradition.

There is one feature that he does reject. Aristotle argued that the first figure was the perfect figure, but that the syllogisms which occur in it could be justified by what came to be called the *dictum de omni et nullo*. This is the maxim that "whatever can be affirmed (or denied) of a class can be affirmed (or denied) of everything included in the class." In the traditional metaphysics this did indeed have a place. According to that tradition, the proposition

(a) S is P

is about substantial forms. But it is true about individuals that

(b) All S are P

in the sense that

(b′) All individuals which are S are also P

Since no substantial forms exist, according to Mill, this latter is of course the only sense that Mill allows. The traditional system has to account for why (a) implies (b)=(b′). The traditional doctrine has it that the forms as active entities cause the individuals of which they are the forms to be in a way that guarantees the truth of (b)=(b′): what is predicable of the universal or form is predicable of the individual substances subordinate to it. Indeed, the metaphysics guarantees not only that (b)=(b′) is true if (a) is true, but ensures that it is necessarily true. Thus, on the traditional view, the *dictum do omni et nullo* expresses a fundamental principle or law about the ontological structure of the universe; as Mill put it, the principle stated that "the entire nature and properties of the *substantia secunda* formed part of the nature and properties of each of the individual substances called by the same name..."[11] But in a world without substantial forms, the *dictum* amounts to nothing more than the principle that what is true of a class of certain objects, is true of each of those objects. It is a trivial proposition that does little more than pleonastically explain the meaning of the word *class*. It has retained its place in logic texts only as a consequence of doing what others did who went before, even though the metaphysics which alone justified it has disappeared.

There is, however, a fundamental principle on which, Mill argues, any logic rests:

[11] *System of Logic*, II, ii, § 2; *CW*, vol. 7, p. 174.

> Every proposition which conveys real information asserts a matter of fact, dependent on laws of nature and not on classification. It asserts that a given object does or does not possess a given attribute; or it asserts that two attributes, or sets of attributes, do or do not (constantly or occasionally) coexist. Since such is the purport of all propositions which convey any real knowledge, and since ratiocination is a mode of acquiring real knowledge, any theory of ratiocination which does not recognise this import of propositions, cannot, we may be sure, be the true one.

This yields two principles, one for affirmative syllogisms — "things which coexist with a third thing, coexist with one another" — and one for negative propositions — "a thing which coexists with another thing, with which other a third thing does not coexist, is not coexistent with that third thing." Nor are these trivial propositions like the *dictum*:

> These axioms manifestly relate to facts, and not to conventions; and one or the other of them is the ground of legitimacy of every argument in which facts and not conventions are the matter treated of.[12]

The point is that every proposition which attributes a property to an individual is either true or false and not both — properties are, in other words, logically and ontologically wholly contained within themselves, separable from and not intrinsically tied to other properties. Contrary to the older tradition which held that one property can be necessarily tied to another property, so that the being of the one implicates the being of the other, everything is what it is and not another thing. This itself is a *matter of fact* about the world, a fundamental law of nature; if the world were not this way, ratiocination would not be possible: all other laws of nature, including the laws of logic, rest on this fundamental fact.

5 TWO SMALLER POINTS

One: Propositional Logic

An argument like

$$\frac{\begin{array}{c}\text{either } A \text{ is } B \text{ or } C \text{ is } D\\ A \text{ is not } B\end{array}}{C \text{ is } D}$$

which has been known as "Disjunctive Syllogism," depends for its validity upon the words "or" and "not", connectives which make compound propositions out of simpler ones. Aristotle studied systematically arguments the validity of which depend upon "all" and "some" and "is", words which occur within simple sentences. But he largely ignored arguments the validity of which depends upon connectives,

[12] *System of Logic*, II, ii, § 3, *CW*, vol. 7, pp. 177-8.

what we now call propositional or sentential logic. In the ancient world the validity of such arguments was studied by the Stoic philosophers. The Stoics and the Aristotelians each argued that they had the correct logic and that the other's logic was incorrect. In one sense they were wrong: we now recognize, since Boole, that these are two parts of what is now the much broader field of symbolic logic. But they were also correct. Aristotle's logic was designed to fit his metaphysics of substantial forms. And the logic of the Stoics was designed to fit their own metaphysics that was at once pantheistic and materialistic. The two metaphysical systems were incompatible, and in that respect so were their logics: the logic of the Stoics did not fit the metaphysics of Aristotle and that of Aristotle did not fit the metaphysics of the Stoics. The metaphysics of Aristotle survived to find a place in the theological philosophy that came after the revival of learning after the fall of the Roman Empire, and with it Aristotle's logic. The Stoic philosophy did not survive, and we now know only fragments of its logical system. Some of this fragmentary knowledge came down to the scholastics, to be explored and even somewhat developed by mediaeval logicians, though the main concern of the latter remained syllogistic. But large parts of the logic of the scholastics disappeared with the eclipse of scholasticism which came when it was replaced by the new science as a method in the search after truth. All that remained were small bits. These survived in texts like that of Aldrich, and were given a minor place in logics like that of Whately.

Their place in Mill's logic is also minor. In fact, he proposes to assimilate them to the logic of subjects and predicates characteristic of syllogistic.

Mill first deals with conjunctions. A conjunction, he argues, is not a new proposition compounded out of the two conjuncts. The word "and" (and the word "but") is but a way of bringing two propositions before the mind at the same time. In other words, "A and B" is taken to be shorthand for "A, B, and it is desired that the two propositions be thought of at once," while "A but B" is taken to be shorthand for "A, B, and note the contrast between them."

Disjunctions received a more complicated treatment. The treatment of conjunctions would suggest a moment of doubt or hesitation about which of the disjuncts is true. But he does allow that disjunctions are genuine propositions. They are genuine propositions but are reducible to hypothetical judgments. "Either A is B or C is D" is taken to be logically equivalent to "if A is not B then C is D, and if C is not D, then A is B." [Mill is correct in this. In modern symbols, "$(\sim p \supset q)\&(\sim q \supset p)$" is logically equivalent to "$(\sim\sim p \vee q)\&(\sim\sim q \vee p)$" which is logically equivalent to "$(p \vee q)\&(q \vee p)$" which is logically equivalent to "$(p \vee q)$."]

He then proceeds to assimilate these hypothetical judgments to the simple categorical judgments of syllogistic logic. He argues that

if A is B then C is D

abbreviates

the proposition C is D is a legitimate inference from the proposition A is B

which is a subject-predicate sentence, where the subject term is "the proposition C is D" and the predicate term is "a legitimate inference from A is B".

Modern logics treat hypotheticals as material conditionals of the form "$p \supset q$" which are false if the antecedent is true and the consequent is false and otherwise true. The material conditional cannot be used to prove a conclusion true using (as we often do) modus ponens if we know it to be true solely because we know the antecedent is false (for then the argument would be unsound); nor can it be so used solely because we know that consequent to be true (for then the conclusion would already be known and the argument would not be needed). Similar remarks hold for material conditionals in their other characteristic use in modus tollens. So a material conditional cannot be used in demonstrating conclusions with modus ponens or modus tollens if the sole evidence we have for its truth is a knowledge of the truth values of its components. But we do use hypotheticals in modus ponens and modus tollens so we must affirm the premise on some other basis, and, more specifically, on the basis of some known connection between antecedent and consequent, a connection which establishes that the antecedent is evidence for the consequent. Mill is drawing our attention to this fact about conditionals. He is, in other words, not seriously at variance then with what goes on in modern symbolic logic. Since the implicit connection between antecedent and consequent will, most often at least, be a law or regularity, he is in effect taking conditionals to be instances of universal conditionals like

Any individual, if it is A then it is B

which, for practical purposes of explanation and prediction, are not different from

Any individual which is A is also B

that is, from the standard categorical form

All A is B

So what Mill says does make sense. Nonetheless, we still have also to say that Mill misses the basic point of sentential logic. It remained for Boole to discover its central place in any complete system of formal logic.

Two: Relations

It was traditional that substances are separable ontologically in this sense, if there are two things a and b, and b ceases to exist, then a will continue to exist unchanged. Now, if there was a relational fact, a is R to b, then they would not be in this sense separable. For, if b ceased to exist then a would cease to have predicated of it the property of being R to b. It was therefore argued that there are no genuine relational facts, that every fact like a being R to b is really two non-relational facts, a is r_1 and b is r_2, where r_1 and r_2 are two non-relational properties, said to be the "foundations" of the relation. It may be true that a is

R to b, but in fact there is no genuine unity. What unity there is is on the side of the mind judging: what unity there is lies in the mind, in comparing a and b in an act of judgment.

This account of relations can be found in Locke, for example. It is also the view of Leibniz. If you have God, then that is a substance that can account for the unity of all things, even in the absence of genuine relations. But God as a substance disappeared under the empiricist critique. Certainly, this entity is not there for Mill.

For many relations, Mill takes Locke's account for granted. This is so especially for social relations like master and servant. In the latter the bond between the two is given by the attitudes of the two persons, where the one has undertaken, or is compelled, to perform certain services for the other.

The empiricists, Locke anyway, were criticized by later idealists such as F. H. Bradley and Bernard Bosanquet for their account of relations. It did not, was the charge, account for genuine unity in the world. If there was an objective being to the relation, then the account would only allow it to be a third thing — in effect, a non-relational fact alongside the other two — which gives not a unity of two things but three separable things, and therefore does not achieve the desired real unity in things. The idealists argued that there could be a genuine unity which included a and b only if these were conceived as parts of a unifying single substance. "a is R to b" would thus in reality have the form "the substantial whole of which a and b are aspects is R-ish" where the predication is again non-relational but of a "higher" and more substantial whole. Since everything is related in one way or another to everything else, there is in the end only one real substance, the Absolute: everything else is but an aspect of this whole.

Mill did not himself confront his idealist critics; they came later in the century. But one can see two things. First, it is clear on empiricist grounds that there are no substances and therefore especially no Absolute. Second, there are, contrary to Locke and as the idealists were to assert, genuine relational unities. Mill certainly holds the latter.

He does allow that some relations are susceptible to a Lockean analysis. And he agrees that a relation cannot be a third thing existing alongside the relata:

> Dawn and sunrise announce themselves to our consciousness by two successive sensations. Our consciousness of the succession of these sensations is not a third sensation or feeling added to these...

Mill agrees with the idealists on this point. However, there is a unity to these that cannot be further analyzed.

> To have two feelings at all, implies having them either successively, or simultaneously. Sensations, or other feelings, being given, succession and simultaneousness are the two conditions, to the alternative of which they are subjected by the nature of our faculties, and no one has been able, or needs expect, to analyse the matter any further.[13]

[13] *System of Logic*, I, iii, § 10, *CW*, vol. 7, p. 69.

Resemblance, too, is a basic, unanalyzable feature of things and of properties.

Mill is here arguing, in effect, that there are certain facts in the world that cannot be represented by propositions consisting of a single subject and a non-relational predicate. "In the same manner, as a quality is an attribute grounded in the fact that a certain sensation or sensations are produced in us by an object, as an attribute grounded on some fact into which the object, jointly with another object, is a relation between it and that other object."[14] Mill's point is ontological: there are relational facts in which the relation is an attribute jointly of two objects. But he does not trace out, or even really notice, the logical implications, that besides sentences of the form

This is F

logic must include sentences of the form

This is R to that

The traditional logic does not admit sentences of the latter sort. It was de Morgan who drew attention to the fact that the traditional logic could not account for the validity of the inference

$$\frac{\text{A horse is an animal}}{\text{A head of a horse is a head of an animal}}$$

The relational predicate "x is of y" adds something with which logic ought to deal — after all, it ought to be able to show why this argument is valid (which it is) — but which has no place in the traditional logic — nor in the formal logic that Mill allowed. Mill had an insight, an ontological insight, that had implications for formal logic, but failed to develop those implications it had for logic, and it was instead de Morgan who was to work out a newer logic that admitted of dyadic as well as monadic predicates. De Morgan's work, along with that of Boole, opened up logic into the broad science that it has become, a science within which the formal logic of Aristotle and Mill is but a fragment. William James and Bertrand Russell were to develop the ontological insight.

6 MILL'S DEFENCE OF DEDUCTIVE LOGIC

Mill accepted what was taken to be a devastating criticism of syllogistic:

> It must be granted that in every syllogism, considered as an argument to prove its conclusion there is a *petitio principi*. When we say,
>
> All men are mortal,
> Socrates is a man,
> therefore
> Socrates is mortal;

[14] *System of Logic*, I, iii, § 10, *CW*, vol. 7, p. 68.

> it is unanswerably urged by the adversaries of syllogistic theory, that the proposition, Socrates is mortal, is presupposed in the more general assumption, All men are mortal.[15]

If the conclusion is already contained in the premises, then deductive inference cannot be a means to *proving* the conclusion; it cannot be productive of new knowledge. To be sure, as Whately emphasized, the conclusion might well be psychologically new, quite unanticipated, but that does not meet the objection, since, if it is true that the conclusion is contained in the premises, then in fact we did know that conclusion, it was just that we did not know that we knew it The better answer, Mill proposes, can be found if we reflect on the principle that "All inference is from particulars to particulars," and "General propositions are merely registers of such inferences already made, and short formulae for making more."[16] It follows that there is a sense in which syllogistic reasoning cannot really be inference. And not being inferences they cannot be expected to inferentially yield new knowledge.

These arguments have often been attacked. But they are in fact straightforward.

First, Mill is making the point that there are no general facts in addition to individual facts. Those in the Aristotelian tradition held that there were general facts over and above individual facts. These were the necessary truths about substantial forms. Mill of course denies that there are such forms and such facts. It might be noted that Russell was later to argue, against Wittgenstein, that there are general facts in addition to individual or atomic facts. Mill is here taking the side of Wittgenstein. But where Wittgenstein argued that a statement of general fact was nothing more than a conjunction of atomic facts, Mill argues that there is something more. If a general fact were a mere conjunction it would be nothing more than a conjunction of statements of facts already known. Generalization is an inference from what we do know to what we can anticipate. From the mortality of Socrates and all other humans hitherto observed we infer the mortality of those now living and those who will live and who have lived, and this inference to these other facts we summarize as it were in the statement of general fact. This statement records not another fact, a general fact, that exists in addition to the individual facts, but simply that the pattern hitherto observed will continue.

What generalization does is *summarize and then anticipate* experience. The rules of syllogistic are rules, not for generating new knowledge, but for interpreting what is recorded in those generalizations. In a syllogism like

All humans are mortal
George Bush is human
―――――――――――
George Bush is mortal

the evidence for inferring the mortality of Bush from his humanity is the set of individual facts in which we have observed mortality to always accompany

[15] *System of Logic*, II, iii, § 2, *CW*, vol. 7, p. 184.
[16] *System of Logic*, II, iii, § 3, *CW*, vol. 7, p. 186.

humanity. The major premise does not add to this evidence, nor does the syllogism record any new data. Rather, their point is to *keep one consistent* in the inferences one draws from the evidence we have, that is, our observations of what has already occurred.

Syllogisms and the rules of formal logic do not produce new truth. Insofar as logic has truth as its end, formal logic has an auxiliary role, that of keeping our inferences consistent. Formal logic is a, or the, *Logic of Consistency*; it is *not* a *Logic of Truth*. Inference begins with observations, and if that inference is warranted then it can be generalized, and is to be presented in that form. Its warrantability is then tested by drawing out its implications that can be tested against further experience. The drawing out of these implications within the broader context of the whole Logic of Truth is the task of formal logic as the Logic of Consistency. What this logic of consistency presupposes are the premises from which it draws its inferences and these are supplied not by formal logic, not by the rules for the syllogism, (nor by rational intuition,) but by inductive inferences. It is inductive inference that is central to the pursuit of matter-of-fact truth: the Logic of Truth is the logic of inductive inference.

7 MILL ON NECESSARY TRUTH, ARITHMETIC ESPECIALLY

To treat a generalization as a sort of conjunction of statements of individual fact, as Mill does, that is, to treat a generalization as a mere contingent truth, is, many argue, to miss the element of unity that must be there if the generalization is to be genuinely explanatory. The later idealists were to make this point, which was of course made earlier in the history of philosophy, by Aristotle against the Megarics, who argued the thesis of the empiricists and of Molière, that a potentiality is the same as its exercise, that is, who argued contrary to Aristotle that were no active powers the exercise of which produces the succeeding event from its predecessor, tying the former necessarily to the latter. Mill of course sides with the Megarics and with Hume in denying that there are any active potentialities or substantial forms and more generally in denying that there are any necessary connections.

There are four cases that we should consider. First there are the so-called essential truths, the real definitions of things. Second, there are the truths of geometry. Third, there are the truths of arithmetic. Fourth, there are the axioms of sciences such as mechanics.

William Whewell, the Master of Trinity College, champion of unreformed universities, and intuitionist in ethics, also championed the idea that science aims at *a priori* truth; he was Mill's opposition on these points. He was followed by the British idealists who championed the same points, though from a slightly different perspective. We shall later examine the response to Mill of the British idealists. The work of Bertrand Russell and G. E. Moore will help us consider what Mill might have said in response to the idealists.

One: Essential Truths

Traditionally, and this tradition survived through to Whately's text, besides nominal definitions there are also definitions of things, so called "real definitions", that explicated the essences of things. The logical structure of the essence or substantial form is reflected it its definition. This definition is said to be "real" because it can be true or false depending on whether it accurately or wrongly reflects that ontological structure. Now, the doctrine of real definitions makes sense only if substantial forms exist. But once they are exorcised through the empiricist critique, there is no longer any reality the logical structure of which is supposedly given by the real definition. All definitions are nominal. The definition of "human" as a "rational animal" is not something that is true or false, it is simply a convention about how to use words: "human" is short for "rational animal."

But we do choose our words for use. The point about "rational animal" is that

$$\text{There are rational animals} \tag{12}$$

and it is convenient to speak more briefly, so we use the concept introduced by the nominal definition

$$\text{‘human’ is short for ‘rational animal’} \tag{13}$$

and re-phrase (12) as

$$\text{There are humans} \tag{14}$$

The general proposition that

$$\text{Human is rational animal} \tag{15}$$

is merely verbal, simply a statement of identity made true by the linguistic convention (13). (15) is not a statement of fact, it is true by convention; that convention is arbitrary — we may define words as we please. But not every definition will please us — not every definition will serve the ends for which we use language, namely, in this context, the statement of facts. (12) is a true statement of fact, and we find it useful to record that facts and facts similar to it using the simple form established by the definition (13). Since definitions are arbitrary, we could equally well define the concept

$$\text{‘Stoogle’ is short for ‘rational stone’} \tag{16}$$

But there are no rational stones, there are no stoogles, and so the concept defined by (16) is of little use as we go about the task of describing the world. We may define concepts as we please, but the concept "stoogle" defined by (16) does not please us: the world being the way it as a matter of fact is, the concept of a "stoogle" is of no use whatsoever in the task of describing the way things stand in the world as we find it.

Mill thus argues that, although there are no real definitions of things, and that all definitions are nominal, there is associated as it were with the definition a statement of fact that establishes the utility of the concept defined in the task of describing the world, a statement of fact to the effect that there are entities falling under the concept defined.[17]

Two: Geometry

Geometry has been a stronghold for the idea that we have *a priori* knowledge of necessary truths about the structure of the world. Plato argues in the *Phaedo* that the geometrical figures of which we are aware in the world of sense experience are imperfect — there are no perfectly straight lines, for example, in our ordinary experience of things. But to judge something is not perfectly straight it is necessary to know what it is for something to be perfectly straight: to judge that something is not-F one must know what it is for something to be F. Since, as we know, we do not obtain the concept of perfect straightness *a posteriori*, we must have that idea through some non-sensory *a priori* means, generally understood to be the intuition of the form of a perfectly straight line. The definitions and axioms of geometry are taken to be exactly true of these forms, and, since the forms are non-sensory or ideal, these truths are necessary. As for applied geometry, this is not exactly true since the world of sense only imperfectly imitates the world of the forms.

Whewell defended this view of geometrical knowledge — it consisted, he argued, of demonstrable truths derived from axioms and definitions as necessary truths. Mill challenged this point of view. There are two points to his critique.

First he challenged the argument that the concepts that form the definitions of geometry, e.g., of a perfectly straight line, could not be derived from sense experience. The Platonic argument takes it to be (what is so) that one cannot judge something to be not-F unless one has the concept of F — one cannot judge a line to be imperfectly straight, that is, not perfectly straight, unless one has the concept of perfect straightness. Mill argues that the concepts of the perfect geometrical figures can, contrary to Whewell and Plato, be derived from things as we sensibly experience them. Consider smooth lines. These come curved, these resemble others of that kind, but some resemble each other in being more curved than others, that is, lines as we experience them have different degrees of curvature. The concept of the perfectly straight line is the one that is negative, it is the concept of a line with *no* degree of curvature. One can similarly form from experience other geometrical concepts, e.g, that of a line with no thickness. And from these one can form the concepts defined in geometry, e.g., the concept of a triangle as a plane figure bounded by three straight lines each perfectly thin. The entities dealt with in pure geometry, the perfect figures, are thus merely

[17]As the Kneales put it, Mill expressed the point that so-called real definitions are "only nominal definitions accompanied by assumptions of matters of fact" in a "downright fashion"; and his influence on this point has been great: "After Mill's book little was heard of the doctrine of real definition." (William Kneale and Martha Kneale, *The Development of Logic* [Oxford: Oxford University Press, 1962], pp. 373-374).

hypothetical and not real. There is no need to suppose that there is beyond the world as we ordinarily experience it a world of ideal or perfect geometrical forms.

Mill's second point is this. Whewell also argued that the truth of geometry had a necessity that could not be obtained by matter of fact truths: unlike matter of fact truths, those of geometry are necessary because their contraries are inconceivable, and our knowledge of them must therefore be *a priori*. Mill agreed that the axioms of geometry is such that their contraries are inconceivable, but denied that Whewell's conclusion follows: to the contrary, the inconceivability of the contrary is compatible with the claim that, insofar as they are true, our knowledge of their truth is derived from experience. To be sure, our knowledge is not derived from something like induction by simple enumeration, nor is their certainty like that of our knowledge that all humans are mortal. It is rather something learned in our earliest experience of the world, and it is reinforced constantly as we interact as incarnate entities with other material things in the world. So profound is our experience of the world as spatial that it is indeed true, as Whewell claims, that the certainty of the axioms of geometry is such that their contraries are inconceivable. But that is compatible with their being ("mere") matter of fact truths.

On the issues concerned with geometry, it is generally now accepted that Mill's positions were right and those of Whewell (and Kant) were wrong: geometry is not a demonstrative *a priori* science. The existence of non-Euclidean geometries showed that the contraries of the Euclidean axioms could conceivably be true and the Euclidean could conceivably be false, and the discovery that the large scale geometry of the world is, as Einstein argued, non-Euclidean showed that the geometry of Euclid, far from being necessary, was in fact false of the real world of ordinary experience: insofar as it is true, it is true only approximately, as Mill claimed, and only as a matter of fact. There is no need to suppose those truths to be necessary nor to suppose that there is a realm of perfect forms to guarantee that necessity.

Three: Arithmetic

Mill argued that the truths of arithmetic were also, like those of geometry, matter of fact. In this hiss views have not generally been accepted.

We may contrast Mill's views with those of Russell.

Russell argued that the truths of arithmetic are deducible from those of logic, and therefore were, like the truths of logic, true necessarily and *a priori* of the world. To establish this he had first to define the concepts of arithmetic in terms of those of logic, and then deduce the axioms of arithmetic from those of logic. Both of these were quite beyond Mill's logic. Yet at times Mill seems to be on the right track.

Thus, Mill argues that numbers are properties of classes. He argues that all classes that resemble the class of Apostles in number have the property of being a 12-class — which is close to Russell's definition of a number as the class of all classes equinumerous to some given class, where equinumerosity is defined (in a non-question-begging way) in terms of there being a one-relationship between the members of the two classes. Mill argued that we know by our sensible experience of the world that two classes resemble each other in being two-ed or in being twelve-ed. Mill also thought of addition much as Russell did: the number a is the sum of the numbers b and c just in case that it is the number of a class which is the union of two disjoint classes, the number of one of which is b and the number of the other is c. But Mill has no way to deal with the concept of "immediate successor" which Russell needs, nor any means to state such necessary premises as "immediate successor is asymmetric", since the concept is essentially relational and Mill has no place in his logic for a logic of relations: it was only with the work of later logicians such as de Morgan and Peirce, and, indeed, Russell himself, that there was any adequate logic of relations.

It is the status of arithmetical truth, however, that we should look at. Mill argues that the laws of arithmetic are empirical, and *a posteriori*, like the laws of geometry. Here he is taking issue, as usual, with Whewell and Plato, and with Russell. The latter argued like the former, that the entities arithmetic talked about were non-empirical, and that the certainty of mathematics derived from this fact. Russell accepted a sort of Platonism, with individuals or particulars known by sense and with general forms or universals known by conception or rational intuition. Arithmetical and other mathematical truths are truths about structural relations among universals, and are known *a priori*. The laws of arithmetic, and also the laws of logic, from which, Russell argues, the former can be derived, are highly general facts about the world, necessarily true because they are grounded in the structures of non-empirical universals.

This Platonism has deep problems. Suppose individual a falls under the concept F. Then it is a fact that a is F. The problem is this: If particulars are known by sense perception and universals by conception, then how does one know that a is F, the fact which involves both the individual and the universal? Moreover, as we have seen, there is the issue about what guarantees that a relation among universals is going to be replicated as it were among particular things in the world of ordinary sensible experience. Those who admitted substantial forms solved this problem by having the active form cause its structure to present in the individuals of which it was the form. That is why, as Mill, saw, the *dictum de omni et nullo* had for them a logical and ontological significance. But disallow the causal activity of the forms, as Russell does, then there is no answer. Mill is correct, the solution is to abolish the forms too: exclude non-empirical entities and make the laws regularities about ordinary matters of fact. Russell generates the problem because he assumes that individual things are known by sense, and the properties of things and their properties are known in another way, by conception. But why assume that? As Mill makes clear, not only individual things, e.g., Jones, are given in our sensible experience of the world, but so are their properties, e.g., when Jones is green, then that fact is known by sense, and that fact includes the property green. Indeed, not only are properties given in sense but so are properties of properties, e.g., the fact that green is a colour we know from our sensible experience of things in the world, and the property of being a colour is a property of the property, also given in experience, of being green. Or, as Mill might put it, the property of being green resembles other properties like red in being a colour, and the fact of resemblance in respect of being a colour is one given to us in our ordinary sensible experience of the world. The point is to eliminate the gap between the world of things and the world of properties or concepts. So Russell's very general laws about abstract objects become very general laws about classes as classes and as entities located ontologically in the world given in sense experience. Laws of number then have a status of very abstract exact generalizations holding of the world of ordinary experience.

If therefore we make Russell's ontology more of a piece with that of Mill, then Mill's account of arithmetic as an exact science has a certain familiarity and even plausibility. But still, his logic is far from that of Russell and entirely inadequate to the task of making clear the logical structure of the laws of arithmetic.

It is worth noting that Mill, like Russell, distinguishes clearly the laws of arithmetic as purely about numbers from the laws of measurement. He insists, rightly, that there is a difference between "$1+2=3$" and "1 litre + 2 litres = 3 litres." In contrast to the exactitude of the laws of pure number, those of applied arithmetic are inexact. But that is as far as Mill goes, or can go. With no logic of relations he cannot recognize that one must, as one says, operationalize or empirically interpret, relations such as "equal in weight" and "less than in weight". Nor, therefore can he see how it is necessary to distinguish the arithmetic "+" of pure arithmetic from "sum" as applied to weights, or whatever one is measuring (or, indeed, distinguishing either of these from the vector addition of forces in mechanics, nor, worse, any of these from mere conjunction, the 'and' of the logicians). The closest Mill can get to a reasonable account of measurement is a relatively vague conception about operationalizing different units of measurement. That is part of what is involved in developing empirical scales of measurement, but far indeed from the whole story.

Mill's account of arithmetic and of applied arithmetic is not as implausible as it is often made out to be; taking into consideration differences in ontology, Mill's view of arithmetic as a set of very general laws about classes as such is not far from that of Russell. At the same time, however, it must be emphasized that, owing to the inadequacies of his formal logic, Mill's is far from the last word and far from the best.

Four: Axioms of Scientific Theories

Whewell developed the view that the axioms of such sciences as mechanics are first known inductively, but then come to be known *a priori* as necessary truths. He suggests that for any science, or any genuine science, one that can be arranged into a system deductively organized into axioms and theorems, the axioms are necessary truths, the contraries of which are inconceivable. Our knowledge of them is in fact innate, though we are not initially conscious of either the concepts or the axioms or their necessity. Only after experience is organized by primitive inductions do we become aware of the ideas within us that provide a conceptual structure for these facts that reveals that those generalizations are rooted in the necessity of that conceptual structure.

Mill objects that the conceptual structure is not provided by the mind, but discovered to be objectively there in the things observed. Whewell's is in fact the old doctrine of forms: the forms in our minds as ideas are the forms in God's mind and things created have within them the necessary structure which they exemplify in the ways they sensibly appear to us. The forms in things are the locus of the power God has put in things to produce appearances in not only a regular but a necessary order. As we pursue our study of that order we gradually awake within ourselves that idea which is identically that order in things — and in the mind of God who created things to have that order. The forms of things that are provided by the mind are forms in things and ultimately forms in the mind of God. The mind may contribute the ideas to the things observed, and in so contributing it contributes something of its own structure to the things of the world. But that structure which it contributes is also the structure God put there in the things. So in contributing its own structure it is in a way finding itself as objectively there in things.

But in the sense required, there are no objective forms. God may play a role in Whewell's account of inductive science and the necessity of the axioms that ultimately becomes present to us as we explore the observable patterns of things, but His presence doesn't really help to render secure Whewell's account. With the absence of forms, however, all Whewell's account of science amounts to is two claims, one that the structure of things as described by the axioms is put there by the mind of the observer or, perhaps, by the mind of the scientific community — which is false, the structure is not contributed by the mind but is there objectively in things, to be discovered by the inquiring mind; and second that the contraries of the axioms are inconceivable — but Whewell here proves no more than can be accounted for by appeal to the strength of psychological associations.

The axioms are, in short, inductive generalizations, but with the evidence such that they become certain enough that they can no longer be counted as "mere" generalizations. As for the concepts we use to express these generalizations, they are formed by the usual methods of definition. In fact, far from being innate, they often find their final form only after the discovery of the laws they are used to express. For, we form concepts to express the laws we discover, and so naturally enough we form accurate concepts of the subject matter of a science only when we have laid the lawful structure out before ourselves.

8 INDUCTIVE LOGIC

One: The Methods of Eliminative Induction

Inductive inference seems at first glance to be radically different from deductive logic: after all, the former is ampliative and fallible where the latter implies only what is already contained in the premises and is necessary. But in Mill's inductive logic, inductive inferences are inter-twined in complex ways with deductive inferences — many of which Mill does not himself see, owing to the limitations of his deductive logic.

The standard notion of inductive inference is the model

$$\begin{array}{l} \text{That } F \text{ is } G \\ \text{That other } F \text{ is } G \\ \text{Yet another } F \text{ is } G \\ \text{All observed } F \text{ are } G \\ \hline \text{so, All } F \text{ are } G \end{array} \tag{17}$$

This is "induction by simple enumeration." Mill, like Bacon and Hume before him, rejected this as too fallible to be relied upon as a valid mode of inductive inference — except, Mill allows, in certain restricted, but important cases. The problem with inferences like (17) is that they don't take account of alternative possibilities — maybe it is but an accident that the observed F's are G, and that really all F's are G' only we have not taken this into account. Since this and other possible alternatives have not been ruled out, the inference (17) is hazardous indeed: just think of the inference of Europeans to all swans are white, which they took to be as certain as anything, ignoring the possibility of black swans (which were in fact later observed by Europeans when some Dutch sailors in the seventeenth century sailing for the East Indies around the Cape of Good Hope were blown off course and landed in what is now Western Australia).

Mill proposed, following Bacon and Hume, that inductive inference, if it was to be certain, had to run through the possible cases and eliminate all but one, what remained would then be certain.

Mill laid out several modes of "eliminative" inductive inferences. The "method of difference" was designed to discover causes in the sense of what are now called "sufficient conditions" — in (17), the conclusion states that F is a sufficient condition for G.[18]

The method takes for granted that there are several possible sufficient conditions for G — let us say F, F' and F''. The task is to eliminate all possibilities but one. We do controlled experiments in the laboratory or rely upon natural experiments

[18] The "method of agreement" is designed for the discovery of necessary conditions. Mill is not fully clear on the distinction between necessary conditions and sufficient conditions, and as a consequence he unnecessarily qualifies the use of the method of agreement.

There are three other eliminative methods described by Mill: all of them follow the basic structure of the method of difference that we are here displaying.

found in nature. One experiment might produce, for example, an instance which was F'' but not G. That would show that the hypothesis that

$$\text{All } F'' \text{ are } G$$

is false, and eliminate F'' from the list of possible causes. In effect — though Mill does not it this way — we have, let us say, three possible cases for the condition which is sufficient for G. These are the hypotheses

(H1) All F are G

(H2) All F' are G

(H3) All F'' are G

Let us suppose we eliminate (H2) and (H3). Mill concludes that (H1) is the true generalization. It is inductive, because it goes beyond the observational data, but it is more secure than any hypothesis arrived at by induction by simple enumeration, since possible competitors have been eliminated. This is Mill's method of "eliminative induction."

Mill's conclusion follows by the rule for disjunctive syllogism:

(D)

(H1) or (H2) or (H3)
Not (H3)
Not (H2)
so, (H1)

This is an inference in deductive logic. So Mill's method of eliminative induction involves not only an inductive but also a deductive step. But Mill does not recognize this as the logical structure of his inference. This is because he does not understand disjunctions, nor the pattern of inference of the disjunctive syllogism.

The disjunctive syllogism (D) is valid, but it will not establish its conclusion as true unless its premises are true. The two minor premises are established by observational data that are counterexamples to the generalization which constitute the alternative hypotheses. What about the major premise?

It might be false, in which case the inference to (H1) is unsound and Mill would be unable to conclude that (H1) is true.

Now, the major premise of (D) might be false because none of the disjuncts is true. So Mill must assume in any inference of this sort that *at least one of the alternative hypotheses is true.* That is, he must assume (for the method of difference for the discovery of sufficient conditions) that *there is a condition the presence of which is everywhere sufficient for the presence of the conditional property*, which in our example is G. In Mill's terminology, he must assume that *there is a property the presence of which is the cause for the presence of the conditioned property.*. This has been called a "Principle of Determinism." It is evident that this Principle is a matter of fact generalization, and therefore, if true, then only contingently true. Mill recognizes the need for such a supplementary premise if his conclusion is to be sound. (Bacon is less clear on the need for this premise; Hume is quite clear that it is needed.) And Mill also recognizes that this supplementary premise, if true, is only contingently true. But he does not really see that it is the logic of the disjunctive syllogism that requires him to assume this extra premise.

It might also be the case that the major premise of (D) is false because all the disjuncts are false but that there is an alternative that has not been included: the list of possible conditioning properties should have included F''' as well as the ones actually considered. That would mean that the hypothesis

(H4) All F''' are G

should be among those considered. In that case we would have

(D′)
(H1) or (H2) or (H3) or (H4)
Not (H3)
Not (H2)
———————————
so, (H1)

But in that case (D′) is not valid and Mill has not established his conclusion that (H1) is true – not all the competitors have been eliminated. If Mill is to conclude that the uneliminated hypothesis (H1) is true because he has eliminated (H2) and (H3), then he must also assume that he has a *complete set of possible conditioning properties*. The assumption that he has such a set has been called a "Principle of Limited Variety." It is clear that such a principle is, like the Principle of Determinism, a matter of fact truth, if it is true, and is only contingently true. Mill is less than pellucid on the need for this supplementary premise. (Bacon is quite clear on the need for this premise; Hume, like Mill, is less clear.)

The structure can be more formally put this way. Let the genus **F** be exhausted by the species F, F', and F''. That is, let it be true that

$$F \text{ is } \mathbf{F},\ F' \text{ is } \mathbf{F},\ F'' \text{ is } \mathbf{F}, \text{ nothing else is } \mathbf{F},$$

so that

$$\mathbf{F} = F \text{ or } F' \text{ or } F''$$

Then the required Principles of Determinism and Limited Variety can be stated as

(L) There is a species f of genus **F** such that all f are G

Mill has sometimes been criticized for not recognizing the need for this premise, and in particular the need for the Principle of Limited Variety. There is a point to this charge, but a charitable reading will make allowance for his vagueness. It was not until the Cambridge philosophers W. E. Johnson, J. M. Keynes and C. D. Broad in the twentieth century and in the light of Russell's full development of formal logic that these matters were finally clarified. It should be allowed, then, that Mill was aware of the need for a premise like (L). Now, given Mill's point that there are no necessary connections, that all properties are logically and ontologically self-contained, then it is clear that this premise, which evidently is a generalization, is, if true, only true contingently and as a matter of fact. It follows that the conclusion that Mill draws from the use of the mechanisms of elimination — in our little example, the conclusion that (H1) is true — is only as certain as this premise (L). The issue is, what evidence could there be for this law?

Mill suggests that background theory can provide the required evidence. (Ultimately, the broadest of these theories is the Law of Universal Causation.) Let F be part of a broader genus $\mathbf{F}$, and let $\mathbf{G}$ be a genus under which the species G falls. Then we might well have a theory like this

(T) For any species g of genus $\mathbf{G}$, there is a species f of genus $\mathbf{F}$ such that all f are g

Knowing that

$$G \text{ is } \mathbf{G}$$

we can deduce (L) from (T). But this only puts the question back a step, what evidence do we have that (T) is true? Since (T) is a generalization which is, if true, only contingently true, (L) can only be as certain as (T), and therefore Mill's conclusion from the process of elimination that (H1) is true can only be as certain as (T). What evidence do we have for theories like (T)?

[As an aside, a remark on Popper is in order. It will likely be recalled that Popper claimed originality in introducing the idea that science progresses through falsification of hypotheses, where hypotheses to be tested are arrived at through guesswork. Popper was hardly correct in his suggestion that he was the first to recognize the importance of falsification. Mill preceded Popper in emphasizing the role of falsification as part of the logic of induction. And, as Mill acknowledged, it was Bacon who introduced this notion. But Popper would have it that the formation of hypotheses has no logic, it is guesswork. For him, the teleology of science is more or less random; at best, it is guided by more or less vague metaphysics or "point of view." For Mill (and Bacon), it is otherwise; they argue that there is a logic to hypothesis formation. Specifically, they notice that research is theory-guided, that is, they notice the importance of abstract generic theories in guiding the formation and selection of hypotheses to be tested.]

Two: Consilience and Colligation

Let us suppose we have discovered several specific laws

(L1) All H_1 are K_1

(L2) All H_2 are K_2

(L3) All H_3 are K_3

We might, for example, suppose that these are Galileo's Law of Falling Bodies, Galileo's Law of Projectile Motion, and Huygen's Law of the Pendulum. Now let us suppose we notice that they in fact share a generic form. Suppose each

$$H_i \text{is } \mathbf{F}$$

Then we have from (L1)–(L3) that each of the causes K_1–K_3 have **F**-ish causes

(L11) There is a species f of genus **F** such that all f are K_1

(L22) There is a species f of genus **F** such that all f are K_2

(L33) There is a species f of genus **F** such that all f are K_3

Each of (L11)–(L33) share a common generic logical form

(f) There is a species f of genus **F** such that all f are g

where "g" is (as we would say, though Mill lacks the logical sophistication to put it this way) a free variable. Then we further notice that each

$$K_i \text{ is } \mathbf{G}$$

which enable us to generalize over all species of genus **G** to obtain our theory (T)

For any species g of genus **G**,
There is a species f of genus **F** such that all f are g

Then, we know already that

$$G \text{ is } \mathbf{G}$$

and deduce that (L)

There is a species f of genus **F** such that all f are G

which provides the Principles of Determinism and Limited Variety that delimit our range of interest to the hypotheses (H1)–(H3) which we are putting to the experimental test.

We arrive at the generalization (T) by generalizing over all the species of a genus. The evidence which has previously confirmed (L1)–(L3) confirms (T), that is, supports the background theory that justifies the Principles of Determinism and Limited Variety that we need to make the eliminative inferences work.

If (L1)–(L3) are such laws as Galileo's Law of Falling Bodies, his Law of Projectile Motion, and Huygen's Law, then we can think of Newton generalizing from these specific laws to the Law of Inertia, the first of his axioms. This axiom has the logical form of a generic theory like (T) — "For any species f of mechanical system there is a force function g such that g is sufficient for accelerations in f." Newton, as a matter of historical fact, records in his *Principia* just this sort of inference from specific laws such as Galileo's Law of Falling Bodies to the generic-level Law of Inertia; the specific-level laws are taken to justify the acceptance of the generic-level law. Mill discusses this sort of inference in the case of one of the central discoveries of the nineteenth century, the Law of Conservation of Energy, which he takes to be a generic-level generalization made from specific-level generalizations about heat energy, mechanical energy, chemical energy, and so on.

We arrive at (T) from (L1)–(L3) when we recognize that the three specific laws share not just the more abstract generic form (f) but in particular the generic logical form

($f\wedge$) a species g of genus $\mathbf{G}$ is such that there is a species f of genus $\mathbf{F}$ such that all f are g

(Again, "g" is a free variable.) Whewell argued that in order to generalize to a theoretical law from simple inductions one had as part of the inference process to "colligate" the facts, that is, discover a concept that is common to the facts in question, and which permits of generalization. As Whewell would have it, this concept is a form that is as it were imposed on the facts. Mill recognizes the need for such a concept. He denies, however, that we impose it on the data observed; rather, the fact that the data fall under the concept is objectively there, discovered by the investigator. Moreover, to locate the concept that applies is not an inference. It is, rather, the recognition of a pattern among the things already known. In terms of our simple example, it is the recognition that the patterns (L1)–(L3) have in common the generic form ($f\wedge$). The identification of this common form colligates these facts, organizes them so that they can be generalized into a fruitful theory. Generalizing, we obtain the theory (T).

Having obtained the theory (T),

> For any species g of genus $\mathbf{G}$, there is a species f of genus $\mathbf{F}$ such that all f are g

we now apply it to areas as yet unexplored, that is, new specific sorts of systems falling under the genus $\mathbf{G}$. We know that

> G is $\mathbf{G}$

and deduce that (L)

> There is a species f of genus $\mathbf{F}$ such that all f are G

which provides the Principles of Determinism and Limited Variety that delimit our range of interest to the hypotheses (H1)–(H3) which we are putting to the experimental test. So, for example, Newton takes his axiom, the Law of Inertia, and extends it to systems not previously explored, e.g,, the planetary system, or sun-comet systems. He deduces that for each of these systems there is a force function that describes accelerations, and therefore determines the motions in the system. The research task is to find the force function that we know, from the theory, is there to be discovered. The theory tells us that this specific law will have a certain generic form, which means we can limit ourselves to searching for a law of this form. Similarly, the Law of Conservation of Energy, once formulated, can be applied to new specific sorts of system, in the search for the specific laws that the theory tells us are there to be discovered and which we know to have a certain generic form.

So, an abstract generic theory like (T) can provide a guide to research in new specific areas, as yet unexplored. It predicts the existence of laws. These laws will have a certain generic form. Experiment will decide among these possible cases, which one is true. Confirmation of this law by successful prediction will confirm — or rather, further confirm — the more general theory. This successful prediction of laws in new areas is the mark of a good theory, Whewell argued. Mill agrees with him: there is nothing in his empiricist account of theories that would count against such a position.

Now notice how the patterns of confirmation go. We have the specific laws (L1)–(L3). These are confirmed through successful prediction. The data that confirm these laws also confirm the generic theory (T) which generalizes from them and from which they can be inferred (assuming the appropriate genus-species relations). This theory is them applied to a new area and yields (H1)–(H3) as possible hypotheses. Each of these derives partial confirmation from (T) — we know that one of them must, given (T), be true, and that other possibilities lack such confirmation and can therefore be excluded. Then, experiment falsifies (H2) and (H3), leaving (H1) alone as acceptable: it has passed the test of experiment and has successfully been confirmed. This confirmation in turn confirms the theory (T) which led to its prediction. (T) is more strongly confirmed, given the successful confirmation of (H1). But if (T) is more strongly confirmed, then so are the specific laws (L1)–(L3) in the different specific areas wherein theory (T) was originally formulated.

This sort of confirmation, by which confirmations in different areas support one another and support the abstract generic theory which binds them together, was called by Whewell the "consilience" of inductions:

> The Consilience of Inductions *takes [lace when an induction, obtained from one class of facts, coincides with an Induction, obtained from another different class. This consilience is a test of the truth of the Theory in which it occurs.*[19]

Consilience was argued by Whewell to be a central feature of inductive inference and support. Mill agrees, but argues, in the way in which we have just outlined, that there is nothing contrary to empiricist principles in recognizing this as a central aspect of inductive support. Whewell explained that "the evidence in favour of our induction is of a much higher and more forcible character when it enables us to explain and determine [i.e., predict] cases of a kind different from those which

[19] W. Whewell, *Novum Organon Renovatum* (London: Parker and Son, 1858), ch. v, Aphorism xiv.

were contemplated in the formation of our hypothesis."[20] Consilience is achieved when one subsumes a specific kind under a more generic kind, where there are other different specific kinds under that genus; there are specific generalizations, but an induction over these to all species within the genus. Whewell claimed that "when the theory ... has included a new range of phenomena, we have, in fact, a new induction of a more general kind, to which the inductions formerly obtained are subordinate, as particular cases to a general population."[21] As Mill put it, the theory is a "law about laws"; in the most general case, the Law of Universal Causation, the theory states that "it is a law, that every event depends on some law" or "it is a law, that there is a law for everything."[22] Whewell's way of stating it is a bit murkier than Mill's but the point is clear: like Mill, Whewell is arguing that generic laws about laws play a central role in science. Such a law, generalized from several specific laws to all species of a genus, permits the prediction of new laws for as yet uninvestigated areas. It determines a Principle of Determinism and a Principle of Limited Variety for some new area, delimiting the range and form of hypotheses that are reasonable candidates to be tested. Each of these will have a prior probability that other possible candidates lack. Experiments then eliminate all of the candidates but one. These data confirm this uneliminated hypothesis. These same data also confirm the theory. This strengthens the support for the theory. With the theory more strongly supported, then the other specific laws subsumed under it, including the specific laws from which it was generalized, themselves receive additional support. The theory provides the linkage that allows inductive support for laws in one specific area to also support specific laws in another area, and to allow the inductive support for the laws in the second area to provide support for the laws in the first area. This is the consilience of inductions. Whewell describes this, but Mill, with his account of the role of the casual principle in confirmation shows why this should be so, why consilience should be so important in the logic of inductive inference.

The crucial point is the abstract generic nature of the theory that binds several specific laws together and so interlocks them that confirmation is transmitted from one specific area to another, and the confirmations of laws in the several specific areas mutually support and enhance one another. The theory itself permits the scientist to colligate the facts in a new area, to discover in the laws of the new area a logical form or concept that is shares with other specific laws in other areas and which admits of generalization which in turn permits the specific laws to be subsumed under the theory and captured and strengthened by the logical connections of that theoretical structure.

It is sometimes suggested that Whewell's account of consilience depends upon his ontology of "natural kinds", where a natural kind is a property of things that determines a "real" set rather than as a group of things arbitrarily lumped together by a person or group of people: a system of natural kinds is a classificatory system

[20]W. Whewell, *Novum Organon Renovatum*, p. 87.
[21]W. Whewell, *Novum Organon Renovatum*, p. 96.
[22]Mill, *System of Logic*, III, v, § 1, *CW*, vol. 7, p. 325.

which as it were "carves nature at its joints" and which thereby places constraints on what sorts of entities can be the subject-matter of laws of nature. So, if a generalization involves natural kinds it is a law of nature, otherwise it is a "mere" generalization. For Whewell there are such kinds. These are the kinds which stand in necessary connections to one another, and where the properties are necessarily connected then there we have a lawful generalization, and not a "mere" matter-of-fact regularity. The necessity of the connection is, according to Whewell, not immediately apparent to the researcher, but becomes so as science progresses, until the laws are fully recognized as necessary axioms. For Mill, however, kinds do not thus resolve themselves into two classes, natural kinds are artificial kinds: there are only kinds, and if they are complex then their definitions are always nominal. These kinds which "carve nature at its joints" are simply those that appear in laws, that is, in matter-of-fact regularities. Those regularities that we count as laws are simply those where we are prepared to infer that the unobserved instances will continue to be as those that have been observed. This does not yet distinguish those regularities that are the suff of superstition ("black cats are unlucky") from those that are scientific, i.e., those that are asserted on rational grounds. So we count as reasonable law-assertions those that are the conclusions of sound inductive inferences. Laws are not determined by "natural kinds"; rather, which kinds are "natural", "carving the world at its joints," are those that appear in statements of law: it is the concept of law which is primary, not the concept of kinds which are somehow more "natural" than others.

The point to be emphasized is that the notion of the confirming power of a "consilience" of inductions does not presuppose any doctrine of "natural kinds" or of essential ties or of necessary axioms. Mill's treatment of theories as forming a generic hierarchy of laws is what is important: it is the abstract generic theories that tie together laws in specific areas that allow inductive support in one area to be transmitted to another area and conversely. Consilience is easily understood in an empiricist-inductivist context, contrary to what Whewell and his followers seem to think.

It is, moreover, safe to say that Mill is more perspicuous in his handling of consilience than is Whewell. Nonetheless, it is also true that Mill does not fully grasp the issues of logical structure. The logical form of the theory (T) which Mill is trying to articulate requires a full grasp of the logic of quantifiers (which he does not have), as well as the idea of a second order logic in which one quantifiers not only over individuals but also over kinds or species (which perforce he also does not have). The limitations of his deductive logic prevent him from fully grasping the role of deductive connections in the consilience of inductions . [It is worth commenting on the frequently made claim that Mill's logic of science makes no allowance for inferences to unobserved entities of the sort physics talks about. This in spite of the fact that Mill makes specific reference to theories of heat that account for heat through the motions of molecules, that is, through the motions of hypothesized unobserved and unobservable entities. Philosophers do make mistakes in construing the positions of other philosophers, but the rule ought to be that of finding a generous reading of their logic of science which manages to allow them to talk about entities that they say they want to talk about. The reading of Mill's logic given above does allow inferences to unobserved entities. Here's how. The theory (T) permits one to deduce the law (L)

There is a species f of genus $\mathbf{F}$ such that all f are G

This mentions an unobserved species f of genus $\mathbf{F}$. It could well be that this species of system consists of minute molecules. For the kinetic theory of heat to which Mill refers, the concept of a molecule is that of a species of billiard ball — a genus which we know in experience — but of a billiard ball smaller than others — the relation of "smaller than" is given in experience — in fact sufficiently small to be in itself undetectable by our sense organs and therefore unobservable. The concept of a molecule is thus one that is acceptable within Mill's empiricist account of concept formation. The species f may therefore be that of a congeries of molecules. So (L) then deals with molecular motion as a cause of heat. But that cause is a set of unobservable entities. Nor is there any need to observe those entities to have evidence that (L) is true. For, we have deduced (L) from the background theory (T), which receives inductive support from elsewhere. Notice, too, the existential quantifier: this means that failure to observe the relevant entities will not lead to falsification. This is sketchy indeed, but clear enough to make the basic point about how infer that there are unobservable entities, and clear enough that this fits with Mill's account of how theories like (T) permit us to infer to laws which we might not be able to confirm — that is, confirm directly by observation — but which are not totally unconfirmed, since background theory (T), which is confirmed elsewhere, provides inductive support.]

Three: The Confirmation of Abstract Generic Theories and the Justification of Inductive Inference

Whewell would, of course argue and did argue that theories like (T), e.g., the axioms of classical mechanics, are true *a priori*, and that that necessity emerges and becomes clear to us only gradually as we use these theories to make inductive inferences. Mill, as we have seen, will have none of that.

What Mill argues is that we do use theories like (T) at the generic level to predict the existence of more specific laws, that is, predict the existence of true generalizations at the specific level, and also argues that the instances of the specific level generalizations that confirm those generalizations also confirm the generic hypothesis (T): the inference to the theory (T) is a sort of induction by simple elimination. Enumeration does not play a role, or rather, an increasingly less important role as one moves down the generic hierarchy of laws. At the specific level one cannot rely on such induction by simple enumeration, one must take account of, and eliminate specific alternatives. But as one moves up the generic hierarchy, the range of alternatives at the generic level becomes smaller.

To the extent that there are fewer possible alternatives, to that extent induction by simple enumeration becomes a more reliable process of inference.

At the most general level, there is the Law of Universal Causation, which states that "it is a law, that there is a law for everything." The principle that, for everything there is a law, is a law about laws, a law at the generic level about laws at a more specific level — it is a generalization like in form to our sketch of a theory (T). This generalization is arrived at by generalizing from more specific cases: "The truth is, that this great generalization is itself founded on prior generalizations."[23] The evidence for this lies in the fact that we really have, so far as we can tell, discovered causes: "The truth that every fact which has a beginning has a cause, is coextensive with human experience."[24] This means, in effect,, that we are relying upon induction by simple enumeration to justify this law on which all other inductions rest. But this is reasonable: at the generic level there are not the variations in nature which, at the specific level, render this an unsafe rule.

> And hence we are justified in the seeming inconsistency, of holding induction by simple enumeration to be good for proving this generalization [that is, the generalization that "everything which begins to exist has a cause"], the foundation of scientific induction, and yet refusing to rely on it for any of the narrower inductions.[25]

There is much more of detail that could, and indeed must be said about Mill's inductive logic, it is rich in its ideas. Like Mill's deductive logic, it has its limitations, and many of those limitations are due to the limitations of the deductive logic. But certainly, it is not as foolish as many have claimed. Equally certainly, it is less dated and less narrow than his deductive logic. It deserves a careful reading by anyone with more than a passing interest in the philosophy of science.

9 THE IDEALIST CRITIQUE OF MILL

One: Inferences in Science

Mill's logic was criticized by the British ideal philosophers later in the nineteenth century and the early twentieth century. F. H. Bradley and Bernard Bosanquet both wrote logic books that attempted to refute Mill.

They were critical of almost all aspects of Mill's philosophy, and all aspects of his logic. In their eye, Mill could do little that was correct. Among other things, they were critical of Mill's account of disjunction and of disjunctive syllogism, and therefore of the role that Mill attributed to negative instances in the scientific method. More specifically, they argued that formal logic alone could not account for the way negative instances functioned in scientific practice.

[23] *System of Logic*, III, iii, § 1, *CW*, v. 7, p. 307.
[24] *System of Logic*, III, v, § 1, *CW*, v. 7, p. 324.
[25] *System of Logic*, III, xxi, § 3, *CW*, v. 7, p. 571.

Here is how Bosanquet analyzed one case. The example is from biology. It concerns a certain plant, the Bee Ophrys; the issue is whether it is self-fertilized or fertilized through insects. It was Darwin's hypothesis that the plant is self-fertilized through the action of wind blowing the pollen masses that develop on the plant until they strike the stigma. The alternative hypothesis was insects were required for fertilization. Note that these are causal hypotheses, and they therefore, as Mill argues, imply regularities. Bosanquet does not acknowledge this point. In any case, in his experiment, Darwin, to confirm his hypothesis, isolated a spike of flowers in water in a room. In this situation there was an *absence of insects* (not-i). This ensured that fertilization by insects was impossible. It also ensured the absence of wind. In this context the pollen masses did not come in contact with the stigma. Thus, in this situation we have both the *absence of wind* (not-w) and the *absence of contact* (not-c). According to Bosanquet, Darwin concludes that his hypothesis is true, that the Bee Ophrys self-fertilizes through the action of wind.

Bosanquet states about this example that

> We have here left the ground of formal logic, in which 'not-w is not c' could only rest on knowledge that 'c is w'. In the process now considered 'c is w' actually rests on the knowledge that 'not-w is not c'. The corroborative power of the negative instance in induction depends on the fact that it has a positive content within the same *ultimate* system as c and w, and, within that system, related by way of definite negation to them.[26]

He is arguing, then, that formal logic cannot account for Darwin's inference. But Bosanquet is wrong. There is in fact a way in which formal logic can deal with the example. Indeed, it is precisely the way in which Mill dealt with such examples.

As Bosanquet understands the statement 'not-w is not c', we have

Free caddices without wind give no contact

From this he infers his conclusion that

c is w = contact is by wind

In the first place we have to recognize that, for the empiricist, the conclusion is not so much 'c is w', as Bosanquet claims, but rather that

wind produces contact

or

Whenever wind then contact

[26]B. Bosanquet, *Logic, or The Morphology of Knowledge*, 2 vols. (Oxford: Oxford University Press, 1888; second edition, 1911; reprinted, two volumes in one, New York: Kraus Reprint, 1968), vol. II, p. 126.

That is, the conclusion to be drawn is that wind is a sufficient condition for contact: this is a causal regularity. The way that Bosanquet states the matter implies that the inference proceeds by contraposition: from 'not-w is not c' we infer 'c is w'. This is a species of immediate inference. But in fact the inference, at least from the point of view of formal logic, is more complicated. Indeed, we can see that it must be more complicated. For, what is observed are particulars things, single instances, where what we want to conclude is a generalization.

Bosanquet in fact notes only one of Darwin's observations. The point is that Darwin also ensured that in the *presence of wind* (w) in the *absence of insects* (i) one has the *presence of contact* (c). He did this by placing a similar spike of Bee Ophrys in the open air in the presence of wind but covered by a net which ensured the absence of insects. Bosanquet's actual account of the inference, as going from 'not-w is not c' to 'c is w', omits this piece of information. But it is crucial for the empiricist analysis of the example. Upon the empiricist account of this inference, there are in fact two pieces of evidence: in both insects are absent while in one both wind and contact are present and in the other both wind and contact are absent. We can summarize these results in a small table, with "a" indicating absence and "p" indicating presence:

	i	w	c
event one:	a	p	p
event two:	a	a	a

And now by a simple application of the *method of difference*, we can infer that

Whenever w then c.

Or at least, we can do so provided that we have grounds for accepting both the *Principle of Determinism*, that there is at least one sufficient condition, and the *Principle of Limited Variety*, that this condition is among a certain determinate set. That is, we need to know a generic statement to the effect that

(*) There is a property f such that whenever f then c, and it is in the set consisting of w and i

This is a *law*; after all, it asserts a general truth. But it also asserts the existence of a property of a certain sort. Because this law also makes an existence claim, it includes the particular quantifier. In fact, it is of mixed quantificational form. Mill sees the need for such an additional principle if his eliminative methods are to work — though his deductive logic is, as we have seen, too primitive to allow him to articulate fully his insights; but he does see, let us emphasize, that such a "law about laws" is necessary. Moreover, again as Mill saw, since the law (*) asserts the existence of a property of a certain sort, it makes an *abstract generic* claim, rather than a specific one: it asserts that *there is* a property of a certain *generic sort* and this property is sufficient for the presence of the conditioned property c.

This law (*) guides the researcher. It asserts that there is a specific law, there to be discovered, and that this law will have a certain generic form. The task of the researcher — Darwin, in this case — is to find the law that (*) asserts to be there.

As for (*) itself, one generally can infer the truth of such an abstract generic law from some background theory of this sort:

(**) For all systems g of generic sort $\mathbf{G}$, there is a property f such that it is of generic sort $\mathbf{F}$ and such that whenever f then g.

From a law of this sort, together with the premises

$$c \text{ is of sort } \mathbf{G}$$
$$\mathbf{F} = \{i, w\}$$

one can deduce the law (*) that guides research. (**) has the same general form as our theory (T), which we talked about above. What (**) asserts is that for all systems of a certain generic sort there are for those systems specific laws of some other generic sort. The justification for accepting (**) is past experience: we have in fact been successful, so far as we can tell, when we have looked at systems of the relevant generic sort, at discovering laws of the sort that (**) would lead us to believe are there. Taken in its most generic form, (**) is of course what Mill referred to as the Law of Universal Causation.

By virtue of our accepting (**), we can infer a law of the sort (*) for the systems in which we are interested, in Darwin's case the Bee Ophrys. The law (*) tells us not only where to look but that if we do look carefully enough then we shall be successful in discovering a law of the relevant sort for those systems. Our prior knowledge of (**) and therefore of (*) thus provides a *teleology* for the research process. The aim is to discover a specific law of a certain generic sort; our knowledge deriving from the theory tells us that the goal *can* be achieved. And our logic tells us that the means towards that goal is the method of difference. The goal, then, is to be achieved through the observational data recorded in the chart of presence and absence.

The empiricist logic that Mill defends draws sharp lines between the statements of individual fact, positive and negative, that are recorded in the chart of presence and absence. These facts are ontologically and logically separable and independent of one another. This means, in particular, that these statements of individual fact do not by themselves imply the statement of law or regularity that is the conclusion of the inference. To be sure, they confirm that law, but it is not they that give it its strongest support. That support comes, rather, from the workings of the eliminative mechanisms, which serve to show that possible alternatives could not be true, that is, show that, given these observational data, none other than the conclusion could possibly be true.

Bosanquet, however, has a rather different account of the process. He considers only the event that we have labelled event two. This event he records as 'not-w is not c' from which he then infers the contrapositive 'c is w'. The latter he takes to be the *causal conclusion* of Darwin's inference. The sharp distinction that the empiricist draws between the statements of individual fact in the premises and the law that is the conclusion is no part of Bosanquet's account. Nor does Bosanquet interpret the example in terms of the eliminative mechanisms. Where the statements of absence do not for the empiricist by themselves imply any law, for Bosanquet in contrast the statement of absence becomes the statement that 'not-w is not c' and from this *alone* the *causal conclusion* that 'c is w' is inferred. Bosanquet as it were *discovers the causal claim* ***in*** the particular events themselves. Nor, I shall now argue, is this at all accidental: Bosanquet is in fact pursuing a point that many have thought to be a fatal criticism of the empiricist account of laws and of scientific inference.

Let us look at something that Bosanquet had said earlier.

Mill argued that a universal judgment

$$\text{All } A \text{ are } B$$

is a statement of individual facts

$$a \text{ is } A \& a \text{ is } B, b \text{ is } A \& b \text{ is } B, \ldots$$

As he put it,

> ... a general truth is but an aggregate of particular truths; a comprehensive expression, by which an indefinite number of individual facts are affirmed or denied at once.[27]

In fact, however, as we have seen Mill argue, it is not a *mere* conjunction: it itself is an inference: the form of language is such that it enables us to record in one statement

> ... all that we have observed, together with all that we infer from our observations....[28]

On this account, a syllogism such as

$$\begin{array}{l} \text{All humans are mortal} \\ \text{Socrates is human} \\ \hline \text{Hence, Socrates is mortal} \end{array}$$

does not provide new knowledge in the conclusion beyond what is contained in the premises. Or rather, insofar as it does provide new knowledge, it does only by inferring the facts of the minor premise and the conclusion from the observed facts that are among those facts recorded in the major premise. What the deductive connections do is guarantee that our thought is consistent; they prevent us from once inferring that all humans are mortal and then denying of the human, Socrates, that he is mortal. As Mill puts it, deductive logic in itself is a logic of consistency, not a logic of truth. It is inductive inference that constitutes the logic of truth.

[27] *System of Logic*, II, iii, § 3, *CW*, vol. 7, p. 186.
[28] *System of Logic*, II, iii, § 3, *CW*, vol. 7, p. 183.

Bosanquet takes up this point.[29] He considers the inference from particulars to particulars:

$$a \text{ is } B, b \text{ is } B, c \text{ is } B, d \text{ is } B; \text{ hence, } e \text{ is } B$$

For example, *a* is a good book, *b* is a good book, *c* is a good book, *d* is a good book, and so *e* is a good book. This, he rightly points out, is not a good inference. In contrast, however, the inference

> Ivanhoe, Waverley and Rob Roy are good books; hence, Guy Mannering is a good book

is reasonable. It is reasonable because "there is a self-evident passage by means of the identity of authorship, which is too obvious to be expressed, but which would form a premise in any explicit statement of the inference."

Bosanquet notes that

> ... it is impossible to state an inference [to particulars from particulars] in a shape that will even appear to be convincing, unless we supply by a second premise the element of unity between the particulars, always operative in the mind, which is necessary to bind the particular differences into the differences *of* a universal.[30]

Note the point: what one needs is "the element of unity between the particulars, always operative in the mind, which is necessary to bind the particular differences into the differences of a universal": there is an element of unity; this binds differences, that is, different differences, into differences within a universal; and this element of unity is operative in the mind. We shall return to these points.

In any case, accepting this, we see that the inference that is supposed to be to particulars from particulars really is

All novels written by Scott are good books
Guy Mannering is a novel written by Scott
———————————————
Hence, Guy Mannering is a good book

where the major premise is justified by the inference

(@) Ivanhoe, Waverley, and Rob Roy are novels written by Scott
Ivanhoe, Waverley and Rob Roy are good books
———————————————
Hence, All novels written by Scott are good books

So far this looks fairly close to Mill's view. In fact, however, Bosanquet qualifies the conclusion of the inference (@) in an important way. Here is one of Bosanquet's examples[31]:

a, b, c, d are rational
a, b, c, d are men
———————————————
Hence, Are all men rational? or, Men may be rational

[29] Bosanquet, *Logic*, vol. II, p. 51f.
[30] Bosanquet, *Logic*, vol. II, p. 51.
[31] Bosanquet, *Logic*, vol. II, p. 51.

The conclusion of the inference (@) is to be taken as a conjecture rather than as something demonstrated.

Bosanquet notes that there is a teleology here:

> Speaking generally, the coincidence of several attributes in one or more objects ... is the starting-point of conscious conjecture and investigation. And this starting point is all that the present form of inference embodies. Conjecture or pure 'discovery' differs only in degree from proof.[32]

Again, this does not sound very different from what Mill might say. But it turns out that the teleology is in fact very different: Mill's account of this teleology has it grounded in abstract generic theories, laws about laws, that guide research. For Bosanquet, in contrast, the teleology is, as it was with Whewell, one that aims at uncovering as it were a real objective necessity in things which is not at first apparent to us, but which becomes explicit as the process of discovery proceeds.

Bosanquet comments on (@) as follows:

> The ground of argument being the characteristic unity of the unanalysed individual object or event, naturally takes the place of the subject in judgment — of the concrete individual which is taken as real — and therefore gives rise to that syllogistic form in which the middle term is the subject of both premises.[33]

Here he again emphasizes the "unity" that one finds in the argument. Only now it is a unity that is in the "unanalysed individual object" rather than, as before, "operative in the mind." Or rather, the element of unity is at once in the subject which the inference is about and operative in the mind of the person making the inference. This view is of course similar indeed to the view of Whewell. In any case, this "unity" that Bosanquet claims we discover in the individual facts is the *ground* of the argument.

As for the notion of a ground, Bosanquet has earlier explained what this is. He has distinguished merely collective judgments from those that are genuinely universal. The former deal with an aggregate that arise from an enumeration[34]. From these are distinguished quasi-collective judgments that are truly general and apply to an indefinite number of individuals, an open class as one would now say. These cannot arise from a mere enumeration, otherwise they could not be truly general, rather than a summary of the already observed. These judgments must therefore "derive [their] meaning from some other source".[35] Among these judgments are those such as "All men are mortal," that is, the sort we have just been considering. These quasi-collective judgments are more than mere summaries of the already observed, but are nonetheless falsifiable by a single counter-example;

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[32] Bosanquet, *Logic*, vol. II, p. 52.
[33] Bosanquet, *Logic*, vol. II, p. 51.
[34] Bosanquet, *Logic*, vol. I, p. 211.
[35] Bosanquet, *Logic*, vol. I, p. 211.

as Bosanquet puts it, they are "helpless in the face of the most trivial exception".[36] Beyond these are the genuinely universal judgments such as "Man is mortal." In practical terms, they are equivalent to propositions like "All men are mortal." However,

> It is obvious that the affirmation of universal connection which we feel in such an instance to be all but warranted is not approached from the side of the individual units, but from the side of the common or continuous nature which binds them into a whole.[37]

Because this sort of universal judgment is warranted by the connection among the attributes it is abstract and therefore hypothetical.[38] The hypothetical judgment contains a consequent and a ground; the latter warrants the former. It does so by virtue of being connected to it, connected internally, in its very being.

> Ground implies a consequent other than, though fundamentally one with, itself.[39]

The two are related as parts in a whole which establishes the connection between them.

> ... the content of a hypothetical judgment is composed of ground and consequent, each referring to something other than itself, and hence essentially a part....
>
> It is only a question of detail how far the system in and by which the nexus subsists, is itself made explicit as a content within the hypothetical judgment.[40].

These nexūs are themselves facts within the world.

> ... every set of relations within which certain nexus of attributes hold good, is itself ultimately a *fact* or datum, relative no doubt within some further totality, but absolute relatively to the inferences drawn within it.[41]

Hence, "all hypothetical judgment rests on a categorical basis".[42].

Thus, Bosanquet proposed that the conclusion of the inference

$$\begin{array}{l} a, b, c, d \text{ are rational} \\ a, b, c, d \text{ are men} \\ \hline \text{Hence, ...} \end{array}$$

was not

[36] Bosanquet, *Logic*, vol. I, p. 211.
[37] Bosanquet, *Logic*, vol. I, p. 211.
[38] Bosanquet, *Logic*, vol. I, p. 234.
[39] Bosanquet, *Logic*, vol. I, p. 239.
[40] Bosanquet, *Logic*, vol. I, p. 238.
[41] Bosanquet, *Logic*, vol. I, p. 241.
[42] Bosanquet, *Logic*, vol. I, p. 241.

All men are rational

but

Are all men rational? or, Men may be rational

The inference from particulars to particulars in effect forms an *hypothesis* which is not ready for warranted affirmation until it is grounded in a categorical basis. At the level where the empiricist stops, that of "Men may be rational," we are, according to Bosanquet, still at a level of ignorance as to the genuine structure of reality. The empiricist would have us stop at the point where genuine inquiry would have to continue.

In effect, then, Bosanquet is arguing that Mill goes wrong in his view of general propositions when he treats them all as quasi-collective judgments. To the contrary, the warrant for truly universal propositions, propositions which cannot be overturned or falsified by a single instance, lies in a nexus that holds between the attributes. As Whewell had also argued, a judgment is truly universal only if it is not falsifiable — only if, in other words, it is somehow true *a priori*.

But Mill had already dealt with this claim. His opponent then was Whewell, not Bosanquet, but the point was the same. Mill insisted that each thing is what it is and not another thing, and in particular, an attribute or property implies nothing about any other property or attribute. This fact, that things and properties are logically self-contained, is a basic feature of the world, given to us phenomenologically. *Mill, in his reply to Whewell, had already replied to Bosanquet.*

Two: The Argument from Ontology

Bosanquet had a further point to make, beyond just re-asserting Whewell's claim that generalizations, to be genuinely scientific, had somehow to be necessary. His claim, following in his argument here the other main idealist, F. H. Bradley, was that this account of the necessity of general propositions or judgments derived from the idealist account of relations.

Bosanquet and Bradley followed Whewell in holding that various generalizations were necessary. Thus, they argued that the relation of contrariety that a property had to other properties was a necessary relation. For example, green is contrary to red and this is part of the being of both, an internal relation that makes the generalization

$$\text{Whatever is red is not green} \tag{18}$$

a necessary truth. Mill of course would argue that it is contingent, an inductive generalization, though, to be sure, no doubt one for which it is psychologically impossible to conceive the opposite. The idealists argued that when one judges that

$$\text{this is red} \tag{19}$$

one is judging, or attempting to judge, that this fact obtains independently of, and separable from all other facts. This *is* what it is, one is saying. But (18) is a necessary truth. So the judgment (19) is not separable from all other facts. One, when asserting it, is saying that it *is* wholly distinct from other facts, yet by the very being of the property judged about, the fact *is not* wholly separable from other facts. One is claiming that the judgment (19) is *wholly true* and is also committed to its being *not wholly true*. The judgment taken in isolation leads to a contradiction, and one achieve the truth about the subject only if one oveercomes this contradiction. The judgment becomes (more) true only if the connection is made explicit as in something like

this is red-as-opposite-to-green (20)

But this, too, is a judgment which aims at being wholly true but is not wholly true, given the relations things have to one another. Ultimately the only wholly true judgment involves the whole of reality:

the Absolute is

and even to put it that way is misleading since it implies a distinction between subject and predicate — between the Absolute and its being — and in the ultimate judgment, the one that is wholly true, even that apparent separation disappears into the whole: the ultimate judgment is not really a judgment, not really a juxtaposition of subject and predicate, but an intuition, an intuition of the Absolute.

This is not the place to deal with the idealist logic and metaphysics in detail. Suffice it to say that Mill does have a reply.

This reply consists in drawing attention to the phenomenological fact that each thing *is what it is and is not another* thing. We saw that Mill defended this principle as the basic axiom on which all formal logic is grounded. His reply to the idealists would consist in re-emphasizing this point. It follows, of course, that statements like (18), taken by the idealists to be synthetic *a priori* general truths, turn out to be contingent inductive generalizations — "merely" contingent inductive generalizations, but, as Mill would also insist, sufficiently deep in the structure of knowledge to be such that their opposites are inconceivable, that is, psychologically inconceivable.

G. E. Moore and, following Moore, Bertrand Russell, were to re-state this point upon which Mill insists and which implies the rejection of the idealist critique.

In his well known essay on the "Refutation of Idealism",[43] Moore makes the point with respect to the idealist claim that in knowing, say, yellow, the knowing and the known are related and are therefore not distinct from each other — implying the idealist point that the object of knowledge is inseparable from the knowing of it and that idealism is therefore true. The idealist, Moore suggests, holds that yellow and the sensation of yellow, that is, the sensing of it, are indistinguishable. Hence, as he puts it, "to assert that yellow is necessarily an object

[43] G. E. Moore, "The Refutation of Idealism," in his *Philosophical Studies* (London: Routledge and Kegan Paul, 1922), pp. 1-30.

of experience is to assert that yellow is necessarily yellow — a purely identical proposition, and therefore proved by the law of contradiction alone".[44] But unlike "A is A", the proposition that yellow is a sensation, that is, yellow is being sensed, is, when true, a substantive truth, and worth defending. Hence, "the proposition also implies that experience is, after all, something distinct from yellow — else there would be no reason for insisting that yellow is a sensation... ".[45] Thus, the very assertion of the idealist, that yellow is inseparable from the experiencing of it, is one that would not be made unless in fact yellow and the experience of it are in fact distinct and therefore separable. The doctrine of relations proposed by the idealists maintains that, since yellow and the experience are indeed related, and therefore also maintains that yellow and the experience are inseparable and therefore not distinct. But to state the doctrine presupposes that yellow and the experience *are* distinct. The very attempt to state the doctrine of relations thus refutes that doctrine. The doctrine of relations thus does not so much establish that there is no distinction as obscure the fact that such a distinction exists.

When, therefore, we are told that green and the sensation of green are certainly distinct but yet are not separable, or that it is an illegitimate abstraction to consider the one apart from the other, what these provisos are used to assert is, that though the two things are distinct yet you not only can but must treat them as if they were not. Many philosophers, therefore, when they admit a distinction, yet (following the lead of Hegel) boldly assert their right, in a slightly more obscure form of words, *also* to deny it.[46]

Moore's argument is clear. What he does do is *insist that things, properties, and relations are distinct, that, while they are related one to another, they are also separable.* Moore urges, in other words, that by itself **relatedness does not imply that distinct relata are logically inseparable**. Moore in effect appeals to the same phenomenological fact to which Mill appeals. We do in fact know yellow; it is presented to us. We also know experience, that is, what an experiencing is; this too is presented to us. These entities that are presented to us are presented as distinct. They are presented as self-complete, and to know the one does not require us to know the other. The entity yellow as we know it, as it is presented to us, is presented as an entity such that there is nothing about it that requires it to be connected inseparably with experience. To identify it as yellow does not require us to identify it as somehow connected to any other property, which would after all be required if it were, as the idealists assert, inseparably connected to some other entity, e.g., experience.

This argument from acquaintance, that we are not presented with the necessary connections claimed by the idealists, and, earlier, by Whewell, is Mill's axiom which he rightly sees as central to the defence of his inductive logic and also to his defence of formal logic. Russell was to build his logic on this same assumption. Moore and Russell provide the response to the idealist critique of the formal logic

[44]G. E. Moore, "The Refutation of Idealism," p. 14.
[45]G. E. Moore, "The Refutation of Idealism," p. 14.
[46]G. E. Moore, "The Refutation of Idealism," p. 15.

of the empiricist; they provide the defence of the logic that Mill was defending. It was in fact a defence that can be found already in Mill's own work. Russell's version of the logic is incredibly more developed than that of Mill. But it rests on the foundations that Mill had already established.

10 THE PHILOSOPHY OF ERROR

Given Mill's aims in writing the *System of Logic*, it should not be surprising that Mill devoted a whole of one book to the fallacies. It was, however, contentious. Like formal or deductive logic, the analysis of fallacies before Whately had hardly progressed beyond Aristotle. Aristotle had given a list of thirteen fallacies and these were repeated in texts down to that of Aldrich. It was no doubt a dreary exercise to study the formal logic of Aldrich, and the fallacies were likely thought a rather frivolous exercise that nonetheless provided a welcome relief from the drudgery of the rules of syllogistic. Things are little improved today: one still finds most texts in introductory deductive logic devoting a chapter or so to the fallacies, and most of these discussions hardly go beyond Aristotle — though, like Whately, they often spice things up with more recent examples. In such a context, then and now, Mill's phrase, "the philosophy of error,"[47] no doubt seems a bit strange. But when one realizes that for Mill, much of human misery and unhappiness is due to error, this description in not at all odd. Would that more take Mill's aim as seriously.

Mill did include among his list of fallacies, Aristotle's thirteen kinds; and his discussion is clearly dependent upon Bacon's treatment of the "Idols of the Theatre" in the *Novum Organon*. But his classification is original and deserves to be better known. The classification consists of five categories derived from a series of distinctions among various kinds of evidence, good and bad, and argument.

The first category, the "fallacies of simple inspection," were forms of thought which disposed a person to believe a proposition without evidence by creating a favourable presumption of truth which permitted one to avoid any appeal to the rules of inductive inference. This sort of fallacy consists of various ways in which a person mistakes a feature of one's subjective way of thinking for an objective feature of things. This is the source of various popular superstitions — one of Mill's examples was the belief that "talk of the devil and he will appear", where an idea is, by itself, supposed to generate a reality — but also is the root of various forms of philosophical "intuitionism" — which included both Whewell's philosophy of science and inductive logic and his principles of ethics. The fallacy is the notion that what is true of "our ideas of things must be true of the things themselves".[48] It is the root of the notion, so common amongst those, like Descartes and Whewell and "modern German philosophy", who would aggrandize the place of pure reason in our claims to knowledge, that if the contrary of a proposition is inconceivable

[47] *System of Logic*, V, i, § 3, *CW*, vol. 8, p. 737.
[48] *System of Logic*, V, iii, § 3, *CW*, vol. 8, p. 751.

then it must be true. This is the source of much bad philosophy.

> I am indeed disposed to think that the fallacy ... has been the cause of two-thirds of the bad philosophy, and especially of the bad metaphysics, which the human mind has never ceased to produce.[49]

It is the root flaw in those systems of thought that are not thoroughgoing empiricist. It might be necessary to devote a whole volume, such as Mill was to write in his *Examination of Hamilton*, to refuting a particular metaphysical system, but here was the root fallacy that lay behind it.

Mill discusses other forms of the fallacy of simple inspection, e.g., the sort of thought that gives rise to mysticism in religion and in philosophy, e.g., Hegelianism (which he no doubt would have applied also to the later British idealists). Then there are other forms of fallacy, e.g., fallacies of confusion (such as arguments involving ambiguous terms — the traditional fallacies of ambiguity) in which the strength of evidence is imperfectly grasped. The traditional *petitio principii* also falls into this category. Then there are various forms of deductive and inductive fallacies. Among the latter are two kinds, fallacies of observation and fallacies of generalization. Fallacies of observation include those of non-observation, e.g., where negative instances of a generalization are ignored, as when one takes an almanac or a psychic to be a good predictor of whatever. Inductive fallacies include such traditional cases as hasty generalization and *post hoc, ergo propter hoc*.

Mill covers all the traditional fallacies and much more besides. There were those, like Whately, who argued that a logic book should present the rules of logic rather than present various ways of breaking them. But Mill's discussion, while partially derivative from the work of Bacon and Bentham, shows how clarity and order can be brought to this subject, which is usually dealt with as a catalogue of tricks to deceive. There is, among those who have followed Boole in developing formal logic, a studied indifference to the fallacies. Mill shows that such indifference is undeserved.

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Mill's treatment of formal or deductive logic is now clearly dated. This is much less true of his inductive logic and more generally his philosophy of science, though it can be said to be nowadays unfashionable. But he also places his discussion in an empiricist metaphysical context. This discussion remains impressive: the criticism of intuitionism in its various forms still commands respect — whether it be in the foundations of logic, or of geometry or of arithmetic — or, for that matter, of the social sciences or of ethics. Mill argued, no doubt correctly, that intuitionist philosophy had, as a "false philosophy," disastrous effects in "morals, politics and religion",[50] and his discussion of logic and the rules for the search after truth

[49] *System of Logic*, V, iii, § 3, *CW*, vol. 8, p. 752.
[50] *Autobiography*, ch. 7, *CW*, vol. 1, p. 233.

should be seen as part of an on-going political battle. Mill is not now included among the list of distinguished logicians — the list that runs from Aristotle (and perhaps Carneades) through Leibniz to Boole and Peirce and on to Frege and Russell does not contain Mill's name.[51] This is a list, however, of those who made logic flourish as a formal science but also of those who made of logic a narrow and therefore a rather impoverished field of study. But in a list of those concerned to defend the search after truth, his name cannot be omitted: his work deserves our attention.

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[51]George Boole's *Mathematical Analysis of Logic*, appeared originally in 1848, just four years after the first edition of Mill's *System of Logic*. Mill was aware of the work of Boole, de Morgan and Jevons that was the beginning of the vast expansion of logic as it became mathematized and symbolic. In one of his letters he wrote that "[Jevons'] speculations on Logic, like those of Boole and De Morgan, and some of those of Hamilton, are infected with this vice" — that is, Mill has said earlier, "a mania for encumbering questions with useless complication, and with a notation implying the existence of greater precision in the data than the question admit of". He furtherr comments that this vice "is one preeminently at variance with the wants of the time, which demand that scientific deductions should be made as simple and as easily intelligible as they can be made without being scientific." [Mill, *Later Letters*, vol. 3, in *CW*, vol. 17, pp. 1862-63]. Mill's objection to the developing symbolic logic reminds one of similar objections from those who have to teach critical thinking to undergraduates: the formalism of formal logic sometimes obscures the tools needed to critically analyze real life arguments rather than contrived examples. Perhaps. But that misses the point about that new logic created by Boole and the others Mill mentions, that it has developed into a formal science in its own right, one of great power and elegance, and one which is studied for its own sake and not for, or not simply for, its applications to the critical assessment of reasoning in ordinary discourse or in science. The editor of a recent edition of Boole's ground-breaking Mathematical Analysis of Logic, John Slater, has noted that

> Boole's work freed logicians, once and for all, from the endless repetition of a few principles, mostly derived from Aristotle, and, by example, challenged them to develop their subject in new directions. Many gifted thinkers, among them John Venn, William Stanley Jevons, Charles Sanders Peirce, Ernst Schröder, Alfred North Whitehead and Bertrand Russell rose to the challenge, and the vast and autonomous subject of modern logic emerged from their combined efforts. It is a glorious success story. (J. Slater, in G. Boole, *The Mathematical Analysis of Logic*, [Bristol: Thoemmes Press, 1998], p. xi)

Mill does not appear in this list. But he did, as we have argued, provide the philosophical and metaphysical context, free of both continental and idealist fuzziness and nominalistic denigration, in which the new formal logic could take off and flourish.

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DE MORGAN'S LOGIC

Michael E. Hobart and Joan L. Richards

1 INTRODUCTION: THE SYMBOLIC TURN IN LOGIC

Looking back on a more than a decade of work in logic, Augustus De Morgan (1806–1871) remembered that from the beginning "in my own mind I was facing Kant's assertion that logic neither has improved since the time of Aristotle, nor of its own nature can improve, except in perspicuity, accuracy of expressions and the like".[1] For Immanuel Kant, writing at the very end of the eighteenth century, the longevity of Aristotle's logic showed that the Greek had essentially gotten it right; for De Morgan it was evidence of stagnation. From his position in the progressive nineteenth century, De Morgan set out in the 1840s to move the study of logic forward by building on the foundation Aristotle had laid. In so doing, De Morgan brought a very specific view of progress to his logical enterprise. "Every science that has *thriven* has thriven upon its own symbols: logic, the only science which is admitted to have made no improvements in century after century, is the only one which has *grown no new symbols.*"[2] De Morgan's logical program, then, entailed creating a set of symbols that would show him the way to move logic beyond its Aristotelian base into an ever-expanding future.

For many, De Morgan's determination to create an operational set of logical symbols has marked him as a pioneer of the symbolic logic of the late nineteenth and early twentieth centuries, and he is routinely credited with the laws that bear his name and with the logic of relations. Beyond this appreciation, the rest of his more than twenty years of logical effort is generally seen to have little importance in the development of logic; most modern readers shrug it off as odd, clumsy, or old fashioned. Thus dismissing the bulk of De Morgan's logic because of its unfamiliarity is myopic at best, however, for it erases the complex dynamic of change that supported the development of modern symbolic logic. And understanding the roots of twenty-first century symbolic logic requires entering the nineteenth-century world in which they developed.

[1]Augustus De Morgan, "On the Syllogism, No. III, and on Logic in general", *Transactions of the Cambridge Philosophical Society*, p. 173, 1858 [OS, p. 74]. (Hereafter, [S3]; see fn. #21 for a complete listing of De Morgan's major logical works and the abbreviations we use in citations.) De Morgan's reference is to Kant's statement: "Logic, by the way, has not gained much in *content* since Aristotle's times and indeed it cannot, due to its nature. But it may well gain in *exactness, definiteness* and *distinctness.*" Immanuel Kant, *Logic.* Translated, with an Introduction by Robert S. Hartman and Wolfgang Schwarz, New York: Dover Publications, Inc., p. 23, 1974.

[2][S3, p. 184] [OS, p. 88] (ital. De Morgan's).

Handbook of the History of Logic. Volume 4: British Logic in the Nineteenth Century.
Dov M. Gabbay and John Woods (Editors)

De Morgan was an erudite scholar and prolific writer, with wide-ranging and catholic concerns; his particular combination of intellectual and moral gravity with high-brow and omnipresent wit runs through not only his many published books, but also the more than 700 articles he wrote for the popular *Penny Cyclopedia,* and the well over a thousand reviews he published in the *Athenaeum.* He addressed subjects ranging from algebra to gymnastics, from chemical change in the Eucharist to astro-theology, but mathematics, logic, and their histories remained his dominant interests throughout his career.

The logic of Augustus De Morgan was primarily rooted in his identity as a Cambridge-educated Englishman of the mid-nineteenth century. Initially, that identity was much more tied up with mathematics than with logic. De Morgan began the serious study of mathematics as a student at Cambridge, where mathematics was the center of a liberal education designed to educate the Anglican clergy.[3] The pairing of mathematics and theology that supported De Morgan's Cambridge education stemmed from a position developed in John Locke's *Essay Concerning Human Understanding.* In this magisterial work, Locke had exempted mathematics and theology from the uncertainties of empirical knowledge by placing them in a separate category of demonstrative knowledge. The mathematical focus of De Morgan's Cambridge education is a testament to the enduring power of Locke's coupling of mathematics and theology as exemplars of certain knowledge.

In Locke's construction, the certainty of mathematical knowledge was attested to by the extra-ordinarily tight fit between symbols and their positive, spatial meanings. The power of this connection is manifest in geometry, where diagrams play a crucial role in illustrating and clarifying proofs. Yet, by the end of the seventeenth century, a great deal of mathematics did not fit the geometrical model of certainty. In the one hundred and fifty years before Locke published his work, algebra had moved from virtual non-existence to the center of mathematical development. At the heart of this transformation lay the creation of numerous and recognizably modern conventions of notation — e.g. such functional symbols as $+, -, \times, \div, =, <, >, \surd, \infty, \frac{dy}{dx}$, plus the use of superscripts $^{(2,3\ldots n,\ldots x,y,z\ldots)}$ for powers and of letters for knowns $(a, b, c, \ldots)$ and unknowns $(x, y, z, \ldots)$.[4] Algebra's power lay precisely in the way its symbolical exuberances broke through the constraints of meaning that held geometry fast.

Throughout the English eighteenth century the problem of algebraic certainty was usually glossed over, but late in the century some of England's political and

[3]For the nineteenth-century development of the Cambridge mathematical education, see: Andrew Warwick, *Masters of Theory: Cambridge and the Rise of Mathematical Physics*, Chicago: University of Chicago Press, 2003. There has been considerable interest in the interaction of mathematics and theology within that education, much of it focused on William Whewell. The articles gathered in Menachem Fisch and Simon Schaffer, *William Whewell: A Composite Portrait,* Oxford: Clarendon Press, 1991, are useful as an initial entré into that material.

[4]For the history of mathematical notation, the magisterial work of Florian Cajori (*A History of Mathematical Notations. Two vols. Vol. I: Notations in Elementary Mathematics; vol. II: Notations Mainly in Higher Mathematics*, New York: Dover Publications Inc., 1993 [c. 1928–1929]) remains unsurpassed.

religious radicals were determined to bring it to the fore.[5] In 1797, the radical Unitarian William Frend, published *A Treatise on Algebra* in which he insisted that all mathematical truth — including algebraic truth — lay in the close tie between symbol and subject matter. Algebra was just generalized arithmetic, and the subject matter of arithmetic had been known since antiquity to be the numbers with which we count the objects around us. There is no such thing as a negative object, Frend insisted, and with that he eliminated negative numbers from algebra. Frend's was an extreme position — to follow his lead was essentially to destroy all algebraic developments since the sixteenth century — but in the Lockean world of Cambridge mathematics his stubborn insistence that the validity of mathematical symbols depended on their interpretations was difficult to refute.

Within five years of Frend's negative-denying treatise, the Cambridge tutor Robert Woodhouse tried to answer its extremism with a re-examination of the connection between mathematics and meaning. Woodhouse's writing was too "difficult and perplexed"[6] for his book have a broad impact, but by the 1810s others moved to free English mathematics from the constraints of the meanings-based Lockean approach. In the 1810s the more effective communicators of the short-lived, but highly influential Cambridge Analytical Society (ca. 1812–1817), set out to introduce the symbology of French analysis into England. In 1816 John Herschel, Charles Babbage, and George Peacock translated Sylvestra François Lacroix's *Elementary treatise on the differential and integral calculus* for the benefit of their countrymen; by the end of the decade, Cambridge students were abandoning Isaac Newton's conceptually rich fluxional notation in favor of the operational power of Gottfried Leibniz's $\frac{dy}{dx}$ notation. This change in symbology carried with it a challenge to English meaning-based mathematics that was deep enough to focus the thinking of a generation of Englishmen.[7]

[5]For a fuller treatment of these developments see Helena M. Pycior, *Symbols, Impossible Numbers, and Geometric Entanglements: British Algebra through the Commentaries on Newton's "Universal Arithmetic"*, Cambridge: Cambridge University Press, 1997.

[6]*Penny Cyclopaedia* (1833–43), s.v. [Augustus De Morgan], "Robert Woodhouse".

[7]For the work of the Analytical Society see Philip C. Enros, "Cambridge University and the adoption of analytics in early Nineteenth-Century England", in H. Mehrtens, *et al.* (eds.), *Social history of Nineteenth-Century mathematics*, Boston: Birkhauser, pages 135–48, 1981; "The Analytical Society 1812–1813," *Historia Mathematica*, 10: 24–47, 1983; Harvey W. Becher, "William Whewell and Cambridge mathematics", *Historical Studies in the Physical Sciences*, 11: 1–48, 1981; Menachem Fisch, "'The emergency which has arrived': The problematic history of 19th-century British algebra — a programmatic outline", *British Journal for the History of Science*, 27: 247–276, 1994. For the influence of French developments in particular on De Morgan's early career, see Maria Panteki, "French 'logique' and British 'logic': on the origns of Augustus De Morgan's early logical inquiries, 1805–1835", *Historia Mathematica*, 30: 278–340, 2003. Classic studies in which nineteenth-century English algebraists are evaluated for their formal content include: Eric Temple Bell, *The Development of Mathematics*, New York: McGraw Hill, 1954; L. Nový, *Origins of Modern Algebra*, Jaroslave Tauer, trans., Leyden: Noordhoff, 1973; Elaine Koppelman, "The Calculus of operations and the rise of abstract algebra", *Archives for History of the Exact Sciences*, 8: 155–242, 1971-72; D. A. Clock, "A New British Concept of Algebra". PhD dissertation, University of Wisconsin, 1964. More recent studies which focus on the complexities of De Morgan's views of algebra include: Joan L. Richards, "Augustus De Morgan, the History of Mathematics and the Foundations of Algebra", *Isis*, 78: 7–30, 1987; Joan L. Richards,

De Morgan was a student during the 1820s, when Cambridge mathematics was delicately balanced between the geometrical and analytical points of view. He counted a number of Analytics, including Peacock, William Whewell, and George Biddel Airy, as his teachers, and finished his education fully equipped to pursue analytical mathematics at its very highest levels. De Morgan's teachers were markedly less successful in the religious goals of their educational efforts, however. Despite the fact that all of them were life-long members of the Anglican Church, De Morgan finished his education facing serious religious doubts. De Morgan must have subscribed to the Thirty-Nine Articles of the Anglican Church in order to receive his Bachelor of Arts degree, but this was the last time that he publicly acknowledged a religious affiliation. Soon after he demurred in signing the Test Act, which was required for taking the higher degree of Master of Arts and becoming a Fellow at Cambridge. Instead, he departed for London, where he found another mentor, one who supported him in his doubts about the legitimacy of the Anglican Church.

William Frend was a Cambridge-educated scholar who, in the 1780s, had left the Anglican Church to become a Unitarian; he was also the man who in the 1790s, had tried to rid algebra of negative numbers. For Frend, the religious and mathematical positions were inextricably bound together. He was led away from the Anglican Church by a close reading of the Bible — that the word 'Trinity' was a Latin word not to be found in the Greek or Hebrew scriptures was an essential argument for his rejection of the doctrine.[8] A decade later, his arguments against the validity of negative numbers flowed from the same literalist impulse. The religious positions Frend shared with a questioning De Morgan were essentially intertwined with the Lockean mathematical program he had so passionately espoused.

Even as Augustus was deepening his personal relations with England's most determinedly restrictive algebraist, his mathematical prowess was being recognized; the young man was just twenty-two years old when, in 1828, he became the first Professor of Mathematics at the newly created London University (later University College London or UCL). The appointment pleased De Morgan's Cambridge teachers who had helped shape one of England's rising mathematicians; it pleased William Frend because the defining feature of the new institution was its religious inclusivity. For most of the next forty years,[9] De Morgan devoted himself to teaching mathematics to classes that indiscriminately mixed Jews, Anglicans, and dissenting Christians.[10]

"The art and the science of British algebra: A study in the perception of mathematical truth", *Historia Mathematica*, 7: 343–65, 1980; and Helena Pycior, "Early criticisms of the symbolical approach to algebra", *Historia Mathematica*, 9: 413–440, 1982; Helena Pycior, "The three stages of Augustus De Morgan's algebraic work", *Isis*, 74: 211–26, 1983.

[8]William Frend, *An Address to the Members of the Church of England and to Protestant Trinitarians in General, Exhorting Them to Turn from the False Worship of Three Persons, to the Worship of the One True God*, London: J. Johnson, p. 7, 1788.

[9]For reasons of academic politics, De Morgan resigned his position at the London University in 1831. In 1838 he accepted a position as Professor of Mathematics at the newly renamed University College London, which he held until 1867.

[10]For De Morgan's appointment and teaching career, see Adrian Rice, "Inspiration or desper-

De Morgan was first and foremost a superb and absolutely devoted teacher: "All I do arises directly out of teaching" he remarked after almost twenty years on the job, "and has in some way or another reference to what can be brought before a class and especially with a view to mathematics as a discipline of the mind."[11] He was well-beloved and respected by his students, many of whom, including J. J. Sylvester, Stanley Jevons, Isaac Todhunter, and Walter Bagehot, went on to distinguished careers of their own. The diversity of this small sample points to De Morgan's success in the primary goal of his teaching, which was to bring the study of mathematics to the heart of a liberal education that would create effective thinkers as broadly interested as he.

In his focus on mathematics as the center of a liberal education, De Morgan was working within the venerable tradition that had shaped English mathematics and education for well over a century. Transposing that education from Anglican Cambridge to the religiously diverse environment of the London University was a significant challenge, but De Morgan gamely rose to meet it. In "An Introductory Lecture delivered at the opening of the Mathematical Classes in the London University [on] November 5, 1828", he argued powerfully for the importance of mathematics for all students. Mathematical study, he insisted, was the best way to teach young people to think. It was "instrumental in furnishing the mind with new ideas, and calling into exercise some of the powers, which most peculiarly distinguish man from the brute creation";[12] it was a "compendious language" that "contains in its very formation, the germ of the most valuable improvements ... and has been from its peculiar structure, a never failing guide to new discoveries".[13] De Morgan's description of mathematics as language whose peculiar strength was its power to move knowledge forward can be seen as the position from which he would develop logical symbology some thirty years later. At the time that he first stated it, however, De Morgan's Frendian outlook meant that it was rather difficult for him to maintain such a progressive point of view.

From De Morgan's very first days as a mathematics professor, his desire to use mathematics as a way to teach his students to think properly meant that foundational questions lay at the very heart of his educational mission; it meant that in all of his classes he had to be absolutely clear about the ways that mathematical symbols related to their meanings. This brought him immediately to the problem of the "impossible" quantities as they were called at the time, the negative and imaginary numbers that Frend had insisted be removed from algebra. The young

ation? Augustus De Morgan's appointment to the chair of mathematics at London University in 1828", *British Journal of the History of Science*, 30: 257–74, 1997, and "What makes a great mathematics teacher? The case of Augustus De Morgan", *The American Mathematical Quarterly*, 106(6): 534–553, Jun/Jul 1999.

[11] Augustus De Morgan to William Rowan Hamilton, April 29, 1854. Quoted in Robert Graves, *Life of Sir William Rowan Hamilton*, London: Longmans, Green & Co., 1889, 3: 479.

[12] Augustus De Morgan, "An Introductory Lecture delivered at the opening of the Mathematical Classes in the London University November 5, 1828", UCL Archives. MS ADD 3, p. 3 (hereafter, "Introductory Lecture").

[13] "Introductory Lecture", p. 3.

De Morgan wanted to recognize that these numbers could be used to generate correct results, even as he honored Frend's position that they were epistemologically illegitimate. In his first publication, a translation of L. P. M. Bourdon's *Algebra*, he essayed tortuously, and in ways that leave modern readers somewhat breathless and bewildered, to explain how these numbers could be at once meaningless and yet useful.[14]

Within the year after De Morgan had thus tried to satisfy the demands Frend represented, one of his Cambridge teachers offered a radically different solution. In *A Treatise of Algebra*, Peacock suggested a novel way to legitimize all of algebra's uninterpretable quantities. Peacock's solution began with a distinction between "arithmetical algebra" and "symbolical algebra".[15] Arithmetical algebra employed signs (+, -, $\sqrt{}$, etc.) in the same sense as common arithmetic, and operations, such as subtraction, were restricted in order to preclude negative and impossible quantities. Symbolical algebra, by contrast, comprised a new and "strictly formal" science, of "symbols and their combinations, constructed upon their own rules, which may be applied to arithmetic and to all other sciences by interpretation."[16]

Having clearly distinguished between these two kinds of algebra, Peacock formulated a "principle of the permanence of equivalent forms" that connected them.[17] Peacock's principle stated that an equation such as $a^2 - b^2 = (a + b)(a - b)$ was legitimate, even in cases where a was smaller than b, because the truth of such equations did not depend on symbolic meanings of the letters *per se*, but rather on the "form" of the arithmetic expressions from which they were derived. So, for

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[14] Consider, for example, the problem: "To find a number which, added to a number, gives for their sum the number a." This perfectly unexceptional problem could be transformed through the formula $b + x = a$ to give $a - b = x$. Given this formulation, what happens if $a = 24$ and $b = 31$? In that case $x = -7$, which is incomprehensible. "But," the text continues, "if we consider the solution independently of its sign, that is, $x = 7$, we may say that it is the solution of the following problem, 'to find a number which, subtracted from 31, gives 24,' in this, that the words 'added to' are supplied by the words 'subtracted from'." Somehow, with this last, head-hurting clause, the problem was solved. Louis Pierre Marie Bourdon, *The Elements of Algebra: Translated from the Three First Chapters of the Algebra of M. Bourdon, and Designed for the Use of Students in and Preparing for the University of London*, trans. Augustus De Morgan, London: J. Taylor, p. 77, 1829.

[15] George Peacock, *A Treatise on Algebra. Vol. I: Arithmetical Algebra; vol. II: On Symbolical Algebra and its Applications to the Geometry of Position*, Cambridge: Cambridge University Press, 1842, 1845 [1^{st} ed., 1830], I: iii–x, II: iii–v, and passim. In vol. II, p. 59, Peacock gave the following "formal statement" of his principle: "Whatever algebraic forms are equivalent, when the symbols are general in form but specific in value, will be equivalent likewise when the symbols are general in value as well as in form." From which it follows "that all the results of Arithmetical Algebra will be results likewise of Symbolical Algebra: and the discovery of equivalent forms in the former science ... will be not only their discovery in the latter, but the *only* authority for their existence: for there are no definitions of the operations in Symbolical Algebra, by which such equivalent forms can be determined."

[16] George Peacock, "Report on the Recent Progress and Present State of Certain Branches of Analysis", *Report of the Third Meeting of the British Association for the Advancement of Science*, London: John Murray, pages 194–95, 1834, cited in Daniel D. Merrill, *Augustus De Morgan and the Logic of Relations*, Dordrecht: Kluwer Academic Publications, p. 187, 1991 (hereafter [ADM]).

[17] Peacock, *Treatise on Algebra*, *loc. cit.*

example, Peacock understood $a^2 - b^2 = (a+b)(a-b)$ as an expression generalized from arithmetic equations like $4^2 - 3^2 = (4-3)(4+3)$; the symbolical generalization obtained by substituting letters for numbers was the "equivalent form" of the arithmetic equation. Peacock's principle of the permanence of equivalent forms guaranteed that as long as the generating arithmetic equation were valid, the equivalent form would be valid as well, even though in other arithmetical instantiations — if $a = 3$ and $b = 6$ for example — the equivalent form might yield negative or imaginary numbers. The presence of such arithmetically illegitimate expressions in symbolical algebra simply indicated that their interpretations were to be found elsewhere; negative numbers could be interpreted through a geometrical number-line, and the imaginary, square roots of negative numbers could be meaningfully interpreted as lying in a two-dimensional plane.

De Morgan was initially shocked by Peacock's vision; "it seemed to us something like symbols bewitched, and running about the world in search of meaning."[18] Over time, however, he came to see its liberating power. De Morgan was never able to persuade Frend of the legitimacy of Peacock's approach — "I am very much inclined to believe that your figment $\sqrt{-1}$ will keep its hold among Mathematicians not much longer than the Trinity does among theologians,"[19] — but the younger man was not to be deterred, and late in the 1830s he exploded into algebra. This is the De Morgan to whom we owe explicit recognition of the basic laws that structure field algebras (he omitted only associativity).

De Morgan's effort to articulate the basic axioms of field theory has earned the respect of many who have seen him as a pioneer of modern algebra. Formal and therefore modern though his work may appear, however, he was always focused on the meanings of the symbols. He compared doing algebra to putting together an upside down jigsaw puzzle; in such a configuration the pieces could be put together following just their forms, but it was the promise of the picture on the other side that justified the effort. Peacock's principle of equivalent forms promised De Morgan that the picture existed; it strengthened his conviction, cited from one of his French predecessors, Jean Le Rond d'Alembert, to "push on and faith will follow". By the 1840s, De Morgan felt this faith justified and the puzzle largely solved: "There is now not a cloud in the heaven of algebra, except in the corner appropriated to divergent series", he wrote to John Herschel in 1844. "I mean that every symbol ... is not merely capable of interpretation, but intelligible on the definitions of the symbols themselves".[20]

His belief that algebra was essentially completed propelled De Morgan to extend Peacock's insight into logic, and in 1847, he published *Formal Logic*. The book does not represent the culmination of De Morgan's logical thought, however; on the contrary, he was already pushing his ideas further when it was still in press.

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[18] [Augustus De Morgan], "Review of George Peacock, *A Treatise on Algebra*", *Quarterly Journal of Education*, 9: 311, 1835.

[19] William Frend to Augustus De Morgan, 22 June 1836. Cambridge UL. Add 7887[29].

[20] Augustus De Morgan to John Herschel, May 1844. #205. *Herschel Archive of the Royal Astronomical Society*, Microfilm.

His most important logical work appeared subsequently as a series of five major essays in the *Transactions of the Cambridge Philosophical Society* from 1849 to 1863. De Morgan recognized the coherence of the series by including "On the Syllogism" (or some variation thereof) in the title of each article; throughout the following we shall use the common bibliographical practice of referring to these essays by number as [S1], [S2], [S3], etc.[21]

De Morgan was certainly not the first to try to bring the symbolical power of mathematics to bear on logic. Already by the late seventeenth century a number of leading intellectuals — including Seth Ward, John Wilkins, Marin Mersenne, Francis Lodowyck, Johann Amos Comenius, and René Descartes — had devoted considerable efforts to devising a new, abstract and symbolic "universal language". Their goal was to create a logic of symbols that they believed would be as powerful as the new mathematics, and by means of which long-standing philosophical and practical disputes could be resolved. Yet by the early eighteenth century the "universal language" movement had failed and had become a historical *cul de sac*.[22] It failed not only because there were as many candidates for a universal language as there were proponents of the idea (rendering nugatory the idea of such a language common to all), but even more significantly because the symbols employed in these efforts were not functional. Rather, they remained ideograms that simply stood for words and, like words, were arranged in classificatory systems.

This means that while in the eighteenth century abstract mathematics forged ahead — providing analytical tools of discovery with an ever more precise array of functional symbols and ever expanding topics and branches — logic ambled along in its own natural language. This helps explain why Denis Diderot and d'Alembert, editors of the *Encyclopèdie*, did not even think it worthwhile to include an article on "Logic" in their great compendium of Enlightenment knowledge: sustained by the new, mathematical sciences and a critical cast of mind, the "*esprit systèmatic*" of reason could do quite well without it.

[21]De Morgan's principle works on logic include: (A) his one book on the topic, *Formal Logic*, London, 1847. Facsimile reproduction, London: The Open Court Company, 1926 (hereafter [FL]); (B) the above-mentioned series of five major articles carrying the title of "On the Syllogism", or some variant (commonly referred to as [S1], ..., [S5]; the full citations for each are: [S1], "On the structure of the syllogism, and on the application of the theory of probabilities to questions of argument and authority", *Transactions of the Cambridge Philosophical Society* [TCPS], pages 379–408, 1849; [S2], "On the symbols of logic, the theory of the syllogism, and in particular of the copula, and the application of the theory of probabilities to some questions of evidence" [TCPS, 79–127, 1851]; [S3], "On the Syllogism, No. III, and on Logic in general" [TCPS, 173–230]; [S4], "On the Syllogism, No. IV, and on the Logic of Relations" [TCPS, 331–358, 1860]; [S5], "On the Syllogism, No. V. and on various points of the Onymatic System" [TCPS, 428–487, 1863]; [S1–S5] are all reproduced in *On the Syllogism and Other Logical Writings*, Peter Heath (ed.), New Haven CT: Yale University Press, 1966; in citing, we shall provide the original first, followed by the Heath edition, listed in brackets as ['OS']); (C) two pieces appearing in 1860, which summarized much of his achievement, "Logic" (written for the *English Cyclopedia*, V, 1860) and "Syllabus of a Proposed System of Logic" (both are reproduced in [OS], and will be referred to in that edition as "Logic" and "Syllabus" respectively); (D) scores of lesser articles and reviews dealing with logic and related topics, which we shall cite as needed.

[22]See Mary M. Slaughter, *Universal Languages and Scientific Taxonomy in the Seventeenth Century*, Cambridge: Cambridge University Press, 1982, passim.

Logic lay dormant in eighteenth-century England as well. Here the problem was its perceived emptiness; logic, Locke asserted, "has been made use of and fitted to perplex the signification of words, more than to discover the knowledge and truth of things."[23] The early nineteenth century brought change in logic as in mathematics, however. In 1826 Richard Whately published his *Elements of Logic*. Whately intended his presentation of Aristotelian syllogistic logic as a textbook for Oxford students, but it was also the first contribution to what in the next several decades developed into a considerable discussion of logic in Victorian England.

At the center of this discussion, De Morgan sought to use Peacock's insight to free logic from its static past. In this he was joined by the young Irish mathematician, George Boole. Boole and De Morgan were friends who corresponded frequently and steadily from the 1840s until Boole's death in 1864; their first books on logic were published within a week of each other. Nonetheless, sharp differences separated their approaches. The most fundamental of these differences lay in their respective readings of Peacock's principle. For De Morgan, who always remained in some sense Frend's disciple and was by this time also his son-in-law,[24] the principle of the permanence of equivalent forms provided a guarantee that valid interpretations could be found for all legitimately generated symbols. By contrast, Boole saw the principle as one "of independence from interpretation in an 'algebra of symbols'."[25] The distinction can be made more succinctly: whereas Peacock's principle freed De Morgan to find myriad interpretations for algebraic symbols, it freed Boole from the need to interpret those symbols at all.

Moreover, for Boole, bringing the insights of algebra to logic entailed not only importing algebra's focus on symbols, but also the functional operations of many symbols themselves. His use of 1 and 0 as class designators, universal and null respectively, and his two-valued algebra based on manipulations of 1 and 0 provided him an extremely robust and functional system. In it he retained many of mathematical functions and symbols (+, −, and the like), even while carefully departing from some conventional algebraic definitions of rules, as with the notion of idempotency, the idea that in logic x times $x = x$, not x^2 (a class times itself cannot create more members that already exist within it). In effect, this allowed him to express and manipulate propositions as equations. Later logicians would build on Boolean 1s and 0s, considering them as "operators" for assigning truth-values in logical conjunction, disjunction, and negation (complementation), which gave birth to an entire propositional calculus and, as is well known after the work of Claude Shannon, digital computing circuitry.[26]

[23]John Locke, *An Essay Concerning Human Understanding*, Twenty-ninth edition, London: Thomas Tegg, p. 349, 1841.

[24]Augustus De Morgan married Sophia Elizabeth Frend on August 3, 1837.

[25]Volker Peckhaus, "The Mathematical Origins of 19^{th} Century Algebra of Logic", (paper delivered January 18, 2003, and made available at: `http://www.uni-Paderborn.de/fileadmin/kw/Institute/Philosophie/Personal/Peckhaus/Texte_zum_Download/hml.pdf`

[26]In his 1938 MIT masters thesis, Shannon, often called the founder of information theory, recognized the linkage between binary systems, pairs of logical (true or false) states, and a "relay-based" system of electrical circuits, which he could exploit formally using Boolean algebra. See

When De Morgan first encountered Boole's notational system he described it as "elegant", but he never showed any desire to follow a comparable path.[27] Rather than importing algebraic symbols into logic, he developed a completely new set of symbols for logical ideas. "My working processes are not so like those to common algebra as to symbols, but more resemble the operations of our heads", he explained.[28] Otherwise stated, algebra inspired in him the idea of a logical machinery, but few details of its actual operations.

Although it was thoroughly functional, De Morgan's system of logical symbols never captured the attention of subsequent generations. At least part of this resistance may be attributed to the essential conservatism of De Morgan's project. Throughout all his symbolic novelties and the innovations in his logic of classes, De Morgan viewed logic as ultimately tied to the world of traditional syllogisms. From the outset his goal was to find a notation that would articulate old as well as new inferences within that world. He may have found Boole's system "elegant", but he was also disturbed that Boole was unable to express key aspects of syllogistic reasoning that his own notation captured.[29]

De Morgan's work was nonetheless highly creative. He saw himself to be extending, as opposed to supplanting Aristotle's syllogisms, but his interpretations of propositions, inferences, and syllogisms led him far beyond the Stagirite. By the end of his life De Morgan had come to see Aristotelian syllogisms as only one subset (onymatic relations) of a more general — and therefore for De Morgan more fundamental — logic of relations. Even more fundamental, De Morgan shared with Aristotle the basic assumption that thought always points to some reality outside of itself. In logic as in algebra, De Morgan was always deeply interested in the

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Arno Penzias, *Ideas and Information: Managing in a High-Tech World*, New York: Simon & Schuster, Inc., p.100, 1989.

[27]Augustus De Morgan to George Boole, 28 November 1847, letter sent, *Boole-De Morgan Correspondence*, p. 26. Also, in [S2, p. 79] [OS, p. 22], De Morgan stated that his "methods" had "nothing in common" with Boole's, whose "mode of treating the forms of logic is most worthy the attention of all who can study that science mathematically, and is sure to occupy a prominent place in its ultimate system". In [S3, p. 183] [OS, p. 87], De Morgan put the point even more bluntly: "The truth is that *I* have not made much use of symbols actually employed in algebra; ..."

[28]De Morgan to Boole, 28 November 1847, draft of a letter not sent, G. C. Smith (ed.), *The Boole-De Morgan Correspondence, 1842-1864*, Oxford: Clarendon Press, p. 24, 1982. See also the apposite remarks of C. I. Lewis, *A Survey of Symbolic Logic*, New York: Dover Publications, p. 43, 1960 (repr. of 1918 ed.): "Mathematical symbols are introduced but without any corresponding mathematical operations. The sign of equality is used both for the symmetrical relation of equivalent propositions and for the unsymmetrical relation of premises to their conclusion."

[29]De Morgan to Boole, 28 November 1847, draft of a letter not sent, *Boole-De Morgan Correspondence*, pages 26–27: "With regard to the syllogistic process, there are unexplained difficulties about v and about division by y." In a recent article, Daniel Merrill shows that De Morgan's "logic of complex terms", which he had sketched in [FL] and which provides the context for this comment, was as far-reaching and robust as the system Boole had published (at virtually the same time) in *The Mathematical Analysis of Logic* (1847), and indeed comprised a "lattice-theoretical formulation of Boolean algebra". See Daniel D. Merrill, "Augustus De Morgan's Boolean Algebra", *History and Philosophy of Logic*, 26: 75–91, May, 2005. Merrill's argument helps explain why De Morgan's reaction to Boole was highly respectful but not overly enthusiastic.

picture behind the puzzle; although his methods were symbolic, the ultimate justification for all of De Morgan's work lay always in the meanings that underwrote his symbolic manipulations.

In this brief introduction to De Morgan's logic, we shall consider three of De Morgan's chief contributions in successive sections. In the first we shall explore the ways he created a purely symbolic and instrumental notation, taking his cues from developments in symbolic algebra, but inventing a completely independent symbolic system. In the second we shall consider the ways he used his notation to rework and extend the world of Aristotelian syllogisms, which led to new interpretations of contraries and contradictories and to considering as valid a set of "strengthened" syllogisms that lay beyond the pale of tradition. Finally, we shall explore the way, in one of his last papers, he expanded his work to embed Aristotle's work into a more expansive logic of relations. Led by the logic of his progressive teaching and research concerns, especially as they revolved around the cultivation of reason, De Morgan's historical accomplishment was to have nudged steadily the concerns of logic from those of natural language to a symbolic, systematic reckoning of common thought.

2 THE "ACTION OF THE MACHINERY"

When in the second of his major logic papers [S2] De Morgan spoke of developing an "*algebra* of the laws of thought", he signaled his intention of approaching the subject with the same inspiration that had propelled algebra beyond its arithmetic roots.[30] He habitually described that process by invoking the metaphor of a machine. In his hands, "thought" would be characterized by the "parts of its machinery" and logic would come to mean the mechanical operations of these various parts. The machine metaphor worked as well to break logic into its form and matter, which specified two intimately intertwined but analytically distinguishable parts of thought. In logic, the "action of the machinery" governs the "form" of thought — its mechanical or "instrumental" operations — as distinct from the matter of thought. And because the machinery can be separated from the matter, we can even "watch the machine in operation without attending to the matter operated on".[31]

Ever as playful as he was serious, De Morgan portrayed the machinery of logic as a "nut-cracker". Take "two levers on a common hinge", he wrote, and "put a bit of wood between the levers to represent ... any ... kind of [edible] nut." The logician calls this the "*form* of *nut-cracking*" and we might even imagine it to be the "*pure form* of nutcracking", until we think about other nutcracking variations — "the screw, the hammer, the teeth, &c." Only then can we detect the really pure form, which applies to all the various forms of nutcracking — that is "strong pressure applied to opposite sides of the nut."[32] In this way, logical form followed

[30][S2, p. 79] [OS, p. 22] (ital. De Morgan's).
[31][S3, pp. 174–179] [OS, pp. 74–78].
[32][S3, p. 174] [OS, p. 75] (ital. De Morgan's).

the same sort of progression De Morgan had earlier come to appreciate in algebra, which he characterised as the "abstraction of the instrument from the material."[33]

De Morgan took pains to explain himself here because in his mind the developments in symbol use and interpretation in algebra suggested comparable advances might be possible in logic. Beginning with his 1830 *Study of Mathematics*, the machine imagery and its conceptual ties to "form" infused and informed De Morgan's mathematical work. There he used the term "form" to mean the manipulation of numerical or algebraic equations according to "mechanical processes".[34] Further, he distinguished between correct and incorrect "forms" in both arithmetic and algebra in order to tackle the problem of negative numbers. To take one of his examples, in arithmetic $3-8=-5$ can be simply transformed into $8-3=5$ using a mechanical manipulation of symbols; likewise, algebraically, "$a-b=-c$ can be written in its true form, or $b-a=c$".[35] For De Morgan, who had yet to assimilate Peacock's principle, these examples illustrated the way that the form of an equation could be reworked and understood in such a way as to retain the "positive" content of mathematics. In both cases the "impossible", negative result of the first equation could be simply eliminated through the action of formal, mechanical processes. When De Morgan embraced Peacock's principle, some five years later, it meant for him the freedom to press on with more sophisticated explorations of the forms, or machinery, of algebra and higher mathematics without fussing excessively over meanings. Peacock's principle promised the receptive, young mathematician that true forms could always be made to yield positive results and that equations could always be translated into their true form.

As he wielded Peacock's principle over the course of the 1830s, De Morgan began to perceive the distinction between matter and form as permeating many and varied levels in the hierarchies of mathematics. He found this distinction even in the "first element of mathematical process" itself, which was "the separation of space from matter filling it, and quantity from the material *quantum*: whence spring geometry and arithmetic, the studies of the laws of space and number."[36] Space was the form of matter and quantity the machinery whereby one counted things (added, multiplied, calculated in various ways). In arithmetic, changing the form of an equation to produce a desirable, positive result required mechanistic reductions, eliminations, and the like on both sides of the equation, even while the material content or meaning or "*quantum*" of the equation stayed the same.

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[33] [S3, p. 176] [OS, p. 77].

[34] Augustus De Morgan, *On the Study and Difficulties of Mathematics*, London, 1831; reprint ed. Chicago: The Open Court Publishing Company, p. 175, 1902 [usually abbreviated as *The Study of Mathematics*, and hereafter 'SDM']. It bears remembering that we get 'algebra' from the title of Al-Khwarizmi's treatise *Hisab al-jabr w'al-muqabala*, which translates as the "calculation of reduction and confrontation". *Jabr* designates the practice of transferring negative terms from one side to the other of an equation and *muqabala* refers to canceling similar terms on both sides. In these and other, comparable instrumental manipulations, algebra carried with it a purely 'mechanical' component from its inception.

[35] [SDM, pp. 102–109].

[36] [S3, p. 176] [OS, pp. 77–78].

Indeed, it was the basic fact that the matter of an equation remained the same regardless its form that allowed one to discern the initial separation of the two. Addition and subtraction tendered the mechanisms of variously combining any number of objects independently of any properties belonging to the objects. Thus equations that included the number seven could be manipulated regardless of whether the objects being considered were seas, sins, or wonders of the world.

With matter and form partitioned in this way it became possible, as Peacock had demonstrated, to adopt the arithmetic form for algebraic equations and concentrate on manipulating them without worrying about their arithmetic (or geometric) meanings or instantiations, thus making "distinctions which are of form in arithmetic become material in algebra".[37] Later, writing for the popular *English Cyclopedia* in 1860, De Morgan illustrated his point. "Pass into algebra, and the differences which are formal in arithmetic become only material: thus $8 + 4 = 4 + 8$ is but one material instance of the form $a + b = b + a$."[38] The hierarchy of form and matter continued as the "lower forms in algebra become material in the algebra of the functional symbol". Moreover, De Morgan explained, "the functional form becomes material in the differential calculus, most visibly when this last is merged in the calculus of operations."[39]

De Morgan's conception here may be illustrated with a simple differential equation or derivative. A derivative takes a function of x, $f(x)$ — to take the particular case of a parabola, $y = x^2$ — and subjects it to other procedural rules, i.e. calculating the changes in the dependent variable y with respect to the independent variable x. So, if $f(x) = x^2$, then, after a series of mechanical operations, it can be shown that the changes with respect to y are twice those of x or $\frac{dy}{dx} = 2x$. During this process the entire function, x^2, becomes the subject matter for the formal, mechanical procedures of the derivative itself. In various passages De Morgan spoke frequently of abstracting the "instrument from the material" in mathematics. From "particular arithmetic" to "universal arithmetic" to "single algebra" to the algebra of "functional symbols" to calculus and beyond, De Morgan saw mathematicians to be constantly, though often unwittingly, separating form and matter.[40]

For De Morgan each separation of form and matter revealed an increasing level of mathematical abstraction. The term 'abstraction' itself derives from the Latin verb *abstrahere*, ("to pull", "drag", or "draw away from") and carries the twofold mental activity of (a) "drawing away from" experience or some portion of it, and (b) "pulling" or "dragging" something — a pattern, a procedure, a class — out

[37] [S3, p. 176] [OS, 77–78].

[38] "Logic", p. 248 . De Morgan's entire treatment of form and matter in mathematics resonated closely with the account of mathematical abstraction earlier developed by d'Alembert and other French mathematicians. See [Panteki, *op. cit.*] for a thorough treatment of the French influences on De Morgan's early career. For d'Alembert's depiction of mathematical abstraction, see Michael E. Hobart, "The Analytical Vision and Organisation of Knowledge in the *Encyclopédie*", *Studies on Voltaire and the Eighteenth Century*, no. 327: 147–175, 1995.

[39] [S3, p. 176] [OS, p. 78].

[40] [S3, p. 176] [OS, pp. 77–78]; see also Merrill ([ADM, p. 100]).

of experience. De Morgan saw the mathematical hierarchy as one of a growing abstraction of identifiable mechanical or instrumental procedures and processes. At each new level, these mechanisms of calculation became the abstraction — both the drawing away and pulling out — from the matter that stayed behind. Recalling the nutcracker, its form entails the mechanism of two levers on a common hinge, while two sticks (of whatever material) comprise its matter. Placing the sticks so as to produce levers turns them from simple materials into a simple machine; it gives them a mechanical form and function. This mechanism becomes more complex when the both levers are situated on a common fulcrum or hinge, thus making a nutcracker. But then the "form" of the hinged levers comes to be seen as only one among several variations of nutcracking mechanisms — screw, hammer, teeth, and the like — which allows one to elicit an even more general and abstract mechanism, the "pure form" of "strong pressure applied to opposite sides of the nut".[41]

In mathematics, the crucial distinction between form and matter meant not only that the mind could separate the two, but that it could critically examine each of them distinctly from one another. This is how the "action of [thought's] machinery" became "more visible in algebra than in other thought".[42] With the machinery out in the open, De Morgan could investigate more closely its properties, the intermeshing of its parts and levers and gears. Thus, as a function of thought itself, abstraction — the drawing away from and pulling something out of experience — made it possible to understand how thought worked by bringing it from nocturnal obscurity, as it were, into the daylight of examination.

De Morgan's understanding of form and matter continued to be fundamental throughout his life's work; it both provided the rationale for his teaching and was the motivating force behind his logical efforts. He taught his students how to think by presenting them with thought's machinery in its most exposed, available, and visible manifestations. First in arithmetic, then through algebra to calculus and beyond, he focused their attention on the mechanical or instrumental forms of thought. Likewise, De Morgan carried the form/matter distinction with him into logic, where his guiding insight would be that logic is to language as algebra is to arithmetic.

Pervading all of De Morgan's thinking about mechanical, mathematical forms is a workable, symbolic system. Whenever he, or his students, changed an equation like $a - b = -c$ into $b - a = c$ they did so by manipulating as and bs, equals signs, plusses and minuses, according to a fixed sets of rules. And as he turned his attention increasingly to logic, one of his central concerns lay with developing a notation that would function comparably. In his 1847 book, *Formal Logic*, he introduced some symbols — the parenthesis, the dot, and the colon — as "abbreviations" for logical processes.[43] These efforts represent De Morgan's first

[41]In [S3, p. 176] [OS, pp. 77–78], De Morgan distinguished between the "abstraction of colleague qualities" and abstraction of the "instrument from the material". The former describes how we devise terms; the latter how we relate them through logical form or machinery.

[42][S3, p. 179] [OS, p. 82].

[43][FL, p. 126]: "Let the following abbreviations be employed:

attempt to use various, essentially diacritical symbols both to express propositions used in traditional, syllogistic reasoning and to introduce distinctions hitherto left blurred.[44] He used these symbols only sporadically, though, and only as shorthand, memory devices, never as instruments of manipulation.

Even before his book was published, De Morgan was pushing ahead with further logical investigations. A major part of the expanded logical program De Morgan developed in his papers focused on experiments with symbol construction and manipulation. In April of 1849, he was deeply engaged in devising a more comprehensive symbolic system; "all inference consists in four parentheses and two dots, at most", he wrote excitedly to his former Cambridge teacher, William Whewell. By that fall the abbreviations of *Formal Logic* were evolving into a carefully defined set of symbols that enabled De Morgan to capture and manipulate instrumentally the propositions used in syllogisms.[45] And by December he was boasting: I have "got a notation" for "my own [logic] so easy that I find getting out the more difficult cases of syllogism easier by *beginning [with] the symbol in my head* — and detecting the symbol of inference — than by thought".[46] The next year (1850), in several passages of [S2], De Morgan introduced publicly his new notation and explained how to read and use it.

The shape of De Morgan's new program can be seen in his demonstration of the advantages it offered over the traditional designations of propositions as A, E, I, O. These vowels refer to the traditional foursome of propositions used in framing syllogisms. A proposition is a statement with two terms — a subject (either universal or particular, all or some), generally denoted by X, and a predicate, generally indicated by the letter Y. Thus 'All humans are animals' connects the subject 'humans' with the predicate 'animals'. It is written 'All X is Y', with 'X' and 'Y' serving as placeholders for different terms, in this case 'humans' and 'animals', and is interpreted as claiming that any being belonging to the category 'humans', also belongs to the class of 'animals'. In traditional logic, A and E are universal propositions, denoting, 'All X is Y' and 'No X is Y' respectively, while I and O denote the particular propositions 'Some X is Y' and 'Some X is not Y' respectively. The letters A and I come from the vowels in the Latin *affirmo* ('I affirm'), making A and I the affirmative propositions, while the E and O stem from *nego* ('I deny') indicating the negative propositions.[47] For centuries the A, E, I, O propositions had been thought to be the building blocks of logical reasoning, but in 1850 De Morgan rejected them as foundational because they were "rather mnemonical than instrumental".

$X)Y$ means 'Every X is Y' | $X.Y$ means 'No X is Y'
$X : Y$ — 'Some Xs are not Ys' | XY — 'Some Xs are Ys' "

[44] [S2, p. 85] [OS, pp. 29–30].

[45] Augustus De Morgan to William Whewell, April 1, 1849. Trinity College, Cambridge University, Whewell Archive. Add Ms. a 202^{113}. Unless otherwise noted, all of the De Morgan-Whewell letters cited in this paper are from this archive.

[46] Augustus De Morgan to William Whewell, December 31, 1849, Add Ms. a 202^{118} (ital. De Morgan's).

[47] ["Syllabus", p. 157]

More to their detriment, the A, E, I, O symbols harbored "compound" meanings. By this, De Morgan meant they confounded quantity with affirmation or negation. The A statement, for example, is both affirmative and universal. These characteristics needed separation. "If A and E had stood simply for affirmative and negative, and two consonants, as B and N, for universal and particular, the distinction of figure might have been symbolized." In this case 'AB' would have symbolized a universal, affirmative proposition, 'AN' a particular affirmative, 'BE' a universal, negative proposition, and so forth . But such development historically had not occured, with the consequence that each proposition, represented by a single letter, continued to possess — confusingly — both quantity and quality (i.e. the qualitative judgement of affirmation or negation).[48] To make logic truly formal, which for De Morgan meant instrumental, required capturing two essential, analytic steps. First, the subject and predicate of a proposition had to be invested with quantity — 'some' or 'all' (particular or universal) — in such a way as to be independent of one another and of the proposition in which they resided. Then, each quantified term had to be logically linked to the other quantified term by means of inclusion or exclusion, that is affirmation or negation.

The instrumental trick lay with capturing these analytical requirements symbolically. For quantification, De Morgan used parentheses: ')' or '('. He had first introduced these symbols in *Formal Logic*, but in [S2] he set about correcting what he saw as a "symbolic want" of his earlier work by developing and explaining them as part of a functional system.[49] Orienting the parenthesis so as to "inclose" a term, as in 'X)' or '(X', denoted that the "name-symbol X" enters the proposition universally, which may be read as 'all X'. Logicians now commonly refer to this as a "distributed" term. To imagine inclosing or inclusion, De Morgan added, picture the parenthesis as part of an oval drawn around the term, and containing all the designated items within it, much like a Venn diagram.[50] Conversely, when the parenthesis excluded a term, as with ')X' or 'X(', it indicated that the "name-symbol X" enters the proposition particularly, which may be read as 'some X'. This is an "undistributed" term in modern parlance. Take for instance the proposition "All men are animals". Letting X stand for 'men', then 'All men' is captured by 'X)'. The proposition affirms that all men are included in the class of animals, but because there may be more animals than men, 'men' is included only in the class of some amimals, which the notation expresses with ')Y' (Y standing for 'animals'). The entire proposition is thus symbolized as '$X))Y$'. To make clear the second step of affirmation and negation, De Morgan introduced dots. An even number of dots or no dots at all denotes affirmation or agreement of the two terms; an odd number of dots, usually one, signifies negation or non-agreement. For example, '$X))Y$' means 'All Xs are [some] Ys', whereas '$X(\cdot(Y$' means that

[48][S2, p. 86] [OS, p. 30]. Along with William Hamilton, among others, De Morgan believed the absence of quantification in the predicate of these traditional propositions rendered the building blocks inadequate for developing syllogistic argument.

[49][S2, p. 91] [OS, p. 36].

[50]The similarity is striking, but De Morgan is here writing long before Venn (1834–1923) was either an adult, or had developed his diagrams.

'Some Xs are not [all] Ys'.

With this symbolic system, De Morgan was at last able to bring to logic the separation of formal from material that had been so powerful in algebra. He did this by dividing the matter, or "name-symbols" $(X, Y, Z$, etc.$)$ from the functional symbols (parentheses and dots) of their quantification and of their affirmation or negation. He even started using the symbols by themselves, without the Xs and Ys, leaving the reader free to fill in the name-symbols: "Thus '$X))Y$' means that all Xs are Ys; '$X(\cdot(Y$' means that some Xs are not Ys: but '))' and '(·(' specify the characters of the propositions; as do also '((' and ')·)'." Moreover, his functional symbols began to show a bit of their flexibility and eventually some robustness, for they could be read "either way: thus '$X))Y$' and '$Y((X$' both denote that every X is Y."[51]

Thoroughly versed in the history of logic, and especially of earlier attempts at using symbolic notation,[52] De Morgan began immediately to pursue the mechanical advantages of his symbolic system. He was especially concerned to demonstrate its superiority over that of his rival, the Scottish logician William Hamilton, who had also been working on quantifying predicates in a proposition. Hamilton had devised a notation that, in De Morgan's view, assumed the forms of predication, whereas his produced them from more primitive, notational bases. This allowed De Morgan greater flexibility and accuracy, and created what he termed the "numerically definite syllogism" (discussed below). In his words, "complete quantification of both terms [subject and predicate] was derived from the algebraical form of numerical quantification."[53]

With these innovations, De Morgan had devised for himself the rudiments of a notation that allowed for the same sort of "mechanical" operations that algebra had provided in mathematics. In years to come, he was to name his new symbology "spicular", "a name first given in derision [by Hamilton], but not the worse for that: it is better than *parenthetic*, which has a derived meaning."[54] The word 'spicular' means 'bracket'; it derives from the Latin '*spica*' which denotes an ear of grain. The curved shape of the ear presumably yields the connection. De Morgan's care in picking the term that would describe his notation reflects its importance; with it he had found the means of resolving problems in traditional, syllogistic reasoning

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[51] [S2, p. 87] [OS, p. 31].

[52] In [S2, p. 87] [OS, p. 32], De Morgan made references to the earlier systems of "Lambert and Euler", which he had also mentioned in *Formal Logic*. Among his other accomplishments, Johann Heinrich Lambert (1728–1777) had devised a pictorial representation of propositions, using lines in a similar fashion as those adopted by De Morgan. Thus 'All X is Y' is represented as two lines Y——y, and X——x:

Y ———————————————— y
X ——————————— x

Visually, one can see the inclusion of X in Y. Similarly, the mathematician Leonhard Euler (1707–1783) had used circles to capture the same visual representation ⊚. Here the outer circle represents Y, the inner circle X; hence 'All X' is included in Y.

[53] [S2, p. 90] [OS, p. 35].

[54] [S3, p. 198] [OS, p. 106]. See also ["Syllabus", pp. 157, 163, 203 n.1].

using "mechanical modes of making transitions."[55]

The first significant results that fell out of the instrumental manipulations of De Morgan's spicular notation were the "erasure rule" and the "transformation rule". These rules can perhaps best be understood by examining a table he presented in his retrospective "Syllabus of a Proposed System of Logic", published in 1860. There, looking back on a decade of logical thought, De Morgan summarized in tabular form the array of valid syllogisms he had generated from his notation in [S2]. With this table, De Morgan presented in complete form what he came to call the "universe of the syllogism" embraced by his functional, spicular notation.[56] We have reproduced the table here as Table 1 (adding labels for reference) as a convenient and useful introduction to his logical system.[57]

To see the "erasure rule" in action, note first that in the Table each set of symbols — a through ff — depicts an argument in syllogistic form. That is, each argument is articulated through two propositions, the syllogism's major and minor premises. For example, the argument labeled s is presented as ')))·)' (we have added some spacing for clarity).[58] To read the argument, insert the "name symbols" of X, Y, and Z, around and between the functional symbols. This produces: '$X))Y)\cdot)Z$' and may be read as a traditional syllogism:[59]

Major premise: All X is Y
Minor premise: Some Zs are not Ys

Now, the reader supplies the conclusion, the inference derived from the premises:

Conclusion: Therefore, Some Zs are not Xs

The "canon" of inference, he wrote, "in the Aristotelian system and in my extension" is to eliminate the middle term from the argument, generally denoted by Y. Using the spicular notation made the elimination mechanical: "*Erase the symbols of the middle term, the remaining symbols shew the inference.*" Thus, he continued, ")))·) gives)·)", or in the expanded version, $X))Y)\cdot)Z$ gives $X)\cdot)Z$.[60]

[55] De Morgan to Boole, 28 November 1847, *Boole-De Morgan Correspondence*, p. 25.

[56] [S2, p. 87] [OS, pp. 31–32].

[57] ["Syllabus", p. 162].

[58] See also [S2, p. 87] [OS, p. 31].

[59] In traditional syllogistic reasoning, this is the form of argument known as "Baroko", in reference to the names used "for many centuries" by logicians. Forms of argument were designated by their mood and figure. The mood referred to the order of names in propositions, while the figure specified the position of the middle term. For example, the argument, 'All Y is Z', 'All X is Y', therefore 'All X is Z' is composed of three A (universal, affirmative) statements, which give it the mood AAA. The middle term is the subject of the major premise and the predicate of the minor, making it a first figure syllogism. It is designated AAA-1, and called "Barbara". The valid forms all carried their own names, names De Morgan took "to be more full of meaning than any that ever were made." Thus did he reaffirm his ties with tradition. [FL, p. 150].

[60] [S2, p. 87] [OS, p. 31] (ital. De Morgan's). De Morgan's example here might seem confusing at first for it reverses the order of the Zs and Ys in the minor premise. But, as he noted, the symbols could be read either way. See Table 2 below for a list of propositions in both his spicular notation and common language.

Table 1. Syllogisms

Premises	Strengthened particular	Minor/Major particular	Universal	Major/Minor particular	Strengthened particular
		-*a*- ()))	-*b*-))))	-*c*-)))(	
Affirmative	-*d*- (( ))				-*e*-)) ((
		-*f*- ((()	-*g*- ((((	-*h*-)(((	
		-*i*- (·((·)	-*j*-)·((·)	-*k*-)·((·(	
Negative	-*l*- (·) (·)				-*m*-)·( )·(
		-*n*- (·))·)	-*o*- (·))·(	-*p*-)·))·(	
		-*q*- ())·(	-*r*-)))·(	-*s*-)))·)	
Affirmative Minor	-*t*- (( )·(				-*u*-)) (·)
		-*v*- (((·(	-*w*- (((·)	-*x*-)((·)	
		-*y*- (·(((	-*z*-)·(((	-*aa*-)·(()	
Affirmative Major	-*bb*- (·) ((				-*cc*-)·( ))
		-*dd*- (·))(	-*ee*- (·)))	-*ff*-)·)))	

In this way one may interpret all thirty-two syllogisms of Table 1 as depicting a valid argument.[61]

De Morgan's "transformation rule" emerged in the context of considering contraries (or complements as these would later be called). For the latter De Morgan used lower case letters (xs and ys, etc.). The contrary of X was thus expressed as x, and interpreted to mean that within the universe of the term and its contrary if all elements fell within the term, X, it would be a contradiction to claim that some fell without, i.e. in x. Conversely, the existence of any or some xs (non-Xs), meant that not all elements in the universe under consideration were X (all Xs). The "rule of transformation", from positive to negative and from universal to particular was the following: "To use the contrary of a term without altering the import of the proposition, alter the curvature of its parenthesis, and annex or withdraw a negative point."[62] For example, 'All Xs are some Ys' equals logically 'All Xs are not all ys', or with the notation: $X))Y = X)\cdot(y$. In the latter expression the parenthesis attached to Y has been reversed, the Y converted to the lower case, contrary y of negation, and the dot of negation added (thus producing the logical double negative, which equals a positive). Rhetorically, this reads "All Xs are not [all] non-Ys".

In [S2], De Morgan demonstrated in detail how his transformation rule generated equivalencies in both universal and particular propositions. The universal propositions begin with $X))Y$ ('All Xs are some Ys' or, an alternative reading, 'Every X is Y'). This yields $X)\cdot(y$ in the manner described above. Now take the next step; reverse the parenthesis on X, change the X to lower case and remove the dot, giving another equivalency: $x((y$. And, once more, change the y to Y, reverse the spica, and now "annex the negative point" for $x(\cdot)Y$. This produces the following array of equivalencies: $X))Y = X)\cdot(y = x((y = x(\cdot)Y$. Using the transformation rule one can manipulate particular propositions in a like fashion to generate comparable equivalencies among them. Begin with 'Some Xs are not all Ys' — i.e. $X(\cdot(Y$ — and proceed step by step as above to net the following: $X(\cdot(Y = X()y = x)\cdot)y = x)(Y$.[63]

The erasure and transformation rules were the functional building blocks of the "universe of the syllogism"; they were the logical machinery now exposed to view. De Morgan's rules permitted him to express any proposition using universal or particular terms and/or their contraries in either the subject or the predicate and to derive a host of inferences in the process. Indeed, he viewed these two rules as the logical equivalents of reduction and cancellation in ordinary algebra, and frequently compared "inference" in logic with "elimination" in algebra: "speaking instrumentally, what is called elimination in algebra is what is called inference in logic."[64]

Building on the mechanisms of the erasure and tranformation rules by filling

[61] De Morgan provided a comparable table of these arguments in [S2, p. 95] [OS, p. 41].
[62] [S2, p. 92] [OS, p. 37].
[63] [S2, p. 92] [OS, pp. 37–38].
[64] [S2, p. 83] [OS, p. 27].

in the name terms as X, Y, x, or y for both the subjects and predicates, and by designating X,Y, x, and y either universal or particular with the parentheses, De Morgan was able to generate an entire table of propositions, which we have reporduced as Table 2, that utilized his new notation.[65] Note here that with the utilization of contraries, all the universals become variations of a single expression,)), while all the particulars derive from (). De Morgan situated these two strings of equivalences prominently on Table 1, with the arguments comprised of universal propositions listed in the central column, while those with particular propositions made up the columns immediately to either side.

Table 2. Propositions

Spicular notation	Proposition expressed in common language
	Universals
$X))Y$	Every X is Y
$x))y$ or $X((Y$	Every Y is X
$X))y$ or $X)\cdot(Y$	No X is Y
$x))Y$ or $X(\cdot)Y$	Everything is X or Y or both
	Particulars
$X()Y$	Some Xs are Ys
$x()y$ or $X)(Y$	Some things are neither Xs nor Ys
$X()y$ or $X(\cdot(Y$	Some Xs are not Ys
$x()Y$ or $X)\cdot)Y$	Some Ys are not Xs

What is more, De Morgan elicited further implications from the mechanisms of these propositions, which were reminiscent of his earlier work on the true form of algebraic equations. For example, he noted, the foregoing abandoned the distinction of affirmative and negative "in the usual sense" because for any affirmative proposition, one could produce its negative equivalent. To wit, the affirmative proposition $X()Y$ ('Some Xs are [some] Ys') could also be expressed in its negative equivalent, $X(\cdot(y$ ('Some Xs are not [all] non-Ys'). Similarly, one could alter one of the quantities in a universal proposition, either from universal to particular or from particular to universal, and the results remained a "true deduction, though not an equivalent". For example, reversing one of the parentheses in $X))Y$ produces both $X()Y$ and $X)(Y$.

Manipulating the spicular notation mechanically not only led to the above list of eight propositions and an entire table of valid arguments, it enabled De Morgan to discover a "canon of validity" that pertained to the entire set of arguments, although he never drew clearly the distinction between a canon of validity and

[65] [S2, p. 91] [OS, pp. 35–36].

a canon of inference.[66] (In fact, De Morgan's Peacockian approach meant that he seldom constructed proofs for his logical inferences and principles — even the theorems that carry his name.) To see how this works, let us recall that a syllogism has three name-terms (X, Y, Z) along with their contraries (x, y, z). These may be expressed as an array: X or x, Y or y, Z or z, because, in De Morgan's view, the alternate forms of the terms (e.g. changing from X to x) have no effect on the "character of the proposition".[67] From this set, there are eight possible variations, using one term out of each pair and combining it with others. We may illustrate this in the following table of our own composition:

Table 3. Terms and Contraries

	1	2	3	4	5	6	7	8
X	×	×	×	×				
x					×	×	×	×
Y	×	×			×	×		
y			×	×			×	×
Z	×		×		×		×	
z		×		×		×		×

Here each column represents a syllogism created by employing one of the terms from each of the pairs, $X - x$, $Y - y$, $Z - z$. From this grid, and assuming the first two terms are distributed (enclosed by the parenthesis), one can derive eight universal syllogisms, those with two universal premises and a universal conclusion, all variations of the universal form:)))) =)). Further, one can produce sixteen particular syllogisms, each with one universal premise and one particular premise and a particular conclusion. The number sixteen derives from the eight syllogisms whose major premise is a universal proposition and is then followed by a particular, plus eight more that begin with a particular and are followed by a universal. The forms followed by the particular arguments are: ((() = () (beginning with the universal premise) and ())) = ( ) (beginning with the particular premise). Finally, the grid also spawns eight further syllogisms, starting with universal premises but yielding particular conclusions. Their spicular form is: (( )) = (). These De Morgan called "strengthened arguments".

For all thirty-two arguments, the "canon of validity" was the same. Remember an inference in a syllogism means eliminating the middle, Y term (the erasure

[66] See [S2, p. 94] [OS, p. 40]. Here De Morgan depicted both the "canon of validity" and the "canon of inference" with perhaps their sharpest contrast. The former he designated as a means of identifying a valid syllogism; the latter comprised the rules for manipulating the symbols in the premises in order to produce one. The "canon" terminology is murky, for, as we note below, the canon of validity entails the erasure and transformation rules that make up inference. Later, in the ["Syllabus", p. 161], De Morgan abandoned "canons" and referred simply to the "test" of validity and the "rule" of inference, a modest linguistic improvement.

[67] [S2, p. 94] [OS, p. 39].

rule). Changing Y into y occurs when one flips both of the parentheses, and therefore does not "affect" the "relative character" of the terms. So, one can use a combination of the erasure and transformation rules to eliminate either the Y or the y term from the argument. If the middle parentheses surrounding the Y term turn the same way — i.e. $)Y)$ or $(y($ —, any two propositions that fit the arrangement yield a valid argument, when at least one of the propositions is universal. And when the middle parentheses turn contrary ways — i.e. $)y($ or (Y) — one must have two universals for a valid argument. The canon of inference that produces the validity in all these cases is simply "strike the middle parentheses" (the erasure rule) and "*two* negative dots, if there be two" (the transformation rule). The remaining symbol exhibits the valid inference.

Taking another look at Table 1, we can see how De Morgan arranged the relations between these groups of arguments. The central column utilizes the universal propositions —)), ((,)·(, (·) — and the eight universal syllogisms they generate (arguments *b*, *g*, *j*, *o*, *r*, *w*, *z*, and *ee*). The columns to the right and left of center display both the particular propositions — (),)(, (·(,)·) — and the sixteen particular arguments utilizing one particular premise (along with one universal premise). The four arguments to the extreme right and left of the central columns (arguments *d*, *l*, *t*, *bb*, *e*, *m*, *u*, and *cc*) are his "strengthened" arguments.

If the spicular notation provided the symbolic counterpart to algebra, De Morgan often employed as well a variety of solid and broken lines, thick and thin, to provide, as he put it, an equally important "graphical representation which is as suggestive as a diagram of geometry".[68] Using an array of lines and bars De Morgan experimented with these "pictorial" representations of arguments, one of which we have provided as Figure 1. To see how the graphics work, we have expanded the argument *s* and added the name symbols.[69]

All the lines (X, Y, Z) combine to represent what De Morgan called the "universe of the syllogism", with each line representing a name or term, and each pair of lines depicting the "universe of the proposition". Here the lines are divided into a dark or shaded portion; elsewhere he used a straight line divided into a continuous and dotted portion (e.g., $X|$ ————· · · · · · ·). The division of lines distributes the "universe of the term" into a "name and its contrary", the name being the darker (or solid) portion of the line and the contrary all the rest (or the dotted portion). (From *Formal Logic* on, De Morgan used "contrary" and "contradictory" as synonyms, on which more below.) To assert a proposition, one need merely inspect visually the shaded and unshaded portions of two lines. Thus in the example, the shaded portion of line X falls entirely within all the shaded portion of Y, capturing the first premise:)), or $X))Y$, or 'All Xs are Ys'. Likewise, the second premise may be seen in the overlap of shaded portions between Y and Z,

[68] [S2, p. 87] [OS, p. 32].

[69] This square is actually taken directly from one of the several sheets of logical diagrams De Morgan had drawn by hand and pasted into his own copy of [FL], this one on page 92. His personal copy of [FL] may be found in the De Morgan collection of the University of London Library.

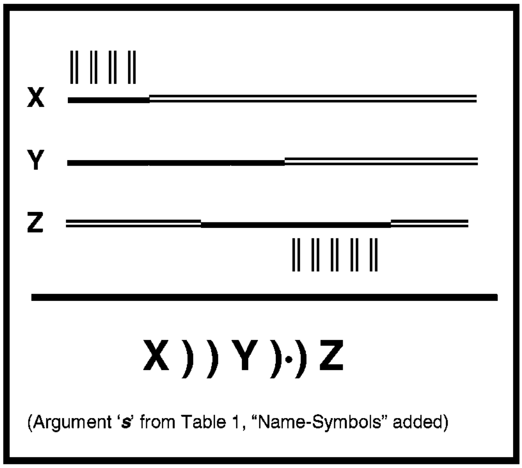


Figure 1.

which nets $Y)\cdot)Z$ (or 'Some Zs are not Ys'). The conclusion of the argument may be discerned by examining the short, vertical lines above X and below Z. These represent individuals embraced within the term as captured by the propositions. In $X))Y$, all the members of X are included in Y and are shown by the vertical lines above all 'All X' (when extended down to the shaded portion of the Y line). In the minor premise, $Y)\cdot)Z$, we see vertical lines beneath that portion of shaded Z which does not overlap the shaded portion of Y, thus depicting those Zs that are not also Ys (again, extending the vertical lines up to Y). The conclusion, then, may be observed by noting that of the vertical lines, some of the Zs (below the Z term line) do not overlap with all the Xs. The erasure rule is thus achieved "pictorially" by merely looking at the vertical lines above X and below Z in each of the syllogism boxes.

As the above account of De Morgan's symbology suggests, it is far more cumbersome to describe logical operations in ordinary language than to observe directly the machinery in action. This, of course, was precisely De Morgan's point. With his algebra-inspired spicular notation and his geometry-inspired diagrams, De Morgan's thinking here forecasts the approach to questions of classes and class extension found later in the work of John Venn, Stanley Jevons, Charles L. Dodgson (Lewis Carroll) and others. In his own lifetime, De Morgan's new symbology led him to reinterpret key points of traditional syllogistic, to which we now turn.

3 THE "UNIVERSE UNDER CONSIDERATION"

De Morgan's new, symbolic and functional spicular notation suffices to secure him an important position in the symbolic turn in the history of nineteenth-century logic. Yet beyond experimenting with new notation, he was keenly aware of new logical meanings his symbols had prompted. Indeed, although we have been looking at his logical machinery as separated from the question of its interpretation, this does not reproduce how De Morgan himself approached the subject. Rather, in his mind the formal and material, symbolic and interpretive dimensions of logic were always intertwined, though, as he always insisted, separable for analytical and pedagogical purposes.

One of the most exemplary manifestations of this interpretive effort may be found in De Morgan's idea of "universe" and its implications for recasting the traditional distinction between 'contrary' and 'contradictory'. For over a century, historians and logicians have been attributing the phrase "universe of discourse" to De Morgan.[70] Theirs is undoubtedly a respectful error, but in fact De Morgan never used this phrase. Instead, he spoke of a "universe under consideration", by which he meant those objects that are expressed by the name terms (X, Y, Z, etc.) and manipulated in his formal logic.[71] It was Boole, not De Morgan, who first employed the phrase "universe of discourse" in describing how to interpret his system. Boole's universe of discourse derives from a process of abstraction and generalization that begins with naming terms "representative of an intellectual operation, that operation being also prior in the order of nature". One begins, so to speak, with the operations of thought, and then proceeds to apply them to the world beyond. Here, the name terms represent either the entire universe of discourse (X) or nothing (0), and "Nothing and Universe are the two limits of class extension".[72] De Morgan's unwillingness to adopt Boole's phrase reveals again his Aristotelian insistence that thought always points to some reality outside itself.

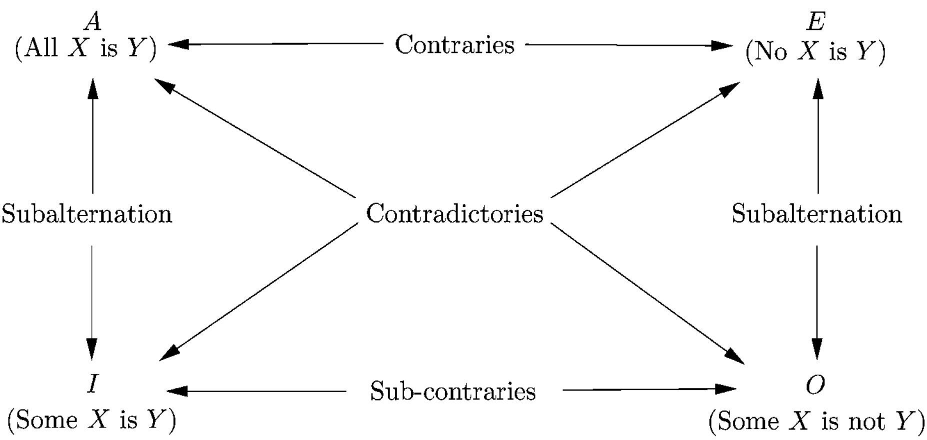
[70]Thus, C. I. Lewis, in his classic [op cit., p. 43], writes that among De Morgan's contributions of "permanent value" is the idea of the "universe of discourse". Lewis then cites a long passage from *Formal Logic* to buttress his claim, but nowhere in the passage does De Morgan use the phrase, nor as we shall see in our analysis below does he embrace the idea in the same sense as Boole and subsequent logicians. Even Peter Heath, in his otherwise excellent introduction, cites De Morgan as proposing a "universe of discourse", giving page two of his own collection (citing De Morgan's [S1]) as a reference, where the expression is nowhere to be found. See "Introduction", in Augustus De Morgan, *On the Syllogism and Other Logical Writings*, Peter Heath (ed.), New Haven CT: Yale University Press, pp. xxv and 2, 1966. Likewise, the Kneales (William Kneale and Martha Kneale, *The Development of Logic*, Oxford: Clarendon Press, p. 408, 1962) cite De Morgan's *Formal Logic* (p. 55) as expressing the "universe of discourse", whereas De Morgan's phrase there is "universe of a proposition". More recently Volker Peckhaus refers to "De Morgan's universe of discourse" in "[op. cit., p. 4], while the author of the *Encyclopaedia Britannica* article on the "History of Logic" claims that De Morgan "introduced" the notion "universe of discourse" that then was used by later Booleans (logic, history of, *Encyclopaedia Britannica* from Encyclopaedia Britannica Premium Service. `http//www.britannica.com/ed/article?tocid=65944`. An exception may be found in John Corcoran's recent "Introduction" to Boole's *Laws of Thought*, New York: Prometheus Books, p. xx, 2003, where he recognizes that Boole authored the expression "universe of discourse" for "the first time in the history of the English language ..."

[71][FL, p. 126].

[72]George Boole, *An Investigation of the Laws of Thought, on which are founded The Mathematical Theories of Logic and Probabilities*, New York: Dover Publications, pp. 43–44, 47, repr. of 1854 ed.

De Morgan developed his idea of a "universe under consideration" in tandem with his reworking of the distinction between contrary and contradictory. In Aristotelian logic a contrary was an apple to a contradictory's orange; they were two essentially different categories of relations between propositions. The contrast between them is most frequently described as part of the Aristotelian "square of opposition". This square which we have reproduced as Table 4, graphically depicts the relations between the four major propositions — A, E, I, O.[73] In this traditional square, contraries, like A and E, connote two propositions both of which cannot be simultaneously true, but both of which can be false. For example, consider 'mammal' as a sub-class of 'animal'. Taken together, the A proposition, 'All mammals are vivipara', and the E proposition, 'No mammals are vivipara', cannot both be simultaneously true, but both can be false. The duckbill platypus renders the first false as does any live-bearing mammal the second. By contrast, contradictories are pairs of propositions that can neither be simultaneously true nor false. As the square indicates, A and O statements contradict one another, as do E and I. Colloquially, this says that if 'All X is Y', it cannot be true that there are some Xs (even one) that are not Y. If it is true that 'all mammals are animals', then there cannot be a single being in the class of 'mammal' that does not also fall in the class of 'animal'. Underwriting these distinctions was the capacious notion of a universe as a "summum genus", filled with beings of all different types, to which names corresponded, and which could be arrayed in a classificatory fashion, a universe whose plenitude was so abundant that contraries were possible.

Table 4. Aristotelian Square of Opposition



In his earliest writings De Morgan appears to have been thoroughly wedded to

[73] See, for example, the standard treatment by Irving M. Copi, *An Introduction to Logic*, 2^{nd} *Ed.*, New York: The Macmillan Company, p. 144, 1961.

the standard segregation of contrary and contradictory. For instance, he devoted Chapter XIV of his 1830 *Study of Mathematics*, to "Geometrical Reasoning" where he grappled with demonstrating how the statements of geometry may be expressed in one of the four traditional propositional forms, A, E, I, and O, and how geometrical reasoning in general might be expressed in standard syllogistic form. Of these "species of assertions" (as he labeled propositions in these passages), the A and E assertions "are called contraries", while the A and O, as well as the E and I are "contradictory".[74]

However, even in this early work, De Morgan was inching toward a later position in which he would collapse the distinction in his notion of "universe under consideration". Again, the mathematical context provided the suggestion. Earlier, we discussed his account of "real" and "incorrect" forms in algebra, in which De Morgan had shown that expressions (like $3 - 8$) that seemed to produce "impossible" quantities (like -5) could be mechanically rearranged $(8 - 3)$ so as to produce a possible (positive) result.[75] He also took another approach. In one of his examples he suggested that the expression $3 - 8$, which produces a negative solution, may be written in a different manner by simply connecting the expression to another number, say 56, which would then yield the unexceptional subtraction $56 + 3 - 8 = 56 - 5$. Here, De Morgan found that changing the mathematical context for framing the original problem led to an acceptable solution. In this example lies the germ of his later idea that enlarging or restricting the "universe" of a putatively intractable expression might render it intelligible.[76]

This germ flowered in *Formal Logic*, where De Morgan announced that "in logic it is desirable to consider names of inclusion with the corresponding names of exclusion". He proposed achieving this end through "inventing the names of exclusion by the prefix not, as in tree and not-tree, man and not-man. Let these be called *contrary* or *contradictory* names." In a footnote he added simply, "I intend to draw no distinction between these words."[77]

Collapsing the long-standing distinction between contraries and contradictories grew out of a problem that De Morgan had found with traditional accounts. Up to his own age, he claimed, logicians had used "such contraries as man and not-man ... [to] mean by the alternative, man and everything else." This practice, however, had "little effective meaning, and no use, in a classification which, because they are not men, includes in one word, *not-man*, a planet and a pin, a rock and a featherbed, bodies and idea, wishes and things wished for." In short, the traditional use of contrary was trivial. Most propositions were much less extensive than the entire universe, and more typically one considered a "range of ideas" as containing "the whole matter under consideration". Further, he remarked, in "such universes, contraries are very common: that is, terms each of which excludes

[74] [SDM, pp. 203–205]; Merrill ([ADM, pp. 26–35]).
[75] See above, p. 294.
[76] [SDM, p. 105].
[77] [FL, pp. 41–42] (ital. De Morgan's).

every case of the other, while both together contain the whole."[78]

Contraries, then, did not embrace the "whole universe", but only one general idea within it. For example, Briton and alien are contraries because every man must be one or the other of the two, and no man can be both. This meant that not-Briton and alien are identical names. The same may be said of other pairings: of "integer and fraction among numbers", peer and commoner among subjects, male and female among animals, and so on. As thus defined, contraries acquire their contradictory status within the universe of the general idea or class term being contemplated (e.g. Briton and alien within the class of men). The "whole idea under consideration *is the universe* (meaning merely the whole of which we are considering parts)." And with "*respect to that universe*", contraries are the names that have nothing in common, but between them they contain the whole idea under consideration (i.e. Britons and aliens have nothing in common with one another, but together comprise all men). To make all this more manageable, De Morgan designated term names by capital letters, their contraries by lower case letters (as with X and x). In this way he formulated what would later be known as logical complementation, with terms and their complements comprising the entire membership of a "universe under consideration".[79]

Restricting the "universe under consideration" was therefore conceptually tied to eliminating the distinction between contrary and contradictory, which De Morgan saw as an important innovation in logic. In passing, it might seem that such a move could turn out to be equally as arbitrary as the usage De Morgan sought to overturn. While certainly true that Briton and alien exhaust the class of men, so too would any pairing of opposing characteristics or features: clad or naked; vocal or mute, men who cap their toothpaste tubes or men who do not (this would entail a further sub-class of men as toothpaste-users or not), and so forth. De Morgan agreed: "We can make a class of any individuals we please. We can imagine a quality by which to describe the class. Any individuals, however capriciously joined, are *logically* a class, ..." His point here was that the universe under consideration depended on the user's designs on the terms (a haberdasher might divide all men into those who wore hats and those who did not), not on whether the terms matched some "real" structure of objects in the outside world. Still, regardless of whether the designs were frivolous or serious, profound or trivial, the terms brought their real 'existence' into logic; as De Morgan declared at the end of his sentence: "... let them be ontologically an unassimilating medley".[80]

Even more telling, this 'existence' extended to contraries, as well as the terms themselves. In fact, whether terms were positive or negative, De Morgan argued, was but "an accident of language". In English one might say "every A is B" while in French the same meaning might be expressed as "no A is b". In short, a term's being contrary has no affect on its existential scope. Consider, he exemplified, the "universe of property". In it "personal and real are contraries, and a definition of

[78] [FL, pp. 45–46] (ital. De Morgan's).
[79] [FL, p. 42] (ital. De Morgan's).
[80] ["Syllabus", p. 167].

either is a definition of the other." At the same time 'personal' and 'real' are not merely the negation of one another; both are "objects from which positive ideas are obtained ... Money is *not* land but it *is* something. And even when the contrary term is originally invented merely as negation, it may and does acquire positive properties." From the foregoing, De Morgan made the point even stronger as he generalized: "... of two contraries, neither must be considered as *only* the negation of the other: except when the universe in question is so wide [i.e. completely unrestricted], and the positive term so limited, that the things contained under the contrary name have nothing but the negative quality in common," a negative quality that in effect becomes trivial, outside of the more restrictive, "universe under consideration".[81]

Terms and their contraries, then, are elicited from a larger universe, and are restricted to lying within the smaller pale being considered. As such, they have positive, 'existential import'. With this perception, De Morgan doggedly stuck to the primacy of being that underlies thought. In his logic, all terms, even negative ones — the contraries, x, y, z, etc. — were charged with existence:

> On looking into any writer on logic, we shall see that *existence* is claimed for the significations of all the names. Never, in the statement of a proposition, do we find any room left for the alternative, *suppose there should be no such things.* Existence as objects, or existence as ideas, is tacitly claimed for the terms of every syllogism.[82]

For De Morgan, thought — even, and especially in its most visible appearances, in the machineries of algebra and logic — always pointed to something positive, something beyond, even if it were nothing.

It is not surprising that De Morgan would have stuck to his logical last with Aristotle. He had never envisioned his own system as doing anything other than "extending" the Philosopher's logic in the technical and symbolic sense. He called his system "an *extension* of Aristotle" because he had propounded "no new laws" beyond Aristotle's and because all of his own syllogisms could be "reduced to an Aristotelian form without any addition except that of contraries [and hence quantification] to the matters of predication."[83]

This last, exception clause brings us to one of the most intriguing innovations in De Morgan's reworking of syllogisms, the "strengthened arguments". In our discussion of Table 1 we noted the eight "strengthened arguments" that lie in the columns on the left and right (arguments *d*, *l*, *t*, *bb*, *e*, *m*, *u*, and *cc*). The "strengthening" in these arguments occurs when a particular premise is transformed into a universal. We can see this by comparing argument *s* (our earlier

[81] [FL, p. 46] (ital. De Morgan's).

[82] [FL, p. 127] (ital. De Morgan's). See also, ["Syllabus", p. 154]: "Remember that in producing a name, the *existence* of things to which it applies is *predicated*, i.e. asserted ..." and "Logic", p. 251: "... every logical term is postulated as having existence in thought or existence in external reality, according to the universe in question. It is only as representing existence that a term is used in logic."

[83] [S2, p. 97] [OS, p. 42].

example) with the strengthened argument u, which lies immediately to the right and slightly lower on the Table. The s argument is $X))Y)\cdot)Z$, which gives $X)\cdot)Z$; the u argument is $X))Y(\cdot)Z$, which produces $X)\cdot)Z$, the same conclusion as s. The middle premise of s is the particular proposition $Y)\cdot)Z$, which reads 'Some Zs are not Ys', while the middle premise in u is the universal expression $Y(.)Z$, ('Everything is Y or Z or both'). The move from s to u "strengthened" the particular, minor premise, by transforming it into a universal. Another example of strengthening can be seen by comparing argument x on the table to argument u. In this instance argument x has a particular, major premise, $X)(Y$, and a universal, minor premise, $Y(\cdot)Z$, which leads to the particular conclusion $X)\cdot)Z$, the same as with argument u. Only now in moving to u, the major, rather than the minor premise is strengthened.

De Morgan's symbolic system showed that this kind of "strengthening" — whether of the minor or the major premise — had no effect on the conclusion. The canon of validity remained the same. Applying the erasure rule to s — that is to $X))Y)\cdot)Z$ — yields $X)\cdot)Z$; applying the erasure rule to u — that is to $X))Y(\cdot)Z$ — also yields $X)\cdot)Z$. From the standpoint of traditional syllogistic, the inferences in the strengthened arguments were problematic, indeed invalid. They are all syllogisms with two universal premises and a particular conclusion, which means they all commit what is commonly called the "existential fallacy", which says a valid syllogism with a particular conclusion cannot have two universal premises. De Morgan acknowledged that such syllogisms are invalid when subject to traditional, Aristotelian analysis: "thus *AEI* in the first figure can be nothing but the invalid mode 'Every Y is Z, no X is Y, therefore some Xs are Zs", he wrote in [S2].[84] However, they were valid in his own system. They were to logic what the negative and imaginary numbers were to algebra. They were the results of the legitimate manipulation of the functional rules of his spicular system, and were, therefore, themselves legitimate.[85] De Morgan's challenge was to find a way to understand them.

De Morgan's defense of the validity of his strengthened syllogisms embraced what we may identify as three separate segments of interpretation that either grew out of his spicular notation or were reinforced by it: (a) the "calculus of opposite relations"; (b) the two novel expressions, $X(\cdot)Y$ ('Everything is X or Y or both') and $X)(Y$ ('Some things are neither Xs nor Ys'); and (c) the subaltern 'existence' possessed by contrary terms. Together these items of interpretation show the ways that De Morgan's Peacockian approach moved him considerably beyond Aristotle, even as it kept him based in the Greek's syllogistic.

De Morgan's "calculus of opposite relations" in symbolic logic was inspired by algebra, where "all oppositions are instrumentally reducible to addition and sub-

[84][S2, p. 85] [OS, p. 29]; [Copi, p. 193].

[85]De Morgan had actually introduced "strengthened" premises for arguments in *Formal Logic* (pp. 70, 104–105) as a consequence derived from his eight basic propositions and his formulation of the "numerically definite syllogism". Tying these strengthened premises to the mechanisms of his new notation, however, gave him a more explicit means of explaining their validity.

traction."[86] This was the core of the "numerically definite syllogism", which he devised to address problems attending the vagueness of traditional quantification. As a mathematician, De Morgan was acutely aware of the way a middle-term's quantity could determine the validity of a syllogism. In criticizing Hamilton's system, which he believed incapable of treating quantity effectively, he introduced a distinction in the quantity terms that marked for him a considerable step forward. Logicians up to his day, he observed, had interpreted the universal 'all' in a "cumular" sense. That is, 'all' referred to the entire, accumulated collection of individuals that fell under a term so quantified, a "completed induction". But this did not exhaust the possibilities in forming terms and deriving inferences, especially as regards the "general propositions of any science [including logic], *as actually proved*". Euclid, for example, intended to show that "all isosceles triangles have equal angles at the bases", but this could not extend inductively to the entire collection of isosceles triangles, which was infinite in number. Rather, Euclid meant 'all' in the sense of 'any one' or 'every'. This usage comprised an "exemplar" interpretation of 'all'. Likewise with the particular 'some'; its exemplar interpretation was "some one", which of course could be expanded to "some two", "some three" and so on indefinitely, with each number specifying the exact amount of quantity. The exemplar reading of quantity terms, in short, gave them a precision previously lacking. This provided their core meaning in logic where often "what is proved is the exemplar proposition".[87] The consequences of these new definitions were far-reaching, for the exemplar readings of 'all' and 'some' opened the door for considering probabilities in the numerically definite syllogism, and probabilities, in turn, made possible the strengthened syllogisms.[88]

[86][S2, p. 83] [OS, p. 26].

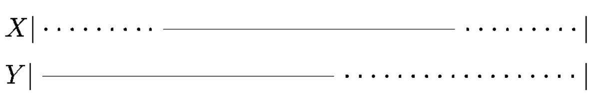
[87][S2, pp. 100–101] [OS, p. 46] (ital. De Morgan's); see also ["Syllabus", pp. 168–169].

[88]De Morgan first introduced the numerically definite syllogism (NDS) in [FL], where he counted it among the major accomplishments of the book (p. x), and, indeed, he devoted an entire chapter to it (Chapter VIII). Without straying too far afield here, we can use his example to illustrate a simple NDS. Let X, Y, and Z be the terms of a syllogism, with "ε the number of Xs in existence, η the number of Ys, and ς the number of Zs, and v the number of instances in the universe [of the syllogism]." Further, let mXY represent that mXs are to be found among the Ys or that mYs are to be found among the Xs. Likewise with nXZ. From the premises mXY and nXZ and assuming only η (the number of Ys) is known, it follows that $mXY + nXZ = (m + n - \eta)XZ$ [FL, pp. 166–168]. See Adrian Rice, "'Everybody Makes Errors': The Intersection of De Morgan's Logic and Probability, 1837- 1847", *History and Philosophy of Logic*, 24: 289–305, 2003. With this formulation it also becomes possible in some cases to create a valid inference from two particular propositions, an anathema for Aristotelian traditionalists. Consider 'Some Ys are Xs', $Y(\,)X$, and 'Some Ys are Zs', $Y(\,)Z$. In traditional syllogistic, nothing follows necessarily about the relation of the Xs and Zs, and, indeed, the inference $(\,)\,(\,) = (\,)$ is not to be found on De Morgan's table of valid arguments (see Table 1). But, by specifying the numbers of Xs and Ys, then a valid inference can follow. Suppose the number of the Ys is η, the number of the Ys that are Xs is a, and the number of Ys that are Zs is b. Then there are at least $(a + b - \eta)$ Xs that are Zs. If the number of ticketholders in an auditorium were 1000, of whom 500 (a) were assigned seats, with the remainder sitting in general admission, and 700 (b) bought items at the concession stand, it would follow that at least $500 + 700 - 1000$, that is, 200, of the assigned-seat attendees bought concession items. See the remarks of Alexander McFarlane, *Lectures on Ten British Mathematicians of the Nineteenth Century*, np, 1916, ("Project Gutenberg" eBook #9942, released, 2003), pp. 15–16.

Beyond interpreting the quantity terms with "numerical" definiteness, De Morgan paid careful attention to the framework in which they operated. The universality or particularity of propositions, for instance, referred to all or some things within the universe of the proposition, whereas applied to terms they referred to all or some things, in the exemplar sense, contained within the terms, or a "portion" of the universe of proposition. But, we recollect, all terms were accompanied by their contraries (complements) so that every term is only a part of the universe under consideration and has an existing contrary within it. These considerations meant it was possible to have a universal proposition in which one of the terms might be particular, e.g. $X))Y$ ('All Xs are some Ys') or a particular proposition in which both the terms might be universals: $X)(Y$ ('Some things are neither Xs nor Ys'). More importantly, they led to the collapse of the distinction between number and kind, between quantity (all or some) and quality (affirmation or negation). In its stead De Morgan proffered a "quantitative" definition of contraries: "the quantitative contrary of 'every X' is 'some xs,' and of 'some Xs,' 'every x'."[89]

With affirmation and negation "in the usual sense" abandoned and subject to an exemplar, quantitative reading, De Morgan observed that within their universes, many expressions, such as $X(\,)Y$ ('Some Xs are Ys'), could be interpreted as possessing degrees of increasing or decreasing probability. To see this, change both the quantities of $X(\,)Y$; this yields $X)\,(Y$, or $x(\,)y$ ('Some things are neither Xs nor Ys'). If the second expression is true, he asserted, then it stands as a "presumption" for the truth of $X(\,)Y$. For, imagine, the more things "there are which are neither X nor Y, the smaller the number of instances in the universe within which all the Xs and Ys are contained." Therefore the greater the probability of $X(\,)Y$, that is, of 'some [number of] Xs' agreeing with or matching 'some [number of] Ys'.

Using De Morgan's diagrams we can more readily view this probability at work. Let the following depict the universe of the proposition $X(\,)Y$:



The X line represents the term X and its complement x, with X denoted by the solid portion and its contrary or complement x by the dotted portion. Likewise with the Y line. The overlap of the solid X and Y lines shows that some Xs match some of the Ys; one can easily see here that the diagram stands as well for $x(\,)y$ ('Some non-Xs are non-Ys'). Now picture the number of dots growing on both the X and Y lines (from either end), increasing the number of instances of $x(\,)y$, and diminishing the scope of the terms X and Y. The ever smaller universe of Xs and Ys increases the probability of the terms matching. Accordingly, by logically linking quantity and quality in the calculus of opposite relations, the payoff lay with the calculus' ability to capture probabilities, and

[89] [S2, p. 92] [OS, p. 37].

therefore to permit comprehending quantification more richly and specifically as matters of degree.[90]

De Morgan took this interpretation of quantification as a specific matter of degree yet further when compiling his "Syllabus" and in so doing presented an additional argument for considering probabilities in particular propositions. There he distinguished "definite" quantity from quantity that is "more or less vague". Definite quantity could be either absolute or relative: "50Xs and no more" was absolute; a fraction, say "2-sevenths (and no more) of all the Xs are Ys" was relative. Moreover, a quantity may be definite at one end of a spectrum, and vague at the other. The only perfectly definite quantities are 'all' and 'none'; these are the signs of total quantity that make propositions universal. The contrary propositions of the universals ('Some Xs are not Ys' and 'Some Xs are Ys') are entirely vague in one direction. They start with "*not-none*" and proceed to add a bit of quantity, so to speak, beginning with "one at least", then continuing to "more", and finally even to "all", the total, definite quantity at the other end of the spectrum. This conception differed from ordinary parlance, he discerned, where "some" often means "*not-none*" or "*not-all.*" In logic it can only mean the former, never the latter. Logically in fact, "*some* may be *all.*" When the logician says 'Every X is Y', he means all the Xs are some of the Ys, and that some of the Ys are all the Xs, but he makes no claim about just how many of the Ys are among the Xs. Indeed, maybe all of them are.[91]

From the above understanding of quantification enriched by probabilities, De Morgan concluded that in those cases where we may move degree by degree through the vagueness, from 'not-none' to 'some' to 'most' to 'all', we do so without affecting the validity of a syllogism. We only make one of the premises in it stronger by increasing the probability of correlating members of the subject and predicate classes, even all the way from 'some' to 'all'. To the extent we increase the probability in the premise, we also strengthen the probability in the conclusion. As a consequence, it becomes possible to have a valid syllogism with two universal premises (one strengthened) and a particular conclusion.

De Morgan was aided in his interpretation of strengthened arguments by two novel statement forms, which he had invented, and which are not found in traditional Aristotelian syllogistic. These were the propositions $X(\cdot)Y$ ('Everything is X or Y or both') and $X)\,(Y$ ('Some things are neither Xs nor Ys'). He first introduced the expressions in *Formal Logic* in the chapter "On Propositions" as

[90] [S2, pp. 91–93] [OS, pp. 35–39]. De Morgan had also done considerable work in the theory of probability itself and was as current with the field as any mathematician in the mid-nineteenth century; he devoted Chapter IX of [FL] to probability, and produced several papers on the subject; his views are treated more fully in Joan L. Richards, "The Probable and the Possible in Early Victorian England" in Bernard Lightman, ed. *Victorian Science in Context*, University of Chicago Press: Chicago, pp. 51–71, 1997. These developments lie beyond the scope of our, more restricted concerns with De Morgan's universe of the syllogism; here we wish to note only how his recasting of the canon of syllogistic validity opened a door to probability, which in turn influenced his account of strengthening syllogisms.

[91] ["Syllabus", pp. 154–156].

part of his discussion of the complement and non-complement relations between X and Y classes. That 'X is a complement of Y' equals 'No non-X is non-Y' and that 'X is a non-complement of Y' equals 'Some non-Xs are non-Ys.'[92] These relations became much easier to see with the spicular notation and the accompanying graphical expressions. The proposition $X(\cdot)Y$ reads literally 'Some Xs are not some Ys'. The universe of this proposition can be expressed with the following diagram:

$X|$ ———————————— . . . . |
$Y|$ . . . . ———————————— |

Again, the X and Y lines represent the terms X and Y and their complements x and y. In the universe of this proposition we can see that there are some Xs that are not some Ys. We can also see the equivalent expression $x))Y$, 'All xs are some Ys'. And we can see the overlapping solid lines, wherein Xs are also Ys. When we take the expression as a whole, then, everything that is in the proposition (all members of X and Y) entails X or Y or both X and Y. Similarly, the expression $X)(Y$ (literally 'All Xs are all Ys') may be graphically depicted to show that 'Some non-xs are some non-Ys', or alternatively stated, 'Some things are neither Xs nor Ys.'

The spicular notation tied both of these novel propositions specifically to De Morgan's conception of quantification, for $X(\cdot)Y$ ('Everything is X or Y or both') was the only universal expression composed of two particular terms, while $X)\ (Y$ was the only particular expression composed of two universal terms. The expressions were not needed in all cases of strengthening arguments. But they combined with the other six propositions to give De Morgan the means of completing the full array of strengthened syllogisms. Attending once again to Table 1, converting a particular syllogism to a strengthened one meant changing the quantity of one term in one of the premises in order to render both premises universal. In arguments s and n this was accomplished by simply altering $)\cdot)$ to $(\cdot)$, a universal premise composed of two particular terms. In a similar fashion it became possible in arguments e and m to derive from two universal premises, composed with universal terms, the conclusion,)(, which although composed of two universal terms remains nonetheless a particular proposition.

The above describes how strengthened arguments were created within De Morgan's system. Still, the question of their validity remains, and leads us to the third point of his interpretive endeavors. De Morgan's central assumption was that changing the form of an expression will not alter its positive content, even though it appears to produce an impossible result. The first figure argument, AEI, we saw, had produced an invalid syllogism in the traditional form. But De Morgan insisted that "every one" of his "syllogisms can be reduced to an Aristotelian form, without any addition except that of contraries to the matters of predication." How could this be? He provided the answer with an example, argument e from Table 1.

[92] [FL, p. 72, 86]; ["Syllabus", p. 168]; Merrill ([ADM, p. 57, 63]).

The argument is $))((=)($, or $X))Y((Z = X)(Z$, which in traditional form is: 'All Xs are Ys'; 'All Zs are Ys'; 'Therefore some things are neither Xs nor Zs'. In addition to committing the existential fallacy, this argument commits the fallacy of the undistributed middle term, Y, and is doubly invalid, so to speak.[93] But using contraries, as well as the erasure and transformation rules De Morgan generated the following, equivalent expression, $X)\cdot(y))z = X)\cdot)z$. And this expression nets a traditional, valid syllogism, "Fesapo" in the time-honored terminology. From this, De Morgan concluded, "the syllogism)) ((=)(can thus be made Aristotelian."[94]

Although De Morgan did not provide it, we can reproduce a proof of a strengthened syllogism's validity, which will reveal a critical linchpin in all these strengthened arguments.[95] To do so we shall need to show that, from the premises, it follows that $X)(Z$, 'Some things are neither Xs nor Zs', or, its equivalent, 'Some xs are some zs'.

Proof:

1. $X))Y$ — Every X is Y (premise)
2. $Y((Z$ — Every Z is Y (premise)
3. $y))x$ — Every y is x (1, contraposition, i.e. restatement using contraries)
4. $y))z$ — Every y is z (2, contraposition)
5. $y()x$ — Some ys are xs (3, subalternation, see below)
6. $x()y$ — Some xs are ys (5, conversion)
7. $x()z$ — Some xs are zs (4, 6, First Figure, AII, "Daripti")

The critical step in this proof is #5, the inference of subalternation. With the traditional, Aristotelian square (seen earlier) this inference allowed one to move logically from the A statement to the I, as well as from the E to the O statement. For Aristotle, universal statements were not empty. Thus 'all men' (understood in the "cumular" sense of 'all') included every being who was a man, and every man gave the "existence" to the term 'all men'. Each man was an existing instantiation of 'all men' or 'man'. Because universal terms were not empty, it followed that particular terms, such as 'some men' were also not empty, and the 'existential

[93] The "fallacy of the undistributed middle" occurs when the middle term in a syllogism, Y, is particular in both premises. For instance, in the syllogism, 'All dogs are animals', 'All cats are animals', therefore 'All dogs are cats', 'animals' is the Y, middle term, but represents only part of the class of 'animals' in both premises (e.g. 'All dogs are [some] animals'). By being undistributed, different parts of the class may be involved in each use and therefore the Y term cannot connect the X and Z terms of the major and minor premises. Traditionally, a valid syllogism requires that the Y term be universal (or 'distributed') in at least one premise. It can then serve as the term through which or by means of which the other two terms of the syllogism can be connected, eliminating Y in the process.

[94] [S2, p. 97] [OS, p. 43].

[95] We wish specifically to thank Professor Merrill for his help in clarifying both this proof and a variety of points pertaining to strengthened arguments.

import' of the general term carried through to the particular term. De Morgan's ploy was to do this with contrary terms as well; the critical inference in step #5 above is from the universal contrary 'every y' to the particular contrary 'some ys'. Once the existential import of 'some ys' is granted, the rest of the proof follows.

The same holds with the other strengthened arguments, including even that original bugaboo, the putatively invalid *AEI* with which we began this excursion. "In my system", he recorded,

> a plain man who sees clearly that some things are proved to be neither men nor mice, were it only because they do not eat cheese, may rest content that his knowledge, even in the form of the light of nature, can be made science, without the necessity of having recourse to the following very venerable, but very unsatisfactory form:
>
> No man is a non-eater of cheese. [proposition E]
> All non-eaters of cheese are other things than mice. [proposition A]
> Therefore some other things than mice are also not men. [proposition I][96]

Throughout his reworking of traditional syllogisms, De Morgan remained convinced that his own system only extended Aristotle's and that any argument in his system could be "made Aristotelian." This required 'existence', which accordingly was not for De Morgan an extrinsic addition to his logic, but an intrinsic component of it, a critical one that made key inferences possible.

4 THE "PRINCIPLE ... OF THE HINGED LEVERS"

De Morgan produced his nut-cracker analogy for logic in the third of his major logic papers, which he wrote in 1858. He devoted much of that paper, [S3], to a review and general treatment of logic as he had understood and developed it to that point. Shortly after, in [S4], published with seemingly little preparation in 1860, he introduced perhaps his most profound and enduring contribution to logic, the logic of relations.[97] In the same year he summarized the central point of the more technical [S4] in a popular article simply titled "Logic" for the *English Cyclopaedia*: the "*pure form* of the proposition, divested of all matter, is the assertion or denial of the following: X stands in relation L to Y."[98]

Here was the nutcracker analogy brought to life in his own work and maturity. Before 1860 De Morgan had thought himself in possession of the "pure form" of logic with his spicularly symbolic reworking of Aristotle. With his table of arguments, he had portrayed the complete "universe of the syllogism" using his

[96] [S2, p. 97] [OS, p. 43].
[97] Merrill ([ADM, p. 115]).
[98] "Logic", p. 252 (ital. added). The *English Cyclopaedia* was the sequel to the *Penny Cyclopedia*, to which De Morgan contributed over 700 articles.

functional, spicular notation, a universe he saw as a "symbolic language [that] gives the expression of the laws of thought in their purest forms." But now he realized he had grasped only the "levers on a common hinge", as it were, and not the pure form of the nutcracker, nor, to exploit the analogy, the pure form of logic. The latter would require yet another step in the process of abstracting and generalizing "the machinery in action" to which he had so far committed himself. Thus, for De Morgan, the logic of relations he presented in [S4], was a major, further move toward the "pure form" of logic — the "principle ... of the hinged levers".

The pathway that led De Morgan to the logic of relations lay through the thicket of interpreting the copula, a problem that engaged him throughout his career. Traditionally, the copula was considered as one of four components of formal logic, along with terms, propositions, and syllogisms.[99] As the etymology of the term indicates (from the Latin *copulare*, "to link"), the copula served to connect the terms of subject and predicate in a proposition; thus in the proposition 'All X is some Y', 'is' stands as the copula. The copula's core definition was "identity" and 'is' and 'is not' provided the words carrying this meaning into propositions. By itself, 'is' also designated affirmation of the identity between subject and predicate, whereas 'is not' signified its denial. De Morgan would later chide logicians for being vague and inconsistent in their restrictive and substantive use of 'is' and 'is not', which took "many meanings in their modes ... and examples."[100] And from the "Preface" to *Formal Logic* on, he believed making the copula "abstract" to be one of his major contributions to logic: "In the form of the proposition, the copula is made as abstract as the terms: or is considered as obeying only those conditions which are necessary to inference."[101]

As with so many of De Morgan's innovations in logic, the inspiration for abstracting the copula "followed the hint given by algebra".[102] Even before De Morgan's more systematic turn to logic, his earliest forays into the subject had dealt with the copula in the context of mathematics. His earliest ruminations arose from an attempt to cast Euclid in syllogistic form.[103] In some early comments in *The Study of Mathematics*, De Morgan experimented with broadening

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[99] ["Syllabus", pp. 153–154].

[100] De Morgan cited the *Port Royal Logic* for his understanding of the traditional, substantive use of the copula [S2, p. 105] [OS, p. 52]. There Arnauld and Nicole had identified three components in a proposition: the subject, the predicate, and an "action of the mind" — affirmation or denial — indicated by 'is' or 'is not'. The 'is' was considered "substantive" because it indicated "the connection ... between the two [substantive] terms of a proposition." See, Antoine Arnauld and Pierre Nicole, *Logic or the Art of Thinking: Containing, besides common rules, several new observations appropriate for forming judgment.* Trans. Jill Vance Buroker, Cambridge: Cambridge University Press, pp. 79–83, 1996.

[101] [FL, p. ix].

[102] [S2, p. 104] [OS, p. 50].

[103] [SDM, p. 203 ff]. The central problem lay with translating *a fortiori* reasoning into syllogistic form. An example of the former would be 'A is greater than B', 'B is greater than C', therefore 'A is greater than C'. This reasoning was critical to Euclid's proofs, but utterly recalcitrant to translation into some meaningful and non-circular or non-question-begging syllogism. See Merrill ([ADM, Chap. 2]).

its definition enough to include 'is equal to' as a second, equally legitimate copula that linked A to B ('A is equal to B'). De Morgan saw himself to be thereby recognizing that there were "two", separate copulas involving 'is', behind which lay the assumption that both comprised a more abstract and general "relation" between the subject and predicate. This was the assumption De Morgan would later unearth and refine in his logic of relations. Implied as well in these developments was the supposition that both senses of the copula could be expressed in the same notation, which in turn required further effort directed toward interpreting the relations between subjects and predicates within his new symbology.

In *Formal Logic*, De Morgan used his analytical scalpel to detach what he later called the "essential from the accidental characteristics of the copula."[104] To do this he devised a "double singular proposition," the most elemental of expressions, which entailed only one instance each of the subject and predicate: "this one A is this one B". De Morgan then identified three features pertaining to the copula of this core proposition. The first was its indifference to "conversion," that is 'A is B' and 'B is A' must have the same meaning; they must "be both true or both false." Second, "the connexion *is*, existing between one term and each of two others, must therefore exist between those two others," such that 'A is B' and 'A is C' must imply 'B is C'. Third, "*is* and *is not* are *contradictory alternatives*, one must, both cannot, be true." These three characteristics — convertibility, transitivity, and contrariety — arose from the "absolute identity" of 'is' and provided the conditions that made "all the rules of logic true". The analogy with algebra drove home the point, for the abstracting from 'is' mirrored just how

> arithmetic was the medium in which the forms and and laws of algebra were suggested. But as now we *invent algebras* by abstracting the forms and laws of operation, and fitting new meanings to them, so we have power to invent new meanings for all the forms of inference, in every way in which we have the power to make meanings of *is* and *is not* which satisfy the above conditions.[105]

Immediately, De Morgan illustrated how other copulas might also meet the essential conditions. Imagine, he proposed, X, Y, and Z as symbols attached to "*material* objects", then let 'is' be placed between two of them (as in 'X is Y'), and let 'is' mean that the "two [objects] are tied together, say by a cord." Now the cord, "tied to", fills all the above conditions: 'X is tied to Y' means 'Y is tied to X'; 'X is tied to Y' and 'Y is tied to Z' imply 'X is tied to Z'; and of 'X is tied to Y' and 'X is not tied to Y' only one can be true.

Not only was 'is' abstracted into the more general conditions of copulas, but De Morgan also noted that this raised the possibility that other copulas might exist, which fulfilled only some of the above conditions. For example "gives" (in the causal sense) is transitive, but not convertible: 'X gives Y' and 'Y gives Z' yields

[104] [S2, p. 104] [OS, p. 50]; [FL, pp. 56–61].
[105] [FL, pp. 57–59] (ital. De Morgan's).

'X gives Z'; yet 'X gives Y' does not produce 'Y gives X'. The same held with other links between subject and predicate, such as the "verbs to bring, to make, to lift, etc."[106] Other connectives might be convertible, but not transitive. For instance, 'converses with', as in 'X converses with Y', converts to 'Y converses with X', but it is not transitive, for X's conversing with Y and Y's with Z do not imply X's conversing with Z. Of course, some links between subject and predicate fail all the conditions. That 'John loves Mary' entails neither that 'Mary loves John' nor, if 'Mary loves Kevin', that 'John loves Kevin'. (To be sure, no one ever seriously proposed that love and logic embodied any sort of mutual, necessary inference, whatever the conditions.) On the basis of these and other examples, De Morgan then defined the "abstract copula" as "a formal mode of joining two terms which carries no meaning, and obeys no law except such as is barely necessary to make the forms of inference follow."[107]

Generalizing and abstracting the copula meant incorporating it into the "action of the machinery". This led to two further, successive innovations before De Morgan would arrive at the logic of relations. The first was his so-called "bicopular syllogism" and its companion, the "composition of relations". The second may be found in his "material" interpretation of the copula 'is'. The former emerged in [S2] as he brought the "abstract copula" to bear on his reworking of traditional syllogistic in tandem with his new symbology. The latter surfaced in De Morgan's response to several of his critics, especially the Reverend Henry L. Mansel.[108] These developments in the 1850s paved the way for his next step in abstraction in two ways. If the copula were part of the logical machinery then the various relations that comprised it might be combined without fussing excessively over their "material" meaning. Moreover, instantiations of these combined relations (including, eventually, syllogisms) would be then understood as "material" in the same way that arithmetic provided an instantiation, but only one, of algebra. Just as the "*forms* born and educated in arithmetic have left their parent and set up for themselves," so too would those "born and educated" in language — especially the traditional copula 'is' — "set up for themselves" in the logic of relations.[109]

Heretofore, De Morgan claimed in [S2], logicians had restricted themselves to a single copula, 'is', when describing inferences, even though it was employed in a variety of ways. In fact, De Morgan had held in *Formal Logic* that syllogisms themselves did not exhaust inferences, even though they typified the instrumentality of reason. "For example, 'man is animal, therefore the head of a man is the head of an animal' is inference, but not syllogism."[110] But now with the 'is' abstracted according to the conditions laid down, the conditions themselves became effectively the markers of the copular connection and hence inference. This

[106] [S2, pp. 106–107] [OS, pp. 53–55]; [FL, p. 312].

[107] [S2, p. 104] [OS, p. 51].

[108] A follower of Hamilton, Mansel had charged De Morgan with abandoning traditional logic's reading of the copula 'is' as a "formal" component of the discipline. De Morgan agreed with Mansel's point and defended himself by claiming the 'is' to be "material" not "formal".

[109] [S2, p. 103] [OS, p. 50] (ital. De Morgan's).

[110] [FL, p. 131].

meant one did not have to rely on any one or number of them, such as transitivity, which up to then had been insisted on "in every kind of logic". One could use any copula whatsoever, because "the perception of relations by means of relations does not require us to use only one relation." Otherwise stated, one did not need a syllogism and its copular 'is' to evaluate a syllogism; indeed, from the perspective of the abstracted conditions, copular relations were themselves abstracted and amenable to other mechanisms. They could be compounded, for example, and it was even possible to introduce different relations in different premises. De Morgan illustrated:

> Every X has a relation to some Y
> and Every Y has a relation to some Z.

From which it followed that "every X has a compound relation to some Z."[111]

As a case in point, De Morgan explained what might happen (logically) in an act of persuasion. "If John can persuade Thomas, and Thomas can command William" we cannot, initially, infer that John can either persuade or command William because of the "intransitiveness of the individual copulae" (i.e. neither 'persuade' nor 'command' is transitive). But let a single word, say 'control', express the "process of gaining an end by persuading one who can command." In this instance we have a legitimate inference: "then John can control William." The inference is valid because the "compound copula" ('control') supplies the transitivity that the individual copulas lacked. "This", De Morgan highlighted, "is the step by which we ascend to the general theory of the copula." And it produces what he called a "bicopular syllogism".[112]

The bicopular syllogism, then, combined two copulas into a "composite copula", which in turn yielded the "instrumental part of inference". De Morgan further described this as "*the elimination of a [middle] term by composition of relations.*"[113] Accordingly, this was a major step toward viewing all copular connectives as but an instantiation of a yet more abstract and general relation. It is fair to say that in [S2] De Morgan had neither worked out all the implications of this direction in his own thinking nor clarified precisely the links between the 'is' of identity and the now "abstract copula".[114] Nonetheless, his insight that copular connectives — which provide inference in logic — can be considered as instrumental, as part of the action of the machinery, carried forward his program of abstracting form from matter in logic, just as he had earlier done in mathematics.

The further separation of logical form and matter manifest in the bicopular syllogism became ever more pronounced as De Morgan responded to Mansel. In

[111] [S2, p. 108] [OS, p. 55].
[112] [S2, p. 108] [OS, p. 56].
[113] [S2, p. 109] [OS, p. 56] (ital. De Morgan's).
[114] For instance, De Morgan tried to explain how a "bicopular syllogism [could] be reduced to a compound process of a unicopular syllogism" ([S2, p. 108] [OS, p. 56]), but the results, at best, "remain problematic". Merrill ([ADM, pp. 69–78]), provides an incisive discussion of the bicopular syllogism and its place in the development of De Morgan's logic of relations.

an 1851 review of De Mogan's *Formal Logic* Mansel charged that in his attempt to make the "abstract copula" intelligible, De Morgan had introduced material beyond the "science of the laws of formal thinking" itself (Mansel's traditional definition of logic). Take "transitivity" for example. For De Morgan this was a condition that could be satisfied by 'is' and by many other connectives as well; an example of the latter might be 'gives' in the causal sense, as we saw above. Mansel responded with counter examples that were of the same form, but obviously invalid inferences:[115]

De Morgan:	X gives Y	Mansel:	Achilles killed Hector.
	Y gives Z		Paris killed Achilles.
	Therefore, X gives Z		Therefore, Paris killed Hector.

With this and many similar examples, Mansel advanced the argument that De Morgan's claims to doing formal logic — viz. the title of his book — collapsed in a plethora of material instrusions into his arguments. The 'is' of identity, was the only legitimate form of the copula in logic. And whatever De Morgan was doing, Mansel concluded, it was no longer the science of formal thinking; his syllogisms themselves were no longer formal, but material.

De Morgan met this charge head on. First, he agreed with Mansel that transitive relations, such as "is equal to", cannot be converted into formal syllogisms using the copula of identity as traditionally understood. Earlier, his rival, Hamilton, had tried, by translating 'A is equal to B' and 'B is equal to C', therefore 'A is equal to C' into the following argument:

What are equal to the same, are equal to each other;

A and C are equal to the same (B);

Therefore, A and C are equal to each other.

But De Morgan confessed he was "quite at a loss" to see how this could be an "expanded form of the first" expression. Buried in Hamilton's syllogism he still saw, rather, the "composition of relation, 'equal of equal is equal', expressing the transitiveness of the copula *equals*." This plus convertibility made the original argument valid, but did so only on the basis of De Morgan's own copular conditions. Then Mansel tried a different tack, claiming that the putatively formal reasoning involved in 'A is equal to B' and 'B is equal to C', therefore 'A is equal to C' was "elliptical" and, therefore, "*as it stands*, material ... and extra-logical." To this De Morgan replied by turning the tables on the traditional distinction between material and formal:

> Matter is opposed by writers [like Mansel] not to form, but to what is recognized as form in the school of Aristotle: the assumption of course being that that school exhausts the forms of thought. Historically speaking, the copula has been material to this day.[116]

[115] Mansel's comments are cited in Merrill [ADM, pp. 93–95].
[116] [S2, pp. 126–127] [OS, pp. 67–68] (ital. De Morgan's).

With this De Morgan stuck to his own version of form. "Transitiveness is the common form: the difference between *equality* and *identity* is the difference of matter." And in case one failed to see the point, he continued: "But the logician who hugs identity for its transitiveness, cannot hug transitiveness: let him learn abstraction."[117] If Mansel and others wanted to make material those arguments that rely on transitivity for their inference, so be it. What's good for the goose is good for the gander, to invoke folk wisdom; the same arguments applied to identity as well. Only by "drawing away from" the 'is' of identity and "pulling" transitivity out of it — that is, by abstracting from it — could one truly perceive the form of logical relations.

With these observations, De Morgan not only furthered his own abstraction toward the logic of relations, but he recast traditional syllogisms as materially, but not formally valid. If the syllogism were to language as, say, algebra is to arithmetic, then the logic of relations would be to the syllogism as the calculus to algebra: a further abstraction of form from matter in the process of reasoning. What was previously formal (in algebra and syllogistic) has now become material (in calculus and the logic of relations). And logic, then, would expose the mechanisms of thought in general, not just those of language.

A further indication that De Morgan was moving well beyond language comes from the opening of [S4] itself. There he introduced the "study of relation in general" by noting that even Aristotle had paid it little heed, being "too much the expositor of common language, too little the expositor of common thought."[118] Logicians since had fared no better, placing relation "among those heterogeneous *categories* which turn the porch of [their] temple into a magazine of raw material mixed with refuse." De Morgan's attack on tradition was by now recognizable. Logicians had affirmed all logical relation as a three-fold reduction "to *identity*, A is A, to *non-contradiction*, Nothing both A and not-A, and to *excluded middle*, Everything either A or not-A." Yet in so doing they had omitted transitivity and convertibility, neither of which could be derived from the triad of tradition without "begging the question" and both of which, therefore, stood "independently of the *three*". Even more troubling, their insistence on the centrality of copular identity harbored a confusion between relation and judgment, for 'is' possessed two meanings. Alone, it meant "*identity affirmed*" and in the phrase 'is not', it meant "only *identity*" (with the 'not' supplying the judgment of denial).[119]

The distinction between relation and judgment gave De Morgan entry into the "analysis of the necessary laws of thought connected with the notion of relation." As with his syllogistic, X and Y would be terms, but L would designate any sort of "relation in which X may or may not stand to Y." To capture judgment, he again invoked his dots, with two being positive and one negative. Thus "let $X..LY$ signify the assertion of the relation, and $X.LY$ its denial." Just after introducing these components, De Morgan asserted that separating relation and judgment

[117] Footnote to [S3, p. 177] [fn to OS, p. 79]; [S4, pp. 338–339] [OS, pp. 217–218].
[118] [S4, p. 331] [OS, p. 208].
[119] [S4, pp. 335–336] [OS, pp. 213–215] (ital. De Morgan's).

tendered an "important step" towards treating "syllogistic inference as an act of combination of relation."[120] In this he once more showed his conservative colors. Despite its next level of formal abstraction, the logic of relations was conceived essentially as a means of enriching the theory of syllogism. It would be Charles S. Peirce's insight to recognize that it had a life of its own, separate from the retrograde intentions of its creator.

De Morgan then set about proposing formal mechanisms of relations, using the "relations between human beings, ... consanguinity and affinity" as appropriate exemplars. Amplifying the above, the expression $X..LY$ means that X is "some one of the objects of thought which stand to Y in the relation L" or otherwise stated that 'X is one of the Ls of Y'. (Likewise, $X.LY$ means that X is "some one of the objects of thought which [do not] stand to Y in the relation L.") The X and Y are, respectively, the subject and predicate of the relation and they are so determined by the relation itself, not by the order of mention, i.e. "Y is the predicate in $LY.X$, as well as in $X.LY$." Composition of relations enters when the predicate term itself is tied to a relation, as with MY. Thus $X..L(MY)$ reads as 'X is one of the Ls of one of the Ms of Y', or, in shorter form, 'X is an L of any M of Y'(noted, too, without the parentheses, as $X..LMY$). Like contrary terms, contrary relations are expressed with the lower case letters: $X..lY$ ('X is not one of the Ls of Y', also recorded as $X.LY$). And De Morgan captured converse statements with a superscript: $X..L^{-1}Y$, interpreted as 'Y is one of the Ls of X'. The following Table 5 depicts both De Morgan's "dot notation", as his expanded symbology is sometimes called, and the basic propositions of his system.[121]

Table 5. De Morgan's System of Relations

1.	$X..LY$	X is one of the Ls of Y
2.	$X.LY$	X is not one of the Ls of Y
3.	$X..L^{-1}Y$	Y is one of the Ls of X (converse of 1)
4.	$X..lY$	X is not one of the Ls of Y (contrary)
5.	$X..LMY$	X is one of the Ls of one of the Ms of Y (composition)
6.	$X..LM'Y$	X is an L of every M of Y (inherent quantity)
7.	$X..L,MY$	X is an L of none but Ms of Y (inherent quantity)
8.	$L))M$	Every L of a Z is an M of that Z (relational inclusion)
9.	$L\|\|M$	$L))M$ and $M))L$ (relational equivalence)

Particulary noteworthy here are statements 5, 6, and 7. Together they provide "three symbols of compound relation", and will become an effective and powerful tool for generating a plethora of new relations, well beyond De Morgan's immediate concerns. In explaining "composition", or compound relation, De Morgan was careful to distinguish between the "*composition* and *aggregation*" of relations, a

[120] [S4, p. 336] [OS, p. 215].
[121] [S4, pp. 341–343] [OS, pp. 220–24]; Merrill ([ADM, pp. 118–119].

distinction he had earlier invoked in discussing terms. With aggregation of terms, a complex term stands for "everything to which *any one or more* of the simple terms applies." Thus, 'animal' is the aggregate of 'man' and 'brute', which are its aggregants. With composition, the complex term stands for "everything to which *all* the simple terms apply"; hence, 'man' is compounded of 'animal' and 'rational', which are its components. Symbolically, aggregate terms are noted as 'X, Y, Z', with commas separating the terms, while compounds are indicated by 'XYZ', with no commas.[122] Now De Morgan extended the distinction to relations. A relation is compounded in the same sense that mathematically X and Y are said to be compounded in XY. (This is now generally referred to as relative product.) In the phrase 'brother of a parent', 'brother' and 'parent' are compounded "in the same manner as *white* and *ball* in the term *white ball*." The notation, $X..LMY$ captures compounded relation. An aggregate of two relations, he remarked, would be expressed by the symbols $X..(L, M)Y$, (comma added), but "at present" he had no need for them. The "compound relation 'L of M' classes with the compound term 'both X and Y'."[123] We should note in passing that "aggregation" and "composition" (compounding) would later be labeled, respectively, logical disjunction and conjunction.

Statements 6 and 7 are also novel in De Morgan's scheme, for they introduce "inherent quantity" as an integral part of the relation itself ("universal quantity ... [as] part of the description of the relation"), not just quantity attached to individual terms. He indicated how this worked in discussing the mechanism converting a compound relation. The dictum is straightforward, but requires a fair measure of parsing: "The conversion of a compound relation converts both components, and inverts their order." Let X be an L of an M of Y for the compound relation ($X..LMY$). Then, in conversion, an M of Y is the converse of X — i.e. $X..L^{-1}(MY)$ — and Y is the converse of an M (M^{-1}) of the converse of an L (L^{-1}) of X. The compounded converse is denoted by $(LM)^{-1}$, which is identical to $L^{-1}M^{-1}$ and the whole expression then reads as $X..L^{-1}M^{-1}Y$. In this conversion, he noted, the "mark of inherent quantity is also changed in place." So, if $X..LMY$, "then Y is an M^{-1} of none but the L^{-1}s of X", which may be expressed as $X..L,^{-1} M^{-1}Y$. De Morgan did recognize the linguistically unwieldy nature of these and other of his theorems. Of the above example, he remarked wryly "a good instance of the difficulty of abstract propositions" and then proceeded to supply a concrete instance: "If X be the superior of every ancestor of Y, then Y is the descendent of none but the inferiors of X."[124]

As was his wont, having described how the expanded symbology functioned, De Morgan then supplied a table with his main theorems (Table 6).[125] Some of these expressions can find their way into English. For instance '$M^{-1}L^{-1}$ is the converse of LM' may be stated as "the composition of the converse of a relation with

[122] ["Syllabus", pp. 180–181].
[123] [S4, pp. 341–342] [OS, pp. 220–222].
[124] [S4, p. 343] [OS, p. 223].
[125] [S4, p. 343] [OS, p. 224].

Table 6. Some Principles of the Logic of Relations

Combination	Converse	Contrary	Converse of contrary Contrary of converse
LM	$M^{-1}L^{-1}$	lM' or L,m	$M,^{-1}l^{-1}$ or $m^{-1}L^{-1,}$
LM' or l,m	$M,^{-1}L^{-1}$ or $m^{-1}l^{-1,}$	lM	$M^{-1}l^{-1}$
L,M or lm'	$M^{-1}L^{-1,}$ or $m,^{-1}l^{-1}$	Lm	$m^{-1}L^{-1}$

the converse of another relation is the converse of the composition of the second relation with the first."[126] Others cannot be so expressed, for English lacks terms that can capture "inherent quantity". As with the spicular notation, De Morgan believed that the action of this machinery was much easier to read directly in the manipulations of its symbols, although in his rush to return to syllogisms, he himself did not expand them very far. Still, these theorems harbor some of the basic principles that subsequent generations of logicians have incorporated into the logic of relations.[127]

As a prelude to reconsidering syllogisms in light of the logic of relations, De Morgan focused on the properties of relations that pertained to the main conditions of his abstract copula: convertibility and transitivity. A convertible relation is one that provides its own converse, "when $X..LY$ gives $Y..LX$", and takes the general form LL^{-1}. A transitive relation occurs when "a relative of a relative is a relative of the same kind", and is "symbolised in $LL))L$, whence $LLL))LL))L$; and so on." Further, a transitive relation has a transitive converse, but not necessarily a transitive contrary. Thus, $L^{-1}L^{-1}$ is the converse of LL, so that $LL))L$ gives $L^{-1}L^{-1}))L^{-1}$. These conditions generated yet another table of results for De Morgan, which we can illustrate with an example. Assume "L is contained in $LL^{-1,}$; l,l^{-1}; $ll^{-1,}$; L,L^{-1}." Now let L signify ancestor and L^{-1} descendant, and the following obtain: "An ancestor is always an ancestor of all descendants [$LL^{-1,}$], a non-ancestor of none but non-descendents [l,l^{-1}], a non-descendant of all non-ancestors [$ll^{-1,}$], and a descendant of none but ancestors [L,L^{-1}]."[128]

With these forms (and their circumlocutory instantiations of lineage), De Mor-

[126]Merrill ([ADM, p. 120]).

[127]A case in point is his "theorem K" (so-called after the 'K' in the syllogisms Baroko and Bokardo), a theorem on which formation of "opponent syllogisms is founded". That theorem reads "if a compound relation be contained in another relation, ... the same may be said when either component is converted, and the contrary of the other component and that of the compound change places" ([S4, p. 344] [OS, p. 224]). In his notation this reads 'If $LM))N$, then $nM^{-1}))l$. See Merrill ([ADM, p. 121]), for proof of this theorem. For an account of how theorem K "constitutes a complete characterization of residuation" in the calculus of binary relations, see Vaughan Pratt, "Origins of the Calculus of Binary Relations", `http://boole.stanford.edu/pub/ocbr.pdf`.

[128][S4, pp. 345–346] [OS, p. 277].

gan had presented the abstract machinery involved in the relations needed for syllogisms. At this juncture he saw no need to go further. The "supreme law of syllogism of three terms" could now be expressed in the most abstract fashion imaginable (to him). "Any relation of X to Y compounded with any relation of Y to Z gives a relation of X to Z." With his revised notation, this read as:

$$\begin{array}{c} X..LY \\ Y..MZ \\ \hline X..LMZ \end{array}$$

Additionally, the conclusion could be altered by means of the above inferences, and expressed in negative form ($X.lM'Z$ or $X.L,mZ$) or as an statement of the composition of L and M: $LM||N$ (with N representing their composition).

From this prototype "unit" syllogism, now cast as a composition of relations, De Morgan proceeded to tabulate the rest of the syllogisms according to their traditional figures, now altered to take into account composition of relations. This was for him the payoff in generating the logic of relations, an abstract account of the "principle ... of the hinged levers." Traditional syllogisms were now considered a material instantiation of the general, formal theory of relations. And, indeed, De Morgan came to the conclusion that the ordinary syllogism, though valid, "does not very frequently contain the act of reasoning." More common and important in logical inference were its other traditional features. "When we examine any book of ordinary reasoning, we find that the onymatic syllogism is not very frequent, the combination of relations much more frequent, and the introduction of composition of terms and transformation of propositions by far the most frequent of all." Terms, aggregates, compounds, and the transformation of propositions — features that have long since entered the lexicon of logical relations as conjunction, disjunction, simple and complex propositions, complementarity, and the like would increasingly command the logician's attention.[129] But not De Morgan's. He saw himself as having achieved a major breakthrough with the logic of relations: "And here the general idea of relation emerges, and for the first time in the history of knowledge, the notions of relation and *relation of relation* are symbolised."[130] But he never returned to it. [S4] represents De Morgan's last significant logical effort.

5 CONCLUSION: THE "NATURAL SYLLOGISM"

From this brief journey into De Morgan's logic, we see that existence beyond thought was the assumption that held together his entire, logical "machine in operation", the "universe under consideration", and the logic of relations. It ascribed to the entirety of his logical thinking the underpinnings of that same positive reality he had grappled with in the mathematics of his early career. In his logic, the "existential charge", so to speak, stood equally as the conceptual pointer from the

[129][S4, pp. 354–356] [OS, pp. 237–239].
[130][S4, p. 358] [OS, p. 246] (ital. De Morgan's).

laws of thought to a parallel, real universe beyond. This charge carried De Morgan through his reworking of the universe of the syllogism and, as well, into his logic of relations: "The admission of relation in general, and of the composition of relations, tends to throw light upon the difference between the invented syllogism of the logicians and the natural syllogism of the external world."[131] The conceptual core of De Morgan's deep conservatism harbored the belief that everything we conceive, think, or imagine in the universe of our logic, sensations, or psychology takes us outside ourselves, into another, parallel universe, that "natural syllogism" of the outside world.

One of De Morgan's star pupils, Stanley Jevons, wrote of his mentor, "there was in fact an unfortunate want of power of generalisation in De Morgan; his mind could dissect logical questions into their very atoms, but he could not put the atoms of thought together into a real system."[132] A fair measure of truth resides in Jevons' assessment, but it masks De Morgan's accomplishments in the context of a major turning point in the history of logic. Throughout the heady and rich years of De Morgan's career, logic would awaken from its "dogmatic slumbers" in no small part because of his keen analytical skills. In the course of devising ever more abstract ways of trying to understand the outside world, De Morgan applied these skills to unearthing and exposing problems that had lain buried in centuries of traditional logical thought. And in so doing he made numerous, often tangential discoveries, including the theorems that bear his name, which merit recording in his own words: "The contrary of an aggregate is the compound of the contraries of the aggregants: the contrary of a compound is the aggregate of the contraries of the components. Thus (A, B) and AB have ab and (a, b) for contraries."[133] Originally devised for De Morgan's logic of classes, these theorems have made their way to the propositional calculus, and in modern notation are written as $\sim (p \cdot q) \equiv (\sim p \vee \sim q)$ and $\sim (p \vee q) \equiv (\sim p) \cdot (\sim q)$. They bear enduring testimony to both De Morgan's analytical brilliance and historical contributions to modern logic.

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[131][S2, p. 111] [OS, p. 59].

[132]W. Stanley Jevons, *Studies in Deductive Logic, a Manual for Students*, London and New York: Macmillan and Co., pp. xii–xiii, 1880.

[133][S3, p. 208] [OS, p. 119]. De Morgan first introduced these theorems in [FL, p. 136].

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BOOLE'S LOGIC

Dale Jacquette

1 ALGEBRAIC ANALYSIS OF LOGIC

The development of modern symbolic logic has involved steady progress from a few extraordinary episodes. Among the handful of outstanding insights and innovations that have contributed most dramatically to the progress of contemporary logic must be included George Boole's algebraic analysis of traditional Aristotelian syllogistic logic. Although Boole's logic was at most a forerunner and not yet a prototype of first-order propositional and predicate-quantificational logic or the so-called functional calculus, Boole, independently of but partially in agreement with parallel advances by Augustus De Morgan, introduced several conceptual breakthroughs that paved the way for the formalizations of mathematical logic as they came to fruition in the work of C.S. Peirce, Gottlob Frege, and, especially, A.N. Whitehead and Bertrand Russell's *Principia Mathematica*.

Boole was trained as a mathematician and in particular as an algebraist. In *The Mathematical Analysis of Logic: Being an Essay Towards a Calculus of Deductive Reasoning* [1847], *The Calculus of Logic* [1848], and *An Investigation of the Laws of Thought on Which are Founded the Mathematical Theories of Logic and Probabilities* [1854], Boole revolutionized the logic of his day, which was an enhanced four-term syllogistic logic that with minor improvements had remained essentially unchanged since its formulation in Aristotle's *Prior Analytics* and popularized in Antoine Arnauld's *The Port Royal Logic* [1662]. Thus, Immanuel Kant, in his *Critique of Pure Reason* [1787], was able to report a mere sixty years before the publication of Boole's *Mathematical Analysis of Logic*, that: 'It is remarkable...that to the present day this [Aristotelian] logic has not been able to advance a single step, and is thus to all appearance a closed and completed body of doctrine'.[1] Kant's pronouncement remained appropriate until Boole discovered how to symbolize logic as a specialized interpretation of a more general algebra of variables and values. Boole afterward echoes Kant's words when in the *Laws of Thought* he relates that: 'In its ancient and scholastic form, indeed, the subject of Logic stands almost exclusively associated with the great name of Aristotle. As it was presented to ancient Greece in the partly technical, partly metaphysical disquisitions of the *Organon*, such, with scarcely any essential change, it has continued to the present day.'[2]

[1] Kant, Preface, *Critique of Pure Reason*, Bviii.
[2] Boole, *Laws of Thought*, p. 1.

Handbook of the History of Logic. Volume 4: British Logic in the Nineteenth Century.
Dov M. Gabbay and John Woods (Editors)

The Aristotelian precedent makes it easier to appreciate the extent to which Boole revolutionized logic. What Boole contributed was a new conception of the basic units of reasoning by which a logical inference is made deductively valid or deductively invalid. When in his first booklength treatment of the subject, Boole speaks of the mathematical *analysis* of logic, his description is meant to be taken literally. Boole breaks logic down into more elementary components than had previously been considered in the Aristotelian tradition in logic, which he argued can be configured algebraically in all possible combinations as representing all predications of qualities and relations to objects and all logical operations on predications. Boole describes his project in precisely these terms, as operations of thought, in the Introduction to his *Mathematical Analysis of Logic*. There he explains:

> It appeared to me that, although Logic might be viewed with reference to the idea of quantity, it had also another and a deeper system of relations. If it was lawful to regard it from *without*, as connecting itself through the medium of Number with the intuitions of Space and Time, it was lawful also to regard it from *within*, as based upon facts of another order which have their abode in the constitution of the Mind.[3]

Boole emphasizes the greater generality of his algebraic interpretation of logic by comparison with syllogistic reasoning. He promotes its advantages as encompassing not only classical syllogistic logic, but the logic of propositions generally in a more universal algebra of symbols.[4] Consider the structure of a standard Aristotelian syllogism. We begin with a simple illustration such as the familiar example from the taxonomy of whales as cetaceans and cetaceans as mammals. The syllogism is:

1. All cetaceans are mammals.
2. All whales are cetaceans.

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3. All whales are mammals.

The inference is deductively valid according to syllogistic logic by virtue of the fact that it contains categorical propositions of a certain sort, belonging to a certain category and arranged in a form that is known to be such that if the assumptions of the inference are true then the conclusion must also be true. The syllogism is valid more particularly because any argument with the AAA-1 form consisting of the proper distribution of major and minor terms is deductively valid, where the 'A' form is the form of a particular type of categorical proposition. We do not need to look more deeply into the internal logical structure of a proposition in Aristotelian

[3]Ibid., p. 1.

[4]Ibid., p. 6. In reflecting on his formalization of logic in the Postscript, Boole qualifies his previous endorsement of his algebraic system over its syllogistic precursor only by strengthening his conclusion, p. 82: 'I have seen reason to change the opinion expressed in pp. 42, 43. The system of equations given there for the expression of Propositions in Syllogism is *always* preferable to the one before employed — first, in generality — secondly, in facility of interpretation.'

syllogistic logic than to determine whether a proposition is superficially of the right form, and we use information about the logical structures of propositions only at this relatively high level of abstraction in deciding which syllogisms are deductively valid and which are deductively invalid. The limited depth of analysis of the logical structures of categorical propositions by virtue of which they make possible deductively valid or invalid categorical syllogisms in Aristotelian syllogistic logic unfortunately implies that as logicians we never gain any further insight into what it is about the internal logical structures of propositions that contribute to the deductively validity or invalidity of the inferences in which they occur.

In some cases, we may be able to translate a proposition like 'I will go wherever you go' into the canonical form of a categorical proposition, if necessary, by introducing concepts that the original speaker of the proposition may not have intended. Here it is not stretching things too much to interpret the original proposition as 'All places to which you go are places to which I will go'. The original speaker does not mention the concept of place, so that in advancing this reformulation we foist onto the proposition an idea that is strictly speaking external to the exact wording of the thought in order to bring the proposition into the domain of canonical propositional forms recognized by syllogistic logic. Some such reinterpretations are no doubt inevitable in any application of logic to colloquial discourse. Logic abstracts from and idealizes reasoning as it occurs in everyday contexts, so there is bound to be some slippage and mismatch in any formalization of ordinary reasoning. We should nevertheless be aware of the fact that we thereby undertake to fit ordinary language into a more rigid mold in which some of its original content may be lost, and other, potentially undesirable content of our own, might be illegitimately added or superimposed. To the extent that classical Aristotelian syllogistic logic requires us to undertake greater liberties and logician's license with the propositions and inferences it considers, to that extent it is less advantageous and less to be preferred, other things being equal, than another more flexible logic that involves fewer or less radical distortions of the natural preanalytical logical structures it is called upon to analyze. Boole understood the advantages of refashioning logic as an algebraic construction of terms, operations, and values, and proceeded to work toward an algebra that would avoid the Procrustean bed of limited regimented structures in syllogistic logic.

Boole proposed abstracting from the underlying grammatical form of subject-predicate sentences by means of which he could algebraically symbolize the combination of any subject term with any predicate term in any categorical proposition. This method permitted him to expose the internal logical structure of propositions and inferences generally, not limiting himself to the particular forms recognized in Aristotelian logic as categorical or hypothetical, primary or secondary, but including any proposition or inference capable of being constructed by means of any combination of terms. Ironically, Boole's algebraic analysis of logic is in this regard more faithful to the Aristotelian concept of a term logic. The propositions and inferences Boole suddenly made accessible to logic in a generalized algebraic theory of logical forms include those involving predications of any number of properties to any number of objects, and any connection of propositions by means of conjunction, disjunction, and if-then conditionals. Where traditional logic distinguishes between categorical and hypothetical propositions and inferences, offering special rules for working with hypothetical syllogisms, Boole recognizes the same distinction, which incorporates both as the distinction between primary and secondary propositions and inferences.[5]

By contrast with traditional Aristotelian syllogistic logic, Boole's algebra formalizes both primary and secondary propositions, and is capable of evaluating the deductive validity or invalidity of categorical and hypothetical inferences. These are based in turn on a proposition's internal logical relations, according to a general theory of the logical terms by which propositions are constituted and their deductively valid assembly into inferences according to their assigned values as the class of all objects (1), or the null class, consisting of no objects (0). He attaches almost mystical Pythagorean significance to the concepts of unity and nullity in algebraic logic. A bivalent logic of the sort he describes in both *Mathematical Analysis of Logic* and *Laws of Thought* is easily mapped onto two propositional truth values, whereby the generality of Boole's logic and its insight into the essential fundamentals of logical relations are again evident.

The first giant step toward modern symbolic logic was thereby taken by Boole. The indispensable idea, without which the further course of symbolic logic would not have been possible, was Boole's concept of a logical operator, which has since come to be known as a Boolean operator. The operators make it possible to understand logical relations combinatorially without restriction to a prescribed number of limited forms, but in any of an indefinitely large number of mathematical combinations involving any choice of predicates. This is the same basic concept that has liberated modern symbolic logic, working instead with unlimitedly many well-formed combinations of truth functions and propositional symbols, constants, predicates, quantifiers and quantifier-bound variables. Boole's method, unlike Aristotelian syllogistic, is not satisfied with an identification of logical form in terms of figure and mood, but provides a much deeper and more versatile mathematical analysis of the internal logical structures of all propositions that can be constructed out of the algebraic combinations of terms by which properties can be predicated of objects and propositions can be combined to form more complex conjunctions, exclusive disjunctions, and conditional statements. In this way, Boole offers greater insight into the fine-grained logical structures of propositions and inferences than could possibly be attained in the syllogistic logic of his predecessors.

[5]Ibid., pp. 48–59. Boole, *Laws of Thought*, pp. 159–184. See Prior, 'Categoricals and Hypotheticals in George Boole and his Successors'.

2 BOOLEAN OPERATORS: NOT, AND, OR

Boole's most important suggestion is that all categorical subject-predicate constructions involve classes of objects that are logically related by a limited number of functions. Boole identifies three logical operators, which have the modified effect of the ordinary language terms, 'not', 'and', and 'or'. Also known respectively as complementation, conjunction, and disjunction, these three logical connectives are the Boolean operators, which can be represented more technically but not yet symbolically as NOT, AND, and OR.

The role of Boolean operators can be understood in Boole's mathematical analysis of propositions and valid inferences in Aristotelian syllogistic logic. The A-E-I-O categorical propositions of syllogistic logic are illustrated again by taxonomic applications: (A) 'All fish are vertebrates'; (E) 'No fish are vertebrates'; (I) 'Some fish are vertebrates'; and (O) 'Some fish are not vertebrates'. These predications express relations holding between designated classes of objects, in this case, of fish and vertebrates. Boole interprets such relaions by means of the three operators, as involving the complementation, conjunction, or (exclusive) disjunction of these classes of objects, and shows how to symbolize the operators as algebraic functions like, respectively, the subtraction or minus sign, multiplication, and addition. Indeed, Boole sometimes refers to conjunction as logical multiplication, and to disjunction as logical addition.

The three Boolean operators can now be characterized more formally. The Boolean operator of complementation is not like negation in the sense of propositional logic. It does not produce Not-P, to be interpreted semantically as the negation of P, meaning that if P is true, then Not-P is false, and if P is false, then Not-P is true. The symbol 'P' in Boole's algebra is not a propositional symbol representing a true or false predication, but is rather a predicate symbol that represents at most only part of a predication standing in need of an object term with which it must be combined in order to produce a true or false proposition. By contrast with contemporary logic, the Boolean operator of complementation does not have the effect of reversing the truth value of a proposition, whatever it is, to which the operator is applied. Boole says clearly in *Mathematical Analysis of Logic*: 'The expression of a truth cannot be negatived by a legitimate operation, but it may be limited.'[6] Instead, the Boolean NOT operator for complementation attaches only to predicate symbols, where it distinguishes a complement class of properties. For example, to write the Boolean 'NOT-Red', in saying that 'Some frogs are NOT-Red', distinguishes the class of all objects that do not have the property of being red, that are, as the terminology directly implies, nonred, or

[6]Boole, *Mathematical Analysis of Logic*, pp. 18–19. See Boole, *Laws of Thought*, pp. 241–242: 'To what final conclusions are we then led respecting the nature and extent of the scholastic logic? I think to the following: that it is not a science, but a collection of scientific truths, too incomplete to form a system of themselves, and not sufficiently fundamental to serve as the foundation upon which a perfect system may rest. It does not, however, follow, that because the logic of the schools has been invested with attributes to which it has no just claim, it is therefore undeserving of regard.'

that belong to the complement class of objects with any properties other than being red.

The difference between negation in modern propositional logic and Boole's class complementation operator is sometimes described as a matter, respectively, of external versus internal negation. The significance of the terminology is that external negation in the sense of propositional logic attaches to entire propositions and has entire propositions in its scope, whose truth value it reverses from true to false or false to true, whereas internal negation never takes an entire proposition as falling within its scope, but attaches only to predicate terms within a proposition in order to designate the complement class of properties represented by the predicate. For this reason, it may be more appropriate to speak of Boole's complementation operator as NON rather than NOT, and to reserve the term NOT for the truth functional definition of external or propositional negation as it is defined in contemporary propositional logic. Such a stipulation would nevertheless probably cause more confusion than clarity, so that, keeping in mind the distinction between Boole's NOT as predicate complementation as opposed to propositional negation, we shall follow established practice by referring to this Boolean operator as NOT. There are further implications of the fact that there is no provision for external propositional negation in Boole's logical algebra, especially for his reduction of Aristotelian syllogistic logic to his algebra of classes, to be addressed further below.

Conjunction or logical multiplication and (exclusive) disjunction or logical addition are similarly interpreted in Boolean algebraic logic. We obtain a more complex term by applying the Boolean operator conjunction or AND to two predicates x, y, of the form, AND(x,y), or, x AND y, which Boole simply writes as xy. In set theoretical terms, this compound term designates the class consisting of all objects that have *both* the property represented by predicate x and the property represented by predicate y. If x = red things and y = round things, then in Boole's logic, xy represents the class of all objects that are both red and round. Disjunction or Boolean logical addition is a somewhat more complicated case, because Boole originally defined this particular operation differently than it has since come to be understood. To speak of Boolean logical addition today is to interpret a Boolean operator that has the effect of disjoining two predicates in such a way that the resulting compound disjunctive predicate OR(x,y), or, x OR y, which Boole writes as $x + y$, represents the class of all objects that have either the property represented by predicate x or the property represented by predicate y, inclusively; that is to say including rather than excluding objects that have both the property represented by predicate x and the property represented by predicate y. The inclusive interpretation of the disjunction or Boolean logical addition operator is considered to be an improvement over Boole's original definition of the operator, which Boole defined exclusively as representing the class of all objects that have either the property represented by one predicate logically added to another or the property represented by the other predicate logically added to the first, but not both. The original exclusive interpretation of Boolean logical addition in $x + y$, where x = red and y = round, as before, designates the class of all objects that are either red or round, but not both, including firetrucks and baseballs, but excluding apples and cherries and crimson beach balls. The more standard inclusive interpretation of Boolean logical addition by comparison, again more closely in keeping with contemporary definitions of the inclusive disjunctive truth function in propositional logic, designates the class of all objects that are either red or round, including those that are both red and round.

The idea of a Boolean operator is that of a function on classes that syntactically combines predicate terms designating classes of objects with particular properties represented by the predicates as input, and produces as output a more complex predicate term that has been constructed out of the simpler component terms, involving negation or NOT, conjunction, logical multiplication, or AND, and disjunction, logical addition, or OR. Despite the algebraic context, we should resist the temptation of conflating the informal use of 'and' in verbalizations of elementary arithemtical operations, as when we say, for example, that '1 *and* 2 is (or equals) 3'. The AND operation for Boole is logical multiplication rather than addition, as we have just seen, and it is the OR operation by contrast that is logical addition. Logical and arithmetical addition, in other words, are not the same, even though Boole uses the same symbol '+' for both, and the two do not univocally map onto the same colloquial uses of similarly suggestive connectives and operations in ordinary language.[7]

In explaining the logic of hypothetical propositions and inferences in *Mathematical Analysis of Logic*, and of secondary propositions such as conditionals and secondary inferences such as hypothetical syllogism more generally in *Laws of Thought*, Boole extends the concepts of logical multiplication and logical addition to provide the rudiments of a truth functional propositional logic. Here again Boole can be seen inching toward but not yet attaining the concept of a truth-functional calculus. Boole allows logical multiplication to hold between complete propositions rather than mere predicate terms, defined as true just in case the propositions are true, and otherwise having the value false. This is precisely the truth functional definition of conjunction in contemporary propositional logic, where p AND q is true just in case p is true and q is true and is otherwise false. Similarly, Boole extends the concept of logical addition to propositions, again defined exclusively, in such a way that p OR q is true just in case either p is true or q is true, but not both, much in the same way that modern propositional logic defines inclusive (rather than exclusive) disjunction by cases in a truth table as true just in case either p is true or q is true, or both are true. Boole, however, nowhere defines a parallel truth functional operator for propositional negation, but resorts instead to complicated equations involving 1 as truth and subtraction from 1, representing the universal class, as falsehood, including in some cases equality with 0. In this respect, Boole's algebra, although it gestures toward, fails to provide adequate coverage of the truth functional foundations of contemporary propositional logic,

[7] See Rosser, 'Boole and the Concept of a Function'.

and in particular does not attain to the concept of propositional negation.[8]

The remarkable thing is that Boole correctly perceives what has since been formally demonstrated that these three operators are sufficient for analyzing the logical structures of all Aristotelian syllogistic logic, and much though not all of what has come to be known as propositional logic. Boole is in this sense deservedly regarded as the founder of modern propositional logic, which can be defined as the logic of Boolean operators as propositional connectives modified for inclusive rather than exclusive disjunction or logical addition, and supplemented by a truth functional operator for propositional negation. The standard theory of propositional logic is not simply the logic of Boole's three operators, differences in the exact interpretation of Boolean logical addition notwithstanding. Modern propositional logic also contains special operators, notably the conditional or 'If-then' and biconditional or 'If-and-only-if' connectives, such as, 'If today is Tuesday, then tomorrow is Wednesday', which, as we have seen, Boole follows the classical Aristotelian tradition in referring to as hypothetical rather than categorical. In *Laws of Thought*, he generalizes his categories even more to distinguish what he terms primary and secondary propositions. Primary propositions include but are not limited to the categorical propositions of syllogistic logic, and are defined as any propositions that express a relation among things, that can be built up by means of the Boolean operators NOT (for predicate complementation only), AND (logical multiplication for predicates and entire propositions) and OR (exclusive logical addition for predicates and entire propositions).

In *Laws of Thought*, by means of his distinction between primary and secondary propositions, Boole approximates the contemporary distinction, respectively, between predicate and propositional logic. Although propositional logic since Boole's day has come to be seen as more fundamental than predicate logic, reversing the order of what Boole for historical reasons considers as primary and secondary, it is important to see that in his later *Laws of Thought* and to a limited extent even in *Mathematical Analysis of Logic*, Boole makes at least two of his operators, AND and OR, do double duty as internal class designators and external propositional connectives. Conditional or hypothetical propositions and syllogisms, such as If p then q, p, therefore, q, can nevertheless be fully analyzed in terms of Boole's three original operators, so that it is appropriate to say that Boole's logical algebra anticipates the later development of symbolic propositional logic for which it provides most of the essential foundations.[9]

No one supposes that Boole invented the concepts or logical-grammatical terms

[8]The discrepancies between Boole's original formulation of his logic and its several refinements in what has since come to be known as Boolean algebra are reviewed in Hailperin, 'Boole's Algebra Isn't Boolean'. See Goodstein, 'Boolean Algebra Since George Boole'.

[9]In *Mathematical Analysis of Logic*, Boole prominently features the reduction of Aristotelian syllogistic logic to his algebra, while in *Laws of Thought*, he emphasizes the development of the system in its greatest generality, and only after numerous illustrations, almost as an afterthought before turning to the theory of probabilities, offers a brief reduction of Aristotelian logic, in Chapter XV, 'The Aristotelian Logic and its Modern Extensions, Examined by the Method of this Treatise', in 226–242.

NOT, AND, and OR. These were already part of thought and language, which could hardly function in their absence. Nor could Boole have possibly made extensive use of these operators if they were not already well established. What, then, can Boole be said to have discovered in defining the three Boolean class operators? What is the nature of his contribution to symbolic logic achieved by the formalization of these connectives that have since come to be known as Boolean operators? Perhaps the major accomplishment of Boole's logical algebra is his identification of the importance of these three operators, his symbolization of them as algebraic operators, and his recognition of their power and usefulness in representing logical relations and designing methods of calculation involving the mathematical properties of a logic of terms after the operators have been rigorously defined. We have yet to see how Boole applies the three Boolean operators, nor are we yet in a position to understand their role in the mathematical analysis of logic. We should nevertheless have already gained an appreciation for the versatility of Boolean operators in describing logical relations more flexibly and with greater generality than the Procrustean forms of classical Aristotelian syllogisms. Boolean operators have in later years established their usefulness in articulating logical commands for computer languages. If we have at least an intuitive grasp of how class complementation, conjunction or logical multiplication and disjunction or logical addition function in ordinary thought and language, then we may be ready to consider Boole's algebra of logic as among the most important chapters in the history of mathematics.[10]

3 BOOLE'S ALGEBRA OF CLASSES AND ELECTIVE OPERATORS

The use of Boolean operators in the mathematical interpretation of classical categorical propositions in Aristotelian syllogistic logic should now be more fully explained. We consider Boole's analysis conceptually and pre-symbolically, while introducing and commenting on some philosophical aspects of his algebraic symbolism. Then we can work systematically through Boole's logical reduction of syllogisms. Boole begins his Introduction to *Mathematical Analysis of Logic* by reminding his readers that:

> They who are acquainted with the present state of the theory of Symbolical Algebra, are aware, that the validity of the processes of analysis does not depend upon the interpretation of the symbols which are employed, but solely upon the laws of their combination. Every system of interpretation which does not affect the truth of the relations supposed, is equally admissible, and it is thus that the same process may,

[10]See the following secondary sources on Boole's algebra; some are formally technical. Bell, *Boolean-Valued Models and Independence Proofs in Set Theory.* Goldstein, *Boolean Algebra.* Hailperin, *Boole's Logic and Probability.* Rudeanu, *Boolean Functions and Equations.* Sikorski, *Boolean Algebras.*

> under one scheme of interpretation, represent the solution of a question on the properties of numbers, under another, that of a geometrical problem, and under a third, that of a problem of dynamics or optics. This principle is indeed of fundamental importance; and it may with safety be affirmed, that the recent advances of pure analysis have been much assisted by the influence which it has exerted in directing the current of investigation.[11]

According to Boole, we can interpret the same abstract formal laws of algebra alternatively as applying to arithmetic, to geometry, or to problems in physics. Nor is there any reason why those should be the only permissible applications. Boole proposes yet another interpretation of algebra that had not previously been developed, applying the principles of algebra to logic and logical inference rather than to number, figure, or to the material properties and causal interrelations of physical substances.

'That which renders Logic possible', Boole continues, 'is the existence in our minds of general notions, — our ability to conceive of a class, and to designate its individual members by a common name. The theory of Logic is thus intimately connected with that of Language. A successful attempt to express logical propositions by symbols, the laws of whose combination should be founded upon the laws of the mental processes which they represent, would, so far, be a step toward a philosophical language.'[12] The fundamental idea of Boole's analysis is that when we say, for an A-style categorical proposition, that 'All whales are mammals', we are saying in effect that the class of all whales is included in the class of all mammals. Boole's logic interprets a general algebra abstracted from its usual application in the analysis of numerical quantity, in which all such class relationships are formalized according to the same basic laws. The advantage of such a method is not only the flexibility, generality, and versatility of expression that an algebraic logic affords, but the fact that a logical algebra can avail itself of all the algebraic techniques that are well-established as previously useful in mathematics.

For those already familiar with contemporary symbolic logic, Boole's notation seems in many ways unorthodox. To understand it fully we need to appreciate its relations to popular techniquess of representing syllogistic relations in his day, and to the state of algebra in the mid-nineteenth century. Algebra has changed considerably since Boole's time, due in no small part to the influence of Boole's

[11] Boole, *Mathematical Analysis of Logic*, 3.

[12] Ibid., 4–5. In his essay, 'The Calculus of Logic', published one year after *Mathematical Analysis of Logic*, in 1848, Boole writes, 196–197: 'These appear to me to be the ultimate laws of syllogistic inference. They apply to every case, and they completely abolish the distinction of figure, the necessity of conversion, the arbitrary and *partial* rules of distribution, &c. If all logic were reducible to the syllogism these might claim to be regarded as the rules of logic. But logic, considered as the science of the relations of classes has been shewn to be of far greater extent. Syllogistic inference, in the elective system, corresponds to elimination. But this is not the highest in the order of its processes. All questions of elimination may in that system be regarded as subsidiary to the more general problems of the solution of elective equations. To this problem all questions of logic and of reasoning, without exception, may be referred.'

investigations. Many similarities nevertheless persist, and the symbolism Boole uses is clear enough once its unfamiliarity is overcome. As we have seen, Boole does not use as explicit operators the symbols we have conventionally represented as NOT, AND, and OR. The operators he requires in his logical algebra are symbolized as in standard arithmetical applications, as '–' or minus or subtraction (in effect, as Boole applies it in logical subtraction to designate the complement class of a class, NOT); simple juxtaposition of class-designating terms to indicate in Boole's case the logical product or multiplication (the set theoretical intersection if any of the designated classes, AND) represented by the terms; and '+' for logical addition or (exclusive) disjunction or exclusive set theoretical union (OR). Using only these simple and exactly defined operations, Boole builds an impressive logical interpretation of a general algebra of symbols.

As a holdover of the syllogistic logic which Boole hopes to replace, he writes, in *Mathematical Analysis of Logic*, X, Y, Z (and seldom needs more than three terms for his purposes) to abbreviate the entire *class of objects* with a certain property. These would be more commonly spoken of today as the *extensions of a predicate* representing the property. Boole's first innovation is to consider a class of symbols, x, y, z, as operators applied to any symbol representing individuals or classes that *selects*, respectively, all of the objects contained in X, Y, Z. Thus, x is a *selection* of all the members that comprise X, and similarly y *selects from* Y, and z from Z. In *Laws of Thought*, Boole dispenses with the capital letters used in the earlier presentation of his logic and begins immediately with *elective symbols* in lower case italics to represent classes of objects. It is not until midway through the book, in Chapter XV, that he turns as a final topic to 'The Aristotelian Logic and its Modern Extensions, Examined by the Method of This Treatise'.[13] By contrast with standard arithmetical algebra, Boole uses only two numerical values in his logical algebra, 0 and 1. He stipulates that 0 is to represent the null or empty class, consisting of no individuals; while 1, perhaps more surprisingly, represents the entire universe of discourse consisting of all individuals.

By juxtaposing elective symbols, for example, in xy, Boole represents the *product* or *logical multiplication* of x and y, by which he means the application to x and y of the operator we have previously denoted as AND. The Boolean operation xy is therefore, as he says in *Mathematical Analysis of Logic*, 'the selection of the class Y, and the selection from the class Y of such individuals of the class X as are contained in it, the result being the class whose members are both Xs and Ys'.[14] If, to pursue the example we have previously discussed from the logic of syllogisms, we were to abbreviate as W the class of all whales, C the class of all cetaceans, and M the class of all mammals, then by an obvious choice of symbols for elective operators, w, c and m, we could write wc to indicate the class of all whales that

[13] Boole, *Laws of Thought*, 226–242. A minor typographic difference is that in *Laws of Thought* the capital letters X, Y, Z, are always italicized, whereas in *Mathematical Analysis of Logic* they are not italicized. I have risked the appearance of inconsistent notation by following the same convention in discussing this aspect of Boole's algebra as he presents it in these two sources.

[14] Boole, *Mathematical Analysis of Logic*, 16.

are also cetaceans. The compound term wc is thus the class of all objects that are both whales AND cetaceans. The class represented by wc happens to include all whales and only some cetaceans, since all whales are cetaceans but not all cetaceans are whales; dolphins providing a counterexample as cetaceans that are not whales. Similarly for wm and cm, and for (exclusive) logical OR addition in $w + c$, $w + m$, and $c + m$. The power of combining symbols to represent ever more complex relations involving classes and complement classes of individuals included in or excluded from other classes and complement classes gives a sense of the greater generality and versatility of Boole's logic.

Class complementation or NOT is handled using minus or logical subtraction from the universal class of all objects, designated as unity by 1. If x is the selection of all objects in class X, then the class of all objects that are not in X is designated by $1 - x.$, as the complement class of all objects other than those selected by x. Combining Boole's algebraic logical symbols in obvious ways, we can represent the selection of all objects that are in class X AND NOT in class Y, for example, as the logical product of x and the complement of y, in $x(1 - y)$ (equivalently, in $(1 - y)x$). The selection of all objects that are in class X OR NOT in class X as the logical addition of $x + (1 - x)$. This, as we should expect, is the universal class or entire semantic domain of Boole's logic, as confirmed by agreement with the ordinary arithmetical interpretation of the algebra, where it is easy to see that $x+(1-x) = 1$. The selection of all objects that are NOT in class X OR in class Y (belonging neither to class X nor to class Y) can likewise be symbolized in Boole's algebra as the logical addition of the complement of x and the complement of y, in $(1 - x)(1 - y)$.

Boole does not propose to present an entirely new algebra, but only to articulate a previously undeveloped interpretation of established algebraic relations. To this end, he advances a set of principles for applying algebra to the mathematical analysis of syllogistic logic. The principles with only minor modifications have remained the essential core of what continues to be known as a Boolean algebra. Boole's logical algebra can be defined as any formal theory that satisfies the following three laws:

Axioms of Boole's Logic

1. $x(u + v) = xu + xv$
2. $xy = yx$
3. $x^n = x$

The first two of Boole's three laws are intuitively correct and related to the usual arithmetical interpretation of algebra. They are respectively laws of distribution and commutation. The third principle is not generally true in numerical algebra and uniquely distinguishes Boole's logical interpretation of algebra.

Principle (1) states that the conjunction of x AND the disjunction of u OR v is identical to the disjunction of the conjunction of (x AND u) OR the conjunction of (x AND v). As an example, consider the fact that the class consisting of all whales AND (all mammals OR all cetaceans) is identical to or the very same class as the class consisting of (all whales AND all mammals) OR (all whales AND all cetaceans). We obtain the same class no matter which way we put them together, and which in either case constitutes the class of all mammals, including all whales and all cetaceans. Principle (2) is equally obvious, implying that the class of all whales that are mammals is identical to the class of all mammals that are whales. As Boole illustrates the principle with a similar application in *Mathematical Analysis of Logic*: 'Whether from the class of animals we select sheep, and from the sheep those which are horned, or whether from the class of animals we select the horned, and from these such as are sheep, the result is unaffected. In either case we arrive at the class *horned sheep*'.[15]

Boole's principle (3), sometimes known as the Index Law, is less intuitive until it is interpreted specifically for the two values of logic, 0 and 1. To say that $x^n = x$ is not true in arithmetical algebra for all values of x and n. It is certainly not the case, for example, that $2^2 = 2$ or that $5^3 = 5$. The principle is nevertheless true even numerically in Boole's logical interpretation of algebra, where the only numerical values that can enter into formulas are 0 and 1. In Boole's logic, moreover, a limited principle of idempotence can be derived, which in the case of logical multiplication in $x^n = xx, xxx, xxxx$, etc., for $x = 2$, 3, 4, etc. This is just the class of all objects selected from class X AND the class of all objects selected from class X, which is identical to the class of all objects selected from class X, or simply to x itself; similarly for any logical product of n iterations of x. Boole cannot include the counterpart form of idempotence, expressed as $x + x = x$, because he interprets logical addtion as exclusive, and since no class excludes itself, any class logically added to itself must be equal to 0 rather than to itself.

Boole's three laws of algebraic logic are minimal but collectively very powerful. They testify to his ability to abstract and axiomatize the essential principles of a theory, relying on parallelisms already known to obtain in more standard arithmetical interpretations of algebra. Among the refinements of Boole's original system that have been deemed expedient over the years are those recommended already in Boole's time by William Stanley Jevons, Charles Sanders Peirce, and John Venn, among others.[16] The improvements notably include reinterpreting Boole's logical addition (OR) as inclusive rather than exclusive disjunction, and expanding the explicit principles of Boolean algebra to include the following now standard axioms, especially the rule of association, which is conspicuously but inexplicably

[15]Ibid., 17.

[16]Jevons, *Pure Logic, or The Logic of Quality Apart from Quantity*; Peirce, 'On an Improvement in Boole's Calculus of Logic'; 'Description of a Notation for the Logic of Relatives, Resulting From an Amplification of the Conceptions of Boole's Calculus of Logic'; and 'On the Algebra of Logic'; and Venn, *Symbolic Logic*. See also Halsted, 'Professor Jevons's Criticism of Boole's Logical System'; and Huntington, 'Boolean Algebras: A Correction'. More recently Hailperin in *Boole's Logic and Probability* has argued that Boolean logic is sound only if Boole's 'classes' are interpreted as multisets rather than sets in the present day conception, where a multiset is so defined that, unlike an ordinary set, it can contain multiple instances of the same objects.

missing from Boole's exposition. Where logical addition is understood inclusively rather than exclusively, the following principles obtain in modern Boolean algebra, and are also derivable from or at least logically consistent with Boole's original system:

Principles of Revisionary Boolean Algebra

Identity:	$1(x) = x$	$0 + x = x$
Boundary:	$0(x) = 0$	$1 + x = 1$
Idempotence:	$xx = x$	$x + x = x$
Commutation:	$xy = yx$	$x + y = y + x$
Distribution:	$x(y + z) = xy + xz$	$x + yz = (x + y)(x + z)$
Association:	$(xy)z = x(yz)$	$(x + y) + z = x + (y + z)$
Complementarity:	$x(1 - x) = 0$	$x + (1 - x) = 1$
Absorption:	$x(x + y) = x + xy = x$	

The complement of set A is sometimes denoted by Boole and later Boolean algebraists as A' or by A with a short horizontal bar directly over it, and logical multiplication represented by the Boolean operator AND as the set theoretical intersection, and logical addition represented by the Boolean operator OR as the set theoretical union, of sets A and B, is conventionally denoted in contemporary Boolean algebra, respectively, as $A \vee B$ and A v B. All of the axioms and theorems of Boole's algebra interpreted for logic on a domain of objects containing at least the two binary elements 0 and 1, and *closed* under the three Boolean operations, which is to say the logic is such that all and only Boolean classes can be generated by their application to any choice of class terms, translate in this way directly into modern set theoretical notation.[17]

Interpreted for arithmetic, logic or set theory, Boolean algebra comprises an elegant set of logical-mathematical relations. The principles of Boole's logic have proved to be intuitively satisfying as axioms for the analysis of logical reasoning, and of enormous utility in theory and practical applications. To understand the full impact of Boole's discoveries on the future course of mathematical logic, we should now consider Boole's comparison of his logical algebra with traditional Aristotelian logic.

4 ANALYSIS OF CATEGORICAL PROPOSITIONS

In reducing traditional logic to logical algebra, Boole's task has three main parts. He must show that his system: (1) makes it possible to express categorical or primary and hypothetical or secondary propositions and categorical and hypothetical syllogisms, including but not limited only to those of traditional Aristotelian logic;

[17]See Bell, *Boolean-Valued Models and Independence Proofs in Set Theory*; Comtet, 'Boolean Algebra Generated by a System of Subsets'; and Stone, 'Subsumption of the Theory of Boolean Algebras Under the Theory of Rings'.

(2) affords general rigorous methods of calculation by which the traditional techniques of syllogistic logic, notably conversion and evaluation of categorical and hypothetical syllogisms as deductively valid or invalid can be accomplished; (3) goes beyond the limitations of traditional logic by offering deeper insights into the logical structures of propositions and inferences. In the final, third, task, Boole must try to explain how it is that syllogistic logic captures only part and not the whole, and in that sense, not the most fundamental logical relations that properly belong to a science of logic as opposed to an incomplete and inadequately systematic collection of scientiic truths about logic, and that his algebraic analysis of logical inference surpasses classical syllogistic logic.

Boole's notation makes it possible first of all to symbolize all four of the classical A-E-I-O categorical propositions for categorical syllogisms. The translations are these:

A:	'All X's are Y's'	—	$xy = x$
E:	'No X's are Y's'	—	$xy = 0$
I:	'Some X's are Y's'	—	$v = xy$
O:	'Some X's are not Y's'	—	$v = x(1 - y)$

To say, as in an A-type categorical proposition, that all X's are Y's, as Boole formalizes it, is to say that the logical product of all objects that belong to class X AND that belong to class Y is identical to the selection of all objects that belong to class X itself; or, in other words, that there are no objects in class X that are not also in class Y. For this reason, Boole offers as an alternative logically equivalent translation of A-type propositions the formula $x(1-y) = 0$. To express an E-type categorical proposition that no X's are Y's is to say that the logical product of the selection of all objects from class X and from class Y is empty or null, here identical to 0.

The interpretation of I-type categorical propositions involves the introduction of a special elective operator v for the class V consisting of the objects (Boole says 'terms') common to classes X and Y. The otherwise unauthorized appearance of this new elective operator is not entirely satisfactory, and has been the subject of complaint by many even of Boole's most sympathetic commentators. The objection is that V and v in Boole's translation merely conjure up a name to stand for what would better be represented as the specific operations by virtue of which a set of objects common to classes X and Y are logically related to the membership of X and Y. There is an easy way to do this that involves a substantial revision of Boole's original notation, and indeed of his conception of logical algebra, if we are permitted to interpret I-type categorical propositions as the *negations* of E-type categorical propositions, as in $x(1 - y) \neq 0$. This states that the selection of objects from class X AND from class NOT-Y is not empty or null, and hence implies that the logical product of these classes contains at least some objects. We should similarly be able to symbolize O-type categorical propositions which say that some X's are not Y's more satisfactorily in Boole's algebra as the negations of corresponding A-type categorical propositions in the unauthorized extra-Boolean expression: $xy \neq 0$.[18]

It is somewhat of a mystery why Boole does not simply avail himself of propositional negation or nonidentity in his algebra, but relies instead on a nominal subterfuge, burying away the logic of 'some' in the unsymbolized definition of class V and its elective operator v for I-type and O-type categorical propositions. It is significant that Boole never uses the nonidentity sign, '$\neq$', anywhere in his system, even though it is commonly found in arithmetical algebra. Boole, as previously mentioned, moreover, explicitly denies that any proposition in his logic can be negated or 'negatived by a legitimate operation'.[19] It would be interesting to know why Boole, who evidently considered and deliberately rejected the possibility, decided to exclude propositional negation from his logic. What specific reasons did he have? The question is philosophically important because the absence of negation is the chief obstacle to regarding Boole's analysis especially of hypothetical or secondary propositions and hypothetical and other related inferences as providing all the essentials of a proto-propositional (and more generally, proto-first-order propositional and predicate-quantificational symbolic) logic. It is possible to define propositional negation or nonidentity only by going beyond Boole's symbolism in ways he would not have allowed. What is needed is a device for denying the truth of an arbitrary proposition, for which we might write 'Not' to distinguish propositional negation from Boole's class complementation operator NOT. Then, moving beyond the limitations of Boole's logic, we could write I-type and O-type categorical propositions, respectively, as $\text{Not}(x(1-y)=0)$ and $\text{Not}(xy=0)$. In this way, we could represent the fact, which Boole's logic can be faulted for not as explicitly and straightforwardly symbolizing, that A and I (and E and O) propositions are contradictories, which is to say negations of one another, in the traditional Aristotelian square of opposition which graphically maps the logical relations among A-E-I-O categorical propositions as contradictories, contraries, subcontraries, and subalterns.

It might appear to follow from $x(1-y) \neq 0$, for example, in Boole's strictly bivalent semantics, that in that case $x(1-y) = 1$, which would disastrous as a formalization or entailment of a formalization of an I-type categorical proposition.

[18]MacHale, 71: 'Boole later acknowledged that *The Mathematical Analysis of Logic* was written too hastily and with twentieth-century hindsight, it is easy to see that it contains a number of flaws. For example, if he had been prepared to write "*some X are not Y*" as $x(1-y) \neq 0$, rather than using the nonelective v in the equation $x(1-y) = v$, he could have saved himself a great deal of trouble. His use of "or" in the exclusive sense of either, but not both, was unnecessarily restrictive and, in addition, his use of the "indeterminate" symbol 0/0 caused a great deal of difficulty.' Grattan-Guinness, 'Logic in Boole's Mathematical Analysis of Logic', xxx: 'However, [Boole] offered no laws which "v" should satisfy, and could not always distinguish between traditional forms of proposition and those involved in the quantification of the predicate (which he did not analyse explicitly); for example, "$vx = vy$" could cover both "Some Xs are Ys" and "Some Xs are some Ys" (pp. 21–22). / Further, contrary to Boole's apparent belief, the solutions found by his methods were not always complete...For example, for the universal affirmative proposition "All Ys are Xs", symbolised as $(1-x)y = 0$, he put forward $y = xv$ as "the most general solution"...but he should have noticed that $x = 0$ was missing from it, and that it did not hold if $x = 0$ and v was a class such that $vy \neq 0$.'

[19]Boole, *Mathematical Analysis of Logic*, 18–19.

Of course, 0 and 1 are not the only values of the identity relation in Boole's logic; nor does it follow from the fact that a logical addition or multiplication is not identical to 0 that it is therefore identical to 1, or conversely. In explaining the logic of conditional hypothetical propositions and syllogisms, in both *Mathematical Analysis of Logic* and in more detail in *Laws of Thought*, Boole introduces the following convention for the negation of a proposition X, writing for it, $1 - x$. Utilizing the same method here, it should be possible in the same way to indicate nonidentity as suggested above for I-type categorical propositions by the sentence, $1 - (x(1 - y) = 0)$, and, similarly for O-type categorical propositions by the sentence, $1 - (xy = 0)$. If Boole had propositional negation, therefore, as from a certain standpoint, without knowing his precise reasons for excluding it, he surely should, then he ought to be able to symbolize E- and O-type categorical propositions as negations of one another.[20]

By the time Boole came to write *Laws of Thought*, he expanded his translations of categorical propositions to include an additional four that are not recognized in traditional Aristotelian syllogistic logic, but that reflect improvements in symbolizing quantifications, and in this case the complementation of major predicate terms that were proposed at about the same time by Hamilton and De Morgan.[21] Boole later writes:

> The course which I design to pursue is to show how these [traditional Aristotelian] processes of Syllogism and Conversion may be conducted in the most general manner upon the principles of the present treatise, and, viewing them thus in relation to a system of Logic, the foundations of which, it is conceived, have been laid in the ultimate laws of thought, to seek to determine their true place and essential character.
>
> The expressions of the eight fundamental types of proposition in the language of symbols are as follows:
>
> 1. All Y's are X's, $y = vx$.
> 2. No Y's are X's, $y = v(1 - x)$
> 3. Some Y's are X's, $vy = vx$.
> 4. Some Y's are not-X's, $vy = v(1 - x)$.
> 5. All not-Y's are X's, $1 - y = vx$.
> 6. No not-Y's are X's, $1 - y = v(1 - x)$.
> 7. Some not-Y's are X's, $v(1 - y) = vx$.
> 8. Some not-Y's are not-X's, $v(1 - y) = v(1 - x)$.[22]

By offering this expansion of syllogistic categorical propositions in his algebra, Boole transcends the limits of syllogistic logic as conceived by Aristotle and the

[20] The relation of propositional negation to Boole's predicate class complementation is considered by La Palma Reyes, Macnamara, Reyes, and Zolfaghari, 'The Non-Boolean Logic of Natural Language Negation'.

[21] Laita, 'Influences on Boole's Logic: The Controversy Between William Hamilton and Augustus De Morgan'.

[22] Boole, *Laws of Thought*, 228.

Scholastic tradition. Boole nevertheless believes that he has correctly identified what is vital to the Aristotelian idea of logic, and that in a way his algebraic approach to logic is more faithful to its intent. As he explains prior to advancing these translations:

> That which may be regarded as essential in the spirit and procedure of the Aristotelian, and of all cognate systems of Logic, is the attempted classification of the allowable forms of inference, and the distinct reference of those forms, collectively or individually, to some general principle of an axiomatic nature, such as the "dictum of Aristotle": Whatsoever is affirmed or denied of the genus may in the same sense be affirmed or denied of any species included under that genus...The idea of classification is thus a pervading element in those systems. Furthermore, they exhibit Logic as resolvable into two great branches, the one of which is occupied with the treatment of categorical, the other with that of hypothetical or conditional propositions. The distinction is nearly identical with that of primary and secondary propositions in the present work.[23]

Far from repenting his choice of the elective symbol v in expressing I and O categorical propositions in *Mathematical Analysis of Logic*, Boole in *Laws of Thought* later expands its use to include formalization of all eight fundamental propositions of syllogistic logic, encompassing the four original Aristotelian A-E-I-O types and the four 'modern extensions' with complemented minor terms proposed by Hamilton and De Morgan. This is the most important difference in Boole's reductions of syllogistic logic in *Mathematical Analysis of Logic* and *Laws of Thought.* Boole explains this general use of elective symbol v near the beginning of his later book, when he is presenting the basic principles of his logical algebra, from which the other applications are easily generalized. He states:

> Let us consider next the case in which the predicate of the proposition is particular, e.g. "All men are mortal." In this case it is clear that our meaning is, "All men are some mortal beings," and we must seek the expression of the predicate "some mortal beings." Represent then by v, a class indefinite in every respect but this, viz., that some of its members are mortal beings, and let x stand for "mortal beings," then will vx represent "some mortal beings." Hence, if y represent men, the equation sought will be $y = vx$.[24]

Boole argues that in this case at least, and in most similar applications involving A-type categorical propositions in traditional syllogistic logic, the assertion that 'All Y's are X's' as in 'All men are mortal', is not meant to express that all Y's are *all* X's or that in this instance that men are the only mortals, since of course

[23]Ibid., 226–227.
[24]Ibid., 61.

many things other than human beings have a limited lifespan and eventually die. To maintain that 'All men are mortal', as Boole understands the assertion, is rather to claim that all men are some among the potentially unlimited number of other things that are also mortal. Such an admission permits Boole to introduce his 'some objects' elective symbol v into all eight translations of the classical categorical propositions and their modern extensions in his logical algebra.

This new insight, which Boole arrives at only later in writing *Laws of Thought*, enables him to symbolize all quantified expressions of enhanced syllogistic logic in a more unified fashion than was possible in *Mathematical Analysis of Logic*. In particular, the revised translations in his later reduction of syllogistic logic embody a more direct way of representing A and O and E and I categorical propositions as explicit contradictories, as the square of opposition requires. If with Boole we interpret $1 - x$ as the logical complement of x, then at least the mystery of the unprecedented elective symbol v in I and O propositions is dissolved, and we can see A and O propositions as contradictories in the sense of involving explicitly complementary classes of objects. Where A propositions are symbolized as $y = vx$ and O propositions as $vy = v(1 - x)$, it is clear that an O proposition involves the complement class $1 - x$ of the class x designated in an A proposition. Here, in lieu of propositional negation, Boole at least offers inklings of a unified analysis of categorical propositions that was not fully achieved until the advent of truth functional calculus. In his later work, Boole arranges things in such a way that a good argument can be made to suggest that the symbolization of (A) 'All Y's are X's' contradicts the symbolization of (O) 'Some Y's are not-X's', in the sense that each translation now makes reference to the complement class of the other translation's major term. Similarly for the class complement contradiction of (E) 'No Y's are X's' and (I) 'Some Y's are X's', involving class x and its complement $1 - x$, which Boole translates, respectively, as $y = v(1 - x)$ and $vy = vx$.

Although Boole does not make the further contention, it is certainly in keeping with his belief that he has at once generalized and offered deeper insight into the logical structures superficially considered in traditional logic, to say that he has thereby accounted very precisely for the sense in which A-type and O-type and E-type and I-type categorical propositions are supposed to contradict one another in the Aristotelian square of opposition. While Boole again does not explain his motivations for rethinking the analysis of categorical propositions from *Mathematical Analysis of Logic* to *Laws of Thought*, it is tempting to suppose that part of his reason may have been to be able to express the logical contradictions that classically obtain between A and O and between E and I type categorical propositions.

Boole's use of elective symbol v nevertheless raises philosophical difficulties, both in *Mathematical Analysis of Logic*, and in its expanded application in *Laws of Thought*. Symbol v functions properly only by virtue of its special interpretation, and in other ways does not have the same symbolic role as other class terms. This is seen among other ways in the fact that from $x = y$ and $y = z$, it follows logically in Boole's system that $x = z$. From the fact that $x = v$ and $y =$

v, meaning that class x and class y are not empty, but contain some indefinite number of objects, however, it does not at all follow logically that therefore $x = y$, as William Kneale has shown.[25] If Boole tries to block the unwanted inference either by restricting identities to class terms, so that $x = v$ and $y = v$ cannot be written in the algebra, the restriction would only serve to further underscore the peculiar nature of v. Moreover, the parallel problem would still remain for the canonical Boolean identities, $x = vy$ and $z = vy$, to symbolize the facts that all x's are (some) y's and that all z's are (some) y's — say, that all whales are (some) mammals and all elephants are (some) mammals — from which it certainly does not follow that all and only whales are elephants, nor that the class of all whales is identical to the class of all elephants. To the extent that v looks like but does not function symbolically as a class term according to general algebraic conversion and transformation rules, Boole fails to satisfy his own requirement that all of logic be interpreted by means of purely algebraic syntactical operations.

Boole's idea that a universal quantification (all) conceals a hidden existential quantification (some) is interesting in formally explaining the standard conclusion of later extensionally interpreted predicate logic that universal quantifications logically imply existential quantifications, but not conversely. While Boole's algebra symbolizes many useful quantified expressions, and in particular recovers for mathematical analysis many quantified expressions that fall completely outside the scope of traditional Aristotelian syllogistic logic, it is not as fully general as Boole declares. It is incapable of adequately symbolizing all mixed multiple polyadic quantifications with overlapping quantifier scopes, including such propositions as 'All whales that swim in some northern oceans live with all sea creatures'. The best that Boole can offer is a formula in which the complex relation of living with all sea creatures is collapsed into a single class term that does not reflect its complex internal structure, contrary to his general rationale for undertaking the mathematical analysis of logic. In general terms, it is the same kind of limitation for which Boole criticizes traditional Aristotelian logic. For quantifications over n-ary relations, we need quantifiers, such as Frege in his *Begriffsschrift* and Whitehead and Russell in *Principia Mathematica* later developed, for which an exact and syntactically unambiguous delimitation of scope can be precisely specified. Only in this way can we refer to a chosen class of all or some objects interrelated in many different ways to classes of all or some particular objects. Boole's algebra approximates the most sophisticated necessary quantificational symbolizations, and does much better than any of his predecessors, but does not deliver a completely satisfactory logic for all quantifications.

5 BOOLEAN REDUCTION OF SYLLOGISTIC INFERENCES

We are finally in a position to see how Boole's logical algebra analyzes syllogistic inferences. Boole, in both *Mathematical Analysis of Logic* and *Laws of Thought*,

[25] Kneale, 'Boole and the Revival of Logic', 163.

demonstrates the method for a number of classically valid syllogisms. For present purposes, it should suffice to continue with the single example we have elaborated concerning the deductively valid cetaceans-whales-mammals syllogism, known, because of its arrangement of three categorical A-type propositions, as a syllogism in Barbara. We can formalize the inference in Boole's original algebra in *Mathematical Analysis of Logic* as follows:

1.	All cetaceans are mammals.	1. $cm = c$
2.	All whales are cetaceans.	2. $wc = w$
3.	All whales are mammals.	3. $wm = w$

As an essential preliminary, Boole makes use of an indispensable technique for *eliminating* certain terms from alternative expressions of categorical propositions. The procedure is essentially the same in both *Mathematical Analysis of Logic* and *Laws of Thought*. Boole abbreviates the class complement $1 - x$ as x', and shows how to eliminate term y from the following expressions, set equal to 0 as the empty or null set:

$$ay + b = 0; a'y + b' = 0$$

The equation for both expressions through which the y term is eliminated states:

$$ab' - a'b = 0.$$

The proof involves only simple arithmetical algebra, which Boole does not bother to spell out. The equivalence can be demonstrated in this way:

$ab' - a'b = \text{unknown}$
$a(1 - b) - (1 - a)b = \text{unknown}$
$(a - ab) - (b - ab) = \text{unknown}$
$a - ab = b - ab = 0.$

The elimination of y is accomplished by $ab' - a'b = 0$, because subtracting the logical product of the complement class NOT-a AND class b from the logical product of class a AND the complement class NOT-b' leaves nothing left over; the complement classes cancel each other out. The resulting formula is algebraically equivalent to the formulas from which the elective symbol y is thereby eliminated when both formulas are revealed as equal to 0. Subtracting ab from a leaves 0 in Boole's logic, as we see when we consider subtracting the class of all whales that are mammals from the class of all whales. Ordinarily, by contrast, subtracting ab from b would not necessarily be equal to 0, as the same instance shows; but the assumption here is that $ay + b = 0$. In that case, ay OR $b = 0$, which is to say that $ay = 0$ AND $b = 0$, from which on the present assumption it follows also that $a = 0$. If there are 0 mammals, then subtracting from the class of mammals the class of all whales that are mammals is equal again to 0. Where these conditions are met, reference to the elective symbol y drops out as logically irrelevant, as Boole argues. Similarly for the assumption that $a'y + b' = 0$.

Boole uses the principle to good effect in both his early and later algebraic treatment of classical syllogisms. We consider its application in each form in turn. If we recall that in Boole's *Mathematical Analysis of Logic*, $xy = x$ (All X's are Y's) is logically equivalent to $x(1-y) = 0$ (There are no X's that are not Y's), then we can equivalently rewrite the symbolization of the cetaceans-mammals-whales syllogism for purposes of calculation:

1.	$c(1-m)=0$
2.	$w(1-c)=0$
3.	$w(1-m)=0$

The equivalence eliminates what in traditional syllogistic logic is technically called the middle term c from the premises of the syllogism, resulting in the expression $w(1-m) = 0$. This is just the conclusion of the syllogism, that there are no whales that are not also mammals. The algebraic manipulations required to justify the inference of this standard syllogism are somewhat more subtle, and Boole is less help than usual in guiding readers who are not already accomplished algebraists but are trying to follow each move. Boole next offers the following shortcut, when he recommends:

> A convenient mode of effecting the elimination, is to write the equation of the premises, so that y shall appear only as a factor of one member in the first equation, and only as a factor of the opposite member in the second equation, and then to multiply the equations, omitting the y. This method we shall adopt.[26]

In the present example, the term to be eliminated is c rather than y, but the very same principle applies. By 'multiplying' two logical formulas, Boole naturally means to use logical multiplication; where $(a = b)(c = d) = (ac = bd)$, and where generally in arithmetical as well as logical algebra, $a(0) = 0$. We begin by converting the above statement of the syllogism into the following form, so that the term c to be eliminated appears as a factor of different members of the two premises:

1. $(1-m)c = 0$

2. $wc - w = 0$

Then we eliminate the middle term altogether, striking it out, according to Boole's shortcut method, leaving us with:

1′.	$(1-m)=0$	
2′.	$w-w=0$	
2a.	$w=w$	(from 2 by algebra)
3.	$w(1-m)=0$	(multiplying 1′ and 2a)

[26]Boole, *Mathematical Analysis of Logic*, 34.

The method appears to work properly. It justifies the conclusion of the syllogism in proposition (3) that all whales are mammals, $wm = m$, or that there are no whales in the complement class of nonmammals. As another example, consider the valid AEE-2 syllogism:

1.	All X's are Y's
2.	No Z's are Y's
3.	No Z's are X's

We symbolize and then convert the premises of the syllogism into the following form with the appropriate distribution of y terms, according to Boole's method:

1. $x = xy$

2. $zy = 0$

Next, we eliminate the y terms and multiply the premises together, to produce a symbolization in Boole's algebra of the validly derived conclusion in this syllogism, that No Z's are X's:

1′.	$x = x$
2′.	$z = 0$
3.	$zx = 0$

Unfortunately, Boole offers no further explanation of why the convenient procedure of eliminating certain terms of a syllogism, reformated first to appear as factors of different members in the premises of a syllogism, and then logically multiplying the resulting premises together, is supposed to result in a valid and never an invalid syllogism's conclusion. Eliminating the middle term in a classical syllogism is a standard method of informally verifying its validity or invalidity in Aristotelian logic. Where traditional logic fails to explain the effectiveness of the procedure, however, it might have been hoped that Boole, in agreement with his promise that his algebraic logic sheds light on what is logically obscure in syllogistic logic, would have lifted the veil and exposed the underlying logical machinery. Boole's method, moreover, is less systematic than the elimination of terms in arithmetical algebra. There, as we know, we can eliminate terms, as appropriate, without limit, for example, by subtracting them from both sides of an equation. In *Laws of Thought*, Boole explains this as one of the most important differences between arithmetical and logical algebra, and maintains that in logical algebra there is no general rule governing the elimination of elective class symbols.[27]

For the sake of comparison, Boole also offers a paradigm for the evaluation of a deductively invalid syllogism, AAA-2, showing that invalid inferences can also be detected by his procedure.

1.	All X's are Y's
2.	All Z's are Y's
3.	All Z's are X's

[27] Boole, *Laws of Thought*, 99–100.

We follow the same method as before. The translation into the logic's notation with y terms occurring as factors of different members of the two equations reads:

1. $x = xy$

2. $zy = z$

Then the elimination of the major term and multiplication of the resulting restatement of premises produces:

$$\begin{array}{ll} 1'. & x = x \\ 2'. & z = z \\ \hline 3'. & xz = xz \end{array}$$

Boole without further explanation reduces the proposition in (3) above to 0 = 0; yet it would appear that he could have done as well to write 1 = 1. The conclusion of the syllogism transformed in the required way in any event is patently not a translation of the AAA-2 conclusion, 'All Z's are X's', indicating that the syllogism fails to deduce a valid consequence.

For the sake of comparing Boole's early and later reductions of syllogistic logic in *Mathematical Analysis of Logic* and *Laws of Thought*, we conclude this section by briefly reconsidering the cetaceans-whales-mammals syllogism in the alternative translation style of Boole's *Laws of Thought*. In the amended symbolization method of *Laws of Thought*, Boole can render the same syllogism in the following way, using the complement class of v in $1 - v$ to indicate that the 'some' class of mammals to which the class of cetaceans belong is not necessarily the same indefinite 'some' class of cetaceans to which the class of whales belong. The translation in Boole's later formalization accordingly states:

$$\begin{array}{ll} 1. & c = vm \\ 2. & w = (1 - v)c \\ \hline 3. & w = vm \end{array}$$

Adapting Boole's method of eliminating terms unchanged from *Mathematical Analysis of Logic*, we arrive at a logically equivalent evaluation of the syllogism's deductive validity. Boole in *Laws of Thought* confidently asserts: 'Nothing is easier than in particular instances to resolve the Syllogism by the method of this treatise. Its resolution is, indeed, a particular application of the process for the reduction of systems of propositions.'[28] Using Boole's formal method, as in the *Mathematical Analysis of Logic* version, we accordingly eliminate term c, and logically multiply the result to obtain $w = (1 - v)vm$, which, alternatively, we can also obtain algebraically by substituting equivalent terms in the two assumptions. Ordinarily, of course, in Boole's logic, $(1 - v)v = 0$, which would make the above proposition equivalent again to $w = m$, meaning that the class of whales is identical to the class of mammals. Here, given the special interpretation of v as designating an indefinite class of some objects that have the property of being mammals, and

[28] Ibid., 231.

of $1 - v$ as a potentially different indefinite class of some objects that have the property of being cetaceans, $(1-v)v$ must correspondingly designate an indefinite class consisting of some objects that have the product of these indefinite classes, which is to say an indefinite class consisting of some objects that are both mammals AND cetaceans. Since assumption (1) declares that all cetaceans are mammals, $(1 - v)v = v$, the indefinite class consisting of at least some mammals. Thus, we obtain the required conclusion in the *Laws of Thought* translation style for 'All whales are (some among other) mammals', $w = vm$, as logically equivalent to the preceding expression, $w = (1 - v)vm$.

In *Laws of Thought*, Boole offers many ingenious examples of the method of evaluating the validity and invalidity of traditional syllogisms and other inferences generally. The applications are fascinating in their own regard, especially those taken from the history of philosophy and scientific argumentation, vividly demonstrating the generality and flexibility of Boole's algebra as a tool for logical analysis. Boole has often been faulted for failing to explain both his choice of symbolizations and formal derivations in sufficient detail, and it must be said that in more complex logically and philosophically interesting cases, Boole's use of his own method, no doubt transparent to him, ranges from the difficult to the extremely difficult for a beginner to follow in a step-by-step way. The examples we have considered here by comparison, while they give an indication of Boole's principles of logical analysis, only hint at and are not fully representative of the most rigorous demonstrations of Boolean logic.

Boole further applies his algebra to the formalization of *conversion* techniques for categorical propositions and syllogisms. Conversion is a traditional logical method whereby the terms in a proposition are transposed, sometimes in logically valid and sometime in invalid ways. For example, beginning with the sentence 'All whales are mammals', we might consider the sentence obtained by converseion which states that 'All mammals are whales'. Terms that are fully convertible in any appropriate context can therefore be regarded as representing logically equivalent concepts. Boole in these applications arrives at formally interesting results, but his conclusions are of concern primarily for the sake of proving that his logic is adequate to and goes beyond the relatively limited combinatorial possibilities of classical syllogistic logic. In the same spirit, Boole reduces conditionals as hypothetical or secondary propositions to the truth functional form of the modern propositional connective, dispensing with the categorical propositions as antecedent and consequent of a conditional as accidental to its correct logical form. In *Mathematical Analysis of Logic*, he argues:

> A hypothetical Proposition is defined to be *two or more categoricals united by a copula* (or conjunction), and the different kinds of hypothetical Propositions are named from their respective conjunctions, viz. conditional (if), disjunctive (either, or), &c.
>
> In conditionals, that categorical Proposition from which the other results is called the *antecedent*, that which results from it the *consequent*.

> Of the conditional syllogism there are two, and only two formulae.
>
> 1st. The constructive,
>
> If A is B, then C is D,
>
> But A is B, therefore C is D.
>
> 2nd. The Destructive,
>
> If A is B, then C is D,
>
> But C is not D, therefore A is not B.
>
> A dilemma is a complex conditional syllogism, with several antecedents in the major, and a disjunctive minor.
>
> If we examine either of the forms of conditional syllogism above given, we shall see that the validity of the argument does not depend upon any considerations which have reference to the terms A, B, C, D, considered as the representatives of individuals or of classes. We may, in fact, represent the Proposition A is B, C is D, by the arbitrary symbols X and Y respectively, and express our syllogisms in such forms as the following:
>
> If X is true, then Y is true,
>
> But X is true, therefore Y is true.[29]

The analysis of these propositions takes Boole well beyond the limited capabilities of traditional logic, and demonstrates the greater flexibility and generality of his algebraic logic over the logic of syllogisms. By defining the truth conditions for conditional propositions, Boole sets the stage for the development of modern symbolic propositional logic as a logic of truth functional connectives. His use of $1 - x$ to represent the falsehood of proposition X nevertheless raises questions in the philosophy of logic to which Boole seems blithely insensitive. The symbolization appears to mark the proposition as detracting from the unity or universe of the logic's semantic domain of objects. It is unclear, moreover, what precise sense such a suggestion might be given, and the analysis at this stage in Boole's theory, although it points roughly in the right direction for the future of symbolic logic, must be seen as philosophically inadequate.

In the final chapter of *Mathematical Analysis of Logic*, on 'Properties of Elective Functions', where he approximates some of problems investigated by logical metatheory that were to enter the field only more than a century later, Boole writes:

[29] Boole, *Mathematical Analysis of Logic*, 48.

> Suppose it is required to determine for what forms of the function $f(x)$, the following equation is satisfied, viz.
>
> $$\{f(x)\}^n = f(x).$$
>
> Here we at once obtain for the expression of the conditions in question,
>
> $$\{f(0)\}^n = f(0),\ \{f(1)\}^n = f(1)...$$
>
> For, as different elective symbols combine with each other according to the same laws as symbols of quantity, we can first expand a given function with reference to any particular symbol which it contains, and then expand the result with reference to any other symbol, and so on in succession, the order of the expansions being quite indifferent.[30]

Boole even contemplates the possibility of a 'contradictory universe' for certain applications of his algebra, for which he says:

> It may happen that the simultaneous satisfaction of equations thus deduced, may require that one or more of the elective symbols should vanish. This would only imply the nonexistence of a class: it may even happen that it may lead to a final result of the form
>
> $$1 = 0,$$
>
> which would indicate the nonexistence of the logical Universe. Such cases will only arise when we attempt to unite contradictory Propositions in a single equation.[31]

The adoption of 0 and 1 as values for truth and falsehood in Boole's system from their original designation as the null and universal classes in the logic's semantic domain is made explicit in the final paragraph of *Mathematical Analysis of Logic*, when he writes:

> In virtue of the principle, that a Proposition is either true or false, every elective symbol employed in the expression of hypotheticals admits only of the values 0 and 1, which are the only quantitative forms of an elective symbol. It is in fact possible, setting out from the theory of Probabilities (which is purely quantitative), to arrive at a system of methods and processes for the treatment of hypotheticals exactly similar to those which have been given. The two systems of elective symbols and of quantity osculate, if I may use the expression, in the

[30]Ibid., 61–62.
[31]Ibid., 65.

> points 0 and 1. It seems to me to be implied by this, that unconditional truth (categoricals) and probable truth meet together in the constitution of contingent truth; (hypotheticals). The general doctrine of elective symbols and all the more characteristic applications are quite independent of any quantitative origin.[32]

Boolean algebra has been important for logic, set theory, and computer science largely because of its binary interpretation on the numbers 0, 1. To equate truth with 1, however, and falsehood with logical subtraction from 1, nevertheless seems peculiar. Even if we consider the universe of discourse to consist of all actually existent facts, it would appear at most to be justified to equate truth with some proper part of the universe, that part consisting only of truth-making states of affairs to which all and only the true propositions in a language correspond. What sense does it make to stipulate that truth is identical to all such facts and to all other classes of objects, chairs, houses, trees, galaxies, DNA molecules, quarks, and the like, that can be represented in the logic?

Boole addresses these questions in *Laws of Thought*. There he offers a more elaborate exploration of the use of 0 and 1 in expressing the truth and falsehood of entire propositions for the same purpose of extending his logical algebra to hypothetical, or what he later refers to as secondary propositions, in particular those involving propositional conjunction, (exclusive) disjunction, and the if-then conditional. Boole admits that in *Mathematical Analysis of Logic* he crudely interpreted 1 as consisting of all states of affairs, so that to logically subtract a proposition from this unity or universal class could be interpreted as considering the class of all states of affairs minus the one represented by the proposition whose falsehood is being affirmed.[33] In *Laws of Thought*, by contrast, he proposes for heuristic reasons only, supplanted later by a more literal interpretation, to treat 1 as the unity of all moments of time, so that logical subtraction from 1 can be understood as indicating that there is no moment of time at which the proposition is true.[34]

Boole suggests as an explanatory expedient a temporal analogy, by which the universe of discourse or semantic domain of secondary propositions, the objects or things with which it is concerned, in the special interpretation of the values 0 and 1 that the logic of secondary propositions requires, consists of moments of time at which propositions are true. He continues:

> Now, in considering any such relations as the above, we are not called upon to inquire ito the whole extent of their possible meaning (for

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[32]Ibid., 82.

[33]Boole, *Laws of Thought*, 176: 'In a former treatise on this subject (Mathematical Analysis of Logic, p. 49), following the theory of [John] Wallis respecting the Reduction of Hypothetical Propositions, I was led to interpret the symbol 1 in secondary propositions as the universe of "cases" or "conjunctures of circumstances"; but this view involves the necessity of a definition of what is meant by a "case", or "conjuncture of circumstances"; and it is certain, that whatever is involved in the term beyond the notion of time is alien to the objects, and restrictive of the processes of formal Logic.'

[34]Ibid., 162–163.

> this might involve us in metaphysical questions of causation, which are beyond the proper limits of science); but it suffices to ascertain some meaning which they undoubtedly possess, and which is adequate for the purposes of logical deduction. Let us take, as an instance for examination, the conditional proposition, "If the proposition X is true, the proposition Y is true." An undoubted meaning of this proposition is, that the *time* in which the proposition X is true, is *time* in which the proposition Y is true. This indeed is only a relation of coexistence, and may or may not exhaust the meaning of the proposition, but it is a relation really involved in the statement of the proposition, and further, it suffices for all the purposes of logical inference.[35]

Boole distances himself from philosophical commitment to a literal interpretation of the truth of propositions in terms of moments or intervals of time during which they are true. Thus, he states:

> I shall avail myself of the notion of time in order to determine the laws of the expression of secondary propositions, as well as the laws of combination of the symbols by which they are expressed. But when those laws and those forms are once determined, this notion of time (essential, as I believe it to be, to the above end) may practically be dispensed with. We may then pass from the forms of common language to the closely analogous forms of the symbolical instrument of thought here developed, and use its processes, and interpret its results, without any conscious recognition of the idea of time whatever.[36]

Surprisingly, Boole never returns to the interesting philosophical issue of how truth and falsehood are to be interpreted if not by reference to the concept of time. Perhaps he thinks it is sufficient to have identified formal relations capable of various interpretations that threaten in their implications to infringe on deeper problems of 'causation', and as such are avoidable in mathematical logic as intractable problems of metaphysics. Perhaps it is enough, as a contemporary logician or formal semanticist might say, to show that the two truth values can be 'mapped onto' 0 and 1. Instead of further exploring these issues, he offers the following series of definitions of truth and falsehood for entire propositions, in terms of which he then proceeds to define the propostional connectives conjunction, (exclusive) disjunction, and the if-then conditional, for the algebra of secondary propositions and logical inferences containing secondary propositions. He writes:

> As 1 denotes the whole duration of time, and x that portion of it for which the proposition X is true, $1 - x$ will denote that portion of time for which the proposition X is false.

[35] Ibid., 163.
[36] Ibid., 164.

> Again, as xy denotes that portion of time for which the propositions X and Y are both true, we shall, by combining this and the previous observation, be led to the following interpretations, viz.:
>
> The expression $x(1 - y)$ will represent the time during which the proposition X is true, and the proposition Y false. The expression $(1-x)(1-y)$ will represent the time during which the propositions X and Y are simultaneously false.
>
> The expression $x(1-y)+y(1-x)$ will express the time during which either X is true or Y true, but not both; for that time is the sum of the times in which they are singly and exclusively true. The expression $xy + (1-x)(1-y)$ will express the time during which X and Y are either both true or both false.
>
> If another symbol z presents itself, the same principles remain applicable. Thus xyz denotes the time in which the propositions X, Y, and Z are simultaneously true; $(1-x)(1-y)(1-z)$ the time in which they are simultaneously false; and the sum of these expressions would denote the time in which they are either true or false together.[37]

The binary algebraic semantics of truth and falsehood are explained by Boole in terms of 0 and 1. We must bear in mind that throughout this discussion Boole for undisclosed reasons requires an account of truth and falsehood of propositions that altogether shuns any recognizable form of truth functional propositional negation. Boole maintains:

> 1st. *To express the Proposition,* "*The proposition X is true.*"
>
> We are here required to express that within those limits of time to which the matter of our discourse is confined the proposition X is true. Now the time for which the proposition X is true is denoted by x, and the extent of time to which our discourse refers is represented by 1. Hence we have
>
> $$x = 1$$
>
> as the expression required.
>
> 2nd. *To express the Proposition,* "*The proposition X is false.*"
>
> We are here to express that within the limits of time to which our discourse relates, the proposition X is false; or that within those limits there is no portion of time for which it is true. Now the portion of time for which it is true is x. Hence the required equation will be
>
> $$x = 0.$$

[37] Ibid., 168.

> This result might also be obtained by equating to the whole duration of time 1, the expression for the time during which the proposition X is false, viz., $1 - x$. This gives
>
> $$1 - x = 1,$$
>
> whence
>
> $$x = 0.$$ [38]

Having defined truth and falsehood of propositions in terms of the durations of time at which a proposition is true, Boole proceeds in his 3rd through 5th 'considerations' to define (exclusive) propositional disjunction, propositional conjunction, and the propositional if-then conditional as they have typically been truth functionally defined in standard propositional logic. Exclusive disjunction is defined as a proposition that is true just in case either but not both of its disjuncts are true; conjunction is defined as a proposition that is true just case both of its conjuncts are true; and the conditional is defined as a proposition that is true just in case where its antecedent or 'if' part is true its consequent or 'then' part is also true, and is otherwise false. He accordingly adds:

> In the laws of expression above stated those of interpretation are implicitly involved. The equation
>
> $$x = 1$$
>
> must be understood to express that the proposition X is true; the equation
>
> $$x = 0,$$
>
> that the proposition X is false. The equation
>
> $$xy = 1$$
>
> will express that the propositions X and Y are both true together; and the equation
>
> $$xy = 0$$
>
> that they are not both together true.
>
> In like manner the equations

[38] Ibid., 169.

$$x(1-y)+y(1-x)=1,$$
$$x(1-y)+y(1-x)=0,$$

will respectively assert the truth and the falsehood of the disjunctive Proposition, "Either X is true or Y is true." The equations

$$y=vx$$
$$y=v(1-x)$$

will respectively express the Propositions, "If the proposition Y is true, the proposition X is true." "If the proposition Y is true, the proposition X is false."[39]

It must be acknowledged that, despite Boole's efforts to forestall philosophical or metaphysical controversy in his interpretation of truth and falsehood for his rudimentary propositional logic of secondary propositions, there are many conceptual difficulties ostensibly entailed by his proposal.

There is in the first place the apparent blatant circularity in Boole's attempt to characterize the truth (or falsehood) of a proposition as the duration of time during which the proposition is true (false). If we knew what it meant for a proposition to be true (or false) at a certain time, then obviously we would not stand in need of a explanation or definition of what it means for a proposition to be true (false). Boole nowhere tries to mitigate or evince any recognition of the problem, but it is hard to see how his algebraic definitions of truth and falsehood escape this sort of conceptual trivialization.

It is also questionable whether Boole's definitions of truth and falsehood can be adequate for propositions that are ostensibly eternal or atemporal, such as propositions about mathematical entities. Boole might want to say that these propositions are true or false at all times, but since abstract entities are themselves atemporal it seems odd to limit their truth conditions in this way. If time is a result of the big bang, and the big bang happened not to occur, then there would be no moments of time at which it is true that $2+3=5$; yet the proposition, many philosophers would argue, is nonetheless atemporally true.

Finally, logicians sometimes claim that sentences in ordinary language are often imperfect, incomplete expressions of entire propositions. To say, simply, 'It is raining', is then shorthand for 'It is raining at time t and place P'. If this is true, then to say that it is true that 'It is raining' on Boole's heuristic temporal analysis of truth conditions is to say that it is true at time t that it is raining. If 'It is raining' is just shorthand for saying 'It is raining at time t and place P', however, then, contrary to Boole's account of propositional truth, it should always be true that 'It is raining', because it is always true that 'It is raining at time t and place P'.

[39]Ibid., 172–173.

It may be possible for Boole to avoid or soften the impact of such criticisms. But Boole does not even seem to recognize the potential difficulties which his concepts of truth and falsity at specific durations of time seem to involve. As a result, it must be said that his attempt to extend his algebra to the truth and falsehood of propositions in his logic of secondary propositions remains problematic. The sticking points in his analysis are reflected in the manifest ambiguity, about which Boole seems altogether unconcerned, by which an expression like xy or $x + y$ can alternatively represent a class of objects in a compound term lacking all truth value, or a propositionally complex proposition that is either true or false, depending on the values of x and y, and in particular on whether or not x and y are class terms or propositions. Since Boole's notation itself does not discriminate between these possibilities, what are we to say if x represents a class of objects (all red things) and y represents the proposition 'All men are mortal'? Are xy and $x + y$ in that case compound class terms or propositions or neither or both? It is a defect of Boole's algebra that it does not anticipate such questions with satisfactorily unambiguous purely formal distinctions.[40]

Even so, it would appear that by defining the truth and falsehood of propositions in terms of his binary values 0 and 1, Boole despite himself equips his algebra with the equivalent of propositional negation. For then it should be possible to define a negation or 'negativing' quasi-Boolean operator Not by stipulating that $x = 1$ just in case or if and only if Not-$x = 0$. If he were to avail himself of this device, Boole would unequivocally have provided all the necessary foundations for classical propositional logic. This he could further streamline at the expense of parallelism with his algebra of classes by defining logical multiplication for propositions in terms of negation and logical addition, or by defining logical addition for propositions in terms of negation and logical multiplication. Such simplifications, and even more radical reductions to a single combined negation and disjunction or negation and conjunction operator are standard in contemporary propositional logic, following what has come to be known as the De Morgan duality equivalences. It would also then be possible for Boole to symbolize categorical propositions more explicitly to indicate that A and O and E and I propositions are logical contradictories in the sense of propositional negation, and not merely in the sense of class complementation. This is a step that for reasons unknown Boole himself was unwilling to take, and that, building on Boole's work, had to await further advances in the evolution of mathematical logic.[41]

[40] See Feys, 'Boole's Methods of Interpretation and Development'; Halsted, 'Boole's Logical Method'; Hooley, 'Boole's Method for Solving Boolean Equations'; Van Evra, 'A Reassessment of George Boole's Theory of Logic'. Lukasiewicz, *Aristotle's Syllogistic from the Standpoint of Modern Formal Logic*, offers an insightful perspective on the role of Boolean algebra in supplanting traditional Aristotelian syllogistic logic.

[41] *Inter alia*, see sources on the subsequent history of symbolic logic, especially Dudman, 'From Boole to Frege'; Feys, 'Boole as a Logician'; and Hailperin, 'Boole's Abandoned Propositional Logic'.

6 BOOLEAN LOGIC OF PROBABILITY AND INDUCTION

In extending his algebraic logic to construct a theory of probability, Boole is chiefly concerned to establish recognized conclusions of statistical analysis within the framework of his algebra. He observes that there are two different approaches to the study of probability, through 'number' and 'logic', and a central point of philosophical interest in his contributions to the topic is his efforts to show that the two apparently distinct ways of understanding probability can be made to coincide.

Although in *Mathematical Analysis of Logic* Boole only hints at the possibility of developing a similar treatment of probability, by the time he came to write *Laws of Thought*, he had devoted much of his thought to its problems, stirred to reflection, as some commentators have conjectured, by De Morgan's work in the field. Thus, in his first book on logic, Boole writes: 'It is in fact possible, setting out from the theory of Probabilities (which is purely quantitative), to arrive at a system of methods and processes for the treatment of hypotheticals exactly similar to those which have been given'.[42]

Boole begins Chapter XVI of *Laws of Thought*, 'Of the Theory of Probabilities', with a convenient statement of fundamental principles, which he attributes to the French mathematician, Siméon Denis Poisson, in his *Recherches sur la Probabilitè des Jugemens*. Boole agrees with the following definitions:

> "The probability of an event is the reason we have to believe that it has taken place, or that it will take place."
>
> "The measure of the probability of an event is the ratio of the number of cases favourable to that event, to the total number of cases favourable or contrary, and all equally possible" (equally likely to happen).[43]

From this starting point, Boole draws the indicated inference that probability so defined must concern the subjective states of knowledge and ignorance of those performing the calculations. He continues: 'From these definitions it follows that the word *probability*, in its mathematical acceptation, has reference to the state of our knowledge of the circumstances under which an event may happen or fail'.[44] Yet Boole thinks that such a conclusion would be incorrect, and that probability is in fact an objective standard of the likely or unlikely conditional occurrence of an event.[45]

With a view toward articulating an objective theory of probabilities, Boole explains his purpose: 'Let us endeavour from the above statements and definitions

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[42]Boole, *Mathematical Analysis of Logic*, Postscript, 82. A defender of Boole on this minor point might argue that in this passage Boole is referring to the theory of probability as it has been arithmetically formalized by other acknowledged theorists, rather than as it might ideally be presented.

[43]Boole, *Laws of Thought*, 244.

[44]Ibid.

[45]Ibid., 244–245.

to form a conception of the legitimate object of the theory of Probabilities'.[46] This he proceeds to do by considering a series of seven principles, which he extracts primarily from the writings of Pierre-Simon Laplace. Together, they offer the most concise conceptual overview of the laws shared by Boole's theory of probability:

PRINCIPLE

1st. If p be the probability of the occurrence of any event, $1 - p$ will be the probability of its non-occurrence.

2nd. The probability of the concurrence of two independent events is the product of the probabililties of those events.

3rd. The probability of the concurrence of two dependent events is equal to the product of the probability of one of them by the probability that if that event occur, the other will happen also.

4th. The probability that if an event, E, take place, an event, F, will also take place, is equal to the probability of the concurrence of the events E and F, divided by the probability of the occurrence of E.

5th. The probability of the occurrence of one or the other of two events which cannot concur is equal to the sum of their separate probabilities.

6th. If an observed event can only result from some one of n different causes which are *à priori* equally probable, the probability of any one of the causes is a fraction whose numerator is the probability of the event, on the hypothesis of the existence of that cause, and whose denominator is the sum of the similar probabilities relative to all the causes.

7th. The probability of a future event is the sum of the products formed by multiplying the probability of each cause by the probability that if that cause exist, the said future event will take place.[47]

Although the algebraic operations Boole mentions in connection with these standard principles of probability are evidently meant to be arithmetical rather than logical, involving numerical addition and multiplication rather than logical addition and multiplication. Already in the first chapter of *Laws of Thought*, on the 'Nature and Design of this Work', Boole previews the convergence of numerical and logical methods in his formal theory of probabilities, distinguishing his approach as unique from previous accounts. He argues:

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[46]Ibid., 245.
[47]Ibid., 249.

> Speaking technically, we must be able to express the event whose probability is sought, as a function of the events whose probabilities are given. Now this explicit determination belongs in all instances to the department of Logic. Probability, however, in its mathematical acceptation, admits of numerical measurement. Hence the subject of Probabilities belongs equally to the science of Number and to that of Logic. In recognising the co-ordinate existence of both these elements, the present treatise differs from all previous ones; and...this difference not only affects the question of the possibility of the solution of problems in a large number of instances, but also introduces new and important elements into the solutions obtained...[48]

Later, Boole claims that it makes no difference whether probability theory is derived as standardly from applied arithmetic, or from the general formal algebraic relations variously interpreted by Boole as logical or arithmetical. He maintains that:

> The Theory of Probabilities stands, as it has already been remarked..., in equally close relation to Logic and to Arithmetic; and it is indifferent, so far as results are concerned, whether we regard it as springing out of the latter of these sciences, or as founded in the mutual relations which connect the two together.[49]

Boole's statement of the seven Laplacian probability principles is now expressed in his algebraic logic. An example is Boole's reference to the complementary probability of the nonoccurrence of an event whose probability of occurrence is p, expressed in the first principle as $1 - p$. This algebraic formula bears a clear and direct analogy to Boole's symbolization of predicate complementarity and the definition of truth and falsehood conditions for secondary propositions in logic. The use of 1 as a term to represent truth in algebraic logic, is further extended in Boole's probability theory as a symbol for the certainty of the occurrence of an event. Boole elaborates the concept in this way:

> When it is certain that an event will occur, the probability of that event, in the above mathematical sense, is 1. For the cases which are favourable to the event, and the cases which are possible, are in this instance the same.
>
> Hence, also, if p be the probability that an event x will happen, $1 - p$ will be the probability that the said event will not happen. To deduce this result directly from the definition, let m be the number of cases favourable to the event x, n the number of cases possible, then $n - m$ is the number of cases unfavourable to the event x. Hence, by definition,

[48]Ibid., 13.
[49]Ibid., 17.

$m/n =$ probability that x will happen.
$n - m/n =$ probability that x will not happen.
But $n - m/n = 1 - m/n = 1 - p$.[50]

Boole then sets the stage to establish the convergence of numerical and logical methods in probability theory. He begins with the following explanation of basic terms, making the projected connection between logic and arithmetic explicit:

> As the investigations upon which we are about to enter are based upon the employment of the Calculus of Logic, it is necessary to explain certain terms and modes of expression which are derived from this application.
>
> By the event x, I mean that event of which the proposition which affirms the occurrence is symbolically expressed by the equation
>
> $$x = 1.$$
>
> By the event $f(x, y, z, \ldots)$, I mean that event of which the occurrence is expressed by the equation
>
> $$f(x, y, z, \ldots) = 1.$$
>
> Such an event may be termed a compound event, in relation to the simple events x, y, z, which its conception involves. Thus, if x represent the event "It rains," y the event "It thunders," the separate occurrences of those events being expressed by the logical equations
>
> $$x = 1,\ y = 1,$$
>
> then will $x(1 - y) + y(1 - x)$ represent the event or state of things denoted by the Proposition, "It either rains or thunders, but not both;" the expression of that state of things being
>
> $$x(1 - y) + y(1 - x) = 1.$$
>
> If for brevity we represent the function $f(x, y, z, \ldots)$, used in the above acceptation by V, it is evident...that the law of duality
>
> $$V(1 - V) = 0,$$
>
> will be identically satisfied.[51]

[50] Ibid., 253.
[51] Ibid., 257–258.

Boole relies as elsewhere generally in his logic on the exclusive interpretation of logical addition. The law of duality for probability theory is invoked in a form precisely isomorphic to that in Boole's logic. Boole asserts that for any predicate term x, $x(1-x) = 0$. This is to say that the logical product of x and the complement of x, in other words, the set theoretical intersection of the extension of a predicate and the complement extension of the predicate, what they have in common, is nothing, null, and hence equal to 0. Boole extends the very same logical duality principle with comparable intuitive justification to the multiplication of probabilities of the occurrence and nonoccurrence of the same event. Boole provides the following table of correspondences for the probabilities of events in the framework of his logical algebra:

EVENTS.	PROBABILITIES.
xy, Concurrence of x and y,	pq.
$x(1-y)$, Occurrence of x without y,	$p(1-q)$.
$(1-x)y$, Occurrence of y without x,	$(1-p)q$.
$(1-x)(1-y)$, Conjoint failure of x and y,	$(1-p)(1-q)$.

> We see that in these cases the probability of the compound event represented by a constituent is the same function of p and q as the logical expression of that event is of x and y; and it is obvious that this remark applies, whatever may be the number of the simple events whose probabilities are given, and whose *joint existence or failure* is involved in the compound event of which we seek the probability.[52]

Boole further extends the method of eliminating terms from algebraic equations in logic to the determination of probabilities for two broad categories of unconditioned events, and in more complicated applications for conditioned events. The analysis of probabilities for conditionally related events is interpreted, as Boole explains, 'according to the rules of the Calculus of Logic'.[53] The overarching formal structure for calculating probability values is thus Boole's logical algebra, in particular with respect to events related by secondary propositional connectives, such as conjunction (and), disjunction (exclusive or), and the conditional (if-then). Having explicated the arithmetical evaluation of probabilities in a logical context, Boole proceeds to bring together logical and numerical approaches to probability theory. He writes:

> It has been stated...that there exist two distinct definitions, or modes of conception, upon which the theory of probabilities may be made to depend, one of them being connected more immediately with Number, the other more directly with Logic. We have now considered the consequences which flow from the numerical definition, and have shown how it conducts us to a point in which the necessity of a connexion with

[52]Ibid., 259.
[53]Ibid., 271.

> Logic obviously suggests itself. We have seen to some extent what is the nature of that connexion; and further, in what manner the peculiar processes of Logic, and the more familiar ones of quantitative Algebra, are involved in the same general method of solution, each of these so accomplishing its own object that the two processes may be regarded as supplementary to each other. It remains to institute the reverse order of investigation, and, setting out from a definition of probability in which the logical relation is more immediately involved, to show how the numerical definition would thence arise, and how the same general method, equally dependent upon both elements, would finally, but by a different order of procedure, be established.[54]

The exact nature of the relation between the numerical and logical aspects of probability theory have yet to be explained. Boole indicates that there is an analogy between the two approaches, and an overlap of common principles that are identical at least in their more generalized mathematical forms, by which one supplements the other. However, he also holds that there are differences of substance as well as differences of interpretation that are of philosophical importance in understanding the formal laws of probability. He continues:

> That between the symbolical expressions of the logical calculus and those of Algebra there exists a close analogy, is a fact to which attention has frequently been directed in the course of the present treatise. It might even be said that they possess a community of forms, and, to a very considerable degree, a community of laws. With a single exception in the latter respect, their difference is only one of interpretation. Thus the same expression admits of a logical or of a quantitative interpretation, according to the particular meaning which we attach to the symbols it involves. The expression xy represents, under the former condition, a concurrence of the events denoted by x and y; under the latter, the product of the numbers or quantities denoted by x and y. And thus every expression denoting an event, simple or compound, admits, under another system of interpretation, of a meaning purely quantitative. Here then arises the question, whether there exists any principle of transition, in accordance with which the logical and the numerical interpretations of the same symbolical expression shall have an intelligible connexion.[55]

Just as 0 is the term indicating for Boole's probability theory the non-concurrence or the occurrence and nonoccurrence of the same event, so 1 represents the certainty of an event occurring. A prime example of an event occurring with certainty is the compound event denoted algebraically as the logical addition of the exclusive

[54] Ibid.
[55] Ibid., 271–272.

occurrence or nonoccurrence of the same event. Boole presents the definition in this way:

> Furthermore, if we set out from the above hypothetical definition of the measure of probability, we shall be conducted, either by necessary inference or by successive steps of suggestion, which might perhaps be termed *necessary*, to the received numerical definition. We are at once led to recognize unity (1) as the proper numerical measure of certainty. For it is certain that any event x or its contrary $1 - x$ will occur. The expression of this proposition is
>
> $$x + (1 - x) = 1,$$
>
> whence, by hypothesis, $x + (1 - x)$, the measure of the probability of the above proposition, becomes the measure of certainty. But the value of that expression is 1, whatever the particular value of x may be. Unity, or 1, is therefore, on the hypothesis in question, the measure of certainty.[56]

Boole now prepares to formulate the numerical probability of any of a series of mutually exclusive equally possible events occurring. He appeals again to algebraic symbolisms to express logical relations between the events in question, relative to which their respective probability values are assigned. He writes:

> The proposition which affirms that some one of these [events] must occur will be expressed by the equation
>
> $$t_1 + t_2 \ldots + t_n = 1;$$
>
> and, as when we pass in accordance with the reasoning of the last section to numerical probabilities, the same equation remains true in form, and as the probabilities $t_1, t_2 \ldots t_n$ are equal, we have
>
> $$nt_1 = 1,$$
>
> whence $t_1 = 1/n$, and similarly $t_2 = 1/n$, $t_n = 1/n$. Suppose it then required to determine the probability that some one event of the partial series $t_1, t_2 \ldots t_m$ will occur, we have for the expression required
>
> $$t_1 + t_2 \ldots + t_m = 1/n + 1/n \ldots \text{ to } m \text{ terms} = m/n.$$

[56] Ibid., 273.

> Hence, therefore, if there are m cases favourable to the occurrence of a particular alternation of events out of n possible and equally probable cases, the probability of the occurrence of that alternation will be expressed by the fraction m/n.[57]

Boole takes advantage of an assumption about the logical relations between alternative mutually exclusive events, which he characterizes by means of logical addition or exclusive disjunction. The logical addition of the events as exclusive alternatives then has an immediate numerical parallel in the quantitative addition of the distinct probabilities of each of the events in the series under consideration, while the probability of any such mutually exclusive event in the series occurring is calculated as the ratio of the number of alternative events divided by the total number of possible events. Boole's result, expressed here by means of his logical algebra, integrated with the standard numerical interpretation of probabilities, fully agrees with the sixth principle of probability derived from Laplace's arithmetical theory, to which Boole is committed. Boole compares the logical and numerical interpretations of probability, and their complicated synthesis in his theory, when he writes:

> Now the occurrence of any event which may happen in different equally possible ways is really equivalent to the occurrence of an alternation, i.e., of some one out of a set of alternatives. Hence the probability of the occurrence of any event may be expressed by a fraction whose numerator represents the number of cases favourable to its occurrence, and denominator the total number of equally possible cases. But this is the rigorous numerical definition of the measure of probability. That definition is therefore involved in the more peculiarly *logical* definition, the consequences of which we have endeavoured to trace.[58]

Boole's demonstration of the convergence of logical and numerical methods in probability theory illustrated here concerns only a limited fragment of the calculations required in standard probabililty evaluations. It is nevertheless suggestive of the general parallel that Boole claims to hold between these alternative integrated interpretations of probability. It is typical of Boole's philosophy of logic and mathematics that he should seek a higher overarching unity of logical and quantitative approaches in probability theory, just as he proposes alternative logical and numerical interpretations of algebra in the mathematical analyis of logic. Boole, in any case, regards the evidence for the convergence of logic and arithmetic in probability calculations in the cases he considers as proving the general thesis. Thus, he concludes:

> From the above investigations it clearly appears, 1st, that whether we set out from the ordinary numerical definition of the measure of probability, or from the definition which assigns to the numerical measure

[57]Ibid., 274.
[58]Ibid.

> of probability such a law of value as shall establish a formal identity between the logical expressions of events and the algebraic expressions of their values, we shall be led to the same system of practical results. 2ndly, that either of these definitions pursued to its consequences, and considered in connexion with the relations which it inseparably involves, conducts us, by inference or suggestion, to the other definition. To a scientific view of the theory of probabilities it is essential that both principles should be viewed together, in their mutual bearing and dependence.[59]

Boole's main purpose in developing a formal theory of probabilities is not merely or primarily to argue for the higher generalization of logical and numerical relations in probability calculations as in universal algebra. His concern is rather to articulate and illustrate a refined and in some ways more systematic treatment of probability than in the work of his predecessors. It is typical of all his work in mathematics, however, that it strives for this kind of simplicity, unity, and generality.

Boole offers a range of elementary applications to advertise the advantages of his formalization of probability theory, which he extends to the analysis of statistics and statistical conditions at increasing levels of complexity. From this vantage point, he then turns to the study of causal connections and the relation of cause and effect in science, and concludes his discussion of probability with a detailed mathematical modeling of social statistics for the probability of collective judgments in practical decision-making. While acknowledging the limitations of using mathematical methods in matters of human choice, Boole with due caution applies his general methods for solving probability equations to judicial deliberations in deciding criminal innocence or guilt by the participation of individual jurors in a jury trial.

Although Boole does not produce any altogether unanticipated mathematical results in probability theory, his generalization of its previously established but less systematically presented central principles, his extremely careful and precise applications of its methods to a wide range of difficult problems, and his efforts to prove the convergence of logical and arithmetical approaches to probability, distinguish his formalization of the theory of probability, inductive reasoning and statistics even from the most sophisticated contemporary expositions of the subject in his day. By choosing to include a highly articulated mathematical theory of probabilities in sequence together with the generalized algebraic logic in *Laws of Thought*, Boole offers a manifest image of the unity and continuity of mathematical methods. The principal modes of reasoning in logic and probability theory are thereby represented by Boole as two sides of the same algebraic coin, for the same inferential law-governed machinery of thought. Symbolic logic and probability theory are adapted to different, but, as Boole believes he has rigorously shown, intimately interrelated cognitive tasks that are united by Boole's multiple interpre-

[59]Ibid., 274–275.

tations of a single underlying universal symbolic algebra of Boolean operators.[60]

7 CONCLUSION

The principles of Boole's algebraic logic have unlimitedly many uses. Boole in his philosophical moments seems to have thought that the extension of his logic to probability theory and its use in what he referred to as the use of logic in the further investigation of the mind were its most important applications. The judgment of history is rather different, and not entirely in agreement with Boole's own assessment.

Probability theory has since been established as an independent mathematical theory with only incidental connections to logic as Boole conceived of it. The use of logic as a tool for the systematic study of thought has equally failed to attract much of a following in the years since Boole's death. The idea that logic describes laws of thought and can be used in turn as a method of psychology has come to be regarded as an objectionable kind of psychologism, which Boolean algebra has managed to survive in spite of itself in the opinion of many contemporary logicians and mathematicians.[61]

That the verdict of history agrees with Boole's expectations that he had discovered a more general and extraordinarily useful analysis of logic, for purposes of expressing thoughts, with rigorous methods of calculation that would carry reason beyond its previously recognized limitations, is largely due to the impressive applications of later generations of Boolean logic in the design, manufacture, programming, and implementation of binary digital computing machines. Developments of Boole's algebra in these areas, which Boole would not have foreseen, and which are in some ways inimical to his concept of logic, are important as evidence for the legitimacy and truth of Boole's systematization of the logical laws of thought.[62] This, after all, is the practical criterion by which Boole himself believed his logical algebra must ultimately be judged, on which grounds he was prepared to see the project succeed or fail.

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[60] A more complete exposition of Boole's probability theory is found in the latter part of Hailperin, *Boole's Logic and Probability*. See also Wilbraham, 'On the Theory of Chances Developed in Professor Boole's "Laws of Thought"', and Hammer and Rudeanu, *Boolean Methods in Operations Research and Related Areas*.

[61] I discuss Boole's psychologism in detail in Jacquette, *On Boole*, Chapter 5, pp. 77-91, and the relevant sections on Boole's use of logic as a method for investigating psychology in Chapter 4, pp. 61-70. See also, Jacquette, 'Psychologism Revisited in Logic, Metaphysics, and Epistemology', and Jacquette, ed., *Philosophy, Psychology, and Psychologism: Critical and Historical Readings on the Psychological Turn in Philosophy*, especially pp. 1-19 and 245-262.

[62] See Grattan-Guinness, 'Logic in Boole's Mathematical Analysis of Logic', xliv: 'On Boole's side, as we have seen, in both logic and for educational practice he thought that the mind was capable of original action, such as grasping general laws from particular cases..., and so he would not have welcomed the association of his logic with the *repetitive* actions of computing; whether Babbagean mechanical or modern electrical.'

We can speak of any area of application to which Boole's logic applies as a Boolean phenomenon. The overwhelming success of Boole's algebra in computer applications and the large array of Boolean phenomena in logic, mathematics, and computing theory, far outweighs the qualified achievements Boole gained in probability theory and psychology, which are not particularly highly regarded today, nor seen as having led to any important new discoveries. All such applications are invaluable in attaining a comprehensive view of all aspects of Boole's logic. Boole's algebra is indispensable to the design and use of modern computing machines, tasks that Boole's logic accomplishes by virtue of several essential insights implicit in Boole's mathematical analysis of logic. They include at least the following enduring and enormously useful aspects of any Boolean logic of Boolean phenomena:

> *First*, Boole pioneered the idea that a binary system of 0's and 1's can be used to represent any values of a logic, which is absolutely vital to modern computer technology.
>
> *Second*, Boole recognized that all such logical operations as needed in order to transform input strings of binary coded information into output strings of binary coded information can be accomplished by mechanical rule-governed methods of substitution and replacement of symbols in an algorithm or step-by-step mechanical procedure, such as he uses in performing the algebraic transformations in his general method of logical inference.
>
> *Third*, as we have emphasized, the three Boolean operators NOT, AND, and OR, and their logic switching gates mechanical implementations, as modified in neo-Boolean algebras, are necessary and sufficient for the logical operations by which the algorithms in computer programming for transforming input information to output, coded by the electrical signals moving through a computer's switching circuits, can be performed.
>
> *Fourth*, Boole's work in logic and probability theory reflects a mathematical insight that is crucial to computer electronic circuit design, programming, and applications.

Any information input to or output from a machine, as is widely recognized, can be minimally coded in a binary system of 0's and 1's. Consider a sentence like the first sentence of this book, and convert its use of a specific sequence of letters of the alphabet, spacing, and punctuation, to a binary string of 0's and 1's, according to an explicit and unambiguous glossary, where, say, $a = 0$, $b = 1$, $c = 11$, $d = 10$, etc. A computer circuit can encode such a message by sustaining a corresponding sequence of electrical signals moving at blinding speed, in which the current is now below .5 volts at a specific instant of time representing each 0 exactly where it is located in the string, and again above .5 volts at a different specific instant of time representing each 1 exactly where it is located at another place in the string.

This is Boole's ruling idea, that logic and arithmetic are alternative interpretations of a common underlying or overarching algebra of symbolic operators, an expectation that finds expression in the programming of computers to perform both number crunching and logical information processing by appropriate adaptations of the same formal symbolic operations. Algorithms of the required sort are mechanically implemented by the logic switching gates of an appropriately designed electronic computer circuit, under the control of an appropriately designed computer program that controls the input to the circuits and determines its mechanical algorithmic transformation into output.

If the project to design a thinking machine that functions entirely in terms of logic switching circuits in its hardware and software programming instructions ever succeeds, as many of its adherents believe it eventually may, then Boole's algebra will have achieved a more remarkable implication for the philosophy of mind than he ever anticipated. In that case, it will have been proven that a mechanical device, a computer with Boolean circuitry and Boolean programming, is capable of thinking, which will represent a most profound breakthrough in mentalistic artificial intelligence. Philosophers for a variety of reasons have been skeptical about the prospects of building such a thinking machine, and Boole himself, as we have seen, would at least initially rebel against the idea that the mind could be understood in purely mechanical terms. For it is one of the conclusions of his application of symbolic logic to psychology that the mind does not always obey the logical laws of thought in the same way that physical phenomena obey the causal laws of nature. To the extent that the simulation of mental activities can be implemented on a Boolean machine, from the function of neural networks analogous to computer switching circuits, to robotic simulacra of the mind's use of memory, perception, logical inference, and other cognitive modalities, to that extent Boole will have inadvertently made his greatest indirect contributions to scientific psychology, laying the mathematical groundwork for meaningful advances in our understanding of the hidden workings of cognitive psychological processes through logic design and computer modeling.[63]

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[63]See, among many other more recent sources, Craven, 'An Engineering Application of Boolean Algebra'; and O'Canainn, 'The Application of Boolean Algebra to Switching Circuits'.

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FRENCH 'LOGIQUE' AND BRITISH 'LOGIC': ON THE ORIGINS OF AUGUSTUS DE MORGAN'S EARLY LOGICAL INQUIRIES, 1805–1835

Maria Panteki

1 INTRODUCTION

1.1 An outline of De Morgan's career in algebra and logic

> Of twenty years of experience as a teacher of mathematics, I may now affirm that the first half of the period established in my mind the conviction that formal logic is a most important preliminary part of every sound system of exact science and that the second half has strengthened that conviction. [De Morgan, 1847][1]

Augustus De Morgan (1806–1871) matriculated at Trinity College, Cambridge, in 1823, graduating a fourth wrangler[2] in 1827. Appointed Professor of Mathematics at the newly founded London University in 1828, he spent his entire career there until 1866, apart from his resignation from 1831 to 1836 as a protest against administrative practices.[3] Deeply concerned with the instruction of elementary mathematics, he perceived in 1831 the utility of Aristotelian logic in the teaching of Euclidean geometry. De Morgan was equally attentive to mathematics and logic, contributing to the advancement of algebra, the extension of syllogistic logic and the founding of the logic of relations. In the ensuing outline we can see the

[1] From a manuscript preface to his *Formal logic* [De Morgan 1847], University of London Library, MS. 775/353.

[2] Cambridge students earned their degree by passing the University Senate House Examinations, known as the Tripos [see fn. 58]. The denomination of "wrangler" was derived from the process of wrangling, that is "the debating method used to test undergraduates before the advent of written examinations" [Crilly 1999, 133]. See also, [Becher 1980b, 4–6], [Durand 2000, 140–141] and [Rice 1997b, 20–24].

[3] On De Morgan's life and career, see [S. De Morgan 1882] and [Rice 1997b]. The latter focuses on De Morgan as a teacher in the context of London mathematics, a role previously overlooked. On his resignation, see [ibid. 87–97]. On De Morgan's publications, see [Smith 1982, 141–147], and on his manuscripts, see [Rice 1997b, 349–357].

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close interaction between his algebraic and logical queries, along with his smooth passage from the lower and more specific to the higher and more general forms of algebra and logic over the years.[4]

Stage 1: From elementary mathematics to algebra and logic (1828–1839) During this period, De Morgan produced the majority of his textbooks on elementary mathematics, as well as his articles on the instruction of mathematics. In his booklet *On the Study and Difficulties of Mathematics* [De Morgan 1831a], cited as SDM, he paid a singular attention to logic, as a prerequisite to the study of geometry. His views were elaborated in his *First Notions of Logic* [De Morgan 1839], a book designed for students of geometry. In 1835 he reviewed George Peacock's *Algebra* [Peacock 1830], eager in promoting this pioneering account of advanced algebra as an indispensable preliminary part of the calculus and mechanics within Cambridge education. In his review, [De Morgan 1835a], he raised links between algebra and logic, thus sealing his life-long interest in the foundations of mathematics and logic. This interest became evident in his article on the "Calculus of functions" (COF), published in the *Encyclopedia Metropolitana* (EM) in 1836.[5] There De Morgan developed his method of abstraction and generalization in mathematics, visible in the ascent from arithmetic to algebra and from algebra to the calculi of functions and operations. Successfully applied in the extension of the domain of algebra, this method would later govern his mature work on logic.

Stage 2: On the foundations of algebra and "formal logic" (1839–1849) De Morgan furnished his treatise on the *Calculus* [De Morgan 1842] by drawing, among other sources, on his COF and the recent development of the calculus of operations.[6] The same sources, including Peacock's book, served as a basis for his four papers "On the foundations of algebra", contributed between 1839 and 1844. These papers, together with his *Trigonometry and Double Algebra* [De Morgan 1849] constitute his basic share in the advancement of algebra, including his near definition of the axioms which characterize the field of complex numbers.[7] De Morgan's project as regards the extension and formalization of algebra was succeeded by his attempt to enlarge the system of Aristotelian syllogistic, a concern rooted in SDM. As a result, he produced his first paper "On the syllogism" [De

[4]The outline is based upon [Panteki 1992, chs. 3, 6] on De Morgan's mathematical and logical contributions. For facility, we divide his career into four stages, each stage roughly equivalent to a decade. However, different divisions may be found in [Pycior 1983] and [Merrill 1990], in connection with De Morgan's algebraic and logical career respectively.

[5]Objecting to his "Treatise" (of 75 quarto double pages in small print) being "locked up" in EM, De Morgan agreed with the proprietors of the encyclopedia to have his article reprinted on its own account [De Morgan 1838, 15]. The outcome of this agreement remains unclear; see also [Todhunter 1876 I, 85]. However, although the volume of EM containing this essay bears the date 1845, there exists a separate offprint, dated 1836.

[6]De Morgan's treatise was of significant impact on the development of the calculus of operations, serving as a basic source for Boole's original contributions in 1844 [Panteki 2000, 173–174].

[7]See [Koppelman 1971, 218–220], [Panteki 1992, sect. 3.9], [Pycior 1983, 221–224], [Richards 1980, 354–357] and [Smith 1981].

Morgan 1846], followed by his book *Formal logic* [De Morgan 1847]. The latter, in conjunction with George Boole's *Mathematical Analysis of Logic* (1847), mark a new era in the development of "algebraic logic".[8] By that time, the Scottish philosopher William Hamilton had accused De Morgan of plagiarism, in connection with the issue of "quantification of the predicate".[9] The accusation gave rise to a controversy between the two men in 1846, which continued well after Hamilton's death in 1856, motivating De Morgan (and Boole) to speculate on the validity and utility of applying mathematical methods to logic.[10]

Stage 3: On the ultimate extension and formalization of logic (1850–1860) This period witnessed the highlight of De Morgan's logical contributions, which consisted of three more papers "On the syllogism" [De Morgan 1850; 1858; 1860a], a booklet on the *Syllabus of Logic* and an encyclopedic article on "Logic", both published in 1860. From amongst these, we single out [De Morgan 1860a] his masterpiece on the "Logic of relations" (LOR), De Morgan's achievement of pure, formal logic. The latter resulted from the method of abstraction and generalization of the copula, which was developed in [De Morgan 1858], amounting to the gradual separation of "form" from "matter". Alluding to his paper on the COF, De Morgan was happy to note that the "form-matter" distinction "exists in all thought", although it is "more familiar" to the mathematician [De Morgan 1858, 82]. The term "Formal logic", which had featured a decade ago in the title of [De Morgan 1847], may be said to have attained its full meaning in [De Morgan 1858],[11] a paper imbued by Hamilton's influence, and one to reveal its debt to [De Morgan 1836].[12]

Stage 4: The formalization of algebra revisited (1860–1866) After his

[8]As recently brought forward in [Valencia 2001], De Morgan reviewed Boole's book in 1848 pinpointing their distinct approach towards the mathematization of logic (see also [Corcoran 1986]). However, despite their varied procedures, both works belong to the realms of "algebraic logic", in so far as they are based on Lagrangian algebras. Algebraic logic should be distinguished from "mathematical logic", as stemming from Cauchy's and Weierstrass' foundations of mathematical analysis [Grattan-Guinness 2000, 570].

[9]According to traditional logic, there are four standard forms of categorical propositions denoted by the letters A, I, E and O. By means of the quantification of the predicate, each form is split into two. For instance, form A "All X is Y", is split into "All X is some Y" and "All X is all Y". On the invention of this doctrine by G. Bentham (1827), T. Solly (1839) Hamilton (late 1830s) and De Morgan (1846), see [Jevons 1873], [Lewis 1918, 33.3], [Panteki 1993, 153, 161–162] and [Styazhkin 1968, 148–157].

[10]On the early stage of the De Morgan-Hamilton controversy see [Laita 1979], the first historian to point out its immediate impact upon Boole. On its later impact upon De Morgan's inquiries, see [Panteki 1992, sects. 6.6–6.7].

[11]On the origins and first systematic use of the term "formal logic", with emphasis on [De Morgan 1858], see [Hodges 2000].

[12]Motivated by [Grattan-Guinness 1988, 74], [Merrill 1990] and [Panteki 1992] questioned the plausible impact of [De Morgan 1836] upon his LOR. The two surveys are complementary, respectively arguing against and for such a significant impact. Moreover, the former presents the LOR from a modern standpoint, which is missing in the latter, while lacking a full-length exploration of its mathematical background, supplied in the latter.

breakthrough with logic (enriched with a final paper "On the syllogism V" in 1862), De Morgan resumed his algebraic inquiries, considering his former project of algebra's formalization from a different angle. Until the mid 1840s, logic had been considered as "a most important preliminary part of every sound system of exact science",[13] primarily connected with the instruction of geometry and algebra. From then onwards, algebraic and functional methods served as powerful tools and devices of utmost utility in De Morgan's advanced logical contributions, leading the way to the LOR. That achieved, it was once again time to reverse the role of the two disciplines, and look for the model for the formalization of algebra in the general methods of pure logic. This newly-conceived project stood at the core of De Morgan's last major contribution to mathematics, his paper "On infinity and the sign of equality" [De Morgan 1865, 180], where he wrote, referring to Peacock's foundation of symbolic algebra: "This is a very near approach to the assertion that algebra is, like logic, a formal science: nothing was wanted but an introduction and incorporation of that distinction between form and matter, which now rules in the definition of pure logic". However, his career was nearing its end, with no scope therefore for the pursuit of this project. His life-long passion for the foundations of mathematics and logic, along with his deep parallel interest in their history, was manifested once again in his *Speech* [De Morgan 1866], which was delivered at the first meeting of the London Mathematical Society (LMS) on 16 January 1865.[14]

1.2 Survey of the literature and scope of our study

> The recollection of these will furnish abundant opportunities for a very important exercise, the detection of incorrect reasoning, an exercise which will be the more instructive, as by the occurrence of great names, in connection with fallacies and misconceptions, the student will perceive that brilliant intellect, unaccompanied by habits of correct thinking, has often led its possessor to the direct path of error, and that if he neglect the constant improvement of the mental faculties, he may perhaps acquire profound knowledge, but will never reason with accuracy. [De Morgan 1828, 24]

The above quotation is from De Morgan's "Introductory lecture", delivered at the opening of his classes at London University on 5 November 1828.[15] Characterized by a "profound mathematical erudition" and a strong educational background, this lecture reveals above all "the importance De Morgan places on logic and reason in the development of the intellect" [Rice 1997b, 72–73]. What were the sources that would inspire a fresh graduate to pay such a singular attention to the import of

[13] See our introductory quote, fn. 1.

[14] On the LMS, see [Rice 1995]. Further on [De Morgan 1866], see [Richards 1987]. On De Morgan's contributions to the history of science, see [Rice 1996].

[15] Brought forward in [Rice 1997b, 67–73], this lecture belongs to University College, London, MS.Add.3, cited as [De Morgan 1828]. Our pagination corresponds to that of the manuscript.

"fallacies" in the establishment of "correct thinking", apparently leading the way to his dual contemplation of logic and geometry three years later?

De Morgan's mature work on logic engaged the attention of historians in the 1870s, ceaselessly continuing to the present day.[16] The aim of most surveys was to record his rather chaotic presentation of logic in a systematic and lucid way, with a view to evaluating De Morgan's share in the development of formal logic.[17] In the 1990s, new light was shed upon the genesis and status of the LOR, as well as its mathematical background, by [Merrill 1990] and [Panteki 1992] respectively. Merrill was the first historian to consider the logical novelties introduced in SDM and the *Notions* [Merrill 1990, chs. 1, 2]. However, an investigation that would account for De Morgan's initiative in reducing geometrical reasoning to syllogistic form is missing from his survey, a fact hardly surprising, however, given the absence of similar concerns in the prevailing British tradition of Aristotelian logic in De Morgan's time.

On the other hand, De Morgan's algebraic contributions took longer to be appreciated than those of his logic. Commencing with [Nagel 1935], historians orientated their research towards an evaluation of De Morgan's papers "On the foundations of algebra" from a modern standpoint, while his broader algebraic approach towards the calculi of functions and relations suffered from near neglect.[18] The 1980s was a turning point in the historiography of early 19th century British algebra, marked by the attempts of H. Pycior and J. Richards to account for the elusive character of algebra at that time, and also bring forward De Morgan's idiosyncratic oscillation between formalism and conceptualism. They did so by stressing the role of extra-algebraic factors, like philosophy and religion, in shaping the views of British algebraists, as well as taking under consideration neglected aspects of De Morgan's work, such as his educational concerns, his debt to the history of mathematics, and his review of Peacock's textbook.[19] Nevertheless, there still remain unexplored areas, as for instance, De Morgan's apparent background in

[16] In chronological order we note the following surveys: [Liard 1878, 71–97], [Halsted 1884], [Lewis 1918, 37–51], [Kneale 1962, 426–428], [Prior 1962, 141–156], [Heath 1966, vii–xxxi], [Styazhkin 1969, 161–169], [Hawkins 1979], [Merrill 1990], [Panteki 1992, ch. 6], [Hawkins 1995] and [Grattan-Guinness 2000, 26–37].

[17] In many histories of logic, "Formal logic" covers both traditions of "algebraic" and "mathematical" logic from mid 19th century onwards. On these terms see fns. 8 and 11.

[18] Among the early studies of British algebra, hinting at De Morgan's allegedly formal approach to algebra, we note: [Nagel 1935], [Clock 1964, 17–105], [Koppelman 1971, 215–220] and [Novy 1973, 189–199]. The last two hinted at the importance of [De Morgan 1836] and its relevance to his work on algebra. De Morgan's study of functional equations in 1836, along with Babbage's earlier work on this branch, were omitted in [Dhombres 1986], a study of the history of functional equations in more than one variable.

[19] Exclusive works on De Morgan's algebra are relatively few; see [Pycior 1983], [Richards 1980, 354–357] and [Richards 1987]. However, we wish to note additionally [Pycior 1981; 1982; 1984; Richards 1991; 1992, Durand 1990; 1996; 2000], and [Fisch 1994; 1999], which offer critical reviews of the studies mentioned in fn. 18, providing new standpoints for the peculiar state of British algebra early in the century. Interestingly enough, these studies largely omit to take under consideration the algebras of operations, functions and logical entities. The latter are briefly noted in [Grattan-Guinness 1997, ch. 9] in the context of a general history of mathematics, and in [Grattan-Guinness 2000, ch. 2] in the context of the history of logic.

French epistemology, or the full import of his review of 1835, which potentially endangered the stability of the curriculum at Cambridge, as supported in William Whewell's treatises on mechanics in 1832.

The aim of our study is to fill in certain of the above-mentioned lacuna in current bibliography. Focusing exclusively on the very early stage of De Morgan's career, between 1828 and 1835, we seek primarily to identify the origins of his interest in the study of logic in connection with geometry. To this end, we draw attention to Condillac' s semiotic philosophy, as elaborated in his *Logique* [Condillac 1780] and in *La langue des calculs* [Condillac 1798]. The latter work, along with S. F. Lacroix's *Essais sur l'enseignement* [Lacroix 1828] — first published in 1805 — were both cited with approval in SDM. Although we lack evidence as to De Morgan's first acquaintance with Lacroix's survey on mathematical education, we will nevertheless show that several of its sources underlined De Morgan's defense of mathematical studies from 1828 onwards, while his SDM was shaped in line with the *Essais*. Last but not least, while the basics of traditional logic as displayed in SDM drew on R. Whately's *Elements of Logic* [Whately 1826], we argue that De Morgan's original attempt to apply syllogistic logic to Euclidean geometry stemmed from Lacroix.

Thus covering De Morgan's educational and epistemological concerns up to about 1833, we conclude by shedding new light on his review, where he demanded the incorporation of logic within the Cambridge curriculum, by holding that it is "an easier science than algebra" and one which "the student must have in one sense, before he can ever become a mathematician" [De Morgan 1835a, 293, 311]. Peacock's foundation of symbolic algebra in 1830 and De Morgan's critical reception of it in 1835 will not be discussed, as the subject has been amply dealt with in recent studies, and does not pertain to De Morgan's interest in logic at the time.[20] We are challenged, though, by De Morgan's polemic against the teaching of mechanics that prevailed through Whewell in the early 1830s, an issue interestingly linked with De Morgan's plea for both algebra and logic to be seen as prerequisites for the study of the calculus and mechanics. This last part of our survey is further motivated by the following question: was it a pure coincidence that soon after the review's publication, Whewell published his *Thoughts on the study of mathematics as a part of a liberal education* [Whewell 1835], a polemic against Continental mechanics, which opened with a comparison between geometry and logic to end with the dismissal of both advanced algebra and logic from a future curriculum?

Thus questioning De Morgan's early links between logic and geometry and between logic and algebra respectively, we wish to stress the subtle blend of his strong background in French epistemology and his Cambridge heritage in mathematics and mechanics, hopefully paving the way for a thorough future study on the mutual development of algebra and logic from 1830 onwards. If occasional lack of sufficient evidence results in tentative arguments we feel entitled to carry on and follow De Morgan's own motto: "We go as far as we can, and we try to see what

[20][Pycior 1981; 1982; 1983] cover a study of Peacock's *Algebra* and De Morgan's reaction to it respectively. See also [Richards 1980; 1987] and [Fisch 1999].

we can: to ask a question is a step in knowledge, and even if there be no answer it is a preparation for an answer” [De Morgan 1858, 105].

1.3 Structure of the paper

> The joint study of diverse sciences is the most appropriate means to discover the method, which must direct the human mind in the search for truth. [Lacroix 1828, 24]

In §2.1 we investigate the mathematical and epistemological background of Lacroix’s *Essais*, pointing out the individual power of its sources, along with clarifying the ambiguous notion of “logique” during his time. We proceed in §2.2 by questioning the multidimensional meaning of “logique” in Lacroix’s own discourse, holding that his peculiar approach to traditional syllogistic logic forms a link between 18th century epistemology and the gradual revival of interest in Aristotelian logic in early 19th century England [§3.1]. This revival was due to Richard Kirwan’s and Richard Whately’s support of the scientific status of logic in 1807 and 1826 respectively. We conclude with the critical reception of Whately’s *Elements* by British logicians and philosophers during the period 1827–1833 [§3.2].

Section §4 covers De Morgan’s Cambridge entourage, dealing in turn with his mathematical background, as largely influenced by the contributions of the Analytical Society [§4.1], Whewell’s oscillation between Newton and Laplace in the teaching of mechanics [§4.2], and De Morgan’s potential as a graduate, with a focus on the influence of his teachers and tutors [§4.3]. In §5 we illustrate Lacroix’s multidimensional impact on De Morgan’s educational writings between 1828 and 1833. We focus first on De Morgan’s views on the status of mathematics and particularly the instruction of arithmetic and elementary algebra, pinpointing numerous similarities between SDM and Lacroix’s *Essais* [§5.1]. We then move on with his speculations on the amount of “reasoning” involved in the teaching of geometry and algebra and his comparative study of the two disciplines in SDM [§5.2], prior to entering into his application of syllogistic logic to geometry in the ensuing chapter of the same book [§5.3].

Having thus argued on Lacroix’s impact upon De Morgan’s early logical inquiries, we question the background and import of De Morgan’s review of Peacock’s *Algebra* [§6]. We begin with Whewell’s reaction to the expansion of the Cambridge curriculum in the early 1830s [§6.1], focusing next on De Morgan’s critical view of the Cambridge educational system, and his emphasis on the utility of both algebra and logic in 1835 [§6.2]. Hinting at the plausible impact of his review upon Whewell’s *Thoughts* [§6.3], we display a summary of our survey, enriched with questions of potential influence for further research on that crucial period of the 1830s, in and around Cambridge University [§7].

2 LACROIX'S PHILOSOPHY OF INSTRUCTION, 1805

2.1 The epistemological background of Lacroix's Essais

> I have always directed my thoughts to the means of presenting scientific results in the most simple manner and the most natural order.
>
> [Lacroix 1828, 168]

An eminent instructor and textbook writer, Lacroix (1765–1843) launched his career in 1779 by inquiring into planetary mechanics, in line with the leading scientists of his time.[21] At Cambridge, he became widely known for the encyclopedic three-volume *Traité du calcul* (1797–1800), the abridged version of which (1802) was translated by the Analytical Society in 1816.[22] The Cambridge dons, however, were largely indifferent to his major work on the didactics of mathematics, his *Essais sur l'enseignement en général, et sur celui des mathématiques en particulier* (1805). Addressed exclusively to instructors, this volume was motivated by Condorcet's plan of 1792, according to which teachers' and students' editions were differentiated for the first time. Lacroix's *Essais* enjoyed several editions in France, but was never translated into English.[23]

In its third, enlarged edition of 1828, upon which we draw, the author retained his admiration for the liberal scientific spirit of the French Enlightenment [Lacroix 1828, 5–38]. His rich teaching and administrative experience were amply manifested in the first part of the *Essais* [pp. 39–167], which included an in-depth survey of the diverse institutions that flourished in France after 1789. The second part focused on the philosophy [pp. 168–230] and practice [pp. 231–344] of mathematical instruction, with emphasis on the teaching of algebra and geometry. Lacroix's discourse was characterized by his immense erudition in the history of instruction, and his awareness of the latter's relevance to logic. As we shall see, he deployed the term "logique" in a multiplicity of ways, its conception ranging from Aristotelian logic to theories of ideas and signs peculiar to the study of language.

In connection with geometry, Lacroix drew almost exclusively on A. Arnauld's and P. Nicole's *La logique, ou l'art de penser* (1662). Referring to it as the "Port-Royal Logic" (PRL), he recommended the study of its last part on "Method", the main ideas of which were derived from Pascal's and Descartes' rules on the

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[21]See [Taton 1959, 127–130] and [Wilson 1980, 280]. Condorcet had also been occupied with the three-body problem a decade earlier [Baker 1975, 8]. On Lacroix, as a textbook writer and instructor, see [Grattan-Guinness 1990, 112–115] and [Schubring 1987].

[22]Lacroix's *Traité* included D' Alembert's limits, the Leibniz–Euler approach and Lagrange's algebraic calculus; see [Grattan-Guinness 1990, 139–142] and [Grattan-Guinness 1992b, 17–20]. On the diffusion of his work at Cambridge, see [Ashworth 1996], [Becher 1995], [Enros 1983] and [Topham 2000].

[23]Commented upon in [Hodgkin 1981, 65–66] and [Richards 1991, 302–303], the historically neglected *Essais* was re-edited in 1816, 1828, 1838 and 1894. In text we draw upon the 3rd edition, providing our own translations of the quoted passages. Our sole evidence for the circulation of this work at Cambridge prior to 1831 remain Whewell' s personal notes from the *Essais*, dated 8 February 1821 [Whewell Papers, Trinity College Library, R.18.98, pp. 14–15].

teaching of geometry.[24] His demand for "simplicity" and "natural order" in teaching stemmed from the PRL theory of clear and distinct ideas and signs, and was consistent with his other major source, Laplace's lectures on algebra delivered at the short-lived École Normale of 1795.[25] The list of authorities cited in the *Essais* included Newton, Leibniz, Locke, D'Alembert, Diderot, Voltaire, Condillac, Condorcet, Lagrange and Monge, with references to the educator Pestalozzi [p. 33] and Rousseau's *Émile* [p. 306, 323–325].

Lacroix recommended the study of D'Alembert's and Diderot's *Encyclopédie* (1751–1772), along with Euler's *Letters to a German Princess* (1768–1772), two sources of opposing views on logic. With no entry on "Logic", but only with entries on issues pertaining to language, the *Encyclopédie* reflected the indifference, or even hostility, of the French "philosophers" towards formal logic (here standing for syllogistic logic).[26] By contrast, Euler's text adapted Leibniz's device of illustrating logical relations through geometrical representation, in line with his firm belief that "the reasoning by means of which we are led to the truths of geometry can be reduced to formal syllogism".[27] Given his collaboration with Condorcet in 1786 on a re-edition of Euler's *Letters*, to serve as a text for Lacroix's course at the Lycée of Paris [Taton 1959, 153–158], we conclude that Lacroix must have been familiar with Euler's ideas on the utility of traditional logic.

There remains, however, an open question as to Lacroix's acquaintance with two other contributions to logic: Condorcet's essay on universal language, intended as a sequel to his *Tableaux historique* (1794); and the syllogistic intonations of the PRL itself. The answer seems to be negative, upon the grounds that Condorcet's adoption of a kind of symbolic logic, echoing Leibniz's ideas, was then still unpublished and apparently unknown [Baker 1975, ch. 6; Granger 1954]. And furthermore, the semi-mathematical treatment of logical conversion and syllogistic which enriched the PRL, had no impact upon Lacroix's predecessors.[28]

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[24]The PRL, was divided into four parts, dealing respectively with "Conception", "Judgement", "Reasoning" and "Method". Including elements of syllogistic logic, the PRL influenced Enlightenment epistemology mainly through its fourth part; see [Auroux 1982], [Baker 1975, 97, 132, 160], [Buickerood 1985, 161, 176, 185] and [Foucault 1970, 52–76]. On the rules for geometry's instruction, see [Kneale 1962, 317] and [Lacroix 1828, 275].

[25]On the École Normale, see [Dhombres 1980] and [Glas 1986]. Laplace's lectures are edited with notes in [Dhombres 1992]. In text we draw from a reprint of them in Laplace's Œuvres completes, cited as [Laplace 1795].

[26]The origin of the term "formal logic" stems from Kant [Hodges 2000]. In text we deploy it in line with Buickerood [1985, 159], as a synonym for Aristotelian syllogistic, so as to distinguish it from other alternative systems on the art of reasoning, attributed to Bacon, Descartes, Locke, Condillac and others. On the *Encyclopédie*, see [Hankins 1985, 163-170] and [Lacroix 1828, 15, 174, 306]. Covering issues on "Etymology" or "Grammar", this work was strikingly devoid of articles on "Logic", as noted by [Aarsleff 1982b, 147–148], [Auroux 1982, 21, 53], [Fraser 1989, 329-330, fn] and [Rider 1990, 114].

[27][Euler 1812 I, 475]. On logic, and what became known as "Euler diagrams", see letters C–CVII at pp. 444–494. See also [Kneale 1962, 349–350] and [De Morgan 1847, 323], where notice is also taken of Gergonne's adoption of these diagrams. On Euler's famous popularization of science, see [Callinger 1976].

[28]Auroux [1982] claims that the PRL anticipated Boole's and De Morgan's algebraization of logic in 1847. However, the PRL did not exert any influence through its unusual treatment of

Another work pertinent to "logique" which was cited by Lacroix, was Condillac's *Logique ou les premiers développements de l'art de penser* [Condillac 1780]. As implied by its title, Condillac's book was indebted to the PRL. However, contrary to the PRL, Condillac was hostile to Aristotelian syllogistic, his conception of "logique" stemming from Locke's *Essay on human understanding* [Locke 1690], a work misinterpreted by Voltaire and his followers as a treatise on logic [Buickerood 1985]. For, Locke alternatively deployed in its last chapter "Of the divisions of sciences", the three terms "Semiotiki" (in Greek), the "doctrine of signs" and "logic", so as to define "the ways and means whereby the knowledge [...] is attained and communicated" [Locke 1690, 309]. Hence the origin of Condillac's "semiotic philosophy", as based on his conception of science as a language, "probably the most influential concept of science developed by the French representatives of the Enlightenment" [Jahnke 1981, 79].[29]

Condillac used the term "analysis" in place of "logique" as the method by which we trace our ideas back to their origin, observing their generation and comparing them under every possible relation. Analysis stemmed from sensual experience and the use of signs. As he claimed, "nature gives us the first lessons of the art of thinking" [Condillac 1780, 45], and ultimately the notion of classification [ibid, 87–112]. His next concern was with linking analysis to language, whereas his formal verdict was based upon the observation that algebra, as handled by Euler and Lagrange, proved that "the progress of the sciences depends upon the progress of their languages" [p. 305]. In brief, algebra was considered as an indispensable analytical method of discovery and a scientific language par excellence.

Condillac's epistemology had a great impact on the classification and nomenclature of science, including Lavoisier's chemical notation [Baker 1975, 87–128; Foucault 1970, 54–76; Gillispie 1960, 200–260; Lacroix 1828, 18–24, 151]. In conjunction with Condorcet's views on social science, it also motivated the movement of "idéologie", a term devised by Destutt de Tracy in 1796 for the new "science of the analysis of sensations and ideas". As it were, "idéologie" became synonymous to "logique" and "universal grammar" exerting influence upon educational policies.[30] What mainly prevailed with those instructors who followed him, was

formal logic, and this was mainly due to its authors, who cautiously warned their reader against any detailed study of syllogistic logic, apparently influenced by Bacon's and Descartes' hostile reaction to Aristotle's logic [Kneale 1962, 319–320].

[29]On the influence of *Logique* upon the scientific language of the enlightenment, see [Albury 1980], [Baker 1975, ch. 2] and [Gillispie 1960, 150–190]. On the term "semiotiki", see [Buickerood 1985, 177]. The PRL was the starting point for both Locke and Condillac, but their conceptions of "logique" differ in many ways. A comparative study of these three works is yet to be undertaken.

[30]The term had no political connotation at the time. According to Tracy, "this science may be called *idéologie* if one attends only to the subject-matter, *universal grammar* if one has reference only to the method, or *logic* if one considers only the goal" [Kretzmann 1967, 390]. In accordance with this conception of philosophy "the traditional chairs of logic and metaphysics in the écoles centrales [schools of secondary education] were replaced in 1795 with chairs of universal grammar" [ibid]. On French institutions of education, see [Grattan-Guinness 1990, ch. 2], [Richards 1991, 299–300] and [Lacroix 1828, 27–36, 56–60]. Another consequence of this movement was the massive production of new textbooks; see [Baker 1975, 391–395], [Dhombres 1980, 145–148], [Rider 1980, ch. 8], [Rider 1990, 132–140] and [Schubring 1987].

their emphasis on the interaction between progress in language and progress in scientific discovery. This was strikingly visible in Laplace’s lectures in 1795.

Laplace compared the derivation of Newton’s binomial formula, a major theorem, with the process with which he arrived at the law of gravity, claiming that “what one observes in analysis happens similarly in nature” [Laplace 1795, 10–12, 35, 135 and 150–156; Dhombres 1992, 14–43]. His students, armed with a copy of Condillac’s book, would be initiated into the virtues of the language of algebra in line with the 7th chapter of *Logique*.[31] Within this context, Laplace discussed Condillac’s two major principles, that of “analogy” and of the “connection of ideas”, [32] which constituted the ground of any comparative study [Laplace 1795, 33–37]. He strongly favored “general methods in teaching” [p. 84], admitting that “algebra furnishes always the best methods” [p. 103]. Lastly, he alluded to his own intricate method of “generating functions”, [33] eager to persuade his audience that “the language of Analysis, the most perfect of all languages, is by itself a powerful instrument of discovery” [p. 156].

Lacroix’s *Essais* was composed at a time when the influence of signs upon mathematical reasoning in France had reached its peak. Indicatively we refer to J. M. Dégerando’s *Des signes et de l’art de penser* (1800), a work which encapsulated the prevailing conviction of leading philosophers and mathematicians (Condillac’s enemies included) on the significance of algebraic signs in scientific inquiry.[34] This conviction was shared by Lacroix, by stressing the import of mathematical language upon the recent “brilliant discoveries” [Lacroix 1828, 6–27] in physical astronomy, notwithstanding the fact that his own tract was devoid of symbols. In a text imbued with his predilection for the system of education followed by Laplace in 1795, Lacroix summarized the latter’s lectures on algebra [pp. 246–273], also promoting Clairaut’s method of instruction based upon historical discovery.[35] Last

[31]A comparison between [Condillac 1780, ch. 7] and [Laplace 1795, 33–37] is worthy of attention. The former dealt with a problem of arithmetic, first using fingers, then spoken words, then numerals, eventually reducing it to the equations $x - 1 = y + 1$ and $x + 1 = 2y - 2$ [pp. 287–303]. On similar lines, Laplace stated an arithmetical problem rhetorically, so as to clarify the symbolical representation of the statement “half the sum of two numbers added to half their difference gives the greater of the two numbers” [pp. 33–34]. On Condillac’s import upon the lectures delivered at the Ecole Normale see also [Dhombres 1992, 20, 35, 142, 165, 185].

[32]The latter principle should be distinguished from Locke’s “association of ideas”. Condillac’s “liaison des idées” was conceived in imitation of the concept of gravity in Newtonian philosophy [Aarsleff 1982b, 199, fn. 1].

[33]Due to the limited duration of the school, Laplace had no opportunity to lecture on the calculus. Nonetheless, he advertised his method [pp. 133, 146, 173], proud of its applications in celestial mechanics and the probability theory. See further [Grattan-Guinness 1990, 161–183], [Hald 1990, 210–212] and [Panteki 1992, ch. 1].

[34]On Dégerando, see [Kretzmann 1967, 388–389] and [Rider 1990, 135–140]. From among Condillac’s most fierce opponents, we single out J. D. Gergonne, editor of the *Annales de mathématiques pures et appliquées* in 1810 [Grattan-Guinness 1990, 135–137, 194]. On the philosophy of symbolic methods, which flourished in France after Lagrange, and their influence upon Cambridge analysts, see [Ashworth 1996, 632–641], [Grattan-Guinness 1988, 73–74], [Grattan-Guinness 1992a, 34–39], [Koppelman 1971], [Panteki 1992, chs. 1–2] and [Sherry 1991].

[35]Clairaut’s *Algèbre*, a starting point for Condillac and Laplace, was re-edited by Lacroix in 1797 [Dhombres 1992, 16], [Schubring 1987, 47]. See also [Schubring 1996, 368] on Lacroix’s

but not least, he recommended the writing of elementary textbooks which would facilitate the perusal of Laplace's *Mécanique céleste* (1799–1805), having already paved the way with his own *Traité*.[36]

2.2 Lacroix on "Logique"

> In order to win a race, it is better to exercise the legs than to reason upon the mechanism of walking. [Lacroix 1828, 305]

Lacroix encouraged instructors to find their own suitable balance between a "very superficial" and a "very rigorous" passage to the "true metaphysics" (i.e. foundations) of mathematics [p. 174]. He was firm in his recommendations when they stemmed from indisputable authorities, and ardently promoted Laplace's "general methods in teaching" [p. 178].[37] However, he expressed ambivalence when dealing with issues for which the current theory and practice of instruction were unable yet to provide guidelines. Such an example concerned the teaching of algebra and geometry, and his dilemma about which of the two subjects should be taught first [pp. 306–307]. Although his answer was far from definite, Lacroix did provide the reader with a flexible approach and a stimulating question.

We wish to note that this issue of priority was preceded by a similar one concerning the priority of geometry versus logic [p. 305]. Lacroix's overall consideration of logic was fragmentary, stemming from three different sources. First of all, he referred to Aristotle and commented upon the role of his logic within the history of philosophy and instruction [pp. 41–59]. Then Condillac's *Logique* entered into his discourse, when he discussed the current philosophy of instruction [pp. 149–152, 228–224]. Finally, he drew on the PRL, strictly in connection with the teaching of geometry [pp. 274–307]. As we shall see, for all his hostility towards the pedantries of scholastic logic, Lacroix paid limited, albeit significant attention to the study of formal, deductive logic, strictly distinguishing it from the latter two sources, which he apparently viewed as two complementary vehicles of methodology . When he referred to Condillac, Lacroix deployed the term "logique" as a synonym for "idéologie", which he placed first in the list of sciences that resulted from the "application of judgment" [p. 149]. To ideology, he stressed, we owe the transformation of sensations into ideas, the combination of ideas into judgments and lastly the derivation from the latter of the rules that govern the search for truth [p. 150]. According to Lacroix, Locke and Condillac had rendered their

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advocation of D'Alembert's attitude towards the foundations of algebra.

[36] See Lacroix's comments on pp. 37, 180. On Laplace's contributions to celestial mechanics, see [Gillispie 1978], [Grattan-Guinness 1990, ch. 5] and [Panteki 1992, ch. 1].

[37] "Give your preference to general methods in teaching", wrote Laplace [p. 84], "and devote yourself to presenting them as simply as possible; you will see that they are nearly always the easiest methods as well". Laplace meant to instill the spirit of research into students, by promoting methods that would serve as potential tools for new discoveries. Upon this "revolutionary" statement, see [Dhombres 1992, 16] and [Richards 1991, 316–317]. In connection with rigor, Lacroix stressed the importance of providing proof of any mathematical result attained by induction [Laplace 1795, 8, 41, 152], [Lacroix 1828, 261].

ultimate service to "metaphysics" (i.e. epistemology) by arguing that first notions are derived from the senses and not from imagination [p. 217]. This achievement pertained, however, only to the first stage of instruction. Lacroix accordingly advised instructors to encourage their pupil to picture his first ideas "in objects of his senses" [pp. 170–l72], but that was the only aspect of Condillac' s epistemology appropriate to instruction in any way.

Lacroix was highly critical of Condillac's method of "Analysis", and maintained that the latter had nothing original to offer the study of mathematics. He noted that by claiming that "algebra was a language" Condillac was not far from the "lucid and precise" notions as furnished in Clairaut's *Algèbre* (1748) [p. 205]. Moreover, he pointed out that this method of analysis was based upon vague definitions and erroneous conceptions. Just as algebra could serve the purposes of a synthetic presentation, similarly, argued Lacroix, geometry was not free from analysis, as in the case of reductio ad absurdum. It was also noted that Condillac himself had built his Logique synthetically, not analytically, quite contrary to his own theory.[38]

Although hardly an admirer of Aristotelian logic, Lacroix criticized Condillac for his neglect of this discipline. Lacroix had indeed condemned excessive preoccupation with the complicated names and rules of syllogistic logic, and had stressed instead the import of Descartes' own rules relating to the process of scientific discovery [p. 47]. However, in the realms of instruction, he twice revealed his appreciation for logic's educational utility, linked with Condillac's principle of the connection of ideas through the aid of signs. He thus stated that "there is no need to neglect the discussion of logical forms, when they are not abused, they can be a very useful exercise for the mind" [p. 15l]. He further recommended Euler's letters on logic, calling them a "brief but illuminating" study of the "diverse forms of syllogistic" [p. 221, fn.]

In line with Laplace, Lacroix regarded algebra as an ideal source for "general methods", useful both in teaching and in mathematical inquiry. However, under the joint influence of Euler and the PRL, it was geometry which served as a paradigmatic model for training the mind in the "diverse forms of reasoning" [pp. 305–306]. As for the teaching of algebra, it sufficed to follow, after Clairaut, the route of historical discovery [pp. 250–252]. For the teaching of geometry, on the other hand, tradition should be avoided, and instructors were urged to follow the PRL instead. Lacroix praised the PRL contributions to the amendment of weaknessess spotted in the structure of Euclid's *Elements*.[39] Were all of his

[38]Lacroix's arguments [pp. 203–226] stemmed from Condorcet's critique of Condillac's erroneous distinction between the "analytic" methods of algebra, as they came to be called, and the "synthetic" methods of geometry [Baker 1975, 110–118]. By convention, analysis assumes the solution of a problem known in unknown terms, while synthesis begins with known principles and leads to the desired result. 18th century mathematicians often associated analysis with algebra and synthesis with geometry, but these connections are very unclear; see [Grattan-Guinness 1990, 135–137], [Otte & Panza 1997] and [Panteki 1992, sect. 1.8].

[39]For instance, Lacroix advised minimum use of the method of reductio ad absurdum and of memorizing propositions, stressing, after the PRL, that the ideal teaching of geometry consisted

recommendations to be taken into account, then [p. 305]:

> Elements of geometry thus treated would become as it were excellent elements of Logic, and would perhaps be the only ones that it would be necessary to study.

In order to support his claim, Lacroix argued that it is more useful to examine whether a geometrical proposition is true or not, than delve into a study of the faculty of reasoning required for this examination. Providing next the metaphor on walking, cited above, Lacroix quoted (in italics) Condillac on the limited effect of theoretical rules in general [p. 305]:

> *Rules are like parapets of bridges, they are not to help a passenger to walk forward but will keep him from tumbling over.*

This quote was apparently borrowed from Condillac's *La langue des calculs*, which was published posthumously by the idéologue P. Laromiguère in 1798. In this work, Condillac elaborated on his method of natural generalization with a view to founding algebra upon arithmetic. This quote originally stems from Leibniz, as a hint against the frustrating rules rules of traditional logic.[40]

Having discussed the limited educational utility of the rules of logic, Lacroix came to question the priority of geometry over algebra. At first he argued that geometry should be taught first, given that it hardly required a knowledge of arithmetic and that it fascinated pupils more than algebra [pp. 306–307]. However, by next considering geometry as a vehicle for "serious forms of reasoning" [p. 307], he realized that its study required a certain maturity on the part of the pupil, and thus it should follow the instruction of algebra. Up to this point, Lacroix had fragmentarily pinpointed few differences between the two disciplines, by viewing them strictly as objects of instruction. A deep comparative study, which would take into consideration the conceptual differences that governed the subject-matter of those disciplines, apparently escaped the purposes of his tract, but not the attention of his followers [§5.2].

Summing up, in line with the PRL and Euler, Lacroix chose geometry as a paradigmatic model of rigorous reasoning, and attributed to formal logic a secondary, albeit not insignificant role. However, for all his limited attention to the latter, he does offer us reasons to assume that he did foresee the decline of that tradition, which, like Condillac, confused formal logic with semiotic epistemology.[41]

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of choosing the right axioms and notions at the exact point when needed. From among the PRL rules, we mention two: "To let no term be obscure" and "To require as axioms only what is perfectly evident" [p. 275]. He also advised instructors to delve into a comparative study of all existing textbooks on this subject [pp. 285–291, 302].

[40] See [Leibniz 1890, 14]. Condillac's *La langue*, studied in [Dhombres 1983], was meant to elucidate elementary algebra, including the legitimacy of negative numbers. However, his method failed in the solution of algebraic equations of the 5th degree and it proved to be the least successful of his books [Auroux 1982, 87]. Warmly recommended to instructors by De Morgan in 1831 [§5.1], this work saw a reprint in 1981.

[41] We may see Lacroix's *Essais* as an anticipation of [Kirwan 1807] [§3.1], since it incorporated

3 LOGIC IN ENGLAND, 1807–1833

3.1 Kirwan (1807) and Whately (1826)

> To Whately is due the title restorer of logical study in England.
> [De Morgan 1860b, 247]

In Britain, at the turn of the century, Aristotelian logic was still held in disdain. Commonsense philosophers retained their admiration for "the logic of Locke", while intellectuals like John Horne Hooke (1736–1812) or Jeremy Bentham (1748–1832) conceived logic as universal grammar in line with the idéologues.[42] Interestingly enough, it was a keen follower of the latter who was the first to argue against the prevailing view that Locke's *Essay* [§2.1] constituted an exact system of logic. We refer to the historically neglected chemist and philosopher Richard Kirwan (1733–1812), and author of *Logick; or an essay on the elements, principles and different modes of reasoning* [Kirwan 1807].[43] Addressed to students of law, *Logick* stressed the importance of a theoretical study of logic nearly two decades before the scientific character of formal logic became diffused through Whately's *Elements of logic*.

Just like Lacroix [§2.2], Kirwan acknowledged Locke and Condillac as "excellent metaphysicians", who, however, had erroneously overlooked the value of syllogism "in legal and theological controversies" [Kirwan 1807, xi]. Eager to make up for this lacuna, Kirwan not only incorporated the basic elements of syllogistic logic into his book [pp. 467–528], but also argued convincingly on logic's utility in directing the mathematician's attention to the absurdities of algebra [pp. iii–v]. Above all, no longer was logic merely an "art", as with the PRL or Condillac [§2.1], but also existed as a "science", the latter term deployed by Kirwan in the sense of a classifying scheme [pp. 1–3].

Possibly influenced by Kirwan, Richard Whately (1787–1863), an Oxford graduate in classics and mathematics in 1808, elaborated on a modern scientific conception of logic in his *Elements*. A first version originally appeared in the *Encyclopedia Metropolitana* in 1823 under the entry "Logic" meeting little response. However, after its publication in expanded form in 1826, it enjoyed several editions, forming

the two competing conceptions of logic that prevailed at the turn of the century, the formal and the facultative, daring to attempt a limited, comparative study. As stressed by Buickerood [1985, 189] a precise understanding of 18th century logic, conceived as an analysis of the cognitive faculties, deserves further study. On the development of syllogistic logic during the 18th century, see [Van Evra 2000, 115–121].

[42]On the stagnation of formal logic in England during the 17th and 18th centuries, see [Dessi 1988, xvi–xvii], [Hamilton 1833, 194–199] and [Van Evra 2000, 116–120].On Locke's impact, see [Aarsleff 1982a] and [Buickerood 1985]. On the politician H. Tooke and the utilitarian philosopher J. Bentham, see [Kretzmann 1967] and [Rider 1990, 113–120].

[43]Born in Ireland of English descent, Kirwan, a close friend of Tooke, was well versed in linguistics and law [Donovan 1850]. In his *Logick*, he drew amply on the PRL, Locke and the ideologues. According to Hamilton [1833, 204], he was the "last respectable writer on logic" in Britain before Whately. However, his contributions to the revival of the study of logic were to be acknowledged but recently; see [Panteki 1992, sect.6.2] and [Van Evra 1984, 9–10].

the starting point for Whately's successors up to Charles Peirce in the 1860s.[44] Whately's own starting point was H. Aldrich's *Compendium* (1691) on syllogistic logic, which he revised, adding a lucid account of fallacies, a chapter "On induction" and other issues related to the "Province of Reasoning".[45]

Logic's "most appropriate office", claimed Whately, "is that of instituting an analysis of the process of the mind in Reasoning", and in this respect, logic is "strictly a *Science*" [Whately 1826, 1]. At the same time he maintained that logic, the "Grammar of Reasoning" [p. 11], is "*wholly* conversant about language" [p. 74], a statement apparently contradicting logic's former definition. His critics hurried to detect traces of a tradition hostile to Aristotelian logic in Whately's connections between logic and language.[46] But Whately's focus was on the syllogism, which he viewed from a novel perspective. As pointed out by Van Evra [2001, 121]:

> No longer, however, was the syllogism merely a way of relating propositions within a given language; now it was a specific abstract thing with a specific role, i.e. to serve as a canonical test of the validity of actual argument, regardless of the language, and regardless of their (actual) form.

This novel perspective is best revealed in Whately's "striking analogy" between algebra and logic. Claiming that just as variables are "arbitrary signs representing numbers in the abstract" [p. 14], he added that:

> So also does Logic pronounce on the validity of a regularly-constructed argument, equally well, though arbitrary symbols may have been substituted for the terms. And the possibility of doing this (though the employment of such arbitrary symbols have been absurdly objected to, even by writers who understand not only Arithmetic but Algebra) is a proof of the strictly scientific character of the system.

Whately alluded to the Scottish philosopher Dugald Stewart who had objected to literal symbols in logic [Van Evra 1984, 11]. Although we lack evidence as to Whately's mathematical background, we consider his arguments as congenial to the algebraic speculations of Woodhouse and Babbage earlier in the century, along with noting that some identical statements were put forward by Boole in 1848,

[44]Hamilton accused Whately of not referring to Kirwan as his precursor [Hamilton 1833, 202–207]. On the impact of the *Elements* upon British logicians, see [Panteki 1993, 341–346] and [Van Evra 1984, 14–15]. By 1848 the book saw its ninth edition in London and New York.

[45]On Aldrich's work, see [Van Evra 2000, 119–120]. On the content of the *Elements*, see [Dessi 1988, xix–xxv], [Merrill 1990, ch. 1], [Van Evra 1984, 9–14] and [Van Evra 2000, 121–122]

[46]By stressing connections between logic and language "as an object", Whately was "introducing semantic ascent to logic, a concept which would later be firmly established by twentieth-century analytic logicians" [Van Evra 2000, 121, fn. 31]. Whately's most severe critics were Hamilton and Solly; see [Hamilton 1833, 208] and [Panteki 1993, 145–146].

while trying to explain the function of his logical variables.[47]

In addition to his lucid account of fallacies [Merrill 1990, ch. 1], Whately's principal contribution consisted in his defense of logic's utility in other fields of inquiry and its theoretical importance. He offered neither any technical innovations, nor even the motivation to extend the limited realms of traditional syllogistic logic. On the contrary, he was happy to claim that "all arguments may be reduced to syllogism" [p. 12], and by "all" he also referred to the inductive argument, which he reduced to a syllogism in Barbara. Whately was aware of the deductive vs. inductive opposition that prevailed during the two previous centuries, with its strong bias in favor of the inductive argument. In his book he meant to show that this controversy arose from the confusion between induction as an "argument" and as a "research process". In the latter case, induction falls outside the provinces of logic, and was thus omitted from his work [Whately 1826, Book IV, ch. 1].

Whately was able to distinguish between logical and physical inquiries. Aware that the conclusion of a syllogism is included in the premises, he acknowledged that the discovery of new truths could not arise from deductive reasoning only. Nonetheless, deductive reasoning — in mathematics or elsewhere — still plays an important role in our cognitive activities, by helping us to discover consequences unnoticed until then. His dubious remarks on induction conceived as an implication, according to recent commentators [Dessi 1988, xxiv; Van Evra 1984, 13], attracted the attention of philosophers of science only after the third edition of the *Elements* in 1829, which was enriched with an Appendix that included an analysis of terms related to political economy.

Whately belonged to the so-called group of "Oriel Noetics" at Oxford, who broadly followed along the lines of D. Ricardo's (1772–1823) theory of political economy.[48] Among the Noetics was the economist Nassau Senior (1790–1864), who, based upon D. Stewart, insisted upon the axiomatic nature of political economy. To this end, Senior wrote in the Appendix to Whately's *Elements* of 1829 [Whately 1848, 230]:

> The foundation of Political Economy being a few general propositions deduced from observation or from consciousness, and generally admitted as soon as stated, it might have been expected that there would be as little difference of opinion among Political-Economists as among Mathematicians.

[47] On Boole, see [Panteki 1992, sect. 8.2]. Connections between English algebra and logic, from the early 19th century, have been fragmentarily studied so far. For instance, see [Sherry 1991] on Woodhouse's debt to Condillac and [Durand 1990] on Peacock's own to Locke. Stewart's nominalistic impact upon Whately is discussed in [Corsi 1988, 42–46 and 150], a book that also informs us about the diffusion of Woodhouse's work on astronomy at Oxford in the early 1820s [ibid, 36–48]. However, Whately's acquaintance with Woodhouse's or Babbage's work is still open to inquiry.

[48] In 1829, Whately was appointed Professor of Political Economy, furnishing by 1831 his *Introductory lectures on political economy*, with the intention of vindicating the usefulness and scientific nature of this new discipline. On the "Oriel Noetics", see [Dessi 1988, xxviii], [Durand 2000, 145], [Corsi 1988, ch. 7] and [Yeo 1993, 102–111].

This specific passage, which reflected the premature deductivism of the Ricardians outraged the Cambridge group, and especially Whewell (1794–1866) and R. Jones (1799–1855) [§3.2].

3.2 The reception of Whately's Elements, *1827–1833*

> As for Whately and his logic you may neglect him or kick him as you like. [Whewell to Jones, 1832][49]

> If Whately be right, Aristotle is fundamentally wrong.
> [Hamilton 1833, 231]

Whately's earliest successor was the botanist George Bentham (1800–1884), nephew of J. Bentham. Motivated by his uncle's manuscripts, which were inspired by Condillac's *Logique*, Bentham produced his *Outline of a new system of logic* [Bentham 1827], which focused on a critical examination of Whately's "last and most improved edition of the Aristotelian system" [Bentham 1827, viii]. Noting that Whately had wrongly confined himself to traditional syllogistic, Bentham set off to extend the latter so as to account for the subtleties of classification in botany. To this end, he introduced his unique novelty, the "quantification of the predicate", thus arriving at an augmented syllogistic scheme of an almost symbolic form. Bentham's innovation was to be acknowledged by De Morgan in 1850, too late to bear any impact whatsoever upon the development of algebraic logic.[50]

Bentham's book sold badly, but this cannot account for the neglect shown to its unique novelty since it featured, together with several new publications inspired from Whately's *Elements*—, in an anonymous lengthy review "Recent publications on logical studies", which was published in the *Edinburgh Review* in 1833. The author of this review was the Scottish philosopher W. Hamilton (1788–1856), who, after a passing commentary on Bentham's critique of Whately [Hamilton 1833, 199–202], focused exclusively upon Whately's work. For all his polemic tone, Hamilton admitted that Whately's *Elements* communicated new life "to the expiring study" of logic [p. 199], a fact proved by the eight new publications on this subject, listed at the opening of his review [p. 194]. This apart, Hamilton objected to nearly every single statement uttered by Whately that deviated from the respectable writings of Aristotle.[51]

Hamilton accused Whately of lacking historical erudition, arguing at length that the "art-and-science" distinction of logic was not new with him [pp. 201–210]. Without approving of Aldrich's *Compendium* as a basis for syllogistic logic [pp. 198, 210–213], Hamilton rejected Whately's links between logic and language,

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[49]See Whewell's letter to Jones on 21.12.1832 [Whewell Papers, Add.Ms.c.51/149].

[50]See [De Morgan 1850, 32]. On Bentham's syllogistic scheme, see [Panteki 1993, 141–143], [Styazhkin 1969, 148–150], [Van Evra 1992]. On the quantification of the predicate see fn. 9.

[51]On Hamilton's review, see [Dessi 1988, xxv–xxviii] and [Van Evra 1984, 14–15]. In a passage where Kant was mentioned as the next eminent authority on logic after Aristotle, Hamilton claimed, "Logic is a formal science" [p. 215]. See also fn. 26.

accusing him of psychologism [pp. 208–9]. Last but not least, he was opposed to Whately’s embrace of induction, as inference, within syllogistic logic [pp. 224–238]. In fact, entering into a comparison between Aristotle and Whately on the latter issue, he reached the verdict that if Whately was taken to be right, then Aristotle was wrong [pp. 212, 231].

While Hamilton criticized Whately’s work from the point of view of formal, deductive logic, Whewell examined it from a diametrically opposed standpoint, that of inductive logic, which underlined his notion of science. Whewell referred in print to Whately’s work in his Bridgewater treatise on *Astronomy* published the same year as Hamilton’s review. Distinguishing between the inductive and deductive modes of thinking, Whewell claimed that these two modes entailed different moral and religious attitudes. Kepler and Newton were the inductive discoverers, while Lagrange or Laplace the mathematical talents, who, unlike the former, were not religiously inspired by their science. He openly downgraded the role of both mathematics and logic, using Whately for his purpose, by holding that:

> [...] all which mathematics or logic can do, is to develop and extract those truths, as conclusions, which were in reality involved in the principles on which our reasoning proceeded.*
>
> * “Since all reasoning may be resolved into syllogisms, and since in a syllogism the premises do virtually assert the conclusions, it follows at once, that no new truth can be elicited by any process of reasoning”. *Whately’s Logic*, p. 223.[52]

Whewell’s polemic against Whately began in 1831, when his close friend Jones quoted in a letter to him Senior’s passage on political economy [§3.1], as appended in the third edition of the *Elements*.[53] According to Whewell and Jones, by supporting Ricardian political economy, the Oriel Noetics wrongly neglected the virtues of induction and the laborious but sure process of ascending from observation to general first principles. The crux of the controversy concerned the notion of the nature of science. Whewell looked down at advocators of the deductive mode of reasoning, calling them “Downward road” people. Whately, in particular, by embracing induction within syllogistic logic, was seen as a severe threat to Whewell’s and Jones’ attempt to explain the nature of the inductive method to the public.[54]

[52] [Whewell 1833, 335–336]. The asterisks denote the footnote, here appended after the text. On *Astronomy* and the inductive-deductive distinction, see [Becher 1991], [Richards 1992, 57–62] and [Yeo 1993, 116–124].

[53] See Jones’ letter to Whewell on 24.2.1831 [Whewell Papers, Add.Ms.c.52/20]. On Whewell’s replies, which amounted to the claim that “The analogy between physical and political or economical sciences is yet to be shown”, see [Todhunter 1876 II, 115–124]. These letters reveal that their sole objection to Whately’s logic concerned his embrace of the inductive argument within syllogistic, deductive logic.

[54] As Whewell wrote to Jones in July 1831: “If you will give me illustrations and examples

As a matter of fact, Whewell and Jones became engaged with the nature of science around 1822. In 1826, Whewell declared his intention to deliver "grand lectures on the principles of induction in mixed mathematics" [Todhunter 1876 II, 71–72], but his plans were postponed until the early 1830s. He was motivated to make his views known on this issue by undertaking to review Herschel's *Discourse on natural philosophy* (1830) and Jones' *Essay on the distribution of wealth* (1831), as well as by revising his treatises on mechanics.[55] By that time, De Morgan had recommended the *Elements* in his educational booklet [De Morgan 1831a, 71, fn.; §5.3], citing the third edition of Whately's book.

4 DE MORGAN'S CAMBRIDGE HERITAGE, 1817–1827

Preface

> Thank heaven that I was at Cambridge in the interval between two systems, when thought about both was in the order of the day even among undergraduates. There are pairs of men alive who did each other more good by discussing x over dx and Newton versus Laplace, than all the private tutors ever do. De Morgan to Whewell, 1861[56]

De Morgan entered Trinity College in February 1823. In 1826, he sat for the strenuous pre-Tripos "disputations", which concerned "Newton's first section, Lagrange's derived functions, and Locke on innate principles" [Morgan 1872, 305], graduating a fourth wrangler in January 1827.[57] In charge of the Tripos[58] were the "examiners" and the "moderators", the latter ranking higher than the former. The moderators were responsible for moderating the discussion involved in the students' disputations, evaluating the classification of wranglers and above all for posing original problems in the Tripos. Privileged, as they were, with these duties, they had a powerful role in potentially influencing the educational system at Cambridge, as De Morgan would note in 1832:

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of the ascending method applied to moral sciences we shall have no difficulty in fighting the 'downward road' people" [Todhunter 1876 II, 125]. On the Oxford–Cambridge controversy, see [Corsi 1988, 150–158] and [Yeo 1993, 12, 93, 102–105].

[55] In the early 1820s, Whewell and Jones talked about the "metaphysics" of science, a notion that gradually developed into what Whewell called in 1837 the "logic of induction". On his reviews and textbooks [§6.1], see [Corsi 1988, 150–158], [Richards 1980, 351–359], [Todhunter 1876 I, 52–57] and [Yeo 1993, 21, 93–99].

[56] From De Morgan's letter to Whewell on 20.1.1861, quoted in [S. De Morgan 1882, 305–306]. Apparently due to a misprint, a dot is missing from x, to denote a fluxion.

[57] Dating back to the Middle Ages, the disputations would be abolished in 1839. Based upon De Morgan's own notebooks as a student, [Rice 1997b, 20–28] offers a detailed account of his student years and graduation exams.

[58] The name "Tripos" originated in the 15th century when, prior to receiving their degree, undergraduates went through an oral examination, seated upon a three-legged stool, known as a tripos; see [Becher 1980b, 1–6], [Becher 1995, 414, 423] and [Rice 1997b, 20–24].

> The *moderators*, or examiners, who are usually younger masters of arts, and come to the matter with the newest ideas going, feel that great scope is allowed, and do not confine themselves to any book or system, further than may appear advisable to themselves. Hence any great improvement is of comparatively easy introduction, it only needs one moderator, who does not fear the appearance of singularity.[59]

Such a moderator was George Peacock, co-founder of the Analytical Society in 1812 with Charles Babbage and John F. Herschel [Enros 1983]. Due to his efforts regarding the Tripos of 1817 and 1819, Newton’s long-standing fluxional calculus was replaced by the differential calculus. However, when De Morgan began his studies, “the old system was still remembered and discussed, and excited much thought about fundamental principles to the great advantage of many” [De Morgan 1865, 146]. The period of his studies was additionally marked by changes in the teaching of mechanics. Thanks to his tutor, William Whewell, students became acquainted with the name of Laplace, and were interestingly induced to discuss the latter’s *Mécanique Céleste* versus Newton’s old-fashioned *Principia*. As he wrote in his letter to Whewell in 1861 — quoted above — De Morgan considered it a great privilege to have been at Cambridge during such an “interval between two systems”, an interval of an enduring impact upon his career. Impressed by the force of his arguments in that letter, we offer an overview of the state of mathematics [§4.1] and mechanics [§4.2] at Cambridge during the period 1817–1827, prior to stressing the role of his Cambridge education and his potential as a graduate [§4.3].

4.1 The aspirations of the analytical society

> The preceding pages have been devoted to a slight account of the history and present state of Analytical Science, that branch of human knowledge, of which Laplace has justly observed “C’est le guide le plus sur qui peut nous conduire dans la recherche de la verite”.[60] [Babbage and Herschel 1813, xxi]

This statement is from the Preface to the *Memoirs of the Analytical Society, for the year 1813*, the unique volume of the journal published anonymously by the Analytical Society (AS) in 1813. The Preface was written by Babbage in close collaboration with Herschel, while the main part of the *Memoirs* consisted of their own original contributions, in line with the work produced by leading mathemati-

[59] The quote is from [De Morgan 1832b, 276], his first review of a Cambridge textbook, namely J. Wood’s elementary *Algebra* [§6.2].

[60] “It is the most certain guide which can lead us in the search for truth”. Quoted without an accent, Laplace’s exact statement has not been traced. However, similar statements can be found in his 1795 lectures [§2], or in subsequent writings, such as his treatise on “Probabilities” of 1812, cited in the Memoirs [p. xii].

cians in France at the turn of the century.[61] The Preface opened with an appraisal of the language of analysis, as shaped and perfected by Lagrange, Laplace and Arbogast. There followed a summary of recent work produced in the realm of differential, finite difference and functional equations, with special attention to "Lagrange's theorem" and Laplace's "method of generating functions", both characterized by a singular analogy between indices of repeated functional operations and exponents. Magnetized by the "peculiar grace of Laplace's Analysis" [p. v], the two authors paid tribute to his contributions in the field of probabilities [p. xii] and celestial mechanics [p. xvi]. But, besides an astonishing erudition on the state of Continental mathematics and mechanics, the volume of *Memoirs* above all reflected the dreams of its authors towards fostering mathematical research in England, by promoting what they conceived as "pure" mathematics over the so-called "mixed" or "applied" or "synthetic" mathematics that had prevailed since Newton's time.

The mathematical curriculum at Cambridge in the 1800s focused on Euclidean geometry and Newtonian fluxions, optics, mechanics and astronomy [Becher 1980b, 1–10]. In other words, mathematics, which involved geometrical reasoning, and were closely linked with intuition and physical concepts. These stood in sharp contrast with Lagrange's "analytics", that is methods stemming from his algebraic calculus, founded upon power-series expansions. Devoid of diagrams, limits and physical concepts, which underlined "synthetic" mathematics, Lagrange's analytics were privileged with a powerful symbolic language, which afforded economical storage of knowledge and the ability towards generalization, thus potentially leading to new discoveries.[62]

According to Lagrange, every function $f(x+h)$ could be expanded in a Taylor series, as follows:

$$f(x+h) = f(x) + ph + p'h^2 + p''h^3 + \dots\dots\dots, \tag{4.1.1}$$

where the symbols p, p', p'' etc. were new functions of x "derived" from f in a certain algebraic manner. Through suitable transformations and by comparing the resulting expansions, Lagrange arrived by induction from (4.1.1) at the formula

$$f(x+h) = f(x) + hf'(x) + \frac{h^2}{2!}f''(x) + \dots, \tag{4.1.2}$$

where $f'(x)$ stood for $df(x)/dx$, $f''(x)$ for $d^2f(x)/dx^2$ etc. Based next upon the analogy between indices of operation, as in $d^2f(x)$, and exponents, as in $(df(x))^2$ (an analogy noted earlier by Leibniz), Lagrange cast (4.1.2) in the symbolic form

[61]On the anonymity and scope of this volume, as well as the papers contributed to it by Babbage and Herschel, see [Enros 1979, ch. 4] and [Panteki 1992, sect. 2.3]. On the drawbacks of its printing, see [Topham 2000]. The significance of the Preface to the *Memoirs* is raised by numerous historians; see [Ashworth 1996, 653] and [Koppelman 1971, 181–184].

[62]On the state of mathematical studies at Cambridge in the 1800s, and the alleged distinction between "analytics" and "synthetics", see [Ashworth 1996, 632–636], [Becher 1980b, 1–10], [Becher 1995, 405–407], [Enros 1981], [Enros 1983] and fn. 38.

$$\Delta f(x) = f(x+h) - f(x) = exp(hdf(x)/dx) - 1, \qquad (4.1.3)$$

which he would generalize for the nth finite difference of $f(x)$. Named after him, this symbolic version of Taylor's theorem proved useful in problems of interpolation and summation (when n attained negative values), leading ultimately to Laplace's method of generating functions in the 1770s.[63]

Eager to provide alternative proof for Lagrange's theorem, L. F. Arbogast furnished his intricate "Calculus of derivations" in his treatise *Du calcul des dérivations* [Arbogast 1800]. While his actual calculus of derivations had very few followers (among them De Morgan), [64] the epistemological basis of his treatise had an immense impact upon the development of the calculus of operations, offering an explicit distinction between "operation" and "function". In a spirit congenial to Condillac and Laplace, Arbogast paid significant attention to the language of analysis, claiming that "The secret of the power of Analysis consists in the happy choice and application of signs, which are simple and characteristic of the things they should repressent" [Arbogast 1800, ii]. Elaborating over this dictum, Arbogast introduced the "separation of the scale of operation" as follows [ibid, viii–ix]:

> This method is generally thought of as separating from the functions of variables when possible, the operational signs which affect this function. Then of treating the expressions formed by these signs applied to any quantity whatsoever, an expression, which I have called a scale of operation, to treat it, I say, nevertheless as if the operational signs which compose it were quantities, then to multiply the result by the function.

After deploying combinatorial techniques in order to prove the formula (4.1.3), Arbogast separated the symbols of operation, like d/dx, from those of quantity, like $f(x)$, so as to obtain the purely symbolic form of Lagrange's theorem,

$$\Delta = exp(hD) - 1, \text{ where } D = d/dx, \qquad (4.1.4)$$

thus giving rise to a study of symbolical methods, whose rigorous foundation was sought after by F. J. Français, F. J. Servois and others from the mid-1810s onwards.[65] At Cambridge, Lagrange's algebraic calculus attracted the attention of

[63]On Lagrange's theory of derived functions, see [Fraser 1987] and [Grattan-Guinness 1990, 129, 161, 195–203]. On the generalization of formula (4.1.3) and its impact upon Laplace and Arbogast, see [Panteki 1992, ch. 1] and fn. 33.

[64]On Arbogast's calculus of derivations, see [Grattan-Guinness 1990, 211–216]. Very few English mathematicians developed this calculus, among them J. West, A. Cayley and S. Roberts, mentioned in [Panteki 1992, ch. 5]. On De Morgan's moderate development of Arbogast's calculus, see [De Morgan 1842, 168–174].

[65]Upon the development of algebraic, symbolical methods in France and England, after Arbogast, see [Koppelman 1971] and [Panteki 1992]. Français' and Servois' foundational studies became known after R. Murphy's and D. F. Gregory's contributions in the 1830s; see [Allaire 2002; Panteki 1993, 136–140; Panteki 2000, 369–178].

Robert Woodhouse, a precursor of the AS. In his *Principles of analytical calculations* [Woodhouse 1803], he discussed the merits and drawbacks of Lagrange's freewheeling use of expansion in the Taylor series, introducing a formal definition of "=" as a link between a function and its expansion, so as to overcome problems of convergence. Although Woodhouse's attempt for a reform at Cambridge failed at the time, his book, along with Lacroix's big treatise on the calculus served as a starting point for the members of the AS in 1811–12.[66]

Herschel launched his research in 1813 with a study of functional equations, treated through an extension of Laplace's method of finite differences. Thereafter, he focused on a combination of the calculus of differences with the calculus of operations, as stemming from Arbogast's method of separation of symbols, while Babbage directed all his energy towards functional equations. Whereas the two friends shared a common passion for analytics, they had distinct preferences as to the specific methods they followed in their papers. For instance, Babbage deployed Monge's algebraic techniques instead of Laplace's method of finite differences, moreover refusing to apply Arbogast's symbolic approach. However, Babbage's freewheeling manipulation of iterated functions, like $ff = f^2$, which betrayed the nature of his algorithmic reasoning, practised later on in computing, stemmed largely from Arbogast's impact upon Herschel. As it were, many of Herschel's novelties in foundations or notational issues were hidden for the most part in the voluminous correspondence between the two friends, which continued ceaselessly from 1812 to 1820.[67]

By 1820 both Babbage and Herschel had left Cambridge, after realizing that their project of fostering the study of analytics had largely failed. To Babbage's dismay, the papers they had contributed to the *Memoirs* had not been reviewed in any British journal [Enros 1983, 37]. Moreover, their subsequent publications received negative reviews, apparently from P. Barlow, teacher in the Military Academy of Woolwich [Enros 1979, 170–193]. The spirit of their research and professionalization of mathematics at large, were foreign to Cambridge's "Liberal education", according to which most graduates, including the members of the AS, sought careers elsewhere.[68] But even for those who did stay, like Peacock, the methods developed by Babbage and Herschel seemed too abstract and general to

[66] On the formation of the AS around 1812, see [Enros 1983]. Woodhouse's textbooks are discussed in [Becher 1980b, 8–10]. On his early algebraic concerns, see especially [Becher 1980a], [Dubbey 1978, ch. 5] and [Sherry 1991]. The latter argues about Condillac's plausible impact upon Woodhouse, while the two former hint at Woodhouse's impact upon Babbage's manuscripts on the "Philosophy of analysis" produced in the early 1820s, serving as a stimulus for [Peacock 1830].

[67] On Babbage's and Herschel's distinct contributions to analytics, see respectively [Grattan-Guinness 1992a] and [Grattan-Guinness 1992b]. On a detailed study of their investigations, largely based upon their correspondence, see [Panteki 1992, ch. 2]. On a novel perspective as to the mechanization of thought entailed in their work, to which Whewell would be strongly opposed, see [Ashworth 1996].

[68] Using Whewell's own words, Cambridge graduates were future "lawyers, or men of business, or statesmen" [Whewell 1835, 40]. On the lack of public institutional encouragement for mathematical sciences in England, see [Ashworth 1996], [Becher 1980b, 1–10], [Becher 1995], [Enros 1981] and [Garland 1980, ch. 3].

be assimilated, let alone form part of the curriculum. Peacock's opposition to their views was evident in the two common contributions of the AS, the translation of Lacroix's abridged textbook on the calculus, published as [Lacroix 1816], and its sequel of *Examples* (1820). After Peacock, [Lacroix 1816] opened with a limit concept, quite in line with the traditional, intuitive approach at Cambridge.[69] Moreover, the main part of the *Examples* was covered by Peacock's share in the differential calculus, while, as a moderator, he introduced into the Tripos Euler's version of the calculus, not Lagrange's theory of derived functions.[70] With Peacock, the textbooks of the AS hardly revolutionized the curriculum, but were seen to fit into it. The marginalization of Babbage's and Herschel's aspirations would become even more striking with Whewell's intervention in the 1820s!

4.2 Whewell between Newton and Laplace

> Instead of balancing the simplicity and evidence of the mathematics of a century ago against the generality and rapidity of modern analysis, it might be better to attempt to combine them. [Whewell 1823, vi]

For all his lack of enthusiasm for abstract analytics, Peacock did enrich the Tripos with infinite series and differential equations, moderately expanding the curriculum in the late 1810s. However, the Tripos would soon undergo new changes, indicative of Whewell's peculiar tendency to combine 18th century mathematics with "modern analysis", as he stated in his above-quoted treatise on *Dynamics* [Whewell 1823]. Due to Whewell's efforts, more questions on mixed mathematics appeared in the Tripos during the 1820s, along with problems of mechanics and physics which required two distinct types of solution: one based upon geometrico-physical reasoning, and another based on analysis.[71]

Whewell matriculated at Trinity in 1812, but never became a member of the AS, nor did he participate in its textbook publications.[72] In 1818 he became assistant

[69] Most Cambridge mathematicians endorsed the intuitive-based limit concept in the calculus; see [Fisch 1999], [Richards 1991], [Richards 1992], [Smith 1980] and [Smith 1984b]. On the social, philosophical and religious factors that differentiated Peacock from Babbage and Herschel, see [Becher 1995].

[70] Indeed, 507 pages of the *Examples* were devoted to Peacock's calculus, 127 to Herschel's calculus of finite differences and only 42 corresponded to Babbage's functional equations. Read by De Morgan as a student, this work would have significant impact upon the revival of the calculus of operations, along Arbogast's and Herschel's lines in the late 1830s (see [Panteki 1992, ch. 4], [Panteki 2000, 169–178]). On Peacock's efforts as a moderator in the late 1810s, see [Becher 1995].

[71] Certain problems required only a geometrical solution (see [Cambridge 1831, 36, 108]), while others required both geometrical and analytical solutions (see [ibid, 139, 144]). Forming a par excellence characteristic of the 1820s, this tendency diminished in the 1830s. Details on the type of problems posed at the Tripos at that time, are included in [Panteki 1992, sect. 3.2].

[72] As stressed in [Becher 1991, 1–2], numerous errors have been committed by historians concerning Whewell's alleged involvement with the AS. In this paper, the author presents a summary of Whewell's multi-dimensional personality and career. Thereafter we draw amply on [Becher 1980b], who provides a step-by-step analysis of Whewell's controversial attitude in connection with the mathematics education at Cambridge for nearly half a century.

mathematics tutor, and in 1823 he was head tutor at Trinity, engaged ambitiously with his plan to update the teaching of mechanics. Whewell' s plan was far from congenial with the spirit of the textbooks produced by the AS, and saw that they gained as limited an audience as possible. [Lacroix 1816] was initially intended to be taught prior to mechanics. Thus, the students would assume a sound foundation of the calculus and learn to think in the abstract, able to proceed from general propositions to specific applications. However, according to Whewell, the student ought to move in the opposite direction, that is from specific examples to general theories. Moreover, only the best students would be encouraged to proceed with the advanced material of modern analysis, which was incorporated in [Lacroix 1816] and its sequel [Becher 1980b, 32–34].

Upon these grounds, he first produced an *Elementary treatise on mechanics* [Whewell 1819], designed so that the students could learn a considerable portion of mechanics before learning the calculus. Opposed to Lagrange's formalism in *Mécanique analytique* (1788), Whewell deployed geometrical diagrams and limits, thus presenting mechanics "as a series of distinct individual constructions, each with its own proof, rather than a series of deductions from general principles" [Becher 1980b, 16]. However, he was not hostile to Laplace's *Mécanique céleste* (MC), a less formal approach to mechanics than Lagrange's, which manifested its author's predilection for applications, not foundations.[73]

It took long for Laplace's MC to be introduced at Cambridge University, although various reforms took place in Scotland and Ireland under its stimulus, thanks to individuals like James Ivory and John Brinkley respectively, well before the foundation of the AS.[74] The lack of public institutional encouragement for the mathematical sciences in England, and particularly the conservatism that ruled Cambridge's adherence to Newton's *Principia*, were at the core of John Playfair's review of MC in 1808 at Edinburgh University. In defense of Laplace's treatise, Playfair argued that not only was it not "threatening religion", but that on the contrary, it reinforced both "the thesis of the existence of an initial design" and "the Aristotelian doctrine of final causes".[75] Playfair's "admirable review" was

[73]Even from a quick glance at the two treatises on mechanics, the reader can see the radically different styles of the two relatively similar analytizations of mechanics. Lagrange's treatise is more lucid, formal and general than Laplace's, focusing on the symmetry of its formulae and avoiding any appeal to intuition. On the other hand, Laplace provides an accumulation of observational and experimental data, omits explanation of his often approximate procedures, eager to show that his solution is confirmed in practice, by replacing the variables with specific values, rather than provide any idea of how he arrived at it in the first place. Some similarities and differences can be found in [Panteki 1992, ch. 1]. On a larger scale, see [Grattan-Guinness 1990, ch. 5]. On Laplace's MC, see also fn. 36.

[74]On Ivory, see [Craik 2000]. On the reforms that took place in Scotland and Ireland, prior to the AS, see [Guicciardini 1989, chs. 7–9] and [Panteki 1987]. Particularly on Ivory's original research into Laplace's potential theory and Brinkley's own on analytics, with which Babbage and Herschel were familiar, see [Panteki 1992, sect. 2.1].

[75]From Playfair's review, partly quoted in [Durand 2000, 142]. On the influential role of religion upon the administrative and educational system at Cambridge, see [Becher 1995] and [Richards 1992]. Particularly on the contrasts between Edinburgh and Cambridge University at the time, see [Ashworth 1996, 639–641] and [Enros 1981, 136–140].

acknowledged in the *Memoirs* [p. ii], but noting that their own business was "exclusively with pure Analytics" [ibid], Babbage and Herschel focused on the latter, [76] letting Woodhouse take the first step towards the diffusion of MC at Cambridge.

Woodhouse furnished an advanced treatise on *Physical Astronomy* [Woodhouse 1818] which he enriched with a comprehensive historical account, still worthy of attention. Referring with praise to Laplace's treatment of the three-body problem, he declared the "superiority of the Analytical over the Geometrical method" [p. lix], proceeding with an elaborate study of the orbital differential equation, indispensable for the study of planetary mechanics. In so doing, he introduced into the curriculum the method of variation of constants, an issue favoured by certain moderators from 1820 onwards.[77] As it were, without incorporating Laplace's theory of the earth's shape, Woodhouse's treatise was nearly the only textbook on physical astronomy to include instances from Laplace's original procedures well until J. H. Pratt's *Mechanical philosophy* in 1836 [§6.1].

In 1823, Whewell furnished his treatise on *Dynamics* [Whewell 1823], as a sequel to his former textbook of 1819, calling students to orientate their studies towards a simplification of Laplace's work [Whewell 1823, v]:

> The student who feels a proper admiration for the system of the *Principia*, ought to look forward to the complete development of it in the *Mecanique Celeste*, as the ulterior subject of his labours; and those who shall simplify the different parts of that work, and reduce them to the level of ordinary readers, as far as they admit it, deserve to be considered as real benefactors to the commonwealth of science.

But that was as far as he went. By failing to include segments of Laplace's MC and to draw the students' attention to a full exposition of the three-body problem, *Dynamics* was definitely a retreat from Woodhouse's *Astronomy.* And even Airy's *Mathematical tracts* [Airy 1826], which included new issues such as the theory of the earth's shape, was in many ways inferior to Woodhouse's treatise, curiously omitting the method of variation of constants and promoting instead only approximate techniques for the solution of differential equations.[78]

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[76] We wish to note, however, that by 1813 Herschel was seriously engaged with the study of MC, and in the *Memoirs*, page xvi was devoted to an extract from this work, concerning the problem of decomposition of forces, which was reduced to a functional equation (see [Panteki 1992, sect. 1.4]). Moreover, plausible applications of their theoretical studies were not omitted from Babbage's and Herschel's projects. As Babbage held in his second paper on functional equations, his calculus would soon develop into a powerful tool for physical discoveries [Babbage 1816, 179–180].

[77] See particularly [Woodhouse 1818, 23, 92–105, 140, 208–211, 271, 400–406]. Instances drawn from his book would form part of the questions posed in the Tripos (see [Cambridge 1831, 10, 16, 78, 139, 142, 162–164, 171–172] and [Panteki 1992, sect. 3.2]).

[78] Like [Whewell 1823, 75], Airy acknowledged Woodhouse's "superior" treatment of the three-body problem, which he omitted from his account. Moreover, he merely mentioned the orbital equation, putting forward, however, only approximate techniques for its solution [Airy 1826, iv, 1–5, 27]. On Airy's *Tracts*, see further [Becher 1980b, 26, 32–34] and [Panteki 2000, 169–174], in connection with his inadequate treatment of the earth-figure equation, a problem favoured by Tripos moderators.

A disciple of Whewell, and senior wrangler in 1823, G.B. Airy was a far better mathematician than Whewell, and a promising astronomer, involved with research on planetary inequalities along Laplace's lines. However, reluctantly following Whewell's suggestions, Airy presented the theory of the earth's shape in accordance with Clairaut's semi-geometrical, semi-analytical style, based on the 18th century theory of hydrodynamics, instead of Laplace's potential theory. Airy's inadequate explanations, and above all his unorthodox combination of "synthetics" and "analytics" would puzzle generations of students [§6.1]. However, the *Tracts* were in full accordance with Whewell's demands, who not only endorsed Airy's book with enthusiasm, but even objected later on to Airy's suggestions to update it.[79]

Whewell's peculiar oscillation between Newton and Laplace, or between tradition and progress, was not only evident in his textbooks and lectures. In his own papers, read for the Cambridge Philosophical Society in the 1820s, he deployed geometrical methods, elementary algebra, trigonometry, differentiation and approximate techniques for the solution of differential equations, looking for simplicity and close contact with physical concepts. As a typical mixed-mathematician, he was concerned with answers, not abstract rigour, and upon these grounds he would deploy, if necessary, even divergent series [Becher 1980b, 16–18]. In 1826 he produced an article "On the mathematical theory of electricity compared with experiment", published in *Encyclopedia Metropolitana* in 1830. Orientated towards applications, [Whewell 1830] focused on the mathematical properties of what became known after Whewell as "Laplace coefficients" (now Legendre functions), implicitly promoting a branch of pure analytics, which would have a decisive impact upon the development of algebraic symbolical method in the 1830s.[80]

4.3 De Morgan's mentors

> We have, in practice, a system, which gives true results; and, to use the words of a writer to whose *Analytical Calculations* elective affinity led me when I was an elementary student, "Since it leads to truth, it must have a logic". [De Morgan 1865, 179]

As a student, De Morgan received tuition from seven men, to whom he remained faithful until the end of his career. According to his wife's *Memoirs* [S. De Morgan 1882, 15–16]:

> He never forgot what he owed to his teachers in the University. These were, as entered in his own book, his college tutor J. P. Higman, Archdeacon Thorp, G. B. Airy, A. Coddington, H. Parr Hamilton

[79] See Whewell's letter to Airy on 11.10.1839, fully quoted in [Todhunter 1876 II, 282] and [Becher 1980b, 26, 34].

[80] On Whewell's article, published anonymously, see [Becher 1980b, 18] and [Todhunter 1876 I, 84–85]. On its impact, see [Panteki 2000, 170–173, 202].

> (Dean of Salisbury), G. Peacock (Dean of Ely), and W. Whewell (afterwards Master of Trinity). With all these gentlemen he kept up a friendship and correspondence during their joint lives.

To these men we could add Woodhouse (alluded to in our opening quote), Babbage and Herschel, with whom at this stage De Morgan was acquainted but through the perusal of their textbooks.[81] Beginning with Woodhouse, the very title and content of De Morgan's paper "On infinity and the sign of equality" [De Morgan 1865], cited above, was highly congenial with the former's research back in 1803.[82] In the same paper, he paid tribute to Peacock, as "a friend whom I so highly value, and to whose thought I have been so much indebted" [De Morgan 1865, 180]. If, moreover, we consider his essay on the "Calculus of functions" (COF) [De Morgan 1836], we have sufficient evidence for supporting the view that De Morgan was a genuine, perhaps the only genuine, follower of all precursors and founders of the AS.[83]

Indeed, in his essay on the COF, De Morgan drew on all the papers furnished by Babbage and Herschel on functional equations, including those incorporated in the obscure volume of the *Memoirs* [Panteki 1992, sects. 3.5–3.9]. Moreover, he was shrewd to perceive the immense historical value of this volume, which has remained basically unread to this day, by writing on his own copy of it in 1858:

> The time will come, when this work will be sought after by the curious, as the earliest indication of the change, which was taking place in English mathematics. I think it is all written by Herschel and Babbage: the preface by Herschel. No more was published under this name.[84]

Furthermore, [De Morgan 1836] manifests his historical orientation, which underlined the majority of his educational, mathematical and logical writings. As we shall see, this inclination was partly due to Lacroix's influence. However, many of his mentors at Cambridge might have provided him with additional stimuli for the study and use of the history of science. For instance, in 1826 Peacock composed a lengthy article on "Arithmetic" published in *Encyclopedia Metropolitana* in 1830,

[81] On De Morgan's tutors and his study of the textbooks issued by the AS, see [Rice 1997b, 24–27, 30–33, 50–52]. Most of his tutors wrote excellent testimonial letters for his appointment at London university in 1828; see [S. De Morgan 1882, 14–17] and [Rice 1997a].

[82] On [De Morgan 1865], see [S. De Morgan 1882, 328–331], [Panteki 1992, sect. 3.9], [Richards 1987, 28–29] and § 1.1. On [Woodhouse 1803] and the definition of equality, see [Becher 1980a] and §4.1.

[83] According to [Richards 1987, 10], De Morgan can be viewed as a "satellite" of the AS. We further support her claim with what follows below in text.

[84] London University Library, L3, [Anal. Soc.]. Well until [Enros 1979], De Morgan appears to have been the only mathematician to have delved into the contents of this journal. It may be worth noting, that Whewell referred to the *Memoirs* in the *British Critic* in 1831 as a meaningless "combination of signs" of "extraordinary complexity" (see [Enros 1983, 37]). Although we know from their correspondence that Babbage undertook to write the Preface, still De Morgan's remark is far from surprising, given the fact that many of the comment concerning French publications actually derived from Herschel, who followed along the lines defined by them in the *Memoirs* more closely than Babbage [Panteki 1992, ch. 2].

while the textbooks and lectures of Woodhouse, Whewell and Airy contained historical comments on physical astronomy.

From the latter, De Morgan also inherited a genuine love of astronomy, with two papers concerning the orbits of the Moon and the Comets published in 1833. But De Morgan was not gifted with enough patience to carry out the lengthy numerical calculations involved in this branch. As he confided to Whewell in 1832: "but by the powers of calculating and the properties of numbers, I protest, I will never work out a planetary perturbation, or the place of a comet, and much obliged do I feel to those who can and will do such things", alluding to Airy, who had just received a medal from the Royal Astronomical Society (RAS) for his paper on planetary inequalities.[85] However, by being himself a member of the RAS from 1830 onwards, De Morgan cultivated his passion for astronomy, contributing numerous historical articles [Rice 1996].

Mechanics and astronomy in fact seem to have been among De Morgan's principal interests during his residence at Cambridge. The recent discovery of a tract he wrote in 1827 on "Statics" reveals Whewell's influence towards the study of celestial mechanics, both through his *Dynamics* in 1823, and his article on "Electricity", which De Morgan read in manuscript in 1826 [Becher 1980b, 18, fn. 68]. As it were, De Morgan composed his manuscript on "Statics" with the intention of producing a sequel on dynamics, ultimately providing the students with a complete course ranging from first principles to Laplace's physical astronomy [Rice 1997b, 53–54]. De Morgan had good reasons for planning such a project, given the rather confusing presentation of mechanics' principles by Whewell and Airy through a combination of old-fashioned synthetic mathematics and modern analysis [§4.2]. However, he never completed the first treatise, and as a result its sequel was never composed either. Notwithstanding the reasons that impeded the publication of De Morgan's tract, it merits attention upon the grounds that it formed the main testimonial for his election at London University in 1828.[86]

De Morgan's tract of 1827 is our earliest evidence of his ability to critically assimilate and combine diverse mathematical and epistemological stimuli, manifesting his enduring interest in the foundations of pure and applied mathematics as well as his talent for teaching. It further justifies his wife's claim that he devoted much time to extensive reading as a student "beyond the bounds marked by his tutors" [S. De Morgan 1882, 15]. Indeed, in it he deployed functional equations, such as

$$f(x+h) + f(x-y) = f(x) \cdot f(y), \tag{4.3.1}$$

which were absent from Babbage's and Herschel's work, while he also drew on Lagrange's principle of virtual velocities, which Whewell had objected to using in

[85] From De Morgan's letter to Whewell on 12.11.1832 [Whewell Papers, Add.Ms.a.202/96]. That year, Airy was awarded a medal from the RAS for his paper on planetary inequalities [Rice 1997b, 103].

[86] On De Morgan's election, see [Rice 1997a] and on his tract, [ibid, 270-271].

his own treatises.[87] Above all, the existence of this tract in the framework of what appears to be a branch of applied mathematics induces us to re-examine certain of his subsequent contributions to education, algebra, the calculus and logic.

De Morgan never came to produce a textbook on mechanics, nor any research work on celestial mechanics. He was largely attentive, however, to issues concerning the first principles of applied mathematics. In 1832, he furnished his elementary *Calculus* [De Morgan l832a, 133], designed for use prior to the students' initiation to mechanics. He also produced a *Spherical trigonometry* in 1834, a subject that became his starting point for the study of astronomy as a student [Rice 1997b, 31, 122, 188–193]. Moreover, we note that part of De Morgan's fame was due to his contributions in the field of probabilities.[88] To this end, he devoted a section of his advanced *Calculus* [De Morgan 1842, 331–340, 746–749] to Laplace's method of generating functions, an issue closely linked to the latter's work on probabilities. Last, but not least, almost half his review of Peacock's *Algebra* in 1835 was a critical overview of the teaching mechanics at Cambridge, which culminated in raising interesting links between mechanics, the calculus, algebra and logic [§6.2].

As we saw, De Morgan was in many ways a faithful disciple of Woodhouse, Peacock, Babbage, Herschel, Whewell and Airy, while the majority of his tutors stimulated his long-standing interest in the foundations of the calculus.[89] By drawing additionally on the original works of Arbogast, Lagrange, Laplace and others, he deviated largely from "the bounds marked" by his mentors at Cambridge, as his wife rightly claimed in her *Memoirs*. Before we proceed to illustrate his debt to the encyclopedist Lacroix, we should add a note concerning his earliest mentor, while at school, his teacher J. Parsons. A former fellow of Oriel College Oxford, Parsons was a close friend of Whately's, talking of him perpetually to his pupils. As De Morgan wrote in his *Paradoxes*: "Before I was sixteen, and before Whately had even given his Bampton lectures, I was very familiar with his name and some of his sayings" [De Morgan 1872, 196]. Although Whately would have his "Logic" published in *Encyclopedia Metropolitana* the very year De Morgan matriculated at Trinity, it is quite likely that he acquired an admiration for Whately's personality through Parsons, long before he got involved with the study of logic.[90]

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[87]On Whewell's objection to Lagrange's principles, see [Becher 1980b, 15–16]. On equations (4.3.1), see chapter 2 of the "Elements of statics", University College Library, MS.Add.27. De Morgan treated this functional equation using Lagrange's method of developing $f(x+h)$ in Taylor series. On this rarely used method, see [De Morgan 1836, 328, 367].

[88]See [De Morgan 1882, 85–93], [Garland 1980, 34], [Rice 1996, 230–231], [Rice 1997b, 96–99] and [Smith 1982, 40–55].

[89]See letters exchanged between De Morgan and Higman in 1847–1848, University College Library, MS.Add.97/5. See also [Smith 1984b] and [Richards 1991].

[90]On Parsons's influence upon De Morgan, see [S. De Morgan 1882, 3-11] and [Rice 1997b 15–20]. The "Bampton lectures" at Oxford University were a series of lectures on theology [Corsi 1988, 16, 67, 76, 100–101].

5 LACROIX'S IMPACT UPON DE MORGAN, 1828–1833

5.1 De Morgan in defense of mathematical studies

> In reality, our senses are our first mathematical instructors.
> [De Morgan 1831a, 2]

After being appointed Professor of Mathematics at London University, De Morgan undertook to defend the study of this branch against common charges and propose efficient methods for its instruction at an elementary level. The ideas contained in embryo in his "Introductory lecture" [De Morgan 1828] — mentioned in § 1.2 — were elaborated on in an ensuing lecture, published as *Remarks on elementary education* [De Morgan 1830]. By that time, he had composed an article "On mathematical instruction" [De Morgan 1831b] and a booklet *On the Study and Difficulties of Mathematics* [De Morgan 1831a], cited as SDM [§1.1]. SDM was published by the Society for the Diffusion of Useful Knowledge (SDUK), founded by Lord Brougham in 1825, in association with London University.[91] As for his article "On mathematical instruction", it was one of the thirty-three articles he contributed to the *Quarterly Journal of Education* (QJE), which was sponsored by the SDUK.[92]

SDM bears striking similarities, in both its structure and content, with Lacroix's *Essais*, while instances from De Morgan's earlier and later writings lead to our viewing Lacroix's tract as a key-link between the Oxford-based revival of syllogistic logic and De Morgan's Cambridge heritage in applied and pure mathematics. In line with the *Essais*, SDM was divided into four parts, concerning the teaching of arithmetic [pp. 4–18], algebra [pp. 18–64], geometry [pp. 65–91] and trigonometry [pp. 91–93] respectively. Alluding to Laplace's lectures at the Ecole Normale in connection with the dangers entailed in the careless use of the method of induction, [93] De Morgan revealed his basic sources, claiming that [p. 63]:

> Both the preceptor and the pupil, but especially the former, will derive great advantage from the perusal of Lacroix, *Essais sur l'Enseignement en géneral et sur celui des Mathématiques en particulier*, Condillac, *La*

[91] The SDUK aimed to "facilitate the education of the working man by means of intelligible books on academic subjects published at affordable prices" [Rice 1997a, 270], [Rice 1997b, 108–110]. According to the latter, De Morgan had proposed submitting his SDM to the SDUK in March 1830. SDM proved very successful, reprinted in the USA in our century [MacFarlane 1916, 21–22], [Grattan-Guinness 1992c, 3] and [Pycior 1983, 213].

[92] During the period of his resignation (1831–1835), De Morgan contributed several book reviews, a variety of scientific commentaries and his most significant writings on instruction, under the editorship of George Long (see comments in [S. De Morgan 1882, 407–414] and [Rice 1997b, 98–111]). Although, as [Corsi 1988, 118] noted, the QJE was almost entirely written by De Morgan, we should add another notable contributor, Baden Powell (1796-1860), Professor of Geometry at Oriel College, Oxford, from 1827. Member of the SDUK from 1830, Powell was a close friend of both Whately and De Morgan [ibid, 5–6].

[93] See [SDM, 63]. De Morgan displayed his own translation of [Laplace 1795, 41], without, however, revealing his exact source. Apparently he became acquainted with Laplace's lectures through Lacroix's *Essais* [§2].

> *langue des Calculs*, and the various articles on the elements of algebra in the French Encyclopedia, which are for the most part written by D'Alembert.

Although this was his only reference to Lacroix's tract, there are indications which lead to the impression that he had gone through it much earlier. In 1828, he asked the beginner "in all his embarrassment" to rely on the word of his instructor [De Morgan 1828, 40], according to D'Alembert's motto: "Go forward and faith will follow".[94] Moreover, in line with Laplace and Lacroix, he argued on the import of analysis in the invention of fertile theories in physical astronomy, claiming:

> Never was the talent of invention so brilliantly displayed as in the various successful attempts by which, from the time of Newton to that of Laplace, all the phenomena of the solar system were mathematically demonstrated to result from the operations of the Newtonian Law of Gravity.[95]

Thus proving that mathematics do not "deaden the imagination", he refuted the view according to which mathematics "destroy the taste for literature" by praising the writings of Pascal, Descartes, Leibniz and D' Alembert equally for "spirit, taste, and beauty" and for "scientific talent" [De Morgan 1828, 30–32]. Questions of priority were not absent from his lecture, which included a comparison between mathematics and natural philosophy [pp. 40–44].

De Morgan showed a great sensitivity to the study of arithmetic, which he called the"groundwork of the mathematics" [De Morgan 1830, 12]. It was perhaps through Lacroix that he also became acquainted with Condillac, furnishing in the title page of his textbook on *Arithmetic* (1830) a quote attributed to him:

> It is not by means of a routine that one becomes educated, but by one's own thinking; it is essential to get into the habit of understanding rationally what one does: such a habit is acquired more easily than one thinks; and once acquired, it is never lost.[96]

In his *Remarks* [De Morgan 1830, 13] he drew once more on Condillac, choosing a quote already deployed by Lacroix [§2.2]:

[94] On D'Alembert's principle, mentioned in [Lacroix 1828, 175] and [Richards 1991, 298], see [De Morgan 1828, 38–40].

[95] De Morgan developed a fascination for Laplace's celestial mechanics through his tutors at Cambridge [§4.3]. However, the tone and poetic style of his lecture on this issue hint at a direct impact of Laplace's and Lacroix's appraisal of enlightenment science and philosophy. One may be surprised today by the erroneous exaggeration of this passage (i.e. "all the phenomena..."); nevertheless, this was exactly the message delivered by those authors at the turn of the century.

[96] As a student, De Morgan read Peacock's article on "Arithmetic" four years before its publication in the *Encyclopedia Metropolitana* in 1830. Interestingly enough, Condillac's *La langue* was found in Peacock's library [Durand 1990, 141–144]. Nonetheless, Peacock did not draw on Condillac's work in his article, so we have no means of establishing his plausible influence upon De Morgan in the direction of Condillac's philosophy of arithmetic. In any case, the passage that follows below in text hints at Lacroix's influence in this direction. On the success of De Morgan's textbook on arithmetic, see [Rice 1997b, 86–87].

> I would even go as far as to say that the science of arithmetic is more easy than the art [...] It has been well observed by Condillac, in treating of this very subject that a rule is like the parapet of a bridge; it may keep a careless passenger from tumbling over, but will not help him walk forward.

He claimed next that algebra may be seen as "an easy generalization of arithmetic" [De Morgan 1830, 14], focusing thereafter on a comparative study between algebra and geometry. Interestingly enough, he did so in a manner congenial, if not identical, to Lacroix's own. He held that "Geometry, which is a science more pleasing to the majority of learners than algebra and which is for the purposes of the many, the more useful of the two, might be taught at an earlier age than is the custom at present" [p. 15]. Noting, however, that the advanced study of geometry in "the most rigorous form" should best be postponed for a later period of instruction, he more or less repeated Lacroix's own arguments [§2.2].

In connection with the teaching of algebra, De Morgan drew on both Lacroix and Laplace by holding that "The new symbols of algebra should not be all explained to the student at once. He should be led from the full to the abridged notation, in the same manner as those were, who first adopted the latter" [1831b, 277], De Morgan followed Clairaut's method based upon historical discovery, recommended by both his mentors. Furthermore, after Euler (see [Lacroix 1828, 170–187, 252]), he proposed the creation on the student's part of a "syllabus of results only, unaccompanied by any demonstration", so that the student would acquire "memory for algebraical formulae, which will save time and labor in the higher departments of the science" [SDM, 63].

Laplace's influence is mostly evident in SDM. In line with the former's lectures on algebra [§2.1], De Morgan initiated the student into algebraic notation through an arithmetical example, with the intention of leading him all the way from numerals to the symbolic formula, by representing the verbal expression of "half the sum of two numbers added to half their [absolute] difference, gives the greater of the two numbers". However, he enriched his material with more examples than Laplace, further offering his own original intermediate step between purely numerical and purely symbolical formulae as follows: "(First No+second No)/2+(First No–second No)/2 = First No" [SDM, 18].

Without yet touching upon the instruction of geometry, we can trace so many similarities between Lacroix's and Laplace's texts on the one hand, and De Morgan's writings on the other, that we are tempted to accuse him of plagiarism. Nonetheless, De Morgan engraved his own synthesis of the French philosophy of education and his training at Cambridge, as revealed, for instance, in his emphasis on exercising the student's familiarity with the binomial theorem through examples, prior to furnishing him with the general proof [SDM, 62]. Lacroix, after Laplace, had stressed the importance of general methods in teaching, along with critically considering the majority of English treatises, which allegedly wrongly introduced pupils to algebra by means of examples alone [Lacroix 1828, 173–176, 203, 264–270]. Between these extremes, De Morgan echoed Whewell by proposing a "middle course", that is by urging instructors to suit "the nature of the proof to the student's capacity" [SDM, 62].

Before switching to De Morgan's views on the teaching of geometry, we wish to clarify a subtle point concerning his debt to the empirical philosophers, Locke and Condillac, in his early educational writings. After them, he insisted that a pupil should be initiated into first principles carefully, after being experimentally trained through "ocular demonstration" [De Morgan 1828, 13–15], by claiming that our senses are our "first mathematical instructors" [SDM, 1–3]. Such instances, however, should not mislead us into regarding the first stage of his career as an exclusive "traditional empiricist stage" [Pycior 1983] which would be abandoned from 1835 onwards.[97] Just like Lacroix, De Morgan drew on these empirical philosophers strictly in connection with the instruction of elementary mathematics and the very early stage of the student's initiation. For, in the same writings [§5.2], De Morgan showed evident traces of his debt to the PRL, foreshadowing statements put forward in 1835, which would be wrongly considered by historians as uniquely characterizing his new, modern approach to mathematics.

5.2 Is reasoning peculiar to geometry only?

> And we may ask, how comes that reasoning, utterly banished from arithmetic and algebra, preserved its place as an essential of geometry?
> [De Morgan 1831b, 268–269]

In his "Introductory lecture", De Morgan claimed that, "the success of every individual in the world must depend on his power of reasoning" [De Morgan 1828, 8]. He went on to defend mathematics against common criticism, concluding, "wherever previously formed habit of abstraction and generalization are valuable, the preparation of mathematical studies is useful in the highest degree" [ibid, 43–44]. A few years later, he argued in more explicit terms that mathematical demonstration is "strictly logical" and that "The same species of logic is used in all inquiries after truth", "logic" standing for "accurate reasoning".[98] But if, in theory, mathematical instruction could nurture the reasoning capacity of the student, what about its efficiency in practice?

De Morgan raised the latter issue in his *Remarks*. He held that mathematical education suffered not merely from inadequate but rather from erroneous methods, as instructors tended to emphasize exercising the faculty of "dry memory" instead of trying to make first principle intelligible to the pupils. He argued that, by directing the students' attention to "rules only, not to the principles on which they are established" instructors had excluded "reasoning and reflection" entirely

[97] On De Morgan's debt to Locke, see [De Morgan 1828, 13–25], [SDM, 1–3], [Pycior 1983, 212–216] and [Rice 1997b, 69–70]. However, not all of De Morgan's early educational views stemmed uniquely from Locke's empirical philosophy [see §5.2].

[98] See [De Morgan 1831a, 3], [De Morgan 1831b, 265]. Ironically, in 1842, De Morgan criticized Whewell for freewheelingly deploying the term "logic" as a synonym for "reasoning"; see [De Morgan 1842, 12–13] and [Todhunter 1876 I, 108].

from arithmetic, which as a science "is the most adapted for the development of these faculties in the young mind" [De Morgan 1830, 12]. Firmly believing that arithmetic was a paradigmatic field for the mind's training in the process of "generalization by induction", he illustrated his point by drawing on the Pythagorean number theory.[99]

In SDM, induction was considered as algebra's "most powerful engine of demonstration".[100] He argued, in fact, that if it is necessary to "learn to reason", then "in no case is the assertion more completely verified than in the study of algebra" [SDM, 60–63]. In brief, neither arithmetic nor algebra were devoid of reasoning. How then, he asked, did reasoning preserve its place uniquely within geometry, or, as it was called, "mathematics" [De Morgan 1831b 268–269]? Moreover, could we presume with certainty that "all who learn geometry will learn to reason correctly" [De Morgan 1833b, 238]?

De Morgan felt induced to account for the instructors' misconception in accepting geometry as the only vehicle for the exercise of mathematical reasoning, and consequently to restore reasoning within mathematical education, commencing with an overview of the defective teaching of geometry. The first misconception, he argued, was partly due to the poor instruction of algebra: "It was probably the experience of the inutility of general demonstration to the very young student that caused the abandonment of reasoning which prevailed so much in English works of elementary mathematics" [SDM, 62]. On a different note, he wrote: "We suspect it arose from the fact of the treatise of Euclid being found already established, and the disinclination to overturn any institution being so great, that this work preserved its place in spite of its truth and beauty" [De Morgan 1831b, 269]. And as for geometry's instruction, De Morgan pointed out many weaknesses that needed serious consideration.

It was firstly noted that the study of mathematics in general was delayed "till what is comparatively so late a period in life" [De Morgan 1830, 11; De Morgan 1831b, 275]. Moreover, when a pupil was confronted with Euclid's *Elements*, he was unable to follow the route of demonstration, as he lacked any preliminary experimental training [De Morgan 1830, 14–16]. De Morgan lamented also the tendency of instructors to demand that the pupil memorize the right order of propositions.[101] Last, but not least, teachers failed to realize that the first book

[99]Namely, that the addition of any successive odd numbers, starting from 1, will always result in a square number [De Morgan 1830, 9]. The Pythagoreans were not mentioned at this point, but the history of mathematics was often a source upon which De Morgan based his views; see [ibid, 8–9], [SDM, 12 and 63]. According to him, arithmetic was considered a most important prerequisite for the study of algebra; see [De Morgan 1830, 12–14], [De Morgan 1831a, 18], [De Morgan 1831b, 266–271]. On De Morgan's own efficiency as a teacher, see [Howson 1982, 80–84] and [Rice 1999].

[100]He dealt at length with the method of induction, mainly in connection with the binomial theorem [SDM, 60–63]. We recall his debt to Laplace's lectures quoted in SDM [§5.1]. In 1838, he distinguished between "mathematical" and "scientific" induction, the former being De Morgan's own term [Rice 1997b, 122].

[101]On lines fairly identical to [Laplace 1795, 20–21], De Morgan wrote: "There seems to be a magic in numbers, which no one can withstand, from Leibniz, who proposed to convert the

of the *Elements* hardly afforded a favorable basis for initiating a student into geometry, commencing, as it were, with certain far from evident definitions and axioms, as well as some "troublesome propositions" [De Morgan 1831b, 275].

In an effort to amend the situation, he furnished a paper "On the method of teaching geometry" [De Morgan 1833a, b], announcing its unusual division into two parts: the first related "to the manner of teaching the terms and the facts of geometry", and the second to "the method of deducing them from one another by reasoning" [De Morgan 1833a, 35]. We might say that what the instructors' methodological framework seemed to lack until then was a proper distinction between the "matter" and "form" of geometry respectively, a distinction successfully deployed in those terms by De Morgan in his mature work on logic.[102] The idea for such a division occurred to him in 1831, in an attempt to advise instructors on how to avoid the dangers entailed in initiating a pupil into geometry by means of Euclid's first book. As he wrote [De Morgan 1831b, 275]:

> It would not be contrary to good logic, to assume the whole of the first book of Euclid, and from it to prove the second, provided that afterwards the first book were proved, without the necessity of taking for granted any proposition in the second. The argument [...] would then stand thus:
>
> If the first book be true, the second is true.
> But the first book is true.
> Therefore the second is true.
>
> The order in which the premises come, does not affect the soundness of the conclusion and provided the pupil understands that the conclusion depends equally on the premises and the reasoning grounded upon them, which are two distinct things, an error in one not necessarily affecting the other, he is perfectly safe, and takes a view of the process of reasoning not generally given to the young.

Representative of De Morgan's idiosyncratic style, this passage deserves attention in so far as he clearly distinguished in it between the "matter" or "premises" and the "form" or "reasoning" of geometry respectively. In so doing, he prepared the ground for the arguments put forward in his review of Peacock's *Algebra* four

King of China to Christianity, by means of binary arithmetic, to the mathematical master of the country school, who measures his pupils' conviction of geometrical truths by their power of recollecting the order in which they come" [De Morgan 1831b, 269]. Further on the abuse of the faculty of memory, see [Lacroix 1828, 285–291] and [De Morgan 1830, 14].

[102] On De Morgan's "form–matter" distinction within logic, see [De Morgan 1858] and comments in [Hodges 2000] and [Merrill 1990, ch. 4]. As argued in [Panteki 1992, sects. 3.4–3.9 and 6.3–6.8], this distinction featured in different terms as a methodological tool for his study of the calculus of functions in 1836, thereafter influencing his study of algebra an logic. But the origin of this distinction stemmed from his concern for geometry in 1831, as noted in text below.

years later, where within a more general context he stressed the difference between the "*certainty of mathematical conclusions*" and the "*correctness of mathematical reasoning*" [De Morgan 1835a, 95]. Although De Morgan's crucial epistemological distinction as well as his singular attention to the importance of reasoning have not escaped historians' attention, we wish to clarify certain misconceptions concerning the true origins of his statements of 1835.

According to Pycior [1983, 217], the distinction between the truth of mathematical conclusions and the accuracy of reasoning was revolutionary, inducing her to claim that by 1835 De Morgan was moving "towards a more modern conception of mathematics [than in the earlier "empirical stage"] which stressed its deductive nature rather than the self-evidence of its concepts and axioms". As we noted above, this distinction was far from new in his review of 1835, but stemmed from his earlier desire to amend the problematic long-standing tradition governing the instruction of geometry. What remains is to consider the stimuli that led De Morgan to develop such a systematic concern for geometry and its reasoning in the first place.

According to Rice [1997b, 73], Kant must have been an influential source in this respect although not "mentioned by name" in De Morgan's writings. Due to lack of substantial evidence we cannot argue for or against this view. However, we wish to draw attention to an interesting note written by De Morgan on a copy of his *Remarks*, found in the Royal Society Library, which runs as follows:

> Most of the opinions contained in this lecture which are opposed to those of Kant's philosophy were found by the author, about four years after it was written, in a pamphlet by Mr. (Dr?) Beddoes entitled "Observations on the nature of demonstrative Evidence with an explanation of the difficulties occurring in the Elements of Geometry by Thomas Beddoes. London J. Johnson, 1793". But the conclusions drawn are different.[103]

In any case, we hold that De Morgan's own debt to Locke and Condillac, through his perusal of Lacroix's *Essais*, must had been quite contradictory to any plausible debt owed to Kant at that stage. Especially in connection to geometry's instruction, the influence of Lacroix and the PRL is too strongly evident in De Morgan's writings of that period to leave an open question as to his sources. Indeed, in line with the PRL, De Morgan asked instructors "To defer every axiom, until that point is arrived at, where it becomes necessary", or "To omit those propositions which are not subsequently useful" [De Morgan 1831b, 276]. Such an influence is further evident in [De Morgan 1833a], which concerned the "matter" of Euclidean geometry, and the ideal order of its instruction (compare with [Lacroix 1828, 274–312]).

[103]The inscription bears no signature, but a comparative study leads to De Morgan as its author. Thomas Beddoes (1760–1808), an Oxford graduate, was a physician and scientist. The question of Kant's influence upon De Morgan at this stage remains open.

The "form" of geometry was considered in chapter 14 of SDM and consequently in [De Morgan 1833b] [§5.3]. Interestingly enough, chapter 14 was preceded by a preliminary one "On the definitions of geometry" [SDM, 65–68], which opened with a comparative study between algebra and geometry. This could have been seen as a rather unusual way of introducing the basic concepts of geometry, had it not been for Lacroix's similar comparison soon after he had briefly associated geometry and logic, as well as for De Morgan's tendency to furnish links between diverse issues in his discourse, a tendency once again largely due to Lacroix's own encyclopedistic style [§1.3, §2 and §5.1].

De Morgan argued extensively on why geometry was easier for the student to grasp than algebra. He supported that the results of elementary geometry "are in many cases sufficiently evident of themselves to the eye" whereas in algebra "many rudimentary propositions derive no evidence from the senses" [p. 65]. Moreover, according to him "there is nothing in the elements of pure geometry comparable, in point of complexity, to the theory of the negative sign, of fractional indices, or the decomposition of an expression of the second degree in factors" [ibid]. Above all, the symbols deployed in geometry were of a "less general nature" than those of algebra. Elaborating on the latter issue, he wrote [SDM, 65]:

> In algebra a general proposition respecting numbers is to be proved. Letters are taken which may represent any of the numbers in question, and the course of the demonstration, far from making any use of the particular case, does not allow that any reasoning, however general in its nature, is conclusive, unless the symbols are as general as the arguments. We do not say that it would be contrary to good logic to form general conclusions from reasoning on one particular case, when it is evident that the same considerations might be applied to any other, but only that very great caution [...] would be required in deducing the conclusions. There occurs also a mixture of general and particular propositions, and the latter are liable to be mistaken for the former. In geometry, on the contrary [...] any proposition may be safely demonstrated on any particular case.

Obviously stimulated by Lacroix, De Morgan's comparative study was more penetrating than his mentor's, as well as original. For, Lacroix firmly believed, after Laplace, that the generality of algebraic methods renders the instruction of algebra straightforward, dispensing with the necessity of furnishing many appropriate examples [Lacroix 1828, 274–275, 299]. Moreover, Lacroix argued that the particular nature of geometry burdened the instructor with the "embarrassing" responsibility of choosing the right examples [ibid]. Both agreed that geometry could ideally exercise the student's capacity in logical reasoning. But whereas for Lacroix the study of geometry sufficed for this purpose [§2.2], for De Morgan [1833b, 238] that was not so:

> Geometry, as it is usually studied, does not teach the principles of reasoning, but applies them [...] to the consideration of the properties

> of space. There is, therefore, no reason to presume that all who learn geometry will learn to reason correctly.

To sum up, it is clear that De Morgan did not identify mathematics with geometrical reasoning. On the contrary, believing, as he did, that this long-standing identification was largely erroneous, he introduced the basics of traditional syllogistic, with a view to arguing on the indispensability of a student's training in logic prior to his acquaintance with the complex nature of geometrical demonstration. Accordingly, chapter 13 ended as follows [SDM, 68]:

> We proceed to the method of reasoning in geometry, or rather to the method of reasoning in general, since there is, or ought to be, no essential difference between the manner of deducing results from first principles, in any science.

5.3 An association of geometry with syllogistic logic

> Whately's Logic [...]. A work, which should be read by all mathematical students. [De Morgan 1831a, 71, fn.]

Apparently acquainted with Whately's *Elements* around 1829, De Morgan explicitly referred to it in SDM above quoted.[104] Among the merits of the *Elements* that plausibly attracted De Morgan's attention at that time was Whately's lucid account of fallacies [§3.1], an issue particularly attended to in Lacroix's *Essais*. Critical towards the prevailing elementary textbooks on mathematics, Lacroix [1828, 176] had urged instructors to prevent their pupils from "the errors of reasoning" and the "seduction of paralogism" through an "analysis of the diverse forms of reasoning". Armed with Whately's study on this issue, De Morgan followed Lacroix's advice in full, devoting to this end chapter 14 of SDM and his paper [De Morgan 1833b].

As a matter of fact, he prepared the ground for his more systematic treatment given in SDM in [De Morgan 1831b, 272], where, after claiming that "It is useless to present reasoning in any shape until the language is perfectly familiar", he wrote in reference to pupils that "they have no acquaintance with the more general part of grammar which is the foundation of the forms of logic", alluding to Whately's link between language and reasoning. Within this context, he noted that pupils often confound the "converse of a proposition" with "the proposition itself' [ibid]. Similarly, he called attention to the fact that "every point in figure A is a point of figure B" does not imply that "every point of B is a point of A" [SDM, 69]. Furthermore, he noted that pupils do not have adequate training in acknowledging a "defect in the method of reasoning, but only by the absurdity of the conclusion", as in the case of "all animals are birds" [De Morgan 1833b, 240]. Such fallacies

[104] In [SDM, 71, fn], he mentioned the third edition of the *Elements*, dated 1829 [§3.1]. Moreover, on the copy of SDM in the Royal Society Library, we read the inscription "1829–1831" in his own handwriting. Thus, he probably began writing this booklet around 1829.

demanded the pupil's acquaintance with the various forms of strict reasoning, geometry serving heuristically for this purpose.

In SDM, he assumed, in line with Whately, that "all reasoning" could be reduced to a number of "single propositions", each of which is divided into the "subject", the "predicate" and the "copula" [SDM, 68]. By means of simple geometrical assertions, he initiated the pupils into the four basic types of propositions (A, E, I, O), noting that these assertions must be derived in one of the following ways: from the definition, from hypothesis, from the evidence of the assertions (axioms) and from proof already given [pp. 69–70]. Drawing on the latter case, he claimed that "no assertion can be the direct and necessary consequence of two others, unless those two contain something in common" [p. 70]. Repeating in different terms an assertion expressed in common language, for instance, as "geometry is useful, and therefore ought to be studied" we may arrive at a "syllogism", such as:

Every thing useful is what ought to be studied.
Geometry is useful, (5.3.1)
therefore geometry is what ought to be studied.

He saw (5.3.1) as an example of modus ponens, the par excellence reasoning scheme deployed in Euclid's *Elements.* This scheme deviates from traditional syllogistic logic, and so also does De Morgan's pioneering acceptance of "is equal to" together with the traditional copula "is" as distinct copulas in his account [SDM, 68, 73–75]. De Morgan was unaware at that time of Thomas Reid's (1710–1796) thesis that traditional syllogism did not suffice for mathematical proof. De Morgan would realize later on the impossible task of casting Euclidean geometry in syllogistic form. Nonetheless, his endeavour to do so for educational reasons paved a way for his original work on the logic of relations in the 1850s.[105]

Drawing on geometry, he illustrated the four figures and the nineteen legitimate moods of syllogistic logic, prior to himself being acquainted with Euler's similar device [§2.1]. For instance, figure I, mood AAA was furnished by the example [p. 72]:

All the circle is in the triangle
All the square is in the circle (5.3.2)
∴ All the square is in the triangle.

Or, indicative of figure III, mood AII, was the following example [p. 73]:

The axioms constitute part of the basis of geometry
Some of the axioms are grounded on the evidence of the senses (5.3.3)
∴ Some evidence derived from the senses is part of the basis of geometry.

[105]Hamilton edited Reid's works in 1846, so there was no possibility of De Morgan's acquaintance with them in 1831 [Merrill 1990, 15–25]. According to [Merrill 1990, ix and 23–25], De Morgan's dual use of "is" and "is equal to" as two distinct copulas was his first major innovation in logic. On De Morgan's mature views on the utopic connections between geometry and logic, see [ibid, ch. 7]. On modus ponens, see [ibid, pp. 10, 18, and 28].

He next defined "Inductive reasoning" and "Reasoning a fortiori". The former proves a universal proposition by separately proving each one of its particular cases. For instance, a figure ABCD "is proved to be a rectangle by proving each of its angles separately to be a right angle [p. 73]. The latter type, he held, was contained in figure I, mood AAA, in a different form:

A is greater than B (or the whole of B is contained in A),
B is greater than C (or the whole of C is contained in B), (5.3.4)
a fortiori A is greater than C (therefore C is contained in A).

Could one indeed reduce all geometrical arguments to one of these types of syllogistic reasoning? As noted above, De Morgan was unaware of the controversy concerning the reduction of relational forms, such as "is equal to" or "is greater than", which involved the transitivity of the copula, to syllogistic form. He confined himself to confirming the alleged adequacy of traditional logic in treating any valid, deductive type of reasoning, by claiming, that "The elements of geometry present a collection of such reasoning as we have just described, though in a more condensed form" [SDM, 73]. To support his claim, he picked up Pythagoras' theorem as "a specimen of a geometrical proposition reduced nearly to a syllogistic form" [ibid]. The reduction he furnished was evidently incomplete, involving propositions such as "AB and BE are equal", or "BG is equal to BC", which were not questioned as to their capacity to be reduced to the form "A is B".[106]

Ironically enough, these incomplete reductions were left as exercises for the student [pp. 73–75]. Nonetheless, De Morgan's joint attention to Aristotelian logic and Euclidean geometry was largely original, including an early perception of what was to become the logic of relations a few decades later. For, as he wrote [SDM, 76]:

> the validity of an argument depends upon two distinct considerations, –1 the truth of the relations assumed [...], –2 the manner in which these facts are combined so as to produce new relations; in the last the *reasoning* properly consists [...]. We are accustomed to talk of mathematical *reasoning* as above all other, in point of accuracy and soundness. This, if by the term *reasoning* we mean the comparing together of different ideas and producing other ideas from the comparison, is not correct, for in this view mathematical reasoning and all other reasonings correspond exactly.

Once again, he stressed the deductive nature of geometrical reasoning in 1833, noting that "we confine the term logic to its strict meaning, not supposing it to have any reference to the truth or falsehood of assertions themselves, but only to

[106] The full reduction of Pythagoras' theorem to syllogistic form — an endeavour first attributed to Dasypodious and Herlinus — requires the full resource of first-order predicate logic [Merrill 1990, chs. 1–2]. On the controversy between Reid and Hamilton over the transitivity of equality, and De Morgan's failure to cast this theorem in syllogistic form, see [ibid, 27–35].

the circumstances under which two of them give us a right to deduce a third" [De Morgan 1833b, 243]. Like its former part, [De Morgan 1833b] was in full accordance with Lacroix's hints, under the influence of the PRL. De Morgan noted that special care could be taken in the use of the method of reductio ad absurdum, aware of the fact that pupils had difficulties in discriminating between contradictory and contrary propositions. Thus, he recommended a minimal use of this method, until the students could understand this difference. Furthermore, students were encouraged to "retrace the steps" in certain propositions, so as to become accustomed to "the analytical method, by which alone they can hope to succeed in the solution of problems" [De Morgan 1833b, 251].

Although De Morgan [1833b] referred to his account of syllogistic logic in SDM, he omitted any details, and in general, his earlier attempt to cast Euclidean geometry in syllogistic form never resurfaced in his subsequent writings on logic. It is also worth noting that it was only in 1833 that he became aware of Euler's logical diagrams [De Morgan 1833b, 239; De Morgan 1847, 323–324]. In conclusion, the message delivered in his article of 1833 was that logic was an important prerequisite for the study of geometry [De Morgan 1833b, 238–239]:

> The principles on which geometrical propositions are established belong to the totally distinct and equally simple science of logic; and since geometry without logic would be absurd, it is desirable that the principles of the latter science should be studied with precision previously to employing them upon the former.

6 CAMBRIDGE CURRICULUM REVISITED, 1827–1835

6.1 Whewell's counter-revolution, 1832

> More than a decade of teaching had convinced [Whewell] that as a young tutor he had taken too analytic an approach. He began his counter-revolution with the same weapon the analysts had used: the textbook. Then he turned to writing pedagogical tracts.
>
> [Becher 1980b, 25]

Soon after declaring to Herschel in 1818 that he "would not be surprised if in a short time we were only to read a few propositions of Newton, as a matter of curiosity" [Todhunter 1876 II, 30], Whewell set off to update the teaching of mechanics at Cambridge. Cautious in taking a middle course between synthetics (Newton) and analytics (Laplace) in 1823, he encouraged his students to study and simplify Laplace's MC. Under his suggestions, Airy's *Tracts* in 1826, an intermediary between Whewell's elementary treatises and MC, fostered the theory of the earth's shape, to incorporate from 1831 the wave theory of light. By 1830, Whewell's article "On electricity" directed the attention of capable wranglers and Tripos moderators to new topics, such as the theory of electricity, magnetism and heat, branches related to his own study of Laplace's coefficients [§ 4.2–4.3]

Thus, largely due to Whewell's own efforts, the curriculum was enriched in 1830 with new physical subjects, a fact that met with his full approval. However, in order to satisfy the demands of such an expansion, the Tripos was accordingly enriched with more questions on pure mathematics, including, for instance, definite integrals, infinite series and advanced algebra [Becher, 1980b, 23–24]. The curriculum's growth in the direction of analytics was inconsistent with Whewell's views of a liberal education. He reacted by revising his former treatises on mechanics in 1832. By arguing on the merits of Newton's *Principia*, along with claiming that the "admirable" treatises of Lagrange and Laplace were not suited for the "common" student, Whewell tried to persuade his audience that the ultimate scope of the teaching of mechanics should be uniquely linked to the mastering of Newton's *Principia*. In full contrast with his initial declarations, Whewell now stimulated his students towards an attempt "to simplify and explain the Third Book of the Principia", rather than the difficult parts of MC.[107]

Whewell's early oscillation between the treatises of Newton and Laplace did not last long. In 1826, he discouraged Airy from including segments from MC in his *Tracts*, and then onwards stood firm against any updating of the *Tracts* in this respect [§4.2]. As it were, Whewell gradually developed an aversion for Laplace's potential theory, and in his own work on tides in the 1830s, for which he received a Royal Medal in 1837, [108] he employed 18th century equilibrium theory, which drew on geometrico-physical procedures. When Airy attacked him for deploying old-fashioned methods, Whewell replied that Laplace's analysis only leads to to differential equations, "which we cannot integrate" [Becher 1980b, 24–26]. Thus, if Laplace's theory did not suit the purposes of Whewell' s advanced physical inquiries, how could it possibly suit those of a liberal, non-professional education?

The intricacy of Laplace's analytical methods, however, formed only one reason that accounted for Whewell's predilection for Newton's work. For all his initial progressive attitude as a tutor and moderator in the 1820s, Whewell did his best to ensure that pure mathematics were subordinate to applied by enriching the Tripos with questions on mixed mathematics [§ 4.2]. However, in 1830 mixed mathematics were prone to give way to pure, and this meant for Whewell fewer chances to diffuse and practice his current views on education, as formulated over the years in conjunction with his views on religion, morality and the philosophy of science. His implicit retreat from MC in 1826, coincided with his intention to deliver "grand lectures on the principles of induction in mixed mathematics" [Todhunter 1876 II, 71–72], a plan that failed at the time [§3.2]. Inductive reasoning for Whewell entailed a strong sense of morality, but such matters were disclosed but to his intimate correspondents, like Richard Jones and the theologian Hugh James Rose.[109] As he wrote to the latter in 1826: "What I hold is that inductive

[107]See [Whewell 1832c, iv]. On Whewell's treatises on mechanics, briefly commented upon below, see [Becher 1980b, 24–26], [Todhunter 1876 I, ch. 2] and [Yeo 1993, 93-99].

[108]On Whewell's work on tides, see [Becher 1991, 13–15], [Todhunter 1876 I, ch. 6] and [Yeo 1993, 53–55].

[109]A close friend of Whewell, Hugh James Rose (1795–1838) was the Christian Advocate at Cambridge; see [Corsi 1988, 28-30, 40–41] and [Todhunter 1876 I, xxii–xxiii].

science is a good thing, and, as all truth is consistent with itself, I hold that if inductive science be true it must harmonise with all the great truths of religion" [ibid, 78].

From 1830 onwards, Whewell decided to progressively unravel his views on the philosophy of science, stimulated by the Oxford–Cambridge debate on political economy, and the books published by Herschel and Jones on related issues [§3.2]. He thus resumed his long-standing interest in the neglected virtues of induction, and proceeded to present some of his views to the public. On a different tone, he included certain of them in the Prefaces to his revised treatises, namely his *First principles of mechanics* [Whewell 1832a], *Dynamics* [Whewell 1832b] and *The free motion of points* [Whewell 1832c]. Here, we draw on his *Dynamics*, where Whewell argued upon the laborious but sure process of ascending from observation to successive levels of abstraction and generalization in mechanics through induction.

After noting that the laws of motion represented a luminous example of a perfect mature science, he wrote [Whewell 1832b, x]:

> We often feel disposed to believe that truths so clear and comprehensive are necessary conditions, rather than empirical attributes, of their subjects: that they are legible by their own axiomatic light, like the first truths of geometry, rather than discovered by the blind groping of experience. And even when the experimental foundation of these principles is allowed, there is still no curiosity about the details of the induction by which they are established.

On the other hand, he claimed, the process of deduction "fills the mind at every step with a confidence of its workings, a consciousness of certainty", so that men feel no fascination in following the "more difficult path" that leads to the ascent of first principles, i.e. that of induction [ibid, xi]. And he added that, by reasoning deductively there can be "no truth contained in the conclusion, which is not involved in the premises: good logic is the one thing requisite; and no name can convert bad logic into good, nor any authority add to the evidence of demonstrated truth" [ibid, xvi].

In line with a passage quoted in §3.2 from his *Astronomy*, he deployed above the term "logic" as synonymous for strict, deductive reasoning, the par excellence characteristic of that entailed in mathematics, and geometry in particular.[110] In many respects, however, his statements on induction were cryptic or inconsistent with those he adhered to from 1831 onwards, and would reveal in later works. By emphasizing the difficulties involved in the inductive process of discovery, he

[110] According to [Becher 1980b, 24], geometry was the only form of pure mathematics that Whewell "found aesthetically satisfying"; see also [Whewell 1837b]. On Whewell's use of the term "logic", see fn. 98, and on his identification of mathematics with deductive syllogistic logic, see also §3.2 and fn. 52. We note that the tone deployed in his educational tract is milder than that used in his ensuing *Astronomy*, "good logic" in the former being indeed "the one thing requisite".

implicitly alluded to a sense of morality, of which analytical methods were devoid due to their mechanical nature. In his *Astronomy* [Whewell 1833, chs.5-6] he would go as far compare the authorities of Newton and Laplace in terms of their religious beliefs, which accounted largely for their distinct approach towards induction. Issues concerning religious faith would not enter the discourse of his educational textbooks. However, in his *Thoughts* he would establish more clearly the position put forward in his treatises of 1832, by claiming that "A scheme of study, which escapes or tries to escape the labor of thinking, will answer none the purposes at which we ought to aim."[111]

On the other hand, his first pedagogical tract [§6.3] would incorporate in brief the position according to which mechanics, unlike political economy, was a mature science, which could be presented as a series of deductions from well-established principles, similar to those of geometry, a position formulated clearly only in his *Mechanical Euclid* [Whewell 1837b]. This opinion was largely inconsistent with the viewpoint promoted in his treatises of 1832, that is that mechanics was an inductive science, which had progressed by confronting a series of empirical problems.[112] However, his predilection for Newton remained resolute ever since. He claimed that the first section of the *Principia* "is eminently instructive with reference to the fundamental principles of the Differential Calculus" [Whewell 1832b, vii]. He also held that "the geometrical method of treating the three bodies might have had its triumphs to point to as well as the analytical" [Whewell 1832c, xii]. And while in 1823, he had called the student's attention to Woodhouse's treatment of the three-body problem [Whewell 1823, 75], in 1832 he declared that [Woodhouse 1818] (let alone MC) was no more a "convenient book" for the undergraduates [Whewell 1832c, iv].

With the exception of Airy, Whewell's followers encouraged his new attitude towards the teaching of mechanics. In March 1832, his former student, the astronomer J. W. Lubbock, praised his new textbooks, approving of his overall approach. In fact, Lubbock argued, to Whewell's satisfaction, that the true discoverers of MC were Clairaut and D'Alembert, since Laplace "did little more" than employ their methods by "taking in terms they omitted".[113] But Whewell's students would not share the same views. The first evidence come in J. H. Pratt's *Mechanical philosophy* [Pratt 1836], a lucid account of the theories of attraction and the earth's shape, in line with Poisson's modification of Laplace's potential theory. The third wrangler in 1833, Pratt expressed the frustration of generations of wranglers, who had struggled hard to understand physical astronomy via

[111]See [Whewell 1835, 42]. On Whewell's aversion to analytics, as a branch of mathematics "insensible to moral evidence", see [Ashworth 1996, 647]. Further on Whewell's views on morality, and Newton's personality, see also [Becher 1991, 18] and [Rice 1996, 211–219]. On the delicate links between science and religion and the hesitancy of Whewell and his circle to express such views in public, see [Corsi 1988, 30, fn. 18].

[112]On Whewell' s inconsistent views on the issue of induction and the philosophy of mechanics, see [Corsi 1988, 153]. The only reader of *Thoughts* to notice Whewell's advocation of the axiomatic nature of mechanics was Hamilton [§6.3, fn. 131].

[113]See Lubbock's letter to Whewell on 2.3.1832 [Whewell Papers, Add.Ms.a.208/85].

Whewell's and Airy's textbooks. Without dismissing the pedagogical virtues of geometry, Pratt criticized his tutors' clumsy combination of analytics and geometry, claiming that "if our course is to be geometrical, let us adhere to geometry, if analytical to analysis; if we are to admit both (the preferable course) let us keep our systems apart; and not have our courses of reading confused, here analysis and there geometry" [1836, iv].

Pratt's treatise drew on Murphy's *Electricity* [Murphy 1833], a work inspired by [Whewell 1830].[114] Robert Murphy, third wrangler in 1829, presented in his book a strictly mathematical study of Laplace's coefficients, which attracted Pratt's attention. Despite its abstract and general style, reminiscent of that fostered by the AS as well as anticipating the forthcoming revival of analytics in 1839 [Panteki 2000, 170–174], Murphy's study was embraced by Whewell, upon the grounds that the results could be experimentally confirmed [Grattan-Guinness 1985, 106]. Pratt would not be the only Cambridge wrangler, though, to criticize Whewell's obsession with mixed mathematics, and his retreat from Laplace. A year prior to the publication of Pratt's book, De Morgan dared criticize the teaching of mechanics at Cambridge, questioning within this context the utility of algebra and logic [§6.2]. For De Morgan, it all boiled down to a sound knowledge of first principles. We might go as far as saying that Pratt's intention of providing a new treatise, which would lead the student "from elementary mechanical principles to the demonstration of celestial phenomena" [Pratt 1836, v], was akin to De Morgan's unfinished project of 1827 [§4.3].

6.2 De Morgan's critical reviews, 1832–1835

> We have thus given an abstract of the history and methods of the most celebrated school of instruction for engineers which have ever existed. Such an institution is the thing most wanted in this country.
> [De Morgan 1831c, 73]

> If Newton had not had the resources of a college fellowship or professorship, we might never have seen the *Principia*; and the same might and still may be said of many others. [De Morgan 1832d, 208]

As a student at Cambridge, De Morgan often deviated from the bounds marked by his teachers. The best proof of his astonishing mathematical erudition is his manuscript on "Statics" in 1827, which drew on English and French texts, hardly recommended to him by his tutors [§ 4.3]. His inaugural lecture at London University, manifested his acquaintance with the history of mathematical education [§1.2, §5.1], while his ensuing writings revealed his immense debt to famous pedagogues and teachers, like Condillac, Lacroix and Laplace. We recall also the impact of Laplace's lectures upon his remarks concerning the instruction of algebra in SDM, a book that imitated the structure and content of Lacroix's *Essais* [§5]. Here we

[114]On Whewell's article, see references in fn. 80. On Murphy and Pratt, see [Allaire 2002, 418–422], [Grattan-Guinness 1985], [Panteki 1992, sects. 3.2–3.3] and [Smith 1984a].

call attention to his reviews of foreign and English systems of education, in which he lamented the latter's lack of professionalization. Our introductory quotes exemplify his attack on the English tradition of liberal education; the former is from his article on the Ecole Polytechnique (EP), [115] and the latter from the epilogue to his paper on the "State of the mathematical and physical sciences in the University of Oxford", which incorporated his first comparative study of Oxford and Cambridge.

De Morgan's tendency to investigate and compare various educational methods may have stemmed from his study of Lacroix's *Essais*, or was at least reinforced by it. Indeed, Lacroix had provided an extensive critical survey of the major educational institutions which had flourished in his country since 1789, revealing his rich administrative and teaching experience; he even referred to the newly established methods for the education of the deaf and dumb, a revolutionary accomplishment of the French Enlightenment [Lacroix 1828, 27–36, 56–60]. This issue inspired De Morgan's article on the "Methods employed for the instruction of the deaf and the dumb" [De Morgan 1832c], holding that their language "is as strict as any in geometry" and should thus be considered useful for the instruction of those who can hear and speak [ibid, 203, 217]. Manifesting his debt to French semiotic philosophy, he cited it for "the student of mathematical symbols" in [De Morgan 1836, 313, fn.].

However, the template for mathematical education would still be that promoted at Cambridge, despite its flaws. De Morgan was above all a Cambridge wrangler, and during the period of his resignation from London University, he was like a "satellite" of Cambridge University.[116] He was very attentive to any new publications or changes in the curriculum, furnishing reviews of the ninth edition of James Wood's *Algebra* (originally published in 1795) [De Morgan 1832b] and Peacock's *Algebra* [De Morgan 1835a]. As we saw in §4.0, in the former he pointed out the freedom enjoyed by Tripos moderators in establishing a potential reform in the curriculum, considering that freedom as one of the most basic advantages of Cambridge's system. Within this context, he alluded to Peacock's reforms, eager to stress that their result was not the mere replacement of the fluxional notation by the differential one: "The question was one of greater importance than appears at first sight, since on the way of settling it, it depended the introduction or non-introduction of the writings of the French and other continental mathematicians" [De Morgan 1832b, 276]. He further contemplated the issue of notation in his article "On the notation of the differential calculus adopted in some works lately published at Cambridge" [De Morgan 1834], indicative of his close contact with the events taking place at Cambridge.[117]

In his review of Wood's book, De Morgan raised only the positive aspects of

[115] In his article, [De Morgan 1831c], De Morgan delved into a study of the administrative and educational policies of the EP [Rice 1996, 205].

[116] See also fn. 83.

[117] In that paper, De Morgan made certain rather eccentric statements concerning the issue of notation, which foreshadowed his attitude in 1835. See particularly [De Morgan 1834, 109]. This article is highly representative of his historical orientation.

Cambridge's system, becoming bolder in his attack against its defects three years later. However, a middle course was taken in his article on Oxford University, in which he exerted a mild critique on Whewell's treatises on mechanics, a unique instance of a direct reference to his tutor. He criticized "the Oxford market of academical distinctions" noting that, contrary to those of Cambridge, Oxford examiners were deprived of the liberty of "representing correctly the various grades of merit, which are found among the candidates of each year" [De Morgan 1832d, 196]. Another issue at stake was the study of Newton's *Principia*, "one of the books which the aspirant for mathematical honours is expected to have studied" [p. 205]. As "it is now universally confessed", held De Morgan, modern methods "are of superior power", noting that there is much in Newton's work, "with which it would be useless for the student to meddle" [ibid]. This last comment was opposed to Whewell's views of 1832.

Indeed, while Whewell had claimed that the first section of the *Principia* was "eminently instructive with reference" to the calculus [Whewell 1832b, vii], De Morgan held that "The Oxford examination comprises only of the three first sections, which is, perhaps, to be regretted, since those sections do not contain by any means the most curious or improving part of the work" [De Morgan 1832d, 205]. De Morgan had produced by that time an elementary *Calculus* [De Morgan 1832a], as preparatory for the student's initiation to mechanics.[118] However, cautious not to undermine his tutor's treatise, he praised [Whewell 1832b] noting that "The most remarkable propositions are introduced, solved according to Newton's method, in their proper places, after the analytical solutions of the same questions" [p. 205].

A third issue deserving attention was the inadequate study of logic at Oxford. De Morgan lamented the fact that "The public examiners have recommended that logic be not absolutely required for the candidates for mathematical honours", adding after Whately: "we cannot see why the theoretical part of a useful science would not be required for those, of whom it must be supposed, that they are better versed in the practical application than their fellows" [De Morgan 1832d, 194]. Still, Oxford's system had at least one merit in comparison with Cambridge's own according to [De Morgan 1833b, 251]:

> Our readers will see that we have throughout advocated the union of the forms of logic with the reasoning of geometry. We are convinced that it would be advantageous to make the former science systematically a part of education. If we except Oxford, there is no place in this country where it is still retained; and unfortunately for the study, it is there more an act of memory about things called moods and figures, than an exercise of reasoning.

[118] In line with his tract on "Statics" [§4.3], De Morgan's textbook, reprinted successfully in USA in 1943, included portions from the calculus of functions, and several examples which illustrated the useful applications of the calculus to mechanics and astronomy [De Morgan l832a, 15, 52–65, 74–77, 96–102].

De Morgan tried to be objective in his views concerning the educational system at Cambridge. As he wrote in his review of Peacock's *Algebra* [De Morgan 1835a, 300]:

> we believe that there is no place of education in the world where the system is carried so far as at Cambridge, and certainly no place where original effort is so much the character of education. A better system of mental training for those, who can bear it cannot be, in our opinion: that a better plan of making the most of the average student might easily be superadded to it, we have no doubt.

The review was published in two parts in January [pp. 91–110] and April [pp. 293–311] respectively. In the first part, he introduced the reader to the peculiarities and utility of algebraic reasoning, commenting upon the instruction of the calculus, mechanics, algebra and logic in connection with the curriculum at Cambridge University. In the second part, he discussed the book under review, for the first time revealing his opinion concerning higher algebra. Here we consider the review exclusively in relation to his defense of both algebra and logic as basic components of a liberal education, and thereby as a potential means towards achieving an educational reform by influencing Cambridge moderators.[119]

Contrary to Whewell, De Morgan viewed students as potential mathematicians, who should acquire a proper method, regardless of its plausible utility in physical applications. For, as he argued alluding to the student: "If he ever wishes to become a mathematician, he must not reject absolutely any proposition because he does not understand it" [p. 95]. Moreover, recommending the perusal of Peacock's difficult work, he wrote [pp. 293-294]:

> If there be a person who cares little for the application of the pure sciences, and much for their methods, he will consider the introduction alluded to as materially increasing their value; if there be another who treats mathematics only as a proper instrument for obtaining and expressing physical truths, he will care but little for it. As a matter of education, we view mathematics almost entirely in the first-mentioned light.

Interestingly enough, De Morgan's discourse was distinctly imbued by his preference of Laplace over Newton in the teaching of mechanics. Indeed, he established links between Laplace's MC, algebra and logic, three subjects at odds with Whewell's educational views. It is of value to follow the reasoning, through which he implicitly attacked Whewell's views on mechanics. Alluding to "the greater part of our elementary treatises" [pp. 97–98], he wrote:

[119]This standpoint is new in this survey, accounting for the tone and scope of Whewell's *Thoughts*, which was published a few months later [§1.2, §6.3]. Historians have so far regarded [De Morgan 1835a] as representative of his new views on algebra and as a critique of Peacock's epistemology; see [Pycior 1983], [Richards 1980] and [Richards 1987]. We wish to add another perspective through which the review might be studied, namely as a motivation towards the composition of his COF in 1836; see [Panteki 1992, ch. 3].

> we consider them as good for the instances they give, and no more; we have never seen independent power obtained by means of them. That which the student afterwards acquires he has to labour afresh; he struggles with an algebraical principle while he is already deep into the Differential Calculus, and gets his first ideas of a common process of numbers out of his treatises on mechanics.

In other words, algebra was seen as an indispensable prerequisite for the study of the calculus, and of Laplace's celestial mechanics. For, it was algebraic principles, not rules, that were needed in order for the student to delve deep into current mechanics. If we take, he wrote, "the whole *Mécanique Céleste* from one end to the other", we notice that while "numerical solution" of equations occurs in few instances only, "many simple principles, not mentioned in our books, are to be applied in almost every page of the work mentioned" [p. 102]. This argument strengthened De Morgan's recommendation of Peacock's *Algebra*, who noted that works such as this, "independently of any other merits", show "what is actually taking place in the world of science, and not new editions of works which have previously appeared on the matters of which they speak" [ibid].

Prior to stressing the educational utility of logic, De Morgan pictured the student confronted with the need to choose between the authority of Laplace and that of his tutor. He solved the dilemma by arguing that "the pupil will do wisely to prefer the former", that is "the greater authority", for: "he may depend upon it, in nine cases out of ten at least, that those who have most reputation* *among the learned* are the best guides to those who have not knowledge to judge for themselves" [p. 94]. The asterisk appended a footnote, where he explained that "We must not be supposed to mean the popular estimate of scientific character. The Newton of the world at large is not the Newton of the philosophers" [ibid].

De Morgan proceeded by arguing that a student will never reason correctly without "the power of rejecting what he thinks wrong" [p. 95], and the basic principles of logic are absolutely essential in helping him to achieve this. To clarify his point, he assumed the case of demonstration: "A is B, B is C, C is D; therefore A is D". If the student is not able to immediately see whether "A is B", he should nevertheless not reject this proposition, but say instead: "If A is B, A is D". Accordingly [p. 95]:

> His reasoning will [...] be perfectly correct: for he must remember that reasoning is not the affirmation or negation of propositions, but the right deduction of them from one another; and that though the *certainty of mathematical conclusions* depends upon that of the fundamental propositions, the *correctness of mathematical reasoning* has nothing whatever to do with that circumstance.

This distinction between the form and matter of mathematical reasoning stemmed from his early inquiries into the instruction of geometry. Moreover, as he had stressed in 1831, reasoning was hardly peculiar to geometry, but present in algebra as well [§5.2]. Realizing the intricacies involved in Laplace's techniques, an author notorious for omitting details of his mathematical procedures, De Morgan mentioned the import of logic straight after his appraisal of MC, so as to reinforce the student's confidence if he ever decided to delve into it. Further, echoing Kirwan [§3.1], and anticipating Hamilton's review of 1836 [§6.3], he claimed:

> The art of reasoning is exercised by mathematics, not taught by it. On the contrary there are principles of other branches of reasoning, which are not employed in most branches of mathematics.[120]

Towards the end of his review, he wrote: "We should very much like to add logic in its most exact form; an easier science than algebra, and which, come by how he may, the student must have in one sense, before he can ever become a mathematician" [pp. 293, 311].

Having been aware of Peacock's treatise since 1832, De Morgan felt induced to furnish excuses for delaying the writing of his review. Firstly, he admitted his neglect of the "higher parts" of algebra, partly "on account of the very great extent and importance of the subject and partly "because it is extremely difficult to draw the line which separates the elements from the higher parts" [De Morgan 1835a, 91]. Finally, he acknowledged "the very great difficulty of forming opinions upon views so new and so extensive" [p. 311]. Indeed, until then he had been occupied only with elementary algebra, providing his students in 1828 with a translation of L. P. M. Bourdon's *Algebra* [Rice 1996, 86–87; Richards 1987, 12–13]. Drawing next on Laplace, he presented the rudiments for the teaching of elementary algebra in his SDM, and made note of Peacock's book in his review of Wood's textbook. There, confronted with Wood's non-rigorous presentation of Newton's binomial formula, he wrote: "The author seems to think he is bound to give either a proof, or something that looks like one. We hold that the less that, which is not a proof, is made to look like one, the better" [De Morgan 1832b, 283]. However, despite the weaknesses of Wood's work, he embraced it, suggesting that the "union" of Wood's and Peacock's textbooks "with some parts of Bourdon's Algebra" would comprise "all that need to be read on this subject by any student" [ibid, 277–278].

Apparently, the publication of Peacock's "Report on the recent progress of analysis" [Peacock 1834] induced De Morgan to re-examine the epistemological framework of his *Algebra* [Peacock 1830]. The "Report" was devoted to an account of the many useful applications of algebraic principles in the field of analysis, including a more detailed elaboration than in *Algebra* on the epistemological foundation of these principles. As it were, Peacock's "Report", cited in De Morgan's review, allowed the latter to overcome his hesitancy in forming opinions "upon views so new and so extensive".[121] It might have also been Peacock's "Report", which

[120]This quote is from his elementary textbook on algebra [De Morgan 1835b, 198], discussed in [Pycior 1982, 399–402] and [Richards 1980, 354–357].

[121]See De Morgan's appeal to Peacock's "Report" in a passage quoted above in text, in connection with the utility of advanced algebra in the study of Laplace's MC [De Morgan 1835a, 102]. In his "Report" [pp. 188-194], Peacock covered the history of algebra from Vieta up to Cauchy; he did not treat the calculi of operations and functions as algebras, thereby omitting

urged Whewell to investigate the nature of algebraic principles in a manuscript book he composed on “The philosophy of the pure sciences” in 1834.[122] Although aware of his colleague’s textbook, Whewell had carefully avoided expressing any opinion for or against it. Given his current interest in the subject, it is possible that he read De Morgan’s review in the spring of 1835, and realized that his revised textbooks on mechanics were no longer a sufficient weapon for defending his educational views. Were moderators to follow up De Morgan’s suggestions, then certainly Whewell’s plans would be under severe threat, and Newton, in particular, would be subordinate to the authority of Laplace.

6.3 Reactions and consequences

> To cultivate logic as an art [...] appears to resemble learning horsemanship by book.
>
> [Whewell 1835, 6]

> It is a great mistake to suppose geometry any substitute for logic.
>
> [Hamilton 1836a, 427]

For Lacroix, the elements of geometry ideally reflected those of logic, and thus “would perhaps be the only ones that it would be necessary to study” [Lacroix 1828, 305]. By adding that “In order to win a race it is better to exercise the legs than to reason on the mechanism of walking” [ibid], he actually held that for educational purposes geometry was superior to logic, since the latter’s rules were of no practical value [§2.2]. According to Kirwan and Whately, however, logic could be of great value in diverse scientific pursuits. Kirwan, in particular, perceived that, though hardly devoid of fallacies itself, logic could serve in detecting the “errors” frequently committed by mathematicians, especially within algebra [Kirwan 1807, iii–v]. Overshadowed by Whately’s systematic presentation of traditional logic, Kirwan’s pioneering account was largely neglected by British mathematicians and logicians, including De Morgan.[123] However, his associations between mathematics and logic largely anticipated De Morgan’s arguments in favor of logic’s educational import.

Lacroix’s *Essais* in conjunction with Whately’s *Elements* formed the starting point for De Morgan’s early educational and logical inquiries. The latter offered him the rudiments of Aristotelian syllogistic logic along with a lucid account on fallacies, while Lacroix’s near identification of geometry with logic induced De Morgan to explore the possibility of rendering geometry in syllogistic form. Notwith-

any reference to Babbage and Herschel. Evidently linked to his indifference for analytics fostered by the latter [§4.1], this omission would not escape De Morgan’s attention see [De Morgan 1840] and [Durand 1990, 140].

[122] On Whewell’s manuscript of 1834, see [Fisch 1994, 270–275] and [Fisch 1999, 166, fn. 3]. Part of that manuscript would be published in Whewell’s *Philosophy* in 1840.

[123] Hamilton formed a unique exception (see §3.2 and [Hamilton 1836a, 440, 452]). Strangely enough, De Morgan never referred to Kirwan in his writings, although his papers “On the syllogism” manifested his deep erudition in the history of logic.

standing the difficulties entailed in this endeavor, De Morgan realized the immense utility of logic within and beyond the boundaries of geometry. For, as he wrote in his article on geometry: "We have said nothing of the other advantages of logic, as they have no relation to the subject of the article" [De Morgan 1833b, 251].

Indeed, two years later, logic would be strongly recommended as a prerequisite for the study of algebra and mathematics at large. For, the reasoning entailed in all mathematical sciences, celestial mechanics included, rested upon a sound knowledge of first principles, which in turn were rooted in deductive logic. Hence he asked moderators to seriously consider the necessity of both algebra and logic forming part of Cambridge's liberal education. A few months later, the same moderators were confronted with the exact opposite request via Whewell's *Thoughts*, that is to consider logic's and algebra's inutility within the context of a liberal education. Whewell' s rejection of logic challenged Hamilton's polemic review of *Thoughts*, and a short-lived controversy broke out between the two men. According to historians, the Whewell–Hamilton debate over the educational and epistemological status of logic was not without consequences for the development of algebraic logic, but none has accounted so far for Whewell's particular arguments against logic in the context of a hastily written tract like the *Thoughts*. What we question here is whether Whewell's tract was meant as a reaction to De Morgan's review of Peacock's *Algebra*. However, before we deal with this question, let us provide the chronicle of the Whewell–Hamilton debate.

A month after the second part of De Morgan's review appeared in April 1835, Whewell announced to Jones his plan of composing *Thoughts*, cautious, however, about disclosing him the true purpose of his tract.[124] Completed by September of the same year, Whewell's 40-page booklet was published in November 1835 and was then sent to his friends, who embraced it with approval.[125] In its opening, Whewell entered into a comparative study between mathematical and logical reasoning, which ended with arguments on the superiority of mathematics over logic [Whewell 1835, 5–8]. According to Hamilton the comparison was groundless and its conclusion was in full opposition to his long-standing views on the import of logic. His response, which was nearly double the length of *Thoughts*, was published in the *Edinburgh Review* in January 1836 [Hamilton 1836a]. In an authoritative style, Hamilton refuted Whewell's views, arguing about all the dangers entailed in any study of mathematics that excluded that of philosophy and logic. For Hamilton, the two disciplines of mathematics and logic offered no possibility for comparison, and hence no scope of mutual relation [ibid, 423]. As

[124]See Whewell's letter to Jones on 26.5.1835 [Whewell Papers, Add.Ms.c.51/183]. Whewell briefly informed his friend that the pamphlet would be based "on the principles of the true philosophy without my telling people more of them than is requisite to be told for the purpose" [ibid]. See also fn. 131.

[125]See Jones' letter to Whewell on 11.11.1835 [Whewell Papers, Add.Ms.c.52/64] and Holland's letter on 29.2.1836 [Whewell Papers, Add. Ms.a.203/31]. Henry Holland (1788–1873) was an eminent physician, glad to have obtained the second edition of Whewell's *Thoughts*. We wish to stress that contrary to his long-standing habit, Whewell had carefully avoided to communicate portions of his tract to his friends prior to its publication.

it were, he expressed a premonition of a potential union effected between the two disciplines, a union visible in De Morgan's SDM, and one to be consolidated firmly in the near future. Whewell replied to Hamilton's accusations by sending a letter to the Editor of the *Edinburgh* Review on 23 January 1836 [Whewell 1836a], which was published together with Hamilton's reply to it [Hamilton 1836b]. The debate culminated with Whewell's second pedagogical tract *On the principles of English University Education* [Whewell 1837c]. Appending both his *Thoughts* and Hamilton's aggressive letter to the end of his *Principles*, Whewell commented upon the latter, pronouncing himself "freed from any obligation to continue the controversy" [Whewell 1837c, 2–3], and the whole issue was not pursued any further.[126]

As is known, the Whewell–Hamilton debate stimulated Boole's algebraization of logic.[127] It reflected the long-standing antithesis between Cambridge and Oxford, engaging "a competition between mathematics and logic around the issue of which could, and should, structure knowledge" [Durand 1966, 446; Durand 2000, 139, 143–152]. However, no commentator has accounted for Whewell's initiative to open his discourse by comparing mathematics to logic.[128] Moreover, no one has noticed that almost a decade before Boole's mathematization of logic, a new *Syllabus of logic* saw the light of publication, a work evidently influenced by the Whewell-Hamilton debate and one to anticipate Boole's pioneering contributions. We are referring to [Solly 1839], a book adorned with a passage from Whewell's *Principles*, devoted ironically enough to the unhappy consequences of a speaker's wrongful use of logic.[129] Solly's book remained neglected until 1847, and came too late to influence the development of algebraic logic [Panteki 1993]. However, notwithstanding the absence of Solly's impact, or the degree of his debt to the Whewell-Hamilton debate, we note that the debate might never have taken place had it not been for Whewell' s endeavor to begin his account by comparing the two disciplines. Let us now seek the reasons that led to the writing of Whewell' s pamphlet. First we will provide a list of De Morgan's educational views and recommendations that might have challenged Whewell's project, and will then raise Whewell's counter-arguments. Drawing on De Morgan's writings up to 1835, [130]

[126] Whewell commented upon Hamilton's letter in the fifth edition of his *Elementary mechanics* [Whewell 1836b, vi–ix], but his full-length reply to the dispute was the *Principles*; see also fn. 134.

[127] Hamilton's review of the *Thoughts* formed one of the motivations for Boole's *Mathematical analysis of logic* in 1847, as argued in [Laita 1979]. According to [Durand 1996, 470–473; Durand 2000, 159–162], Boole's mature *Laws of thought* in 1854 was also plausibly imbued with Whewell's philosophy of science. In general, Whewell's philosophical speculations became widely known after his debate with Hamilton, bearing considerable impact upon his contemporaries, including Charles Peirce; see [Agassi 1991, 355, 365; Becher 1991, 16; Fisch 1994, 253, fn.]

[128] According to [Yeo 1993, 218] Whewell's argument "was not with logic but with recent views on mathematics teaching". But can we differentiate "logic" from "the recent views on mathematical teaching" in 1835?

[129] The passage runs as follows: "A single fault of logic may shew that the speaker has no distinct apprehension of the force of demonstration; and when this judgement is formed of him, he immediately appears to sink below the standard point of cultivation and connexion of thought" [Whewell 1837c, 19].

[130] The background discussion of these topics lies in §4.0, §4.3, §5 and §6.2.

we distinguish the following:

1. His elaborate defense of mathematics against common charges.

2. A genuine concern for elementary mathematics (arithmetic, geometry, algebra, trigonometry and the calculus) and accordingly an empirical approach in the early stage of their instruction.

3. His attention to fallacies and his arguments on the insufficiency of geometry in teaching the principles of sound reasoning to students. Hence his plea for logic as a prerequisite to the study of geometry.

4. The recommendation of Lacroix's *Essais* and Whately's *Elements* to both students and instructors.

5. His critical approach to English liberal education and his comparative study of the systems followed at Oxford and Cambridge.

6. His attribution to algebra of a degree of reasoning equivalent to that traditionally entailed in geometry. Moreover, his serious consideration of algebra as an indispensable prerequisite to the study of the calculus and mechanics.

7. His predilection for Laplace over Newton in celestial mechanics, and his admiration for French institutions, such as the Ecole Normale and the Ecole Polytechnique.

8. Last, but not least, we note his acknowledgment of the authority possessed by Cambridge moderators, and his direct appeal to them in 1835 in an attempt to influence the shaping of the University's curriculum to meet with his educational standpoint.

Whewell's *Thoughts* relates to these points thus:

1′ Whewell held that mathematics was viewed either as "a most admirable mental discipline" or as a habit of thought that makes the mind "captious, disputatious, over subtle, over rigid" [p. 3], considering it rather "obvious" that in his discourse it was to be viewed "as an example and exercise of exact reasoning" [p. 4]. Moreover, at the end he claimed that his purpose had been to speak "of the study of mathematics as a logical and philosophical discipline of the mind" [p. 45]. However, his arguments in defense of this view hardly covered one page of his tract (see point 3′). So when accused by Hamilton for being inconsistent with his statements, he replied that the main scope his book was to contemplate only "*what kind* of mathematics is the most beneficial part of a liberal education" [Whewell 1836a, 271]. Thus, in a work devoid of any reference to the history of mathematical instruction, Whewell accomplished his task in a rather unorthodox way; that is, by dismissing in turn the teaching of mathematics based (a) on arbitrary definitions [pp.

11–31], (b) on experience [pp. 32–35] and (c) on general procedures [pp. 35–39]. As it were, points (a) and (c) constituted the basis of algebra, and analytics at large.

2′ With the exception of geometry [pp. 11–16] and instances from the calculus [pp. 16–18], elementary mathematics had no place in Whewell's discourse. Moreover, Whewell was concerned with issues of instruction only when related to purely physical notions (see point 6′). In the context of issue (b) above, he dismissed the empirical approach in education and hinted at his position on the philosophy of mechanics, which should be regarded in line with geometry, as an axiomatically grounded science [p 33–35],[131] — a strong indication of his overall indifference for the instruction of elementary mathematics. What mattered to him was mechanics, and to this end the only required branches of mathematics were those deployed in Newton's *Principia.*

3′ According to Whewell, a student of mathematics, compelled to fix his attention on the conditions upon which demonstration depends, is presented with "the most natural fallacies, which he sees exposed and corrected" [p. 6]. Moreover, he becomes accustomed to long chains of deduction and the "usual forms of inference", thus learning continuity of attention and coherency of thought. Aware that "all depends upon his first principles", the student "flows inevitably from them", certain of the "necessity and constant identity of the conclusion legitimately deduced from them" [p. 7]. These arguments constituted his sole vindication of mathematical studies against common charges. At the same time, these advantages were juxtaposed with the properties of pure logical reasoning, in order to prove geometry's superiority in teaching accurate reasoning. As he claimed, logic does not familiarize us with "trains of strict reasoning", since it regards special deductions "only as examples of forms of arguments"; and if a fallacy exists, the student is provided with rules "by which it may be condemned and made more glaringly wrong" [p. 7]. Thus mathematics (always conceived as geometry) enables one to form "logical habits better than logic itself" [pp. 7–8].

4′ As Hamilton observed [Hamilton 1836a, 412], Whewell did not refer to any authority in order to sustain his arguments. However, Whewell was well

[131] Accepting more or less Whewell's points (a) and (c), Hamilton raised six distinct objections against Whewell's philosophical approach as regards issue (b) (Hamilton 1836a, 414, 416–417]. He repeated his arguments in his "Notes" [Hamilton 1836b, 274], to which Whewell alluded to in [Whewell 1836b, viii]. Consequently, Whewell exposed his views on the axiomatic nature of mechanics in the "Remarks on mathematical reasoning and on the logic of induction", appended to his [Whewell 1837b, 143–182]. From Jones' reaction to Whewell's "Remarks" (see Jones' letter to Whewell on 13.5.1837 [Whewell Papers, Add.Ms. c.52/69] where he called Whewell's *Euclid* a "pocket-pistol"), it becomes crystal clear that Whewell had feared this reaction since May 1835, and had been cryptic all along about issue (b), and also that Jones had overlooked this issue when going through his friend's pamphlet after its publication.

acquainted with Lacroix's *Essais*, [132] upon which he apparently drew. Note, for example, the similarity between Lacroix's metaphor on running and Whewell's on horse-riding, deployed with the purpose of proving the inutility of logic's rules [p. 6]:

> For reasoning — a practical process — must, I think, be taught by practice better than by precept, in the same manner as fencing or riding [...]. It is desirable, not so much to define good arguments, as to feel their force; not so much to classify fallacies as to shun them; just as the horseman tries to obtain a good seat rather than to describe one, and rather avoids falling than consider in how many ways — he may fall.

In this way, Whewell ridiculed the educational utility of logic, along with refuting the opinion according to which logic was considered necessary for the proper study of geometry. And such an opinion was but peculiar to De Morgan's early writings.

5′ As the title of his tract shows, *Thoughts on the study of mathematics as a part of a liberal education*, Whewell took for granted the traditional system of education at Cambridge. In full accordance to it, he viewed students as future "lawyers, or men of business, or statesmen" [p. 40], contrary to De Morgan's consideration of them as potential mathematicians [§6.2]. In the opening pages of his tract, he appeared eager to enter into a critical overview of the prevailing systems of English liberal education. Indeed, by posing the question "what is the best instrument for educating men in reasoning?" [p. 5], he alluded to Cambridge and Oxford University respectively by claiming [ibid]:

> There are two principal means, which have been used for this purpose in our Universities; the study of Mathematics and the study of Logic. These may be considered respectively as the teaching of reasoning by practice and by rule.

Upon these grounds, he was quick to reject the teaching "of reasoning by rule" [points 3′–4′], and proceeded with still more rejections in connection with the teaching of mathematics [see point 1′ and points 6′–7′ below].

Noting also the Cambridge-Oxford controversy [§3.2], could we infer from points 3′–5′ that what Whewell had meant to do was to downgrade the whole

[132] As mentioned in fn. 23, Whewell had been aware of Lacroix's *Essais* at least since February 1821. It is quite possible that he recommended it to De Morgan, but this is a purely tentative assumption. Interestingly enough, when attacked by Hamilton for lack of historical erudition on mathematical education [Hamilton 1836a, 412], Whewell asked him to propose titles of works on logic, which could serve as "rival instruments of education" to mathematics [Whewell 1836a, 271–272]. Hamilton found Whewell's request "misplaced" and declined to provide him with related bibliography [Hamilton 1836b, 274].

educational system at Oxford University? Neither the rest of the tract, nor his correspondence at the time (including that with Hamilton) exhibits such an interpretation. His rejection of logic as a plausible candidate for entering the curriculum can be accounted for only as a reaction against De Morgan's plea in 1835. In fact, Whewell's ultimate aim was to persuade moderators about the utmost importance of mixed mathematics in a liberal education [points 6′, 8′], and the ways he invented to sustain this view appear less arbitrary and incomprehensible if seen in opposition to the structure and content of De Morgan's review. The message delivered in the latter was closely linked with De Morgan's vision; a sound foundation in the principles of pure mathematics, which would ideally afford the student's initiation into Laplace's most wanted work on celestial mechanics. And to this end, logic was an indispensable tool. However tentative, this standpoint illuminates a major part of Whewell' s *Thoughts*, in which the author had to refute each one of De Morgan's arguments in order to achieve his goal.

Whewell's pamphlet may be viewed additionally as a vehicle of his current philosophical position on the axiomatic nature of mechanics, as hinted at in the context of issue (b), the main novelty of his pamphlet (point 2′). In fact, his attention to logic seems hardly irrelevant to this latter goal if examined under the light of his *Euclid* [Whewell 1837b].[133] Still, it is hard to discern whether *Thoughts* had been intended as a predecessor of *Euclid*, or if it served a posteriori to sustain the philosophical position in the latter work. But we hold that De Morgan's review offered at least the incentive for his sudden plan of May 1835, and in particular the inspiration for the arguments exposed in points 3′–5′.

6′ Contrary to De Morgan, for Whewell algebraic and symbolical methods afforded no scope for the exercise of the faculty of reasoning, since a student trained in viewing problems in an algebraic manner would miss the whole point of comprehending physical notions. Indeed, as he claimed in the context of point (a) against arbitrary definitions, by viewing physical notions in an algebraic way "it will be no wonder if his notions always remain mere algebraical abstractions, without mechanical value or meaning" [p. 26]. Furthermore, algebra was rejected within the context of point (c), as representing a general mathematical method [point 7′]. Lastly, by addressing moderators he wrote [p. 44]:

[133] No matter what might have been Whewell's initial motivation to open his *Thoughts* with a comparative study between mathematics and logic, this study was far from accidental or trivial. Whewell's publications in 1837 prove that his views concerning the comparison of the two disciplines were further consolidated after his short-lived debate with Hamilton. Best proof for our claim is found in the "Remarks" appended to his *Euclid* (fn. 131), where Whewell repeated the same arguments in favour of mathematics in a more sophisticated tone, in order to proceed from an opposition between logic with geometry to his ultimate comparison between geometry and mechanics, which culminated with his novel theory on the "Logic of Induction". On the latter, we refer the reader to [Fisch 1991, 57; Richards 1980; Richards 1988, 20–27; Todhunter 1876 I, chs. 7–10 and Yeo 1993, 160].

> [...] let a knowledge of some portions of Mechanics and Hydrostatics be introduced among the requisites for a degree; and if necessary let the knowledge of Algebra be required no longer, for I can hardly believe that this part of our mathematical teaching is of much value in any point of view,

further arguing that the most "instructive", "simple" and "philosophical" branches of mathematics after plane geometry, were mechanics and hydrostatics [p. 45].

Later evidence can account for his utter rejection of algebra as an impulsive reaction to the potential threat stemming from De Morgan's review. For, happy to note in 1837 that his request for mechanics and hydrostatics had been met with approval by the moderators, Whewell admitted the utility of some parts of algebra in the study of mechanics, going as far as speaking with admiration of Peacock's *Algebra*, his first public reference to that work.[134]

7′ Whewell's opposition to general procedures in teaching in point (c) [pp. 35–39] may be viewed as a polemic against the preference for analytics used by the AS at Cambridge, French mathematics and mechanics at large. Indeed, alluding to Lagrange and Laplace, he wrote [pp. 36–37]:

> The great geniuses of the mathematical world have always delighted in the widest generalizations, because they have by nature possessed this distinctness of particular knowledge, and have thus been able to perform the ascent to generalities and the descent to particulars with a secure rapidity. But these are feats of strength and agility, which it is not given to all to imitate. The talent of generalization is the last, which is developed in the mathematical students.

Whewell also elaborated on his former arguments of 1832, stressing again that the first sections of *Principia* were the best introduction for the student's

[134]In both his *Euclid* and the *Principles*, Whewell acknowledged the fulfillment of his demands concerning the inclusion of more applied mathematics in the lower examination for a degree [Whewell 1837b, vii; Whewell 1837c, 17]. The first chapter of *Euclid* was devoted to instances from Wood's *Algebra*, while in his *Principles* we read: "We ought, therefore, to include in our course, not only pure mathematical sciences, geometry, arithmetic, and algebra [...]" [Whewell 1837c, 16]. Furthermore, in the "Additional thoughts on the study of mathematics" [pp. 177–186], which followed the reprint of the *Thoughts*, he argued for the utility of algebra in the realm of analytic geometry, praising accordingly Peacock's treatise [p. 179]. By incorporating such opposing views on the utility of the algebra (as in the reprint of *Thoughts* and the "Additional thoughts"), Whewell's *Principles* is a much more controversial work than his initial educational pamphlet, and merits further attention, especially as regards Whewell's final reply to Hamilton. On Whewell's ambivalent approach to symbolic algebra, see additional comments in [Fisch 1994, 270–275] and [Richards 1980, 350–353].

acquaintance with the calculus, hiding between the lines his newly conceived "Theory of Fundamental Ideas" (FI).[135]

8′ Whewell's views on mixed mathematics and Newton were not new in this pamphlet [§4.2, §6.1]. But here for the first time he addressed moderators directly. Indeed, in the final pages of the *Thoughts* he wrote: "But whether such a system of mathematical study shall or shall not prevail in this University, must depend entirely upon our Examiners" [p. 43].

His message is summarized thus: "It is highly for the interest of the University that the over disposition to analytical generalizations should not be fostered; that a clear acquaintance with first principles in all subjects should be demanded, and that to each subject its own proper principles should be assigned" [p. 42]. He thus insisted that each subject be treated by particular methods, whereas De Morgan, although aware of the differences between diverse branches, such as algebra and geometry [§5.2], fostered the view of common reasoning underlying all branches of mathematics, emphasizing logic's utility upon these grounds.

7 EPILOGUE

> When the study of mathematics revived at the University of Cambridge, so also did the study of logic. The moving spirit was Whewell [...]. Doubtless De Morgan was influenced in his logical investigations by Whewell. [MacFarlane 1916, 29]

> The correspondence with Dr. Whewell, which had begun soon after the pupil left Cambridge, related at first to Mathematical questions. But when Mr. De Morgan began to make the application of Mathematical principles to Logic, Dr. Whewell was naturally one of the first to whom his ideas were communicated. [S. DeMorgan 1882, 112–113]

Our survey focused on the origins of De Morgan's early involvement with logic, the latter ranging from accurate mathematical reasoning, equally present in geometry, arithmetic and algebra, to Aristotelian syllogistic logic. The roots of his relevant concerns were sought after in French semiotic philosophy, the current English tradition of Aristotelian logic, his Cambridge background in mathematics and mechanics, and lastly in his own critical reviews and writings on the ideal instruction of mathematics and mechanics. Indeed, the study of the latter manifested an extraordinary melange of diverse stimuli, indicative of De Morgan's

[135] In his text Whewell vindicated the Newtonian "conception" of "limit" [Whewell 1835, 12-20], where "conception" stood for what was called later on a "fundamental idea" (FI). On the origins of his theory of FI, see Whewell's letter to Jones on 21 August 1834 [Whewell Papers, Add.Ms.c.51/175]. On Whewell's FI and his philosophy of the calculus (which would be of significant impact upon De Morgan), see [Becher 1980b, 27–30; Fisch 1991, 36, 56–58; Richards 1987, 23–29 and Yeo 1993, 13, 189, 192, 216].

capacity to critically assimilate ideas stemming from a variety of educational systems and institutions, such as the Ecole Normale, the Ecole Polytechnique, Oxford and Cambridge.

For all his critical disposition to its flaws, the educational system of Cambridge University would have an enduring impact upon his foundational concerns. As he confessed to Whewell in 1861, he had been fortunate to have studied there in the "interval between two systems" [§4.0]. During the transitional period of the 1820s, students were challenged by questions such as "fluxions or differentials" and "Newton or Laplace", thus induced to form their own distinct opinions concerning the unsettled foundations of the calculus and mechanics. Among them De Morgan retained for over "thirty years" the conviction that he could adopt "any one of the systems in which infinity is explained" [De Morgan 1865, 146]. But such questions of priority, and epistemological debates at large, would remain in the order of the day in and around Cambridge after his graduation in 1827.

Due to the Cambridge–Oxford debate on political economy in 1831, the long-standing controversy over "deduction or induction" re-emerged, while later on Tripos wranglers and moderators were called to choose between "geometrical or analytical procedures", "pure or applied mathematics" and "mathematics or logic" at large. By viewing the implications of such questions through De Morgan's eyes, we have hopefully shed light not only on the neglected origins of his early logical inquiries, but also on the latter's dynamics. In what follows below, we present a summary of our survey, elucidating its structure, along with raising questions that may contribute to a more comprehensive future study of the joint development of mathematics and logic in England of the 1830s.

Our survey was based upon two comparative studies: one of De Morgan and Lacroix, and another of De Morgan and Whewell, as it was these two men who drew De Morgan's attention to the instruction of mathematics and mechanics respectively. Under their dual influence, De Morgan became ardently involved with the study of the first principles of the mathematical sciences, eventually raising the import of logic as a most indispensable prerequisite for them. The role of his mentors occasionally proved to be subtle and obscure, a fact that accounts for the length of our inquiry. As a matter of fact, our second comparative study turned out to be less explicit and exhaustive than the first, due to lack of substantial evidence.

Geometry and mechanics are the key words underlying our two comparative studies respectively. After having read Lacroix's *Essais*, De Morgan was above all inspired by the critique of the PRL against the traditional teaching of Euclid's *Elements*, and Lacroix's near identification of geometry with formal logic. The latter's impact was mostly evident in De Morgan's SDM, which imitated Lacroix's *Essais* in both its content and structure. Additionally, armed with Whately's *Elements*, De Morgan ventured to cast Pythagoras' theorem into syllogistic form. In so doing, he took the first step towards the extension of syllogistic logic, while along parallel lines he inquired into the wider educational import of logic [§2, §3.1, §5, §6.2].

In connection with Whewell, we pointed out his peculiar oscillation between Newton and Laplace [§4.2], arguing that in conjunction with Airy they motivated De Morgan's project on "Statics" in 1827 and his enduring passion for astronomy and its history [§4.3]. As it were, De Morgan gradually switched his attention from the principles of mechanics to those parts of elementary mathematics with which a student should be well acquainted prior to his study of that branch [§4.3, §5]. Although a Cambridge outsider, De Morgan kept well informed about both the curriculum's expansion as well as Whewell's counter-revolution [§6.1]. He reacted accordingly through his numerous critical reviews, eventually asking moderators to seriously consider higher algebra and logic as basic components of the curriculum [§6.2]. Lastly, we contrasted De Morgan's and Whewell's views on the educational import of logic in 1835, thus hinting at the apparent instant import of De Morgan's early logical investigations [§6.3]. But this last part of our investigation appears to be in want of further clarification.

How far are we entitled to assume that Whewell was acquainted with De Morgan's contributions by 1835? Moreover, was there indeed any ground for a mutual influence between the two men as regards the field of logic? Interestingly enough, it was exactly the latter question, which, posed long ago, led us to discover the close affinity between Lacroix's and De Morgan's work. That question had stemmed from MacFarlane's and Sophie De Morgan's claims, partly quoted in the opening of our epilogue. So, at first we built our inquiries assuming that De Morgan derived "an interest in the renovation of logic" from Whewell [MacFarlane 1916, 20], an assumption further reinforced by S. De Morgan's evidence as regards the communication between former tutor and pupil on logical issues in the 1840s [S. De Morgan 1882, 113]. Relatively soon, though, it became clear that both of De Morgan's biographers failed to distinguish sharply between inductive and deductive logic. This is best manifested in MacFarlane's claim that Whewell was the "moving spirit" in the renovation of logic, through his two treatises on the *History* and *Philosophy* of the inductive sciences in the late 1830s [MacFarlane 1916, 29]. But before we proceed, let us not wholly dismiss MacFarlane's somewhat inaccurate claim, given some of the presently acknowledged consequences that Whewell's debate with Hamilton had upon the development of formal logic [§6.3].

Under the stimulus of De Morgan's biographers, we began searching for any traces concerning Whewell's own logical concerns, digging up his voluminous correspondence and thus coming across his notes on Lacroix's *Essais* and his confessions to Jones in connection with Whately.[136] Moreover, puzzled with a passage from his revised *Dynamics* in 1832, where he spoke of "good logic" being "the one thing requisite" [§6.1], and the hostile tone in which "logic" entered his discourse

[136] In connection with Lacroix and Whately, see respectively fn. 23 and the letters cited in fns. 49, 53 and 54. Aware of the fact that we have treated Whewell's involvement with induction very tangentially [§3.2, §6.1], we direct the reader's attention to Jones' influential attempts to distill into his friend a satisfactory definition of the process of induction, a topic to enter their correspondence from 1822 onwards. See, particularly, Jones' lengthy letter of 7.3.1831 [Whewell Papers, Add.Ms.c.51/26], a letter imbued by his prominent study of Aristotle and Bacon on the issue of induction.

in his *Astronomy* a year later [§3.2], we were induced to consult all his publications until 1837, aiming to understand Whewell' s conception of "logic". As a result, we focused on his *Thoughts*, realizing the import of his poorly studied debate with Hamilton.[137] A great deal of this investigation has been unfortunately considered irrelevant to the main scope of our survey, and is thus here omitted. Nevertheless, it gave rise to our overview on French "Logique" and British "Logic" at the turn of the century [§2, §3], inducing further attention to the diverse interpretations of the term "logic" in England of the 1830s.

In connection with induction, we refer to De Morgan's relevant chapter in his *Formal logic* [De Morgan 1847, 211–226], a chapter that may well have surfaced in imitation of Whately's *Elements* [§3.1]. In its opening we read that "there is not much to say upon the genuine meaning of the word [induction], in any system of formal logic" [De Morgan 1847, 211]. De Morgan's indifference for inductive logic is further revealed in the letters exchanged between him and Whewell from 1846 onwards. These letters, highly indicative of their distinct predilection for deductive and inductive logic respectively, manifest that the main reason De Morgan communicated his current logical inquiries to Whewell was to consult him strictly on issues concerning the nomenclature of logic.[138] Thus, if Whewell was of any impact upon De Morgan's mature work on logic, the degree of his influence appears rather insignificant, and far from pertaining to the period under investigation.

Focusing now on the period from 1823 up to 1831, we have no evidence as regards a plausible way through which Whewell might have instilled in his pupil a passion for logic, in the wider meaning of the term. We recall in particular that Whewell's plan to deliver lectures on induction in 1826 had been postponed for the early 1830s [§3.2, §6.1]. There might have been, however, a subtle way by which Whewell implicitly attracted De Morgan's attention to Whately's third edition of the *Elements* (1829), the edition that triggered the Oxford–Cambridge debate on political economy in 1831 and the one recommended to the reader of SDM [§3.2]. At the time, Whately was on friendly terms with Baden Powell, an active member of the SDUK from 1830, and a contributor to the QJE. Being in close collaboration with De Morgan, Powell possibly acted as a channel between Oxford, Cambridge and De Morgan in the early 1830s, reinforcing in particular the latter's high estimation for Whately.[139]

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[137] Aspects of the Whewell–Hamilton debate have been discussed in [Durand 1996, 450–459; Durand 2000, 144–152; Garland 1980, 39–43; Laita 1979, 47–49; Panteki 2000, 191–192; Richards 1988, 20–27; Todhunter 1876 I, 94–95; Yeo 1993, 218–219]. However, a full account of this epoch-making debate has yet to be written.

[138] See the letters fully quoted in [S. De Morgan 1882, 194–201, 228–229, 302–308, 315–320], the earliest dated 21.10.1846. We have traced two more letters, dated 3 and 5.10.1846, in which De Morgan announced to Whewell his plan of composing a work on "formal logic" [Whewell Papers, Add.Ms.a.202/104–105]. In fact, a third letter of 30.4.1844 [ibid, a.202/100] reveals that their discussion upon philosophical issues dated back at least to 1844.

[139] On De Morgan's early acquaintances with Whately's name, see fn. 90. Further on Powell, see fn. 92 and [Corsi, 1988, 37–39, 44], a work wholly dedicated to Powell's scientific and theological contributions. In his article on Oxford University, De Morgan mentioned Powell's preceding article on the "defects of the scientific education given at Oxford" with appraisal [De Morgan

It thus seems unlikely that Whewell influenced his former pupil in any direct way in connection with logic. However, a further comparative study of their writings entice the view that Whewell was well aware of De Morgan's educational contributions, and affected by them. Both men had strong views as regards the status of geometry; Whewell admiring Euclid beyond any criticism, and De Morgan being distinctly under the influence of the PRL, moreover differentiating his position from Lacroix's as to the alleged sufficiency of geometry in teaching the principles of strict, accurate reasoning. It is more than probable that Whewell read De Morgan's SDM, and was confronted from its first pages with De Morgan's initial empirical approach towards the instruction of mathematics, geometry included. For, in his *Euclid*, he repeated De Morgan's arguments concerning Euclid's axiom on straight lines so as to draw conclusions opposed to those put forward by De Morgan. Interestingly enough, Whewell's presentation of Euclid's axiom on straight lines was to be critically commented upon in De Morgan's advanced treatise on the *Calculus*.[140] We are thus tempted to hold that De Morgan and Whewell "seem to have played a game with each other quoting Euclid passages, and not citing each other (so that nobody previously has noticed)".[141]

Nevertheless, there does exist one evidence as to the diffusion of De Morgan's early contributions, namely Peacock's "Report", delivered in 1833. Peacock noted De Morgan's translation of Bourdon's *Algebra* in 1828 [Peacock 1834, 287] with appraisal, addressing De Morgan [ibid] as:

> a gentleman whose philosophical work on Arithmetic and whose various publications on the elementary and higher parts of mathematics, and particularly those which have reference to mathematics education, entitle his opinion to the greatest consideration.

We regard Peacock's acknowledgement of De Morgan's work as far from trivial, given his total absence of reference to the notable contributions of William Wallace, Babbage and Herschel in analytics.[142] Peacock's "Report" became widely known, in and outside the realms of Cambridge, as was proven in the case of Wallace

1932d, 191]. Whewell and Jones kept expressing to one another their contempt for the "immoral" theory of the Ricardians on political economy much earlier than 1831; see Jones' letter to Whewell on 27.11.1827 [Whewell Papers, Add. Ms.c. 52/15]. De Morgan seemed already aware of the importance of this branch in 1828, referring to political economy in his [De Morgan 1828, 2]. We hold that it is more than plausible that the ensuing Cambridge–Oxford debate was known to him at the time, and he had a stimulating impact upon his philosophical speculations.

[140] Compare [SDM, 2] with [Whewell 1837b, 148–149], and consequently the latter with [De Morgan 1842, 6]. It is quite striking that the "Introductory" chapter of De Morgan's *Calculus* above cited would contain remarks on Euclidian geometry, as well as lengthy comments against Whewell's abuse of the term "logic" [De Morgan 1842, 12–13, fn.], without naming him in either case. See further comments in fn. 98. We wish also to clarify a significant detail: although the *Calculus* was being published in parts from 1836, the above-cited "Introductory" chapter was found to have been dated from 1842.

[141] Interestingly enough, although the preceding arguments presented in our text had not been included in any of the former versions of this paper, our concluding comment in quotes was offered by an anonymous referee.

[142] In connection with Babbage and Herschel, see fn. 121. On Wallace (1768–1843), a Scott

[Panteki 1987]. Moreover, since Peacock appeared so well versed in De Morgan's contributions by 1833, it might not be a bold conjecture to assume that so must have been other Cambridge dons, and in particular Whewell, who by 1832 had begun communicating with De Morgan on mechanics and astronomy.[143]

Focusing on the issue of logic, the Whewell–De Morgan relationship will remain a mystery. While our study so far largely refutes MacFarlane's claim concerning Whewell's alleged influence upon De Morgan's work on logic, instances from their correspondence induce us to believe that there might have been a deeper degree of understanding between them, despite the contrasting directions of their research. For example, De Morgan wrote to Whewell on 12 July 1850 [S. De Morgan 1882, 212–213]:

> I have to-day got Sir W. Hamilton's system for the first time in a full and acknowledged form [...]. I and Boole come in, without being named, for a lecture against meddling with logic by help of mathematics. Pray get this work and read it carefully.

And we ask: how far was Whewell, a fierce opponent of formal logic, able or willing to help his former student deal with Hamilton's attack, especially on an issue upon which Whewell himself would have had the same objections as Hamilton?

Resuming De Morgan's logical contributions, we underline that his account of traditional, syllogistic logic, enriched with the copula "is equal to" and the "à fortiori syllogism", was elaborated upon in the *First notions of logic* in 1839, subsequently constituting the first chapter of his *Formal logic* in 1847. None of his publications after 1831 incorporated his partial reduction of Pythagoras' theorem in syllogistic form, an impossible assignment, as we now know, without first order predicate logic [Merrill 1990, 15]. Realizing the impossibility of this project, De Morgan wrote in his fourth paper "On the syllogism" that geometry "is of little, though some, account for technical exercise in the syllogism" [De Morgan 1860a, 241]. As a matter of fact, he never ceased to associate these two branches, and in his *Speech* to the LMS he again raised the import of logic in geometry, furnishing instances from Euclid's *Elements* as examples of "bad reasoning" [De Morgan 1866, 5], which were in want of a sound knowledge of logic's first principles. It is hardly surprising upon these grounds that his *Notions* was meant, according to its subtitle, as "Preparatory for the student of geometry". What is quite surprising, though, is that it had been additionally destined to form an Appendix to his *Arithmetic*.

Indeed, as De Morgan wrote in his third paper "On the syllogism" [1858, 116, fn. 1]:

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mathematician, see [Panteki 1987], an annotated commentary upon Wallace's letter to Peacock in 1833, in which he lamented the latter's indifference to his contributions in the introduction to analytics in early 19th century Britain.

[143] See De Morgan's letters to Whewell on 12.11.1832 and 13.4.1833 [Whewell Papers, Add.Ms. a.202/96–97].

> Nineteen years ago I wrote my *First Notions of Logic*, intended, as the preface states, ultimately to become an appendix to my *Arithmetic*. I had not then any glimpse, so far as my memory serves, of the numerical syllogism: and I doubt if I could have given any very distinct account of my reason for appending the common syllogism to a book of numbers. But it may be that my now confirmed notion of the usual form of syllogism being arithmetical was germinating.

De Morgan's numerically definite syllogism was conceived in 1846, to be further expanded in his *Formal logic*, thereafter bearing significant import on the shaping of his LOR.[144] As an indicative example of this scheme, we append the following:

$$\begin{aligned}&m\ X\text{s are } Y\text{s}\\&n\ Y\text{s are } Z\text{s}\\&(n+m-s)\ X\text{s are } Z\text{s},\end{aligned}\tag{7.1}$$

where s is the number of Ys given in advance. If De Morgan could not account for the exact origins of this scheme, we are hardly entitled to offer any interpretation other than rephrasing his last sentence: i.e., that it was not only geometry which apparently inspired his early logical inquiries, but that he was also aided by his parallel inquiry into establishing the science of arithmetic upon as firm foundations as geometrical reasoning itself [§5.2]. De Morgan's numerical scheme has been largely overlooked by recent commentators of his mature logical contributions. Overstressed in [MacFarlane 1916, 29] as the most "remarkable" aspect of his *Formal logic*, De Morgan's arithmetical scheme had been the par excellence issue at stake in his notorious debate with Hamilton, offering De Morgan with a conspicuous opportunity to develop his purely formal logical scheme.[145]

De Morgan's *Notions* deserve a final comment before we close our survey. Far from constituting an original piece of work, when compared to SDM and the ensuing *Formal logic*, the *Notions* acquire a distinct place in the history of English formal logic when viewed in conjunction with Solly's *Syllabus of logic*. Both works were composed by former Cambridge students, both addressed students of mathematics, and both appeared early in 1839, not to mention their common anticomformistic religious standpoints, which were radically opposed to those enjoyed by their Cambridge tutors, and Whewell in particular.[146] For all their distinct mathematical illustrations and procedures, the most striking feature of these two

[144] On De Morgan's arithmetical scheme [including (7.1) below in text], see [De Morgan 1846, 17–19; De Morgan 1847, 142–145; De Morgan 1860b, 258–263], a novelty amply dealt with in [Panteki 1992, sect. 6.6]. On a different note, see [Rice 1997b, 2–3].

[145] A study of De Morgan's numerically definite syllogism, and of the relevant implications of the De Morgan–Hamilton debate on the issue of predication, is strikingly absent in [Merrill 1990, 91]. De Morgan's own writings hint clearly at the import of this debate in his own work, stressing that Hamilton's predicated scheme "accidentally" overlapped with his own [De Morgan 1850, 32–35, 42, 49; De Morgan 1860b, 258–263]. De Morgan's evidence is taken under consideration in [Panteki 1992, ch. 6].

[146] Solly was elected a scholar at Gonville and Caius College in 1835, but being a Unitarian he left in 1837, without obtaining a degree, staying around Cambridge until 1839 [Panteki 1993,

works was their timing, as well as their ultimate goal. Published only a few months after the appearance of De Morgan's *Notions*, Solly's *Syllabus* answers to all of De Morgan's aspirations in 1835, along with rendering true all the fears and premonitions of Whewell and Hamilton, his distant mentors.[147] For, drawing on the newly furnished foundation of algebraic symbolical methods by D. F. Gregory, Solly held that the connection between symbolic algebra and logic would help the student acquire a sound foundation of logic's indispensable principles [Solly 1839, iii].[148]

Upon these grounds, we hold that any future comprehensive study of the development of formal logic in England during the first half of the century should incorporate a brief comparative study of Solly's and De Morgan's textbooks of 1839, further exploring their distinct connection with the Whewell-Hamilton debate over the epistemological and educational status of mathematics and logic. Such a survey should also include a reference to De Morgan's neglected work on the COF in 1836, and the latter's debt to the "form–matter" distinction stemming from L. Carnot's *Réflexions sur la métaphysique infinitesimale* (1797), whose author was mentioned with appraisal in De Morgan's review within the context of Peacock's novel jargon: "Except perhaps Carnot, we know of no writer, who has dwelt upon the meaning of his phrases" [De Morgan 1835a, 310-311].[149]

But here our study has reached its limits. We have shown amply that De Morgan played the role of a visionary, a critic and a catalyst during the crucial period of the 1830s within the wider sphere of influence of Cambridge University. In accordance to our final suggestions, we repeat his words while addressing the LMS, holding that indeed "We want a great deal of study of the connection of Logic and Mathematics" [De Morgan 1866, 4].

133]. Best source for De Morgan's upbringing and his religious standpoint, is [S. De Morgan 1882, 1–18]. Solly's work would ideally fit in the context of [Richards 2002], a paper with which we became acquainted while finishing off our epilogue. Her comparative study of Whewell's and De Morgan's different views on mathematics and logic in terms of their distinct theological beliefs [ibid, 144–145, 150] is viewed as complementary to our survey, which has treated such matters tangentially.

[147]Solly's work was published a few months after the appearance of De Morgan's *Notions*, but independently from it, as implied from their first communication in 1847 [Panteki 1993, 134–135]. On the significance of these two works as regards the Whewell–Hamilton debate, see also [Panteki 2000, 190–193]. While Whewell's apparent influence upon the shaping of Solly's mathematical and philosophical speculations (which drew heavily on Kant) is hinted at in [Panteki 1993, 143, 152, fn. 23], we wish to add another valuable source, traced later, in connection with Hamilton, a close friend, as it were, of the Solly family; see [H. Solly 1893 I, 327; II, 65].

[148]On Gregory's pioneering contributions to the realms of symbolic algebra, see [Allaire 2002]. On Gregory's apparent impact upon Solly, see [Panteki 1993, 137–140, 149–151, 163–165; Panteki 2000, 190–193].

[149]Carnot's work and its evident impact upon [De Morgan 1836], and consequently upon the development of the LOR, has been extensively studied and discussed in [Panteki 1992, chs. l, 2, 3, 6, 9]. We suggest here that a further epistemological comparative study of the foundational principles put forward by Carnot, Babbage, Peacock and De Morgan is worthy of investigation, being closely linked with the latter's logical contributions from 1846 onwards.

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The story of this survey started a long time ago. My initial intention was to demonstrate the impact of De Morgan's treatise on the calculus of functions in 1836 upon the shaping of his logic of relations in the 1850s. This proved a lengthy exercise and so I decided to focus on the largely unexplored origins of his logical concerns. After two more versions and thanks to the help of the journals' editors and the anonymous referees I arrived at the present result.

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LEWIS CARROLL'S LOGIC

Amirouche Moktefi

It is well known that Augustus de Morgan's and George Boole's works on logic in the mid-nineteenth century opened the way to the mathematisation of logic. We know much less about the work of de Morgan and Boole's immediate followers, like William Stanley Jevons, Hugh MacColl and John Venn. Historiography is almost silent about the work of the majority of Britain's logicians in the 1880s and 1890s. Given that there was no Boolean tradition properly speaking, the historian is confronted with a multitude of logicians with different approaches and individual conceptions. The study of the main authors of that period is thus necessary for a better understanding of the status of logic during that very confusing intermediary period where the old syllogistic was still taught, the new Boolean logic not yet established, and Russell's mathematical logic soon forthcoming [Grattan-Guinness, 1988; Anellis and Houser, 1991; Peckhaus, 1999].

Lewis Carroll was one of those writers on logic obscured by contradictory influences, balanced between the old logic and the new trends. Unlike John Venn, who had at Cambridge some close colleagues and students (like John N. Keynes, William E. Johnson, and E. C. Constance Jones) who were acquainted with Boole's work, Lewis Carroll worked at Oxford almost alone in that line. His colleagues (like Thomas Fowler and John Cook Wilson) were conservative in their approaches to logic. In his *Symbolic Logic*, he described himself as "an obscure Writer on Logic, towards the end of the Nineteenth Century." [Carroll, 1958a, p. 184]. The object of this chapter is to shed a light on Carroll's work and legacy, and to discuss how he became involved in logic, the main elements of his logical theory, notably his symbolism, his diagrammatic scheme, his approach to the elimination problem and his work on hypotheticals. We will also discuss his relationship with the logicians of the time and assess his main contributions to logic.

Lewis Carroll's "factual" biography is quite easy to summarise. Charles Lutwidge Dodgson (the real name of Lewis Carroll[1]) was born on 27 January, 1832, at Daresbury (Cheshire, England). He took his BA at Christ Church, Oxford, in 1854 and

[1]Charles L. Dodgson used "Lewis Carroll" as a pseudonym for the first time in 1856. The editor of the magazine *The Train*, Edmund Yates, where he published some poems, asked him to choose a pen name. Dodgson suggested first *Dares* (for Daresbury, his native village). However, on Yates' demand, he sent him a new list of suggestions. On 11 February 1856, Dodgson recorded in his diaries: "Wrote to Mr. Yates, sending him a choice of names, 1. *Edgar Cuthwellis* (made by transposition out of "Charles Lutwidge"), 2. *Edgar U. C. Westhill* (ditto), 3. *Louis Carroll* (derived from Lutwidge = Ludovic = Louis, and Charles), 4. *Lewis Carroll* (ditto)." [Wakeling, 1993, p. 39]. On 1 March, 1856, "*Lewis Carroll* was chosen." [Ibidem]. It is well known that Dodgson wanted to save his anonymity all throughout his life and that he always denied in public being the same as Lewis Carroll, the author of the *Alice* tales. This fact led to the development

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was appointed mathematical lecturer the next year. He remained at Christ Church for the rest of his life, and died on 14 January, 1898 at Guildford (Surrey, England). Though his many biographers report almost similarly the major events of his life, there are disputes about his personality, his social life, and the significance of his work.[2] The richness and complexity of his life might be illustrated by his numerous activities and his impressive bibliography.[3] He was a mathematical teacher, the author of children's tales, and photographer. He was also a very prolific letter-writer and puzzle-maker, and regularly wrote pamphlets and letters to periodicals on matters as various as vaccination, teaching sciences at the university, child actors, and vivisection.

Carroll's fame is due mainly to his fictional works, essentially the *Alice* books — *Alice's adventures in wonderland* (1865) and *Through the looking-glass* (1872) — and some other books of fiction, which appeal essentially to children, such as *Phantasmagoria and other poems* (1869), *The hunting of the snark* (1876), *Sylvie and Bruno* (1889) and *Sylvie and Bruno concluded* (1893). As a mathematician, the work he published for the most part under his real name dealt with geometry, algebra, arithmetic, trigonometry, and political theory; but the majority of his books and pamphlets were concerned with Euclidean geometry, on which he wrote several textbooks. His mathematical works included the *Elementary treatise on determinants* (1867), *Euclid and his modern rivals* (1879), *Principles of Parliamentary representation* (1884), *New theory of parallels* (1888), *Pillow problems* (1893), and some others.[4] The logical works appeared later. They include two

during the first half of the twentieth century of the dual personality myth, with on the one side Charles Dodgson the dull teacher of mathematics and on the other Lewis Carroll the fantastic author of child tales [Lebailly, 1996]. Less known is the fact that Charles Dodgson used many other pseudonyms, like B. B., R. W. G., etc. One particularly widespread fable is that Queen Victoria, having enjoyed the Alice tale, asked for the author's other books and thus received Dodgson's mathematical works (there are many variants of this story). Lewis Carroll denied the story in the second edition of *Symbolic logic* [Bartley, 1986, p. 49]. On Carroll's use of his pseudonym, see also [Gattégno, 1976, pp. 229-231; Humberstone, 1995]. In this chapter, we preferred the use of the pseudonym Lewis Carroll for he signed his logical works with that pseudonym.

[2]There are numerous biographies of Lewis Carroll, but they generally deal very little with Carroll's non-fictional works. His nephew published the first of these as early as 1898 [Collingwood, 1967]. The best available one is [Cohen, 1995]. For a discussion of the Carrollian myth and of how Carroll's biographers dealt with it, see Karoline Leach's recent provocative work [Leach, 1999, pp. 15–60].

[3]The standard bibliography of Lewis Carroll's works is [Williams-Madan, 1979]. It should however be used with caution for it contains many errors and omissions [Heath, 1982]. For a bibliography of Carroll's contribution to periodicals, see [Lovett, 1999].

[4]Lewis Carroll's work in mathematics is today well recognized in at least two areas: determinants and political theory. On determinants, he invented a new rule for the evaluation of determinants by condensation [Dodgson, 1867; Abeles, 1986; Rice and Torrence, 2007]. In political theory, he published several pamphlets dealing with the issues of voting, proportional representation, choice theory and elections, etc. This work has been first recognised by Duncan Black [Black ,1958; McLean-McMillan-Monroe, 1996]. Carroll's pamphlets on the subject have been recently republished in one volume as [Abeles 2001]. As geometer, Lewis Carroll is essentially remembered for his defence of Euclid [Dodgson, 1885], his new theory of parallels [Dodgson, 1890] and his numerous textbooks. Lewis Carroll also made some contributions to probabilities

textbooks, *The Game of Logic* (1886) and *Symbolic Logic* (1896), two contributions to the journal of philosophy *Mind*: "A logical paradox" (1894), and "What the Tortoise said to Achilles" (1895), as well as some other minor works, circulars, and notes.

1 ON THE WAY TO LOGIC

1.1 Early interests in logic

Although all Carroll's published works in logic appeared after 1885, his interest in the subject was much older. In a letter dated 29 December, 1891, to his nephew Collingwood, he wrote that his interest in logic was forty-years-old:

> "At present, when you try to give *reasons*, you are in considerable danger of propounding fallacies. Instances occur in this little essay of yours; and I hope it won't offend your *amour propre* very much, if an old uncle, who has studied Logic for forty years, makes a few remarks on it" [Collingwood, 1967, p. 299].

Carroll's diaries confirm his early interest in logic. In the 13 March, 1855 entry, he includes logic in his reading plan: "Second *Logic*, finish Mill and dip into Dugald Stewart" [Wakeling, 1993, p. 74]. Again, on 6 September, 1855, he records: "Wrote part of a treatise on Logic, for the benefit of Margaret and Annie Wilcox" [Wakeling ,1993, p. 129]. This is the first reference to his own work on logic.

Certainly, logic was not yet his main interest, but it was probably never completely absent, as attested to by the numerous logical references in his diaries and letters, and in his other literary and mathematical works.

In fact, more than his logic textbooks, Lewis Carroll's fame among logicians is due mainly to his fictional works, particularly the two *Alice* tales. In 1918, Philip E. B. Jourdain annexed to each chapter of his book *The philosophy of Mr. B*rtr*nd R*ss*ll* an extract from Carroll's literary works, particularly from the *Alice* books [Jourdain, 1991, p. 335-342]. A. J. Ayer, C. D. Broad, P. Geach, G. E. Moore, W. V. Quine, B. Russell, G. Ryle, L. Wittgenstein, and many others also refer to the *Alice* books and its characters (Humpty-Dumpty, the Cheshire Cat, the Mad Hatter, the Tweedle brothers, etc) in their works [Heath, 1974, p. 247-249]. Quickly, commentators presented *Alice* as the work of an "unconscious" logician who is more ingenious than what we will find later in the logic textbooks. The view that without his mathematical and logical avocation, Lewis Carroll would be unable to write the *Alice* books is today largely accepted. It was well explained by Peter Alexander in his "Logic and the humour of Lewis Carroll", where he claims that:

[Seneta, 1993; Dale 1999, pp. 447-464], cryptology [Abeles, 2005b], and recreational mathematics [Gardner, 1960]. Many anthologies of Carrollian puzzles and problems appeared [Fisher, 1973; Wakeling, 1992; 1995a; Gardner, 1996]. Lewis Carroll's mathematical pamphlets were published in one volume as [Abeles, 1994a]. For more information on Carroll as mathematician, see also [Eperson, 1933; Weaver, 1956; Beale, 1973; Seneta, 1984].

> "[I]f Lewis Carroll had not been a logician as well as an artist the "Alice" books would have been much less convincing and aesthetically satisfying than they are [...] Now, in my view, it was Carroll's training in logic which showed him how to construct a setting within which inconsistency would appear inevitable, and so convincing; or more precisely, showed him how to *use* a common fairy-tale setting to contain more than any normal fairy-tale ever contained." [Alexander, 1944, p. 551].

This view is perfectly defensible.[5] However, there is a tendency in some commentaries to overestimate the logical (and more widely mathematical and philosophical) background of the Alice tales. It has even been written that Alice was a treatise of logic, and that Lewis Carroll, by writing it, wanted to provide lessons in correct reasoning to his children readers. There is no evidence for that. It is more convincing to think that there are no such morals in the Alice books. Remember how, in the tale, Carroll makes fun of the lessons British children learn in their schools. One could go further and say that the success of the book is partly due to the non-existence of a moral. In a radio program, commenting the *Alice* book, Bertrand Russell confirms this viewpoint: "... When I was young, it was the only children's book that hadn't got a moral. We all got very tired of the morals in books." [Russell, 1996, p. 522-523].

Carroll's early logical avocation is also indicated in his mathematical writings where he gave a particular importance to the logical structure of the arguments. His teaching at Oxford coincided with a wide debate in British schools and Colleges on the use of Euclid's *Elements* for teaching geometry. Until the mid-nineteenth century, Euclid's book had been the standard textbook for teaching geometry in England. But in the 1860s, a number of mathematics teachers questioned the adequacy of Euclid's *Elements* and called for it to be replaced by other texts [Brock, 1975; Richards, 1988, pp. 161–198; Moktefi, 2007]. In *Euclid and his modern rivals*, Dodgson collected together the main textbooks which were intended to supersede Euclid. He meticulously analysed them. Then he refuted them and loudly claimed the superiority of Euclid's book for teaching geometry [Dodgson, 1885]. For Euclid's defenders, Euclid's *Elements* was a textbook of logic as well as of geometry. Teaching Euclid was an instrument for training the reasoning faculty. W. H. Brock wrote that:

> "[T]he more serious argument in favour of Euclid [...] was formal; Euclid did not teach geometry but orderly thinking. It was an educational masterpiece because it concentrated on manipulating *things* deductively (and not merely symbols of things, as in algebra). Such mind-training would prove useful in later life and in science." [Brock, 1975, p. 26]

[5]There are at least two annotated editions of Lewis Carroll's *Alice* books where the readers may find a good deciphering of the logical structures employed by Carroll in his tales [Heath, 1974; Gardner, 2001].

Carroll's work on geometry led him to discuss the validity of arguments, and in fact, many of the concepts he will introduce later in his logical works already appear in his geometry books.

There are other connections which help us understand Lewis Carroll's interest in logic. In his private writings he often linked logic with religious thought. As early as 2 February 1857, he recorded in his diaries how correct reasoning is important for religious belief [Wakeling 1995b, p. 18]. Much later, he insisted in many of his letters on the importance of logical argumentation in sermons. For example, in a letter to his nephew S. D. Collingwood, he wrote that:

> "The bad logic that occurs in many and many a well-meant sermon, is a real danger to modern Christianity. When detected, it may seriously injure many believers, and fill them with miserable doubts." [Collingwood, 1967, p. 301]

Later, he planned to publish a book on religious matters from a logical viewpoint. In a letter to his publisher, he described it as:

> "... an attempt to treat some of the religious difficulties of the day from a logical point of view, in order to help those, who feel such difficulties, to get their ideas clear, and to see what are the logical results of the various views held. Venn's Hulsean Lectures, which I have just met with, called *Characteristics of Belief*,[6] is very much on those lines, but deals with only *one* such difficulty." [Cohen-Gandolfo, 1987, p. 319]

Unfortunately the book never appeared. However, Carroll's letters to an agnostic give an idea of the kind of questions he would have discussed in this book [Abeles, 1994a, 5-8].

1.2 The Game of logic

In his later years, Carroll focused solely on logic. This revival is shown by the reappearance of entries of logical content in his diaries. On 25 May 1876, he recorded: "Have been writing a good deal today about Logic in algebraical notation (Boole's plan but with an addition which occurred to me the other day, the representation of "some a are b" by "ab not equal to 0,"[7] instead of "$va = b$"): today I further improved it by making it "$ab > 0$," which exactly expresses the logical truth. I have been putting "Barbara" etc. into this notation." [Wakeling, 2001, pp. 463-464]. This is a rare proof that Carroll knew Boole's work before the 1880s. Another reference to Boole occurs on 20 November, 1884 when he wrote:

[6] In his introduction, Venn explains that he treated the question of belief with a logical approach: "The method of treatment here adopted is logical and not metaphysical, and on the field of logic, as a great authority has told us, people of the most opposite schools may meet and shake hands." [Venn, 1970, p. vi].

[7] This is how the entry appears in the recent edition of Carroll's diaries by Edward Wakeling. However, in an early study of Carroll's logic, Wakeling published the same journal entry with the symbol "$\neq$" instead of the expression "not equal to". [Wakeling, 1978, p. 14].

"In these last few days I have been working at a Logical Algebra, and seem to be getting to a simpler notation than Boole's." [Wakeling, 2004, p. 153]. This entry is the first of a long and regular list, which testifies that logic became an important preoccupation for Carroll and occupied a good deal of his time. On 29 March 1885, he made a list of his "literary projects" and included "A symbolical logic, treated by my algebraic method." [Wakeling, 2004, p. 180]. The idea of writing his first logic textbook, entitled *The Game of logic*, occurred to him in 1886, recorded in his 24 July diary's entry:

> "The idea occurred to me this morning of beginning my "logic" publication, not with "book I" of the full work "Logic for Ladies" but with a small pamphlet and a cardboard diagram, to be called *The Game of Logic*. I have during the day written most of the pamphlet." [Wakeling, 2004, p. 285]

The pamphlet became a book, which appeared the same year under the suggestive title *The Game of Logic*. However, since Carroll was not satisfied with the printing quality, he condemned this edition, and a new one appeared the next year [Imholtz, 2003a]. The title of the book and its preface are perfectly clear about Carroll's aims in publishing it. It is presented essentially as a game where, thanks to its use of a board and counters, the players could find it amusing to draw conclusions from a set of premises. But more than a game, Carroll conceived the book to popularise logic and thought it could be a source of instruction too:

> "A second advantage, possessed by this game, is that, besides being an endless source of amusement (the number of arguments, that may be worked by it, being infinite), it twill give the Players a little instruction as well. But is there any great harm in *that*, so long as you get plenty of amusement?" [Carroll, 1958b, unpaged preface]

The book is divided into four chapters: The first explains briefly the laws of the game, the second is a collection of problems, the third gives their answers and the fourth is a list of problems without answers. Each copy of the book is accompanied by an envelope containing a diagram on a card, and nine counters, four red and five grey. In order to make his game accessible to a large public, Carroll took special care when writing it. He signed it with his "literary" pseudonym to guarantee better publicity. The style is very familiar and the problems (where we find a variety of fabulous characters) are often funny.

The Game of logic had a mixed reception from reviewers. An anonymous review published in *The Literary World* is very instructive on the difficulty of understanding the book as it swings between seriousness and fun. Its author compares Lewis Carroll to Dickens' *Dr Blimber!* He asks how such a book (on such a subject) could interest children:

> "We confess to having spent some minutes in trying to make out just how children are to be persuaded to enjoy Mr. Lewis Carroll's new

book, *The Game of Logic*, with its accompanying diagrams and red and grey wafers [...] We seem to see some pale little Dombey junior bending a puzzled brow over the book, and trying to convince himself that it is fun and a game, and not hard work under a thin disguise; but a sturdy boy, not of the little Paul order and not educated by Dr. Blimber, would, we are inclined to think, spurn *The Game of Logic* as a stupid sham, black rabbits, greedy rabbits, pink pigs, and all, and clamor for some play that is really play, or else some study that is really study, on the principle that two things, each good in itself, often make when mixed a third thing which is neither good nor desirable." [Anonymous, 1887]

Thus, despite all Carroll's hard work, it is difficult to say that *The Game of Logic* enjoyed a large success, and it seems that logic fascinated neither children nor adults.

1.3 Symbolic Logic

Ten years after the publication of *The Game of Logic*, Carroll published his second book on the subject, entitled more seriously *Symbolic Logic*. It was planned to publish it in three parts. However, only part one, subtitled *Elementary*, appeared in 1896 and had three other printings within a year. Carroll again introduces his diagrammatic method, but in a more complete and precise way than *The Game of Logic*. In his successive prefaces and introductions, Carroll insists on the importance of logic both as a source of instruction and a mental recreation.

Symbolic Logic. Part 1 contains eight parts called "books". Lewis Carroll introduces first the important logical concepts of things and their attributes (Book I) and propositions (Book II). Then he introduces his biliteral and triliteral diagrams (Books III and IV). The next books deal with syllogisms (*Book* V), the method of subscripts (*Book* VI) and sorites (*Book* VII). *Book* VIII is a collection of examples with answers and solutions. Finally, an appendix, addressed to teachers concludes the book.

The book seems to have been well received by its reviewers. An anonymous review in the *Educational Times* described it as "a *tour de force* of originality, throwing light on its subject from fresh angles."[Anonymous, 1896, p. 316]. However, it obtained only little attention from logicians.[8] Rather more appreciated for

[8]One notable exception was Hugh MacColl who reviewed Carroll's book for *The Athenaeum*. The review was anonymous, however thanks to the *Athenaeum* marked copies owned by City University library (London), one can identify Hugh MacColl as its author. The review was critical but appreciative [Maccoll, 1896]. Later on, in a letter sent to Bertrand Russell, dated 17 May 1905, he reported how his reading of Carroll's *Symbolic logic* encouraged him to return to logical investigations after about 13 years of abstention: "... for... twelve or thirteen years, I devoted my leisure hours to general literature. Then a friend sent me Mr. Dodgson's ('Lewis Carroll's') *Symbolic Logic*, a perusal of which rekindled the old fire which I thought extinct. My articles since then I believe to be far more important from the point of view of general logic than my earliest ones..." [Astroh-Grattan-Guinness-Read, 2001, pp. 93-94]

its humorous examples, its problems were largely reprinted, adapted and imitated in modern logical manuals. Its scientific content, elementary as Lewis Carroll himself stated it to be, drew little attention, despite the fact that it contained many interesting inventions.

Carroll was still working on parts 2 and 3 of his *Symbolic Logic*, intended to be sub-titled "advanced" and "transcendental", when he died on January 1898. In a letter to his sister Louisa, dated 28 September, 1896, he expressed the importance he gave to these sequels, and even abandoned other projects in order to accomplish the logic books first:

> "So I have decided to get Part II [of *Symbolic Logic*] finished *first*: and I am working at it, day and night. I have taken to early rising, and sometimes sit down to my work before 7, and have 1 1/2 hours at it before breakfast. The book will be a great novelty, and will help, I fully believe, to make the study of Logic *far* easier that it now is: and it will, I also believe, be a help to religious thoughts, by giving *clearness* of conception and of expression, which may enable many people to face, and conquer, many religious difficulties for themselves. So I do really regard it as work for *God*." [Cohen-Green, 1979, p. 1100].

Unfortunately, Lewis Carroll died before accomplishing that promising work. Logicians thought that the works were lost. Only few believed that the manuscripts and galley proofs survived. Peter Geach was among them, as is shown by his letter to the *Times literary supplement*, published on 26 December, 1968:

> "At the time of Lewis Carroll's death, his *Symbolic Logic*, Part 2, existed in proof; but apart from a small fragment in the library of Christ Church, Oxford, these proofs have disappeared. It is possible, however, that some complete set of them may exist somewhere; Lewis Carroll used to send round his work in proof for his friends' comments. It would be a great service to scholarship if this work could be found." [Geach, 1968]

It was however only in 1977, that the American philosopher, W. W. Bartley III, published large surviving fragments of the second part of Lewis Carroll's *Symbolic Logic* [Bartley, 1986]. The book, which also reproduces part I, contains the galley proofs discovered by Bartley, and many other manuscripts, notes and letters on logical matters. Bartley's edition is often said to be the best presentation of Carroll's logic ever to be produced. Some authors referred to it as if it was a definitive edition. However, this claim can be challenged. In effect, Bartley's edition is more a collection of surviving manuscripts than a "real" part II.

In an advertisement for part I, Carroll describs briefly the contents of parts II and III. The former would discuss, among other subjects, "propositions of other forms", "triliteral and multiliteral propositions", hypotheticals and dilemmas. Only a few of these subjects appeared to any extent in the volume published by Bartley, which, of course, didn't include anything of the expected contents of part III as announced by Carroll (Analysis of a proposition into its elements, numerical and geometrical problems, the theory of inference, the construction of problems, "and many other *Curiosa Logica*"). Otherwise, among the eight books contained by Bartley's reconstitution of Part II, two books (IX and X) are in fact extracts from the appendix of part I, and three are exclusively collections of problems and their solutions (Books XIII, XIV and XXII). The three remaining books are a collection of puzzles (Book XXI), a book on symbols and logical charts (Book XI written essentially by the editor) and a presentation of the method of trees (Book XII which is probably the most interesting book of part II). Also, there is a gap between chapters XIV and XXI. Finally, the books themselves are probably somewhat different from what Carroll himself would have published. Not only were the galley proofs published here sent to friends to be corrected, but one could note that the books of part II are too long and irregular comparative to the books of part I. When we remember that Carroll planned to publish later the three parts of his *Symbolic Logic* in one volume,[9] one could suppose he intended to give the chapters somewhat equal weight. So, in spite of the high quality of Bartley's editorship, historians of logic must always remember when using the book that it is not exactly part II of *Symbolic Logic*, but rather a collection of Carroll's surviving logical papers, including letters, pamphlets and other manuscripts, some which were intended for part II.

In his introduction (and in the various articles he published prior to the publication of the book), Bartley claimed a higher place for Lewis Carroll among logicians [Bartley, 1972; 1973; 1986]. But his enthusiasm is not shared by everybody. Peter Geach for example, though recognizing the richness of Carroll's problems and puzzles, accused Bartley of having absurdly exaggerated Carroll's merits [Geach, 1978]. Peter Alexander is more severe:

> "It is not the fault of the Editor, who deserves our thanks, that this book is likely to disappoint the Carroll-addicts, among whom I count myself, who have an interest in logic. It reveals Carroll as less inventive, less able to profit from the available literature and less philosophically acute than the "Alice" books lead one to expect." [Alexander, 1978, p. 350]

Some other historians of mathematics were more positive. They expected that this new publication would widen Lewis Carroll's reputation. For example, Ivor Grattan-Guinness concluded his review with optimism:

> "Lewis Carroll subtitled *Symbolic Logic* 'A fascinating mental recreation for the young'. I trust that this edition will help stimulate a long overdue re-appraisal of Carroll as a logician suitable for the atten-

[9]In a letter to his publisher Macmillan, dated 1 February, 1893, Carroll writes: "My idea is to divide the work into 3 Parts, viz. "Elementary," "Advanced," and "Higher," and to publish them *separately*, in paper covers (or perhaps stiff covers, like picture-books), and also the 3 Parts in one volume, in cloth." [Cohen-Gandolfo, 1987, p. 290].

tion of the adults, and not just as a puzzle-setter for juvenile minds." [Grattan-Guinness, 1979, p. 653].

Though Bartley's edition has been globally well received, it is not certain that it has changed Carroll's reputation as a "logician for children", among both logicians and Carrollian scholars. In effect, it is generally assumed that he was an "unconscious" logician, that he considered logic as a game, and that he intended his work for children. It is clear that these generally received ideas harm an objective understanding and a correct appreciation of Lewis Carroll's work as a logician [Moktefi, 2005, pp. 139-140]. Let us now look at the content of the logical works themselves.

2 CLASSES AND PROPOSITIONS

2.1 Things and their attributes

Symbolic Logic opens with a chapter of definitions on things and their attributes. The universe contains things. Things have attributes. Any attribute, or any set of attributes, may be called an "adjunct". Classes are the result of a mental process called classification "in which we imagine that we have put together, in a group, certain Things. Such a group is called a 'Class'." [Carroll,1958a, p. $1^1/_2$]. Note that Carroll constructs his logical concepts by mental processes. There are two important concepts, which permit us to understand fully Carroll's logic of classes: The universe of discourse and the division by dichotomy. Both were widely discussed by nineteenth century British logicians.

The notion of universe of discourse seems to have been first introduced by Augustus de Morgan [DeMorgan, 1966, p. 2] and then to have been used by Boole and his followers [Coumet, 1976, pp. 182-186]. In Carroll's logic, the Universe is the class "Things" obtained when we have put together all things [Carroll, 1958a, p. $1^1/_2$]. The Universe of discourse, however, is the Genus of which the two terms of a proposition are specieses[10] [Carroll, 1958a, p. 12].

Example: In the proposition: "No one takes the *Times*, unless he is well-educated.", the subject is "persons who are not well-educated", the predicate is "persons taking the *Times*" and the Universe of discourse (Univ.) is thus "persons". [Carroll, 1958a, p. 15]

The notion of division was also widely known in nineteenth century British logic [Keynes, 1906, pp. 441-449]. Lewis Carroll defines it as "a Mental Process,

[10]The notions of "Genus" and "Species", together with "peculiar", and "Differentia" are introduced by Carroll in the chapter II on classification: "We may think of the class "Things," and may imagine that we have picked out from it all the things which possess a certain Adjunct *not* possessed by the whole Class. This Adjunct is said to be 'Peculiar' to the Class so formed. In this case, the Class "Things" is called a 'Genus' with regard to the Class so formed: the Class, so formed, is called a 'Species' of the Class "Things": and its peculiar Adjunct is called its 'Differentia'." [Carroll, 1958a, p. $1^1/_2$]

in which we think of a certain Class of Things, and imagine that we have divided it into two or more smaller Classes" [Carroll, 1958a, p. 3]. Division by dichotomy is the particular case when we divide a class into two (and only two) distinct complementary subclasses: x and not-x. Here is Carroll's definition of Dichotomy:

> "If we think of a certain Class, and imagine that we have picked out from it a certain smaller Class, it is evident that the *Remainder* of the large Class does *not* possess the Differentia of that smaller Class. Hence it may be regarded as *another* smaller Class, whose Differentia may be formed, from that of the Class first picked out, by prefixing the word "not"; and we may imagine that we have *divided* the Class first thought of into *two* smaller Classes, whose Differentiae are *contradictory.* This kind of Division is called 'Dichotomy'." [Carroll, 1958a, p. $3^1/_2$]

Division and particularly dichotomy surely have taken an important place in the history of logic, particularly British symbolic logic in the nineteenth century. One can link it to such fundamental issues as Boole's law of duality, Jevons's logical Alphabet and Venn's compartmental view of logic. From this viewpoint, Carroll may really be defined as one of Boole's followers. Unlike many of them, Carroll went even further and claimed a perfect symmetry between the two classes (x and not-x) resulting from dichotomy, and regarded them on the same footing in his logical system. This matter is discussed in the third part of his appendix addressed to teachers, in which he condemned the Logicians' "*morbid* dread of negative Attributes" [Carroll 1958a, 172].[11] Carroll argues that:

> "Under the influence of this unreasoning terror, they plead that, in Dichotomy by Contradiction, the *negative* part is too large to deal with, so that it is better to regard each Thing as either included in, or excluded from, the *positive* part. I see no force in this plea: and the facts often go the other way [...] For the purposes of Symbolic Logic, it is so *much* the most convenient plan to regard the two sub-divisions, produced by Dichotomy, on the *same* footing, and to say, of any Thing, either that it "is" in the one, or that it "is" in the other, that I do not think any Reader of this book is likely to demur to my adopting that course." [Carroll, 1958, 172]

We will see later how Carroll gave an important role to the symmetry between a term and his negation, in his diagrammatic representation of propositions.

2.2 Propositions

Propositions are introduced in book II (that is chapter II) of *Symbolic logic.* When in normal form, a Carrollian proposition, as in traditional logic, consists of four parts: the sign of quantity ("some" or "no" or "all"), the name of subject, the

[11] On Lewis Carroll's treatment of negation, see also [Englebretsen-Gilday 1976].

copula, and the name of predicate. However, Carroll distinguishes only three kinds of propositions according to their sign of quantity [Carroll, 1958a, p. 10]:

1. Particular (proposition in I): "Some x are y".
2. Universal negative (proposition in E): "No x are y".
3. Universal affirmative (proposition in A): "All x are y".

There is no particular treatment for propositions in O in Carroll's logic, given that he didn't distinguish between affirmative and negative particulars. In effect, he considered that the propositions in O "Some A is-not B" could be reduced to the I form "Some A is not-B". [Carroll, 1958a, pp. 171–172]

He also distinguished two kinds of propositions according to what they assert: propositions of existence and propositions of relation. Proposition of existence have the class "existing Things" for its subject and asserts "the *Reality* (i.e. the real *existence*), or else the *Imaginariness*, of its Predicate." [Carroll, 1958a, p. 11]. Its sign of quantity is "some" or "no".

Examples:

"Some honest men exist"

"No men 50 feet high exist"

Propositions of relation "assert that a certain *relationship* exists between its Terms" [Carroll, 1958a, p. 12]. In the first part of *Symbolic Logic*, he discusses only propositions of relation where the terms are specieses of the same Genus (which is the Universe of discourse) and "each of the two Names conveys the idea of some Attribute *not* conveyed by the other" [Carroll, 1958a, p. 12]. Propositions of existence's signs of quantity are "some", "no" and "all".

Examples:

"Some apples are not-ripe fruit".

"No not-brave persons are persons deserving of the fair".

"All men who do not know what 'toothache' means are happy men".

Lewis Carroll defines propositions of relation in A as a double proposition equivalent to the conjunction of two propositions (in I and E). Thus the proposition "All x are y" is equivalent to the two propositions "Some x are y" and "No x is not-y" [Carroll, 1958a, pp. 17-18].

Example:

"All bankers are rich men" is equivalent to the two propositions:

1. "Some bankers are rich men".
2. "No bankers are poor men".

This equivalence leads Carroll to his decision on the existential import of propositions. Since universal affirmative propositions contain "necessarily" particular propositions, Carroll adopts the view that I and A propositions assert the existence of their subject while E doesn't [Carroll, 1958a, p. 19 and pp. 165-171]. Though Carroll's view is different from the modern use, he however carefully defends the view that such a choice is a matter of convenience and that "every writer may adopt his own rule, provided of course that it is consistent with itself and with the accepted facts of Logic." [Carroll, 1958a, p. 166]. It is even possible that Carroll later changed his mind on the question, as is suggested by some late private writings recently rediscovered [Bartley 1986, pp. 34-35; and Abeles 2005a, p. 45].

2.3 Symbolism

Most of Carroll's rules for symbolic representation of propositions appears in the introductory chapter to the sixth book. However, some rules were introduced earlier for classes. Terms are represented by lower case letters (x, y, z, etc.) and their negation can be represented simply by adding a small accent mark. Thus not-x may be represented by x'. Propositions in which the terms are represented by letters are said to be abstract while they are said to be concrete when the terms are represented by words [Carroll, 1958a, p. 59]. In order to translate a concrete proposition (Example: "Some soldiers are brave") into abstract form, we may simply determine the universe (men) and symbolize the terms (x = soldiers and y = brave) to obtain the abstract form "Some x are y".

Carroll provided an easy method to represent propositions by reducing all of them to propositions of existence[12] which are easily represented. The symbolic representation of propositions follows Carroll's method of subscripts. All one has to do is to index a subscript to the symbols of the classes in order to indicate their existence or emptiness. For instance, given the term x, the subscript 1 indicates that there are some x. Thus, x_1 means that "Some existing things have the attribute x" or more briefly that "Some x exist". Such a proposition is called an entity. In the same way, the subscript 0 annexed to x means that there are no x. Thus, x_0 means that "no existing things have the attribute x" or more briefly that "No x exist". Such a proposition is called a nullity. Also, Carroll introduces the dagger sign '†' which means "and". For implication, he uses the reversed paragraph sign '⁋' and writes that it should mean: "would, if true, prove" [Carroll 1958a, p. 70].[13] Finally, he uses the therefore symbol ∴ as a solution indicator for logic problems.

This method allows him to represent the various propositions he has already defined. "Some x are y" is equivalent to "some xy exist" which can easily be

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[12]Some authors observed similarities between Carroll's method and that of Franz C. Brentano [Church, 1960, p. 264]. However, there is no evidence for any influence between the two logicians.

[13]Carroll already made the same use of the reversed paragraph sign in the first edition of *Euclid and his modern rivals* [Dodgson, 1879] but dropped it in the second edition [Dodgson, 1885].

represented as xy_1. In the same way, "No x are y" is equivalent to "No xy exist" and could be represented as xy_0. Finally, in order to represent the proposition "All x are y", Carroll assumes it to be equivalent to the double proposition "some x exist" and "no xy' exist".[14] He thus represented it as $x_1 \dagger xy'_0$, which can also be shortened to "$x_1y'_0$", given that "*each* Subscript takes effect back to the *beginning* of the expression." [Carroll, 1958a, p. 72].

This is the method that Carroll uses to represent classes and propositions in the first part of *Symbolic logic*, where letters were used only to represent terms. Among all the symbolisms that flourished in the second half of the nineteenth century, Carroll's is not the easiest to manipulate. It has however its own advantages, notably, unlike many other early symbolisms, its clear distinction between terms and propositions, at least in this first part of *Symbolic Logic* [Quine, 1977, p. 1018; Geach, 1978, p. 124].

In the surviving writings that Bartley used to reconstruct the second part of *Symbolic Logic*, Carroll seems to have used letters to represent propositions too. The matter is not clear owing to the non-completeness of the surviving material, and to Carroll's use of the same letters also for terms in the same material, for reasons of convenience. We lack a clear understanding of how Carroll jumps from terms to propositions, and the surviving material doesn't actually help us. Propositions are also represented by capital letters in Carroll's two contributions to *Mind*, which we will discuss later. We know that he also introduced other symbols described by Bartley in the first chapter of book XI. For disjunction, Carroll would introduce the sign '§' to symbolize "or" in the nonexclusive sense, while equivalence is represented by the triple-bar (≡). Thus, "$a \equiv b$" should be interpreted as "a ¶ b † b ¶ a" [Bartley, 1986, p. 256].

It is difficult to evaluate the status which Carroll gave to symbolism. Like the majority of the key-issues of the book, he didn't comment upon his choices, not even in the appendix addressed to teachers. The question should have been important for him given that he entitled his book *Symbolic Logic*, which is somewhat too serious for a book intended for a wide public and signed with his pseudonym. Quine thought that some of the uses that Carroll made of symbols suggest that "the algebraic notation is for him less a medium of calculation than a shorthand. The thought is borne out as we read on: he is given to testing implications by descriptive rules rather than by algebraic transformations." [Quine 1977, p. 1018]. One may adhere to Quine's judgment, but one should also keep in mind that Carroll's use of symbols is unevan throughout his textbook. The passages in which he clearly searches for methods for solving problems suggest a more "automatic" treatment. Carroll's symbolism was not adopted by any other logician after him. His diagrammatic representation had more success.

[14]Note that Carroll's view here is slightly different from the earlier definition of propositions in *A* [Carroll, 1958a, pp. 17-18] which, according to him, contains "superfluous information" [Carroll, 1958a, p. 72]. See also [Englebretsen, 1983, pp. 40-41].

3 THE DIAGRAMMATIC REPRESENTATION OF LOGICAL PROPOSITIONS

3.1 Logic diagrams

Logic diagrams constitute the central object of Lewis Carroll's logical writings published during his lifetime. He first published them in *The Game of Logic* (1886), before explaining their use more extensively ten years later in the first part of *Symbolic Logic*. Of course, the use of diagrams in logic is much older [Davenport, 1952; Baron, 1969; Gardner, 1983]. Though it is difficult to locate with precision the introduction of such methods in logic, one can assert with more confidence that it is with the Swiss mathematician Leonhard Euler's *Lettres à une princesse d'Allemagne*, published in 1768, that the use of logic diagrams became more rigorous and more popular [Euler, 2003]. However, It became obvious with Boole's work that Euler's diagrams were hardly adaptable to solve some more complicated problems involving more than three terms. A new diagrammatic scheme was thus invented by Cambridge logician John Venn. He first published them in 1880 [Venn, 1880b], and expounded them in more detail later in his *Symbolic logic* [Venn, 1971]. Venn's major innovation was to represent classes first and then propositions. One begins by drawing a diagram representing all the possible combinations of the n terms involved in the problem (that is 2^n combinations). Then, in order to represent the information contained in the premises of the problem, we introduce graphical devices to express the emptiness or non-emptiness of the compartments. For instance, we shade the compartment xy to represent the proposition "no x is y". However, Venn's scheme suffered from some deficiencies such as its ambiguity in the representation of existential and disjunctive propositions, a matter that would be more satisfactorily treated later by Charles S. Peirce [Peirce, 1933, pp. 307-315].

Carroll's diagrams are Venn-type diagrams, in the sense that he too represents classes first, then propositions with graphical devices. Still, his approach is quite different from that of Venn. He starts by representing the universe of discourse by a square (figure 1). We divide the square horizontally to obtain two subdivisions corresponding respectively to x and not-x (figure 2).[15] If we would like to add a second term y, we have just to divide again our square vertically. We thus obtain four subdivisions corresponding respectively to the sub-classes xy, x not-y, not-xy and not-x not-y (figure 3). Carroll calls this diagram a biliteral diagram.

[15]This diagram (figure 2) is called by Lewis Carroll a mononomial diagram in the proofs of an early table of contents of the book, surviving in the Berol Collection of Lewis Carrol (Fales Library, New York University) as 2B/12/511C.

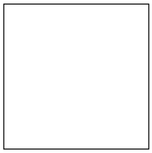
Figure 1

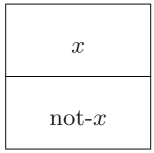

Figure 2

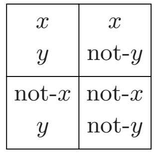

Figure 3

To represent a proposition, all we have to do is to put a grey counter (or the symbol '0') on a cell if the corresponding class is empty, and a red counter (or the symbol 'I') if it is not empty. In order to memorise the functions of the coloured counters, Lewis Carroll composed the following verses, which appear in *The Game of Logic* on the unnumbered page opposite the title page [Carroll, 1958b]

"See, the Sun is overhead,
Shining on us, FULL and
RED!
Now the Sun is gone away,
And the EMPTY sky is
GREY!"

For example, to represent the proposition "No x are y", which asserts that the sub-class xy is empty, we should simply put a '0' on the corresponding cell (figure 4). In the same manner, to represent the existential statement "Some x are y" which asserts that some x y exist, we have just to put the symbol 'I' in the corresponding cell (figure 5).

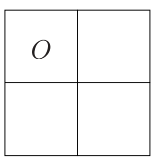

Figure 4

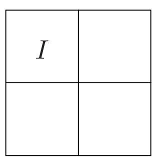

Figure 5

The representation of affirmative universal propositions follows Carroll's theory which asserts that a proposition "All x are y" is equivalent to the conjunction of "Some x are y" and "no x are not-y". Thus, to represent the proposition "All x are y", all one has to do is to represent on the same biliteral diagram the two propositions "Some x are y" (by putting a red counter or 'I' on the cell xy) and "No x are not-y" (by putting a grey counter or '0' on the cell x not-y). We obtain a diagram representing the proposition "All x are y" (figure 6).

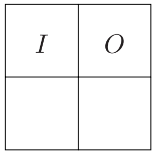

Figure 6

The biliteral diagram allows us to represent propositions involving two terms. For three terms, as one needs to treat syllogisms, Carroll introduced a new diagram by inserting a small square inside the universe, dividing dichotomically each of the subdivisions of the biliteral diagram (figure 7). The cells inside the small square correspond to the affirmation of the third term (let it be m) and the cells outside the small square correspond to the negation of that third term (not-m). One thus obtains the triliteral diagram with eight cells corresponding to the eight combinations of the three terms: $xym, xym', xy'm, xy'm', x'ym, x'ym', x'y'm$ and $x'y'm'$ (figure 8).

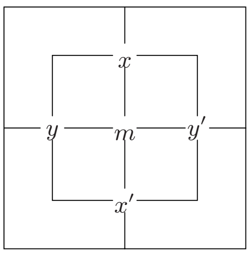

Figure 7

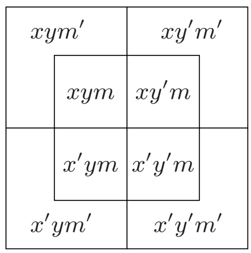

Figure 8

Suppose now that we would like to represent the existential proposition "Some x are m". This proposition is equivalent to the disjunction of the two existential propositions "Some x are ym" and "Some x are $y'm''$. So it asserts that at least one of the compartments xym and $xy'm$ is not empty. To represent this situation, Carroll introduces the following graphical convention: To put a red counter (or the symbol 'I') on the boundary between two cells means that at least one of them is not empty. So to represent the proposition "Some x are m" all we have to do is to put a red counter (or the symbol 'I') on the boundary between the cells corresponding to xym and $xy'm$ (figure 9).

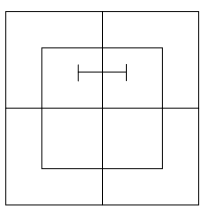

Figure 9

With his inimitable humour, Carroll observed that:

> "Our ingenious American cousins have invented a phrase to describe the condition of a man who has not yet made up his mind *which* of two political parties he will join: such a man is said to be "sitting on the fence." This phrase exactly describes the condition of the Red Counter" [Carroll, 1958a, p. 26]

Many modern authors have claimed for Carroll's diagrams a higher place in the history of logic diagrams, even their superiority over the more widely used Venn diagrams. This involves essentially two major points: the representation of the universe of discourse, and the construction of diagrams for more than four terms. It is interesting to note that on both matters it was Lewis Carroll himself who first claimed the superiority of his diagrams.

3.2 Enclosing the universe of discourse

Carroll's first criticism of Venn's diagram concerns the universe of discourse. In the appendix addressed to teachers, he writes:

> It will be seen that [in Venn's two-terms diagram], of the *four* Classes, whose peculiar Sets of Attributes are $xy, xy', x'y$, and $x'y'$, only *three* are here provided with closed Compartments, while the *fourth* is allowed the rest of the Infinite Plane to range about in!
>
> This arrangement would involve us in very serious trouble, if we ever attempted to represent "No x' are y'." [Carroll, 1958a, p. 175]

The representation of the universe of discourse is often considered as one of Carroll's most important contributions to logic. In fact, he was far from being the first to enclose the Universe. One must not forget that the representation of the universe is an issue that is prior to Venn diagrams themselves and can be quite naturally asked for in Euler diagrams too. It is thus not surprising to find that, even before Venn published his own diagrams, another British logician, Alexander Macfarlane, drew Euler diagrams with a closed figure (square) around to delimit the universe [Macfarlane, 1879, p.63]. Later on, in a paper published in 1885, Macfarlane introduced his own diagrammatic scheme called the *logical spectrum*, in which the universe is again clearly enclosed. Macfarlane's comment is explicit: "Let the universe be represented by a rectangular strip" [Macfarlane, 1885, p. 287]. One may add that Venn himself occasionally represented the universe [Venn, 1880b, p. 17]. Finally, Allan Marquand also represented the universe as early as 1881 and even used the same graphical convention as would Carroll later on: "Conceiving the logical universe as always more or less limited, it may be represented by any closed figure. For convenience we take a square" [Marquand, 1881,

p. 266].[16] It is thus obvious that Lewis Carroll was not the first to represent the universe, though most modern authors refer to him.

In spite of this priority myth, the question of representing the universe is important for understanding Carroll's method of construction. As we have seen, Carroll proceeds by a dichotomic (and symmetric) division of the Universe. This involves not only a logical symmetry between a and not-a, but also a spatial (and visual) symmetry between the figures representing a and not-a. On the contrary, Venn proceeds more by the selection of the members of the class a, leaving the members of not-a outside. Thus, although one can add a square representing the universe *a-posteriori* on a Venn diagram, that doesn't make it a Carroll diagram. An early version of the Carroll diagram confirms that Carroll's approach differs radically from Venn's (figure 10).

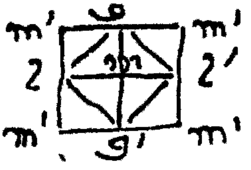

Figure 10

This figure appears in Carroll's diaries entry of 29 November, 1884 with the following comment:

> "Devised a way of working a syllogism as opposite, Universe divided into 8 categories, e.g. the upper right corner triangle is $(gl'm')$ i.e. "g, not l, not m", where "m" is the middle term, "g" is the greater, i.e. the "major" term, and "l" the less, or "minor"."[Wakeling, 2004, p. 155]

This early triliteral diagram illustrates clearly the symmetric division process used by Carroll. Also, it shows that, even for a limited number of terms, Carroll initially didn't try to represent all classes by closed continuous figures, unlike Euler and Venn. It is only later that Carroll changed his diagram. Thus, by their method of construction, Carroll's diagrams differ from Venn's. On this point, his diagrams seem closer to the rectangular diagrams of Marquand [1881] and Macfarlane [1885].[17]

[16] Marquand's notes of Peirce's course, conserved with the Marquand papers in Princeton University Library, include many diagrams with enclosed universes. Such a diagram, dated 1880 is reproduced in [Anellis, 2004, p. 59].

[17] Other rectangular diagrams, albeit without any established relationship with those of Lewis Carroll, have been invented since, such as those of Thomas D. Hawley [1896] and William J. Newlin [1906]. One may add to this family the more recent Karnaugh tables used in computer science for the simplification of propositions and logic circuits [Karnaugh1953].

3.3 Logic diagrams for more than three terms

The second major criticism that Carroll made regarding Venn diagrams, concerns their inability to treat problems involving a higher number of terms (more than five terms) [Hamburger-Pippert, 2000]. We have already seen how, thanks to his triliteral diagram, Carroll represented syllogistic arguments involving three terms. However, in the light of the new Boolean logic, there was no reason to stop at three terms. One might look at the appendix to teachers in the first part of *Symbolic Logic* for an idea of Lewis Carroll's process of constructing logic diagrams for more than three terms. In effect, he didn't represent any diagram for more than three terms in *The Game of Logic*. The appendix was written to give a first idea of what would appear in the second part of *Symbolic Logic*:

> "This last Diagram [the triliteral diagram] is the most complex that I use in the *Elementary* Part of my 'Symbolic Logic.' But I may as well take this opportunity of describing the more complex ones which will appear in Part II". [Carroll, 1958a, p. 176]

As we have seen, Carroll died before ending the preparation of the second and third parts of his work, and unfortunately, there is no mention anywhere of these diagrams in the fragments published by Bartley in 1977. Fortunately, the appendix exposes the general lines of the extension process. For four terms (a, b, c, d), Carroll improves on the triliteral diagram. He transforms the central square into a rectangle and adds another rectangle for the fourth term d in such a way as to obtain sixteen continuous subdivisions corresponding to the sixteen combinations of the four terms (figure 11).

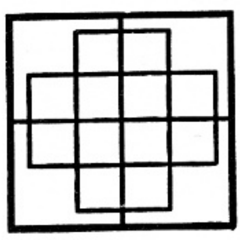

Figure 11

For five terms $(a,\ b,\ c,\ d,\ e)$, in *Symbolic Logic* every compartment of the 4-terms diagram is divided into two parts without attending to the continuity of the resulting classes (figure12). It is clear that this 5-class diagram differs from all the others. In effect, the fifth class e is not represented by a continuous figure. Carroll was aware of the disagreements that such a choice involves. He writes:

> "Here, I admit, we lose the advantage of having the e-Class all *together*, "in a ring-fence", like the other 4 Classes. Still, it is very easy to find;

and the operation, of erasing it, is nearly as easy as that of erasing any other Class." [Carroll, 1958a, p. 177]

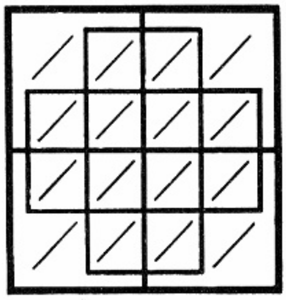

Figure 12

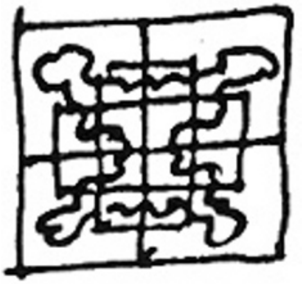

Figure 13

The interesting point here is that Carroll knew how to represent a continuous fifth class, but he preferred ignoring it. In effect, such a diagram (figure 13) appears in his diaries entry of Tuesday 27 November 1888:

> "Devised a Logic Board, with which *five* attributes can be worked. (I had made the adjoining one for four attributes, last Saturday). The new one has an additional zigzag line working in and out so as to divide each one of the existing 16 compartments". [Wakeling, 2004, p. 434]

Clearly, Carroll prefers to use a regular though non-continuous figure rather than a continuous but non-regular figure. That may be explained by the fact that it is easier to locate the compartments when the figures are regular; this is an important feature when it comes to manipulating the diagram. There is another important historical lesson that one should take from that 27 November, 1888 entry. It contains in effect the first reference in Carroll's writings to logic diagrams for more than three terms. We have seen that he invented his diagrams about four years earlier and that he published in the meantime his *Game of Logic* in which one finds no diagram for more than three terms. This late interest in the problem of extension suggests again that Carroll may have ignored Venn's work when he was working on his own diagrams. The question of drawing diagrams for more than three terms is essential for Venn and appears in all his early writings on the subject. This question was also important for Marquand and Macfarlane, who both justified the invention of their diagrams by the necessity of having diagrams which could deal with more terms than Venn's did.[18] So Carroll's motivation

[18]Allan Marquand began his *Philosophical Magazine* paper by explaining that he wanted to construct logic diagrams that are more extendible than Venn's. After listing briefly some limitations of Venn's scheme, he writes that: "It is the object of this paper to suggest a mode of constructing logical diagrams, by which they may be indefinitely extended to any number of terms, without losing so rapidly their special function, viz. that of affording visual aid in the solution of problems" [Marquand, 1881, p. 266]. Macfarlane proceeds in the same way. After explaining that it is impossible to draw a four-term diagram with four circles, he introduces his

for inventing his diagrams seems completely different from that of his post-Venn contemporaries.

There is still a third version of the 5-terms diagram, which is known through a set of leaflets that Carroll printed presumably in 1896 [Carroll, 1896], one of which represents "quinqueliteral diagrams" as in (figure 14).

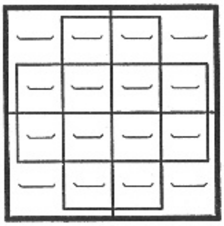

Figure 14

Although it again privileges the regularity of the diagrams and sacrifices their continuity, this version, which Carroll didn't use in his textbook, has the advantage of constituting a better transition to the forthcoming diagrams (for more than five terms). One can use them more easily to draw immediately a 6-terms diagram (figure 15), whereas one has to draw a new diagram to jump from the other version of the 5-terms diagram (figure 12) to the 6-terms diagram (figure 15).

For six terms, Lewis Carroll simply inserts a biliteral diagram in each compartment of a quadriliteral diagram. For instance, for six terms (a, b, c, d, e, h), he inserts a biliteral diagram (corresponding to the terms e and h) in each compartment of a quadriliteral diagram (corresponding to the terms a, b, c, d) in order to obtain the 64 combinations of six terms (figure 15). The classes e and h are represented by discontinuous figures. The same method is used in order to represent respectively 7-term and 8-term diagrams by inserting respectively a triliteral and a quadriliteral diagram in each cell of a quadriliteral diagram (respectively figures 16 and 17).

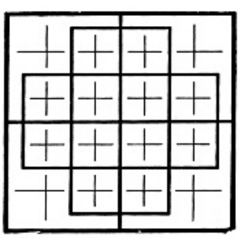

Figure 15

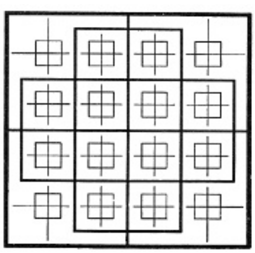

Figure 16

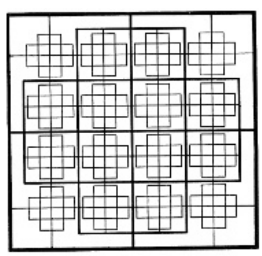

Figure 17

method which, by contrast, could work with any number of terms. He writes: "Another method, which I propose to call the *logical spectrum*, is capable of representing quite generally the universe subdivided by any number of marks" [Marfarlane, 1885, p. 287].

The 8-term diagram is the most complex that one can find in Lewis Carroll's works. However, for more than 8 terms, Carroll gives some hints on how to draw a corresponding diagram. For nine terms $(a, b, c, d, e, h, k, l, m)$, he suggests the use of two eight-terms diagrams (for the terms a, b, c, d, e, h, k, l): one for the m part and the other for the not-m part. That is like putting an 8-term diagram in each compartment of a one-term diagram. For 10 terms $(a, b, c, d, e, h, k, l, m, n)$, Lewis Carroll suggests simply to put an 8-term diagram in each compartment of a biliteral diagram, the four cells of which correspond to the four combinations offered by the ninth and tenth terms (m and n). Though he didn't go further, it is easy to imagine a general method for constructing a Carroll diagram for any number of terms. An example of such a generalisation has been proposed by Anthony J. Macula. He furnishes an algorithm for constructing a Carroll diagram (he called a "Lew k-gram") for any number k of terms:

> "By iterating, one can construct a Lew k-gram for any k: If k is a multiple of four, then one constructs the Lew-gram on $k+1, k+2, k+3$, and $k+4$ many sets by placing a Lew k-gram in the skewed boxes of the Lew 1-gram, Lew 2-gram, Lew 3-gram, and Lew 4-gram, respectively". [Macula, 1995, p. 271]

Although it is not sure that Carroll would have used this particular algorithm, it is easy to verify that Macula's method works not only for Carroll's diagrams but also for Venn's. Macula defends the superiority of Carroll diagrams on this point:

> "Have you ever tried to draw a Venn diagram, depicting all possible intersections, using four, five, or six sets? It can be quite cumbersome [...] Lewis Carroll provides an easy method for drawing set diagrams depicting all possible intersections. Using his method, one can actually draw these diagrams for ten or more sets. The only limitations are the patience of the drafter and the size of the paper on which the diagram is drawn" [Macula, 1995, p. 269].

Carroll himself suggested the superiority of his extension process in comparison with Venn, observing that Venn couldn't go beyond six terms while he himself has diagrams for up to ten terms. In fact, a close look at the diagrams shows that Carroll and the majority of historians of logic who shared this view, are hardly fair to Venn. First, like Venn, Carroll didn't use continuous figures for more than four terms, and his method of extension is actually not very different from that of Venn. Second, although he drew diagrams for up to eight terms, he never represented any proposition on such diagrams and didn't explain how to do it. One should remember that the discontinuity of the figures would necessitate the treatment of existential propositions as disjunctives, an option that Carroll doesn't discuss. Finally, we should keep in mind that nowhere in his published works did Carroll solve a problem involving more than three terms with his diagrammatic method,[19]

[19]There are few manuscript notes where Carroll used diagrams for more than three terms in order to solve logic problems. For instance, such a diagram is used in a set of logic notes in the

whereas Venn does much better. The main advantage of Carroll's diagrams is visual. They remain symmetric and conserve an apparent unity, thanks to the representation of the universe and the symmetry of its dichotomy division. That makes Carroll diagrams for more than three terms easier to draw than Venn's, but not necessarily better and easier to use.

3.4 The historical place of Lewis Carroll's logic diagrams

In our review of Carroll diagrams, we have seen two indications (based on two entries from his journal) of Carroll's ignorance of Venn diagrams (and those of Marquand) when he was inventing his own. First, he proceeds by division rather than by selection, and doesn't initially take into account the continuity of all the classes. Second, he didn't initially envisage the construction of logic diagrams for more than three terms, an issue which is essential in Venn's and Marquand's writings. It is difficult to give a definitive answer to the question. Of course, Venn's priority is established, but there is no evidence concerning a possible influence. Historians of logic generally assume without discussion that Venn was Carroll's point of departure. We have already seen that Carroll's interest in logic was much older and that he knew the work of Boole earlier. There are of course many similarities in their respective works, both entitled *Symbolic Logic*, though Venn's was published fifteen years before Carroll's. The main point was of course that they both invented logic diagrams and gave them a high place in their books. There is however no further evidence that Carroll's diagrams were "improvements" of Venn's, as it is generally assumed. There is no mention of Venn diagrams in Carroll's works before 1896, when he compared his own diagrams with those of Venn.[20] In the *Game of Logic* (1886) where he first published his diagrams, Carroll makes no mention of Venn.[21] So, there is no evidence, either historical or

Houghton Collection, Morgan Library, New York (AAH545). Another such a diagram occurs in Lewis Carroll's notebook, conserved with Carroll's mathematical manuscripts in the Parrish Collection, Princeton University library (Box 1, Folder 1).

[20]Carroll himself listed some resemblances and differences between his diagrams and those of Venn but didn't present his own as a modification of those Venn : "My method of diagrams *resembles* Mr. Venn's in having separate compartments assigned to the various Classes, and in marking these compartments as *occupied* or as *empty*; but it *differs* from his Method, in assigning a *closed* area to the *Universe of Discourse*, so that the Class which, under Mr. Venn's liberal sway, has been ranging at will through Infinite Space, is suddenly dismayed to find itself "cabin'd, cribb'd, confined", in a limited Cell like any other Class! Also I use *rectilinear*, instead of *curvilinear*, Figures; and I mark an *occupied* Cell with a 'I' (meaning that there is at least one Thing in it), and an *empty* Cell with a '0' (meaning that there is no Thing in it)" [Carroll, 1958a, p. 176].

[21]Surprisingly, Venn didn't mention Carroll diagrams anywhere in his published works. He didn't mention them in his second (revised) edition of *Symbolic Logic*, which appeared in 1894, while he referred to Marquand diagrams published after the first edition too. There is no copy of Carrol's *Game of Logic* in Venn's rich collection of books on logic as it is described in its 1889 catalogue when he presented it to Cambridge University Library [Venn, 19889]. We know, however, that Venn knew about Carroll's diagrams immediately after their publication for the first time in *The Game of Logic*. In effect, Venn wrote in 1887 a letter to *Nature* [Venn, 1887] where he replied to some remarks, regarding the representation of existential statements, made

conceptual, to say with full confidence that Carroll was aware of Venn diagrams when he invented his own.

Carroll diagrams have some undeniable advantages over Venn diagrams. The universe is enclosed and the classes *a* and *not-a* are symmetric. His method for representing existential statements is more clearly explained and more intuitive for some disjunctive statements (when it suffices to put the counter on the border between the cells). For problems involving more than four terms, his diagrams are more regular and would have been more interesting had he invented an effective method for representing disjunctive propositions. Carroll's use of board and counters also has pedagogical advantages, given that one need not draw the diagrams for each problem and that the counters can be simply "slipped on".

These interesting features didn't make Carroll's diagrams popular however. Though known and cited with respect by logicians and historians of logic all through the twentieth century, they were seldom used. One notable exception is Peter Geach's *Reason and Argument* [Geach1976]. The absence of visual logic itself in the logical works at the beginning of the twentieth century may explain the little interest accorded to Carroll's diagrams as well as other diagrammatic schemes, including Venn diagrams, which were not so widely used by his successors as one might suspect today. In recent years, there is a growing interest in logic diagrams, both in philosophy [Shin1994] and mathematics [Edwards2004], and it may be hoped that this new work will shed new light on the place of diagrammatic reasoning and its place in the history of logic.

4 THE ELIMINATION PROBLEM

4.1 Syllogisms

Carroll defines a syllogism as a trio of biliteral propositions of relation, where:

> "1) all their six Terms are Species of the same Genus.
>
> 2) every two of them contain between them a pair of codivisional Classes,
>
> 3) the three Propositions are so related that, if the first two were true, the third would be true." [Carroll, 1958a, p. 56].

The eliminated term is called "Eliminand" while the two others are "Retinends". To represent symbolically syllogisms, Carroll represents each of its three propositions in an abstract form then writes them all in a row, with the symbol "†" between the two premises, and "⁋" before the conclusion.

by Alfred Sidgwick in his review of Carroll's *Game of Logic* in the same journal [Sidgwick, 1887].

Example: The syllogism

No x are m'

All m are y

$\therefore$ No x are y'

is represented as: $xm'_0 \dagger m_1y'_0 ⁋ xy'_0$.

The first method used by Lewis Carroll to solve syllogisms is the diagrammatic method. Though it is possible to use logic diagrams to solve elimination problems involving more than three terms, Lewis Carroll didn't introduce any such example in his published works, even if he exposed possible diagrams for the representation of more than three terms, as we have seen in the preceding section. Syllogisms are thus the most complex problems that Carroll treated with his method of diagrams.

For that purpose, one has to reduce first the premises of the syllogism into abstract form (let x and y be the retinends, and m the eliminand). The premises should be represented together on the same triliteral diagram. Then one has to ascertain what proposition is represented in terms of x and y [Carroll, 1958a, p. 60], either directly on the triliteral diagram or more suitably by transferring the information on a biliteral diagram. To transfer the information, it is necessary to observe two rules for each of the four quarters of the triliteral diagram (containing each two cells), corresponding respectively to the four compartments of the biliteral diagram. The first rule is that if the triliteral diagram's quarter contains an "I" in either of his two cells, then it is occupied, and thus the corresponding compartment of the biliteral diagram is marked with a "I". The second rule asserts that if the triliteral diagram's quarter contains a "0" in each of its two cells, then it is certainly empty, and thus the corresponding compartment of the biliteral diagram is marked with a "0" [Carroll, 1958a, pp. 53-54].

Example: Take the following problem that Carroll submitted to Venn in order to compare their respective diagrammatic methods [Carroll 1958a, pp. 179-183].

1. No philosophers are conceited
2. Some conceited persons are not gamblers

To translate the syllogism into an abstract form, we take the universe as "persons", x = philosophers, m = conceited, and finally y = gamblers.

The two premises became:

1. No x are m
2. Some m are y'

Let us now represent the two propositions on a triliteral diagram. The first proposition asserts that "No xm exist", so we put a '0' on the xm compartments to assert that they are empty. The second proposition asserts that some my' exist, and thus that the my' compartments are occupied. Given that the xmy' is already empty, we put a 'I' on the only available compartment, that is $x'my'$. This gives the following triliteral diagram (figure18):

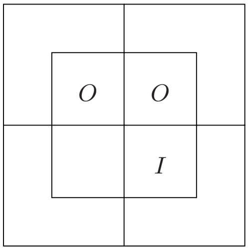

Figure 18

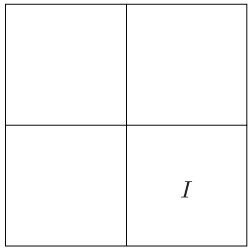

Figure 19

Examining the information given in the diagram as to x and y, either directly from the triliteral diagram or more suitably by transferring the information into a biliteral diagram (Figure 19). We observe that the $x'y'$ compartment is occupied. Hence, the conclusion is "Some $x'y'$ exist", i.e. "Some x' are y'", which is translated in a concrete form to obtain the final conclusion: "Some persons, who are not philosophers, are not gamblers" [Carroll, 1958a, p. 183].

The second method used by Carroll to solve syllogisms is based on the distinction of three figures for all syllogisms; it suffices to determine the figure to which a given syllogism corresponds. The following table constructed by Carroll [Carroll, 1958a, p. 78] presents three formulae (corresponding to three different forms of pairs of premises) to which Carroll reduces syllogisms.

Figure I	$xm_0 \dagger ym'_0$ ⁋ xy_0	"Two Nullities, with Unlike[22] Eliminands, yield a Nullity, in which both Retinends keep their Signs. A Retinend, asserted in the Premisses to exist, may be so asserted in the Conclusion."
Figure II	$xm_0 \dagger ym_1$ ⁋ $x'y_1$	"A Nullity and an Entity, with Like Eliminands, yield an Entity, in which the Nullity-Retinend changes its Sign."
Figure III	$xm_0 \dagger ym_0 \dagger m_1$ ⁋ $x'y'_1$	"Two Nullities, with Like Eliminands asserted to exist, yield an Entity, in which both Retinends change their Signs."

Example: Let the following pair of propositions be the premises of a syllogism [Carroll, 1958a, p. 79].

> "All cats understand French"
>
> "Some chickens are cats"

Let the Universe be "creatures", m = cats, x = understanding French, and y = chickens.

[22]Two letters are said to be "like" (respectively "unlike") when they have the same signs (respectively opposite signs) [Carroll, 1958a, p. 70].

The pair of premises is represented in abstract form by: "$m_1x'_0 \dagger ym_1$". We have an Entity and a Nullity, with like eliminands (m). According to the second figure, it yields an Entity in which the Nullity-Retinend (x') changes it sign (and becomes x). Thus, the conclusion is "xy_1", which gives in concrete form the final conclusion:

> "Some chickens understand French".

In the appendix addressed to teachers, Carroll insists on the advantage of his system. Instead of the traditional nineteen forms of syllogisms, "each with its own special and exasperating Rules", Carroll proposes only three forms, "each with a very simple Rule of its own", to which one can reduce a given problem [Carroll, 1958a, p. 183].

4.2 Sorites

Sorites are inferences involving more than two premises. Lewis Carroll defines the problems proposed by Sorites as follows:

> "Given three or more Propositions of Relation, which are proposed as Premisses: to ascertain what Conclusion, if any, is consequent from them." [Carroll, 1958a, p. 87].

Carroll's definition follows the general object of the elimination problem which British nineteenth century symbolic logicians considered as a fundamental problem of logic [Bartley, 1986, pp. 22-23]. John N. Keynes, for instance, wrote:

> "The great majority of direct problems involving complex propositions may be brought under the general form, *Given any number of universal propositions involving any number of terms, to determine what is all the information that they jointly afford with regard to any given term or combination of terms.* If the student turns to Boole, Jevons, or Venn, he will find that this problem is treated by them as the central problem of symbolic logic." [Keynes, 1906, p. 506]

In order to find what conclusion is consequent from the premises of a Sorite, Lewis Carroll proposes in the first part of *Symbolic Logic* two methods. The first one is the method of separate syllogisms. The rule is to select two premises which together can be used as the premises of a syllogism and to find their conclusion. Then, one has to find a third premise which together with that conclusion can be used as premises of a second syllogism. And so on, until all premises have been used, the last conclusion will be the conclusion of the sorite.

Example: Let us take the following set of premises [Carroll 1958a, p. 88]:

Concrete form	Abstract form	Subscript form
1) All the policemen on this beat sup with our cook;	1) All k are l	$K_1 1'_0$
2) No man with long hair can fail to be a poet;	2)No d are h'	dh_0
3) Amos Judd has never been in prison;	3) All a are c'	3) $a_1 c_0$
4) Our cook's 'cousins' all love cold mutton;	4) All b are e	4) $b_1 e'_0$
5) None but policemen on this beat are poets;	5) No k' are h	5) $k'h_0$
6) None but her 'cousins' ever sup with our cook;	6) No b' are 1	6) $b'1_0$
7) Men with short hair have all been in prison.	7) All d' are c	7) $d'_1 c'_0$

Dictionary: The universe is "men"; a = Amos Judd; b = cousins of our cook; c = having been in prison; d = long-haired; e = loving cold mutton; h = poets; k = policemen on this beat; l = supping with our cook.

This sorite can be solved by the following process:

8) $K_1 l'_0 \dagger k'h_0$ ⁋ $l'h_0$ (premises 1 and 5)

9) $l'h_0 \dagger dh'_0$ ⁋ $l'd_0$ (the conclusion of 8 and premise 2)

10) $l'd_0 \dagger b'l_0$ ⁋ db'_0 (the conclusion of 9 and premise 6)

11) $db'_0 \dagger b_1 e'_0$ ⁋ de'_0 (the conclusion of 10 and premise 4)

12) $de'_0 \dagger d'_1 c'_0$ ⁋ $e'c'_0$ (the conclusion of 11 and premise 7)

13) $e'c'_0 \dagger a_1 c_0$ ⁋ $a_1 e'_0$ (the conclusion of 12 and premise 3)

The conclusion is $a_1 e'_0$, which should then be translated into abstract form ("All a are e") and finally into concrete form to give the final solution: "Amos Judd loves cold mutton".

Lewis Carroll's second method for solving sorites is the method of underscoring. It is in fact a kind of variation on the separate syllogisms method, where one has simply to mark the eliminated letters (letters of unlike signs) by underscoring them, with a single score under the first letter and a double score under the second. For instance, if we have the pair of premises: $xm_0 \dagger ym'_0$, after underscoring we obtain: $x\underline{m}_0 \dagger y\underline{\underline{m'}}_0$. The underscored terms can thus be dropped [Englebretsen, 1989, p. 30]. Before underscoring, Carroll recommends omitting the subscripts:

> "In copying out the Premisses for underscoring, it will be convenient to *omit all subscripts*. As to the "0s" we may always *suppose* them written, and, as to the "1s", we are not concerned to know *which* Terms are asserted to *exist*, except those which appear in the *Complete* Conclusion; and for *them* it will be easy enough to refer to the original list." [Carroll, 1958a, p. 91]

Example: Let us solve the sorite discussed above. We place its seven premises in a row as follows:

$$\begin{array}{ccccccccccccc} 1 & & 2 & & 3 & & 4 & & 5 & & 6 & & 7 \\ k_1 l'_0 & \dagger & dh'_0 & \dagger & a_1 c_0 & \dagger & b_1 e'_0 & \dagger & k' h_0 & \dagger & b' l_0 & \dagger & d'_1 c'_0 \end{array}$$

Then, in order to proceed in the manner of separate syllogisms, we place the propositions in the requisite order to facilitate the elimination process. This might be, for example, the following order:

$$\begin{array}{ccccccccccccc} 1 & & 5 & & 2 & & 6 & & 4 & & 7 & & 3 \\ \underline{kl'} & \dagger & \underline{\underline{\underline{k'}}}\underline{h} & \dagger & \underline{d}\underline{\underline{\underline{h'}}} & \dagger & \underline{b'}\underline{\underline{l}} & \dagger & \underline{\underline{b}}e' & \dagger & \underline{\underline{\underline{d'}}}\underline{c'} & \dagger & a\underline{\underline{c}} \end{array}$$

Then by successive elimination, we obtain the final conclusion as $e'a_0$. Given that a is given as existing in the third premises, we add a_1. The final solution is "$a_1 e'_0$", which gives in concrete form "All a are e", is the same conclusion obtained earlier with the separating syllogisms method.

4.3 The method of trees

In the second part of *Symbolic Logic*, Carroll introduces other methods for solving logical problems. The method of barred premises, for instance, is an extension of the underscoring method for solving problems involving multiliteral propositions [Abeles, 2005a, pp. 36-38].[23] More interesting is the method of Trees. Lewis Carroll invented this method on 16 July, 1894.[24] That day he recorded in his diary:

> "Today has proved to be an epoch in my Logical work. It occurred to me to try a complex Sorites by the method I have been using for ascertaining what cells, if any, survive for possible occupation when certain nullities are given. I took one of the 40 premisses, with "pairs within pairs," and many bars, and worked it like a genealogy, each term proving all its descendants. It came out beautifully, and much shorter than the method I have used hitherto. I think of calling it the "Genealogical method."" [Wakeling, 2005, p. 155]

The Tree method is a kind of *Reductio ad Absurdum* argument. For instance, first suppose the Retinends to be an Entity. Then we deduce from that assumption an absurd result, which means that our initial assumption was false and thus that the aggregate of the Retinends is a Nullity.

Example: Here is an elementary example using the tree method for biliteral propositions [Bartley, 1986, pp. 283-285]. Note that Carroll also uses this method for triliteral and multiliteral propositions. Let us take the following premises:

[23]Mark R. Richards identifies another extension of the underscoring method, which he calls the method of barred groups. It is used by Lewis Carroll in a manuscript from the Dodgson family collection [Richards, 2000].

[24]Francine Abeles [1990] suggests that Carroll might have been inspired, for developing his method of trees by Peirce and his students' work *Studies in Logic* [Peirce, 1883].

$$\begin{array}{ccccccccc} 1 & & 2 & & 3 & & 4 & & 5 \\ b'_1a_0 & \dagger & de'_0 & \dagger & h_1b_0 & \dagger & ce_0 & \dagger & d'_1a'_0 \end{array}$$

a, b, d and e are the four Eliminands, while c and h are Retinends.

Let us suppose ch to be an entity (i.e. "Some thing having the attributes c and h exist"). So, ch is called the Root of the tree.

ch

From the fourth proposition "ce_0", we conclude that c and e are incompatible. Thus, if a thing has the attribute c it should have also the attribute e'. Thus, we put e' under ch in the tree, with the reference-number 4 (which refers to the fourth premise). The tree becomes:

	ch
4.	e'

From the third proposition "h_1b_0", it follows that h and b are incompatible. If a thing has the attribute h, it should have also the attribute b'. Accordingly, we add b' under ch in the tree, with the reference-number 3. The tree becomes:

	ch
3,4.	$e'b'$

This means that the thing having the attributes c and h must also have the attributes b' and e'. One now looks for b' and e' in the premises. They appear in the first and second propositions: "b'_1a_0" and "de'_0". It follows from them that the thing having the attributes b' and e'should also have the attributes a' and d'. Thus, $che'b'd'a'$ is an entity. The tree becomes:

	ch
3,4.	$e'b'$
1,2.	$d'a'$

If we look at the premises for d' and a', they appear together in the fifth proposition "$d'_1a'_0$". This asserts that d' and a' are incompatible and thus that any thing having the two attributes is a nullity. It follows that $che'b'd'a'$ is a nullity. This result is represented in the tree as follows:

	ch
3,4.	$e'b'$
1,2.	$d'a'$
5.	o

From the forgoing it follows that *ch* is a nullity, i.e. "ch_0". We now return to the premises to examine whether *c* or *h* is given as existing. In the third proposition, *h* is given as existing, ie: "h_1". So, the final conclusion is: "$ch_0 \dagger h_1$", i.e. "$h_1 c_0$". Hence, the complete tree is:

	ch
3,4.	$e'b'$
1,2.	$d'a'$
5.	o
$\therefore$	$h_1 c_0$

The final solution is "All *h* are c'".

Bartley claims a high place for Carroll's method of trees in the history of logic and sees in it anticipations of the Beth's semantic tableaux published in 1955 [Bartley, 1986, p. 32]. More recent authors defend a similar viewpoint [Okashah, 1992; Abeles, 2005a; 2007]. They demonstrate how Carroll's eliminating methods hold some of the key ideas of modern decision procedures, thanks to their algorithmic process.[25]

5 THEORY OF HYPOTHETICALS

5.1 Lewis Carroll's contributions to Mind

In addition to his textbooks and various pamphlets and circulars, Lewis Carroll made some logic contributions to periodicals, on the problem of hypotheticals. The best known are his two articles in the philosophical review *Mind*: "A logical paradox" (1894) and particularly "What the Tortoise said to Achilles" (1895). Both have been widely reprinted, commented and discussed by logicians and philosophers throughout the twentieth century. They are generally considered as Carroll's best contributions to logic. Bertrand Russell, for example, discusses both these works in his *Principles of Mathematics* [Russell, 1937]. In 1942, during a radio program with Mark Van Doren and Katherine Anne Porter, Russell assessed Lewis Carroll's logical work as follows:

> "I think he was very good at inventing puzzles in pure logic. When he was quite an old man, he invented two puzzles he published in a learned periodical, *Mind*, to which he didn't provide answers. And the providing of answers was a job, at least so I found it." [Russell, 1996, p. 525]

And later on:

[25]See [Abeles, 1994b] for other instances of algorithms and mechanical processes in Carroll's works.

> "His works were just what you would expect: comparatively good at producing puzzles and very ingenious and rather pleasant, but not important... None of his works was important. The best work he ever did in that line was the two puzzles that I spoke of..." [Russell, 1996, p. 528]

Though Carroll's *Mind* papers (notably the Achilles and the Tortoise paper) are widely known, it is curious to note that commentators haven't yet agreed on what is the moral of the story. They tend to share only the view that Carroll himself didn't intend his texts to have a single interpretation and that he himself was not fully conscious of their importance. Braithwaite opines that "[i]n both these papers in *Mind* Lewis Carroll was ploughing deeper than he knew. His mind was permeated by an admirable logic which he was unable to bring to full consciousness and explicit criticism." [Barithwaite, 1932, p. 176]. J. F. Thomson wonders whether Carroll intended the Achilles and the Tortoise dialogue to have any moral at all:

> "The extreme eccentricity of the behaviour of both of the characters may well make us wonder whether Lewis Carroll knew what he was up to in writing the story. Certainly it cannot be merely taken for granted that he intended to advance some moderately clear thesis or theses about inference but chose to do so in a veiled and cryptic way. It is just as likely that the story is the expression of perplexity by someone who was not able to make clear to himself just why he was perplexed." [Thomson, 1960, p. 99].

Even the Carrollian scholar W. W. Bartley III, is drawn to a similar view. Writing about the Achilles paper, he observes: "As for the story itself, I do not share the view that there is one clear interpretation of it and its intended moral. It seems most plausible to understand it as an attempt on Carroll's part to express some difficulties he felt but could not adequately explain..." [Bartley, 1986, p. 468].

As we are here merely concerned with Lewis Carroll's logic, we will not discuss the various (sometimes curious) interpretations of Lewis Carroll's *Mind* papers. We will try rather to examine these writing with a view to understanding his position on hypotheticals. A look at Carroll's private papers shows that he was seriously working on a theory of hypotheticals during the 1890s. His two *Mind* papers were surely neither "unconscious" writings nor jokes. They result largely from this work and from the correspondence he, in parallel, privately maintained with many contemporary logicians, to whom he sent copies of his problems and replies to their answers. His diaries of 1894 show that he gave a particular attention to the problem of hypotheticals during that year, during which both *Mind* papers were written. They are parts or steps of a methodical and conscious search for a theory of hypotheticals.

Though letters, manuscripts and diaries entries exist on the subject, there are unfortunately no traces of that work in the surviving parts of his forthcoming book *Symbolic Logic*. We know that he intended to include a discussion of hypotheticals in the book's second and third parts. In the advertisement for the first part, Carroll briefly describes the contents of the second and third parts. In the former, he includes "hypotheticals" as one of the subjects investigated, while the latter would contain a "theory of Inference". Later on, in the appendix addressed to teachers, he again announces that the forthcoming second part of the book would discuss "the *very* puzzling subjects of Hypotheticals and Dilemmas" [Carroll, 1958a, p. 185]. However, no such writings appear in Bartley's rediscovered galley proofs. Only versions of the *Mind* papers were included in a chapter on logical puzzles.

5.2 The Barbershop problem

Lewis Carroll's first contribution to *Mind* reports a debate that opposed him to John Cook Wilson. Wilson was appointed Wykeham Professor of logic at Oxford in 1889.[26] We know from their abundant correspondence that all throughout the 1890s, the two men debated logical matters, as well as problems of geometry and probability.[27] The dispute that leads to the barbershop problem began around 1892 and culminated in 1894. The discussion must have been very passionate as one can see from Carroll's numerous entries in his journal. On 21 January, 1893, he records: "Also I have worked a good deal at Logic, and am still unsuccessfully trying to convince the Professor of Logic (J. Cook Wilson) that he has committed a fallacy" [Wakeling, 2005, pp. 50-51]. On 5 February 1893, he adds "Heard from Cook Wilson, who has long declined to read a paper, which I sent January 12, and which seems to me to *prove* the fallacy of a view of his about Hypotheticals." [Wakeling, 2005, pp. 52-53]. One has to wait until 1 February 1894 for more development: "Then I got, from Cook Wilson, what I have been so long trying for, an *accepted* transcript of the fallacious argument over which we have had an (apparently) endless fight. I think the end is near, *now*." [Wakeling, 2005, p. 124].

Lewis Carroll wrote successive versions of the problem on which he disagreed with Wilson, and sent them to many of Britain's leading logicians, collected their answers, compared them, and responded. The list includes notably Thomas Fowler, J. A. Stewart, Bartholomew Price, John Venn,[28] James Welton, F.H.

[26]In this election, Cook Wilson prevailed over Venn. However, his election was not unanimously well received. In a letter to Samuel Alexander, F. H. Bradley commented as follows: "I do not quite know what to think of Wilson's election. I think that probably they may have done the very best thing, but it would be difficult for them perhaps to defend their choice by anything they could state. However that does not matter. Wilson ought to do uncommonly well if he will give up Aristotle's text. Otherwise -" [Keene, 1999, p. 41].

[27]There are more than forty letters from Carroll to J. C. Wilson, together with some (hardly readable) letters from Wilson to Carroll, in the John Cook papers owned by the Bodleian library (Oxford) as part of a bequest from John Sparrow. The earliest Carrollian letter is dated 5 June 1890, while the latest is dated 17 May 1897. There are curiously no surviving letters between 1892 and 1894. Some letters from that period were, however, briefly quoted by A. S. L. Farquharson, Wilson's posthumous editor in 1926 [Wilson, 2002, pp. xli–xliii].

[28]Venn was the first to discuss the problem in print in the second edition of his *Symbolic Logic*, where he called it the *Alice problem* [Venn, 1971, p. 442]. There is a surviving letter on the matter in the Venn papers at Gonville and Caius College, Cambridge. However, it doesn't

Bradley, and Henry Sidgwick. We also know from a manuscript note that John N. Keynes also discussed the problem [Keynes, 1894]. W. W. Bartley III counted (and published) at least eight versions of the problem, the earliest being dated in April 1894 [Bartley, 1986, p. 465]. There seems however to be at least one earlier version dated March 1894. A copy of it sent by Carroll to Bartholomew Price was already published (in a "slightly modified" form entitled "Going out") by Edward Wakeling in a compilation of Carrollian games and puzzles [Wakeling, 1992, pp. 20-21], where it has been identified by Mark Richards as an early version of the Barbershop problem [Wakeling, 1992, pp. 67-68].

The contradictory responses that Carroll collected from his "logical friends", as he called them, encouraged him to write a new version of the problem on 03 May 1894 and to send it to the journal *Mind* for publication.[29] In a note annexed to the problem, he explains that:

> "The paradox, of which the foregoing paper is an ornamental presentment, is, I have reason to believe, a very real difficulty in the Theory of Hypotheticals. The disputed point has been for some time under discussion by several practised logicians, to whom I have submitted it; and the various and conflicting opinions, which my correspondence with them has elicited, convince me that the subject needs further consideration, in order that logical teachers and writers may come to some agreement as to what Hypotheticals *are*, and how they ought to be treated." [Carroll, 1894a, p. 438]

The paper appeared in the July issue of the same year [Carroll, 1894a]. After its publication, the problem was discussed in *Mind* by other logicians like W. E. Johnson [1894 and 1895], Alfred Sidgwick [1894 and 1895], and later on Hugh MacColl [1897, pp. 501-503; and 1900a, pp. 80-81], E. C. C. Jones [1905a and 1905b], John Cook Wilson [1905a and 1905b] and finally Bertrand Russell [1905, pp. 400-401]. Even after its publication, Lewis Carroll continued to write other versions of the problem and to correspond about it with other logicians.[30] A last version (also not recorded by Bartley) appeared posthumously in the *Educational Times* [Carroll 1900] and received two replies, one from Hugh MacColl [1900b] and the other from H. W. Curjel [1900].

The version which appeared in *Mind*, entitled "a logical paradox", is mainly written as a dialogue between two uncles Jim and Joe, disputing about a barbershop. As with all the other versions, where the two characters are called Nemo and

---

contain a discussion of the barbershop problem itself. Lewis Carroll simply gives permission to Venn to use the problem and to include it in his new book, and asks him not to reveal his real name in connection with his pseudonym [Dodgson, 1894].

[29]The manuscript of *A Logical Paradox* is in the Parrish Collection of Lewis Carroll, Princeton University Library, Princeton (Box 9, Folder 2).

[30]It seems that Lewis Carroll also sent off-prints of his paper to his friends and colleagues. A copy of the *Mind* paper was for instance enclosed by Carroll in a letter to his friend Bell, dated 21 November 1896 (conserved in the Berg Collection, New York Public Library). Many copies of the off-print are known in various collections though it is not always possible to identify to whom they were sent.

Outis, Carroll doesn't say frankly which position he himself accepts. The problem is the following. Let there be a barbershop where Allen, Brown, and Carr work. There are two rules:

1. All three should not be out of the shop at once. This implies that: If Carr is out, then (if Allen is out, then Brown must be in).

2. If Allen is out, then Brown must be out.

Since "If Allen is out, then Brown must be in" and "If Allen is out, then Brown must be out" are contraries, and given (2), Carroll concludes that:

3. Not- if Allen is out then Brown must be in.

From (1) and (3), by *modus tollens*, Carroll concludes that: "Carr must be in". This result is paradoxical since Carr can be out when Allen and Brown are both in, without violating rules (1) and (2).

Lewis Carroll's version published posthumously in the *Educational Times* formulates the same problem more briefly:

> "It is given that (1), if C is true, then, if A is true, B is not true; and (2), if A is true, B is true. Can C be true? What difference in meaning, if any, exists between the following propositions? – (1) A, B, C cannot be all true at once; (2) if C and A are true, B is not true; (3) if C is true, then, if A is true, B is not true." [Carroll, 1900]

Bertrand Russell discussed the *Mind* version in his *Principles of Mathematics*. Russell offers what is today's commonly admitted solution to the problem, which turns on what is commonly known as the paradoxes of material implication. Carroll's argument assumes that the two propositions "If Allen is out, then Brown must be in" and "If Allen is out, then Brown must be out" are contraries and thus incompatible, while in fact they can both be true, in which case they simply imply "not-Allen is out". In effect, the two propositions "P then Q" and "P then not-Q" can be both true when P is false, then "principle that false propositions imply all propositions solves Lewis Carroll's logical paradox" [Russell, 1937, 18]. Russell's interpretation is adopted by his immediate followers, and it closed for a while the debate on the Barbershop problem. One had to wait until 1950 for a new reading by Arthur W. Burks and Irving M. Copi who linked the problem with the matter of causal implication [Burks-Copi, 1950].

The note that Carroll annexed to his *Mind* paper (and which is curiously excluded in the majority of modern reprints), shows how Carroll asks explicitly in his problem the question of the legitimacy of the material interpretation of implication. He, for instance, asks: "Can a Hypothetical, whose protasis is false, be regarded as legitimate?" [Carroll 1894, p. 438]. Though he didn't reveal explicitly his opinion in the *Mind* text, there is more evidence to believe, as Bartley shows, that he accepted material implication and thus, that he took up the cause of Outis rather than Nemo, and Jim rather than Joe, that is, that Carr can leave his shop

[Bartley, 1986, 448]. Lewis Carroll's correspondence confirms this view as one can observe in his letters to an unidentified "Sir" (now in the Berol Collection, Fales Library, New York) where he explicitly wrote that "Nemo" was a "friend" of his. Thus, Lewis Carroll was "Outis" and knew that the Barber shop argument was not valid. This is notably what is shown in a kind of primitive truth table that Carroll included in the seventh version published by Bartley, where he lists all possible combinations of A, B and C, their truth or falsity depending on respectively Allen, Brown, and Carr's being out of or in the barbershop. Carroll concludes from that that there are cases where Carr can be out without violating any of the rules already listed [Bartley, 1986, p. 465].

5.3 What the Tortoise said to Achilles

Contrary to the barber shop problem, we know little about the genesis of the Achilles and the Tortoise problem. In 1974, Ivor Grattan-Guinness drew attention to an 1874 Lewis Carroll manuscript owned by Christ Church library, where he discusses the Zeno paradox, and suggested a possible link between the two texts [Grattan-Guinness, 1974, p. 16]. The idea that this manuscript would be an early version of the *Mind* paper has also been proposed [Williams-Madan, 1979, p. 82; Abeles, 1994b, p. 104; Cohen, 1995, p. 501]. However, a look at the content of the manuscript and the twenty years separating them indicate no such a relationship.[31] In fact, the manuscript in question entitled "*An inconceivable conversation between S. and D. on Indivisibility of Time and Space"* concerns Zeno's original paradox and is thus closer to a number of Carrollian references to that paradox. It is more convincing to relate Carroll's own Achilles and Tortoise problem to his 1894 work on hypotheticals. There is a manuscript version of the problem, dated 23 August 1894, which Carroll sent to the editor of *Mind,* G. F. Stout.[32] After a brief correspondence between Stout and Carroll, the paper appeared finally in the April issue of the next year [Carroll, 1895].[33]

"What the Tortoise said to Achilles" formulates the following problem. Imagine Achilles and the Tortoise, the two famous Zeno characters, discussing Euclidean Geometry after their famous race. Take the following inference:

[31] John Gattégno, who first published that manuscript, misdated it and thought that it was written on 22 November 1894 [Gattégno, 1976, p. 305].

[32] The manuscript of *What the Tortoise said to Achilles* is in the Parrish Collection of Lewis Carroll, Princeton University Library, Princeton (Box 9, Folder 7). Lewis Carroll's correspondence with G. F. Stout is in the same collection, among Carroll's mathematical manuscripts (Box 1, Folder 6).

[33] There is a widespread misunderstanding about the date of publication of the Achilles and the Tortoise dialogue among Carrollian scholars. Carroll's standard bibliography says that the text was printed presumably in December 1894 [Williams-Madan, 1979, p. 190]. But there is no *Mind* issue in December! The same error appears in the annotations of Carroll's diaries [Wakeling, 2005, p. 161], as well as in certain Carrollian biographies [Gattégno, 1976, p. 10] and studies [Gardner1996, p. 72]. The confusion is due to Carroll himself who printed a copy of the text with the inscription "Reprinted from *Mind* for December, 1894" [Carroll, 1894b]. See also [Goodacre, 1994; Imholtz, 2003b].

(A) Things that are equal to the same are equal to each other.

(B) The two sides of this Triangle are things that are equal to the same.

(Z) The two sides of this Triangle are equal to each other.

The Tortoise claims that he accepts the premises as true but doesn't accept the Hypothetical:

(C) If A and B are true, Z must be true.

Then, the Tortoise asks Achilles to force him "logically" to accept Z. Carroll shows that even if the Tortoise admits the truth of the hypothetical proposition (C) and incorporates it as a premise in the inference, he is still not obliged to accept Z because he still might deny the validity of the new Hypothetical:

(D) If A and B and C are true, Z must be true.

And so on, the Tortoise might claim the necessity of further hypothetical statements, *ad infinitum*.

Bertrand Russell also discusses the Achilles and the Tortoise problem in his *The Principles of Mathematics* [Russell, 1937, p. 35]. He claims that Carroll's story illustrates the necessity of his fourth indemonstrable principle asserting that: "A true hypothesis in an implication may be dropped, and the consequent asserted" [Russell, 1937, p. 16]. After Russell, the Achilles and the Tortoise problem of infinite regress has been widely discussed by philosophers and logicians.[34] Generally, the accepted "point of the story" is that an inference should not include its own Hypothetical proposition in its set of premises. One sees this view defended by Gilbert Ryle: "The principle of an inference cannot be one of its premises or part of its premises. Conclusions are drawn from premises in accordance with principles, not from premises that embody those principles. The rules of evidence do not have to be testified to by the witnesses." [Ryle, 1963, pp. 306-307].[35] The abundant literature that was aroused by Carroll's problem gives little attention to Carroll's own conception of hypotheticals. It is true that the text itself is not explicit enough on what Carroll meant. As we are here concerned by Lewis Carroll's logic, it is necessary to turn to Carroll's private papers to fathom his own interpretation.

When the editor of *Mind*, received Carroll's contribution, he immediately sent him a letter (dated 24 August 1894) asking for clarification of the moral of the story. Stout asked Carroll whether his paper should not consider the "difference between affirming A and affirming the truth of A" [Bartley, 1986, p. 471]. Carroll

[34]The Achilles and the Tortoise problem is discussed by, among others, W. J. Rees [1951], D. G. Brown [1954], J. F. Thomson [1960], W. W. Bartley III [1962], John Woods [1965], William A. Wisdom [1974], Barry Stroud [1979], Simon Blackburn [1995], Pascal Engel [1998], etc.

[35]Ryle defends the same position in [Ryle, 1946]. Stephen Toulmin, after an explicit reference to Carroll's problem, defended a similar view on the laws of nature: "The conclusions about the world which scientists derive from laws of nature are not deduced from these laws, but rather drawn in accordance with them or mnferred as applications of them. . . " [Toulmin, 1967, p. 91]

responded on 25 August, 1894 that his "... paradox does *not* attempt to draw any distinction between these 2 processes: it turns on the fact that, in a Hypothetical, the *truth* of the Protasis, the *truth* of the Apodosis, and the *validity of the sequence*, are 3 distinct Propositions." [Bartley, 1986, p. 472]. He then applies his regression to the more familiar "Socrates' mortality" example: If we grant that: 1) "All men are mortal, and Socrates is a man", but not 2) "The sequence "If all men are mortal, and if Socrates is a man, then Socrates is mortal" is valid", then we do not grant 3) Socrates is mortal.

This rare testimony shows that Carroll expressly considers the relation of implication between the antecedent and the consequent in a hypothetical and takes it as a relation of content and not of truth-value. For example, given the proposition: "If A then B", Carroll holds that the validity of the sequence (that is the hypothetical proposition itself) doesn't depend on the truth-value of A and B but on the significance of A and B themselves. This Carrollian interpretation of the Achilles and the Tortoise problem may deceive modern logicians and philosophers, who will find it less ambitious than is usually claimed. It has, however, more historical coherence in regard to the work of some of Carroll's contemporaries, notably W. E. Johnson, E. C. C. Jones, and Hugh MacColl [Prior, 1949; Rahman, 2000].

5.4 A "workable" theory of hypotheticals

Lewis Carroll's two contributions to *Mind* both deal with the problem of hypotheticals. The Barbershop problem suggests that Lewis Carroll knew about and accepted material implication. However, the Achilles and the Tortoise paper shows that he also understood the difficulties that occur when we adopt such an interpretation of implication. This being so, not only should we acknowledge Carroll's complete understanding of the importance of the matter he was discussing, but we should also remember that both contributions to *Mind* were steps or intermediary results of a larger work in progress on this subject, developed by Carroll around 1894. We have no traces of the final conclusion which he planned for in the second part of *Symbolic Logic*. Assuming that he actually wrote it, it seems neither to have been published nor to have survived. Two entries from Lewis Carroll's journal dated on December, 1894 indicate some further developments and confirm the early directions suggested by the *Mind* papers.

On December 11th, 1894, he writes in his diary:

> "I am giving all my time to Logic, and have at last got a workable theory of Hypotheticals — to represent "a ⁋ b" by "$ab'_0 \dagger a_1 \dagger b'_1$", meaning by "$_0$", "cannot exist", and by "$_1$", "can exist"." [Wakeling, 2005, p. 184]

As we have seen in our discussion of Carroll's symbolism, ab'_0 means "No a are not-b" and a_1 means "some a exist", and in consequence "$ab'_0 \dagger a_1$" (which is equivalent to "$a_1 b'_0$") means "all a are b". However, Carroll adds "b'_1" which means that "some not-b exist". This is introduced probably to express contraposition, for if "all a are b" then "all not-b are not-a" and thus, according to Carroll's interpretation of A propositions, "some not-b exist". Bartley reports some other writings where Carroll interprets propositional implication as a relation of class inclusion. One source is a Carrollian notebook where one finds, for example, "α ⁋ xyz" interpreted as "$\alpha_1(xyz)'_0$". Another interesting manuscript quoted by Bartley makes the matter even clearer:

> "Denoting a term which asserts the possession of some property (such as 'straightness') by a single letter (such as a), I shall denote the term which denies it by not-a, or, yet more briefly by a'. And I shall denote the logical copula 'is,' which asserts that the possession, or non-possession, of some property, is necessarily followed by the possession, or non-possession, of some other, by the symbol ⁋. Thus, if a stands for 'human' and b for 'mortal,' the time honoured proposition "all men are mortal" may be abbreviated into a ⁋ b..." [Bartley, 1986, pp. 256-257].

The combination of these references confirms that particular Carrollian interpretation of hyotheticals, with the omission of the superfluous "b'_1". Finally, it is interesting to note the introduction of the modality in Carroll's definition of the subscripts in the journal entry where "a_1" stands for "a can exist" rather than "a exists".

Ten days after recording this "workable" theory of hypotheticals in his journal, Lewis Carroll was still working on the subject and recorded on 21 December, 1894:

> "My night's thinking over the very puzzling subject of "Hypotheticals" seems to have evolved a new idea — that there are *two* kinds, (1) where the Protasis is *in*dependent of the Hypothetical, (2) where it is dependent on it." [Wakeling, 2005, pp. 185-186]

There are no further records on hypotheticals in Carroll's diaries on the subject of hypotheticals. One cannot determine whether Carroll went further in this work and what results, if any, he obtained. However, from the discussion of the Barbershop and the Achilles and the Tortoise problems, and the numerous private documents mentioned above, it may be said that Carroll was developing a theory of Hypotheticals, which can partially be reconstructed from his papers. He tended to interpret implication materially but felt uneasy with it and expressed some difficulties in that interpretation. All that confirms Bartley's view that Carroll's work on hypotheticals ""sounds" like a foretaste of what was to come a few years later and from other logicians as strict and causal implication" [Bartley, 1986, p. 448]. Interestingly, C. I. Lewis and Arthur Burks, both refer to Carroll on the this matter, though in opposite ways. Lewis, in the exposition of his system of strict implication, inserts a surprising and unexpected footnote: "Lewis Carroll wrote a *Symbolic Logic*. I shall never cease to regret that he had not heard of material implication." [Lewis, 1918, p. 326]! Burks is more explicit and mentions Carroll's

Barbershop problem in the exposition of his system of causal implication [Burks, 1951, pp. 377-378]. On a more "social" level, Carroll's work on hypotheticals made him known to his contemporary logicians, chiefly thanks to the Barbershop problem, which was discussed by Britain's leading logicians up to 1905. Later on, the Achilles and the Tortoise dialogue brought him posthumous fame among logicians and philosophers more than any other of his logical writings.

6 CONCLUSION

A few years after Lewis Carroll's death, Russell published his *Principles of Mathematics*, and the science of logic took a new direction again. Like the majority of his contemporary British colleagues, Carroll's work quickly became out-dated. His only logical contributions, which one still meet(s) with in modern textbooks, are his logic diagrams and *Mind* problems. Modern logicians still quote the Alice's books and use the voluminous Carrollian collection of logical problems for teaching. For historians of logic, however, Lewis Carroll's logical work is a rich source of information for a better understanding of the state of logic toward the end of the nineteenth century.

Though many features of Carroll's work are still close to Aristotelian logic, (he even dedicated his *Symbolic Logic* to the "memory of Aristotle"), he was conscious of the fact that recent work in Britain (from Boole to Venn) opened a new and more effective approach to the subject, and himself claimed to belong to this new trend. In the appendix to teachers, he describes the whole syllogistic system as "an almost useless machine, for practical purposes, many of the conclusions being incomplete, and many quite legitimate forms being ignored" [Carroll, 1958a, p. 183]. In the preface of *Symbolic Logic*, he considers symbolic logic more helpful and easier that the old formal logic, which he found to be obscure and cumbrous [Carroll 1958, p. xiv].[36] In a letter to his publisher Macmillan, dated 19 October, 1895, he is even more explicit on the state of logic at that time:

> "[T]his book [That is his *Symbolic Logic*] is *not* offered as a "school book." In the present state of logical teaching, it has *no* chance of being "adopted" as "a school book," as it would be for no use in helping its readers to answer papers on the Formal logic, which is the only kind taught in Schools and Universities. It teaches the real *principles* of

[36] Note that contrary to many other logicians, Lewis Carroll differentiated between symbolic and formal logic, formal logic being that which was still taught in British schools and universities (that is the traditional Aristotelian logic) while symbolic logic is the logic that John Venn, for instance, worked (that is the new Boolean logic). Historians of logic generally consider Venn to be the first to use the expression "symbolic logic" in his works, notably in the first edition of his *Symbolic Logic* published in 1881. In fact, Venn used that expression at least two years earlier [Venn 1879, p. 580], where its use suggests an even earlier invention. Though it seems true that it was through Venn's writings that the expression itself attained a wide publicity, the oldest known use in print of the full expression "symbolic logic" that came into our knowledge occurs however in a paper by the forgotten logician and early Boole champion, George Bruce Halsted [Haslted, 1878, p. 83].

> Logic, and it enables its readers to arrive at *conclusions* more quickly and easily that Formal Logic, but it does not enable any one to answer questions in the form at present demanded. I have no doubt that Symbolic Logic (not necessarily *my* particular method, but *some* such method) will, *some* day, supersede Formal Logic, as it is immensely superior to it: but there are no signs, as yet, of such a revolution." [Cohen-Gandolfo, 1987, p. 323]

Symbolic logic was not yet established, and it surely was less recognised at Oxford than at Cambridge [Grattan-Guinness, 1986; Marion, 2000]. Oxford's Professor of logic, John Cook Wilson strongly opposed the new logic, on the ground that, among other things, it was mathematics, not logic, and that "[i]n comparison with the serious business of logic proper, the occupations of the symbolic logician are merely trivial." [Wilson, 2002, p. 637]. Wilson was not the only anti-mathematical logician, as Venn designated them "without offence" [Venn, 1971, ix]. Symbolist logicians, pleading for the generalisation of symbols to represent operations in logic, were in the minority. And Lewis Carroll was surely one of them.

Carroll's logical work shows also that he was acquainted with the logical players of his time. In his *Symbolic Logic*, he mentions George Boole, Augustus de Morgan, William S. Jevons, John N. Keynes, John Venn and members of The John Hopkins University (Charles S. Peirce and his students). His private library, contained copies of the main logical works of the time. In addition to the above mentioned authors, he also owned copies of the works of Bernard Bosanquet, F.H. Bradley, Thomas Fowler, Rudolph H. Lotze, Henry L. Mansel, J.S. Mill, James Welton, Richard Whately, and many others [Lovett, 2005]. But, of course, like his contemporary British logicians, he ignored the work of the German logician Gottlob Frege.[37]

Reading Lewis Carroll reveals conflicting influences from both traditional and modern logicians. It also shows some interesting inventions which deserve more attention from logicians and historians of logic. One cannot however fully understand Carroll's work and its importance without paying attention to the effort he made to make his work accessible to a wide public. He himself taught logic in many Schools at Oxford. In the preface of *Symbolic Logic*, he proudly claims it as "the very first attempt (with the exception of [his] own little book, *The Game of Logic*, published in 1886, a very incomplete performance) that has been made to *popularise* this fascinating subject." [Carroll, 1958a, p. xiv]. Carroll's work should be understood as the work of an author of logic who wrote to be read.

[37]Frege's work was almost ignored in Britain before Russell. Venn wrote in 1880 a dismissive review of Frege's *Begriffsschrift* for the journal *Mind*, which he concluded by: "I have not made myself sufficiently familiar with Dr. Frege's system to attempt to work out problems by help of it, but I must confess that it seems to me cumbrous and inconvenient." [Venn, 1880a, p. 297]. On the general reception of Frege's work, see [Vilkko, 1998].

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JOHN VENN AND LOGICAL THEORY

James Van Evra

1 INTRODUCTION

John Venn (1834–1923) lived in what for logic were interesting times. He was in his early teens when George Boole's *Mathematical Analysis of Logic* and Augustus De Morgan's *Formal Logic* appeared, and he died eight years after the publication of volume 3 of the first edition of *Principia Mathematica.* His remarkable life thus spanned the development of logic from the beginning of its sustained involvement with mathematics through the appearance of the now standard version of quantification theory.

Though Venn is best known for the widely recognized diagram that bears his name,[1] his primary contribution to logic lies in the critical commentary on contemporary work in the foundations of the subject that he produced over a period of thirty years. Venn was ideally suited to the task; A meticulous scholar and antiquary, he had both a wide ranging command of mathematics and logic and a thorough grasp of the history of the formal sciences.[2] He also made contributions to probability theory and to areas now included in the social sciences.

The nineteenth century was a period of significant change in logic generally, and especially in British logic. At its beginning, texts in the subject were constrained more by tradition than self conscious theory, and Aristotle's name continued to be invoked more as authority than carefully considered source. As mid century approached, however, theory began to intrude in the form of analogical connections between logic and mathematics proposed by Boole, De Morgan and others. The introduction of mathematical links produced, in turn, a marked division of opinion about the nature of the subject. On one side, there were those, often more traditionally inclined, who considered logic to be an immediate representation of human mental processes (a view that Venn later called the "conceptualist" theory of logic).[3] In their view, to mathematize logic was to introduce something inherently foreign into the subject, for there is nothing overtly mathematical in the way we reason. On the other, those more abstractly inclined, including Boole, De Morgan, Venn and others saw no necessity in maintaining a recognizable connection

[1]As the time of this writing, for instance, a web search of "Venn diagram" produced references to 1,050,000 www sites containing the term.

[2]His collection of works on logic, housed in Cambridge, remains one of the best in existence. See [Venn, 1889].

[3]Among those mentioned by Venn who held such a view are Lambert, Jevons, Spalding and Baynes.

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between logic and apparent thought processes. As Venn put it, The interpretation of propositions in terms of classes... "may not be the most fundamental in a Psychological sense; but when... we are concerned with logical methods merely, this does not matter." (5)[4] For them, logic was a science, and mathematics was a powerful tool for the representation of the formal basis of logical inference.

When *Symbolic Logic* appeared in 1881, the debate had been ongoing for more than thirty years. In large part, the book presents both an account of symbolic logic (Venn's term) that brings it up to date by including post-Boolean contributions made by C. S. Peirce, Ernst Schröder, and others, as well as a defense of it against ongoing criticism from those still maintaining the older point of view. In the book, he uses the common or traditional logic as a foil by first describing how various topics are handled in the older logic, followed by a reasoned justification for differences in the way symbolic logic deals with the same topics. The effect is a topic by topic redefinition of logic set against a traditional background.

2 BIOGRAPHY

Venn was born on 4 August 1834 to a family of intellectuals prominent in the evangelical movement in the 18th century. After early private schooling, he entered Gonville and Caius College, Cambridge in 1853. His ability in mathematics quickly became apparent. He was elected mathematical scholar in 1854, was sixth wrangler in the mathematical tripos of 1857 (the highest ranking in his college), and as a result was elected fellow of the college. With the exception of a period from 1858 to 1862 during which he served as an active member of the clergy, his life was spent within its confines, beginning as lecturer in Moral Sciences on his return, and culminating in his tenure as president from 1903 until his death. He was elected fellow of the Royal Society in 1883.

Venn's major philosophical works include *The Logic of Chance* (1866), *Some Characteristics of Belief, Scientific and Religious* (1869), *Symbolic Logic* (1881), and *Principles of Empirical or Inductive Logic* (1889). As the titles make clear, his interests were not confined to symbolic logic narrowly conceived. Though obviously influenced by Boole and De Morgan in the narrower field of formal logic, his work in probability and philosophical psychology reflect another influence on his thought, i.e. that of John Stuart Mill.

From the mid-1880s to the end of his life, Venn's interest turned to antiquarian research. During that period, he published historical works on his college and a roster of Cambridge alumni from its founding to 1900. He died on 4 April 1923 in Cambridge.

[4]Page references are to [Venn, 1894].

3 VENN'S CONCEPTION OF LOGIC

Venn's commentary on logic is largely a detailed development and defense of George Boole's conception of the subject.[5] He supports Boole on major points, including a commitment to extensionalism, to the exclusive interpretation of disjunction (in opposition to W.S. Jevons' intentionalist, inclusive reading)[6], and to treating the syllogism as a systematic elimination of middle terms. He did not intend it, on the other hand, to be a "mere commentary on Boole," for despite his defense of the Boolean conception on major points, in those cases in which he disagreed with Boole, (for instance on Boole's handling of existential commitment (436)), he was just as quick to point out flaws in Boole's approach. Also, while he identified Boole as the "main originator" (436) of symbolic logic, he also recognized that "It would certainly seem that Boole had no suspicion that anyone before himself had applied algebraic notation to logic." (xxix)[7] Hence Venn's aim was also to complete the picture by placing Boole's achievements in a larger historical context by recognizing the contributions of Leibniz, and the eighteenth century logicians who followed him (especially Lambert, but also Segner, Ploucquet, and von Holland).

Symbolic logic arose in part as a result of changes occurring within logic itself,[8] but also as a result of a broader change that occurred in thinking about formal science in Britain in the early nineteenth century. Mathematics in Britain had long been settled in its own tradition, centered on Newton's conception of the calculus and a strict arithmetical interpretation of algebra. In 1806, Three Cambridge undergraduates, George Peacock, Charles Babbage and John Herschel,[9] formed the Analytical Society with the specific goal of promoting the more abstract continental version of the calculus by replacing the "dot-age" of Newton's fluxions with the "d-ism" of Leibniz' functional notation. Later, in his 1830 *Treatise on Algebra*, Peacock turned his reforming attention to algebra. There he introduced an abstract ("symbolical") conception of the subject by separating algebraic structure from interpretation, thus permitting more than one interpretation of an algebraic expression while maintaining equivalence of formal operations, i.e. whatever is true under one interpretation remains true under the other (this is his "principle of the permanence of equivalent forms").[10] While Peacock did not mention logic, and limited the extension of interpretation of algebraic symbols to geometry, the later

[5]According to A. N. Whitehead [1898, 115n], "The task of giving thorough consistency to Boole's ideas and notation, with the slightest possible change, was performed by Venn in his 'Symbolic Logic.'

[6]Jevons [1864, p. 4] claims that his work is "...founded on that of Professor Boole... The forms of my system may, in fact, be reached by divesting his system of a mathematical dress, which...is not essential to it."

[7]Venn was quick to deny that such a characterization of Boole amounts to a criticism, given the modest resources available to Boole during his early years.

[8]See, e.g. my [1984].

[9]And joined later by William Whewell and others.

[10]Curiously, while Venn mentions the principle (431n) he neither attributes it to Peacock, nor mentions him elsewhere in the book.

algebraic logicians brought the ideal of an abstract system to logic. Both Boole and Venn describe such a system as being largely uninterpreted, and bounded only by a few elementary laws (such as transitivity, commutativity, etc).[11] In terms of this ideal, Venn begins with the idea that there is "no vested right in the use of $+$ and $-$." (xiv) The standard operators can thus be assigned one interpretation in arithmetic, and another, formally similar, but not identical interpretation in logic. It is this idea that forms the basis for the creation of analogical links between algebra and logic.

The major advantage of such links, according to Venn, was that it permits logic to be at once both essentially separate from mathematics, while being ("accidentally," as he puts it) dependent on it.. However, Venn was also aware of problems such association brings. Like Boole, for instance, he realized that the inverse operations of subtraction and division are not readily interpretable in logic without significant auxiliary assumptions (which Venn discusses at length; cf. pp. 73-96). He was also aware that even the seemingly simplest cases demand a clear grasp of the scope and limits of the analogy. Thus he points out that although the operations are formally similar, arithmetic addition and class aggregation are not identical; "We do not," he says, "add together the English, French, Germans and so forth in order to make up the Europeans." (54)

3.1 An Example of Venn's Analysis: Existential commitment

Venn's treatment of existential commitment in logic serves as a good example of his style of analysis. While some of the greatest logicians (e.g. Aristotle, Leibniz and Peirce) were heavily involved with questions of ontology, Venn points out that "Many logicians, if not a majority of them have ... passed the subject by entirely..." (141). The reason for such neglect, he suggests, was the widespread acceptance of conceptualist view of logic, mentioned above, in terms of which logic is confined to a study of the relations between mental concepts. That being the case, there is no need to question the referential status of X or Y, for they can only refer to mental entities. Rejecting such a view, Venn posed the key existential question for logic by asking whether, from a strictly logical point of view, uttering 'All X is Y,' "asserts or implies" that there are such things as X or Y, i.e. "do such things exist in some sense or other?"(141) He attacks the problem by using a method that reflects the influence of Hume and anticipates W. V. O. Quine's method of semantic ascent, i.e. rather than confronting substantive metaphysical questions about the sorts of things that *exist*, he focuses instead on criteria used for the acceptance or rejection of existence *claims*. That is, regardless of how differently existence is construed from one universe of discourse to another, and hence how widely criteria vary for the verifiability of such claims, the logician's interest lies in the fact that acceptance or rejection remains a uniform possibility

[11] Although both Boole and Venn were aware of the difficulties facing such a conception, i.e. the non-commutative algebras recently devised by William Rowan Hamilton.

and a real distinction, regardless of how questions about the nature of existence or its verification are settled.

Venn confronts the original question from three perspectives: common language, the older common logic, and symbolic logic, asking how each in turn deals with the question of implied commitment on the utterance of each of the four standard forms of proposition. His answer is that generally speaking, common language does entail such commitment, given the fact that ordinarily, we do not utter propositions without believing that their subjects and predicates refer. As he puts it, "Broadly speaking, the statement that All X is Y does imply that there are Xs, and consequently indirectly that there are Ys." (145) The same holds true for I and O propositions. E propositions, on the other hand, are slightly more problematic, given that it is not clear that the predicates in such statements need refer if they are universally denied subjects. But all of this is only true generally speaking, for there are always problem cases. Claims about the future, for instance, or talk of ideal states are ontologically problematic regardless of the form of proposition in which they occur. What Venn sought, by contrast, was an analysis of existence claims for logic that avoids all such problems.

To arrive at such a view, Venn next turns to (traditional) logicians. After discussing their penchant for conceptualism and the problems it brings, he notes that while some attempt to tie questions of existential import to hypothetical propositions (151), that will not work either, for the distinction between hypothetical and categorical propositions is merely one of expression, i.e., as other logicians had noted, there is no real distinction between the two.

What Venn suggests instead is a (remarkably modern) view "...almost necessarily forced on us by the study of Symbolic Logic," (157) that "*the burden of implication of existence is shifted from the affirmative to the negative form.*" (159; ital his) All questions of the existence of the subject or predicate are, accordingly, reformulated as follows: given the proposition 'All x is y' containing two class terms, there are four ultimate (i.e. possible) classes involved: xy, x not-y, y not-x and not-x not-y. "Now what we shall understand the proposition 'All x is y' to do is, not to assure us as to any one of these classes (for instance xy) being *occupied*, but to assure us of one of them (viz. x not-y) being *unoccupied*." (158; ital his) Hence "we are led to the following result ... ; that in respect of what such a proposition affirms it can only be regarded as conditional; but in respect of what it denies it may be regarded as absolute."(159) It is this form, he says, that renders conclusions about existence "appropriate and unambiguous." (158)

Given this analysis, Venn goes on (correctly) to conclude that within the square of opposition, the old assumption that A and E propositions imply I and O propositions fails, for if none of the subject terms refer, the former are true while the latter are false. Similarly, the old relations of contrariety between A and E and subcontrariety between I and O also fail, for if the subject terms fail to refer, the former can both be true and the latter both false.

3.2 Diagrams

In his introduction to the diagrams that would later bear his name, Venn is quick to acknowledge that diagrams similar to his had often been used in the past, and particularly the two figure diagram devised by the mathematician Leonard Euler.[12] Venn saw nothing inherently wrong with the older diagrams, in fact he thought that an Euler style diagram was perfectly adequate for the representation of the *conclusion* of an argument. What he sought, on the other hand, was a way of going further by serially representing each of the steps in a complex series of inferences within a single diagram. Beginning with the representation of one premiss using two figures, that is, one can then add premises by adding figures one by one. In the end, various conclusions can be seen in the finished diagram.

Venn sought supreme generality in the figures and their interpretation. He suggested that any closed figures can be used in the diagrams, and any number of figures can be incorporated into one diagram (although, as he pointed out, diagrams containing more than five figures are rarely needed in the representation of actual inferences). In addition, and consistently with the larger aims of symbolic logic, Venn regarded the figures as having no fixed interpretation, but rather understood them, prior to interpretation, to be merely figures enclosing spaces. They may then be taken to represent either classes, in which case a diagram represents the relation between classes as such, or as representing propositions.[13]

4 CONCLUSION

Given the sophistication of his logical analysis, and in particular the close proximity of his conclusions to results in truth functional sentential logic and first order logic that are now taken for granted, it is clear that Venn stands at the apogee of nineteenth century algebraic logic. Yet however close his results were to current logical theory, he remained on the term logic side of the divide separating it from the logic of Frege and Russell and Whitehead. His understanding of the scope of logic is particularly evident in his reaction to Frege's *Begriffsschrift.* Venn refers to Frege's work twice, once early in *SL*, where he makes the (ironically prescient) remark that it "has no reference to any symbolic predecessor except a vague mention of Leibnitz."(xxx) He did not, that is, recognize that the reason for the lack of reference is that Frege's work presents a new conception of the subject that depended on no such ties. Venn's other reference to *Begriffsschrift* occurs in a final chapter entitled "Historical notes," in which he compares thirty logicians on their respective formulations of *E* propositions. Of Frege, he says "Here again we have an instance of an ingenious man working out a scheme — in this case a very cumbrous one — in apparent ignorance that anything better of the kind had ever

[12]Venn remarks (110n) that a canvass of sixty books produced 34 that employed diagrams.

[13]An informal discussion of mathematical properties of the diagrams, together with a helpful bibliography, can be found in Edwards [2004]. A more formal analysis can be found in Ruskey and Weston [2005].

been attempted before.” (493/4) Once again, Venn interprets Frege from the point of view of the logic of terms. Further, he makes no mention of Frege’s conception of quantification. For Venn, propositions were still interpreted as complexes of terms, which in turn remained the primary carriers of meaning, and although he recognized truth functional properties of compound propositions, he did not recognize truth functions as such. While he did not make the leap, however, the algebraic style of logic was far from moribund, instead going on to find application in the twentieth century work of Thoralf Skolem, and later in the work of Alfred Tarski.

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WILLIAM STANLEY JEVONS AND THE SUBSTITUTION OF SIMILARS

Bert Mosselmans and Ard Van Moer

1 INTRODUCTION

William Stanley Jevons was born in Liverpool on 1 September 1835. He went to University College School in London in 1850, at the age of fifteen, and in 1851 he became a student at the University College. He studied chemistry under Graham and Williamson, two pioneers in the development of atomic theory and the theory of molecular motion. Chemistry remained important during Jevons' life, and he even published two papers on Brownian Motion in 1870 and 1878. Another major influence at University College was Augustus De Morgan (1806–1871), with his courses on mathematics and logic. Jevons' own approaches to scientific method, probability, logic and mathematics were influenced by De Morgan. Jevons had also a lively interest in botany, which probably stemmed from his mother. Jevons' interest in political economy is not surprising, given his non-conformist intellectual and family background, but it can also be explained by the context of economic development he lived in, with both its dark and good sides. Jevons left University College without taking his degree and went to Australia to become an assayer at the Australian Mint.

Jevons sailed from Liverpool on 29 June 1854, and he arrived in Melbourne on 6 October. At first Jevons found the assaying business exciting: he experimented and even wrote an article on 'Gold Assay' which appeared in Watt's *Dictionary of Chemistry* [1864]. However, after April 1855 it became a sinecure and he devoted much more time to other scientific investigations. Jevons' 'science of man' project entailed an interdisciplinary utilitarian approach to different aspects of individual and social life. His work covered many different areas, as is shown by the bibliography collected by Inoue and White [1993]: railway policy, meteorology, protection, land policy, cloud formation, gunpowder and lightning, geology, etc. Jevons established a detailed meteorological account of Australia and studied the city of Sydney, and not surprisingly the problem of sanitation received a central place in these investigations. Another study in this context is Jevons' work on 'division of labour' and 'classification of occupations': Jevons wanted to investigate how the interaction of different kinds of labour resulted in 'the industrial mechanism of society'.

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Jevons left Australia in 1859 and returned to University College in London to complete his education. The early 1860s are important for Jevons' intellectual development, and he reports in his diary that he received significant insights in both economics and logic: a 'true comprehension of value' and the 'substitution of similars'. In 1863 *A Serious Fall in the Value of Gold* was published, and for the first time Jevons became recognised as a political economist. Jevons published two important theoretical works: *The Theory of Political Economy* [1871] and *The Principles of Science* [1874]. He drowned in 1882, at the age of 47.

2 THE LAWS OF THOUGHT

Jevons evades the question about the nature of the Laws of Thought when he argues that they are true both "in the nature of thought and things". This question is difficult to decide, and should be investigated by psychologists rather than by logicians. Logicians simply assume the truth of the Laws of Thought and apply these laws in a variety of different circumstances. In this application the truth of the Laws of Thought may be manifested. The profession of the logician is similar to the mathematician's: the latter develops the formal laws of plurality, but does not investigate the nature of unity and plurality [Jevons, 1874, p. 8].

The question remains whether the Laws of Thought are in the human mind (and hence subjective) or in nature (and hence objective). Since science is in the mind and not in the things, scientific laws must be in accordance with the laws that formed them (i.e. the laws of the human mind). This seems to suggest that the Laws of Thought are purely subjective, and only verified in the observation of the external world. However, it is impossible to prove the fundamental laws of logic by reasoning, since they are already presupposed by the notion of a proof. Hence, the Laws of Thought must be presupposed by science as "the prior conditions of all thought and all knowledge". Furthermore, our thoughts cannot be used as a criterion of truth, since we all know that mistakes are possible and omnipresent. Hence, we need to presuppose objective Laws of Thought in order to discriminate between correct and incorrect reasoning [Jevons, 1874, pp. 6–7].

It follows that Jevons (with a reference to Herbert Spencer) tends to regard the Laws of Thought as objective laws. The Laws of Thought are "the prior conditions of all consciousness and all existence", are objectively true in both nature and the mind, but the subjective mind can make mistakes in reasoning. Ultimately, logic deals with both thoughts and things. Directly, the logician deals with symbols, just like the mathematician does. However, these symbols are only instruments of reasoning; and since the outcome of the reasoning must be verified, it follows that symbols and signs only make sense when they correspond to the thoughts and things that they are supposed to express [Jevons, 1874, pp. 7–8].

3 THE SUBSTITUTION OF SIMILARS

Jevons' conception of the Laws of Thought can be summarised by his dictum "Substitution of Similars", the origin of which can be traced in his (published) personal journal. On 18 December 1861 Jevons writes in his journal that he spent the last two years working chiefly on philosophical subjects. "For a year perhaps I have entertained hopes of performing a general analysis of human knowledge, in which the fallacies of words would be as far as possible avoided — and *phil. would be shown to consist solely in pointing out the likeness of things*" [Black, 1973, Vol. 1, p. 179, original emphasis].

The fundamental mental powers for knowledge acquisition are the power to discriminate, to detect identity and to retain. Discrimination occurs in every act of perception, otherwise we would not be able to differentiate between perceptions. Even more important seems the power to identify, or the ability to detect "common elements of sameness" in (past and present) perceptions. The latter power is present in different individuals to a greater or a lesser extent, and "furnishes the true measure of intellect" [Jevons, 1874, pp. 4–5].

The fundamental laws of thought are threefold: the Law of Identity, the Law of Contradiction and the Law of Duality. The first law is described as "Whatever is, is" and implies that a thing is always identical with itself. Jevons does not provide a definition of the concept of "identity" and suggests that the meaning of the word should be explained by providing an example. The second law is the classical law of contradiction: "A thing cannot both be and not be". Jevons traces this law back to Aristotle and argues that a demonstration for "self-evident truths" like this law is not required. (Jevons also argues that all truths cannot be demonstrated, since that would imply an infinite chain of demonstrations). The third law, which originates from Aristotle as well, is known as the law of the excluded middle: "A thing must either be or not be." Jevons suggests that these three laws are merely different aspects of one and the same Law, although it seems impossible to express this Law in less than three lines (Jevons did not live to see the invention of the Sheffer stroke) [Jevons, 1874, pp. 5–6].

The absence of a clear definition of 'identity' is striking, especially since Jevons recognises that there are different kinds and degrees of sameness. "Sameness or identity presents itself in all degrees, and is known under various names; but the great rule of inference embraces all degrees, and affirms that *so far as there exists sameness, identity or likeness, what is true of one thing will be true of the other*" [Jevons, 1874, p. 9, original emphasis]. Jevons recognises that the main problem is to point out a "sufficient degree of likeness or sameness". The simplest form of inference is making use of a pattern, proxy, example or sample. If the sample "exactly represents the texture, appearance, and general nature" of a certain commodity, then what is true for the sample will also be true for the commodity as a whole. This approach evades defining similarity, as it presupposes that the sample is an 'exact representation' of the commodity whereas it is unclear under what conditions that would be the case [Jevons, 1874, p. 9].

Jevons argues that the same mode of reasoning is applied when magnitudes are concerned. In order to compare magnitudes of different objects, we do not need to compare the objects directly (e.g. the height of two churches), but we can do it indirectly using a standard. In this case the sample would be length, but since lengths can be used in calculations, the proxy does not need to be as large as the object. Jevons provides a "beautiful instance of representative measurement": Sir David Brewster used liquids with the same refractive power as a mineral, which is shown by the fragment becoming invisible when put into the liquid. It would then be possible to determine the refractive power of the mineral by measuring the refractive power of the liquid [Jevons, 1874, pp. 10–11].

All processes of inference are based on the principle of substitution. All knowledge is derived from sensual experience, which implies that all knowledge is inductive. Deduction is the inverse process of induction, and both rely upon the nature of identity. "Our first task in this work, then, must be to trace out fully the nature of identity in all its forms of occurrence. Having given any series of propositions we must be prepared to develop deductively the whole meaning embodied in them, and the whole of the consequences which flow from them" [Jevons, 1874, pp. 11–12].

Jevons denotes terms by capital letters A, B, C, etc. and their negative counterparts by small italic letters a, b, c, etc. (see below). The relation of identity or sameness is represented by the sign '='. This sign is already customary in many disciplines (including philology and chemistry), although mathematicians such as De Morgan argue that its use should be restricted to mathematics. Jevons argues that there is an analogy between logical propositions and mathematical equations, and hence the use of the same sign in both cases is justified. The common verb 'is' (or 'are') is too ambiguous. The expression '$A \sim B$' indicates that A and B are not identical with each other; '$A \; \S \; B$' indicates that there exists any relation between A and B, which includes but is not restricted to the relations of equality or inequality. The general formula of logical inference implies that from $A = B \; \S \; C$ we can conclude that $A \; \S \; C$. "In whatever relation a thing stands to a second thing, in the same relation it stands to the like or the equivalent of that second thing". Moreover we can replace a part of a whole by its equivalent and leave the whole unchanged: "Same parts samely related make same wholes". Jevons concludes that making identical copies of parts of a certain object, for instance a house, must result in a new house similar to the original one. The houses would be only "numerically different" [Jevons, 1874, pp. 14–19]. This indicates that the theory of number has an important role to play — see section 10 for an investigation of this topic.

4 TERMS

Jevons defines the concept 'term' in his *Pure Logic*: "*Term* will be used to mean *name*, or any combination of names and words describing the qualities and circumstances of a thing" (PL 7). Jevons distinguishes between the extent and the intent of a term or a name. "The objects denoted form the *extent* of meaning of the term; the qualities implied form the *intent* of meaning". It follows that extent and intent of meaning are negatively correlated: when the intent of meaning increases, the extent of meaning diminishes [Jevons, 1874, pp. 24–27]. In other words, when more qualities are added to the meaning of a term, less objects will correspond to that meaning.

Abstract qualities, denoted by abstract terms, originate when objects are compared and resemblances and differences identified. Abstract terms possess only one kind of meaning. Substantial terms, such as 'gold', denote substances. These terms are not abstract since they denote visible bodies, but they share with abstract terms that they are one and the same everywhere, irrespective of shape or size. The combination of the simple terms A and B is written as AB. The second law of thought — that a thing cannot both be and not be — is represented symbolically by $Aa = 0$. Nothing is hence denoted by the symbol '0', which in logic means "the non-existent, the impossible, the self-inconsistent, the inconceivable" [Jevons, 1874, pp. 27–32].

Jevons lists several "special laws" which govern the combination of terms. The "Law of Simplicity" implies that a term combined with itself has no effect, hence $A = AA = AAA =$ etc. The law of commutativeness indicates that the order of the combination does not matter, hence $AB = BA, ABC = ACB = BCA =$ etc. Throughout the text Jevons refers several times to Boole's use of logical terms and symbols [Jevons, 1874, pp. 33–35].

5 PROPOSITIONS

The truths of science are expressed in the form of propositions. "Propositions may assert an identity of time, space, manner, quantity, degree, or any other circumstance in which things may agree or differ" [Jevons, 1874, p. 36]. Simple propositions $A = B$ express the most elementary judgment regarding identity. Jevons remarks that these simple propositions have no recognised place in Aristotle's logic, although it is impossible not to use them in scientific reasoning (which Aristotle does in at least one place). Aristotle's conclusion that singulars cannot be predicated of other terms is rejected (as it obviously blocks the road towards the quantification of the predicate) [Jevons, 1874, pp. 36–39]. Instead, Aristotle used what Jevons calls 'partial identities' as the foundation of his system. Aristotle's principle of inference implies that what is true of a certain class, is also true of what belongs to that class. Following the quantification of the predicate, several logicians (including Boole) opted for what Jevons calls the "indeterminate adjective" 'some', represented symbolically by 'V'. Jevons rejects the usage of indeterminate symbols and suggests that $A = VB$ (All As are some Bs) should be written as $A = AB$. Propositions of this kind express an identity between a part of B and the whole of A [Jevons, 1874, pp. 40–42]. Propositions of the kind $AB = AC$ express limited identities, since they imply $B = C$ within the limited sphere of things called A [Jevons, 1874, pp. 42–43].

The law of duality implies that everything must either belong to a certain class or to its opposite. Hence, whenever we define a certain term, we are simultaneously defining its corresponding negative term. The symbolic representation of a negative proposition is $A = b$. Given that (according to Jevons) all logical inference consists of the substitution of equivalents (and hence that "difference is incapable of becoming the ground of inference") we should not transform affirmative propositions into their negative counterpart [Jevons, 1874, pp. 43–46]. Due to the law of commutativeness $A = B$ and $B = A$ express the same identity. It is also possible to conclude $B \sim A$ from $A \sim B$, although the relation of differing things is not wholly reciprocal (A does not necessarily differ from B in the same way as B from A — for instance, if A is smaller than B, it follows that B is larger than A) [Jevons, 1874, pp. 46–47].

Propositions can be interpreted in two ways — the qualitative meaning of a name denotes the intent of meaning, whereas the quantitative meaning denotes the extent of meaning. Jevons argues that the primary and fundamental meaning is the qualitative or intent of meaning. Every creation or destruction of individuals belonging to a certain class, denoted by a certain term, changes the extent of meaning of that term, whereas the intent of meaning may remain fixed [Jevons, 1874, pp. 47–48].

6 DIRECT INFERENCE AND DEDUCTIVE REASONING

Direct inference consists of applying the 'substitution of similars' to certain premises in order to arrive at logical conclusions. Immediate inference is the simplest form of inference: from $A = B$ we can infer $AC = BC$ (by substituting B for A in the right side of the identity $AC = AC$). Inference with two simple identities entails that from $B = A$ and $B = C$ we can conclude that $A = C$. According to Jevons the analogy with Euclid's first axiom ("things equal to the same thing are equal to each other") is "impossible to overlook". Jevons discusses several other forms of inference:

- with a simple and a partial identity ($A = B$ and $B = BC$ imply $A = AC$);
- of a partial from two partial identities ($A = AB$ and $B = BC$ imply $A = ABC$);
- of a simple from two partial identities ($A = AB$ and $B = AB$ imply $A = B$);
- of a limited from two partial identities ($B = AB$ and $B = CB$ imply $AB = CB$);
- and miscellaneous forms of deductive inference.

Jevons indicates that traditional syllogistic forms such as Barbara, Celarent, Darii etc. can be represented easily in his logical system. It is also convenient to represent more complicated cases, such as inferences derived from more than two premises. Jevons insists that logical fallacies are impossible when we strictly stick to the rules of the 'substitution of similars' [Jevons, 1874, pp. 49–65].

7 DISJUNCTIVE PROPOSITIONS

Direct inference and deductive reasoning are fairly simply, but disjunctive propositions are a "more complex class of identities". They are employed in the inverse process of class formation, i.e. "distinguishing the separate objects or minor classes which are constituent parts of any wider class". In other words, disjunctive propositions are used whenever an abstract term is 'developed' in its constituent parts or subclasses — whenever the extent of meaning of a term is explored. In order to represent disjunctive propositions, Jevons suggests to use the symbol ' $\cdot|\cdot$ '. The words 'and' and 'or', used in everyday language, can denote exclusive or inexclusive alternatives: Bacon died either in 1284 or 1292 (it can be only one of these years), but an unselfish despot should be either a saint of philosopher (but he could also be both). Where Boole used 'and' to discuss exclusive alternatives, Jevons' symbol ' $\cdot|\cdot$ ' refers to alternatives that are not exclusive. The law of commutativeness holds true for this symbol: $A \cdot|\cdot B = B \cdot|\cdot A$ [Jevons, 1874, pp. 66–71].

The law of unity, $A \cdot|\cdot A = A$, reveals an imperfect analogy between mathematics and logic. In his earliest work on logic Jevons used the symbol '+' instead of ' $\cdot|\cdot$ ', but in *The Principles of Science* he recognises that the analogy between logic and mathematics is imperfect. Using the symbol '+', Jevons' law of unity would imply $x+x=x$. Indeed Jevons writes: "In extent $x+x$ means all xs added to all xs (...)" and if we "take all the xs there can be no more left to add to them" [Grattan-Guinness, 1991, p. 25]. Jevons and Boole corresponded on this issue (see also section 10). Boole, obviously bored with Jevons' letters, writes: "To be explicit, I now however reply, that it is not true that in Logic $x + x = x$ though it is true that $x + x = 0$ is equivalent to $x = 0$" [Grattan-Guinness, 1991, p. 30]. The replacement of '+' by ' $\cdot|\cdot$ ' does not, however, end all problems. In his review of *The Principles of Science*, Robertson [1876, p. 21] was highly critical: "Mr. Jevons is (...) anxious to extrude [the particular symbol +] from logic; but I do not see why it does not tell with equal force against the use of the symbol =, the true fount and origin of the evil against which he finds it thus necessary to protest". The 'imperfect analogy' between logic and mathematics remains problematic in Jevons' work, which can also be seen in Jevons' theory of number (see section 10). The unexclusive disjunction is especially helpful when negating combined terms, since the negative of ABC is $a \cdot|\cdot b \cdot|\cdot c$ (the negative of malleable dense metal is everything which is not malleable, not dense or not a metal, where the 'or' is used nonexclusively) [Jevons, 1874, pp. 71–74].

Considering the law of duality, Jevons first introduces the "fundamental logical axiom that *every term has its negative in thought*". Jevons then rewrites the laws of thought using the newly introduced symbolic apparatus:

Law of identity $A = A$
Law of contradiction $Aa = 0$
Law of duality $A = AB \cdot|\cdot Ab$.

Jevons then considers some examples and then proceeds to inference by disjunctive propositions. Disjunctive terms can be combined with simple terms: $A(B \cdot|\cdot C) = AB \cdot|\cdot AC$. Two disjunctive terms can be combined following $(A \cdot|\cdot B)(C \cdot|\cdot D \cdot|\cdot E) = AC \cdot|\cdot AD \cdot|\cdot AE \cdot|\cdot BC \cdot|\cdot BD \cdot|\cdot BE$. Jevons' system also includes the modus tollendo ponens, i.e. from $A = B \cdot|\cdot C$ it follows that $Ab = Bb \cdot|\cdot Cb$, which (by non-contradiction) implies $Ab = Cb$ [Jevons, 1874, pp. 74–80].

8 INDIRECT INFERENCE

Indirect inference or indirect deduction consists of pointing out "what a thing is, by showing that it cannot be anything else". According to Jevons this is an important method, since "nearly half our logical conclusions rest upon its employment". The simplest form of indirect inference starts from $A = AB$. The law of duality implies that $b = Ab \cdot|\cdot ab$, or by substitution $b = ABb \cdot|\cdot ab$. Since $ABb = 0$ (a contradiction), it follows that $b = ab$. Hence, if a metal is an element, it follows that a non-element is a non-metal. Jevons refers to this conclusion as the "contrapositive proposition" of the original. The contrapositive proposition can be employed in syllogisms such as Camestres, Cesare and Baroko [Jevons, 1874, pp. 81–86].

The contrapositive of a simple identity $A = B$ is $a = ab$, and since $A = B$ implies $B = A$ it also follows that $b = ab$. The two contrapositives taken together let us conclude that $a = b$. The method of indirect inference can be used to describe a class of objects or a term, given certain conditions. The class is first of all 'developed' using the law of duality, then alternative expressions taken from the premises are substituted, and finally all contradictory alternatives are scrapped. The remaining terms may be equated to the term in question [Jevons, 1874, pp. 86–90].

Next Jevons introduces the logical alphabet — a series of combinations that can be formed with a given set of terms. For instance, A and B produce the four combinations AB, Ab, aB and ab. Jevons' table is reproduced below.

Using the logical alphabet, logic becomes simply an exercise of fully developing all terms and eliminating the contradictory terms. However, when the amount of letters increases, the amount of possible combinations becomes considerable. Jevons considers some techniques and devices to facilitate these endeavours, such as a 'Logical slate' (the logical alphabet engraved upon a school writing slate). Nevertheless, when more than six terms are involved, it becomes almost impossible to solve the problem [Jevons, 1874, pp. 91–96]. To facilitate this kind of reasoning Jevons developed a logical abacus, which operates on simple mechanical principles. It can be seen as one of the first computers.

Figure 1. The Logical Alphabet

I.	II.	III.	IV.	V.	VI.	VIII.	
X	AX	AB	ABC	ABCD	ABCDE	ABCDEF	
	aX	Ab	ABc	ABCd	ABCDe	ABCDEf	
		aB	AbC	ABcD	ABCdE	ABCDeF	
		ab	Abc	ABcd	ABCde	ABCDef	
			aBC	AbCD	ABcDE	ABCdEF	
			aBc	AbCd	ABcDe	ABCdEf	
			abC	AbcD	ABcdE	ABCdeF	
			abc	Abcd	ABcde	ABCdef	
				aBCD	AbCDE	ABcDEF	
				aBCd	AbCDe	ABcDEf	
				aBcD	AbCdE	ABcDeF	
				aBcd	AbCde	ABcDef	
				abCD	AbcDE	ABcdEF	VIII (cont.)
				abCd	AbcDe	ABcdEf	aBcDEF
				abcD	AbcdE	ABcdeF	aBcDEf
				abcd	Abcde	ABcdef	aBcDeF
					aBCDE	AbCDEF	aBcDef
					aBCDe	AbCDEf	aBcdEF
					aBCdE	AbCDeF	aBcdEf
					aBCde	AbCDef	aBcdeF
					aBcDE	AbCdEF	aBcdef
					aBcDe	AbCdEf	abCDEF
					aBcdE	AbCdeF	abCDEF
					aBcde	AbCdef	abCDeF
					abCDE	AbcDEF	abCDef
					abCDe	AbcDEf	abCdEF
					abCdE	AbcDeF	abCdEf
					abCde	AbcDef	abCdeF
					abcDE	AbcdEF	abCdef
					abcDe	AbcdEf	abcDEF
					abcdE	AbcdeF	abcDEf
					abcde	Abcdef	abDeF
						aBCDEF	abcDef
						aBCDEf	abcdEF
						aBCDeF	abcdEf
						aBCDef	abcdeF
						aBCdEF	abcdef
						aBCdEf	
						aBCdeF	
						aBCdef	

9 INDUCTION

Induction is the inverse process of deduction, but it is a much more complicated mode of reasoning. Whereas deduction consists of deriving several results from certain given laws, induction attempts to do the opposite. Induction proceeds according to certain rules of thumb, trial and error, and past attempts. Jevons draws an analogy with mathematics: whereas it is fairly simple to differentiate very complex functions, it may be almost impossible to integrate even relatively simple expressions. General laws are usually rather simple and it is therefore relative easy to derive conclusions from them. Starting from a finished result is much more complicated, since several laws are mixed up in the production of the result [Jevons, 1874, pp. 121–127].

Induction of simple identities becomes very complex as soon as more than just a few terms are involved. Induction of partial identities starts from a certain premise in disjunctive form $A = B \cdot|\cdot C \cdot|\cdot D \cdot|\cdot \ldots \cdot|\cdot P \cdot|\cdot Q$, and then we need propositions that ascribe a certain property to all individuals: $B = BX, C = CX, \ldots, Q = QX$. Substituting and rearranging yield the desired result $A = AX$. According to Jevons this is the most important scientific procedure, as "a great mass of scientific truths consists of propositions of this form $A = AB$" [Jevons, 1874, pp. 127–131].

Jevons distinguishes between perfect and imperfect induction. Perfect induction can be done when all individuals belonging to a class can be examined; otherwise we need to resort to imperfect induction. Although most scientific endeavours will encounter uncertainty and hence require imperfect induction, in some cases perfect induction is very useful, i.e. whenever our investigation is limited to a restricted amount of observations (e.g. all the bones of a certain animal, all the caves in a mountain side). Although it can be argued that this perfect induction consists of nothing more than a summing up of already known information, it still leads to an abbreviation of mental labour, which is important for knowledge acquisition. Jevons' section on induction ends with a brief discussion of the problem of induction — we can never be sure to predict the future based on past knowledge. In order to continue the discussion of this topic, Jevons needs to bring in principles of number and theory of probability [Jevons, 1874, pp. 146–152].

10 PRINCIPLES OF NUMBER AND PHILOSOPHY OF MATHEMATICS

Jevons' principles of number reflect his insistence that mathematics should be based on logic, not the other way round. Grattan-Guinness [1988] identifies two traditions in the interaction between mathematics and logics: the algebraic tradition includes Boole and De Morgan; the mathematical tradition Peano and Russell. Boole gave mathematics priority over logic, whereas Russell tried to found mathematics on Peano's mathematical logic. Jevons occupied a somewhat contradictory position in between of these opposites: he tried to found mathematics on logic, but his form of logic was inspired by the works of Boole and De Morgan.

Jevons attempts to define 'number' by counting 'units' in space or time. When counting coins, every coin should receive a proper name: we should count $C' + C'' + C''' + C'''' + \ldots$. The coins are equal to each other (they all belong to the class C); they are different only because they reside on different points in space. Before counting, we should reduce all identical alternatives; the remaining 'units' reside on different points in space and time. "A unit is any object of thought which can be discriminated from every other object treated as a unit in the same problem" [Jevons, 1874, p. 157]. The concept of 'unit' encounters some severe difficulties, as Frege notes : "If we use 1 to stand for each of the objects to be numbered, we make the mistake of assigning the same symbol to different things. But if we provide the 1 with differentiating strokes, it becomes unusable for arithmetic" [Frege, 1884, p. 50, translated by J. L. Austin]. In other words, we can only add up Cs that are identical, but they cannot denote different things if the same symbol C is used. Jevons was unable to resolve this contradiction. Jevons' problem to define 'similarity' blocks the establishment of a genuine definition of a 'unit', as Frege's criticism shows.

Moreover, similarity as such does not provide a satisfactory explanation, as Hempel and Oppenheim [1948, p. 323] note: "The same point may be illustrated by reference to W. S. Jevons' view that every explanation consists in pointing out a resemblance between facts, and that in some cases this process may require no reference to laws at all and 'may involve nothing more than a single identity, as when we explain the appearance of shooting stars by showing that they are identical with portions of the comet'. But clearly, this identity does not provide an explanation of the phenomenon of shooting stars unless we presuppose the laws governing the development of heat and light as the effect of friction. The observation of similarities has explanatory value only if it involves at least tacit reference to general laws". Jevons did not bridge the gap between particular entities and the abstract notion of number, or between particular facts and a general law. Jevons had a logical positivist *attitude* (the project of a Unified Science, the reduction of the laws of thought to one fundamental expression, logic as the base of knowledge), but he used a logic from the algebraic tradition whereas a mathematical logic would be required.

Neurath [1983, p. 67] praises Jevons' mechanical logic: "All this [physicalistical expression of equivalence] could be developed experimentally with the help of a 'thinking machine' as suggested by Jevons. Syntax would be expressed by means of the construction of the machine, and through its use, logical mistakes would be avoided automatically. The machine would not be able to write the sentence: 'two times red is hard'." On the other hand, Neurath [1970, pp. 1–27] criticizes Jevons for not applying his reasonings to all the sciences: "But neither Mill nor other thinkers of similar type [including Jevons] applied logical analysis consistently to the various sciences, thus attempting to make science a whole on 'logicalized' basis."

11 COMPARISON WITH BOOLE

It is an interesting exercise to compare Jevons' ideas about logic with those of George Boole. It has already been mentioned above that Jevons often used the same kind of notation as Boole did. Both wrote 'AB' when referring to the conjunction of A and B, and wrote '$A + B$' when referring to the disjunction of A and B. However, Boole and Jevons were not the first who realized that there are certain similarities between the multiplication of numbers and the logical conjunction, or between the addition of numbers and the logical disjunction. In algebra, it was already customary to write '$A.B$' or just 'AB' to indicate the multiplication of A and B. And in ordinary language, it is perfectly normal to write 'a big black box' to refer to a box that is big and black. So a simple succession of symbols was used in algebra for the multiplication of numbers, and in ordinary language for the conjunction of certain properties.

Boole, however, realized that the analogy between algebra and logic runs much deeper than this. It is not only possible to use the same kind of notation for algebra and logic, it also possible to use similar rules in algebra and logic. For instance, Boole uses the following premises in his 'algebra of logic' [Kneale and Kneale, 1962, p. 412].

1. $xy = yx$
2. $x + y = y + x$
3. $x(y + z) = xy + xz$
4. $x(y - z) = xy - xz$
5. If $x = y$, then $xz = yz$
6. If $x = y$, then $x + z = y + z$
7. If $x = y$, then $x - z = y - z$

It is obvious that there is, for each of these seven rules, an analogous rule in numerical algebra. In Boole's algebra of logic, however, these rules express logical truths. In this algebra of logic, the third premise, for instance, would be interpreted as 'the class of the objects that belong to class x, and that belong either to class y or to class z, is identical to the class of the objects that either belong to class x and class y, or belong to class x and class z'. In the fourth premise, the expression $y - z$ should be interpreted as 'the class of all objects that belong to class y, but do not belong to class z'.

However, Boole also uses an eighth premise in his algebra of logic:

8. $x(1 - x) = 0$

The symbol '1' in this premise refers to what De Morgan called the 'universe of discourse', the class of all objects that are under discussion at a certain moment.

The symbol '0' refers to the so-called 'null class', which contains no objects at all. It follows that the expression '$1 - x$' refers to the class of all the objects under discussion that do not belong to class x. So premise (8) should be interpreted as the logical truth that there is no object that belongs to class x and does not belong to class x.

The rule $x(1 - x) = 0$ is not generally true in numerical algebra. So it would seem that the analogy between algebra and logic is not perfect and breaks down at this point. It should be noted, however, that premise (8) is true in a so-called 'two-valued algebra'. This is an algebra in which there are only two numbers or values, viz. 0 and 1.

Now suppose that a new rule is introduced to the system:

9. Either $x = 1$ or $x = 0$

In numerical algebra, this rule is equivalent to premise 8. So the system is not really changed if we interpret it as a system about numerical algebra. It is still possible to interpret the system as describing a two-valued algebra. Indeed, in this interpretation, rule (9) states explicitly that an arbitrary variable can have only two values, viz. 0 and 1.

But how should the new system, including rule (9), be interpreted as a logical system? It is not true that every class is either the universe of discourse or the null class. (In fact, this would only be true if the universe of discourse contains but a single object, and in this case, the development of an elaborate logical system would seem rather pointless.) Boole suggests another logical interpretation: $x = 1$ means that proposition X is true, and $x = 0$ means that proposition X is false. In this way, Boole's algebra of logic can be interpreted in terms of the 'truth-values' of propositions.

It should be mentioned, however, that Boole did not include rule (9) in his system. He just used the system with eight premises and realized that it can be interpreted in many different ways: in terms of classes of objects, in terms of the truth-values of propositions, and as a two-valued algebra. In his book *The Laws of Thought* he points out that there is still another possible interpretation, namely an interpretation in terms of probabilities. This interpretation will not be discussed here.

Boole wrote two monographs on his algebra of logic: *The Mathematical Analysis of Logic* [1847], and *The Laws of Thought* [1854], or, more precisely, *An Investigation of the Laws of Thought, on which are founded the Mathematical Theory of Logic and Probabilities.* The titles of these texts are, however, somewhat misleading.

The title '*The Mathematical Analysis of Logic*' might seem to imply that Boole regarded logic as a branch of mathematics. This is, however, not the case. Although Boole used mathematical symbols (such as '+', '=', '0' and '1') in his algebra of logic, and although certain inference rules in his system can be interpreted most easily as rules about numbers (see, for instance, premises (5), (6) and (7) above), he did not think that logic should be founded on mathematics. At first sight, Boole's system seems to be an indication that logic (as a theory about the truth-values of certain propositions) is just mathematics restricted to two values, viz. 0 and 1. It should be realized, however, that it is, in this transition from mathematics to logic, necessary to re-interpret almost all symbols used in Boole's system. The interpretation of the symbol '1' in numerical algebra (as the number one) seems to be unrelated to the interpretation of this symbol in logic (as the truth-value 'true' of a proposition). Boole was, unlike Jevons, not a reductionist. He did not want to reduce logic to mathematics. He merely pointed out a deep analogy between mathematics on the one hand, and logic on the other hand.

The title 'The *Laws of Thought*' is also somewhat misleading. Although Boole himself might have believed that he was describing certain 'laws of thought', it is, in retrospect, one of Boole's greatest achievements that he freed logic from all sorts of epistemological and psychological considerations. By developing his calculus of logic, Boole showed that it is possible to study logic as a separate science, without reference to thought processes. The fact that his algebra of logic can be interpreted in many different ways and that no fixed meaning can be attached to the inference rules of this algebra, is an indication that Boole's system cannot be used to describe universal 'laws of thought'.

12 THEORY OF PROBABILITY, STATISTICS AND ECONOMETRICS

Given the problems that we encountered above, the role and importance of Jevons' system of logic and philosophy of mathematics seems to be limited. It seems to be limited to a pedagogical aspect: Jevons' writings on logic, such as his *Elementary Lessons in Logic*, were widely used as textbooks and saw numerous reprints, up to decades after his death. This appraisal would not, however, do justice to Jevons' most important achievement: the introduction of statistics and econometrics in the social sciences and the use of empirical data.

Stigler [1982, pp. 354–7] argues that statisticians in the first part of the 19th century were concerned with the collection of data, but not with analysis. The data suggested too many different causes, and the hope to establish a Newtonian social science using statistics faded away. Statistical journals published tables and numbers, but graphical representations and analysis remained absent. Jevons' interest in empirical economic work is probably derived from meteorology, another field in which he was active and for which he collected data and drew diagrams. In 1863 Jevons' use of empirical methods in economics resulted in a first practical survey: A *Serious Fall in the Value of Gold*. This survey studied the influence of Australian and Californian gold discoveries of 1851 on the value of gold. For this purpose he compared the prices since 1851 with an average price drawn from the previous fluctuation of 1844–50, in order to eliminate fluctuations of price due to varying demand, manias for permanent investment and inflation of credit. The investigation showed that prices did not fall to their old level after a revulsion, which indicates a permanent depreciation of gold after the gold discoveries. Prices rose between 1845–50 and 1860–62 by about ten per cent, which corresponds to a depreciation of gold of approximately 9 per cent [Jevons, 1884, pp. 30–59]. Stigler [1982, pp. 357–61] states that Jevons' methodology is remarkable and novel for his time. The survey computes, for 39 major and 79 minor commodities, the ratio of the average 1860-2 price to the average 1845-50 price. A diagram with a logarithmic scale reveals that 33 of the 39 major commodities and 51 of the 79 minor commodities encountered a rise in price. The 9 per cent gold depreciation is calculated using a geometric mean of the price changes. The use of the geometric mean prevents that large values receive disproportionate weights. In 1863 the choice of the geometric mean relies on intuition, and in later publications on inadequate explanations. One of these explanations is statistical, saying that multiplicative disturbances will be balanced off against each other using the geometric mean. There is however no empirical verification of this 'multiplicative disturbances' hypothesis. Stigler [1982, pp. 362–4] regards the absence of a probabilistic analysis and the measurement of the remaining uncertainty in the averages as an anomaly in Jevons' work. But it should nevertheless be seen as a milestone in the history of empirical economics, because his conceptual approach opened the ways for a quantification of uncertainty and for the development of statistics for the social sciences.

Aldrich [1987, pp. 233–8] denies that Jevons has no interest in probability. Quite the contrary is the case, as his *Principles of Science* contains an elaborate discussion of probability. Jevons did not use the laws of probability to describe the behaviour of empirical entities, but rather as rules for the regulation of beliefs. Probability enters when complete knowledge is absent, and it is therefore a measure of ignorance. Aldrich [1987, pp. 238–51] argues that Jevons used probability in two main patterns of argument: in the determination whether events result from certain causes or are rather coincidences, and in the method of the least squares. The first approach entails the application of the 'inverse method' in induction: if many observations suggest regularity, then it becomes highly improbable that these result from mere coincidence. An application of this principle can be found in *A Serious Fall*, where Jevons concludes that a large majority of commodities taken into consideration show a rise of price, and therefore a rise in exchange value relative to gold. The 'inverse inductive method' leads to the conclusion that a depreciation of gold is much more probable than mere coincidences leading to the rise of prices. The second approach, the method of the least squares, appears when Jevons tries to adjudge weights to commodities (giving more weight to commodities that are less vulnerable to price fluctuations), and when he tries to fit empirical laws starting from an a priori reasoning about the form of the equation. These methods show at least some concern for probability and the theory of errors. But Jevons worked on the limits of his mathematical understanding, and many ideas that he foreshadowed were not developed until decades after his death. *A Serious Fall* is not so much remembered for its limited use of probability theory, but rather for its construction of index numbers. In his *Principles of Science* Jevons refers several times to Adolphe Quetelet. Elsewhere I elaborate on Quetelet's influence on Jevons' writings [Mosselmans, 2005].

13 CONCLUSION

We conclude that Jevons' work occupied a somewhat contradictory position in the history of logic and philosophy of science. He had a logical positivist attitude, tried to build up a unified science and wanted to reduce mathematics to logic. But the form of his logic (taken from Boole) was not compatible with these ambitions, as can be seen in his failed attempts to define concepts such as 'similarity' and 'number'. His works on logic were widely used as textbooks, but he did not found a 'school' in the history of logic. This may also be partly explained by his early death. Jevons' most important contribution to the history of science lies in the introduction of statistics and empirical methods in the social sciences. Contemporary research in applied economics and econometrics owes a lot to this methodological pioneer (a 'logical positivist *avant la lettre*').

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HUGH MACCOLL AND THE BIRTH OF LOGICAL PLURALISM

Shahid Rahman and Juan Redmond

INTRODUCTION

Hugh MacColl (1837–1909) was a mathematician and logician who was born, raised and educated in Scotland and after a few years working in different areas of Great Britain moved to Boulogne-sur-Mer (France), where he developed the greater part of his work and went on to become a French citizen. Hugh MacColl was well known in his time for his innovative contributions to logic. Despite the fact that one can hardly say that his work satisfies the standards of rigour of Frege's philosophy of mathematics and logic it might represent one of the first approaches to logical pluralism. His first contribution to the logical algebras of the 19^{th} century was that his calculus admits not only a class interpretation (as in the algebra of Boole) but also a propositional one. Moreover, MacColl gave preference to the propositional interpretation because of its generality and called it *pure logic*. The main connective in his pure logic is the conditional and accordingly his algebra contains a specific operator for this connective. In *Symbolic Logic and its Applications* (1906) MacColl published the final version of his logic(s) where propositions are qualified as either certain, impossible, contingent, true or false. After his death his work suffered a sad destiny. Contrary to other contemporary logicians such as L. Couturat, G. Frege, W.S. Jevons, J. Venn, G. Peano, C. S. Pierce, B. Russell and E. Schröder, who knew MacColl's work, his contributions seem to have received neither the acknowledgement nor the systematic study they certainly deserve. Moreover, many of his ideas were attributed to his successors; the most notorious examples are the notion of strict implication, the first formal approach to modal logic and the discussion of the paradoxes of material implication normally attributed to C. I. Lewis. The same applies to his contributions to probability logic (conditional probability), (relational) many-valued logic, relevant logic and connexive logic. Less known is the fact that he also explored the possibilities of building a formal system able to handle reasoning with fictions. The latter seems to be linked to his formal reconstruction of Aristotelian Syllogism by means of connexive logic.

Two main factors might have been determinant for the fact that his work fell into oblivion. One is related to technical issues and the other to his philosophical position.

Handbook of the History of Logic. Volume 4: British Logic in the Nineteenth Century.
Dov M. Gabbay and John Woods (Editors)

The strong influence of the logistic methodology triggered by the work of Frege immediately after MacColl's death accounts for the first factor. Indeed, the logistic methods of presenting a logical system as a set of axioms closed under a consequence relation initiated by Frege and further developed by Peano, Russell and others rapidly replaced the algebraic methods of calculation of the 19^{th} century employed by MacColl.

The second factor relates to his philosophy of logic. MacColl's philosophical ideas were based on a kind of instrumentalism which was extended beyond both of the mainstream paradigms of formal logic in the 19^{th} century, namely mathematics as logic (logicism) and logic as algebra (Boole's algebraic approach). It is interesting to note that his philosophical position could be linked to French conventionalism and instrumentalism such as developed by his younger contemporaries Henri Poincaré and Pierre Duhem and the American pragmatism of Charles Saunders Peirce rather than to empiricism or logicism. MacColl was probably the first to explicitly extend conventionalism and instrumentalism to logic.

Beyond his scientific contributions, MacColl had literary interests. Following the spirit of the century, he published two novels, *Mr. Stranger's Sealed Packet* (1889) and *Ednor Whitlock* (1891) and the essay *Man's Origin, Duty and Destiny* (1909). The first is a novel of science fiction, namely a voyage to Mars. In fact it was the third novel about Mars to be published in English. In his two last works MacColl discusses the conflicts between science and religion and the related problems of faith, doubt, and unbelief.

It is impossible to resist the temptation to compare MacColl's contributions to science with his literary incursions. MacColl penned both one of the most conservative Victorian books on science fiction of his time and one of the most innovative proposals on logic of the 19^{th} century.

Let us trace back, briefly, the revival of the interest in MacColl's work. During the nineteen-sixties Storrs McCall (1963–1967) drew attention to MacColl's work particularly in relation to the paradoxes of material implication, connexive logic and MacColl's reconstruction of Aristotle's Syllogism.[1] Between the eighties and nineties at the Erlangen-Nürnberg Universität, Christian Thiel and his collaborators rediscovered the work of our author in the context of a research project on the social history of logic in the 19^{th} century. The project led to the first systematic historic explorations into the work of Hugh MacColl by Christian Thiel [1996], Volker Peckhaus [1966] and Anthony Christie [1986; 1990]. The Erlanger group motivated Michael Astroh to deepen the research on Hugh MacColl's logic and philosophy of language. [1993; 1995; 1996; 1999a; 1999b], as then did Shahid Rahman [1997a; 1997b; 1999; 2000]. Together Michael Astroh and Shahid Rahman conceived a workshop, finally organized in Greifswald, on "Hugh MacColl and the Tradition of Logic" with the participation among others of Stephen Read, Peter Simons, Volker Peckhaus, Göran Sundholm, Christian Thiel and Ian Woleński. The workshop yielded a special volume of the *Nordic Journal of Philosophical Logic*, edited by Michael Astroh and Stephen Read [1999]. This volume constitutes the

[1]See too Spencer [1973].

main secondary source of our paper.

BIOGRAPHICAL NOTES

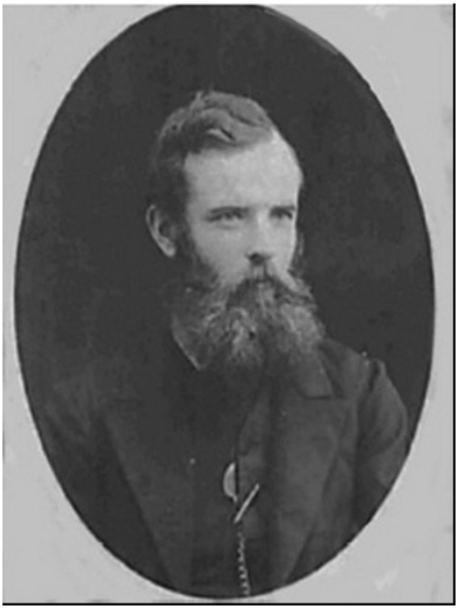

Figure 1. MacColl in 1886[2]

Unfortunately there are no precise records about MacColl's life. We will rely on the information contained in MacColl's second marriage contract, signed by himself. We follow here the strategy used in the biography of MacColl published in 2001 by M. Astroh, I. Grattan-Guinness and S. Read which in addition to Rahman's paper [1997b] contains the main contents of the present biographical outline.

Hugh MacColl was born in Strontian, Argyllshire, on 11 or 12 January 1837. He was the son of John MacColl and Martha Macrae. Hugh was their youngest child. He had three brothers and two sisters. The father was a shepherd and tenant-farmer from Glencamgarry in Kilmalie (between Glenfinnan and Fort William) who had married Martha Macrae in the parish of Kilmalie on 6 February 1823.

The early loss of Hugh's father had a strong impact on the whole family. The father was 45 and the little Hugh only three. After he passed away, the mother and all the children moved on to Letterfearn and from there to Ballachulish. It was only from that moment that the children learned English, since their mother only spoke Gaelic. In 1884 Hugh's elder brother Malcolm, then aged nine, was

[2]by courtesy of Michael Astroh.

living alone with his aunt and grandmother. Malcolm did so well at school that a wealthy lady paid for him to attend a seminary in Dalkeith, near Edinburgh, where schoolteachers were trained. He taught in different places and was ordained priest in St Mary's, Glasgow by the Bishop of Glasgow in August 1857.

At this time Malcolm was supporting his younger brother Hugh in his studies. But unfortunately this aid finished before Hugh completed them: his brother became involved in a dispute which divided the Episcopal Church in 1857-1858, and since he refused to support the Bishop's position he was consequently dismissed from his post.

From 1858 Hugh took up various positions as a schoolmaster in Great Britain, until he left the country a few years later. In 1865 he moved to Boulogne sur Mer (France) and settled there for the rest of his life. The reasons why he left his country are unknown but it is not difficult to imagine economic motivations. If we take account of the immense emigration from Great Britain in the 19^{th} century, it is not so difficult to imagine changing country a sensible option. At that time Boulogne-sur-Mer was a prospering town with close economic and cultural links with Britain. Thus, a pleasant place for people who left Great Britain.

Before MacColl left he married Mary Elisabeth Johnson of Loughborough in Leicestershire. She moved with him to France, where in April 1866 their first daughter Mary Janet was born. All five children, four girls and a boy, were born there.

During these years the economic situation of the MacColls was very vicissitudinous and often precarious. Hugh worked principally as a private instructor teaching mathematics, English and logic.

In contrast with this description, an obituary published in *La France du Nord* on 30 December 1909 depicts MacColl as a teacher of the *Collège Communal.* However it has not been possible to identify MacColl as one of the members of the teaching staff because he did not appear in the college's advertisements and brochures.

But the economic restrictions and the growing family did not prevent him from continuing to study. He prepared for and took a BA in Mathematics as an external student at the University of London in 1876.[3]

From his frequent publications in different scientific journals and the interchange of letters with same of the most famous men of science of his time, we can presume that MacColl hoped for recognition of his achievements in logic. We can imagine too that he hoped for a university post. The latter is on record in a letter addressed to Bertrand Russell in 1901 in which MacColl, at the age of 64, recommends himself as a lecturer in logic.[4]

His first wife Mary Elisabeth died on 2 February 1884 after a long disease. Three years later, on 17 August 1887, MacColl married Mlle Hortense Lina Marchal, who came from Thann (Alsace). For MacColl it was an economic and social improvement of his condition, especially due to the regular financial position of

[3]University of London. 1877. *Calendar for the Year 1877.* London: Taylor and Francis.
[4]MacColl [1901c].

his new wife's family. Hortense and her sister Mme Busch-Marchal were running a well-known '*pensionnat de jeunes filles*' in a selected area of Boulogne. The parents of Hortense were living on a private income, while her brother Jules Marchal and her brother-in-law Gustave Busch were shopkeepers at Boulogne. The couple built up a harmonious family life with a solid economic basis.

In the successive years after his first wife's death, MacColl abandoned his customary publications for a literary interest. In 1888 and 1891 he published two novels: *Mr Stranger's Sealed Packet* and *Ednor Whitlock*, respectively. The former is a work of science fiction.

Even though in standard books of history of logic one can rarely find a systematic study or even description of his work, in his time his scientific contributions were widely discussed. This is testified by his publications and the scientific interchange with among others, Bertrand Russell and Charles Sanders Peirce. Actually, at least from 1865 onwards MacColl contributed to different prestigious journals such as: *The Educational Times and Journal of the College of Preceptors*, *Proceedings of the London Mathematical Society*, *Mind*, *The London, Edinburgh and Dublin philosophical Magazine and Journal of Science* and *L'Enseignement Mathématique*.

His ideas were discussed and quite often severely criticized as well. A very fruitful discussion took place between MacColl, Russell and T. Sherman on the existential import of propositions. Unfortunately, many influential logicians coming from Boole's tradition such as W. S. Jevons, were hostile to MacColl's innovations. In an article from 1881, Jevons criticized the propositional formulation of the conditional "if. . . , then. . . " presented by MacColl in the paper "Implicational and equational logic" from the same year.[5] MacColl writes there

> *Friendly contests are at present waged in the 'Educational Times' among the supporters of rival logical methods. I hope Prof. Jevons will not take it amiss if I venture to invite him to enter the lists with me, and there make good the charge of "ante-Boolian confusion" which he brings against my method.*

The answer from Jevons came without delay:

> *- It is difficult to believe that there is any advantage in these innovations* [...]. *His proposals seem to me to tend towards throwing Formal Logic back into its Ante-Boolian confusion* [...] *I certainly do not feel bound to sacrifice my peace of mind for the next few years by engaging to solve any problems which the ingenuity and leisure of Mr. MacColl or his friends may enable them to devise.*[6]

Actually, the point is, precisely in relation to the conditional, that MacColl was searching for a logical formulation of the conditional compatible with the notion of

[5]MacColl [1880o], paragraph 43.
[6]Jevons [1881], p. 486.

hypothetical judgements (of the philosophical tradition [Rahman, 2000]. A notion of conditional which people like Jevons and Venn, as we will discuss further on, wanted to get rid of.

Gottlob Frege too knew about the work of MacColl (but unfortunately not the contrary). He compares his *Begriffsschrift* with the works of Boole and MacColl in the paper *Über den Zweck der Begriffsschrift*,[7] where he criticizes the Achilles' heel of MacColl's project, the lack of precise notion binding the propositional with the first-order level.

Ernst Schröder, who in his famous *Vorlesungen über die Algebra der Logik* quotes and extensively discusses MacColl's contributions, first had quite a negative impression of MaColl's innovations though later he seems to have changed his mind, conceding that MacColl's algebra has a higher degree of generality and simplicity in particular in contexts of applied logic. What Schröder definitively rejects is MacColl's propositional interpretation of the Aristotelian Syllogistic.

C. Ladd-Franklin, in those days a very well-known disciple of Peirce, accuses the British scientific community of having ignored MacColl's contribution to the formalization of universal propositions with help of a conditional:

> *The logic of the non-symmetrical affirmative copula, "all a is b", was first worked out by Mr. Maccoll. Nothing is stranger in the recent history of Logic in England, than the non-recognition which has befallen the writings of this author.* [...], *it seems incredible that English logicians should not have seen that the entire task accomplished by Boole has been accomplished by Maccoll with far greater conciseness, simplicity and elegance. . .*[8]

In fact, most of the positive comments did not come from Great Britain: in his *Formulaire de Mathématique* of 1895 G. Peano acknowledges his debt to MacColl's propositional logic. The case is similar to G. Vailati and L. Couturat's lines on MacColl published in 1899 in volume VII of the *Revue de métaphisique et morale.* Couturat works out MacColl's propositional logic, though he leaves out his modal and probability logic:

> *Ceci n'est vrai que pour les propositions à sens constant, qui sont toujours vraies ou toujours fausses, mais non pour les propositions* **à sens variable**, *qui sont tantôt fausses, en d'autres termes, qui sont* **probables**. *C'est ce qui explique la divergence entre le Calcul logique que nous exposons ici et le* **Calcul des jugements équivalents** *de M. MacColl, fondé sur la considération des probabilités***.**[9]

The most influential reception of MacColl's work is certainly the one contained in C. I. Lewis' development of strict implication and formal modal logic.

[7] Frege [1882], p. 4 or p. 100 in Angelleli's edition of 1964.
[8] Ladd-Franklin [1889].
[9] Couturat [1899], p. 621.

But at the end of the 1890s, the London Mathematical Society refused to publish further contributions by MacColl. Searching for another way to present the final developments of his logic, he attended to the *First International Congress of Philosophy*[10] in Paris (1901) and in further publications of *L'Enseignement Mathématique*[11]. A few years later he published an extended English version in a book: *Symbolic logic and its Applications* (1906). Three years after that he published *Man's Origin, Destiny and Duty*,[12] an essay with his ideas about science and religion. As we will see in our appendix the image of MacColl we can draw from these works of literature contrasts with the image of the innovative and tolerant logician of his scientific work. Sadly, as is so often the case in the history of science, his ideas on politics, society and ethics did not stand on the same high level as his open-minded spirit in logic.

MacColl died in Boulogne on 27 December 1909 as a French citizen. Hortense died on 13 October 1918.

1 THE ELEMENTS OF MACCOLL'S PHILOSOPHY OF LOGIC AND LANGUAGE

> *There are two leading principles which separate my symbolic system from all others. The first is the principle that there is nothing sacred or eternal about symbols; that all symbolic conventions may be altered when convenience requires it, in order to adapt them to new conditions, or to new classes of problems* [...]. *The second principle which separates my symbolic system from others is the principle that the complete statement or proposition is the real* unit *of all reasoning.*
>
> (Symbolic Logic and Its Applications, 1906a, pp. 1-2).

> *Symbolical reasoning may be said to have pretty much the same relation to ordinary reasoning that machine-labour has to manual labour* [...]. *In the case of symbolical reasoning we find in an analogous manner some regular system of rules and formulae, easy to retain* [...], *and enabling any ordinary mind to obtain by simple mechanical processes results which would be beyond the reach of the strongest intellect if left entirely to its own resources.* (Symbolic Logic I [1880p], S. 45).

> *Mais la logique symbolique fait pour la raison ce que fait le télescope ou le microscope pour l'œil nu.*
>
> (La logique symbolique, 1903d, p. 420)[13]

As already mentioned in the introduction MacColl's philosophy is a kind of instrumentalism in logic which led him to set the basis of what might be considered

[10] MacColl [1901f].
[11] MacColl [1903e], MacColl [1904j].
[12] MacColl [1909a].
[13] Cf. Frege's almost identical remark in his *Begriffsschrift*, xi.

to be the first pluralism in logic[14]. The point condensed in the epigraph amounts to the following: it could well be that in some contexts of reasoning the existing argumentation demands a type of logic which is not applicable in others. When constructing a symbolic system for a particular type of logic, the corresponding expressions in use should therefore be taken into careful consideration. Actually this position of his is based on a pragmatic notion of statement and proposition where the communicative aspect of signs (signs used to convey information) is at the centre of his philosophy. This aspect of MacColl's philosophy of language leads him to explore the possibilities of a formal system sensitive enough to capture the nuances and features of natural language. The result of these explorations are impressive: the use of restricted domains to render the formal counterpart of the grammatical notion of subject, the use of terms for individuals and individual concepts, the distinction between the negation of propositional formulae and the negation of predicates, the use of both a predicate for existence and a predicate for non-existence, the criticism of the paradoxes of material implication in the framework of modal logic and strict implication, many-valued logic, probability logic, relevant logic, connexive logic. Unfortunately, most of his ideas were not developed thoroughly and many others remained in a state which makes them difficult to grasp. One of the main reasons for the unfinished character of his work is linked to the lack of an appropriate technical framework able to implement his various ideas. Not only did MacColl not know the axiomatic approaches to logic, he did not realize the insight provided by quantifiers (and bound variables), despite the fact that his formal language contains operators which come very close to the notion of (restricted) quantifiers. Moreover, there is a fundamental tension in his notion of statement and proposition between a semiotic and pragmatic approach and a semantic one which is not solved and which in some passages confronts the reader with a hard interpretative task. In general it is quite difficult to render a systematic description of his work which is subject to unexpected and manifold changes. Nevertheless, some critics might judge more mildly if we came to a better understanding of his instrumentalist conception of formal language. MacColl's formal language is essentially context-bound in the sense that the logical notation cannot be read independently of the informal context in which the formal instruments are to be applied. MacColl's language is not a universally applicable ready-made notation, but rather a flexible system conceived so as to adapt to different contexts and built over a tacit (metalogical) background of knowledge of the environment involved. It is, in fact, a framework rather than a system. We will continue to use the word *system* following MacColl's own use, but it is important to always remember that MacColl's logical language is indeed a formal framework. When he is studying natural language the framework takes on the features of a formal grammar — though his main approach is that of a logician rather than that of a linguist.

In the following paragraphs we will try to outline the path that led MacColl from his notion of statement to his various proposals for innovating logic.

[14]Cf. Grattan-Guinness [1999].

1.1 Two Basic Notions in MacColl's Ideas on a Formal Grammar: Statements *and* Propositions

MacColl presents the final version of his system of logic in his book *Symbolic logic and its applications* of 1909 which will be the main source of our discussion.

In MacColl's view a *statement* is any *sound*, *sign* or *symbol* employed to convey information, and a *proposition* is a statement which, in regard to form, may be divided into two parts respectively called subject and predicate. It seems that according to MacColl a symbol is a sign in an artificial language and expressions for propositions are the main symbols of the artificial language. Here *artificial* includes natural language, that is, *artificial* is meant here as a cultural product, and *symbol* is meant here as a code. Moreover the artificial language where the expressions for propositions are embedded is conceived here as being provided with some grammatical structure. A proposition expresses more specifically and precisely what the simpler statement express more vaguely and generally. Every proposition is thus a statement but not the contrary It is clear that in any case (according to MacColl) we can convey the same information with both of them. For example, if someone asks us if we would like to smoke a cigar, we can answer by shaking the head: a statement, or with a proposition: "I don't smoke cigars".

> *I define a statement as any sound, sign, or symbol (or any arrangement of sounds, signs, or symbols) employed to give information; and I define a proposition as a statement which, in regard to form, may be divided into two parts respectively called subject and predicate.* [...] *A nod, a shake of the head, the sound of a signal gun, the national flag of a passing ship, and the warning "Caw" of a sentinel rook, are, by this definition, statements, but not propositions. The nod may mean "I see him"; the shake of the head, "I do not see him"; the warning "Caw" of the rook, "A man is coming with a gun", or "Danger approaches"; and so on. These propositions express more specially and precisely what the simpler statements express more vaguely and generally.*[15]

MacColl's theory of statement is rather a complicated one and stands in relation to his theory of the evolution of language in human culture. This has been studied thoroughly by Michael Astroh, who discussed the relations between MacColl's notion of statement and the linguistic traditions of his time.[16] Furthermore, MacColl recasts the traditional theory of hypotheticals in the terms of his definition of statement and *propositions of pure logic* (roughly: valid propositions).[17] For our purpose let us retain here the idea that a statement is a chain of signs used to convey information and which can become what MacColl calls a *proposition*. The latter assumes a Subject-Predicate structure. This subject-predicate structure actually represents the relation between a restricted domain (MacColl's subject)

[15] MacColl [1906a], pp. 1-4.
[16] Astroh [1999b] and Astroh [1995].
[17] Cf. Sundholm [1999], Rahman [1998], [2000].

and a predicate (MacColl's predicate) defined over this domain.[18] The insight that the grammatical notion of subject corresponds to the concept of restricted domains of a formal grammar is, in our opinion, one of MacColl's major contributions. This restricted domain (subject) can be a *qualified* one, that is, qualified by means of the introduction of a term, or an *unqualified* one. The latter consists only of the domain and a predicate defined over some or all of the elements of the domain. *Unqualified* domains (subjects) are thus MacColl's version of existential and universal quantification. Actually this type of quantification does not necessarily assume ontological commitment.[19] To avoid confusion we will call MacColl's notion of *unqualified subjects, quantified propositions.* Furthermore, within quantified propositions (unqualified subjects) we will distinguish *A-propositions* (universal propositions) from *I-propositions* (*existential propositions*). Indeed the basic expressions of MacColl's formal language are expressions of the form

$$H^B$$

where H is the domain (subject) and B a predicate. He gives the following example:

H :	The horse
B :	brown
H^B :	The horse is brown

MacColl remarks that the word 'horse' is a *class term.* To distinguish between the elements of the class denoted by H we may either

i) introduce numerical suffixes: H_1, H_2, etc.

or

ii.1) attach to each individual a different attribute, so that H_B would in this case mean "the brown horse" or H_W "a particular horse which won the race".

ii.2) introduce a sub-class (proper or otherwise), so that H_W would in this case mean "all those horses which won the race".[20]

Class terms seem to correspond to our modern notion of restricted domain for the scope of quantifiers such as found nowadays in formal grammars. Here MacColl closely follows natural language. Actually, he also assumes a universal (tacit)

[18]As mentioned above, MacColl seems to think that the passage - within a given human community - from the stage where information has been somehow conveyed by means of a statement, to the articulation of an adequate proposition - which assume subject-predicate structure – is a sign of a higher degree of the evolution of that human community. Furthermore, MacColl believes, that in a higher degree of evolution this passage will be introduced by conventions into the linguistic stock of the source language. (cf. MacColl [1906a], pp. 3-4).

[19]Cf. Rahman/Redmond [2005].

[20]MacColl's use of the natural language articles "the" and "a(n)" is hard to follow. Sometimes MacColl uses them to distinguish between "all" and "some" and sometimes to distinguish between the quantified expression and a particular instance of these expressions.

domain, which as we will discuss further on includes non-existents, but at the object level of his logical language only some restrictions (*portions*) of that universe will be expressed. Those *portions* (restrictions of the universal domain) constitute the domain the propositions are about.

> *Let S denote our Symbolic Universe or "Universe of Discourse" consisting of all the things S_1, S_2, &c*, real, unreal, existent, *or* non-existent, *expressly mentioned or tacitly understood in our argument or discourse. Let X denote any class of individuals X_1, X_2, &c., forming a* portion *of the Symbolic Universe S.*[21]

MacColl's use of suffixes — as described in i and ii.1 — seem to correspond to our modern-day notion of *individual term* (including terms for individuals and individual concepts).

> *On the other hand, S_B and H_k* [...]. *These are not complete propositions; they are merely qualified subjects waiting for their predicates.*[22]

Symbols such as B, H, W ... can be used both as a class term or as suffixes. While explaining their different uses MacColl speaks of the difference between an *adjectival* and a *predicative* use of those symbols.

> *Thus the suffix w is adjectival; the exponent s is predicative* [...].
>
> *The symbol H^W, without an adjectival suffix, merely asserts that a horse, or the horse, won the race without specifying which horse of the series H_1, H_2, &c.*[23]

The expression H^W is in fact ambiguous: It might mean be both "*the horse of the series*" (such as in "the horse is an animal") which must be read as the universal proposition "*every horse of the series* ":

> *"The horse has been caught".* [...] *asserts that every horse of the series H_1, H_2, &c., has been caught.*[24]

It might also, however, mean a *portion* of the restricted domain and therefore read as the particular proposition "*a horse has been caught*", that is, "some of the horses [of the series] have been caught".

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[21] MacColl [1906a], pp. 1-4.

[22] MacColl [1906a], p. 5.

[23] MacColl [1906a], p. 5. The interpretation of this passage is not so straightforward. Here we chose to understand H^W as a quantified proposition, which seems to be compatible with MacColl's uses in the rest of the book. Another choice would be Russell's interpretation (and criticism); he seems to understand MacColl's expression as propositional functions (see MacColl's reply to Russell in MacColl [1910c,d]). Russell's reading might be supported by the fact that MacColl stresses that those expressions do not have existential import. The problem with Russell's reading is that it does not match with most of the explanations and commentaries given by MacColl himself.

[24] MacColl [1906a], p. 41.

In order to remove any ambiguity posed by the expression MacColl uses two special exponents, namely ε, for universal, and θ for particular.[25] These devices yield the expressions for the A- and I-propositions respectively

$$(H^W)^\varepsilon \text{ and } (H^W)^\theta$$

Unfortunately MacColl uses the same exponents as modal operators. Presumably he thought of them as a kind of general quantifying expressions (see chapter 2.1 below). The expression $(H^W)^\varepsilon$ indicates that the predicate W applies to all of the elements of the domain H. Dually, θ indicates that the predicative applies to a sub-class of horses.

This is the closest MacColl comes to the notion of (restricted) quantifier, whereby, with some hindsight, the expressions ε and θ could be seen as the second order interpretation of quantifiers propagated by Frege. Sadly, MacColl did not use explicitly bounded variables for individuals. This prevented him from realizing the insight that the notion of quantifier gives.

Another interpretation MacColl gives to adjectivals is that of sub-classes (possibly with only one element) of the domain. Thus, say, the adjectival H (for *horse) in* A_H represents that sub-class from the domain A (animals) containing either all of those animals which are horses or some of the animals which are horses. Here we meet MacColl's notion of *unrestricted* (and *restricted*) use of predicates: if the sub-class contains all those animals which are horses, then the use of the adjectival is said to be unrestricted (we might write $A_{(H)u}$) and dually for the restricted sub-class containing some of animals which are horses (we might write $A_{(H)r}$).

Moreover, we might combine this with the exponents for quantification in the following way: In *All horses won the race* $(H^W)^\varepsilon$ the predicate *won* is said to be unrestricted, as opposed to *All brown horses won the race* $(H_B^W)^\varepsilon$, which is a different device MacColl uses to express the I-proposition *At least some of the horses (namely, all those which are brown) won the race.* In fact, MacColl proposes replacing any indication of a specific adjectival with r, which works here as a variable over adjectivals (or sub-classes)[26] to obtain: $(H_r^W)^\varepsilon$ *At least some of the horses (namely, all those that are elements of a given sub-class) won the race*, that is, *Some horses won the race.*

Quite often, if the predicative is unrestricted and part of an A-proposition MacColl omits the exponent ε. If there is an unrestricted adjectival he also omits the explicit indication of this assumption — that is, in such cases MacColl omits to introduce the expression u.

MacColl points out that the use of B (for *brown*) as an adjectival in H_B^W assumes H^B. Clearly, if it is possible to select an individual which is brown or a sub-class of brown individuals of the class H of horses, this assumes that the class of horses is a sub-class (proper or otherwise) of the class of brown objects.

It is important to see that in MacColl's system the classes involved in expressions such as H_B^W are never empty and thus the propositions expressed have no

[25] Cf. MacColl [1906a], pp. 40-41.

[26] Notice that this comes very close to the medieval theory of restricted *suppositio.*

ontological commitment. Notice that since MacColl's universal domain includes non-existent objects neither H^B nor H_1^B necessarily have ontological commitment. With this important proviso we could say that the adjectival use of a logical predicate (or predicates) *when they are used as expressions for individual-terms* (that is, used in the sense ii.1)) seem to relate to the modern-day term-use of a definite description — recall that nowadays in the case of definite descriptions we implement their term-use with the help of the iota operator. A thorough study of the evolution of Russell's theory of definite descriptions in relation to his discussion with MacColl is still lacking. Perhaps, since MacColl's *adjectivals* lack ontological commitment, these expressions are better understood as non-rigid designators rather than Russellian-definite descriptions.

Most importantly MacColl's adjectival devices have some advantages over Frege-Russell's first-order logic. Indeed, take

The fast turtle is brown (*the turtle which is fast is brown*)

In standard first-order logic we translate such an expression with the help of a conjunction: one individual of the domain is fast, a turtle and brown. This translation expresses something which is generally false. It may well be that the turtle in question is fast (for a turtle), but even fast turtles are slow creatures in the animal kingdom. MacColl's logical language allows the more accurate translation

$$(T_f)^B$$

Where the expression T_f signalizes that from the universe of turtles we picked up the one which is fast and of which the predicate B (being brown) can be said to hold.

N-places predicates and second-order predicates can be embedded in MaColl's formal language (presumably) in the following way. A predicate L(oves) like in *Hugh loves Hortense* can be treated in MacColl's notational system as an expression which, when applied to an individual constant (suffixes of the form 1, 2...) which designate elements of the domain H of human beings, results in a one place predicate. This one-place predicate expresses the property of loving the individual designated by the suffix 1 (: Hortense).

$(H_1)^L$(where H is the domain of human beings, L stands for *loving* and 1 designates *Hortense*)

Let us now, write the result of this operation, namely the predicate *loving Horstense* as a new predicate, namely $L1$. And this predicate can in turn be applied to the individual constant which designates Hugh, as a result of which we have a formula that says that the individual designated by the suffix 2 (: Hugh) has the property of "*loving the individual 1 (Hortense)*:

$(H_2)^{L(1)}$(where H is the domain of human beings, $L(1)$ stands for *loving Hortense*, and 2 designates *Hugh*)

Also second-order predicates can be expressed within MacColl's framework. Take a one-place second order predicate like in *Red is a colour*. The predicate C(olour) is treated as an expression which when a applied to an element of the domain yields the one-place predicate R(ed) which results in a formula expressing the proposition *Red is a colour*. That is, we build first the first order predicate R(red)

> $(O_1)^R$(where O is the domain of objects, R stands for *red*, and 1 designates a given red object)

and then we predicate of R that it is a colour

> $((O_1)^R)^C$ (where O is the domain of objects, R stands for *red*, 1 designates a given red object and C stands for *colour*)

Another, simpler, possibility would be to capture the result of the second order construction with the help of the use of adjectivals.

> $((P)_R)^C$ (where P is the domain of one-place predicates, R stands for one specific element of P, namely the one place predicate *red*, and C stands for *colour*)

This possibility is simpler but does not show (formally) the second-order structure of the expression. Most probably MacColl would prefer this last formal expression adding informally the intended interpretation.

As mentioned already, MacColl's formal approach to natural language strongly resembles to the translation devices of modern formal semantics such as DRT and DPL. MacColl tried to reflect the distinctions of natural language in his formal system rather than the other way round, as for example Frege would do. This relates to his instrumentalism, where context sensitivity plays a crucial role.

The connection of MacColl's notational system with the way of thinking in modern formal grammar becomes even stronger when we consider the ways he deals with negation. In fact, MacColl's logical language contains two symbols for negation. The first one is an external negation which affects the whole expression (*de dicto*) such as in $(A^B)', (A_B)'$ and while the second affects only the predicate (*de re*) such as in A^{-B}, A_{-B}.[27] The second one requires some structure of the predicate. Let us test once more MacColl's notational system in relation to standard first-order logic with the help of the following example:

> *Smoking is unwise*

In this example the problem lies not only with the second-order properties involved here but also with the translation of *unwise* because in standard first- and second-order logic the negation applies to formulae expressing propositions not to predicates. Frege and Russell explicitly rejected negative predicates. However, in many natural languages there is a productive process allowing words for negation to combine with expressions of various types, including predicates. Modern formal grammars provide some structure to the predicates (usually via the λ-operator). MacColl did not have such a device and was severely criticized because of negated

[27] Cf. Rahman [1999] and Rahman [2001].

predicates. However, MacColl's notation can reflect the natural language structure of the negative predicates of our examples. More precisely, the negation of an adjectival allows MacColl to capture expressions such as *unwise*, *uncaught* and so on in his formal language with the negation+predicate structure of natural language. Accordingly, in MacColl's notation the formal analysis of the sentence above renders the formula:

> $((P)_S)^{-W}$ (where P is the domain of one-place predicates, S stands for a specific element of P, namely the one-place predicate *to smoke* and -W stands for the negation of *wise*)

Certainly a shortcoming of this translation is that it does not show the difference between a second-order and a first-order predication. But this can be implemented in the way described above while discussing the formulation of second-order predicates.

Notice that the negation of a predicative as opposed to the negation of the whole propositional formula where this predicative occurs has strong similarities with Russell's theory of the two scopes of negation as applied to predicates on definite descriptions. Indeed Russell's distinction between

1. It is not the case that (there is one and only one individual that is now the King of France and this individual is bald)

2. There is one and only one individual that is now the King of France and this individual is not bald

can be reflected in MacColl's notation in the following way:

1*. (H_K^B)' (where H is the domain of individuals, K: the present king of France and B: bald)

2*. $(H_K)^{-B}$

Unfortunately MacColl does not realize the full expressive power of the distinctions of his own notational system and sees formulae such as 1* and 2* as equivalent.[28]

A particularly difficult part of MacColl's system relates to the way he introduces quantified propositions with ontological commitment. In this context he introduces the predicate 0 which might be negated *de re*. Unfortunately the way he combines this with expressions for quantified propositions is rather cumbersome (see 2.2 below).

Let us conclude this section with the following remark: the use, within a formal language, of restricted domains of quantifications, the distinction between an adjectival and predicative function of predicates, the negation of the latter, the introduction of a non-existence predicate, the distinction between quantified expressions with and without ontological commitment are bold ideas, despite the

[28]MacColl [1909a], p. 5.

manifold uses and distinctions MacColl accords to his notational system sometimes without any warning.

Let us now present some details of MacColl's logic of non-existence

1.2 MacColl's Logic of Non-Existence

1.2.1 The Symbolic Domain and its Dynamics

The most influential approach to the logic of non-existents is certainly the one stemming from the Frege-Russell tradition. The main idea is relatively simple and yet somehow disappointing: to reason with fictions is to reason with propositions which are either (trivially) true, because with them we deny the existence of these very fictions, or otherwise they are false in the same trivial way. The problem is the *otherwise*: Every proposition (unless it is a negative existential one) which contains fictional terms and which you may come to assert is false. For example, if, relative to a given domain, Pegasus is an empty name, then the sentences "Pegasus has two wings" and "Pegasus has three wings" both express false propositions relative to this given domain, though the sentence "Pegasus does not exist" expresses a true proposition.

The justification for this way of tackling the problem is pretty straightforward too: in science we are interested in talking about what counts as real in our domain. Do you want to reason with the help of a mental experiment where counterfactual propositions other than negative existential ones are asserted? Consider, then, that the objects of your mental experiment are elements of your domain and then apply the good old first-order logic. That is, reason as if the world described by your fiction were real, and for this nothing else as standard classical logic is needed. At this point one might start to suspect that something has gone wrong here. Quite often, when we introduce fictions we would like to establish connections between two domains sorted in different ontological realms. In other words, one point of reasoning with counterfactuals is to be able to reason within a structure which establishes relations between what has been considered in our model to be real and not real. The challenge is indeed to reason in a parallel fashion. Moreover, such reasoning requires an understanding of how the flow of information between these parallel worlds works. In fact, Frege's way is not exactly the one the Russell tradition propagated, and in the history of the development of modern logic one finds some dissidents to the solution mentioned above. One of the most important dissidents was indeed MacColl. It is in regard to the notions of existence and arguments involving fictions that MacColl's work shows a deep difference from the work of his contemporaries. In fact, he is the first to attempt to implement in a formal system the idea that to introduce fictions in the context of logic amounts to providing the logic not only with a many- (ontologically) sorted language but also with devices for establishing connections between the different ontological realms. Nevertheless, his dynamic approach to the logic of fictions might motivate new and deeper researches of his work, particularly in intentional contexts.

To achieve his target of a logic of non-existences MacColl first introduces two *mutually complementary and contextually determined* classes:

- the class of existents and the class of non-existents. He calls the class of real existents "e" containing the elements: $e_1, e_2, \ldots$. Every individual of which, in the given circumstances one can truly say "it exists" belongs to this class.

- the class of non-existents, "the null class 0". That he calls this class the *null class*, although it is actually full, is unfortunate. For it contains objects $0_1, 0_2, \ldots$ which correspond to nothing in our universe of admitted realities. *Objects such as centaurs and round square, belong to this class.* The notational mistake of MacColl's in calling the class of non-existents "null class" opened the door to criticism such as that of Bertrand Russell and diverted the scientific community's attention from one of MacColl's most original contributions. Actually Frege uses a null class too in the context of fictive entities. Moreover Frege even uses this class as an object.[29]

and then a third one,

The Symbolic Universe which includes the former two.

Furthermore, if a set is included in the set of non-existents, then its complement is included in the set of existents and vice versa. Thus we have the following structure of the domain:

e: *Real existence.* The elements are also elements of the class S but not of the class 0.

0: *Non existence or only symbolic existence.* The elements are also elements of the class S but not of the class e.

S: *Symbolic existence.* The elements are also elements of the classes 0 and e.

MacColl introduces these classes into the object language by means of the corresponding predicatives, e.g. $(H_3)^S$, $((H_1)^B)^S$, $((H_1)^B)^0$, $(H_2)^e$ (the horse 3 is an element of the symbolic class, the horse 1 is brown and is an element of both the symbolic and the non-existent classes and the horse 2 is brown an element of the class of non-existents). The notational system also allows the classes to

[29]Recall that for Frege an expression with sense may have no denotation. Thus, if we assume compositionality of denotation, those sentences which contain expressions without denotation will not have any denotation either. Now, for Frege the denotation of an assertion is its truth-value. Thus, if a given assertion contains any fictional (or empty) term, this assertion will lack a truth-value. But this would, in Frege's view, ban this kind of assertion from the paradise of science. Moreover we could not even truly assert that fictional or empty terms do not exist. If, on the contrary, we would like to assert negative existentials about fictional entities, Frege's solution is to assume that each different fictional term denotes the same null-class (***Grundlagen der Arithmethik***, paragraph 53). This device has the consequence that any proposition containing fictional entities is false unless it is a negative existential. MacColl thinks of a null class too, but it is neither empty nor does it necessarily produce false propositions. More precisely, since MacColl's null class is not empty it allows different non-existent entities as its elements.

be introduced too as the domains of discourse: e.g. $(S_3)^H$, $((0_1)^H)^B$, $((S_1)^H)^B$, $(e_2)^H$ (the element 3 of the symbolic class is a horse, and so on).[30]

As already mentioned, if the suffixes are adjectivals representing classes, and they are elements of one of the classes 0 and e, MacColl's assumption that the complements of these adjectivals are elements of the complementary ontological classes amounts to the following: if the class e containing as its only element the horse, say *Rocinante*, then the complement of any other class of horses of 0 is a sub-class of e. The sense of the other direction of the class inclusion seems harder to grasp. Indeed, if the singleton Rocinante is a sub-class of the class of horses H, then MacColl's assumption requires that the complement of this singleton be a sub-class of e. This same assumption makes it difficult to understand suffixes as individuals; a theory of complements of individuals is due.

It is interesting that MacColl assumes that these domains interact, or more precisely that there is an interaction between the symbolic and the other two ontological classes. In fact, MacColl's view seems to be more epistemic and dynamically oriented than ontological. For instance, assume that at a given point of an argument, the following proposition is asserted:

$(H_3)^S$ (horse 3 has a symbolic existence)

This proposition might have been asserted because at the moment of the assertion, the context lacked precise information concerning the ontological status of the horse at stake. But in a further state of information fresh data about the ontological status of the object in question might arrive. This might allow a more precise assertion such as asserting that the brown horse we are talking about does not really exist. MacColl provides some examples of this dynamics in cases of deception which link the dynamics of his symbolic universe with contexts of intentionality.[31] Examples such as:

The man whom you see in the garden is really a bear.

The man whom you see in the garden is not a bear.

The examples are interesting and challenging. MacColl takes the point of view of an observer which asserts the above propositions and studies what happens with the ontological assumption implied by these propositions. He concludes that the ontological status of the individual man is that of not-existent in the first example and existent in the second. MacColl did not analyze the dynamics produced within the sentence:

The object that you see in the garden and that you think is an existent man is really a existent bear.

Still, MacColl's examples are exciting and deserve detailed further exploration.

[30] MacColl [1905o], pp. 74-76 and [1906a], pp. 76-77.
[31] MacColl [1905o], p. 78.

1.2.2 Propositions With and Without Ontological Commitment

MacColl attempts to capture in his formal language quantified propositions with and without ontological commitment.

Let us come once more to the expression of an I-proposition concerning MacColl's paradigmatic brown horse:

$((H_r)^B)^\varepsilon$ (*At least some of the horses are brown*)

that in MacColl's view does not commit itself to the existence of horses. Thus the universal expression

(H_r^B)' (*It is not the case that there is a brown horse =no horse is brown*)

In order to obtain expressions for I-propositions with ontological commitment, MacColl introduces, as explained above, a non-empty symbolic universe including existent and non-existent objects. Furthermore, on page 5 of his book MacColl introduces the predicate of non-existence 0 in the context of formulating expressions for I-propositions which might be negated *de re*. With the help of the non-existence predicate we obtain expressions such as

$(H_c)^{-0}$

which, according to MacColl, reads *Every one of the caught horses is existent.* Or *At least some of the horses (namely, all those which were caught) are existent.*[32] MacColl employs here the unrestricted adjectival C as standing for the sub-class of all caught horses and the negation of the non-existence predicate as the assertion that any such class is included in the set of (non-non-) existent objects. Thus the formula $(H_c)^{-0}$ expresses an I-proposition with ontological commitment. A more explicit formulation would be

$((H_r)^e)^\varepsilon$ (*Some horses are existent*)

At least some of the horses (namely, all those which are elements of a given sub-class) are existent.

In fact, MacColl uses a special case of the latter notation in his reconstruction of traditional syllogism where he omits the exponent ε:[33]

$(X_Y)^e$

At least some of the X (namely, all those which are elements of Y) are existent.

That is,

Some of the X are Y (and are existent).

[32] MacColl [1906a], p. 5.
[33] MacColl [1906a], p. 44.

Similarly for the O-propositions:

$(X_{-Y})^{e}$

Some of the X are not Y (and are existent).

MacColl also introduces in his system expressions for A-propositions with ontological commitment, such as

$(H_{-c})^{0}$

Which accordingly expresses:

Every one of the non-caught horses is non-existent.

But according to MacColl, this implies (recall MacColl's assumption of complementarity)

Every horse that has been caught exists, or *Every horse has been caught (and is existent)*[34]

Actually, MacColl's notational system has a more direct way of achieving I-propositions with ontological commitment, namely

$((H_{r})^{e})^{\varepsilon}$ (*Some horses are existent*).

The notion of propositions as conveying information seems to furnish the motivational background of all his conception of a logic where data from the context might have logical consequences. In the section above we outlined how data about the ontological status of the subjects on which propositions are built might determine the set of consequences which could be drawn. It is an interesting fact that MacColl attempted to formulate a logic where data of any kind might have logical consequences. The result of these explorations of his yielded the basis of modern formal modal logic and the start of the philosophy of the conditional.

2 THE PHILOSOPHY OF THE CONDITIONAL

MacColl's central notion is that of the conditional. He not only acknowledges that this connective holds a privileged place in his logic, he also makes the conditional the center of his philosophy. This triggered a whole series of reflections on the conditional the repercussions of which never faded since MacColl's first challenges on the logical meaning of material implication. Moreover, many of todays discussions involving the pragmatic aspects of meaning can be seen as sharing MacColl's central philosophical background, namely, the basic units of meaning in logic are to be linked to convey information. MacColl did not work out thoroughly his theory of meaning, but he approached it from diverse angles, all of which seem to be connected to the idea that the passage of information must yield a logic where the income of data might make a difference.

[34] MacColl [1906a], p. 5.

2.1 The Invention of The Modal Logic T: MacColl's Criticism of Material Implication

Let us start with two quotes of Stephen Read who puts the historical point succinctly[35]:

> *The received wisdom is that strict implication was invented and developed by the American logician C. I. Lewis.* [36]
>
> *C. I Lewis repeatedly exempts MacColl from criticisms of his predecessors in their accounts of implication. They had all taken a true implication, or conditional, to be one with false antecedent or true consequent. MacColl uniquely, and correctly in Lewis' view, rejected this account, identifying a true implication with the impossibility of true antecedent and false consequent. Lewis's development of the calculus of strict implication arises directly and explicitly out of MacColl's work.*[37]

Indeed *the received wisdom* on the origins of formal modal logic is wrong — and Lewis was far from innocent in the propagation of this mistaken wisdom.[38] The contributions of MacColl in this field came at a time when the long ancient Greek, Arabic and Medieval traditions were put aside due to the suspicion of psychologism. In fact MacColl understood his work as building a bridge between the philosophical and the new mathematical approach to logic[39] and discussed the main ideas of his modal logic in an epistolary interchange principally with Bertrand Russell. The first paper containing the final version of his modal logic was presented in Paris (1901) at the *1*ère *Congrès International de Philosophie. Logique et Histoire des Sciences*,where among others Couturat, Frege, Peano and Russell were members of the scientific committee. Further publications on the same subject were published in France between 1903 and 1904. Eventually the most developed formulation of MacColl's logic was presented in this book of 1906 *Symbolic logic and its applications*. But let us come back to his reflections on material implication:

> *For nearly thirty years I have been vainly trying to convince them that this assumed invariable equivalence between a conditional (or implication) and a disjunctive is an error, and now Mr. Shearman's quotation supplies me with a welcome test case which ought, I think, to decide the question finally in my favour. Take the two statements "He is a doctor" and "He is red-haired", each of which, is a variable, because it*

[35]See too Rahman [1997b].
[36]Read [1999], p. 59.
[37]Read [1999], p. 59.
[38]Cf. Rahman [1997b] and Read [1999], p. 59.
[39]*The writer of this paper would like to contribute his humble share as a peacemaker between the two sciences, both of which he profoundly respects and admires. He would deprecate all idea of aggression or conquest* [...]. *Do not Englishmen and Scotchmen alike now both "glory" as George III said he did, "in the name of Briton"? Why should not logicians and mathematicians unite in like manner under some common appellation?* (MacColl [1880p], p. 46-47).

> *may be true or false. Is it really the fact that one of these statements implies the other? Speaking of any Englishman taken at random out of those now living, can we truly say of him "If he is a doctor he is red-haired", or "if he is red-haired he is a doctor?" Is it really a certainty that either "all English doctors are red-haired", or else "all red-haired Englishman are doctors?"*
>
> [...]
>
> *Thus, Mr. Russell, arguing correctly from the customary convention of logicians, arrives at the strange conclusion that (among Englishmen) we may conclude from a man's red hair that he is a doctor, or from his being a doctor that (whatever appearances may say to the contrary) his hair is red.*[40]

MacColl's strict implication grew out of his dissatisfaction with the material implication contained in the text (and similar) quoted above. His notion of conditional is the known definition of strict implication: "A implies B" is understood by MacColl as "it is impossible that A and not-B" (in MacColl's notation $(AA')^{\eta}$):

> *Let W denote the first proposition and E the second. It is surely an awkward assumption (or convention) that leads here to the conclusion that "either W implies E or else E implies W". War in Europe does not necessarily imply a disastrous earthquake the same year in Europe; nor does a disastrous earthquake in Europe necessarily imply a great war the same year in Europe.*
>
> [...] *with Mr Russell the proposition "A implies B" means* $(AB')^{\iota}$, *whereas with me it means* $(AB')^{\eta}$.[41]

MacColl's modal language is conceived as development of the "Boolian" values 'true" and "false" which he thought to be not general enough and even not a faithful description of human reasoning abilities.[42] MacColl introduced the further

[40]MacColl [1908b], p. 152.

[41]MacColl [1908c], p. 453.

[42]MacColl seems to have taken the ability to drive conclusions by means of strict implication as a sign of a higher degree of evolution of men over animals. The following text is probably aimed against the Boole and followers who were the mainstream and had no connective for the conditional:

> *Brute and man alike are capable of concrete reasoning; man alone is capable of abstract reasoning* [...]. *The brute as well as is capable of the concrete inductive reasoning* [...] *that is to say, from experience -often painful experience- the brute as well as man can learn that the combination of events A and B is invariably followed by the event C* [...]. *We have seen that from two elementary premisses A and B, brutes as well as men can* [...] *draw a conclusion C. But no brute can, from the two implicational premisses* [...] *draw the implicational conclusion A:C* [it is impossible that A not C]. *It is evident that the latter is not only more difficult, but also that it is on a higher and totally different plane. In the former the two premisses and the conclusion are all three elementary statements* [...] *while the whole reasoning is a simple implication. In the latter, the two premisses and the*

modalities "certain" (or "necessary"), "impossible" and "contingent" (or "neither impossible nor necessary". One motivation for this was the probabilistic interpretation. In fact on page 7 of his book of 1906 we find the following presentation:

Truth	τ	A^τ : A is true in a particular case or instance.
False	ι	A^ι : A is false in a particular case.
Certain	ε	A^ε : A is always true (true in every case) within the limits of our data, so that its probability is 1.
Impossible	η	A^η : A contradicts some datum or definition, so that its probability is 0.
Variable	θ	A^θ : A is possible but uncertain (A is contingent). It is equivalent to $A^{-\eta}A^{-\varepsilon}$: A is neither impossible nor certain; A is possible but uncertain. The probability is neither 0 nor 1, but some proper fraction between the two.

On page 14 MacColl adds the modality "possible" (π) defined as not-impossible, and explains that this modality indicates that the probability is not 0 but might be 1 or less than 1.

On the same page we find the resolution of the internal (de re) negation of these operators. MacColl does not make use here of two different signs one for de re negation and one for de dicto negation (as he did before for the negation of predicates). To introduce the distinction here MacColl makes use of the difference between the adjectival and predicative positions. We will not follow the latter device but simply change the positions:

Internal Negation of Truth	$(A')^\tau$	A^ι
Negation of False	$(A')^\iota$	A^τ
Internal Negation of Certain	$(A')^\varepsilon$	A^η To be distinguished from the external negation (A^ε)': $A$is not necessary
Internal Negation of Impossible	$(A')^\eta$	A^ε To be distinguished from the external negation (A^η)': A is not impossible
Internal Negation of Variable	$(A')^\theta$	A^θ To be distinguished from the external negation (A^θ)': A is not contingent
Internal Negation of Possible	$(A')^\pi$	It is possible that not$A$ To be distinguished from the external negation (A^π)': A is impossible

conclusion are all three implications, while the whole reasoning is an implication of the second order. MacColl [1902i], pp. 367-368.

These modalities combine with propositional logic in the following way:

$A+B$	disjunction
A'	negation
AB	conjunction
$A{:}B$	strict implication
$(AB')^{\eta}$	Definition of strict implication

MacColl repeatedly claims the superiority of his logic over the "Boolian Logicians" (Jevons, Schröder, Venn *et al.*) on the grounds of its greater success in solving specific problems of logic and mathematics, particularly in relation to probability. In fact, MacColl explains explictly that his logic had its origin in problems of probability.[43] Theodor Hailperin, who studied MacColl's application of logic in probability, links him to the first results on conditional probability.[44] Later on MacColl gives a more general interpretation to his modal logic. Unfortunately the interpretation of his expressions for modalities always oscillate between values, predicates, operators and variables for propositions of the corresponding kind. Peter Simons claims that MacColl modalities do not yield a many-valued functional logic and are better understood as probability logic.[45] Werner Stelzner and Ian Woleński prefer the operational interpretation.[46] Read reconstructs MacColl's modal logic within the style of his times, that is, as an algebra suitable for diverse interpretations.[47] We would like to recall that, as already explained in 1.1, MacColl's modalities are also used as operators to produce quantified expressions. It looks as if sometimes, in order to achieve generality, MacColl considered the data over which the modalities are defined as of any kind: individuals, contexts, probabilities and so on. However, in the following, we will in principle take modalities as exponents to stand for operators over formulae and the non-exponential use of these modalities to stand for formulae. Nevertheless, Read shows that the

[43] *It may interest some of the readers to be told that this method owes its origin to a question in probability (No. 3440), proposed by Mr. Stephen Watson in the Educational Times. My solution of this question, with an introductory article, entitled "Probability Notation", was published in the Educational Times for August, 1871; and in this introductory article may be seen the germs of the more present method. Shortly after this I gave up all mathematical investigations, and my thoughts did not again revert to the subject till two or three months before the appearance of my article on "Symbolical Language" for July, 1877.* [...] *I noticed that this "Symbolic Language", as I called it, might also be employed, without change or modification, and with unerring certainty, in tracing to their last hiding-place the limits which often escape so mysteriously from the mathematician's grasp when he ventures to change the order of integration or the variables in a multiple integral. I still looked upon the method, however, as an essentially mathematical one - grafted on a logical stem, but destined to yield mathematical fruit, and mathematical fruit only.* (MacColl [1878d], p. 27).

[44] According to Hailperin, unlike with Boole and Peirce, MacColl's notation clearly distinguishes between an argument (event, proposition, class) and its probability (cf. Hailperin [1996], pp. 132-134).

[45] Cf. Simons [1999].

[46] Cf Stelzner [1999], Woleński [1999].

[47] Cf. Read [1999].

following theorems of MacColl's modal logic amount to the logic nowadays known as T.

In fact MacColl assumes following validities on modalities[48]:

1. $(A + A')^{\varepsilon}$
2. $(A^{\tau} + A^{\iota})^{\varepsilon}$
3. $(AA')^{\eta}$
4. $(A^{\varepsilon} + A^{\eta} + A^{\theta})^{\varepsilon}$
5. $A^{\varepsilon} : A^{\tau}$
6. $A^{\eta} : A^{\iota}$
7. $A^{\varepsilon} = (A')^{\eta}$
8. $A^{\eta} = (A')^{\varepsilon}$
9. $A^{\theta} = (A')^{\theta}$

It is interesting to compare formula 2 with formula 1. In the second formula the truth of the proposition involved is brought into the object language. Stelzner interprets this device of MacColl's as the introduction into the object language of names for contexts.[49]

Formula number 4 (and its dual 5) correspond(s) to the axiom characterizing the nowadays well known normal modal logic T. Moreover, as pointed out by Read, MacColl explicitly rejects S4-axiom

$A^{\varepsilon}{:}A^{\varepsilon\varepsilon}$ [50]

and accepts the normality axiom

$\eta^{\eta} = \varepsilon$[51]

where the exponential notation signalizes the use of the modality as an operator

All this confirms Read's reconstruction of MacColl and even the claim that the system T was penned by MacColl far before it received the name T. Read's claim opposes the reading of Storrs MacCall who thinks that MacColl's logic corresponds to one of the non-normal logics. Up to this point of the discussion Read's argument seem to be the right one nevertheless, as we will say further on there might be some kind of compromise. The point is that MacColl did not have only one system and this was particularly the case when he tackles the issue of the conditional. Let

[48] MacColl [1906a], p. 8.
[49] Cf. Stelzner [1999].
[50] Cf. MacColl [1896c], p. 13 and Read [1999], p. 579.
[51] Cf. MacColl [1906a], p. 13 and Read [1999], p. 74.

us thus come back to the start and present MacColl's solution to the paradoxes of material implication in the framework of his modal logic.

MacColl realizes that his strict implication avoids the paradoxes of material implication mentioned above. That is,

1. $(A : B) + (B : A)$
2. $B : (A : B)$
3. $A' : (A : B)$

are certainly non-valid.

Notice that if A and B are formulae with the structure = necessity operator+propositional variable, then the disjunction (1) is not valid either. We know today that this particular case of the disjunction characterizes the logic S.4.3, which, when formulated in the framework of a Kripke semantics, amounts to assuming a reflexive, transitive and linear frame. However if A stands for a contradiction and/or B for a tautology then all of 3 turns out to be valid. In fact, MacColl points out that nevertheless instances of the following are valid in his system:

4. $A : \varepsilon$
5. $\eta : A$

where ε stands for a formula expressing an arbitrary necessary proposition and similarly for η. Clearly the following are valid too:

6. $B^{\varepsilon} : (A : B)$
7. $A^{\eta} : (A : B)$

Moreover the following is also valid:

8. $\eta : \varepsilon$

such as the instantiation

9. $(BB') : (A + A')$

MacColl concedes that they might give the impression of *paradoxical-looking formulae.* But they are not. MacColl simply states, as do many modern-day modal logicians, that they have to be accepted because the meaning which follows from the definition of strict implication is not paradoxical.[52] On the very same page where he concedes the validity of the formulae above he explicitly rejects

10. $A^{\eta} : A^{\varepsilon}$[53]

[52]MacColl [1906a], p. 13.
[53]MacColl [1906a], p. 13

MacColl's argument for this rejection is hard to follow, but if we put all his explanations on this issue together[54] it looks as if he rejects substitutions such as

11. $(AA')^{\eta} : (AA')^{\varepsilon}$

The construction of this formula seems to follow from the difference between the predicative position of the modalities defined over a class-term and their interpretation as formulae. His main explanation is based on the remark that formulae such as 10 assert that *every impossibility of the class η is also an individual of the class of certainties ε which is absurd.*[55] This would make the following formula hard to understand:

12. $(A + A')^{\eta} : (A + A')^{\varepsilon}$

On the one hand it should be accepted because the consequent (antecedent) is valid (a contradiction), furthermore the subject-formula of the antecedent $(A + A')$ is the same as the subject-formula of the consequent (and apparently contained in the class ε). On the other hand, the reading of this formula in terms of classes proposed by MacColl would render the antecedent hard to read: $A + A'$ is not an element of the class of impossible formulae. Still, one could argue that if the latter is the case, then 12 must be valid.

Another possibility is to come to the conclusion that the construction rules for formulae implicit in 8 and 10 do not allow formulae such as 12 and the two below to be built from them:

13. $(AA') : (AA')$

14. $(A + A') : (A + A')$

Does MacColl accept 4), 5) and 8) under the assumption that the scope of the modalities of antecedent and consequent is not the same formula (see 2.2.1.2 below)? Certainly, this does not necessarily mean that MacColl rejects 13 and 14, only that they do not follow from 8.

The predicative use of a modality is not an operator after all but a kind of a metalogical predicate over propositions. Actually this seems to be related to MacColl's use of the theses of connexive logic — which cannot be proved in the standard versions of the normal modal logic T — and to his methods of eliminating the redundant formulae in an implication. In MacColl's connexive logic, according to our reconstruction, neither 13 nor 14 are valid. Moreover the use of the metalogical predicates "true" and "false" in $(A^{\tau} + A^{\iota})^{\varepsilon}$ seem to be linked to metalogical restrictions on the connexive conditional.[56] Here we find ourselves at the start of another story.

[54]MacColl [1906a], pp. 13, 78-79.

[55]MacColl [1906a], 78.

[56]See the operator V and F in section three below. With help of these operators the formula $(A^{\tau} + A^{\iota})^{\varepsilon}$could be read as expressing that it is valid that the proposition A is either logically contingently true or false (Cf. MacColl [1906a], p. 16).

2.2 Towards Connexive Logic and Relevance

MacColl recasts the philosophical tradition of connexive logic in his formal system. Connexive logic is strongly linked to his criticism of material implication because in connexive logic the following substitution of the disjunction $(A : B) + (B : A)$ is not only not valid, it is invalid

$$(A' : A) + (A : A')$$

Indeed, connexive logic contains as theses the negation of the disjunction. That is

$$(A' : A)'$$
$$(A : A')'$$

Furthermore, connexive logic is at the center of MacColl's reconstruction of the syllogism which allows him some generalizations on the notion of sub-alternation by means of the derived theses

$$(A : B) : (A' : B)'$$
$$(A : B) : (A : B')'$$ [57]

To embed these theses in a logical system is not a trivial task. MacColl achieved this by introducing some metalogical restrictions on the modalities of the antecedent and the consequent which are linked with our remarks about his identification of necessary with valid propositions. Let us first briefly present a more general description of connexive logic.

2.2.1 Connexive Logic

Many of the discussions about conditionals can best be put as follows: can those conditionals that involve an entailment relation be formulated within a formal system? The reasons for the failure of the classical approach to entailment have usually been that they ignore the *meaning connection* between antecedent and consequent in a valid entailment. One of the first theories in the history of logic about meaning connection resulted from the stoic discussions on tightening the relation between antecedent and consequent of conditionals, which in this context was called $\sigma\upsilon\nu\alpha\rho\tau\eta\sigma\iota\varsigma$ (connection) and played an important role in the history of philosophy.

This theory gave a justification for the validity of what we today express in standard classical logic through formulae such as $\neg(a \to \neg a)$ and $\neg(\neg a \to a)$ (let us here, and below, use the standard terminology symbol '$\to$' for the material implication).

Let us first discuss two examples which should show what the ideas behind connexive logic are. The first example is a variation on an idea of Stephen Read's [1994], who used it against Grice's defence of material implication. The second is based on an idea of Lewis Carroll's.

[57] MacColl [1906a], pp. 49-65, 92-93.

The Read Example

This example shows how a given disjunction of conditional propositions, none of which is true, is, from a classical point of view, nevertheless valid. Imagine the following situation:

Stephen Read asserts that dialogical relevance logic is not logic any more. Suppose further that Jacques Dubucs rejects Read's assertion.[58] Now consider the following formulae where a, b, stand for propositional variables:

1. If Read was right, so was Dubucs: $(a \rightarrow b)$

Now (1) is obviously false. The following proposition is also false:

2. If Dubucs was right, so was Read: $(b \rightarrow a)$

Thus, the disjunction of (1) and (2) must be false:

3. $(a \rightarrow b) \vee (b \rightarrow a)$

From a classical point of view, however, this disjunction is valid and this seems counterintuitive. Recall MacColl's criticism of the material implication based on counterexamples of precisely this form of disjunction.

If we reformulate (3) in the following way:

4. $(a \rightarrow \neg a) \vee (\neg a \rightarrow a)$

the truth-functional analysis of this disjunction, which regards the disjunction as valid, shows how awkward such a theory can be. Certainly, the disjunction is not intuitionistically valid, but the intutionistically valid double negated version is not plausible either.

The point of connexive logic is precisely that this disjunction is invalid. Thus, in connexive logic the following holds:

5. $\neg((a \rightarrow \neg a) \vee (\neg a \rightarrow a))$

or

6. $\neg(a \rightarrow \neg a)$

and

7. $\neg(\neg a \rightarrow a)$.

Proposition (6) is known under the name *first Boethian connexive thesis.* Number (7) is the *first Aristotelian connexive thesis.*

Actually we should use another symbol for the connexive conditional:

8. $\neg(a \Rightarrow \neg a)$ (*first Boethian connexive thesis*)

9. $\neg(\neg a \Rightarrow a)$ (*first Aristotelian connexive thesis*).[59]

[58] The content of these examples is fictional.

[59] At this point it should be mentioned that the connexive theses are given various names in the literature. What we call the first Boethian thesis is often referred to as the Aristotelian thesis and what we call the second Boethian thesis is often called simply the Boethian thesis.

The Lewis Carroll Example

In the 19th century Lewis Carroll presented a conditional which John Venn called *Alice's Problem* and which resulted in several papers and discussions. The conditional is the following:

10. $((a \rightarrow b) \wedge (c \rightarrow (a \rightarrow \neg b))) \rightarrow \neg c$

If we consider $a \rightarrow \neg b$ and $a \rightarrow b$ as being incompatible the conditional should be valid. Consider, for example, the following propositions:

11. If Read was right, so was Dubucs: $(a \rightarrow b)$

12. If Read was right, Dubucs was not: $(a \rightarrow \neg b)$

They look very much as if they were incompatible, but once again, the truth-functional analysis does not confirm this intuition: if a is false both conditionals are true. Boethius presupposed this incompatibility on many occasions. This motivated Storrs MacCall to formulate the *second Boethian thesis of connexivity*:

13. $(a \Rightarrow b) \Rightarrow \neg(a \Rightarrow \neg b)$ (*second Boethian connexive thesis*)

Aristotle instead used proofs corresponding to the formula:

14. $(a \Rightarrow b) \Rightarrow \neg(\neg a \Rightarrow b)$ (*second Aristotelian connexive thesis*)

which is now called the *second Aristotelian thesis of connexivity.* Aristotle even showed in *Analytica Priora* (57a36-b18) how the first and second Aristotelian theses of connexivity are related. Aristotle argues against $(a \Rightarrow b) \Rightarrow (\neg a \Rightarrow b)$ in the following way: from $a \Rightarrow b$ we obtain $\neg b \Rightarrow \neg a$ by contraposition, and from $\neg b \Rightarrow \neg a$ and $\neg a \Rightarrow b$ we then obtain $\neg b \Rightarrow b$ by transitivity, contradicting the thesis $\neg(\neg b \Rightarrow b)$.

The problem with this logic is that if we embed the first Aristotelian connexive thesis in a logic such as standard classical logic, then, because of the validity of

$$\neg(\neg a \rightarrow a) \rightarrow \neg a$$

by modus ponens, for any a (and we obtain as a theorem) $\neg a$

It gets worse if we embed the Boethian thesis, because similarly we obtain a.

It should now be clear, that the connexive theses cannot be incorporated all that easily.[60] As quite often with his work, MacColl did hit the right ideas, but he did not explore them systematically.

[60]Routley and Montgomery [1968] studied the effects of adding connexive theses to classical logic.

2.2.2 MacColl's Connexive Logic and his Explorations on Relevance

2.2.2.1 Connexivity and Sub-alternation MacColl was the first to attempt to embed the connexive theses in a formal system. In his papers *The Calculus of Equivalent Statements* he gives the following condition for the second Boethian thesis:

> *Rule 18- If A (assuming it to be a consistent statement) implies B, then A does not imply B'* [i.e. not-B].
>
> *Note.- The implication α:β' asserts that α and β are inconsistent with each other; the non implication $\alpha \div \beta$' asserts that α and β are consistent with each other.*
>
> [...] *α is a consistent statement – i.e., one which may be true.*[61]

MacColl introduces the metalogical requirement of consistency into the object language while stating the second Boethian thesis of connexivity. That the proposition A is a consistent one, explains MacColl in the footnote quoted above, means that A is possibly true. That is, no logical contradiction follows from the assumption of the truth of A. In the final versions of his logic he makes use of the modality of contingency. More precisely, MacColl requires that antecedent and consequent of the connexive conditional must be contingent propositions.[62] Now, contingency means, here, a formula which is neither a contradiction nor a validity. The interpretation of the modality here is the one we mentioned above in relation to the different interpretation MacColl gives to modalities in a predicative position. The modalities at issue here introduce metalogical properties into the object language, and this might support the idea that he came close to considering a non-normal logic alongside the logic T. Actually MacColl warns explicitly that his modalities might be read either as *formal* (not bound to any data or information) or *material* (bound by data or information).[63]

As mentioned above, MacColl makes use of the connexive theses to formulate a general version of sub-alternation with and without ontological commitment. As also mentioned above, Aristotle uses the connexive theses to prove some properties of his syllogistics. It is interesting to note that MacColl's reconstruction of Aristotle's logic yields a Syllogistic without ontological commitments. In order to put the point clearly, let us combine the nowadays standard quantifier notation with MacColl's propositional one. With such a combination sub-alternation will be then formulated as follows:

$$\forall x(Ax : Bx) : \forall x(Ax : B'x)'$$

Clearly, MacColl's version of sub-alternation is valid if we assume the validity of $(A : B)$:$(A : B')'$.Furthermore, the validity of this version of sub-alternation does not assume any ontological commitment.

[61] MacColl [1877p], p. 184.
[62] MacColl [1906a], pp. 45, 49-65, 92-93.
[63] MacColl [1906a], p. 97.

Once more, the main idea here, so far as we have reconstructed MacColl, is that the connexive conditional is a kind of strict implication with the two additional restrictions that there is no contradiction in the antecedent and no tautology in the consequent. This way of producing several logical systems seems to be typical of MacColl's style. In fact he also explored the idea of eliminating *redundancies* occurring in an implication in order to tighten the relations between antecedent and consequent.

2.2.2.2 Towards Relevance and Concluding Remarks As early as 1878 in the third paper of his series *The Calculus of Equivalent Statements* MacColl seems to be interested in developing methods to eliminate redundancies of a given formulae. MacColl states there in relation to eliminating redundancies occurring in a disjunction (he calls it *indeterminate statement*):

> *Any term of an indeterminate statement may be omitted as redundant when this term, multiplied by the denial of the sum of all its co-terms, gives the product 0.*[64]

Actually, as pointed out by MacColl further on, the method is based on the idea of transforming the disjunction into a material implication, where the presumably redundant disjunct constitutes the antecedent. If the resultant material implication is valid, then that disjunction at stake is indeed redundant.

Given $A^0 + B + C$, the material implication $(A^0(B + C)')'$ is valid. In fact this is one case of the paradoxes of material implication discussed above. Dually, we obtain the other case of the paradoxes of material implication if it happens that one of the factors is materially implied by one or more of the others. Thus, take A^1BC, the resulting material implication $(BC(A^1)')'$ is indeed valid.

Now, one could certainly use the same method to obtain a tighter notion of implication, that is, a notion of implication without redundancies. This seems to be the background for his diverse methods of obtaining the *strongest premise* or/and the weakest conclusion.[65] Rahman (1997a) suggested that MacColl's method of eliminating redundancies contains the following concept of relevance. Recall that in this section and in the sections below we come back to standard classical notation and use '$\rightarrow$'.

Let us call a set of (labelled occurrences of) propositional variables *truth-determining* for a formula A iff the truth value of A may be determined as true or false on all assignments of true or false to the set. Let us say further that (the labelled occurrence i of) a propositional variable in A is *redundant* iff it is known that there is a truth-determining set for A that does not contain (the labelled occurrence i of) this propositional variable. Thus, clearly, the set $\{a\}$ is not truth-determining for $a \rightarrow b$, but the set $\{a_1,a_2\}$ is truth-determining for $a \rightarrow a$.[66]

[64]MacColl [1878d], p. 17.
[65]MacColl [1906a], pp. 27-33.
[66]Cf. Rahman [1998], p. 37.

According to this all of the following formulae which contain redundancies should be rejected in the context of inference:

$$
\begin{array}{ll}
((a \to b) \to a) \to a & a \to (b \to a) \\
((a \to a) \to a) \to a & a \to (a \to a) \\
a \to (b \vee \neg b) & a \to (a \vee b) \\
a \to (a \vee \neg a) & a \to (a \vee a) \\
(a \wedge \neg a) \to b & (a \wedge b) \to a \\
(a \wedge \neg a) \to a & (a \wedge a) \to a
\end{array}
$$

In his last papers and in his book, MacColl combines this concept of redundancy with his strict implication. Notice that in this context MacColl's rejection of formula 11 (in 2.1 above) could be explained because of containing redundant *occurrences* of the antecedent.

It is challenging to fasten all the threads together and produce a connexive relevant logic which meets MacColl's suggestions. A further philosophical task would be to connect all this with the notion of information contained in MacColl's notion of statement. The notion of redundancy understood as loss of information seems to be a good candidate for a start.

Let us conclude our paper with a work in his honour. A reconstruction of the notion of connexive logic in the proof-theoretical style which started to flourish some years after his death.

3 TABLEAUX FOR MACCOLL'S CONNEXIVE LOGIC[67]

3.1 Introduction

MacColl and more recently R. Angell [1962], S. McCall (from 1963 to 1975), and C. Pizzi [1977; 1993; 1996] searched for a formal system in which the validity of the connexive formulae could be expressed. New results have been achieved by R. Angell [2002], M. Astroh [1999a], Cl. Pizzi/T. Williamson [1997], G. Priest [1999], S. Rahman/H. Rückert [2001] and H. Wansing [2005]. Wansing also penned a very thorough overview on connexive logic in the Stanford Encyclopedia of Philosophy.

The present paper presents a new tableau system for connexive logic as an attempt to push forward Hugh MacColl's original ideas and as a non-dialogical version and revision of Rahman/Rückert [2001]. In this context we recall (see 1.2 above) that Stephen Read showed that MacColl's modal logic amounts to an algebraic form of the modal system **T** and contests Storrs MacCall claim that MacColl's logic corresponds to what is known since Lewis as the LS3 logic. My aim is to show that there might be some kind of compromise. Indeed MacColl had the system **T** but when he is talking about the connexive conditional, which is essential to his reconstruction of syllogism, MacColl prefigured a logic which involves metalogical properties of the system **T** and of what seems to be related to

[67]This section has been written mainly by Shahid Rahman.

what we nowadays call non normal modal logics (worlds where tautologies might fail).

Furthermore we provide the main ideas for a Kripke-semantics for these tableaux. We follow here MacColl's idea that the connexive conditional is a a kind of strict implication with the two additional conditions that there is no contradiction in the antecedent and no tautology in the consequent. That is, the worlds (of the corresponding models) where the antecedent and the consequent are evaluated, do contain neither contradictions nor tautologies.

In the present reconstruction of MacColl's connexive logic in the context of tableau systems we will make use of two operators with the intended interpretations "α is not a logical truth" and "there is at least a model where α is true". These operators will yield a kind of implicit modal logic. In the third part of the paper we introduce a Kripke-style semantics where two distinguished sorts of sets of worlds take care of logically contingent true and logically contingent false formulae. At the end of the paper we will sketch the way how to combine MacColl's connexive logic with his notion of logical relevance of the preceding section (2.2.1.2).

Contrary to the main-stream style we will go from the proof-theoretical semantics to the model-theoretic semantics. Philosophy is contentious, to use the words of Graham Priest, but actually, this was the way that nowadays relevant logic was developed. Moreover we think that the dialogical interpretation is a sufficient framework to implement the semantic intuitions behind such difficult issues as relevance and "logical connection".

3.2 Tableaux for Connexive Logic

The present tableau system is based on a first formulation of a dialogical connexive conditional introduced by Rahman [1997a] in his *Habilitationsschrift* and further developed in Rahman/Rückert [2001].

It uses two pairs of signs, namely $\{\mathbf{O}, \mathbf{P}\}$ and $\{\bullet, \circ\}$.

The intended interpretation for the pair $\{\mathbf{O}, \mathbf{P}\}$ is *Opponen*t, *Proponent* and for the pair $\{\bullet, \circ\}$ is *burden of the proof of validity*, *no burden of the proof of validity*. If you do not like this dialogical interpretation change it for *true*, *false*; *denial*, *assertion* or whatever. Actually it is the second pair, namely $\{\bullet, \circ\}$ which corresponds to the well known $\boldsymbol{f}$, $\boldsymbol{t}$ signs ((left)right-sequent-rules) of standard tableau systems (sequent calculus).

The tableau rules must then include the combinations of the two pairs of signs. For the sake of simplicity the rules will be formulated for $\mathbf{X}$ and $\mathbf{Y}$, where these letters ($\mathbf{X}\neq\mathbf{Y}$) are slots for $\mathbf{O}$, $\mathbf{P}$. For the standard logical constants we thus have the following set of rules:

$(\mathbf{Y}^\circ)$-***Cases***	$(\mathbf{X}^\bullet)$-***Cases***
$\dfrac{\Sigma, (\mathbf{Y}^\circ\alpha \vee \beta}{\Sigma, (\mathbf{Y}^\circ)\alpha \mid \Sigma, (\mathbf{Y}^\circ)\beta}$	$\dfrac{\Sigma, (\mathbf{X}^\bullet)\alpha \vee \beta}{\begin{array}{c}\Sigma, (\mathbf{X}^\bullet)\alpha \\ \Sigma, (\mathbf{X}^\bullet)\beta\end{array}}$
$\dfrac{\Sigma, (\mathbf{Y}^\circ)\alpha \wedge \beta}{\begin{array}{c}\Sigma, (\mathbf{Y}^\circ)\alpha \\ \Sigma, (\mathbf{Y}^\circ)\beta\end{array}}$	$\dfrac{\Sigma, (\mathbf{X}^\bullet)\alpha \wedge \beta}{\Sigma, (\mathbf{X}^\bullet)\alpha \mid \Sigma, (\mathbf{X}^\bullet)\beta}$
$\dfrac{\Sigma, (\mathbf{Y}^\circ)\alpha \to \beta}{\Sigma, (\mathbf{X}^\bullet)\alpha \mid \Sigma, (\mathbf{Y}^\circ)\beta}$	$\dfrac{\Sigma, (\mathbf{X}^\bullet)\alpha \to \beta}{\begin{array}{c}\Sigma, (\mathbf{Y}^\circ)\alpha \\ \Sigma, (\mathbf{X}^\bullet)\beta\end{array}}$
$\dfrac{\Sigma, (\mathbf{Y}^\circ)\neg\alpha}{\Sigma, (\mathbf{X}^\bullet)\alpha}$	$\dfrac{\Sigma, (\mathbf{X}^\bullet)\neg\alpha}{\Sigma, (\mathbf{Y}^\circ)\alpha}$

The closing rules are the following

- A tableau for $(\mathbf{X}^\bullet)\alpha$ (i.e. starting with $(\mathbf{X}^\bullet)\alpha$) is closed iff each branch (including those of each possible *subtableau*) is closed by means of either the occurrence of a pair of atomic formulae of the form $((\mathbf{Y}^\circ)a$, $(\mathbf{X}^\bullet)a)$ or of a *special closing rule*. Otherwise it is said to be open.

The reasons for including clauses on *subtableaux* and on *special closing rules* will be given in the next section.

The Operators **V** *and* **F**

As mentioned above, the present reconstruction of MacColl's connexive conditional makes use of the following operators: the *satisfiability* operator **V** and its dual the operator **F**. The operator **F** is related to the well-known *failure operator* of Prolog.[68] Let us introduce the corresponding tableau rules for them.

The operator **V**

The intended interpretation of this operator is "there is at least a model where α is true". In the context of a tableau system the intended interpretation of the occurrence of formula $\mathbf{V}\alpha$ in a branch is "there is an **open (sub)tableau** for α".

Actually, one could see any tableau for α as a finite sequence of subtableaux such that the first tableau is a single-point one, whose origin is α, and the other

[68]Gabbay [1987] used this operator for modal logic. Hoepelmann and van Hoof [1988] applied this idea of Gabbay's to non-monotonic logics. Finally Rahman ([1997a], chapter II(A).4.2), introduced the **F**-Operator in the formulation of semantic tableaux and dialogical strategies for connexive logic.

members of the sequence are obtained by application of the tableau rules. We will however, only label explicitly the subtableau opened by the **V** and **F** operators. To keep track between tableau and subtableau we will make use of a system of labels: If the branch where one of both operators occurs carries the label **i**, then the subtableau has the label **i.1**.

More generally, the intuitive idea is that a label **i** names a subtableau and **i**A tells us that A is to be evaluated at the subtableau **i** names. Moreover, our labels will be finite sequences of positive integers such as 1.1.1 and 1.1.2.

DEFINITION 1.

- A *label* is a finite sequence of positive integers. A *labelled formula* is an expression of the form iφ, where **i** is the label of the formula φ.
- If the label **i** is a sequence of length >1 the positive integers of the sequence will be separated by periods. Thus, if **i** is a label and an **n** is a positive integer, then **i.n** is a new label, called *an extension of* ***i***. The label is then an *initial segment* of **i.n**.

Let us assume that the expression $(\mathbf{P}^{\bullet})\mathbf{V}\alpha$ occurs in a branch with the intended interpretation:

> "the proponent who in this branch has the burden of the proof of validity states that there is an open tableau for α".

This formula will generate a subtableau for $\mathbf{P}^{\circ}\alpha$ with the intended interpretation

> "the proponent who in this subtableau does not have the burden of the proof of validity states that there is an open (sub)tableau for α".

The tableau rules for the operator **V** must include the combinations of the two pairs of signs {**O**, **P**} and {$\bullet, \circ$}:

$(\mathbf{Y}^{\circ})$-***Cases***	$(\mathbf{X}^{\bullet})$-***Cases***
$\dfrac{n, (\Sigma, \mathbf{Y}^{\circ})\mathbf{V}\alpha i.}{n+1, \Sigma, (\mathbf{Y}^{\circ})\alpha i.1}$	$\dfrac{n, \Sigma, (\mathbf{X}^{\bullet})\mathbf{V}\alpha i.}{n+1, \Sigma, (\mathbf{X}^{\circ})\alpha i.1}$

"n" is the number of the step in the (sub)tableau **i** where $\mathbf{V}\alpha$ occurs.

The conditions of the closing of the **whole** branch (the branch which starts with the main formula of the whole tableau and which end with a subtableau with the label **i.1**) should be now clear:

- The branch of (sub)tableau **i** where $\mathbf{V}\alpha$ occurs is **open at step** n if the subtableau **i.j** is closed and dually if the subtableau is open then **i** will **be closed at n**.

Examples

In the following examples n indicates the point in the branch of the tableau **i** where **V** occurs and 1.i is the subtableau generated by an application of the rule **V**.

EXAMPLE 1.

$$\begin{array}{c} \vdots \\ n(\mathbf{P}^{\bullet})\mathbf{V}\alpha \wedge \neg a\mathbf{i} \\ (\mathbf{P}^{\circ})a \wedge \neg a\mathbf{i.1} \\ (\mathbf{P}^{\circ})a\mathbf{i.1} \\ (\mathbf{P}^{\circ})\neg a\mathbf{i.1} \\ (\mathbf{O}^{\bullet})a\mathbf{i.1} \end{array}$$

The branch of **i** *is* **open at** n *because the subtableau* **i.1 closes** *with* $\{(\mathbf{P}^{\circ})a, (\mathbf{O}^{\bullet})a\}$

Notice that the subtableau corresponds to the standard tableau for $\boldsymbol{t}\ a \wedge \neg a$.

EXAMPLE 2.

$$\begin{array}{c} \vdots \\ n(\mathbf{P}^{\circ})\mathbf{V}\neg a\mathbf{i} \\ (\mathbf{P}^{\circ})\neg a\mathbf{i.1} \\ (\mathbf{O}^{\bullet})\ \mathbf{i.1} \end{array}$$

The branch is **closed at** n *because the subtableau is* **open**

Notice that the subtableau corresponds to the standard tableau for $\boldsymbol{t}\neg a$.

The operator **F**

The operator **F** is the dual of **V**. The intended interpretation of this operator is "the formula α is not a logical truth". In the context of a tableau system the intended interpretation of the occurrence of formula $\mathbf{F}\alpha$ in a branch is "there is **no closed (sub)tableau** for α"

More precisely let us assume that the expression $(\mathbf{P}^{\bullet}\mathbf{F})\alpha$ occurs in a branch with the intended interpretation:

> "the proponent who has in this branch the burden of the proof of validity states that there is no closed tableau for α".

This formula will generate a subtableau for $\mathbf{O}^{\bullet}\alpha$ with the intended interpretation:

> "the Opponent states that there is a closed (sub)tableau for α where he has the burden of the proof of validity".

The tableau rules are the following:

$(\mathbf{Y}^{\circ})$-***Cases***	$(\mathbf{X}^{\bullet})$-***Cases***
$\dfrac{n, \Sigma, (\mathbf{Y}^{\circ})\mathbf{F}\alpha\mathbf{i}}{n+1, \Sigma, (\mathbf{X}^{\bullet})\alpha\mathbf{i.1}}$	$\dfrac{n, \Sigma, (\mathbf{X}^{\bullet})\mathbf{F}\alpha\mathbf{i}}{n+1, \Sigma, (\mathbf{Y}^{\bullet})\alpha\mathbf{i.1}}$

Notice that this operator produces the change from $X^?$ to $Y^?$ and vice versa

- The branch of (sub)tableau **i** where **F**α occurs is **open at step n** if the subtableau **i.j** is closed and dually if the subtableau is open then **i** is **closed at** n.

EXAMPLE 3.

$$\begin{array}{c} \vdots \\ n(\mathbf{P}^{\bullet})\mathbf{F}a \vee \neg a\mathbf{i} \\ (\mathbf{O}^{\bullet})a \vee \neg a\mathbf{i.1} \\ (\mathbf{O}^{\bullet})a\mathbf{i.1} \\ (\mathbf{O}^{\bullet})\neg a\mathbf{i.1} \\ (\mathbf{P}^{\circ})a\mathbf{i.1} \end{array}$$

The branch is **open at** n *because the subtableau* **closes** *with* $\{(\mathbf{P}^{\circ})a, (\mathbf{O}^{\bullet})a\}$

EXAMPLE 4.

$$\begin{array}{c} \vdots \\ n(\mathbf{P}^{\circ})\mathbf{F}\neg a\mathbf{i} \\ (\mathbf{O}^{\bullet})\neg a\mathbf{i.1} \\ (\mathbf{P}^{\circ})a\mathbf{i.1} \end{array}$$

The branch is **closed at** n *because the subtableau is* **open**

The Connexive Conditional

Tableau rules for the connexive conditional

As we understand it, MacColl's reformulation of the connexive conditional comprises the following restrictions on the strict implication:

1. The antecedent must be logically contingent (not inconsistent)
2. The consequent should not be tautological.

3. The strict implication is to be thought of in a frame with reflexivity. That is, the material implication holds in the world where the strict implication holds.

These three conditions can be expressed in the framework of an implicit modal logic and in the language of the tableau system described above by means of the operators **V** and **F**:

- $\alpha \Rightarrow \beta$ is connexively valid if the antecedent (of the strict implication) yields an open subtableau, i.e. there is at least one model where the antecedent is true (that is, if $\mathbf{V}\alpha$ holds). The idea here is that ***ex contradictione nihil sequitur*** (nothing follows from contradiction). Similarly:

- $\alpha \Rightarrow \beta$ is *disconnexive* if the consequent (of the strict implication) yields a closed subtableau, i.e. there is at least one model where the consequent is false (that is, if $\mathbf{F}\beta$ holds). The idea here is that ***ex quodlibet verum nequitur*** (there is no proposition from which tautological or assumed truth follows).

This yields the following tableau rules

$(\mathbf{Y}^{\circ})$-***Case***	$(\mathbf{X}^{\bullet})$-***Case***
$\dfrac{\Sigma, (\mathbf{Y}^{\circ})\alpha \Rightarrow \beta\mathbf{i}}{\begin{array}{c}\Sigma, (\mathbf{Y}^{\circ})\alpha \rightarrow \beta\mathbf{i}\\ \Sigma, (\mathbf{Y}^{\circ})\mathbf{V}\alpha\mathbf{i}\\ \Sigma, (\mathbf{Y}^{\circ})\mathbf{F}\beta\mathbf{i}\end{array}}$	$\dfrac{\Sigma, (\mathbf{X}^{\bullet})\alpha \Rightarrow \beta\mathbf{i}}{\Sigma, (\mathbf{x}^{\bullet})\alpha \rightarrow \beta\mathbf{i} \quad \vert \quad (\mathbf{X}^{\bullet})\mathbf{V}\alpha\mathbf{i} \quad \vert \quad (\mathbf{X}^{\bullet})\mathbf{F}\beta\mathbf{i}}$

Thus, the application of the rule of the connexive conditional for the $(\mathbf{X}^{?})$-case produces a conjunction with three members, namely the formulae with the two operators **V** and **F** and the standard *material implication*. We will now introduce rules which will have the effect of producing an implicit modal logic in the following sense:

- The subtableaux produced by $\mathbf{V}\alpha$ and $\mathbf{F}\alpha$ will contain, the subformulae of $\mathbf{V}\alpha$ and $\mathbf{F}\alpha$, material implications and no other formula than the subformulae of these implications. Think of the subtableaux as worlds where the only formulae which will be carried from the upper branch are precisely the corresponding material implications. Thus it is as every implication on the upper tableau works as a strict implication.

- The subtableau produced by **V** and **F** are different. That is though the subtableaux have a common ancestor (namely the material implication) one is not accessible to the other.

To implement this we use an idea similar to the deleting device of intuitionistic logic: the formulae **V** (or **F**) will generate subtableaux which contain no other formulae than the subformula of the **V**-formula (or **F**-formula), and the standard conditionals (material implications) of the upper section. The subtableaux generated by the operators **V** and **F** will start with a set $\sum_{[\rightarrow]}$ where "$\sum_{[\rightarrow]}$" reads as follows:

- $\sum_{[\rightarrow]}$-rule:
  If α is the subformula of a **V**-formula (or a **F**-formula) and $\sum \cup \alpha$ the start of the corresponding subtableau, then replace in $\sum \cup \alpha$ **every** connexive conditional occurring in the upper tableau as the main connective by the corresponding material implication, change the **burden of the proof** of the material implication(s) if necessary and according to the rule for the operator at stake. No other formula is an element of $\sum_{[\rightarrow]}$.

In other words, each subtableau will at its start only contain either the antecedent or the consequent of the connexive conditional and the corresponding material conditional. The emphasis on *every conditional* will be made clear in Example 7.

$(\mathbf{Y}^{\circ})$-***Cases***	$(\mathbf{X}^{\bullet})$-***Cases***
$\dfrac{\Sigma, (\mathbf{Y}^{\circ})\mathbf{V}\alpha\mathbf{i}.}{\Sigma_{[\rightarrow]}, (\mathbf{Y}^{\circ})\alpha\mathbf{i.n}}$	$\dfrac{\Sigma, (\mathbf{X}^{\bullet})\mathbf{V}\alpha\mathbf{i}.}{\Sigma_{[\rightarrow]}, (\mathbf{X}^{\circ})\alpha\mathbf{i.n}}$
$\dfrac{\Sigma, (\mathbf{Y}^{\circ})\mathbf{F}\alpha\mathbf{i}.}{\Sigma_{[\rightarrow]}, (\mathbf{Y}^{\circ})\alpha\mathbf{i.m}}$	$\dfrac{\Sigma, (\mathbf{X}^{\bullet})\mathbf{F}\alpha\mathbf{i}.}{\Sigma_{[\rightarrow]}, (\mathbf{X}^{\circ})\alpha\mathbf{i.m}}$

The main tableau starts for α with $(\mathbf{P}^{?})\alpha$

- Starting rule for strategies for connexive logic:
  We assume that a tableau for α starts with $(\mathbf{P}^{?})\alpha$. Thus, a closed tableau (a tableau with all its branches closed) for α proves that α is valid.

Though the practice of closing branches is straightforward the precise formulation of the adequate closing rules is quite tricky because it must include cases where the branch ends with a subtableau and branches where subtableaux do not occur.

- **R-C**: Closing rules for Ω (Ω is either **V** or **F**): Let us assume a tableau the origin of which is $(\mathbf{P}^{\bullet})\alpha$. If a branch of such a tableau ends with a subtableau generated by either a $(\mathbf{P}^{\bullet})\Omega$-formula or a $(\mathbf{P}^{\circ})\Omega$-formula then the whole branch is closed iff the subtableau does not close. A branch of the same tableau which does not end with a subtableau of the form described before[69] is closed iff it ends with a pair of the form $(\mathbf{O}^\circ)a$, $(\mathbf{P}^\bullet)a$. Otherwise it is open.

- Notice that we consider that a branch includes the entire sequence of subtableaux generated:

In the following examples the numeration to the left of the formula indicates the step in the proof while the number to the right labels the tableau and subtableau:

EXAMPLE 5.

1. $(\mathbf{P}^\bullet)\neg(a \Rightarrow \neg a)\mathbf{1}$
2. $(\mathbf{O}^\circ)a \Rightarrow \neg a\mathbf{1}$ (negation rule on 1)
3. $(\mathbf{O}^\circ)a \rightarrow \neg a\mathbf{1}$ (left-rule for connexive conditional on 2)
4. $(\mathbf{O}^\circ)\mathbf{V}a\mathbf{1}$ (left-rule for connexive conditional on 2)
5. $(\mathbf{O}^\circ)\mathbf{F}\neg a\mathbf{1}$ (left-rule for connexive conditional on 2)
6. $(\mathbf{O}^\circ)a\mathbf{1.1}$ ($\mathbf{V}$ rule on 4)
7. $(\mathbf{O}^\circ)a \rightarrow \neg a\mathbf{1.1}$ ($\sum_{[\rightarrow]}$-rule on 4)

Now at the subtableau **1.1**, the standard rule on the material implication $(\mathbf{O}^\circ)a \rightarrow \neg a$ applies:

8. $(\mathbf{P}^\bullet)a\mathbf{1.1}$	|	9. $(\mathbf{O}^\circ)\neg a\mathbf{1.1}$
	|	10. $(\mathrm{P}^?)a\mathbf{1.1}$

The (unique) branch for the tableau ends with a subtableau generated by a $(\mathbf{O}^\circ)\mathbf{V}$-formula. The two branches of the subtableau (**1.1**) close with $(\mathbf{O}^\circ)a$,$(\mathbf{P}^\bullet)a$. Thus, according to **R-C, t**he whole branch and tableau **is closed.**

The proof for $\neg(\neg a \Rightarrow \neg a)$ is very similar but makes use of the other operator:

EXAMPLE 6.

1. $(\mathbf{P}^\bullet)\neg(\neg a \Rightarrow a)\mathbf{1}$
2. $(\mathbf{O}^\circ)\neg a \Rightarrow a\mathbf{1}$ (negation rule on 1)
3. $(\mathbf{O}^\circ)\neg a \rightarrow a\mathbf{1}$ (left-rule for connexive conditional on 2)
4. $(\mathbf{O}^\circ)\mathbf{V}\neg a\mathbf{1}$ (left-rule for connexive conditional on 2)
5. $(\mathbf{O}^\circ)\mathbf{F}a\mathbf{1}$ (left-rule for connexive conditional on 2)
6. $(\mathbf{P}^\bullet)a\mathbf{1.1}$ ($\mathbf{F}$ rule on 5)

[69] That is, if it ends with no subtableau or if it ends with a subtableau generated by a $(\mathbf{O}^\bullet)\Omega$-formula, or $(\mathbf{O}^\bullet)\Omega$-formula.

7. $(\mathbf{O}^{\circ})\neg a \to a\mathbf{1.1}$ ($\sum_{[\to]}$-rule on 5)

Now at the subtableau **1.1**, the standard rule on the material implication $(\mathbf{O}^{\circ})\neg a \to a$ applies:

8. $(\mathbf{P}^{\bullet})\neg a\mathbf{1.1}$ | 9. $(\mathbf{O}^{\circ})a\mathbf{1.1}$

10. $(\mathbf{O}^{\circ})a\mathbf{1.1}$ |

The two branches of the subtableau **1.1** generated by $(\mathbf{O}^{\circ})\mathbf{F}a$ close with $(\mathbf{O}^{\circ})a$,$(\mathbf{P}^{\bullet})a$. Thus, according to **R-C,** the (unique) branch for the whole branch and tableau **is closed.**

EXAMPLE 7.

In the following example the full strength of the $\sum_{[\to]}$-rule is put into action. This rule allows "carrying" **every connexive conditional** as a material implication into the generated subtableaux.

1. $(\mathbf{P}^{\bullet})(a \Rightarrow b) \Rightarrow \neg(a \Rightarrow \neg b)\mathbf{1}$

2. $(\mathbf{P}^{\bullet})(a \Rightarrow b) \to \neg(a \Rightarrow \neg b\mathbf{1}$ | 3. $(\mathbf{P}^{\bullet})\mathbf{V}(a \Rightarrow b)\mathbf{1}$ | 4. $(\mathbf{P}^{\bullet})\mathbf{F}\neg(a \Rightarrow \neg b)\mathbf{1}$

At this stage it should be clear that the development of the **V** and **F** formulae (3, 4) will produce closed branches, because antecedent and consequent of the connexive conditional are logically contingent.

Let us see what happens in the outmost left branch (2) if we develop the rule of the material implication.

LEFT (LEFT) BRANCH

5. $(\mathbf{O}^{\circ})(a \Rightarrow b)\mathbf{1}$ (material implication on 2)

6. $(\mathbf{P}^{\bullet})\neg(a \Rightarrow \neg b)\mathbf{1}$ (material implication on 2)

7. $(\mathbf{O}^{\circ})a \Rightarrow \neg b\mathbf{1}$ (negation on 6)

8. $(\mathbf{O}^{\circ})a \to b\mathbf{1}$ (left-rule for connexive conditional on 5)

9. $(\mathbf{O}^{\circ})\mathbf{V}a\mathbf{1}$ (left-rule for connexive conditional on 5)

10. $(\mathbf{O}^{\circ})\mathbf{F}b$(left-rule for connexive conditional on 5)

11. $(\mathbf{O}^{\circ})a\mathbf{1.1}$ (**V** rule on 9)

12. $(\mathbf{O}^{\circ})a \to b\mathbf{1.1}$ ($\sum_{[\to]}$-rule on 5)

13. $(\mathbf{O}^{\circ})a \to \neg b\mathbf{1.1}$ ($\sum_{[\to]}$-rule on 7)

14. $(\mathbf{P}^{\bullet})a$...$\mathbf{1.1}$ | 16. $(\mathbf{O}^{\circ})b\mathbf{1.1}$
| 17. $(\mathrm{P}^{\bullet})a\mathbf{1.1}$ | 18. $(\mathbf{O}^{\circ})\neg b\mathbf{1.1}$
| | 19. $(\mathbf{P}^{?})b\mathbf{1.1}$

The subtableau closes because of the pair(s) $((\mathbf{O}^\circ)a$, $(\mathbf{P}^\bullet)a)$ and $((\mathbf{O}^\circ)a$, $(\mathbf{P}^\bullet)a)$. Thus, according to **R-C,** the whole branch **is closed.** As already mentioned the other branches (3, 4) close too, thus the whole tableau closes.

The following example displays the case of a formula that is *dangerous* for connexive logic. It is the formula which indicates that if you add both of the connexive theses the logic will explode into triviality. Indeed if one adds $\neg(\alpha\rightarrow\neg\alpha)$ and $\neg(\neg\alpha\rightarrow\alpha)$ to classical logic, then the following holds

$$\neg(\alpha \rightarrow \neg\alpha) \rightarrow \alpha$$
$$\neg(\neg\alpha \rightarrow \alpha) \rightarrow \neg\alpha$$

Let us prove that this type of trivialization will not happen in our system. Indeed we show here that a tableau for $\neg(\alpha \Rightarrow \neg\alpha) \Rightarrow \alpha$ does not close. The proof for the dual $\neg(\neg\alpha \Rightarrow \alpha) \Rightarrow \neg\alpha$ is very similar.

EXAMPLE 8.

1. $(\mathbf{P}^\bullet)\neg(a \Rightarrow \neg a) \Rightarrow a\mathbf{1}$

2. $(\mathbf{P}^\bullet)\neg(a \Rightarrow \neg a) \rightarrow a\mathbf{1}$ | 3. $(\mathbf{P}^\bullet)\mathbf{V}\neg(a \Rightarrow \neg a)\mathbf{1}$ | 4. $(\mathbf{P}^\bullet)\mathbf{F}a\mathbf{1}$
(right-rule for '⇒' on 1)

Branches 3 and 4 yield closed branches. The outmost right branch (4) closes because $(\mathbf{P}^\bullet)\mathbf{F}a$generates a subtableau which will remain open.

To show that branch 3 closes is a little more complicated. The subtableau which it generates will start with $(\mathbf{P}^\circ)\neg(a \Rightarrow \neg a)$, and follow with $(\mathbf{O}^\bullet)a \Rightarrow \neg a$. The three branches of this subtableau will close and cause the whole branch to close. Indeed, the first branch of the subtableau containing the material implication $(:(\mathbf{O}^\bullet)a \rightarrow \neg a)$ will be open by the standard rules and this will yield the closing of the branch. The second branch of the subtableau containing $(\mathbf{O}^\bullet)\mathbf{V}a$, will generate a subsubtableau where the formulae $(\mathbf{O}^\circ)a$, $(\mathbf{O}^\circ)a \rightarrow \neg a$ will yield the closure of the branch with $(\mathbf{O}^\circ)a$, $(\mathbf{P}^\bullet)a$. The case can be shown to be similar for the last branch of the subtableau containing $(\mathbf{O}^\bullet)\mathbf{F}\neg a$.

Unfortunately we are not through yet. Moreover, there is a branch which will cause the whole tableau to be open. Let us show this for the branch containing 2 $(\mathbf{P}^\bullet)\neg(a \Rightarrow \neg a) \rightarrow a$:

LEFT (LEFT) BRANCH

5. $(\mathbf{O}^\circ)\neg(a \Rightarrow \neg a)$ **1** (material implication on 2)

6. $(\mathbf{P}^\bullet)a\mathbf{1}$ (material implication on 2)

7. $(\mathbf{P}^\bullet)a \Rightarrow \neg a\mathbf{1}$ (negation on 6)

8. $(\mathbf{P}^?)a \rightarrow \neg a\mathbf{1}$ | 9. $(\mathbf{P}^\bullet)\mathbf{V}a\mathbf{1}$ | 10. $(\mathbf{P}^\bullet)\mathbf{F}\neg a\mathbf{1}$
(right-rule for connexive conditional on 7)

The application of the standard tableau rule on 8 yields $(\mathbf{O}^{\circ})a$. $(\mathbf{P}^{\bullet})\neg a$ which closes with 6 $(\mathbf{P}^{\bullet})a$. We will develop the middle branch (9) and open the subtableau **1.1**.

11. $(\mathbf{P}^{\circ})a$**1.1** (**V** rule on 9)
12. $(\mathbf{P}^{\circ})a \to \neg a$**1.1**($\sum_{[\to]}$-rule on 6)

13. $(\mathbf{O}^{\bullet})a$... **1.1**	14. $(\mathbf{P}^{\circ})\neg a$**1.1**
	15. $(\mathrm{O}^{\bullet})a$**1.1**

The subtableau generated by the formula $(\mathbf{P}^{\bullet})\mathbf{V}a$ closes because of the pair(s) $((\mathbf{P}^{\circ})a$, $(\mathbf{O}^{\bullet})a)$. Thus, according to **R-C,** the whole branch and tableau are **open**.

3.3 Kripke-Style Semantics for Connexive Logics

Here we can at last spell out the model-theoretical semantics. The idea is very close to Routley/Meyer semantics for relevant logic where a ternary accessibility relation is introduced for the conditional **Rwiwjwk**, such if the conditional is true at **wi** and the antecedent true at **wj**; then the consequent of the conditional is true at **wk**. Moreover, as already mentioned the whole idea seems to come very close to Priest's "Negation as Cancellation and Connexive Logic" [1999], though the present work was developed independently and arouse as a result of a dialogical approach to connexive logic already suggested in Rahman [1997a].

The point of MacColl's connexive logic as understood here is that MacColl's use of the contingency operators is ambiguous: sometimes it is the standard modal use (a proposition which is possible but not necessary in a model set in the framework of modal logic) and sometimes he is rather thinking of *logical contingency* (a formula which is non valid and non-contradictory). We introduce two sets of worlds $\mathbf{W}^{\theta t}$ and $\mathbf{W}^{\theta f}$ in order to implement the latter idea.

Given a model $\langle W, R, v\rangle$ for standard modal logic extended as $\langle \mathbf{W}, \mathbf{W}^{\theta t}, \mathbf{W}^{\theta f}, \mathbf{R}, \mathbf{R}^{*}, \mathbf{v}\rangle$ with

1. two sets of worlds $\mathbf{W}^{\theta t}$ and $\mathbf{W}^{\theta f}$, $\mathbf{W}^{\theta t} \cap \mathbf{W}^{\theta f} = \emptyset$, such that the formula φ is said to hold at $\mathbf{w}^{\theta t}, \mathbf{w}^{\theta t} \in \mathbf{W}^{\theta t}$, iff the valuation v of the model renders φ true at $\mathbf{w}^{\theta t}$. Dually, the formula φ is said to hold at $\mathbf{w}^{\theta f}, \mathbf{w}^{\theta f} \in \mathbf{W}^{f}$, iff the valuation v of the model renders $\neg\varphi$ true at $\mathbf{w}^{\theta f}$.

Tautologies do not hold in worlds from $\mathbf{W}^{\theta f}$ and dually $\mathbf{w}^{\theta t} \in \mathbf{W}^{\theta t}$ does not contain contradictions. Indeed, let us assume that φ is $a \vee \neg a$, then φ does not hold at $\mathbf{w}^{\theta f}$**,** $\mathbf{w}^{\theta f} \in \mathbf{W}^{\theta f,}$because there is no model with a valuation which renders $\neg\varphi$ true at $\mathbf{w}^{\theta f}$.

2. a quaternary relation $\mathbf{R^*}$ between worlds such that $\mathbf{w} \in \mathbf{W}, \mathbf{w}^{\theta t} \in \mathbf{W}^{\theta t}, \mathbf{w}^{\theta f} \in \mathbf{W}^{\theta f}$
   $\mathbf{R}^{*}\mathbf{w}\mathbf{w}\mathbf{w}^{\theta t}\mathbf{w}^{\theta f}$
   (that is, the relation $\mathbf{R^*}$, establishes that w is accessible to w, and that from w there is access to $\mathbf{w}^{\theta t}$ and to $\mathbf{w}^{\theta f}$).

Intuitively, the idea is that the connexive implication $\alpha \Rightarrow \beta$ is true in a given world iff the material implication is true in that world; and there are two accessible different worlds, in one of which the antecedent and the consequent are true and in the other of which the negation of the antecedent and the negation of the consequent are true:

$$\alpha \Rightarrow \beta \text{ is true at } \mathbf{w} \in \mathbf{W}$$
$$\text{iff } w \in W, \mathbf{w}^{\theta t,} \mathbf{w}^{\theta f} \text{ such that } \mathbf{R}^{*}\mathbf{w}\mathbf{w}\mathbf{w}^{\theta t}\mathbf{w}^{\theta f}$$

and all of the following conditions hold

1. $v_w(\alpha)$=0 or $\mathrm{v}_w(\beta)$=1
2. $v_{w\theta t}$ (α)=1
3. $v_{w\theta t}$ (β)=1
4. $v_{w\theta f}$ $(\neg\beta)$=1
5. $v_{w\theta f}$ $(\neg\alpha)$=1

Validity is defined over normal worlds. Clearly, no valid formulae (other than contingencies) could hold in the non-normal worlds.

It should be clear what happens if in the antecedent we have a contradiction or if in the consequent we have a tautology. The connexive conditional fails because of the failure of conditions 2 and 4 respectively.

EXAMPLE 9.
Let us show that $\neg(a \Rightarrow \neg a)$ is valid.

Let us assume that it is false at **w**
then $(a \Rightarrow \neg a)$ is true at **w.**

The relevant conditions are 2 and 3 — we leave the others to the reader. According to these conditions, the following must also be true at $\mathbf{w}^{\theta t}$:

$$a$$
$$\neg a$$

but this is contradictory: both cannot be true at $\mathbf{w}^{\theta t}$. Thus, there is no countermodel to $\neg(a \Rightarrow \neg a)$.

EXAMPLE 10.
Let us show that $\neg(a \Rightarrow \neg a) \Rightarrow a$ is not valid and does not make our system trivial.

We will only show the main parts of the proof.

Let us assume that $\neg(a \Rightarrow \neg a) \Rightarrow a$ is false at **w.**

Because of condition 1, $\neg(a \Rightarrow \neg a)$ must be true and a must be false at w hence $a \Rightarrow \neg a$ must be false at **w**.

If $a \Rightarrow \neg a$ is false, then it is false that some of the truth conditions given above hold in relation to this formula. Indeed this is the case. Let us take once more the conditions 2 and 3: according to these conditions a and $\neg a$ should be true at $\mathbf{w}^{\theta t}$. But this cannot be. Hence it is false that conditions 2 and 3 hold, and therefore the formula is not valid.

Proofs of Soundness and Completeness

The proofs can developed from the following considerations:

1. Subtableaux for **V** and **F** correspond to worlds of the sets $\mathbf{W}^{\theta t}$ and $\mathbf{W}^{\theta f}$.
2. The condition $\mathbf{W}^{\theta t} \cap \mathbf{W}^{\theta f} = \emptyset$ corresponds to the fact that the subtableaux for **V** and **F** will always be in different branches.
3. Given an open branch of a tableau which does not contain a subtableau and interpretation can be read off from the branch in a quite standard way: If $(\mathbf{O}^{\circ})a$ occurs at a node on the branch, assign a the Boolian valuation 1; if $(\mathbf{P}^{\bullet})a$ occurs assign a the value 0.
4. If there is an open branch which starts with the thesis and ends with a subtableau and $(\mathbf{X}^{\circ})a$ occurs at a node on the branch of the subtableau, assign 1 to a; if $(\mathbf{Y}^{\bullet})a$ occurs, assign a the value 0.
5. If a given branch starting with the thesis and ending with a subtableau generated by either an $(\mathbf{O}^{\circ})\Omega$-formula or an $(\mathbf{O}^{\bullet})\Omega$-formula is open, then there is at least one world which according to 1 is an element of one of the sets $\mathbf{W}^{\theta t}$ and $\mathbf{W}^{\theta f}$ such that it satisfies the values described in 4.
6. If a given branch starting with the thesis and ending with a subtableau generated by either a $(\mathbf{P}^{\circ})\Omega$-formula or a $(\mathbf{P}^{\bullet})\Omega$-formula is open, then there is no world which according to 1 is an element of one of the sets $\mathbf{W}^{\theta t}$ and $\mathbf{W}^{\theta f}$ such that it satisfies the values described in 4.

3.4 Relevance and Connexivity

Many proponents of relevant logic might think that the preceding systems for connexive logic are not satisfactory, on the grounds that there are intuitively correct principles concerning the conditional that they do not validate and that there are others that they should validate. For the first group we have $(\alpha \vee \neg \alpha) \rightarrow (\alpha \vee \neg \alpha)$ which is not valid in the systems displayed above, and for the second group we have the paradoxes of material implication. In fact, one of MacColl's motivations was to solve the paradoxes of material implication. As detailed before, MacColl tried to develop a method by means of which no formula should contain (occurrences of) propositional variables which are truth functionally redundant for the truth conditions of the formula involved (see 2.2.1.2).

Thus, e.g., $a \to (a \vee a)$ and $a \to (a \vee b)$ will be "eliminated" from the set of validities because in the first conditional the first occurrence of a can be substituted by any other propositional variable and this substitution is redundant for establishing its truth condition. A similar argument can be put forward for the occurrence of b in the second conditional. To implement this in a tableau system is simple; roughly speaking, every occurrence of a propositional variable has to be used to close a branch. Actually two systems are possible, one in which *every propositional variable* of the main formula has to be used to close a branch (this allows $(a \vee \neg a) \to (a \vee \neg a)$ and $a \to (a \vee a)$ to be valid) and another system which requires that every *occurrence of a propositional variable* has to be used in the sense described above. In the second system the formulae $(a \vee \neg a) \to (a \vee \neg a)$ and $a \to (a \vee a)$ will not yield closed tableaux.

The point is how to combine this relevance logic with connexive logic in such a way as to have a conditional that it is both connexive and relevant.

The idea is quite simple from the point of view of tableaux: if the formula at stake is relevantly valid, then the connexive conditional expressing this formula holds. If the connexive conditional expresses a formula that is one of the Aristotelian or Boethian axioms, then the formula at stake is valid.

Let us write $\alpha \rightarrowtail \beta$ for a relevant conditional in any of the relevant systems mentioned above. The connexive conditional will arise through adding metalogical conditions on this conditional by means of the operators **V** and **F**.

However, here we will assume the weaker system (where different occurrences of propositional variables can make a difference — see 2.2.1.2) for the simple reason that connexive logic then arises as a conservative extension of this type of relevant logic, in the sense that every formula valid in this relevant logic will be valid too in the fragment where the connexive formulae have been added. It is particularly interesting that the infamous trivialization formulae for connexive logic, namely $\neg(\alpha \to \neg\alpha) \to \alpha$, and $\neg(\neg\alpha \to \alpha) \to \neg\alpha$ do not even hold relevantly. Here we assume tableaux with labels for worlds as standard in these type of proof systems for modal logic.

$(\mathbf{Y}^\circ)$-***Case***	$(\mathbf{X}^\bullet)$-***Case***
$\dfrac{\Sigma, (\mathbf{Y}^\circ)(\alpha \Rrightarrow \beta)\ \mathbf{wi}}{\Sigma, (\mathbf{Y}^\circ)\alpha \rightarrowtail \beta \quad \Sigma, (\mathbf{Y}^\circ)\mathbf{V}\alpha\mathbf{wi} \quad \Sigma, (\mathbf{Y}^\circ)\mathbf{F}\beta\mathbf{wi}}$	$\dfrac{\Sigma, (\mathbf{X}^\bullet)(a \Rrightarrow \beta)\ \mathbf{wi}}{\Sigma, (\mathbf{X}^\bullet)\alpha \rightarrowtail \beta\mathbf{wi} \mid \Sigma, (\mathbf{X}^\bullet)\mathbf{V}\alpha\mathbf{wi} \mid \Sigma, (\mathbf{X}^\bullet)\mathbf{F}\beta\mathbf{wi}}$

Here we have assumed a truth-functional semantics for the relevant part of the implication. A fully-fledged model-theoretic semantics for this type of connexive logic is still an open problem, but the following considerations should help:

- $\alpha \Rrightarrow \beta$ is true at $\mathbf{w} \in \mathbf{W}$ iff
  $w \in W, \mathbf{w}^{\theta t,} \mathbf{w}^{\theta f}$ such that $\mathbf{R^*ww}\mathbf{w}^{\theta t}\mathbf{w}^{\theta f}$ is defined just as before

and all of the following conditions hold
$v_w(\alpha \rightarrowtail \beta) = 1$. That is, $v_w(\alpha) = 0$ or $v_w(\beta) = 1$ and there is no propositional variable which is not an element of the truth-determinant set for $\neg\alpha \vee \beta$.
$v_{w\theta t}(\alpha) = 1$
$v_{w\theta t}(\beta) = 1$
$v_{w\theta f}(\neg\beta) = 1$
$v_{w\theta f}(\neg\alpha) = 1$

- Given an open branch of a tableau which does not contain a subtableau and interpretation can be read off from the branch in the following way: If $(\mathbf{O}^{\circ})a$ occurs at a node on the branch, assign a the boolian valuation 1, if $(\mathbf{P}^{\bullet})a$ occurs assign a the value 0. If there is a pair $(\mathbf{O}^{\circ})a$, $(\mathbf{P}^{\bullet})a$ and the branch is open then there is a truth-determinant set for the thesis such that at least one (occurrence) of a propositional variable is not an element of that set.

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APPENDIX

MACCOLL ON LITERARY FICTION

The planet Mars has always been a disturbing presence. As early as the ancient times, because of its provocative movement across the sky, or more recently for the material similarities with our planet. In fact, there are a lot of troubling similarities: its magnitude, its solid surface and atmosphere, the probability of the presence of water. All these characteristics have led to thoughts of the perhaps not so remote possibility of life on Mars. Moreover, in 1877 the American astronomer Asaph Hall (1829-1907) had discovered two moons of Mars, Deimos and Phobos. But there is another fact, most interesting from the point of view of how fiction might have consequences for our real world. The Italian astronomer Giovanni Virginio Schiaparelli (1835-1910) reported that with the help of a telescope he had observed groups of straight lines on Mars. He called these lines "channels" in

Italian but they were erroneously (or maybe intentionally) translated as "canals". This translation might support the speculation about the presence of intelligent beings who built these "canals". Not surprisingly, this inaccurate translation of the report increased popular expectations and fantasy.

As already pointed out by Stein Haugom Olsen in his thorough paper on MacColl's literary work (1999), all these elements made that planet very popular in the days Hugh MacColl decided to write science fiction. In addition to *Man's Origin, Destiny, and Duty* and his work on logic, MacColl also published two novels, *Mr. Stranger's Sealed Packet* (Chatto & Windus, London, 1889) and *Ednor Whitlock* (Chatto & Windus, London, 1891). In *Mr. Stranger's Sealed Packet*, MacColl was also something of a pioneer in his choice of subject: a voyage to the planet Mars. Some years before him, another writer explored the possibility of a journey to outer space: Jules Verne. The French writer chooses the moon as his destination in the novels *De la Terre à la Lune* from 1865 and *Autour de la lune* from 1870. The success of Verne's novels, which was a public affair in France, might have inspired MacColl to expand the idea to Mars. Actually MacColl was not the first but the third to propose a novel in English involving such a kind of adventure. In 1880 Percy Gregg published *Across the Zodiac* (in which an inhabitant of the Earth uses negative gravity to travel through space, discovering on Mars a Utopian society that is highly advanced technologically and that practices telepathy); in 1887 Hudor Genone published *Bellona's Bridegroom: A Romance.* (in which, again, an inhabitant of the Earth discovers on Mars an ideal Anglophone society that rejuvenates instead of growing old). Of course MacColl was not the last. A few years later the most popular book on the field was published: *The War of the Worlds* (1898) by H.G.Wells.

The striking fact is that it is not a fantasy work, as in Edgar Allan Poe's *Adventures of Hans Pfaall* (1835), but science fiction in the style of Verne, although, sadly, in MacColl's work the pedagogical aim of popularizing science is to the detriment of the literary quality.

Indeed, a first approach to his novels will be disappointing for a modern reader of science fiction. In fact MacColl was not really inspired at the time to imagine a "different world". In *Mr. Stranger's Sealed Packet* he actually was projecting onto Mars the world around him. In this respect the novel does not differ much from the first two works of science fiction published about Mars. The point is that he did not have really a literary approach to science fiction but understood the gender rather as a means to illustrate or make popular natural science. Unfortunately the reader notices this from the very start. What is surprising for the reader who knows MacColl from his logical writings is his very conservative of thinking about alternative worlds. In his logic, MacColl conceives worlds where all sort of fictions, including contradictory objects, have their place, but this is not the case in the world of his science fiction.

The main person of his work of science fiction, Mr. Stranger, carries out the wish of his dead father that he should dedicate himself exclusively to science. His father had been working on science and he continues developing his father's theories and discoveries, building a vehicle that produces artificial gravitation. With this anti-gravitational spaceship he visits Moons of Mars — already discovered in the real world — and then the red planet itself. Though the planet is red its inhabitants are bluish. Mars itself is like the Earth, and the Martians are humans transferred to Mars in a prehistoric disaster when the proximity of the red planet allowed the gravitational transfer of a large number of people. The Martians behave like humans, but there is a Utopian constituent in their description. Their society is very rational, with a rigid and superior moral, with a uniform and harmonious structure. There is no illness, no social conflict, and it completely lacks the technology of war.

In both of MacColl's novels we recognize issues typically representative of the Victorian era, namely the conflicts between religion and science. It is a period in which the reexamination of centuries of assumptions began because of the new discoveries in science, such as those of Charles Darwin and Charles Lydell. It was in the order of the day to hold discussions about Man and the world, about science and history, and, finally, about religion and philosophy. This inescapable sense of newness resulted in a deep interest in the relationship between modernity and cultural continuities. MacColl reflects this interest through his novels. Though the two novels are different, they share a number of motifs and concerns and anticipate a number of the arguments MacColl was later to present in *Man's Origin, Destiny, and Duty*.[70]

MacColl shows conservative attitudes and opinions when it comes to these fundamental questions, particularly when he writes about the role of men and women, the family, marriage and so on. In this period in Great Britain, the emphasis on female purity was allied to the stress on the homemaking role of women, who helped to create a space free from the pollution and corruption of society.[71]

On Mars Mr Stranger meets a family and is welcomed into it. He falls in love with the daughter of the family and marries her. His new wife has all the characteristics a Christian man of the Victorian era in England could expect. She is obedient to her husband, compassionate, a loving woman with enough emotional strength and wisdom to become the guardian of the central values of the family and the (Victorian) society in general. She dies upon a visit to the Earth because of her intolerance to the bacteria there. Quite a common issue in fiction: the fictional alternative world is pure and the Earth is impure. The fictional persons die when they encounter the Earth. The Earth, the point where fiction meets reality, makes the fictional persons die. As in *Don Quijote*, the fictional character dies of "reality". Unfortunately MacColl did not make very much out of this exciting feature of fiction. Once more, the reader who knows MacColl from his writings on the logic of fiction might expect the same author to explore the logical and literary possibilities resulting from the intersection between inhabitants of different worlds, since MacColl does suggest some thoughts on this possibility in his logic of fiction, where some domains contain objects of the real world and of the

[70] See Cuypers [1999] and the remarks of the already mentioned paper of Olsen [1999].

[71] Cf. Cuypers [1999] and Olsen [1999].

purely fictional world. But if the same reader reads MacColl's fictional literature he will be utterly disappointed. MacColl doesn't apply MacColl here.

As already mentioned, the novel has an educational aim: instructing his audience in the possibilities opened up by science. MacColl makes full use of known scientific theories and facts to make Stranger's story as plausible as possible. Not unsurprisingly, this defeats the literary value of the whole enterprise.

Ednor Whitlock is not a novel of science fiction, but presents similar characteristics. The beginnings are sufficient to give us an idea of what is going on: Ednor, a young lad of nineteen, looking for shelter from the rain in the local library, by chance picks up an issue of the *Westminster Review*. There he finds a paper and becomes absorbed by the arguments against his religious beliefs. His faith is undermined by becoming aware of the new scientific ideas and the historical criticism of the Bible. MacColl's own beliefs are concerned in this tale. This novel is technically simpler than the other but develops more carefully the thesis that unbelief causes immorality.

Another characteristic we want to point out here is the relationsip MacColl had with Germany and with the German culture. In the first place, he did not know the German language – thus he could not read the work of the German thinkers. He stressed this himself in a letter to Bertrand Russell:

> *...unfortunately all German works are debarred to me because I do not know the language, so that I know nothing of Cantor's and Dedekind's views on infinity.*[72]

But it seems hard to believe that MacColl simply did not read German thinkers because he did not know the language, especially since MacColl, who lived and worked in France for many years, had experience of how to handle situations where different languages are in use.

A role might have been played by some social and political prejudices against Germans shared by British and French society. As already remarked by Cuypers and Olsen, the work *Ednor Whitlock* give us some indices for this hypothesis. In the novel the German motif is presented by means of the Reverend Milford and Fräulein Hartman. The later is an unpleasant character in the novel that MacColl equips with unattractive qualities that are clearly connected with her German ancestry. The former develops arguments in support of theism.

These social and political prejudices were quite often based on what was taken as an attack on Christian faith. Actually, Germany and German universities were the source of the "German Higher Criticism", due especially to the scientific rationalism that became so popular in Germany as a result of Ernst Haeckel's (1834-1919) efforts to turn Darwin's science into a popular movement. He created the conditions for Darwinism to engage a wider public. In fact, Haeckel appears as the main target of attack in *Man's Origin, Destiny, and Duty*, the last book of MacColl's that summarized all his points of view about the conflict between religion and science.

[72] MacColl [1909c].

It is sad that these prejudices prevented him from reading the German mathematicians and logicians of his time. It could have provided him with the instruments he needed to accomplish his various innovative proposals in logic.

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THE IDEALISTS

David Sullivan

1 BRITISH IDEALISM IN HISTORICAL CONTEXT

1.1 The 'revolution' in philosophy

> "What might be called the 'official story' of recent British philosophy, largely written in the 1950s, refers to a 'revolution' which occurred at the beginning of the century ... in the 1950s the idea of a revolution, a radical change in the style of philosophizing, was in the air."[1]

A discussion of the philosophical idealists, once prominent in the British context, typically invokes a strikingly heterogeneous group of thinkers, all born exclusively between 1834 and 1869, and nearly all dead before the conclusion of hostilities in World War II. This group — with two distinct cohorts — is sometimes said to include, *inter alia*:

> E. Caird (1835–1908), T. H. Green (1836–1882), J. Ward (1843–1925), W. Wallace (1844–1897), R. L. Nettleship (1846-1892), F. H. Bradley (1846–1924), J. Watson (1847–1939), B. Bosanquet (1848–1923), H. Jones (1852–1922), J. Bonar (1852–1941), D. G. Ritchie (1853–1903), W. R. Sorley (1855–1935), J. H. Muirhead (1855–1940), R. B. Haldane (1856–1928), A. Seth [Pringle-Pattison] (1856–1931), H. Rashdall (1858–1924), J. S. Mackenzie (1860–1935), G. F. Stout (1860–1944), J. A. Smith (1863–1939), H. C. Sturt (1863–1946), J. M. E. McTaggart (1866–1925), H. W. B. Joseph (1867–1943), H. H. Joachim (1868–1938), and W. R. Boyce Gibson (1869–1935).

Of course this list — as in the case of any such list — is not exhaustive and some of the individuals included therein might admit of certain important, descriptive qualifications. Also, as British Idealism was a growing, evolving movement, another potential concern is that as we move from the earliest cohort to the later one, we correspondingly move from some variant of easily recognizable idealism to various more specialized sects of neo-idealism. In all events, this list ends with, among

[1] Manser, [1983, p. 1].

Handbook of the History of Logic. Volume 4: British Logic in the Nineteenth Century.
Dov M. Gabbay and John Woods (Editors)

other things, a figure remembered primarily as the commentator and translator of Eucken and Husserl (i.e., Boyce Gibson).[2]

In any event, a laundry list of thinkers does not itself constitute a movement. But just what the movement identified as 'British Idealism' was, however, is something neither its proponents nor its critics were in a position to come to any meaningful agreement about.[3] There is truth, of course, to Russell's caricature, which painted a portrait of a metaphysical theorist who affirms that "[e]very apparently separate piece of reality has, as it were, hooks which grapple it to the next piece; the next piece, in turn, has fresh hooks, and so on until the whole universe is reconstructed"[4] and also in Moore's riposte that in asserting the spiritual nature of the universe, modern idealism asserts "(1) that the universe is very different indeed from what it seems, and (2) that it has quite a large number of properties which it does not seem to have."[5] Such well-known quotations do indeed, neatly and cruelly, highlight certain important and quite typical emphases. But, in point of fact, British Idealism — as in the parallel situation of continental neo-Kantianism — names less a coherent school, with an established and agreed-upon doctrine, than a general movement, defined often in the main more by its shared enemies.

British Idealism, which is typically viewed in Anglo-American circles as a peculiar aberration, might appear less problematic if one came to see in it a further permutation or flowering within the larger program of post-Kantian idealism. Here, by the label of 'post-Kantian,' one could thereby encompass not just the immediate successors to Kant but all the various movements deriving in some sense from Kant, in the post-Enlightenment until early Modernism. This was the view first given voice by Royce:

> By the term post-Kantian idealism, we name a group of philosophical movements which grew out of the study of Kant's doctrine, and which are, therefore, closely related to it, but which are usually, in one or another respect, opposed to certain of Kant's most characteristic tendencies. These movements form a varied collection, and cannot be described as the work of any single school of mutually agreeing thinkers ... A list of those who, with more or less obvious justice, might be called post-Kantian idealists, would include Cousin, Strauss, Fechner, Lotze, von Hartmann, T. H. Green, Bradley, and even Martineau, despite his pronounced hostility to Hegelianism. And, in a measure, most of our own American pragmatists could be viewed as the outcome of the same movement. Where such varieties of opinion are in question, there is no longer any reason to speak of a school at all. Post-Kantian idealism,

[2]In addition, it should be noted that there are also a handful of academics following them — e.g. R. F. A. Hoernlé (1880–1943) R. G. Collingwood (1889–1943), G. R. G. Mure (1893–1979), M. Oakeshott (1901–1990) — who form not so much a continuation of that movement but instead a kind of "idealist epilogue," one that runs right up to the end of the twentieth century.

[3]I will use 'British Idealism' in preference to 'British Hegelianism' or any other variant.

[4]Russell, [1964, p. 142].

[5]Moore, [1959, p. 1].

> viewed in its whole range of manifestation, is not any one theory so much as a tendency, a spirit, a disposition to interpret life and human nature and the world in a certain general way — a tendency, meanwhile so plastic, so manifold, so lively, as to be capable of appealing to extremely different minds, and of expressing itself in numerous mutually hostile teachings ... Post-Kantian idealism was prominent among the motives that lead Europe into those revolutionary political activities which centered about the year 1848 ... [and it] has so transformed both the theology and the practical methods of the non-Roman portion of Christendom .... [that] I think it fairly likely that future historians will look back on the history of idealism as being that of the dissolution of the classic Protestantism.[6]

While, at first sight, this might be thought an unhelpful over-generalization, in this brief excerpt Royce does advance several interesting points worth bearing in mind. First, he emphasizes that philosophy in both the German- and English-speaking world passed through a phase of active politicization and controversial religious criticism before completely withdrawing from these areas of inquiry altogether. Second, he suggests that these thinkers, although commonly grouped together, can scarcely be seen to form a school in the orthodox sense. And, third, he speculates that these thinkers might more properly be addressed as a 'movement' or a 'tendency', one with a special affinity for (or connection to) certain theological disputes peculiar to Protestant Europe. To discover which of these is true of British Idealism will require further investigation.

Within this larger intellectual context, our special interest concerns the contributions of the idealist thinkers to the historical development of logic. But, within this narrower vein, we quickly encounter an unpleasant reality: *these thinkers produced very little that today would readily be recognized as a contribution to logic.*[7] Furthermore, this is true is despite a penchant for composing works that prominently feature the word 'logic' in their title — the most famous examples being Bradley, *Principles of Logic* (1883) and Bosanquet, *Logic; or the morphology of knowledge* (1^{st}: 1888, 2^{nd}: 1911).[8] But this appearance of paradox is, in large part, a result of their being considered only in retrospect, as the bulk of their productions preceded the modernist divide in philosophy. Philosophical modernism laid down a division that can be — on both the analytic and the continental side of the equation — summed-up or crystallized in the word 'revolution'. The modernist thinkers of the twentieth century saw themselves as revolutionaries, a

[6]Royce, [1919, pp. 1–3].

[7]By the time that Mace wrote his logic, this sort of characterization was possible: "No serious discussion of logical problems is possible without raising philosophical and epistemological issues, but writers in whose works these issues are prominent and explicit may be described as philosophical logicians" Mace, [1933, p. 15].

[8]Only Bradley and Bosanquet receive (brief) treatment in the article entitled "History of Logic" in the 1967 *Encyclopedia of Philosophy* (v. 4, pp. 549–550). The books published in the twentieth century by Joseph and Joachim — that lie outside our proper purview — receive no mention and these figures were also not accorded individual entries in the *Encyclopedia*.

situation that climaxed in the high modernist works of the teens and twenties.[9] Hence, it will be worthwhile to consider the somewhat amateurish attempts at intellectual history inspired by the analytic movement.

The revolution that was meant to be effected by the philosophical modernists was that given voice in their radical new works.[10] Although usually not accounted the same monumental status as Wittgenstein's *Tractatus* (1921/22), Carnap's *Der Logische Aufbau der Welt* (1928) is perhaps less idiosyncratic and, thus, more representative of the programmatic impulses of this academic, philosophical modernism. Carnap's larger agenda — his connections with modernism in art and architecture and with progressivism in politics — have been widely documented.[11] (Precisely the same sorts of connections can also be made with other prominent figures associated with logical positivism, most notably, perhaps, Neurath.) The logical positivists started new journals and issued polemical manifestos, all with an eye to creating a radical new approach to the theory and practice of philosophy. Of course, the seamless progress of the movement was sharply interrupted by the rise of European fascism, which necessitated exile for a large number of these same thinkers.

Later on, after the Second World War, the academic heirs of the positivist movement sought consolidation, thereby making certain their account with world history. In so doing, however, prominent individuals (and their younger promoters) mis-remembered, distorting the significance of the earlier work, by down playing its connection with prior accomplishments. The end result was often a complete mystification of the previous century of thought in the minds of their contemporary readers. A central symptom of this distortion is the almost omnipresent concept of a 'gap' or 'lull' in philosophical development in the nineteenth century, one that required a re-establishment of philosophy on a new and firmer foundation. Consequently, when analytic philosophers inquired into the philosophical situation after Kant, they typically saw nothing but an incomprehensible, gaping hole.[12] Into this blank space, they projected a malaise that required nothing less than a new and revolutionary methodology for doing philosophy, one that changed the very self-conception of philosophers and their own enterprise. (Such seismic shifts can be monitored by, among other things, the official examinations set for graduates; as Manser notes, in the case of Oxford: "in 1942–43 they were such

[9]In what follows, we shall restrict our discussion solely to the analytic side, although a similar gap to that between Kant and Frege, is identifiable on the continental side, as well: "most philosophers between Hegel and Heidegger ... [are considered] either *epigoni* or *precursors* — to the extent that they are not forgotten altogether" [Schnädelbach, 1984, p. 2].

[10]Schnädelbach enumerates Wittgenstein, Lukács, and Heidegger and their famous works from the twenties (p. 1). It might be noted that these three are the most prominent students of Frege, Simmel, and Husserl, who flourished earlier, within the "neo-Kantian paradigm" (see [Sullivan, 2002]).

[11]See [Gallison, 1990], *et al.*

[12]And, in the English-speaking context, an even larger *lacuna* emerges, one that successfully stifled historical scholarship for decades: "Why bother to produce any evidence for what is perfectly clear to everyone: that between Hume and Russell there is nothing whatever of interest in British philosophy?" [Richter, 1964, p. 138].

as could have been answered by those trained in 1900. By 1946 a very different attitude was needed."[13])

Equally symptomatic is an associated tactic that tries to breach this gap by locating the "re-founding" of respectable philosophy in the works of philosophically-oriented scientists and mathematicians, leaving the intervening academic philosophers completely out of the picture. Accordingly, to authors such as Reichenbach, the tale is thus a charmingly simple one: philosophy lost its way after Kant, separating itself from science and pursuing unjustifiable metaphysical speculations (i.e., Hegel and the Hegelians). This unscientific philosophy perished, after coming to an inevitable dead-end. Only the scientifically infused and inspired reflections of other thinkers, unconnected with the deadening hand of academic philosophy, could create the conditions for philosophy to be re-born.[14] And their successors, the logical positivists were thus the true heirs to this tradition of correct philosophizing, one that was in close accord with the results of scientific research.

Furthermore, another significant, but nonetheless mythic, portrayal central to the self-conception of analytic philosophy, involves an even larger chasm, one seemingly evident to everyone: in the case of logic, between Aristotle and Frege, *not one single step of progress had been made.* (Of course, even Kant had affirmed this view, announcing that logic was a fixed and closed science, akin to Euclidean geometry. Hence, from that perspective, everything that *could* be said on the topic *had* been said.) In analytic mythology, until the revolutionary discoveries of Frege, logic persisted in a long, historically determined lull. Only Frege — whose unique ideas sprang solely from his own cranium, as Athena arose from the head of Zeus — provided the impulse to logic that made it relevant once more to the new philosophy. It is thus these very mythic pictures, tainted with the modernist "revolutionaries" mis-rememberings of the past (in the course of justifying their present and future), that have passed uncritically into the vast number of unexamined, tacit beliefs of most working philosophers.

Such views were subsequently buttressed by the multiple accounts that appeared after the war. These books formed part of the literature emerging to consolidate academically the new movement now baptized "analysis"; they include the following well-known titles, for instance:

> *Readings in Philosophical Analysis*, ed. H. Feigl & W. Sellars, Appleton-Century-Crofts (New York, NY: 1949)
>
> *Philosophical Analysis: A Collection of Essays*, ed. M. Black, Cornell UP (Ithaca, NY: 1950)

[13] Manser, [1983, p. 3]. See the examination papers reproduced in Manser, [1983, pp. 25–27].

[14] See [Reichenbach, 1961]. Reichenbach sermonizes: "... the philosophy of the systems ends with Kant ... The history of philosophy, which up to the time of Kant manifested itself in the form of philosophical systems, should be regarded as continued after Kant not by the pseudosystems of the imitators of a great past, but by the new philosophy that grew out of the science of the nineteenth century and was continued in the twentieth century" [Reichenbach, 1961, p. 122].

Logic and Language, First Series, ed. A. Flew, Blackwell (Oxford: 1950)

Logic and Language, Second Series, ed. A. Flew, Blackwell (Oxford: 1953)

Essays in Conceptual Analysis, ed. A. Flew, Blackwell (Oxford: 1956)

All of these volumes continued to serve as textbooks for several decades to come[15] and to this list also must be added some of the first retrospective or quasi-historical volumes:

The Revolution in Philosophy, A. J. Ayer *et al.*, Macmillan (London: 1956)

English Philosophy since 1900, G. J. Warnock, Oxford UP (London: 1958)

Just as the logical positivists repressed the academic philosophy (i.e., neo-Kantianism) in which much of their learning and thinking had been formed, the English-speaking analysts, for similar reasons, downplayed the interest (and vitality) of the academic philosophy (British Idealism) that had flourished just before the rise of analysis. Part and parcel of this approach is nothing less than *the casting of nineteenth-century thinking as not really philosophy at all.* But in this way, once again, a clear view of the nineteenth century is occluded and it appears only as an anomalous moment in the otherwise coherent story of the progress and development of philosophy. Consequently, all this undigested (and undigestable) material may safely be ignored, with attention focused solely on the dramatic re-birth of the discipline post-Frege.[16]

Unfortunately, one stumbling block the post-war writers had to contend with was *just how long it took some philosophers to catch on to the new order of things.* Again, a paradoxical approach, combined with a sometime torturous re-characterization of events, is the result. Take Warnock's book, for example, and consider the following short excerpts:

> In 1924 and 1925 there appeared two bulky volumes with the general title *Contemporary British Philosophy.* They make strange reading today. Their title itself, even then was doubtfully appropriate ... it should have been clear by 1925 that the scene had changed, was changing and continued to change.[17]
>
> The dominant doctrine that these big men [appearing in the aforementioned volumes] represented ... has been called Absolute Idealism ... It would be extremely difficult to say what this species of philosophy exactly was. Perhaps indeed it was nothing at all exactly.[18]

[15] As my own undergraduate education in the late 1970s confirms.
[16] Or *post*-Helmholtz, *post*-Mach, *post*-Russell, or whatever.
[17] [Warnock, 1958, p. 3].
[18] [Warnock, 1958, p. 4].

> ... the state of British philosophy in the early years of the present century was itself highly unusual and full of novelty. It bred, no doubt, its own revolutionary illusions. To see in it now a tradition is certainly a mistake ... It was in fact an exotic in the English scene, the product of a quite recent revolution in ways of thought due primarily to German influences ... Hume and Berkeley would have been sadly puzzled by the pages of Bradley, to say nothing of Hegel's. But either might have conversed quite naturally with Moore, and with Russell too, at least in his less technical moments.[19]

While there are a number of pertinent observations that could be adduced here, it is important to note at least one: namely, that British philosophy *before analysis* thought of *itself* as contemporary and even revolutionary. But this self-conception was, for the analysts, dead wrong: British Idealism was a foreign transplant, a hothouse tropical flower that could not flourish in the native soil and clime. Because, once again, this abstruse metaphysics had come to a dead end, only the true revolutionaries that followed could both enliven and re-connect philosophy with its living tradition.

In any event, it is undisputed that a clear dividing line is attained once we come to that next generation, that of revolutionary modernists proper — or those figures who were simultaneously to dominate much of the twentieth century and to occlude and obscure our view of the nineteenth century; that is, B. Russell, 1872–1970 and G. E. Moore, 1873–1958. Indeed, although we tend to think of the early part of the twentieth-century as the heyday of analysis (as evidenced in the quote by Warnock), this may yet be another artifact of being viewed from the perspective of the present (in the rear view mirror, so to speak). As already noted, the volumes entitled *Contemporary British Philosophy* from the mid-twenties, while including contributions from both Moore and Russell, is overwhelming dominated by thinkers who could be included under the rubric of "British Idealist."[20]

Given the circumstances embodying these prior assumptions, it is no surprise that the British Idealists were mostly ignored by those who came after them. It would be natural, hence, to assume that the only possible interest in the British Idealism would be that of a perverse antiquarian kind, suitable only for intellectual history. And so it remained for a goodly number of decades. But recently, this intellectual opprobrium has shifted due to interest guided by concerns about the very origins of analytic philosophy itself. Recent scholarship has revealed that, for instance, in the case of Russell, the connection with the old-fashioned academic thought may have been more important than previously acknowledged.[21]

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[19] [Warnock, 1958, pp. 7–8].

[20] There are numerous anecdotes of the tenacious survival of British Idealism in some form or other; of John Anderson, a student of Henry Jones at Glasgow, it is claimed: "Indeed, as late as 1941, Anderson still lectured extensively on Green at Sydney University, exerting a strong personal influence on a number of students ... He expressed himself in the language and style of Green, Bradley, Bosanquet, and Caird, although his basic sympathies owed more to Scottish common-sense philosophy" [Vincent, 2006, p. 489].

[21] Cf. *Idealism and the Emergence of Analytic Philosophy* by P. Hylton [1990] and *Russell's*

For, although analytic philosophy was on the ascent after 1910, idealism was nowhere near its end. (This is easily documented by perusing pre-World War II volumes of *Mind* or *Proceedings of the Aristotelian Association.*) Furthermore, despite the blanket assumptions of many of the practitioners of analytic philosophy, there is little philosophical work that can be pointed to as providing a definitive *refutation* of idealism — *pace* Moore. For although the idealists may have been guilty of various logical mistakes (although no more so than any other figures in the history of philosophy), there is no single document or unique methodology that finishes off idealism as a philosophical program. Rather, interest gradually shifted to new topics and approaches so that idealism was not so much refuted as merely superseded.[22] (Hence, the unconscious living-on of idealist tendencies, even within its announced enemy, is perhaps less remarkable than at first glance.) The question may at the very least be raised: did modernist "revolutionaries" build upon the earlier advances of the previous generation? The following is a continuation of, and also a very small contribution to, that line of discussion.

1.2 Why logic mattered to British Idealism: the logic question

> "... the fact remains that Bradley and Bosanquet ... turned direct to Germany for their inspiration, not merely to Kant and Hegel but to Herbart, Lotze, Sigwart, and Ueberweg. Bosanquet, indeed, did little more than acclimatise German logic in England ...."[23]

For individuals with a reference point rooted firmly in the twentieth century, the very phrase "the new logic" names, first, the logistic or predicate calculus codified by *Principia Mathematica* (1910-13) and, second, the critical historical developments that followed directly in the first half of the twentieth century (involving the well-known names of Gödel, Tarski, etc.). Strangely, however, even a superficial examination of the historical record reveals that the very phraseology of "the new logic" first appears in the middle of the *nineteenth century*, predating the appearance of even that other founding document, the *Begriffsschrift* (1879). But the existence of an active project devoted to the "renovation" of logic, undertaken in the nineteenth century, expressly contradicts the perspective of the philosophical radicals of the twentieth century who tried to portray nothing but one big, continuous (and empty) chasm between Aristotle and Frege.

Of course, even the great Kant himself had already declared that since Aristotle not one step of progress had been achieved, nor would it. So what could motivate the post-Kantian thinkers, obviously inspired by Kant, to second-guess him on this issue? An answer, should it be formulable, would have to be located in the seminal events in philosophy *after* Kant. Here, numerous intellectual historians have noted

Idealist Apprenticeship by N. Griffin [1991], for two pertinent examples.

[22]For more on this topic see [Baldwin, 1984].

[23][Passmore, 1966, p. 159].

another fracture or line, centering around the year 1835. Indeed, after the deaths of Hegel and Goethe, absolute idealism itself underwent a rapid transformation, with the Hegelian movement fracturing into different, competing sects; also other oppositional views quickly came to the fore, including "late idealism" and "materialism" — both of which were sometimes presented as stark alternatives. It was within this peculiar intellectual ferment — in this emerging Hegel-critique — that an attempt to revamp the place and position of logic began.

This notion of a nineteenth-century project devoted to the revivification of logic might seem *prima facie* incredible unless the following phenomena are acknowledged: first, the mighty proliferation of works in logic after the death of Hegel; and, second, the fact that the situation with regard to logic in post-1835 involved new forays into a middle territory between what came to be perceived as two polarizing and untenable positions. Indeed, one of the most famous and influential of the early works — Trendelenburg's *Logische Untersuchungen* (1 840) — sought to revivify Aristotelian logic precisely in this middle ground by standing firmly against the two extremist positions, identified by the schools of Hegel and Herbart respectively. This book thus deserves a prominent place amongst the German-language volumes devoted to logic that saw the light in this time-period. Indeed, in this circumstance, Trendelenburg's influence can scarcely be over-estimated:

> Trendenlenburg stimulated anew the debate on the question how must logic be constituted in order to be able to lay claim to be a philosophical discipline. It was precisely a matter of understanding logic as philosophy and as science. The former appeared missing in formal logic, the latter he denied to Hegelian logic ... His own answer to this question was that logic must gradually stake out its entire domain through continuing specialized research. It is a philosophical discipline when it is oriented to the investigation of the philosophical assumptions in the sciences.[24]

Köhnke begins his monumental work on neo-Kantianism with Trendelenburg – who he dubs the "Great Unknown" — and asserts that, with the publication of the *Logische Untersuchungen*, "the modern movement in epistemology and scientific theory that was to attain its culminating point in neo-Kantianism" truly began.[25]

With Trendelenburg, logic was now to be recast as the *basic* science — a new "*philosophia fundamentalis*"[26] — for the following reason: although we may, licitly, speak of methodologies peculiar to the special sciences, the *factum brutum* remains that

> ... these various different procedures are only an expression of *a single mode of thought* which, assuming many shapes, clings and conforms to its object in order to lay hold of and comprehend it. In the sciences all

[24] [Frank, 1991, p. 248].
[25] [Köhnke, 1991, p. 11]. Note too the striking modernity of the modest title: "Logical Investigations."
[26] [Köhnke, 1991, p. 22].

> that happens is that this single mode of thought is given different kinds of stimulus to the invention of ever new skills and artifices to which the object has to submit as though captive ... method is what makes the science a science. And when the methods appear in the object of interest of the sciences but are not given by it but have their universal and common foundation in the thought that is working through the object of interest, this leads us to the task of seeking out their origin in the nature of thought.[27]

The individual sciences distinguish themselves by their individual objects of investigation and by their specialized methods of inquiry. But, as they all depend upon thought *simpliciter*, the science of thought (or, logic) must underlie (and, in a sense, underwrite) all of these aforementioned sciences. Hence the need to investigate and re-vivify the doctrine and practice of logic appears paramount and also the prospect for logicism, as a foundationalist program, revived.

The historical reality of this renewed interest in logic has recently come to light in academic scholarship, most notably in a seminal article by Hartwig Frank "Reform efforts of logic at mid-nineteenth century in Germany" (1991); it has also has received a book-length treatment in Risto Vilkko's monograph, *A Hundred Years of Logical Investigations: Reform Efforts of Logic in Germany 1781 — 1879* (2002). Vilkko's assessment of our contemporary situation speaks to the 'gap' situation, emphasized above:

> Many historians and philosophers of logic have claimed that during the nineteenth century, before Gottlob Frege's *Begriffsschrift* (1879), there was a period of stagnation or even of decline in the field of logic not only in Germany but also elsewhere in the Western world ... Statements about the 19th century as a non-creative period in the history of logic can be shaken by mere reference to a number of significant first editions that were published in Germany in the field of logic during the decades of suggested decline.[28]

Now, of course, one important proviso must be inserted here is that these "logics" typically included material that today we would parcel out to epistemology and the philosophy of science. For instance, Lotze's *Logik* (1871) provides, perhaps, a paradigmatic case of this state of affairs: "The first part ... deals with pure logic; the second part ... with methodology or the philosophy of science; and the third part ... with scepticism and other epistemological issues."[29] Our failure to acknowledge this crucial difference typically stems from the historically

[27] Cited in [Köhnke, 1991, pp. 20-21].

[28] Vilkko, 2002, pp. 11-13. Frank's assessment is even sharper: "... I will argue that the work on logic in Germany in the years between 1830 and 1880 may claim an absolutely independent place in the history of logic, a place that may be briefly termed 'the generation of ideas by reform projects.' We cannot then avoid the assumption that such a *Zeitgeist* had some influence on the great reformer in logic, Frege" [Frank, 1991, p. 247].

[29] [Vilkko, 2002, p. 14].

anachronistic assumption that "since there were ... only a few logicians who took the term 'logic' to mean roughly the same as we do know, then hardly anything important happened in the field of logic during that time."[30]

While not an exhaustive catalog, by any means, the following gives a good indication of the flourishing of texts and important articles on "logic" during the years 1835 to 1870 (with only some, not all, of the subsequent editions noted):

1835–1843
Drobisch, *Neue Darstellung der Logik* (Leipzig, 1^{st}: 1836; 2^{nd}: 1851; 3^{rd}: 1863; 4^{th}: 1875)
Schleiermacher, *Dialektik* (Berlin, 1839)
Trendelenburg, *Logische Untersuchungen* (Leipzig, 1^{st}: 1840; 2^{nd}: 1862; 3^{rd}: 1870)
J.E. Erdmann, *Grundriss der Logik und Metaphysik* (Halle, 1^{st}: 1841; 2^{nd}: 1843; 4^{th}: 1864)
Beneke, *System der Logik als Kunstlehre des Denkens* (Berlin, 1842)
Trendelenburg, *Erläuterungen zu den Elementen der aristotelischen Logik* (Berlin, 1^{st}: 1842; 2^{nd}: 1861)[31]
Trendelenburg, *Die logische Frage in Hegels System. Zwei Streitschriften* (Leipzig, 1843)
Lotze, *Logik* (Leipzig, 1843)

1844–1852
H. Grassmann, *Die Ausdehnungslehre* (Berlin, 1^{st}: 1844, 2^{nd}: 1862)
Harms, *"Von der Reform der Logik und dem Kriticismus Kants, Ein Entwurf"* in *Jahrbücher empirischen Wissenschaften*, 1/1846, pp. 128-164
Rosenkranz, *Die Modifikationen der Logik, abgeleitet aus dem Begriffs des Denkens* (Leipzig, 1846)
Chalybäus, *Entwurf eines Systems der Wissenschaftslehre* (Kiel, 1846)
Trendelenburg, *"Geschichte der Kategorienlehre. Zwei Abhandlungen"* in *Historische Beiträge zur Philosophie* (Berlin, 1846-67)
Prantl, *Die Bedeutung der Logik für den jetztigen Standpunkt der Philosophie* (Munich, 1849)
K. Fischer, *Logik und Metaphysik, oder Wissenschaftslehre* (Stuttgart, 1^{st}: 1852; 2^{nd}: 1865)
Ulrici, *System der Logik* (Leipzig, 1852)

1853–1861
Steinthal, *Grammatik, Logik und Psychologie* (Berlin, 1855)
Prantl, *Geschichte der Logik in Abendlande* (Leipzig, 1855-1870)
Ritter, *System der Logik und der Metaphysik* (Göttingen, 1856)
Prantl, *Ueber die zwei ältesten Kompendium der Logik in deutscher*

[30] [Vilkko, 2002, p. 14].
[31] This is edition that was suggested for *student use* at Oxford on the topic.

Spracher (Munich, 1856)
Ueberweg, *System der Logik und Geschichte der logischen Lehren* (Bonn, 1st: 1857; 3rd: 1868, 5th: 1882)
Schmid, *Entwicklungsgeschichte der Hegel'schen Logik* (Regensburg, 1858)
Rosenkranz, *Wissenschaft der logischen Idee* (Königsberg, 1858-1859)
Ulrici, *Compendium der Logik* (Leipzig, 1st: 1860; 2nd: 1872)

1862–1870
Jevons, *Pure Logic* (London, 1864)
Rabus, *Logik und Metaphysik* (Erlangen, 1868)
Biedermann, *Kants Kritik der reinen Vermunft und die Hegelsche Logik* (Prague, 1869)
Trendelenburg, *Kuno Fischer und seine Kant* (Leipzig, 1869)
Fischer, *Anti-Trendelenburg: eine Gegenschrift* (Jena, 1870)
Rosenkranz, *Erläuterungen zu Hegel's Encyklopädie der philosophischen wissenschaften* (Berlin, 1870)
Ulrici, *Zur logischen frage* (Halle, 1870)
Bain, *Logic* (London, 1870)

Towards the end of this period, "the new logic" — and the associated slogans of "*die Reform der Logik*" and "*die logische Frage*" — were so well established that they began to become the subject of further reflection in various synoptic and retrospective works, such as Rabus, *Die neuesten Bestrebungen auf dem Gebiete der Logik* (1880) — a volume that was among the first to include Frege within its scope of review.

While the "new logic" does not name a single view, it does accurately describe a kind of research program that agreed upon certain principles, while disagreeing about others. For instance, the 1911 Encyclopedia Britannica article, in the section entitled "General Tendencies of Modern Logic," expostulates the situation for "the logic of our day" as follows:

> In the first place, it tends to take up an intermediate position between the extremes of Kant and Hegel. It does not, with the former, regard logic as purely formal in the sense of abstracting thought from being, nor does it follow the latter in amalgamating metaphysics with logic by identifying being with thought. *Secondly, it does not content itself with the mere formulae of thinking, but pushes forward to theories of method, knowledge and science; and it is a hopeful sign to find this epistemological spirit, to which England was accustomed by Mill, animating German logicians such as Lotze, Duhring, Schuppe, Sigwart and Wundt.* Thirdly, there is a determination to reveal the psychological basis of logical processes, and not merely to describe them as they are in adult reasoning, but to explain also how they arise from simpler

> mental operations and primarily from sense.[32]

Accordingly, the new logic is (1) seemingly agnostic on the realistic division or the idealistic identification of mind and world; (2) fundamentally epistemologically-oriented, concerned with both method and discovery; and (3) prone to psychological reductionism. The question that required immediate further discussion, of course, was whether (2) demands (3): or, *psychologism or anti-psychologism*. Indeed, this is where many practitioners of the new logic *disagree*, whether the epistemological orientation of logic demands cashing-out by experimental results in psychology *or not*.[33]

As a pertinent example of this new orientation, the Britannica article references the preface to the second edition of Sigwart (1888). While noting that Sigwart's views have a psychologistic bent, as the quoted remarks make clear, the article's author also stresses the significance of epistemology and methodology for a proper system of logic:

> "Important works have appeared by Lotze, Schuppe, Wundt and Bradley, to name only the most eminent; and all start from *the conception which has guided this attempt. That is, logic is grounded by them, not upon an effete tradition but upon a new investigation of thought as it actually is in its psychological foundations, in its significance for knowledge, and its actual operation in scientific methods*." How strange! The spirit of every one of the three reforms above enumerated is an unconscious return to Aristotle's *Organon*. Aristotle's was a logic which steered, as Trendelenburg has shown, between Kantian formalism and Hegelian metaphysics; it was a logic which in the Analytics investigated the syllogism as a means to understanding knowledge and science: it was a logic which, starting from the psychological foundations of sense, memory and experience, built up the logical structure of induction and deduction on the profoundly Aristotelian principle that there is no process from universals without induction, and none by induction without sense. Wundt's comprehensive view that logic looks backwards to psychology and forward to epistemology was hundreds of years ago one of the many discoveries of Aristotle.[34]

Again, the principles of this peculiar orientation are very apparent: this new movement is, on their own description, nothing but the rediscovery of *the true meaning of Aristotle*: in steering between the Scylla of Kantian-Herbartian formal logic, on the one hand, and the Charybdis of Hegelian metaphysical logic, on the

[32] [Case, 1911, p. 885] (my emphases).

[33] On the very emergence of a new issue, that of psychologism vs. anti-psychologism, cf. [Kusch, 1995].

[34] [Case, 1911, p, 885] (my emphases). The quotation is from the preface to the second edition (SL: x). In the preface to the first edition, Sigwart acknowledges his "obligation" to Trendelenburg, Ueberweg and Mill, all of whom "have died while the book was being planned and carried out" (SL: ix).

other, the intermediate position is that which approximates logic as an *organon*, an instrument for "understanding knowledge and science."[35]

Another important characteristic of the new logic, as highlighted by the Britannica article, concerns the priority of judgement over concept. For instance, the article continues to remark that the new logic's "healthy" emphasis on *judgement* represents a "recovery from Hume's confusion of beliefs with ideas and the association of ideas, and the distinction of the mental act of judging from its verbal expression in a proposition."[36] As further evidence, it then goes on to cite several representative theories of judgement under discussion, characterizes them in the following summary fashion; judgement is such that

a. It expresses a relation between the content of two ideas, not a relation of these ideas (Lotze).

b. It is consciousness concerning the objective validity of a subjective combination of ideas, i.e. whether between the corresponding objective elements an analogous combination exists (Ueberweg).

c. It is the synthesis of ideas into unity and consciousness of their objective validity, not in the sense of agreement with external reality but in the sense of the logical necessity of their synthesis (Sigwart).

d. It is the analysis of an aggregate idea (*Gesammtvorstellung*) into subject and predicate; based on a previous association of ideas, on relating and comparing, and on the apperceptive synthesis of an aggregate idea in consequence; but itself consisting in an apperceptive analysis of that aggregate idea; and requiring will in the form of apperception or attention (Wundt).

e. It requires an idea, because every object is conceived as well as recognized or denied; but it is itself an assertion of actual fact, every perception counts for a judgment, and every categorical is changeable into an existential judgment without change of sense (Brentano, who derives his theory from Mill except that, he denies the necessity of a combination of ideas, and reduces a categorical to an existential judgment).

f. It is a decision of the validity of an idea requiring will (Bergmann, following Brentano).

g. Judgment (*Urteil*) expresses that two ideas belong together: by-judgment (*Beurtheilung*) is the reaction of will expressing the validity or invalidity of the combination of ideas (Windelband, following Bergmann, but distinguishing the decision of validity from the judgment).

[35] [Case, 1911, p. 885]. According to Köhnke, "... the Aristotelianism of the nineteenth century can be understood as an aspect of the interest which ... led to the various attempts made in the 1820s and 1830s to effect an epistemological transformation of logic" (Kôhnke, 1991, p. 23).

[36] [Case, 1911, p. 885].

h. Judgment is consciousness of the identity or difference and of the causal relations of the given; naming the actual combinations of the data, but also requiring a priori categories of the understanding, the notions of identity, difference and causality, as principles of thought or laws, to combine the plurality of the given into a unity (Schuppe).

i. Judgment is the act which, refers an ideal content recognized as such to a reality beyond the act, predicating an idea of a reality, a what of a that; so that the subject is reality and the predicate the meaning of an idea, while the judgment refers the idea to reality by an identity of content (Bradley and Bosanquet).

k. Judgment is an assertion of reality, requiring comparison and ideas which render it directly expressible in words (Hobhouse, mainly following Bradley).[37]

Strangely, despite their apparent disunity, these approaches, which include Bradley and Bosanquet, are *all are characterized as similarly modern due to the fact that they concern complete contents of thought (and not mere ideas) and to the fact that the judgement itself is not conceived of as built-up out of more primitive, atomic parts.* Hence, another important aspect of the new logic is its clear opposition to empiricist approaches, favoring instead a holistic view that distinguishes amongst the component parts of judgement via a process of de-composition (or the very view commonly associated with Frege).

The volumes by Lotze and Sigwart were, of course, translated by the British Idealists (and their spouses). And although Trendelenburg's works were well-known, they received no such treatment. However, the point of view espoused by Trendelenburg could rely on another relevant mode of transmission via T. M. Lindsay's translation (1871) of the third edition (1869) of Ueberweg's *System of Logic and History of Logical Doctrines*.[38] In the preface to the first edition (1857), Ueberweg also seeks to situate his volume within the diverging streams of existing literature, identifying, first, explicitly with the tradition initiated by Schliermacher, namely that which "... sought to explain the forms of thinking from science ... [and to establish] their parallelism with the forms of real existence." This approach may, once again, be characterized as holding "a middle place between the subjectively-formal and the metaphysical Logics," a position that is

> ... at one with the fundamental view of Logic [*der logischen Grundansicht*] which Aristotle had. The subjectively-formal Logic — that promulgated by the schools of Kant and Herbart — puts the forms of thought out of all relation to the forms of existence. Metaphysical Logic, on the other hand, as Hegel constructed it, identifies the two kinds of forms, and thinks that it can recognise in the self-development of pure thought the self-production of existence. Aristotle, equally far

[37] [Case, 1911, p. 886] (There is no item *j*.)

[38] Passmore claims that Ueberweg's book "was a widely-read text-book, both in Germany and in England" [Passmore, 1966, p. 159n].

> from both extremes, sees thinking to be the picture of existence [*das Abbild des Seins*], a picture which is different from its real correlate and yet related to it, which corresponds to it and yet is not identical with it (USL: xi (s. iii)).

This telling soon neatly recapitulates the Trendelenburg line: the middle territory in logic includes a swath of thinkers (more or less aligned with Schliermacher's view) including Ritter and Vorländer, on the near side, and Trendelenburg and Lotze, on the far one. Indeed, we may conclude that, excepting the already acknowledged followers of Kant-Herbart or Hegel, "... the whole post-Hegelian labours in the province of the doctrine of thought and knowledge [*auf dem Gebiete der Denk- und Erkenntnisslehre*] ... occupy a common middle place between the opposites" (USL: 72 (s. 57)).

Ueberweg's book, while maintaining a spirit of independence, also "proceeds in the direction denoted by the labours of these men" (USL: xii (s. iv)). Accordingly, Ueberweg defines logic as "the science of the regulative laws of human knowledge [*die Wissenschaft von den normativem Gesetzen der menschlichen Erkenntniss*]," where these laws denote "those universal conditions to which the activity of knowledge must conform in order to attain to the end and aim of knowledge" (USL: 1 (s. 1)). Logic is thereby fully coordinate with epistemological investigations. Hence, logic as "the doctrine of knowledge [*Erkenntnislehre*]," constituting the "middle position" and occuping "the mean between" the two extremist doctrines mentioned above. In all events, the main point is clear: logic was being busily re-conceptualized in the time period after the death of Hegel, and just leading up to the British Idealists. Whether or not this theorization was subsequently fruitful is not as important as the fact that logic was a matter of lively discussion and debate in these intervening years.

1.3 Why logic mattered to British Idealism: the rehabilitation of philosophy

"The study of philosophy was becoming a serious affair."[39]

In the search for the closest thing to a group manifesto unique to British Idealism, one could turn quite naturally to the 1883 volume dedicated to the memory of Green, the *Essays in Philosophical Criticism*. (It should be observed first that the contributors had decided (implicitly) that the essay form was the appropriate venue for philosophical thinking of the sort they favored.) One might also note that the salient term in the title was 'criticism' (and, consequently, not any number of other possible terms that come to mind.) Caird, in his introduction, insisted however squarely upon emphasizing "a certain community of opinion ... [that] may be described as an agreement as to *the direction* in which inquiry may most fruitfully be prosecuted" (EPC: 1, my emphases):

[39]BCW, V: 200.

> The writers of this volume agree in believing that the line of investigation which philosophy must follow, or in which it may be expected to make the most important contributions to the intellectual life of man, is that which was opened up by Kant, and for the successful prosecution of which no one has done so much as Hegel (EPC: 2).

To follow in the footsteps of Kant and Hegel, from Caird's perspective, implicates a singular purpose of vision but not an attachment that involved anything like slavish discipleship:

> The only important question now is, not whether we are disciples of Hegel, – the days of discipleship are past, — but whether we recognize the existence of a living development of philosophy, and especially of that spiritual or idealistic view of things in which philosophy culminates — a development which begins in the earliest dawn of speculation, and in which Kant and Hegel are, not indeed the last names, but the last names in the highest order of speculative genius, *i Maestri di color che sanno* (H: 224).

For Caird, the British Idealist project entailed neither the importation of thought from another time and place nor an imposition of scholastic slavishness to that same thought. Instead, a living (and growing) relation to this thought is required — much as Green himself had embodied — one showing "that it is possible to combine a thorough appropriation of the results of past speculation with the freshness and spontaneity of an original mind" (EPC: 3).

If, following this argument, there are no serious grounds for expecting a singular representative of British Idealism — because British Idealism represents more tendency and an approach than any orthodoxy of scholastic doctrine — the most famous British Idealist, F. H. Bradley, could scarcely be thought to provide an apt example. For, although the very mention of the term 'British Idealist' typically brings to mind the pre-eminent personality of Bradley, this is rarely useful. (In particular, one might note that Bradley, especially, is *sui generis*.) However, even Ryle — in his programmatic introduction to the *Revolution* volume — was forced to acknowledge, first, the importance of Bradley as a thinker and, second, the existence of certain central affinities between Frege and Bradley: namely, anti-psychologism, epistemological holism, logical form (as distinct from grammar), a recognition of the truth-value as intrinsic, and a burgeoning insight into semantics[40] — i.e., some of the very issues arising first in the discussion of the new logic.

Without exploring here any of these areas of potential overlap now, a tangential question now forces itself upon thoughtful observers: it asks for an explanation of this state of affairs, given the seeming radical differences in intellectual context (despite the nearly overlapping births and deaths of these two thinkers). The answer — it will be suggested here but not established — is to be found in the

[40]See [Ayer, 1957].

fact that both thinkers did indeed share the same intellectual backdrop, namely the one provided *to* the generation of Green, Bosanquet, Frege, Bradley and Royce *by* the previous generation of Trendelenburg, Mill, Lotze, Lange and Ulrici. This earlier group included both German logicians — J. E. Erdmann, Prantl, Sigwart — but also important figures on the English side, including Mansel, Jowett, and J. H. Stirling. We may label these thinkers "the second generation after Kant" (b. 1795–1830), who functioned effectively as the teachers of "the third generation" (b. 1834–1869) that followed.

The postulation of a second generation naturally leads to speculation about the philosophical agenda of that group. But part of the answer has already been provided: for, first and foremost, there is the approach centered on logic, one sufficiently broadened to include questions of epistemology and scientific methodology. But, of course, why else did 'epistemo-logic' suggest itself as the *philosophia fundamentalis*? The appeal of this view arose, it may be hypothesized, because of the great advances in the special sciences at the same time that philosophy itself seemed to be less and less relevant: "In the course of the nineteenth century, a change of function occurred in philosophy and in its relations to other disciplines that frequently found resonance externally as a sharp rejection of any philosophy at all."[41] Various sciences began, one by one, to detach themselves from philosophy altogether. Simultaneously, the formerly superior position occupied by philosophy within the university system was also under attack:

> Up until the 1850s, "lawyers and physicians had to hear philosophical seminars and undertake an examination in the so-called *Tentamen philosophicum* in logic and psychology." This kind of examination was later abolished and replaced by a *Tentamen physicum* — in Prussia through a ministerial command of February 19, 1861. Now "it was only was only theologians, philologists, and mathematicians for whom some philosophical disciplines (in particular history of philosophy) existed."[42]

In the face of these pressures, philosophy underwent a kind of disciplinary crisis that has been codified by some as 'the rehabilitation of philosophy':

> The phrase 'the rehabilitation of philosophy' refers to the essential features of the attempt to allot to philosophy, in a scientific age, a domain of problems which would be independent of the special sciences ... The best known form of the rehabilitation of philosophy is a theory of knowledge which first appeared in the fifties of the last century.[43]

The philosophers most famously implicated here are the neo-Kantians, the school that "rehabilitated philosophy as a whole in the form of the theory of knowledge by

[41] [Pester, 1991, pp. 242-43].
[42] [Pester, 1991, p. 242].
[43] [Schnädelbach, 1984, p. 103].

attributing to this discipline the function of a basis for philosophy and science."[44] Here it is important to note that the work of many "Hegelian" thinkers also played a role in this movement: "Karl Rosenkranz arranged an edition of Kant from 1838 onwards, and in 1860 there appeared the first edition of the book on Kant by Kuno Fischer, which went through many editions and influenced the general interpretation of Kant until well into our own century."[45] And, of course, the lack of any firm distinction between Kantianism and Hegelianism is characteristic of the British Idealists.

The increasingly *academic* discipline of philosophy required rehabilitation because it had undergone a profound identity crisis in the intellectual furor in the years after the deaths of Hegel and Goethe.[46] This crisis arose in part because, after Hegel, there sprung forth a confusing panoply of differing systems whose very existence posed yet another dilemma: "the increasing incongruity between the constantly rising number of philosophical systems on offer, on the one hand, and the claim to scientific character and validity which each of these systems made for itself, on the other."[47] Hence, there existed a dazzling multiplication of competing systems while, simultaneously, the sciences were producing unparalleled results *in complete isolation from philosophy*. Subsequently, a series of questions were posed, from within and without the philosophical profession, concerning the exact relation of philosophy and the sciences:

> How could philosophy assure its own scientific character in relation to the spectacular technical and material achievements of the special sciences? More fundamentally, what was, in the end, genuinely "scientific" about philosophical discourse? Was it subject matter? Method? A rigorous adherence to internal rules of scientific logic? In an era of post-Hegelian crisis, one asked: What was the principal relationship of philosophy to science (as a model) and to the sciences (as specific forms of research)?[48]

The favored response to these questions, not unconnected with the figuration of logic as basic epistemology, was the "concept of philosophy as *the* science." This idea, it has been asserted,

> ... became increasingly dominant from the nineteenth century to the present. This took place, not on the basis of the inner wealth and original impulses of the philosophizing, but rather — as in neo-Kantianism — out of perplexity over the proper task of philosophy. It appears to

[44] [Schnädelbach, 1984, p. 106].
[45] [Schnädelbach, 1984, p. 105].
[46] The notion of "pervasive transformations in the entire bourgeois culture after the deaths of Hegel and Goethe" can be, of course, attributed to a variety of factors, including "the industrial revolution, the ascent of the natural sciences and medicine, and the entry of the working class into the arena of world history" [Pester, 1991, p. 239], just to name the most obvious.
[47] [Pester, 1991, p. 93].
[48] [Bambach, 1995, p. 23].

> have been deprived of this perplexity because the sciences have occupied all fields of reality. Thus, nothing was left for philosophy except to become the science of these sciences, a task taken up with increasing confidence, since it seemed to have the support of Kant, Descartes, and even Plato.[49]

Of course, the crisis of philosophy did not occur in a vacuum: it stood out against parallel developments in the sciences and in the productive capacity of society as a whole. For after "a period of relative cultural uniformity spanning the decades between 1770 and 1830" there came "the beginning of the age of science, of historical culture, of realism and 'disillusionment' ... [that heralded] the beginning of the crisis of European humanist civilization."[50] In this way, 'crisis' itself came to be identified with the very process of the movement from the post-Enlightenment to Modernism and, thence, of Modernism itself.

In a different way, some intellectual historians prefer to emphasize a correlative "resurrection of philosophy" in the years after 1870.[51] Of course, such labeling does nothing to explain the underlying causes of these shifts (this discussion will have to take place elsewhere). The point at hand is simply the widespread acknowledgement that it was "a historical fact that at this time a profound transformation had taken place in academic philosophy."[52] For our purposes, we need record that it was the first cohort of the third generation that were the "first beneficiaries" of the resurrection of philosophy.[53] Numerous opportunities arose in the German context because of a hiring crisis that demanded the filling of numerous vacancies left open during the 1850s and also because of the "very rapid extension of the new universities undertaken by the new German Reich," including the establishment of a new German university at Strasbourg.[54]

The "boom in academic positions of the 1870s" is evidenced in the statistic that "between 1860 and 1880 the number of teachers in the philosophical faculties of Germany almost doubled."[55] It was also exhibited in the changed behavior of the younger academics

> When, after securing his doctorate in 1871, Friedrich Paulsen rejected his teacher Trendelenburg's suggestion that he should consider taking the senior teachers' examination he was not only acting quite differently from the way in which the preceding generation of those who had studied in the 1850s and early 1860s had acted: though an impecunious son of a peasant he could immediately afterwards venture to enter on an academic career. As the decline in the number of *Privatdozenten*

[49] [Heidegger, 1988, p. 10]. This, perforce, cuts both ways: (1) philosophy as the supervisory science; and (2) philosophy as the discipline freed from the constraints of any particular method.
[50] [Schnädelbach, 1984, p. 3].
[51] See [Köhnke, 1991, pp. 198ff].
[52] [Köhnke, 1991, p. 198].
[53] [Köhnke, 1991, p. 201].
[54] [Köhnke, 1991, p. 202].
[55] [Köhnke, 1991, p. 203].

> after 1870 shows, in the entire history of the German university there has probably never been a time when conditions were for favourable for achieving at least an assistant professorship so soon after receiving a doctorate and license to lecture as they were in the first years of the new Reich.[56]

In the British context, the socio-economic situation was importantly different. Furthermore, the aforementioned questions about philosophy and science were further complicated as a consequence of their being embedded within larger issues of university reform — issues that themselves were further tinged by the politics of the varying Anglican church parties (and, in particular, because of the rising ascendance of the Broad Church party within certain university circles). In opposition to the pedagogical approach of the High Church party, two entirely different attitudes (embodied by Jowett and Pattison, repectively) began to emerge; these new trends were clear by 1865, when Jowett observed that, at present

> ... there is a great change in education at the Universities, especially at Oxford. When I was an undergraduate we were fed upon Bishop Butler and Aristotle's *Ethics*, and almost all teaching leaned to the support of the doctrines of authority. Now there are new subjects, Modern History and Physical Science, and more important than these, perhaps, is the real study of metaphysics in the Literae Humaniores school — every man for the last ten years who goes in for honours has read Bacon, and probably Locke, Mill's *Logic*, Plato, Aristotle, and the history of ancient philosophy. See how impossible this makes a return to the old doctrines of authority.[57]

A corresponding shift in attitude amongst the students is found in the petition presented to Green by his students, proposing his leadership of an "essay society." Signed by seven undergraduates (including both A. C. and F. H. Bradley), the petition from 1872 began:

> What some people feel the need of now in Oxford: (1) belief in principles, instead of the present eclecticism; (2) earnest effort to bring speculation into relation with modern life instead of making it an intellectual luxury, and to deal with various branches of science, physical, social, political, metaphysical, theological, aesthetic, as part of a whole instead of in abstract separation; (3) co-operation instead of the present suspicious isolation; (4) fearlessness in expression of opinions amongst men who really have opinions, instead of the present deadly reserve.[58]

The earnest tone of the petition is itself an indication of a number of things. But clearest among its explicitly elaborated sentiments in the view that philosophy

[56][Köhnke, 1991, p. 204].
[57]Cited in [Richter, 1964, p. 143]. For more information, see [den Otter, 1996, pp. 36-44].
[58]Known as MS 5 June 1872, cited in [Richter, 1964, p. 159].

needs to be pursued both freely, soberly, and in a way which betokens professional competence. Of course, in Green himself one encounters the first modern exemplars of the professional (and professorial) philosopher: indeed, it is sometimes said that Green was "certainly the first Fellow of his College and possibly the first of his University to conceive of himself as a professional philosopher."[59] Now whatever view Green actually held of himself, one critical fact remains: Green, unlike prior academics, did not take orders. Thus, we come face to face with another structural shift in the English universities away from the clerical and tutorial towards the secular and professorial.[60]

Yet it is precisely in this context that the turn to Hegel by English academics appeared to be wildly out of step, so to speak, with the present-day movements and the directions of the times. This seemed so even to some contemporary observers. James, for example, worried aloud, the pages of *Mind*, in 1882, suggesting that the revival (and further spread) of "Hegelism" was contraindicated by its contemporaneous demise on German soil. If this were indeed the case, then the British Idealists had done nothing but dig their own (historical) grave. But the focus for the Idealists, I would suggest, was on Kant and Hegel as logical theorists of the first order: it was their transcendental approach that afforded philosophers the possibility of a rigorous methodology for their discipline, as sharpened and strengthened in the "new logic" debate of the post-Hegel period.

Some have postulated that this 'mis-step' was made possible because of the insularity of British thought, which remained "largely oblivious to the sustained critique of his thought that had taken place in Germany (since Hegel's death)". Only because of this myopia had "they found it possible to resurrect a form of ... Hegelianism at just the time when this critique had made such a position 'defunct' on the Continent."[61] That such a commonplace will not stand, however, is documented in both the lives and the writings of the earliest idealists. The relevant parties were fully cognizant of the ins and outs of the Hegel debate in the decades preceding them. Many of them made academic trips to the German universities. Some Idealists provide abundant citations or reviews of contemporary literature.[62] And, finally, there are the translation projects, such as that of Lotze, started by Green and finished by Bosanquet. Such factors should readily establish that the British Idealists were not unawares of the contemporaneous Continental debates.

Furthermore, as discussed in the next section, the British Idealists' view of Hegel was much more complex (and interesting) than the merely cartoon version of Hegel as an *apriorist* bent upon offering superfluous explanations of some reality

[59] [Richter, 1964, p. 140].

[60] Likewise, a complete historical accounting would have to take note of the contemporaneous emergence of professional journals such as *Mind* (1876) in Britain — and, on the continent of the *Vierteljahrsschrift für wissenschaftliche Philosophie* (1877) — that stand as another signpost of important changes underway.

[61] [Stern, 1994, pp. 297-98].

[62] For instance, Caird reviewed Rosenkranz, Tredelenburg, Kuno Fischer, Emil Arnoldt and J. B. Meyer in the years 1870-71.

and blithely ignoring other aspects of that same reality whenever it suited him. Stern, in his article "British Hegelianism: A Non-Metaphysical View?" (1994), takes pride of place as the first to voice this possible approach. But although Stern is correct in emphasizing the positive effects of the more traditional sources of Hegel criticism (e.g., Schelling, Feuerbach, *et al.*), it is important to give at least equal weight to the literature directly centering on the debates about the new logic and about the methodology of philosophy (e.g., Trendelenburg, Lotze, the Herbartians, etc.) as seminal sources for re-interpretation.[63]

Furthermore, in England, the earliest expositor of Hegel's *Logic*, the second generation's J. H. Stirling (1820–1909) and his curious volume, *The Secret of Hegel* (1st edition: 1865), may well be the inspiration of this new, non-metaphysical orientation. For Stirling, Hegel's *Begriffe* had their origins in Kant's categories, albeit now freed from the Kantian implication of transcendental subjectivity:

> Stirling argued that Hegel's development of Kant centers around the notion, or *Begriff*, and is an attempt to go beyond Kant by showing that the categories of the understanding are not merely abstract universals awaiting content, but categories which are "reciprocated" or "externalized" in nature ... For Kant the categories are abstract universals, or pure concepts, which require the act of judgment in order to subsume particulars under them. In Hegel, however, the categories are neither abstract nor completely concrete, but are moments in the activity of speculative reason whereby abstract understanding progresses to concrete knowledge.[64]

Consequently, the type of thought that utilizes these entities outstrips the merely theoretical understanding, one which is incapable of unifying its world. By contrast, philosophical inquiry demands the categories for its rightful development:

> True philosophical thought, Hegel argued involves the employment of speculative reason and is, according to Sterling, concrete because the *Begriff*, or notion, is the universal which particularizes itself. Or, to put it another way, the *Begriff* is the active process by which thought determines itself and gives itself content. By "concrete" Hegel means thinking which deals only with universal types and is not restricted to partial, dependent, or externally determined objects, or, indeed, to anything sensuous. "The task before us," Hegel wrote, "consists not so much in getting the individual clear of the state of sensuous

[63]Bradley in 1907 and again in 1920 implicated this psycho-epistemo-logical ferment: "So far as I remember, such writers as Bain & Lotze & again the Herbartians, such as Volkmann & again Waitz (who is more or less Herbartian), helped me the most. The Hegelian psychology on desire & will is also far from negligible & I think I learnt a good deal from that, especially as I came to that before Lotze & the Herbartians" (BCW, V, p. 60) and "Certainly as to my psychological views I owe the most to Hegel, and, next to Herbart, or rather to his followers Waitz, Drobisch and Volkmann. I am however indebted to a considerable extent to Lotze" (BCW, V, p. 244).

[64][Stormer, 1979, pp. 49-50].

> immediacy" as it is in "actualizing the universal and giving it spiritual vitality, by the process of breaking down and superseding fixed and determinate thoughts" (*Phenomenology*, Baillie, trans., 94).[65]

So perhaps it was this sort of non-metaphysical viewpoint that allowed the British Idealists to see and to characterize Hegel as the heir to and clearest expositor of the Kantian legacy (which is way, among other things, they often spoke the two names in the same breath): Hegel was simply the best and most up-to-date version of the Kantian philosophy. Furthermore, as the Idealists turned primarily to Hegel's logic, for the historical reasons elaborated above, Hegelian philosophy, for them, was not in the main religion, phenomenology, art, ethics but logic. Finally, some British Idealists were quite explicit that Hegel had "to be done over again" and that parts of his theory were utterly dispensable — viz. the dialectic (cf. EPC: 63).

To see this, we must next turn to the actual words of these Idealists themselves. Consequently, in what follows, substantial quotation from the authors is required so that they may speak in their own voice, given that these voices are not oft heard or recognized (and the volumes from which these are taken rarely consulted or made available). Only three idealists — prominent in the nineteenth century — will be investigated and then only portions of their doctrines (of a logical nature) will be discussed. An entire survey of their philosophies is not possible, nor even an entire survey of their logical doctrines. Instead, an intensive deliberation of a small subset of their views will, hopefully, prove to be more revealing. We will see first that the debate generated by the logic question and the pressures resulting from the rehabilitation of philosophy, resulted in a Hegel conceived, by the British Idealists, less as an arch-metaphysician and more as a comrade-in-arms of the various thinkers critical to the gestation of the neo-Kantian paradigm.[66]

2 CASE STUDIES OF INDIVIDUAL IDEALISTS

2.1 The 'non-metaphysical interpretation' of Hegel

> "... the great problem as to the relation of the human to the divine ... is the greatest theme of modern philosophy."[67]

While difficult to assay now, the nineteenth century was a century obsessed with religion and, in particular, with the relation of religion to philosophy and to science. And although the basic circumstances characteristic of the nineteenth century involved a continuation of the project initiated by the rise of modern science — the demythologization of the human and natural world — the British Idealists were, in the main, optimistic that, by means of the insights of German Idealism,

[65] [Stormer, 1979, pp. 44-45].
[66] For more on the neo-Kantian paradigm, see [Sullivan, 2002].
[67] ETGP II: 360.

"a way *through* the modern principles of subjective freedom ... to a reconstruction of the intellectual and moral order on which man's life had been based in the past" (H: 3) was possible. For if it were not, on the view of the Broad Church party, the fruits of radical individualism could be nothing less than the inevitable rebirth of either irrationalism or authoritarianism; for, as Edward Caird remarked, it is precisely

> ... those men who have most deeply been imbued by the modern spirit of subjectivity, which knows no authority but itself and opposes its own inner light to all external teachings of experience, have not infrequently been driven in the end to save themselves from the waywardness and vacuity of mysticism by subjecting themselves to the outward rule of an authoritative Church (H: 208).

The Idealists, as theological liberals, hoped instead that "emancipation from the weight of the past," rather than sweeping away everything of value, would only more clearly reveal "the permanent basis of human faith and hope, the eternal rock on which all human beliefs and institutions are built" (H: 1). Naturally, the earlier attitude, from the childhood of man (myth and poetry), is forever gone: the "first immediate awe and reverence ... has passed away from the world" (H: 112). But the "prosaic world" of "finite science" can be overcome by "neither poetry nor religion" but only, instead, by the offices of *philosophical criticism*, "by awakening science to a new consciousness of its presuppositions" (H: 114-115).

As the earlier quote makes clear, a not-so-subterranean enemy in these polemics is the High Church party, in general, and those who subsequently converted to Catholicism (Newman *et al.*), in particular. Bosanquet, for example, accuses subjective psychologism itself of giving rise to "the doctrine of the 'Grammar of Assent', which makes assent or affirmation both absolute and irrational" (KR: 116). Caird acerbically adds that Newman's work "asserts the right — in the general impossibility of finding sufficient evidence for any kind of religious truth — to treat insufficient evidence as if it were sufficient" (H: 17). This or any other attempt to restore faith through a "sacrifice of reason," Caird insists, will end only in perfect "slavery" (H: 209). We must move instead through "negation" to "reaffirmation," from mere "emancipation from the weight of the past" to "the permanent basis of human faith and hope" (H: 1). But in all events, we must acknowledge that the "cure for the diseases of rationalism and skepticism" cannot be found in "implicit faith" (H: 209).

For Caird, because the nineteenth century must be conceived as "a movement through negation to reaffirmation, through destruction to reconstruction," the significance of Hegel's approach is sketched out along similar lines; Passmore comments that

> ... his [Hegel's] philosophical method, the dialectic, is, as Caird sees it, a method of reconciliation. For him there are 'no antagonisms which cannot be reconciled' — there must always be a higher unity within which antagonistic tendencies will each find a place. Thus if religion

> and science appear to be irreconcilably opposed, this can only be an appearance; in reality they *must* form part of a higher unity.[68]

This helps make the general purport of such philosophical investigations clear; for Caird specified, in his own words, that "the task of philosophy is to gain, or rather to regain, such a view of things as shall reconcile us to the world and to ourselves. The need for philosophy arises out of the broken harmony of ... a spiritual life, in which the different elements or factors seen to be set in irreconcilable opposition to each other ..." (ELP: 191).

Now while the topic of religious belief provides a readily apparent subtext, the larger story of Caird's Hegel includes placing him within the wider context of just what Kant accomplished and, more importantly, how Kant's work was successfully corrected by the work of the post-Kantian idealists. (Ironically, the Kant that emerges is much closer to that of the continental neo-Kantians than anything like the historical Kant.) As Caird neatly summarized it, the main advance made possible by Kant can be glossed as follows: "... neither time nor space nor the facts of experience conditioned by them exist for us, except as elements of an experience which is organised according to the categories. *This is the essential truth which Kant had to express*" (ELP: 405, my emphases). The central motif of Kantianism is that there can be no experience without the priori existence of that which lies outside of experience, the pure concepts of the understanding or the categories.

Nevertheless, there are also several difficulties planted amongst the constructive contributions of Kant's philosophy and, indeed, on one construal, his key advance also comprises his central defect; for about the Kantian philosophy, it could be said that:

> Its main merit is, that it shows that experience rests on something which, in the ordinary sense, is beyond experience; or, what is the same things in another point of view, that it brings out the relativity of being to thought ... in so far as it shows that reality as known is phenomenal ... Its weakness lies in this, that it does not carry the demonstration to its legitimate result; it still retains the idea of a "thing in itself" (H: 121).

Experience depends upon the existence of cognitive elements that are not found in experience. But in retaining the uncognizable *noumenon*, such thinking embodies, however, "an absolutely irreconcilable dualism" that cannot overleap the structural divides between "[s]ense and understanding, necessity and freedom, the phenomenal and the real self, nature and spirit, knowledge and faith" (H: 121-122). Into this very breach steps Hegel. Hegel's purposes, thereby, is nothing less than "to show that the kingdoms of nature and spirit are one, in spite of all their antagonisms ... that this antagonism itself is a manifestation of their unity" (H:

[68] Passmore, 1966, p. 53.

128). But to attain this reconciliation involved, perforce, breaking "with all the ideas of logical method that had hitherto ruled the school" (H: 129).

The first step in this break with logical tradition is a crucial one: it was the realization that thought itself had, from the time of Aristotle onwards, been pigeon-holed as "always distinction, determination, the marking off of one thing from another" (H: 134). But thought is not just division for it also unites: "thought is *not only* distinction, it is at the same time *relation*. If it marks off one thing from another, it, at the same time, connects one thing with another" (H: 134-35). Thus Caird emphasizes the following epistemo-logical principle:

> If, therefore, we say that everything — every intelligible object or thought as such — must be differentiated from all others, yet we must equally say that no object or thought can be absolutely differentiated; in other words, differentiated so as to exclude any identity or unity which transcends the difference. An absolute difference is something which cannot exist within the intelligible world ... (H: 135).

Hence, co-requisite with the "reform of logic" was "a new conception of knowledge" (H: 148), for from the "old analytic logic" could issue only "such a theory of knowledge [that] is, as it were broken in pieces against the idea of self-consciousness, in which the true unity ... is seen to be essentially complex or concrete" (H: 148-149). The true understanding of self-consciousness presupposes and contains "all the categories by which science and philosophy attempt to make the world intelligible" (H: 150). And to exhibit these truths is the function of logic in Hegel's *oeuvre.*

As previously noted, while our Hegel is, most commonly, the Hegel of the *Phenomenology*, the Hegel of the British Idealists is clearly the Hegel of the greater (and lesser) *Logic*. And what Hegel's singular contribution to logic was, according to British Idealism, is a major component of Caird's 1883 monograph. After some long biographical re-countings, and a sketchy history of Kant and post-Kantian thought, Caird takes pains to emphasize that the logical universal is not empty and abstract — "[u]niversality is readily confused with emptiness" (H: 155; cf. H: 151-157). This is followed by a *précis* of the greater *Logic* appears on pp. 164-166, as a forward to the extended discussion, which develops that argument, on pp. 167-183.

On Caird's reading, the *Logic* possesses a triadic structure, one that ascends via each of its three main sections. The first division — that concerns "categories like Being, Quality, Quantity" (H: 164) — is the level of ordinary or "common sense" (or, perhaps, of unreflective, passive observation): here, reality is conceived of being composed of individual things "standing each by itself, determined in quality and quantity, but as having no necessary relations to each other" (H: 165). The second division – which involves categories such as "Force ... Substance ... and Effect" — re-conceptualizes the world "as an endless aggregate of essentially related and transitory existences, each of which exists only as it determines and is determined by the others, according to universal laws" and moves us, subsequently, to the stage of scientific or reflective thought. But this too must be ultimately superseded in a third standpoint, one that truly deserves the name "philosophical thinking"; for it alone is dominated by certain special categories "such as those of final cause and organic unity" (H: 165). The division portrayed seems, hence, to mirror that between common sense, scientific understanding, and speculative reason.

In the final, *philosophical* stage, the domain of multiplicitous objects is now painted as one where "each and all of which exist only in so far as they exist for intelligence, and so far as intelligence is revealed or realised in them" (Ibid.). For Caird, from the "simplest and most unsophisticated consciousness of things" we are lead, by way of "the scientific or reflective consciousness," to "the categories of *Ideal Unity*" (H: 166). Accordingly we must acknowledge that

> Science is the *truth* of common-sense ... and philosophy is the *truth* of science ... because it is science and something more. This something more ... is not merely something externally added to what went before; it is a vital growth from it, — a transformation which takes place in it, by reason of latent forces that are already present. In this way self-consciousness — the last category or point of view — is seen to sum up and interpret all that went before (H: 166)

Of greatest interest in this Cairdian reading is both the peculiar centrality of the *Logic* and, therein, the special insistence upon the notion of *category.* Kant, on this view, had stopped short of the highest categories, resting instead with "the categories of reflection — categories like causality and reciprocity — ... [as] the last scientific determination of nature" (H: 190). Kant had also left them subjectivized by merely establishing "that the categories are only forms of expression for the unity of self-consciousness in relation to the world of objects" (H: 184). But these categories must be brought to truth in the highest categories in a way that "involves a complete inversion of ... [the scientific] way of thinking" (H: 192); we come, thereby, to contemplate necessarily

> ... the world as an organism in which even what is termed by distinction the inorganic is a vital part or organ. The partial prevalence of this mode of thought is shown by the tendency ... to regard human society as an organism, — a whole in which there is some kind of unity or self which is present in every part, — and not as a mere collection of units externally related to each other (H: 192).

This holistic and teleological conception is both reinforced and embodied in the notion of the highest category, also describable as "the last category," the one that

> ... contains and implies all the other categories; and, in another way, it has been shown to be implied in each and all of them. For what the whole 'Logic' has proved is, that if we take the categories seriously, abstracting from all subjective associations, and fixing our attention on

> their objective dialectic, — or, in other words, if we leave the categories to define themselves by the necessary movement of thought through which they carry us, — they lead us in the end to this idea of self-consciousness as their ultimate meaning or truth (H: 183).

The final category is, thus, 'self-consciousness'. But now if we have rejected Kant's deduction, by what means are the categories to be procured so that this path is possible? According to Caird, Hegel insisted that this question too demanded an holistic (and an historical) approach:

> ... the categories must be considered in themselves and in their relation to each other, — rather than in relation to the objects to which they are applied or in which they are realised ... Hegel, in short, is, in his 'Logic,' simply seeking to prove that these different categories are not a collection of isolated ideas, which we find in our minds and of which we apply now one, now another, as we might try one after another of a bunch of keys upon a number of isolated locks; he is seeking to prove that the categories are not instruments which the mind *uses*, but elements in a whole, or the stages in a complex process, which in its unity the mind *is* (H: 157).

In this manner, Caird proposes to confute the spectre of Hegel as the *apriorist* metaphysician *par excellence*: *viz.,* as a thinker involved in nothing other than "an a priori construction of the world" (H: 195), where his *Logic* was supremely "the groundwork ... for an attempt to construct nature *a priori*, and without reference to facts and experience" (H: 157). This bogey is defeated, however, once we acknowledge the status of the categories as "the forms of thought implied in all existence," whose relations, one to the other, are of "an imminent relativity or necessary connection with them, so that the other categories spring out of it the moment we attempt to confine it to itself" (H: 161).[69]

If these factors are not kept squarely in mind, it will be impossible to block the alternative approach that seeks to comprehend "what a thing is, ... *in itself*, apart from all relation to other things or the mind" (H: 160). Such a seemingly common-sense approach, when "logically worked out to its consequences"

> ... leads directly to the conclusion that the reality of things, — that which things are *in themselves*, — is unknown and unknowable. *For all existence is but the manifestation, and all knowledge but the apprehension, of relations; and the attempt to strip a thing of its relations must therefore end in reducing it to a caput mortuum of abstraction of which nothing can be said* (H: 160-61, my emphases).

The attempt to arrive at "bare particulars," stripped of all relations, leads to a philosophical impasse, an *aporia* that demands, ultimately, Hegel's vision that

[69] Although he is gradually forced to admit that in many Hegelian utterances we find expressions that "seem to be breaking through the very limits of language" (H: 181).

> ... there is not merely formal process of intelligence — no process of intelligence which is not also a determination of its object by categories; – and the advance from less to more perfect knowledge is a continual transition from one category to another by which that determination is changed, and made more complete and accurate. While, therefore, knowledge is ... a process in which the mind is continually bringing the object more and more within the net of its categories, and changing its aspect, till all its strangeness has disappeared, and it has been made one with the thought that apprehends it. Thus the investigation of the object turns out to be at the same time the evolution of the mind in relation to it; and the highest category by which *it* is determined is at the same time the discovery of its essential relativity to the mind for which it is, and the recognition that in thus dealing with an object, the mind is really dealing with itself — or, in other words, with something that forms an essential element in its consciousness of self (H: 186-87).

The development of mind or intelligence is coeval with the construction of our picture of external reality, for Hegel. But a few of the details of that process, in its guise of "triadic" structure first mentioned above, require further elaboration.

In the beginning, it seems, cognizers take objects *as they are*, "as they lie before us in perception" (H: 167). This, unreflective, common sense, however, by access to the Eleatic paradoxes, soon gives rise to a skepticism that casts doubt upon our heretofore unperturbed sense of reality. This is where the "scientific or reflective consciousness" enters the picture, beginning, as it were, "with the negation of the immediate reality of finite things," inspired to uncover "some deeper ground or principle" (H: 168). But within this schema, such "explanation can never be complete" because

> [t]he categories used are such as substance and accident, force and expression, inner and outer being, cause and effect. In each of these cases we have an essential relation of two terms of such a kind that, though the explanation of the second term is always sought in the first, yet the first term has no significance except in relation to the second ... Thus we explain the accidents by referring them to the substance; but the substance has no meaning apart from the accidents (H: 171-72).

This epistemological circle, which results from "unresolved dualism ... left by the application of the scientific categories," reveals "the necessity of a reinterpretation of the results of science by other higher categories" — and it is this very task that "constitutes the peculiar work of philosophy" (H: 173) according to Hegel's *Logic*. Employing an organic example, Caird speculates aloud that:

> The life of the body is not a principle that dominates over dead members, and uses them as instruments to realise itself; it is *in* all the members, so that each of them in turn may be regarded as means and

> end to the others. There is, no doubt, a unity of the whole that subordinates all the parts, but it only subordinates them, so to speak, by surrendering or imparting itself to them, and giving to them a certain independent life, — a life which, though embraced in a wider circle, is still centered in itself (H: 178-79).

Thus we may now underwrite our critical, aforementioned principle: "The reality is the universal, which goes out of itself, particularises itself, opposes itself to itself, that it may reach the deepest and most comprehensive unity with itself" (H: 180-81). The universal as concretized in reality is simply an alternative way of expressing the truth that "the world is an organic unity" (H: 181). It is the concept of the universal that condenses together what we have identified as the holistic and the historical.

But although this approach requires a view of "the world is an organic unity," this is not the same as claiming that the world ought "to be interpreted on the *analogy* of the living body, or of a plant or animal" (H: 181). Rather, "the conception of an ideal or self-determining principle, with which we begin this third stage of the Logic ... will be seen to find its further final form and expression only in *self-consciousness*" (H: 181-82). Self-consciousness is hence equivalent to "the last category, [that] contains and implies all the other categories" (H: 183). The conclusion?

> ... it follows that the objective world is and can be nothing but the manifestation of intelligence, or the means whereby it attains the fullest realisation of itself. Thus it is proved that there is a *spiritual* principle of unity, — a principle of unity which is renewed in every conscious self, — underlying all the antagonisms of the world, even its apparent antagonism to spirit itself (H: 185).

Thus it should be clear now that there is an important affinity here with the contemporary *non-metaphysical interpretation*, or the view that "...reads Hegel's philosophy as a non-metaphysical theory of categories."[70] For the doctrine of Hegel as a "category theorist" places him as the continuator of Kant's transcendental project (but without the background realist frame of reference) and philosophy is, thereby, revealed to be nothing less than "reason's own hermeneutic." In studying the world we discover ourselves and *vice versa*. Or, as Caird himself styled it, "... the perfect revelation of what the object is, is also the return of intelligence to itself, or rather the discovery that in all its travels, it has never really gone beyond itself" (H: 187).

2.2 The logic of the British Idealists: Green on relations

" ... in the open scroll of the world, of the world,

[70] Bole, 1994, p. 103. The may be other important impulses as well. Seth's opening essay in the *Essays on Philosophical Criticism* is entitled "Philosophy as Criticism of Categories" (EPC: 8-40).

however, as written within and without by a self-conscious and self-determining spirit."[71]

No discussion of idealism in Britain could ignore T. H. Green.[72] But, as with Caird, it is also the case that Green produced no "logic," although lectures on the subject appeared in his posthumous, collected works (GCW II: 157-306). These were, in fact, notes of lectures delivered at Balliol in 1874-75 and divide into two sections: first, a consideration of a formal logician (Mansel) and, then, of "another" view (as exemplified by Mill). One might imagine that, given Green's abhorrence for empiricism, he sides squarely with the former. *But this is not the case*: in opposition to the formal logic of Hamilton and Mansel, Green stands with what he conceives to be *the majority view* of "Mill, Kuno Fischer, Sigwart, Ueberweg" that we have already encountered: "logic is the science of the method of knowledge" (GCW II: 158). On this point, all modern logicians, including Green, are united although "their views of *what* the method of knowledge is vary according to the difference in their notions of what the object of knowledge is" (Ibid.).[73] Yet, in all events, by logic we explicitly do *not* mean merely an empty account of "those 'forms of thought' in conforming to which we think *correctly*" (GCW II: 160).

Instead of attacking Mill directly, Green attempted to parry the distortions inevitably caused by Mill's epistemological foundation: accordingly, he sometimes discredited Mill only when he had fallen into certain misconceptions imposed by empiricism — primarily, *abstractionism*. As an example, in Green's discussion of the "categories" in Mill (GCW II: 207ff), Green states that the categories are, in truth, " ... the relations or formal conceptions (which comes to the same, since conception constitutes relation), without which there would be no knowledge and no objective world to be known. They are not the end but the beginning of knowledge ..." (GCW II: 207). Their very existence contrasts with the illusory impression according with "the false notion that the essential of thought is abstraction": on this account, "they are things" and "are really apart from the objects of ordinary knowledge and experience, and are known by abstraction from these" (GCW II: 207). But on the wrong-headed approach, the categories are merely a higher-level *abstraction*: the "summa genera' of things" (GCW II: 207).

But, unlike Caird, rather than emphasize categories (or even concepts), Green's preferred term of art is 'relation.'[74] Accordingly, for Green, the standard proposition is generally describable as one that "expresses ... 'the thought of an object under relations'" (GCW II: 216). His argument *in nuce* is that it is impossible to

[71]GCW, III: 119.

[72]Despite this fact, Green is surely among the most maligned of the idealist thinkers: it is often said that his arguments are faulty or fail to establish what he wants them to establish. C. D. Broad famously labeled him "thoroughly second rate."

[73]The adoption of Mill *on logic* by the British Idealists is also made evident in some opening remarks of Bosanquet: "the reform of Logic *in this country* dates from the work of Stuart Mill, whose genius placed him, in spite of all philosophical short-comings, on the right side as against the degenerate representatives of Aristotle" (LMK, I, p. vii).

[74]Most likely to reinforce its active (and verbal) connection to Kant's synthetic unity of apperception: see PE: 35 and also [Hylton, 1990, p. 32].

conceive of the object without multitudinous relations and just as impossible to think of reality as a system of relations without real objects as the given *relata*: "We cannot reduce the world of experience to a web of relations in which nothing is related" (PE: 45). For, ultimately, "It will be through it [the understanding or our capacity for relating things] that there is *for us* an objective world" (PE: 17).[75]

Furthermore, the terminology of 'relations' names not merely perceived differences in sensation but rather always involve a conscious awareness of the self and of the various objects of that self's thought. A consciousness devoid of thought and composed instead of 'mere feeling' would not "present its feelings to itself as permanent felt objects, — does not retain its feelings as objects still there for thought when they have ceased to be felt, and for the same reason is not conscious of a relation of unlikeness as a *relation*" (GCW II: 217n). Mere differences in sensation are not sufficient for the apprehension of relations or of the objects of those relations: these are products of thought that presuppose some permanence and coherence in the stream of our perceptions such that they might be considered experience, in the Kantian sense.

Green initiates his most detailed discussion of relations with the claim that if we have objective knowledge of even the simplest natural phenomena, then we must admit that our cognitions of these simple, material objects "consist in, or are determined by, relations between the objects of that connected consciousness which we call experience" (PE: 13):

> If we take any definition of matter, any account of its 'necessary qualities,' and abstract from it all that consists in a statement of relations between facts in the way of feeling, or between objects that we present to ourselves as sources of feeling, we shall find that there is nothing left. Motion, in like manner, has no meaning except such as is derived from a synthesis of the different positions successively held by one and the same body; and we shall try in vain to render an account to ourselves of position or succession, of a body or its identity, except as expressing relations of what is contained in experience, through which alone that content possesses a definite character and becomes a connected whole (PE: 13-14).

To Green, concepts such as the 'motion' of a material object would be meaningless without multiple, successive positions or places occupied by said object: thus relations can always be said to subsist between the object and itself, between the object and other objects, and between the object and the comprehending consciousness. Call this feature of reality the *ubiquity of relations*.

Given the *ubiquity of relations*, the next question for Green becomes: "What then is the source of these relations, as relations ... of that which exists for consciousness?" (PE: 14). This question may in turn, he insists, only be answered

[75]The failures of Kantian dualism, as Green sees it, follows, in large measure, the discussion in the previous section by Caird. See also [Hylton, 1990, pp. 36ff].

"through analysis of the conditions which render this [very] experience possible" (PE: 18). This does not, however, pose a psychological, a genetic or a historical question (PE: 20ff), nor does it admit either of any obvious naturalistic explanation:

> It seems necessary, then, to admit that experience, in the sense of a consciousness of events as a related series — and in no other sense can it help to account for the knowledge of an order of nature — cannot be explained by any natural history, properly so called. It is not a product of a series of events. It is not developed by a natural process out of other forms of natural existence ... Nature, with all that belongs to it, is a process of change: change on a uniform method, no doubt, but change still. All the relations under which we know it are relations in the way of change or by which change is determined (PE: 22).

This very *irreducibility of relations* entails "that a form of consciousness, which we cannot explain as of natural origin, is necessary to our conceiving an order of nature, an objective world of fact from which illusion may be distinguished" (Ibid.). For while on a broadly construed "Kantian" view, reality is "already determined by thought, and existing only in relation to thought" (PE: 38), without further elaboration, such a view faces with a number of philosophical challenges. "But," Green contends, "the idealism which interprets facts as relations, and can only understand relations as constituted by a single spiritual principle, is charged with no such outrage on common-sense. On the contrary, its very basis is the consciousness of objectivity" (PE: 39). All this is, of course, leading to Green's ultimate conclusion that "... nature in its reality ... implies a principle that is not natural" (PE: 56).

Now, to some extent, such arguments presuppose an earlier *reductio*, from the critical introduction to the edition of Hume (cf. GCW I, pp. 89ff). As with Descartes, we start, in our thought experiment, from the basic case of our unimpeded perception of simple material bodies and their phenomenal properties; but, Green insists, the primary qualities, such as 'place' and 'position', reveal too that the "body is thus a complex of relations" although for the empiricist "all [are], according to Locke's doctrine of relation, inventions of the mind" (GCW I, p. 90).[76] But thereby empiricism is rendered swiftly self-refuting, as the requirements of empirical knowledge are either unreal or cannot be accounted for on empiricist principles.[77] But Green is insistent upon this point: "a consistent sensationalism must be speechless" (GCW I: 36). An empiricist might object here, complaining that "mere individuality, exclusive of all relation" is readily apparent in "[t]he simple 'this' and 'that'" (GCW I: 35), querying

> ... surely it is possible to pick out things as real without bringing in their relational context? At the very least can we not just point

[76]Hume's more logical account only ramifies the difficulties for empiricism (GCW I, pp. 189ff).
[77]See [Brink, 2003; Hylton, 1990], for different reconstructions.

> them out using simple demonstratives, 'this,' 'that,' 'here,' and 'now'? Green thinks not. Even here, he argues, our attempt to locate a given thing is implicitly relational; "if we say that it is the mere 'this' or 'that' — the simple 'here' and 'now' — the very 'this' in being mentioned or judged of, becomes related to other things which we have called 'this,' and the 'now' to other 'nows.'[78]

Or, more simply, even a 'this' must contrast with a 'that' and a 'now' with a 'then.' Hence, although relations are, as Locke feared, the work of the mind, "the work of the mind is real" (PE: 27). The alternative is a form of empiricistic skepticism: if a body is a complex of relations and relations are a work of the mind, then a material body is only the work of the mind.

In arguing that experience itself depends upon relations, Green far outstrips Kant's more moderate idealism. We cannot inquire into the nature of the real (cf. PE: 27-29). Instead, we may only acknowledge "that there is an unalterable order of relations ... is the presupposition of all our enquiry into the real nature of appearances; and such unalterableness implies their inclusion in one system" (PE: 29-30). Hence, on Green's construal, "[t]he real thing, then, is individual because universal: i.e. its individuality lies in its relation to all other things, which is a one in all, the common element in all, a *universal*; it lies in this relation, this mere difference from all other things, as *particularised*" (GCW II: 189): what is really real — what lays claim to the greatest degree of reality — is hence neither an 'abstract universal' (GCW II: 188) nor a mere individual:

> I have the conception of a flower, and upon the occurrence of a sensation, which I interpret by means of this conception, I judge 'there is a real flower'; but the flower is *really* much more than the relations which I had previously conceived *plus* the present relation to sense. But this 'more' still lies in relations which can only exist in a conceiving mind, and which my mind is in the process of appropriating. The great mistake lies in regarding a conception as a fixed quantity, a 'bundle of attributes.' In truth a conception, as the thought of an object *under relations*, is from its very nature in constant expansion (GCW II: 190).

To this Green adds that "... [t]he ordinary definition of an object is available only for rhetorical purposes" (GCW II: 190), which is to imply that it is only available as a mere act of arbitrary stipulation.

Such remarks must not be considered dispensable: for as the object and its properties are in constant flux, so to is the mind in the modes of apprehension. What remains secure through all these acts of perception are relations: call this the *mystery of relations*.[79] And acknowledging this necessity, the *mystery of relations*, will assist us in moving from our ordinary but false views (GCW III: 56-7) to the

[78] [Mander, 2001, p. 57]. The quotation is from Green (GCW I: 36) and the missing single quotes have been restored.

[79] Mander introduces this manner of speaking [Mander, 2001, p. 59].

true, namely that the very individual itself is "not *merely* individual" (GCW III: 70):

> The individual has thus transformed itself into the universal in virtue of its particularity or definite relations ... But it is not *merely* individual ... *As known, it is in implicit relation to all things* ... It is an individual universalised through its particular relations or qualities ... It [the universal] must be that which is the negation of all particular relations so as to be determined by the sum of them. In virtue of this negative relation, as identical with itself in exclusion of all things, it is individual. It is a universal individualised through its particularity (GCW III: 70, my emphases).

There are a number of connected epistemological doctrines that characterize this Hegelian view, that also connect it with a number of neo-Kantian (and hence non-Kantian) positions. First and foremost, the denial of the *noumenon* or thing-in-itself. Second, the lack of any real distinction between intuition and thought: "the opposition between intuition and thought, as between presentative and representative, is fallacious" (GCW II: 192). But this discussion is designed to swiftly bring the main enemy into range: "the false doctrine of abstraction" (GCW II: 192).

Understood correctly, then, thought is (really) "a process from the more abstract to the more concrete" (GCW II: 193). The false perspective is, by contrast, rooted in the enlargement of a true distinction between mere sensation and thought proper into an erroneous divergence "between the sensible thing (feeling as determined by its conditions) and the work of thought" (GCW II: 192): "All thought must be conscious" and in human knowers "the preliminary or 'unreflective' stage of knowledge is indefinitely abridged by language" (GCW II: 194). There can be no such thing as an abstract universal — but only "a universal individualised through its particularity" (GCW III: 70) — because, as we have already seen, each individual may be universalised and each universal may be individualized.

Green's Hegelian perspective is only reinforced in the acknowledgement that "What Hegel had to teach was, not that thought is the *prius* of things, but that thought *is* things and things *are* thought" (GCW III: 144-45). On this basis, we may embrace the conclusion that the real world is essentially a spiritual world, which forms one inter-related whole because related throughout to a single subject. Rather than rendering the world irreal, Greenian idealism emphasizes that

> ... *the world, which alone we know or can know, consists in relations to consciousness and in relations of those relations.* Space, time, matter, motion, force, are not indeed modes of consciousness, but apart from consciousness they would not be. We use words without meaning when we talk of a time when as yet consciousness was not, of an endless space without a mind, for time and space alike are abstractions from relations between phenomena. They are creatures of reflection upon related presentations to consciousness which can be related to each

> other only in virtue of their equal relation to a single subject of the presentation. Neither the relations of succession and externality, nor the empty forms which we construct by abstraction and substantiation of them, are possible except as resulting from the unity of a thinking consciousness (GCW III: 228-29, my emphases).[80]

This last quotation reveals a number of things, not least of all just how far Green's account has moved from anything like orthodox Kantianism. But then what is the source of this new emphasis on relations, such that the world itself may be said to consist in them? Kant is not known for the doctrine that *to be consists in relations.* Neither indeed is Hegel although the opening sections of the System of Logic contains much relation talk and despite the fact that the *Zusatz* to section 135 of the Encyclopedia Logic does state that:

> Essential correlation is the specific and completely universal phase in which things appear. Everything that exists stands in correlation, and this correlation is the veritable nature of every existence [*Das wesentliche Verhältnis ist die bestimmte, ganz allgemeine Weise des Erscheinens. Alles, was existiert, steht im Verhältnis, und dies Verhältnis ist das Wahrhafte jeder Existenz*] (HEL: 191)

This seems to say only that relations merely name the mode of appearance for beings. But Green's claim is seemingly stronger and removed from any pretence of phenomenalism: to exist is to stand in relations. And this means that the world of objects *consists* in relations.

The source of the new emphasis on relations may very well be found elsewhere, namely in the thinker who most developed this notion in the nineteenth century, Lotze. Lotze famously attacks Kantian-Herbartian thesis that "being" is equivalent to "position without relation" and asserts instead that being is relation: "Real Being ... can never be arrived at by this bare act of *Setzung*, but only by the addition in thought of those relations, to be placed in which forms just the prerogative which reality has over cognitability" (MP: 44). Lotze is indeed the only nineteenth-century thinker who placed great (and novel) importance upon relations and Lotze consistently espoused an idealism — not a realism — about relations. This emphasis on relations is caught up in his polemic against Herbart's conception of the world as composed on self-subsistent elements called "Reals": the concept of a thing utterly *outside* of relations, subsisting on its own, is, Lotze believes, philosophical nonsense — a bit of nonsense that is clearly not evidenced by our ordinary experience. Hence, no terms without relations; but these relations are, nevertheless, thought-things and not realities — for if they were real, with things dependent upon them, then they, in turn, would depend, for their reality, on some *third* thing, thus producing a vicious infinite regress (MC, II: 620).

Some interpreters, following earlier readers, decline to attribute any such connection, e.g.: "My own — tentative — view is essentially that of J. T. Merz.

[80]Although this unified, thinking consciousness is ultimately revealed, on Green's account, to be that of the absolute or God.

Green owed no deep, formative influence to Lotze."[81] Merz himself hypothesizes that Green's thinking emerged in parallel but otherwise divergent tracks with Lotze's. But he offers no evidence for this view and, furthermore, it flies in the face of Green's keen interest in Lotze. As we know that Green himself had provided the translation of "[t]he whole of Book I (Ontology) and the chapter 'Of Time' (Book II, chapter iii)" (Bosanquet's introduction to the *Metaphysics*, MP: v), Green must have had an intimate knowledge of precisely these portions. Indeed, Green told Bosanquet directly: "The time that one spent on such a book as that (the 'Metaphysic') would not be wasted as regards one's own work" (Bosanquet's introduction to the *Logic*, L: v). However, while it may remain a disputed question — about the exact nature and extent of Lotze's influence upon Green — the possible connections are nevertheless unavoidable and striking.

Lotze was well-known for espousing, among other things, a strong idealism about relations, including, most particularly, spatial relations: space was to be identified with spatial relations. Yet in his version of spatial idealism, space is objective but nonetheless utterly irreal. Two separate quotations from the *Metaphysics* make this clear. First, Lotze insists we refrain from ascribing to space any substantial existence separate from the objective nature of the spatial relations in which things appear to us (MP: 246-47, § 108).[82] Yet, despite this fact, Lotze 'objectivity' is not equivalent to 'reality' (or, better, 'actuality'): hence space can be objective without being materially actual (although perceptually effective) and is not, consequently, a mere 'nothing' (MP: 258, § 113).[83] With regard to space, Lotze suggests that any attempt to buttress its undeniable centrality for our cognition, by lending to it a special metaphysical character, is attempt that is simply doomed to failure. But nothing in this discussion can undermine the reality of relations.

Nevertheless, although objective, relations are creations of mind; the relevant statement from the *Metaphysics* reads:

> ... concerning the nature of all 'relations' [*die Natur aller Beziehungen*] ... they only exist either as ideas in a consciousness that imposes them [*als Vortellungen in einem beziehenden Bewusstsein*], or as inner states, within the real elements of existence [*als innere Zustände in*

[81] [Thomas, 1987, p. 62].

[82] "... we are inclined to supply to space, which at first we took for a mere tissue of relations [*nur ein Gewebe von Relationen schein*], some substratum of properties, indefinable of course, but still such as to serve for a substantive support to these relations. We gain nothing by doing so ... Therefore we must abide by this; there is simply nothing behind that tissue of relations [*es ist gar Nichts hinter jenem Gewebe von Relationen*] which at starting we represented to ourselves as space ...."

[83] "Men will go on repeating the retort; that it is impossible to doubt the reality of space [*die Wirklichkeit des Raumes*] ... But are we denying this reality? ... space would lose nothing of its convincing reality [*Wirklichkeit*] for our perception if we admitted that it possesses it only in our perception. We long ago rejected the careless exaggeration which attaches to this idea; space is not a mere semblance [*blosse Erscheinung*] in us, to which nothing in the real world [*im Reelen*] corresponds ...."

> *der realen Elementen*], which according to our ordinary phrase stand in the relations [*in ihnen stehen*] (MP: 247, § 109).

Indeed, one of the later chapters in the *Metaphysics* is entitled "On the Mental Act of 'Relation'," which is the Englished version of "*Von dem beziehenden Vorstellen*" or "the relating capacity/activity of ideation." Of course, there must be something in the real world to which these relations correspond: but whatever these things are, they are not relations!

Green himself encapsulates the dilemma in his observation that "That 'all reality lies in relations' will more readily be admitted than that 'only for a thinking consciousness do relations exist'" (GCW, II, p. 179). That reality consists in relations is to assert the reality of relations, while to claim that relations exist only for a thinking consciousness is to assert the ideality of relations. Here Green stands with Lotze. But in his further insistence upon the irreducibility, ubiquity and mystery of relations, Green creates an epistemological and logical framework upon which to root his seemingly outlandish metaphysical pronouncements. And this impulse far outstrips the more modest program of Lotzean metaphysics that, by mediating between Hegel and Herbart, seeks to discuss metaphysical problems individually in order to remove the semblance of contradiction apparent in those discussions, traditionally conceived. But metaphysics has no unique method by whose application these apparent tensions can be swiftly dissolved. Rather a patient and sober consideration of both sides is required in any attempt to reach resolution. And, to the degree that it is applicable, this perspective is much more descriptive of Bradley than of Green.

2.3 The logic of the British Idealists: Bosanquet on universals

> "... it is the strict fundamental truth
> that love is the mainspring of logic."[84]

Bosanquet — as in the case of Bradley — overlaps both in chronological years and philosophical emphases with Frege. Often Bosanquet is portrayed as a mere camp follower of Bradley. But the intellectual dependence of Bosanquet upon Bradley has been overworked. After all, it is important to recall that Bosanquet began his own publishing career with an attack on Bradley's reliance upon the, as he deemed it, "reactionary logic" of Sigwart and Lotze (KR: vii); in this fashion, Bosanquet hoped to bring Bradley back to the Hegelian party proper. In reality, Bosanquet stood in a much closer relation to Green than Bradley (it was, after all, Bosanquet who completed Green's project of translating Lotze). And, most importantly, for our present purposes, Bosanquet — unlike either Caird or Green — has the acknowledged status of a writer who returned to the question of the nature and status of logic on multiple occasions, in differing venues. He thus claimed, in sympathy with Greenian idealism, that by the very term 'logic' "we

[84] PIV: 341.

understand with Plato and Hegel ... the impulse towards unity and coherence ... by which every fragment yearns toward the whole to which it belongs" (PIV: 340).

Yet before publishing any volume devoted specifically to logic, Bosanquet first addressed the topic in his contribution to the Green *Festschrift*: "Logic as the Science of Knowledge" (EPC, pp. 67 — 101). The title and the very first sentence are most telling: "The Science of Knowledge' is a title which everyone concedes to logic" (p. 67). And he reminds us, in a footnote, that Mill is to be numbered among *the everyone* who actually matter: for although Mill's philosophical positions are "fundamentally untenable," "his services to logic" are commendable as their effects have produced "a more healthy influence than any other logician since Hegel" (EPC, p. 67n)! (Although, included, as well, in this company are Lotze, Sigwart and Jevons.) For Bosanquet, the status of logic is clear, as the science of those thoughts that are knowledge (= *Erkenntnistheorie*). And this orientation carries over to what we may call the 'greater' (1888) and the 'lesser' (1895) logics.

Passmore provides the following introduction to Bosanquet's larger logic (which is, on Passmore's account, much more Hegelian and much less Lotzean than Bradley's):

> His *Logic* (1888) bears the subtitle *The Morphology of Knowledge*. That summarizes its contents. The Logic is an attempt, in the manner of Hegel and of Lotze — even though in opposition to Lotze's metaphysics — to depict the stages through which thought passes from the simplest form of judgment ('this is red') to that complex disjunctive in which is exhibited the concrete universal, the universal which is a systematic interrelation of its constituent parts.[85]

Now by 'morphology' it seems that Bosanquet hoped to establish a connection between Hegel and Darwin, both of whom provide epigraphs to the volume. But, more to the point, Passmore's synopsis forces us to encounter a concept, already implicitly discussed by both Caird and Green although not explicitly developed by them: that curious hybrid, the concrete universal. To explore this concept more fully requires a very short return to the history of Hegel-reception in Britain.

It is often suggested that Hegel was, after all, introduced to Britain via Stirling's aforementioned volume, *The Secret of Hegel*. Since many wags and wits had observed, slyly, that "the secret has been well kept,"[86] by the time of the second edition, Stirling felt compelled to state things just a bit more succinctly:

> The secret of Hegel may be indicated at shortest thus: As Aristotle — with considerable assistance from Plato — made *explicit* the *abstract* Universal that was *implicit* in Socrates, so Hegel — with less considerable assistance from Fichte and Schelling — made *explicit* the *concrete* Universal that was implicit *in* Kant (SH2: xxii).

[85][Passmore, 1966, p. 167].
[86]See [Passmore, 1966, p. 49].

So, the secret of Hegel was nothing but the very uncovering of the concrete universal implicit in the Kantian synthetic *a priori.* There is little evidence, as yet however, that this verbiage names a truly technical term. This might lead one to assume that the "concrete universal" was a kind of *portmanteau* concept, so to speak, denoting only a possible sublation of well-known dualistic oppositions: abstract vs. concrete, unity vs. plurality, identity vs. difference, thought vs. intuition, universal vs. particular, thought vs. reality/being, part vs. whole, and one vs. many, for instance.

Unfortunately, Bosanquet never gives anything like a sufficient definition of the term although he does approach it, on several occasions, by varying methods of approximation; the beginning of the second volume of the greater *Logic*, he cryptically divulges that

> Logic is little more than an account of the forms and modes in which a universal does or does not affect the differences through which it persists. I can only point out that all turns on the distinction between the abstract or powerless and the concrete or dominant universal. To interpret the latter by the former, *to reduce all universals to marks . . . is a fatal tendency of popular logic.* A very elementary example of a relatively concrete universal may be found in the nature of a geometrical figure, say of a circle or a triangle. The given arc is not simply repeated, it is continued according to a universal nature which controls its parts, and with a result which though involved in the given arc is yet outwardly and as an actual content distinct from it. This is clearer if instead of a circle we take an ellipse, in which the given fragment of the curve cannot in any sense be said to be simply repeated without change in constructing the remainder. There is something in the curve as given which is capable of dictating a continuation and completion of its outline distinguishable from the given arc or fragment itself. Just so with a triangle — given two sides and an angle, we can find the third side and remaining two angles . . . [and, as such] is then *the basis of* mediate judgment or *inference* (LMK, II: 2-3, my emphases).

Accordingly, the concrete universal can be neither a universal composed of discrete marks, garnered by abstraction, nor a merely abstract definition or criterion. It is, however, accurately characterized as, first, *an identity pervading manifold differences* and, second, as *the basis for further inference.*

The example employed by Bosanquet seems to have been chosen in order to connect the concrete universal more with mathematical construction in pure intuition than with "bloodless concepts" and also illustrative of the fact that there can be many, manifestly different kinds of, say, triangles that, nevertheless, have something in common. Ultimately, however, the idea rests upon the proper construal of the role of the characteristic marks in composing (the definition of) the concept. A "*mere* mark," Bosanquet complains, "conveys nothing" (LMK, II: 3). What we require instead is a mechanism by which "we could infer all sort of . . .

consequences ... and these consequences would not be the same for all the objects, but would be modified by their nature." Only then, Bosanquet exclaims, would "the universal ... be an identity pervading different manifestations" (LMK, II: 3).

The concrete universal must be a unity or an identity made possible only by means of difference and, as such, comprehend both the individual in-itself and the particular reflective identifications of that individual (made possible by going out-of-itself); and it must do so concretely and not via any mechanism describable as abstraction. Furthermore, the point may also be to suggest that a "mere mark" may name an accidental or inessential property that in no way determines the nature or essence of those things, whereas "being an arc of the circle" both names an abstract property that without which a thing would not be what it is and instantiates that very same property. Therefore, a concrete universal, such as "humanity," is and includes these things as fully actualized in a unity made possible only by difference.[87]

However, Bosanquet then proposes that proper names (which are "designative and not definatory") — or rather their referents — seem to afford *the simplest example* of the concrete universal. He then observes that

> ... the reference of a proper name is a good example of what we called a universal or an identity. That which is referred to by such a name is a person or thing whose existence is extended in time and its parts bound together by some continuous quality — an individual person or thing and the whole of this individuality is referred to in whatever is affirmed about it. Thus the reference of such a name is universal, not as including more than one individual, but as including in the identity of the individual numberless differences — the acts, events, and relations that make up its history and situation (BEL: 65).

So, if a coherent referent is implicated in our use of a proper name, then it is possible to see, Bosanquet believes, that and how "individuals may be universal in a sense which does not depend upon abstraction" (LMK, I: 68).

To further illustrate this point, consider the singular sentence employed by both Bosanquet and Lotze (and Hegel, HEL: 233, § 167), "Caesar crossed the Rubicon":

> If I say 'Caesar crossed the Rubicon,' I start with an individual Caesar, whose continued identity extended through a certain space of time and revealed itself in a variety of acts, and I exhibit his identity in one of the acts and moments — its differences — through which it persisted. What I mean by the affirmation is that he, *the* Caesar who had before conquered Gaul, and who was afterwards murdered on the Ides of March, displayed his character and enacted part of his history

[87] Mure complains "We cannot join the empiricists in endeavouring so completely to absorb and exhaust the universal in its instances [such] that only singular individuals emerge as real. Our universal takes its nature from the universe" [Mure, 1978, p. 71].

> by crossing the Rubicon. This is a clear case of exhibiting an identity in difference (LMK, I: 99, note to Lotze, *Logik*, § 58).

Although the man who crossed the Rubicon was not yet the man murdered on the Ides of March, in utilizing the proper name 'Caesar', I implicitly include all of these events and relations in making reference to (the historical) Caesar: *the* Caesar (the actual referent of the term 'Caesar') is an identity encompassing differences. (This sharply contrasts with Lotze's view. In the note to the *Logik*, Lotze asserted *the exact opposite*, namely that the predicate "crossing the Rubicon" characterises only that-Caesar-who-is-crossing-the-Rubicon and "not the Caesar who lay in the cradle, or was asleep, or was undecided what to do." Consequently, "in no subsequent moment of his life either can he be the subject intended by this judgment" (L: 85, § 57).) Surely then, in this peculiar construal of self-identity, as Mander observes, "the strangeness (we might even say, the perversity) of the doctrine stands out sharply."[88]

It is in the context of a discussion of traditional doctrines of intension and extension in Book I (LMK, I: 46-71) that the Hegelian notions of individual, particular and universal are brought into play. After recapitulating the Lotzean account concerning naming and objectification (LMK, I: 8-46),[89] Bosanquet begins an extended attack on the traditional doctrine of the "inverse ratio" of intension and extension, claiming that his prior examples of proper and collective names (LMK, I: 46-58) "are enough to show that not every variation of intension involves a corresponding variation of extension, or *vice versa*" (LMK, I: 58). Furthermore, he insists that

> If we look at a real individual into which other individuals enter as constituent parts, are we prepared to say that the containing whole (e.g., the state as compared with the citizen) has the less meaning or intension of the two? The old logic would retort here that the extension of 'state' is made up of particular 'states,' not persons, or that that of 'nation' or 'army,' consists in the several nations and armies, not in individual men and soldiers ... But this would only meet the objection at the cost of narrowing the idea of universality to that of mere abstraction, in contrast with the sense — synthesis of differences — in which we have taken it throughout. Moreover, *even the aggregate of men, nations, or animals which is indicated by the abstract universal name has in virtue of that universal a common nature which is a germ of concreteness.* A crowd is not an army, but it has in it always the elements of a mob. As we saw above, collective names mark a mere half-way house from aggregation to individuality and it is a purely arbitrary procedure when examining the nature of universals, to restrict our notice to such as have attained to no higher embodiment than as an aggregate of particulars. But in fact our prejudices would cause us to

[88] [Mander, 2000, p. 295].

[89] For a fuller discussion of Lotze's approach to naming and objectification see [Sullivan, 1991].

> neglect a concrete nature if any such were apparent in the aggregate (LMK, I: 65, my emphases).

Here he first emphasizes that, under the traditional doctrine ("the old logic"), extensions are, after all, not composed of the individuals falling under those attributes but, rather, are made up of further, narrower extensions: e.g., "armies" is made up of "the French army" and "the German army" and not individual soldiers.

But, further, "collective names" (as opposed to "proper names") — or "[a] name or idea which, while involving a number of identical parts, is not truly predicated of each part singly" (LMK, I: 57) — are typically excluded from consideration as merely compromise formations, locatable somewhere between individuals and mere aggregations; e.g., as "regiment" is certainly made up of soldiers, but the term cannot be predicated of (applied to) any individual soldier, then varying the extension should have no effect on the intension. To believe otherwise stems from an acceptance of "the Aristotelian definition by genus and species, ... [where] the generic attributes were contained together with others in the definition of the species" (LMK, I: 58).)

The only way out of this impasse Bosanquet can envision is to abandon the traditional instinct "to conceive of all universality as arising by way of abstraction" (LMK, I: 66) — where abstraction means "mere neglect or omission of attributes" (LMK, I: 65). Bosanquet then complains that:

> It is easy to say that animality is common to men and beasts, while rationality belongs to men only, and in place of it animals have either instinct or nothing, and that animality is the intension of the class that includes beasts and men, which each of these subclasses has a separate and additional intension. *But in fact the animality of men is quite different from the animality of beasts, and is not an attribute common to both in the sense in which a tree-trunk is the common support of two of its branches.* While on the other hand the thorough modifications which distinguish the intelligence of man from that of animals do not suffice to dissociate them beyond identification; and the class-conception which simply omits all reference to intelligence is an inadequate class-conception for men and animals. *Therefore the notion or abstraction which is to include both men and animals must on the one hand provide for a variable animality; must be considered, that is, not in light of a fixed mark but as a scheme of modifiable relations; and must, on the other hand, find room for some reference to intelligence, and not simply strike it out as a mark in which the kinds to be classified are not the same. Prima facie* then the content of the superior class-conception is made up of the very same elements as those of the conception nearer to individual reality, only that it must represent each attribute schematically, by limits of variation, instead of embodying a fixed system of amounts or values (LMK, I: 66-67, my

emphases).

To unpack this last, lengthy excerpt is now the task at hand.

First, as Mander has already identified,[90] a gloss of Bosanquet's remark that "the work of abstraction should be represented *not* as selective omission *but* as constructive analysis" (p. 69), may be found already in Lotze's *Logik*:

> ... we do not get the universal image of animal by comparison, if we leave out of our minds entirely the facts of reproduction, self-movement, and respiration, on the ground that some animals produce their young alive, others lay eggs, others multiply by division, that some again breathe through lungs, others through gills, others through the skin, and that lastly many move on legs, others fly, while some are incapable of any locomotion. On the contrary, the most essential thing of all, that which makes every animal an animal, is that it has some mode or other of reproduction, of motion, and of respiration. In all these cases, then, the universal [*das Allgemeine*] is produced not by simply leaving out the different marks [*Merkmale*] $p^1$and p^2, q^1 and q^2, which occur in the individuals [*Einzelfälllen*] compared, but by substituting for those left out the universal marks P and Q, of which p^1p^2 and q^1q^2 are particular kinds (L: 41-42, § 23).

The Lotzean recipe for the formation of universals — "that of determining the element which maintains itself in the same instance under changed conditions" — contrasts with the typical logical approach, "that of bringing out the common element in different instances when at rest" (L: 40-41, § 22):

> Of the true universal ... which contains the rules for the entire formation of its species, it may rather be said that its content is always precisely as rich, the sum of its marks precisely as great, as that of the species themselves; only that the universal concept, the genus, contains a number of marks in a merely indefinite and even universal form; these are represented in the species by definite values or particular characterisations, and finally in the singular concept all indefiniteness vanishes, and each universal mark of the genus is replaced by one fully determined in quantity, individuality and relation to others" (L: 52, § 31).

Lotze introduces this discussion on the heels of his distinction between, not the abstract and the concrete universal but, what he calls the 'first' and the 'second' universal. The 'first universal' arises "in the comparison of simple ideas" (L: 30, § 14): "This first universal, therefore, is no product of thought, but something which thought finds already in existence" (L: 31, § 14). The various instances of the first universal "is a more or a less of a common sensible element [*eines fühlbaren Gemeinsamen*], which in itself, undetermined by any degree, is no object of perception [*nicht anschaubar ist*]" (L: 32, § 16). When our consciousness presents "to

[90] [Mander, 2000, p. 300].

itself and compare[s] the ideas [*vorstellbaren ... vorzustellen*]" this is only "an act of so comparing them to grasp the common element which our sensation testifies them to contain" and not something detachable, to be made "the material of a new and equally perceptible idea [*dem Inhalt einer gleich anschaulichen neuen Vorstellung*]" (L: 32, § 15).

The second or true universal, by contrast, must be a product of thought "that is giving affirmative position to the object-matter, that of distinguishing it negatively from all others, and that of estimating by quantitative comparison its differences and resemblances" is itself "a new logical operation" (L: 35, § 19). By contrast, "[t]he first universal, as we saw, can only be experienced in immediate sensation [*unmittelbarer Empfindung*]" (L: 36, § 19). But the second operation of thought proper "separate[s] the merely coincident amongst the various ideas which are given to us, and ... combine[s] the coherent afresh by the accessory notion of a ground for their coherence [*durch den Nebengedanken des Rechtsgrundes seiner Zussamengehörigkeit*]" (L: 37, § 20). The necessity of this added thought, in Lotze, seems to speak to the requirements laid down by Bosanquet for the universal: it must possess explanatory force and provide the basis for further, valid inference.

Without, as in the case of Green, asserting a direct borrowing, we should note that the idea of such an intellectual connection was already suggested by J. S. Mackenzie, who claims that the "technicalities" of Hegel's presentation are improved in Lotze's 'simpler language."[91] Despite this difference, "the Hegelian doctrine of the concreteness of the true universal" receives "a definite account" in Lotze's work.[92] If correct, we now know why the Hegelian 'concrete universal' shares an important connection the Lotzean 'second universal.' As Mander has expressed it: "Mere differences, mere conglomerations of distinct and unconnected elements, do not then correspond to genuine universals ... To qualify as a universal, a group must possesses some unifying explanation, some account of why they all belong together."[93] Accordingly, the Lotzean warning is validated: "We may class cherries and flesh under the group *ikl* of red, juicy, edible bodies, but we shall not suppose ourselves thereby to have arrived at a generic concept" (L: 52, § 31). As the proper concept of the universal demands something more, the Lotzean 'second universal' and the idealist 'concrete universal' is intended to addess these deficiencies.

3 CODA

3.1 Vagaries of the post-Enlightenment

> "Just point in these days to the picture of some huge baboon, and — suddenly — before such enlightenment — superstition is disarmed, priests confess their imposture, and the Church

[91]UO: 189.
[92]UO: 189, 190.
[93][Mander, 2000, p. 305].

sinks — beneath the Hippocampus of a Gorilla!"[94]

Whatever the ultimate provenance of the concept of a 'concrete universal', one can see that it is another attempt, somewhat analogous to 'relations' for Green, to think through certain logico-metaphysical difficulties. Universals, relations and categories are *logical* entities, characteristic of mind or reason. Furthermore, each of these concepts — the "higher categories," the "relational whole," and the "concrete universal" — possesses one importantly shared sense: they are all (or they directly imply) "unions of *differences*."[95] As Mander elaborates it, in the case of the concrete universal:

> Or rather we might say that they are unions of qualitative diversity as well as quantitative diversity. This ambiguity explains Bosanquet's rather ambivalent attitude to abstract universals, as varyingly either inferior or impossible. For he thinks that *true universals need to bring together diversity*. If you count numerical diversity as a real, if lesser, sort of diversity, then abstract universals are inferior universals. If on the other hand you count it no diversity at all, then unions of abstract sameness do not even deserve the title of universals.[96]

Because true universality in this sense (the encompassing of diversity) outstrips mere generality, we have implicitly returned to the other shared motif: a consistent attack on any construal of thought (or its operations) as abstract.[97] It is no surprise then that the idealist perspective marks the death knell of numerous traditional logical doctrines, for example: "it is clear that immense deduction must be made from the traditional doctrine that Intension and Extension vary inversely" (LMK, I: 68). This rejection follows because "individuals may be universal in a sense which does not depend upon abstraction" (LMK, I: 68) and, furthermore, "the work of abstraction should be represented not as selective omission but as constructive analysis" (LMK, I: 69). In this way, Bosanquet envisions, "the advance from abstractness to concrete individuality would have grounds in historical fact" (LMK, I: 71).

This logical doctrine of the British Idealists thus appears as a somewhat heady *mélange* of Hegelian and Lotzean beliefs, as further developed out of the wider context of the reform of logic debate. First and foremost, as we have seen, a *leitmotiv* of the British Idealists is that the universal or conceptual is not merely empty and abstract but always meaningful and composed of cognitive content. This ramifies their rejection of traditional "formal" logic although they use slightly different terminology and emphasize slightly different aspects. They focus on correlative notions of systematicity or holism, while at the same time exposing the inadequacies of the traditional logic. This, alongside the distinction between the

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[94] [Sterling, SH2: xxxv].
[95] [Mander, 2000, p. 302].
[96] [Mander, 2002, p. 302n], (my emphases).
[97] Bosanquet later writes: "The true office of thought ... is to build up, to inspire with meaning, to intensify, to 'vivify'" (PIV: 58).

first and the second universal, is Lotzean. But it is retranslated into the language of Hegel, and the terminology of the individual, particular, and universal, on the one hand, and the abstract and the concrete, on the other.[98]

A further detail that might merit discussion here returns us to the affinities listed earlier between Frege and Bradley. (As noted above, no explanation for these correspondences was proffered by Ryle.) Strikingly, however, few have noticed the strong affinities between Frege and Bosanquet. Although their two works — *The Foundations of Arithmetic* (1884) and *Logic* (1888) — are only separated by a few years, cross-pollination is unlikely. But the perceivable parallelisms are comprehensible against the shared backdrop and influence of such texts as Lotze's *Logic* in the context of the wider reform of logic discussion. While it is neither necessary nor practicable to give a complete catalogue, within their proto-semantical reflections, one might safely emphasize a firm distinction between logical and grammatical form and an insistence upon the priority of judgement over concept that issues in something like a "context principle." Bosanquet's commitment to holist doctrines is readily apparent in his theory of judgement: for him, the essence of

[98]It may also be that other Hegelian reflections were influential here. For Hegel, 'abstract' and 'concrete' themselves can be seen as quasi-technical terms, ones that are somewhat disconnected from their common-sense application in which they might stand in for, respectively, 'world or reality' and 'mind or mentality': instead, their proper analogues might better be stated as 'one-sided', on the one hand, and 'multi-faceted', on the other. Wallace illustrates this point by quoting directly from Hegel's essay "*Wer denkt abstrakt*?":

> A murderer is, we may suppose, led to the scaffold. In the eyes of the multitude he is a murderer and nothing more. The ladies perhaps may make the remark that he is a strong, handsome, and interesting man. At such a remark the populace is horrified. "What! a murderer handsome? Can anybody's mind be so low as to call a murderer handsome? You must be little better yourselves." And perhaps a priest who sees into the heart, and knows the reasons of things, will point to this remark as evidence of the corruption of morals prevailing among the upper classes. A student of character, again, inquires into the antecedents [*den Gang*] of the criminal's education [*Bildung*]: (omitted in the translation: "he finds bad instruction in his history," [*findet in seiner Geschichte schlechte Erziehung*,]) he finds a wrong set of relations between father and mother; or he finds out that this man has suffered severely for some trifling offence, and that under the bitter feelings thus produced he has spurned the order of society [*die bürgerliche Ordnung*], and cannot support himself otherwise than by crime. No doubt there will be people who when they hear this explanation will say "Does this person then mean to excuse the murderer?" ... By abstract thinking, then, is meant that in the murderer we see nothing but the simple fact [*dies einfache Qualität*] that he is a murderer, and by this single quality annihilate all the human nature which is in him" (HEL: lxxix-lxxx).

Wallace glosses this, which is found in his early translation (1874) of the *Encyclopedia Logic*, as follows: "A concrete notion is a notion in its totality, looking before and after, connected indissolubly with others. An abstract notion is one withdrawn from everything that naturally goes along with it, and enters into its constitution" (HEL: lxxix). Consequently, as already noted, in the case of universals, "[t]he thesis is not really that some universals are abstract, others concrete, but rather that the idea of abstract universals belongs to an inadequate view of the world better replaced by that of concrete universals" [Sprigge, 1994, pp. 439-440]. The earlier question is thereby rendered none at all: there are not abstract universals and concrete universals; rather, there are only the falsely construed universals and the correctly attained ones.

judgement is to connect a present sensation with "a definite organised system" or "real world" (LMK, I: 77). Hence, "[t]he ultimate subject of the perceptive judgement is the real world as a whole" (LMK, I: 78). Or, as phrased more pithily: " 'Judgment' as the consciousness of a world" (BEL: 21).

Along similar lines, Bosanquet first cautions that the "formative elements of language are significant" (LMK, I: 40) yet not so *qua* logical. Bosanquet insists on the freedom of logical thought from language:

> I can see no ground for restricting the *logical* conception of language to written or spoken words. We must not argue from the possibility of educating the deaf and dumb (cp. Lotze, Logik, sect. 6) that 'the logical operation in the mind is independent of the possibility of linguistic expression.' It is unfortunate that the German "Sprache' and 'sprachlich' make this inference appear a truism (LMK, I: 16).

Nevertheless, "[a] name has meaning only in a sentence or by suggesting a sentence. The sentence is the significant unit of language" (LMK, I: 40). Bosanquet hypothesizes that "to isolate a single word from the sentence" (LMK, I: 10) or "the isolation of the significant name from its context" (LMK, I: 11) must have been unusual (if not impossible) in primitive speech. But rather than thinking of judgements as composed of concepts, "I think of the concept as existing only in the act of judgment" (LMK, I: 34). Subsequently, we can identify knowledge with judgement: "Knowledge is a judgment, an affirmation" (BEL: 23). But, more importantly, "*Knowledge* is the medium in which our world, *as an interrelated whole*, exists for us" (BEL: 22). But this is possible only because "the world of objective reference and the world of reality are the same world" (LMK, I: 5).

Subjective idealism, or solipsism, is encapsulated in the Schopenhauerian utterance: "the world is my idea." That is to say, insofar as anything can become an object of my perception, it becomes so only as an idea belonging solely to me. Objective idealism turns subjective idealism on its head. While the stage of subjective idealism may be useful in the advance of the philosophical dialectic, one must insist as well that the contrapositive is true, that: "My idea is (of) a world." Hence, my 'idea' properly understood is not the product of a random stream of presentations or a haphazard chaos; it is, rather, of an organized, regular whole or system — what Kant denominated as *experience*.[99] Mind makes the world and "From the standpoint of a theory of knowledge ... we may talk indifferently of the one or the other [mind and world]" (EPC: 13-14).

From the perspective of objective idealism, on some readings, the British Idealists *were* neo-Hegelians because they *were* neo-Kantians. Of course, we have already seen that the British Idealists were prone to speak of Kant and Hegel in the same breath and to counterpoise both to the representative names of British empiricism; as Green had expressed it:

> ... the true result of Hume's philosophy was the demonstration of the bankruptcy implicit in the empiricistic principles of Locke and

[99] See BEL: 4ff for a fuller discussion of these points.

> Berkeley, that Kant and Hegel had pointed the way to an acceptable alternative account of experience and knowledge, and that nineteenth-century British philosophy ... was consequently anachronistic, since it sought to continue along the bankrupt lines laid down by Hume, ignoring the work of Kant and Hegel (PE: v).

But although most stop there, assuming that the idealistic impulse was to be explained primarily as the proper medicine to an existing ailment — and that was the robust position of British empiricism, one which could write and act as though Kant had never even existed and which had both positivist and psychologistic characteristics — this may not be (genetically) accurate. Bradley later reflected that:

> In the latter half of the sixties what authority Mill had had at Oxford was much impaired. The study of philosophy was becoming a serious affair. There were lectures &c. on German philosophy, and also by Green on Hume. I don't think that *any* of the younger teachers (who made any mark) followed Mill. In the early seventies this movement advanced rapidly. I don't think that by the middle of the seventies Mill counted as anything (BCW, V, p. 200).

If the influence of the second generation is acknowledged and investigated, then despite the numerous and important differences between British Idealists and the neo-Kantians, it might be concluded that "[n]evertheless, it is most true and noteworthy that *British Hegelianism is, in a sense of its own, Neo-Kantian*."[100] (There are numerous other references to the British Idealists as neo-Kantians, including the 1883 review in *Mind* of *Essays in Philosophical Criticism*, which stated that "this volume is no unworthy witness to the presence of the 'Neo-Kantian' school as a party and a power."[101])

Whether it is indeed proper to characterize the British Idealists as analogues of the continental neo-Kantians cannot be explored here. But if the hypothesis advanced here about the shared formative influence of the second generation is accepted, then although these two movements were not blood siblings, one might concluded that they were indeed first cousins. (Hence, if the suggestion by Royce were to be adopted, we would subsume both movements under the broader tent of post-Kantian idealism.) But both movements shared similar concerns and faced related problems. And, as alluded to above, one looming difficulty coalesced around the term 'science'. These issues included not just the debate on materialism but also the caustic sting of Darwin's *The Origin of Species* (1859). Unfortunately, this was soon followed by a plethora of disturbing publications, beginning with *Essays and Reviews* (1860), and including Colenso's *The Pentateuch and the Book of Joshua Critically Examined* (1862), Huxley's *Man's Place in Nature* (1863), Renan's *Vie de Jésus* (1863), and Strauss's revised *Das Leben Jesu* (1864).

[100] [Mackintosh, 1903, p. 87].
[101] [Hodgson, 1883, p. 580].

For although the impact course between science and religion had already been charted in the very emergence of modern philosophy, even by the time of the Enlightenment there were a number of mitigating factors that prevented a head-on collision. First, science was still too weak as a social force to make any serious challenge to religion as a guide to living, as a way of life. Second, there existed a serious current of Enlightenment thinking, one proposing that the basic truths of religion (if not all of its trappings) — in particular, its morality — could be recouped by and for reason. These thinkers had assumed that religion could be eventually subsumed under science or reason in such a way that the basic truths of religion would be shown to be reasonable (e.g., Kant and Christian morality). But despite the efforts of Feuerbach and others, in the post-Enlightenment, not only did the projected unification not occur but rather the split, and the looming contradictions, worsened.

Although such concerns might seem remote from the British Idealists — and their logic — they were given polemical voice by Stirling in his introduction to Hegel's *Logic*. There one discovers that the Enlightenment, for Stirling, may be summed up as "that general movement which has been named Aufklärung, Free-thinking, the principle of which we acknowledge to be the Right of Private Judgment" (SH2: li). Although Kant was a full-fledged participant in this movement, "he saw the necessity of a *positive* complement to the peculiar *negative* industry to which, up to his day, both movement and principle had alone seemed adequate" (SH2: li). In other words, the destructive character of the Enlightenment needed to be supplemented with a positive program unless the prospect of "universal scepticism" were to be accepted. From Sterling's perspective, the baby (humanity) had been stripped of the last vestiges of the old order; but something must nevertheless take its place:

> Now, Kant, saw a great deal of this — Kant saw that the naked baby would not do; that, if it were even necessary to strip off every rag of the old, still a new would have to be procured, or life would be impossible. So it was that, though unconsciously to himself, he was led to seek his *Principles*. These, Kant came to see, were the one want; and surely, if they were the one want in his day, they are no less the want now. Self-will, individual commodity, this has been made *the principle*, and accordingly we have turned to it, that we might *enjoy ourselves alone*, that we *might live to ourselves alone*, that the I might be wholly the I unmixed and unobstructed; and, for result, the I in each of us is dying of inanition — even though we make (it is even *because* we make) the seclusion to self complete — even though we drive off from us our very children, and leave them to corrupt at Boarding-schools into the one common model that is *stock* there. We all live now, in fact, *divorced from Substance*, forlorn each of us, isolated to himself — an absolute abstract unit in a universal, unsympathising, unparticpant *Atomism*. Hence the universal *rush* at present, as of maddened animals, to material possession; and, this obtained, to material ostentation, with the

> hope of at least buying sympathy and bribing respect. Sympathy! Oh no! it is the hate of envy. Respect! say rather the sneer of malice that disparages and makes light. Till even in the midst of material possession and material ostentation, the heart within us has sunk into weary, weary, hopeless, hopeless ashes. And of this the Aufklärung is the cause. The Aufklärung has left us nothing but our animality, nothing but our relationship to the monkey! It has emptied us of all essential humanity — of Philosophy, Morality, Religion. So it is that we are *divorced from Substance*. But the animality that is left in the midst of such immense material appliance becomes disease; while the Spirit that has been emptied feels, knows that it has been only *robbed*, and, by very necessity of nature, is a craving, craving, ever-restless void (SH2: lii-liii).

So, there is one thing wanting: objective principles that might redeem the subjective alienation that each individual was now disposed to live. And this is precisely what both Kant and Hegel sought, "for Hegel is but the continuator, and, perhaps, in a sort the completer, of the whole business inaugurated by Kant" (SH2: liv). And so in the very phraseology of 'private judgement' the emphasis cannot be upon 'private' but upon 'judgement':

> The subject, then, must not remain Formal — he must obtain Filling, the Filling of the Object. This subject is not my true Me; my true Me is the Object — Reason — the Universal Thought, Will, Purpose of Man as Man. So it is that *Private* Judgment is not enough: what is enough is Judgment ... Self-will shall work out, shall *realise* Self-will — that is, effect a true will of any kind — by following the Universal Will (SH2: lv).

Obviously, in Stirling's hysterical reference to the "monkey" we can locate an angry response, among other things, to Darwin and to the challenges to Biblical chronology from the geological and biological record. In this social ferment, in England, the church parties formulated responses. In England, both the High and the Broad Church movements responded.[102] On the Broad Church side, one discovers the beginnings of the de-mythologisation project in Green's ambition to

[102]While the High Church party advocated nothing less, in the minds of its Broad Church opponents, than the unthinking (and unthinkable) return to authority, these seemingly sectarian issues had epistemological consequences; for, as Pattison remarked

> The controversy on "private judgment" involved, if it did not elucidate, the question of reason v. authority. The dispute as to the merits of the Reformation was no a mere theological quarrel, it inevitably carried the thoughts of the disputants to *the ultimate criterion of belief* (Pattison, 1876, p. 86, my emphases).

In Bosanquet's final diagnosis, the separation of the act of assertion from the content asserted therein could only be motivated by a subjectivism that construed "judgment as an irrational and arbitrary activity"(LMK, I: 378). He adds: "It is not surprising that in the 'Grammar of Assent' ecclesiastical interest should have thrown itself zealously on the side of such a conception" (LMK, I: 378).

preach a sermon in which the essential truths of Christianity were presented in non-theological (philosophical) language. Jowett noted this prospect, hesitantly, in his diary:

> G[reen] wants to write a sermon in which the language of theology is omitted – a Christian discourse meaning the same thing in other words.
>
> The attempt is worth making, but it requires great genius to execute it. The words will seem thin, moral, unitarian ... Yet something like this is what the better mind of the age is seeking — a religion independent of the accidents of time and place.[103]

Some of these attempts remain recorded in Green's lay sermons. But these linger as mere words on a page, devoid of "the speaker himself ... addressing himself to the practical needs of his hearers"[104] — a point not lacking in significance once it is acknowledged that "God is not to be sought in nature ... but in man himself" (GCW III: 265), a further conclusion that Green did not shrink from:

> To say then that God is the final cause of moral life, the ideal self which no one, as a moral agent, is, but which everyone, as such an agent, is, however blindly, seeking to become, is not to make him unreal. It *is*, however, (and this may seem at once more presumptuous and less reasonable) in a certain sense to identify him with man; and that not with an abstract or collective humanity but with the individual man. Let us consider in what sense. An assertion of identity, it must be remembered, not only admits of but implies difference or change ... Whatever unity of principle or law runs through any process of change, there the different objects which result from the process at its several stages have a real identity with each other, though they may be as different as the oak from the acorn or the complete animal from the embryo; and on the recognition of the difference depends the significance of the assertion of identity. We need not be frightened then from the doctrine that man is identical with God on the ground that it makes God 'no more than' man ... The whole force of the doctrine lies in the interpretation of the identity claimed for man with God as an identity of self with self ... in the process constituting the moral life according to our interpretation of it, the germ and the development, the possibility and its actualisation, are one and the same consciousness of self. That in virtue of which I am I, and can in consequence so set before myself the realisation of my own possibilities as to be a moral agent, is that in virtue of which I am one with God (GCW III: 225-226).

This passage, which mixes logic and theology in a way congenial to the British Idealists, perhaps finally drives home the fact that *these concerns are not our*

[103] As quoted in [Richter, 1964, p. 99].
[104] [Ward, 1967, p. 59].

concerns: likewise, the logic of the British Idealists is *not* our logic. Of course, from the Idealist perspective logic and theology were not disjoint, as evidenced in the long passage ascribed to Mr. Grey in his response to Robert Elsmere's spiritual crisis:

> 'I know it, I have gone through it ... I know very well ... to him who has once been a Christian of the old sort, the parting with the Christian mythology is the rending asunder of bones and marrow. It means parting with half the confidence, half the joy, of life! But take heart,' and the tone grew still more solemn, still more penetrating. 'It is the education of God! Do not imagine it will put you farther from Him! He is in criticism, in science, in doubt, so long as the doubt is pure and honest doubt, as yours is. He is in all life, in all thought. The thought of man, as it has shaped itself in institutions, in philosophies, in science, in patient critical work, or in the life of charity, is the one continuous revelation of God! Look for Him in it all ... All things change, — creeds and philosophies and outward systems, — but God remains!'
>
> " 'Life, that in me has rest,
> As I, undying Life, have power in Thee!' "[105]

God is in logic for He is the "wide-embracing Love" that, as 'the mainspring of logic,' "[c]hanges, sustains, dissolves, creates, and rears."[106] Strange, strange indeed. But to do justice to these reflections — irrespective of their value or utility — demands taking them on their own terms. But to do that, likewise, requires (minimally) that they be situated in the philosophical context, one with both a 'before' and an 'after'.[107] The thought of the British Idealists still awaits just such a comprehensive re-consideration, if the common assumption of a meaningless interregnum is to be refuted.

ACKNOWLEDGEMENTS

This chapter should have been much better than it is. By way of excuses, that do not touch upon the numerous inadequacies of the author, one could emphasize that there is little reliable literature on the British Idealists, even less on their logical doctrines directly, and most of the better part of that on Bradley. Nevertheless,

[105] [Ward, 1967, pp. 355-356].

[106] The lines quoted by Mr. Grey (previous quotation) and here come from Emily Brontë's poem "No Coward Soul is Mine" (1848).

[107] Among the further questions that remain, one concerns whether the British Idealists (and the continental neo-Kantians), should be construed as either the precursors or as the first wave of philosophical modernism, and indelibly colored, nevertheless, by the previous generation of post-enlightenment thinkers. To full answer this, one would need to know in what ways they advanced or hindered the revolutionary approaches of "high modernism."

there has been an emerging literature, since the 1984/85 "turning point," without which this essay, impoverished as it is, could not have been written. Special thanks are due to Alan Richardson for his advice and encouragement. Naturally all errors and misrepresentations remain mine and mine alone.

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BRADLEY'S LOGIC

William J. Mander

1 INTRODUCTION

The history of logical thought in Britain contains few texts more central than *The Principles of Logic* by F.H.Bradley. The date of its first appearance (1883) heralds a sharp and profound turn in logical thinking in Britain as, almost single-handedly, it shifted the main patterns of thought — which prior to that had been either empiricist in the fashion of John Stuart Mill or deriving from the Scottish common-sense philosophy of Sir William Hamilton — in an idealist direction, one inspired by Kant and Hegel yet quite transcending them. That some twenty years later the work of Russell, Whitehead and Frege precipitated another equally radical and widespread shift, one of whose marks was a near-complete neglect of what had gone before, has served to hide this fact. Yet the vast majority of philosophical work dating from this turn of the century period is written in an idealist style for which Bradley's *Principles* is the pre-requisite starting point.

However, it is not only with respect to the *history* of logic that Bradley's thought deserves our attention, for his logical contributions retain great interest and value in their own right. Running through his ideas modern logicians are able to find many specific insights and analyses of value and illumination to their own efforts, but perhaps the greatest lesson which his logic has to teach us lies in the profound difference of his approach to our own. Studying his logic reminds us how many different types of thinking have borne — and still could with equal right claim — the name of logic other than simply that narrow and specific discipline with which we are familiar today. Yet for all that it can teach us, the distance between Bradley's approach to logic and that with which we are familiar make him hard reading today. The following essay aims to ease that burden and to explain his basic logical ideas.

1.1 Bradley's conception of Logic

A helpful place to begin is with Bradley's conception of the *purpose* of logic. The objective of logic, claims Bradley, is

> "to set out the general essence and the main types of inference and judgment, and, with regard to each of these, to explain its nature and special merits and defects.... The degree in which the various types

Handbook of the History of Logic. Volume 4: British Logic in the Ninteenth Century.
Dov M. Gabbay and John Woods (Editors)

> each succeed and fail in reaching their common end, gives to each of them its respective place and its rank in the whole body. Such an exposition is in my view the main purpose of Logic." (PL 620)[1]

The thought that it is the business of logic to classify and analyse types of thinking is familiar, less so that it should be given such an evaluative role, that it should rank forms of thought for their effectiveness in reaching their aim. For what is that aim? And where does it come from? And how is success to be measured? The key to answering these questions is to understand that for Bradley logic is the science of *knowledge;* it is concerned with determining how adequate forms of thought are to express and convey *truth.* The relevant rules here are partly internal to the practice of thinking itself — thought in its very nature sets its own goals and critical standards — but they stem also in part from deeper metaphysical questions about the nature of thought and reality, which raises complex questions regarding the relation between logic and metaphysics. To Bradley there is no absolute distinction between logic and metaphysics — as he himself says, "I am not sure where Logic begins or ends." (PL ix) — but to the degree that metaphysical questions may be kept at bay room may be made for a discipline designated Logic.

For Aristotle, who founded the logical system which dominated throughout most of philosophical history, logic was something static. It was a fixed categorization of the forms of thinking and reasoning. The idealists had a very different understanding of logic. On their view, the various forms of thought form a scale; an ascendance of forms, one succeeding the other from the lowest up to the highest. The limits of the scale were measured in truth and individuality, from the barest and least true to the fullest and most true. This scale was moreover something *teleological*, a fundamentally dynamic structure, with each form leading to and even bringing about the next. For thought was understood as itself seeking something, even as possessing its own inner drive towards truth, consistency, and completeness, a drive that leads it onwards from unsatisfactory expressions to more satisfactory ones. The extent to which this developmental structure was a notional one, or a real historical process in the psychological history, either of the individual or of the species, was generally left unclear, but its centrality in the idealists' overall conception of logic is paramount.

Although, by its means our different patterns of thinking can be arranged in some sort of hierarchy, Bradley holds that logic as a subject has no unique starting point — no privileged set of concepts or axioms — nor any one correct route through its matter. (PL 597) There is no foundation upon which is gradually built in stages the whole edifice, rather we should think of logic as a connected whole, such that wherever we start we in some way bring in everything else. Indeed, in a sense it is circular in that everything supports and is supported by everything else, in the end turning back on itself, while nothing can be supported by anything

[1] All references to Bradley's works are given in the text by the following abbreviations: AR = *Appearance and Reality* (2^{nd} edition), PL = *The Principles of Logic* (2^{nd} edition) , ETR = *Essays on Truth and Reality*, CE = *Collected Essays.*

outside the system as a whole. In this way, although Bradley starts the *Principles* by discussing the notion of judgment, he is not thereby claiming that this is somehow a more basic notion for logic than any other (PL 1) for, he admits,

> "any one of the three, judgment, inference, and ideas, can be plausibly shown as preceding the others. But really, here as elsewhere, what in every sense comes first is the concrete whole, and no mere aspect, abstracted from the whole, can in the end exist by itself." (PL 640)

Such a coherentist attitude is one that might not seem so strange today, given a general suspicion of foundationalism and a widespread recognition of the arbitrariness of alternative axiomatisations, but one big difference between Bradley's approach to logic and that of contemporary thinkers is that he had no interest in merely formal logic. In contrast to the modern discipline, which virtually defines logic as a concern with the formal properties of thoughts, Bradley regards it as impossible to model logic on mathematics, and his use of symbols is as sparing as it is lacking in system. Bradley devotes an entire chapter of his book to exposing the inadequacies of purely formal logic (Book III, Part II, Chapter I) urging that It is not possible to draw anything except a relative distinction between form and content (PL 524, 532), that the possibilities of thought and reason exhaust any fixed list of schemata (PL 521), and that little of logical interest depends on solely on form — for the ultimate test is always one of the adequacy of our scientific or philosophical thinking to fact or reality itself. This general attitude is further visible in his attack on what was, perhaps, the most highly developed purely formal logic of his day, William Stanley Jevons' equational logic.[2] He devotes an entire chapter to refuting Jevons (Book II, Part II, Chapter IV) but rather than consider the formal adequacy if the system — indeed he concedes that it is well able to model much of our reasoning (PL 370, 377) — he focuses in on the metaphysical question of whether propositions really assert identity as Jevons suggests. Bradley's lack of interest in this side of matters is, of course, the one respect in which his logic most rapidly dated, for the great advances in logical thinking in the years after his time have been precisely in its formal aspects.

1.2 Influences

Bradley's logic did not simply emerge in a vacuum and it is useful to consider some of the influences, both positive and negative, which shaped his thinking. We can begin with Hegel. There has been much debate concerning the extent and nature of Hegel's influence on Bradley, but whether or not one wants to mark Bradley as merely an 'Hegelian' (as some have done) or to suggest a more complex relation, there can be no dispute that Hegel exercised an absolutely central influence on Bradley's thought about logic. This can be seen in his advocacy of such things as the notion of identity-in-difference, the concept of concrete universal, the view

[2]Jevons [1874]. For a more detailed discussion of Bradley's treatment of Jevons see Griffin [1996, pp. 217–230].

that association involves universals, his distinction between the genuine and the spurious infinite, the view that thought begins in the unity of experience, and the use of holistic coherentism as a standard. On the other hand, however, *The Principles of Logic* is certainly not in any obvious sense an 'Hegelian' logic; it is no *a priori* attempt to determine a hierarchical scheme of categories for thought, nor does it contain explicitly dialectical triads of reasoning. Indeed in anticipation of the charge Bradley says in his Preface to the book

> "Assuredly I think him a great philosopher; but I never could have called myself an Hegelian, partly because I cannot say that I have mastered his system, and partly because I could not accept what seems his main principle, or at least part of that principle. I have no wish to conceal how much I owe to his writings; but I will leave it to those who can judge better than myself, to fix the limits within which I have followed him. As for the 'Hegelian School' which exists in our reviews, I know no one who has met with it anywhere else." (PL x)

The 'main principle' here is Hegel's panlogisism, the rejection of which placed Bradley at some distance from his fellow British idealists, many of whom certainly were deserving of the term 'Hegelian' (Richard Burdon Haldane, for example.) Here it is appropriate to discuss another key influence on Bradley's logic, Hermann Lotze to whom Bradley admits, of recent writers, he owes the most (PL ix). Though also an idealist (of a sort) and not without debts to Hegel, Lotze rejected Hegel's key identification of thought and reality, and here Bradley found a point of connection, for he too thinks it necessary in the end to draw a clear distinction between these two.[3] Further crucial points of influence are to be found in Lotze's worries about relations and his strong emphasises on organic unity, both picked up by Bradley. Even if the worry about relations only becomes explicit in Bradley's *Appearance and Reality* published ten years after the *Principles*, there can be no doubt that both function as constant underlying themes in the *Principles* also.

Another key influence on Bradley's thought was Johan Friedrich Herbart, Lotze's predecessor at Gottingen and Kant's successor at Konigsburg. Herbart rejected not only Hegel, but idealism too, nonetheless Bradley was much influenced by him. We shall see below, for example, how one of Bradley's key arguments for the hypothetical nature of judgment, its inability to refer directly to reality, originates from Herbart.

I have mentioned something of the positive influences, but just as important on Bradley's thought were the negative ones, those thinkers he reacted against. In particular Bradley sets himself against the whole empirical psychological tradition

[3]It is worth noting that Bradley's interest in Lotze was not unusual. Now largely forgotten, Lotze was very influential in the 1880's in Britain. Even more effort was put into the translation of his work as into that of Hegel — all his main writings were translated during this period. The fact that he was taken so seriously by all the British idealists shows that they fully understood the problematic nature of the Hegelian identity, and thus that the difference between them and Bradley was not perhaps so great as all that. For more on Lotze and British Idealism see Devaux [1932] and Kuntz [1971, pp. 48–68].

in Logic, first clearly formulated in Britain by Hume, but developed by Hartley, Bentham, James and John Stuart Mill, and Alexander Bain.[4] According to this view, ideas are atomistic psychical states whose intentionality is a matter of their resemblance to the objects which they are supposed to represent. The ways in which they follow one another in our thinking history is to be explained in terms of habits — 'psychical bonds' or 'associations' — set up in us by repeated experience. Against this widespread view, Bradley insists (as we shall see below) that thoughts are first and foremost logical entities, and the connections between them logical ones, not psychological. In so far as this shapes his entire view, we must regard his logic as in a very real sense born out of opposition to psychological empiricism.

Opposition to psychologism constitutes another point where we can trace the influence of Lotze. For he too, was also strongly anti-psychologistic in his understanding of concepts and logical processes. Concept formation is a function of thought proper not the psychological processes of abstraction or synthesis, while justification derives from the logical rather than the psychological basis for our thinking. In this connection it is worth noting, too, the influence of Lotze on Frege, one of the other great modern critics of psychologism. Not least through its also being a hallmark of Fregean thought, anti-psychologism is the one area of Bradley's logic that has won him greatest praise among modern logicians.

But a word of warning should be sounded least we overestimate Bradley's opposition to psychologism. For while certainly he opposes those empirically minded thinkers who conflate logic and psychology, he nonetheless is not to be found among the class of those who regard logic as a merely formal science unconnected with human thought. His general conception of logic is not that it is the study of pure abstract timeless forms, but rather that it is the study of mental acts and functions; it is judgments and inference which we study, not propositions and entailment. But both judgments and inferences are things which occur in the conscious mind. Bradley makes the point forcibly with regard to truth, the ultimate concern of logic. He insists, "truth in the end is not truth unless it is thought, and so is actually thought by this or that mind, and therefore is thought at some one time." (PL 704n)

1.3 The Principles of Logic

Speaking as I have done so far of Bradley's 'logical views,' there is one further point that needs to be made before we examine the *Principles of Logic* itself. This concerns the evolution of Bradley's logical thought. The *Principles of Logic* was first published in 1888. But Bradley's opinions did not remain unchanged. There were many developments which found their way into subsequent books and articles. Of particular significance in this regard was Bernard Bosanquet's book *Knowledge and Reality*, a detailed criticism of the *Principles*, published just two

[4]It should not be thought, of course, that the assimilation of logic to psychology is a uniquely British phenomenon. Another important proponent of the view was Christoph Sigwart, a figure to whom Bradley regards himself as both indebted and disagreeing.

years later in 1885. While broadly sympathetic to Bradley's approach, Bosanquet nonetheless found much also to criticise in it. In the end, but not until 1922, there appeared a second edition of the *Principles* incorporating Bradley's various changes of mind. This second edition leaves the basic text unchanged, but has a great many footnotes and twelve terminal essays, vastly expanding the whole. Many of the changes are minor, but many too are substantial, for by that time, and in no small part as the result of Bosanquet's criticisms (PL viii), Bradley had come to feel that there were numerous places in the first edition where he had gone badly wrong. Inevitably this must complicate any presentation of his views, and the most important of the second edition changes will be noted as our discussion progresses.

2 JUDGMENT

2.1 General comments

The Principles of Logic deals with two broad subjects — judgement (Book I) and inference (Books II and III, each themselves divided into a first and second part.) Following Bradley's order we may begin our account with a consideration of the first chapter of Book I in which Bradley sets out his general conception of the nature of judgment.

Bradley's choice of unit, the *judgment*, is unfamiliar today, but quite deliberate. It is chosen in contrast to the *sentence*, because Bradley holds that sentences — which are essentially grammatical units — often fail radically to capture the underlying logical structure of the thoughts behind them. (We shall encounter numerous examples of this failure as our discussion proceeds.) That such failure can occur is a commonplace in modern logic, with credit for first seeing it clearly often given to Frege and Russell, however Bradley has as much claim as they to be included in this list.[5] Notwithstanding this agreement, Bradley's selection of unit remains at variance with theirs, for he also chooses judgment in contrast to the *proposition*. Propositions, while admittedly logical rather than psychological creatures, are rejected by Bradley as illegitimate abstractions; too far removed from the realities of belief and knowledge to which they are supposed to contribute. For it is a key element of Bradley's view that judgment can not be understood in isolation from the actual context in which it is made. Unlike a proposition, a judgement is an *act*, specifically a mental one.

Bradley describes judgment in the following way: "Judgment proper is the act which refers an ideal content... to a reality beyond the act." (PL 10) [6] His thought is that in judgment we abstract out some aspect or feature of the world

[5]Indeed with respect to certain analyses (such as the treatment of universal statements as conditionals) a case can be made out for Bradley's influence on Russell.

[6]"[A]ffirmation, or judgment, consists in saying, This idea is no mere idea, but is a quality of the real. The act attaches the floating adjective to the nature of the world, and, at the same time, tells me it was there already" (PL 11).

we encounter and assign it objective reference to some part or parts of reality; we characterise some object or other by predicating our ideas of it. This statement needs some unpacking.

The word 'ideal' in the phrase 'ideal content' is simply the adjective of idea,[7] so what Bradley is claiming in his definition here is that judgment involves the manipulation of ideas. But what are ideas? It is crucial to understand Bradley correctly on this matter. He distinguishes between two senses of 'idea'; between, as we might variously put it, the symbol and the symbolised, the image and its meaning, the psychological and the logical idea. (PL 6)

As individual psychical occurrences, ideas "are facts unique with definite qualities... the same in all points with none other in the world" (PL 4). They are particular existences which occur at specific times, determinate yet fleeting events in the ceaseless flux that is our inner mental history.

But, Bradley goes on, these private mental events can have a meaning as well as a nature, the psychological ideas can come to be logical ideas also. This occurs when they are treated as *symbols*, as signs of an existence other than themselves. Indeed, he insists that, in the logical sense, "Ideas are not ideas until they are symbols." (PL 2) Ideas for logic have meaning, they refer beyond themselves. They do not properly 'exist,' in the manner of some event having a location in space and time, they are facts no more inside our heads than outside them, (PL 7) rather they are universals such that more than one mind may without contradiction think one and the same thought. Making the point in a slightly different way Bradley distinguishes within any psychological idea between its nature or content and its actual existence — its 'what' and its 'that' as he calls them. In becoming an idea for logic the content is separated notionally from its existence and referred to some other reality. In this way an idea, considered in its logical aspect, is, he thinks, a kind of "parasite cut loose." (PL 8) It does not exist in its own right.

Bradley's purpose in drawing this distinction is to insist that logic is interested in ideas only as meanings and not as mental events, and in so claiming he is setting himself sharply against the British empiricist tradition. For that tradition, from Locke, Berkeley and Hume through to Hartley and John Stuart Mill regards ideas — the subject matter of logic and philosophy — precisely as those things which psychological introspection reveals to us, the dated mental events which make up our private stream of consciousness. Yet Bradley sets himself against all this, urging that "In England at all events we have lived too long in the psychological attitude." (PL 2) This is a radical attack, and the resultant conception of ideas one of the most characteristic features of his logic. But immediately, we can draw at least two corollaries distancing him from his psychologistic predecessors.

First of all, it follows that an idea is not, as traditional empiricism has often thought, some sort of a *copy* or *image* of what it represents. Ideas do not posses meaning by copying the objects or impressions which they signify, for they do not possess meaning *in their own right* at all. Rather they are *given* meaning by being made to stand for something else. Yet this relation of representation is

[7][Stock, 1984].

one of artificial custom, as can clearly be seen from one of the examples which Bradley uses to illustrate it, viz. the signification of flowers. (PL 3) That some flowers stand for love and others for hope is clearly a matter of convention, not resemblance. Of course, images may come before the mind when we think or understand, and they may even resemble the object of our thought, but that is not what gives them their meaning.

A second corollary concerns our ideas of universals. For the traditional empiricist, who believes that our ideas copy our impressions, and that all of our impressions are of particular existences, our acquisition of universal concepts is a puzzle. The struggles of Locke and Berkeley on this issue are familiar. Bradley, however, insists that it is simply a "false assertion, that merely individual ideas are the early furniture of the primitive mind" (PL 35) and so for him the problem is non-existent. Unlike mental images, logical ideas are not particular, either in their being or in their signification, but rather universal from the start. We may use a particular word or image, but insofar as we or others could use an identical word or image on another occasion to refer to the same thing, the idea we use is a universal. And what we pick out too is something inherently universal. (This second point will be argued in more detail below.)

Given that judgments are made up of ideas referred away to reality it might be wondered why Bradley's logic has only two main divisions, instead of the traditional three: terms, propositions, inferences. But this is quite deliberate and, in fact, even to speak of ideas in this way as the 'components' of a judgment is potentially misleading. For on Bradley's view judgments are prior to ideas. As logical entities, as things possessing truth or falsity rather than just psychological existence, there can be ideas only in so far as they are signs, but ideas can be signs only in so far as actually used to refer to things, that is to say, in so far as they figure in the context of judgment. Thus although we may consider their nature, ideas have no independent reality, and there exists only a conceptual difference between them and the judgments in which they occur. In this way Bradley anticipates another axiom of modern logic, the 'context principle' — the doctrine that a word has meaning only in the context of a proposition. Often attributed to Frege, this principle has in fact been held by many before him, including many of the British idealists.[8]

To say that ideas occur only in judgments does not, of course, imply that it is impossible to say something by means of a single word or idea — for surely we may usefully cry out such warnings as 'Wolf!' or 'Fire!' etc. The correct conclusion to draw is rather that such interjections may in fact be, contrary to their appearance, complete assertions or judgments. Since they can readily be the "vehicle of truth and falsehood," (PL 57) they are to be understood as shorthands for such assertions as 'Here is a wolf' or 'There is a fire.' They are implicit judgments whose subject is the unspecified present environment, which they then qualify by the attribute in question.

[8]It can, for example be found in both T.H.Green and Bernard Bosanquet. See Manser [1983, pp. 60–4].

If ideas are universals, it must be asked how we ever come to connect them with reality itself for that, to Bradley's mind, is something quite individual and particular. The answer to this question is: through sense perception. Bradley is insistent that it is in present, waking awareness, and in that alone, that we encounter reality. It our sole point of cognitive connection with reality. He says "we seem to find contact with reality and to touch the ground nowhere, so to speak, outside the presented." (PL72) In consequence, our judgments, to be judgments of reality, must all of them be judgments of perception. Bradley insists that "All judgements predicate their ideal content as an attribute of the real which appears in presentation." (PL 50) [9]

Sometimes this occurs directly, and we simply describe what is perceptually given to us. But of course Reality vastly exceeds our limited perception of it; it is never fully given, and often hardly given at all. And thus in most cases our judgments refer only *indirectly* to perceptually given reality. The qualitative content of our judgment "is not attributed to the given as such; but by establishing its connection with what is presented; it is attributed to the real which appears in that given." (PL 72) Though a continuity with what is given, we attach it to one and the same world as that we encounter in perception. (We will encounter several examples of this indirect attachment below.)

This link between judgment and conscious perceptual awareness is crucial for Bradley, and it should be noted that it goes both ways. Not only does judgment essentially involve perceptual awareness, but all perceptual awareness essentially involves judgment. There can no more be data without interpretation, than interpretation without data. (ETR 204) There is no pure sensuous given about which we then theorise or judge. Rather, from the first, perception involves judgment; so that, notwithstanding the possibility of our also considering them separately in thought, these are in reality but two sides of a single phenomenon.

Perception plays two roles in Bradley's theory of judgment. Not only does it provide us (as we have just seen) with reality to which our ideas are referred, but it serves too as the source from which those ideas are drawn. For all his insistence on their logical character, it is significant that Bradley never uses the term 'concept' to characterise the tools of our thinking. (Just as at the next level up he rejects 'propositions' in favour of actual judgments) his ideas are not abstract platonic entities, but retain a residual tie to the psychological. A logical idea or sign, he says, consists of a part or aspect of the content of some mental state "cut off, fixed by the mind and considered apart from the existence" of that state. (PL 4) But the contents we abstract and refer away are precisely those given to us in actual perception. Here once again we see that it would not do to overestimate Bradley's anti-pschologism. Though not themselves psychological entities, ideas, on Bradley's conception, retain an essential connection with psychological phenomena. Insofar as the logical idea is an abstraction from the psychological idea, it continues in at least that sense to *depend* upon it.

[9] "We escape from ideas, and from mere universals, by a reference to the real which appears in perception" (PL 69).

While noting the basic point that Bradley does not distance himself wholly from the 'psychological attitude' is easy enough, settling the precise sense in which our ideas depend upon the psychological (or psychical, as Bradley calls it) is much harder. In holding perception as the source of our ideas, it is tempting to think of them as abstractions from mental images, but that would seem to imply that in judgment there must always be present a mental image (even if the idea itself is not to be identified with the image.) But if that ever was a requirement of the theory, it was certainly one that Bradley found himself wanting to draw back from in the second edition. For he argues there that in his original presentation he had too much exaggerated the role of images. (PL 38n8) Yet if not involving an image, the sense in which an idea is nonetheless something abstracted or cut off from what is given in psychical life remains both unspecified and unclear.

Because it typically involves numerous ideas, judgment is usually thought to possess a structure, and prior to the twentieth century this was always thought to be subject-predicate in form. That is to say, all judgments were held to attribute some predicate to some subject. Bradley rejects this account of the structure of judgment. He has two points of difference with it.

First of all he argues that it is better to regard the whole judgment as *single* ideal content attributed to the subject of the judgment. For a single judgment like 'The wolf eats the lamb' can be divided up in many different ways and, depending on the fineness of the divisions, be thought to contain any number of different ideas. (PL 11) It makes more sense thinks Bradley to hold that "Any content whatever which the mind takes as a whole, however large or however small, however simple or however complex, is one idea, and its manifold relations are embraced in an unity." (PL 12) The notion of 'one idea' here is, unlike the 'simple idea' of empiricism, the notion of something which, though unified, may be possessed also of internal complexity. Bradley's thought here also undermines any distinction we might be tempted to make between simple and complex propositions. There are no atomic or elementary thoughts. We can, thinks Bradley, divide up thoughts how we like. All that is objectionable "is our then proceeding to deny that the whole before our mind is a single idea." (PL 11)

Particularly criticised is the received view that all judgments resolve into the single uniform scheme; the subject predicate form: '*S* is *P*'. It is says Bradley a mere "superstition" (PL 13, 21, 50) to hold that in judgment we use one idea (a copula) to connect two others (a subject and a predicate). We do not. One strategy which he employs to make this point is simply to list types of propositions which it is very hard to analyse in this way, such as 'A and B are equal' or 'The soul exists.' (PL 13) But he has a deeper, more theoretical, argument as well. Thinking about links, such as conjunction or the copula, which, it might seem, we use to combine our ideas, it needs to be remembered that these

> "relations between the ideas are themselves ideal. They are not the psychical relations of mental facts. They do not exist between the symbols, but hold in the symbolized. They are part of the meaning and not of the existence" (PL 11).

In the judgment 'The wolf is eating the lamb,' the idea of 'eating' does not simply connect the two ideas, 'wolf' and 'lamb.' It is rather a third idea which along with them we refer to the situation itself. And the same holds with such words as 'and' and 'is.' They are not ancillary to our thoughts, but part of what we are thinking.

Bradley's second point of difference with the subject-predicate form of judgment follows from the first; it concerns the subject of our judgments. Subject-predicate judgments can be regarded as attributing some idea or predicate to their subject. But if, as Bradley thinks, the whole judgment forms one single idea, of what subject is this idea being predicated? Instead of each picking out different subjects, Bradley suggests that all judgments should be read as having the same subject — reality as a whole, to which they attribute their entire content. Thus the judgment 'S is P', instead of picking out S and saying that it is P, we say of reality as a whole that it is 'S-P ish,' or as he later puts it "Reality is such that S is P." (PL 630) In this sense all judgments have denotation and all are existential (ETR 426n) for all refer to something that cannot fail to be present, namely, reality itself.

It might be thought that, if not the element indicated by the subject term, the subject of judgment ought to be, not reality as a whole, but just reality as it appears to us. Bradley, however, would disagree with that. Although the real is something which appears in perception, it is not identical with the merely momentary appearance (PL 50, PL 70–2). "The real, which appears in perception, does not appear in one single moment" (PL 54) he says. Momentary presentation is an unreal abstraction, which transcends itself; it is but a fragment of a much wider reality. The real appears in, but is not identical with, such momentary presentation. It gives us knowledge *that* things are, but not *what* they are.

Understood in this way, the subject, we might say, does not belong to or fall within the content of our judgment at all. It is "never an idea" but "always reality" itself (PL 81), that to which all of our ideas are referred. This way of putting the matter, however, is not entirely happy in so far as it suggests that we can regard the subject of the judgment (reality as a whole) as something completely separate from our ideas of it. Yet this cannot be completely right for it amounts, in effect, to a kind of dualism which in the ultimate (metaphysical) analysis Bradley would reject. And in the second edition his statement is more nuanced; thought is always continuous with, and never ultimately opposed to, the underlying reality from which it emerges.[10]

His distinctive position with respect to the subject of judgment was one which Bradley felt called for modification in the second edition of the *Principles*. Without denying that it is reality as a whole that is always the 'ultimate subject' of our judgments, Bradley came to feel that this common referent is not quite the undifferentiated whole it first seems. Rather judgments have also what he vari-

[10]See Ferreira [1999, p. 45]. For example in the first edition he says, "[I]n every judgment there is a subject of which the ideal content is asserted. But this subject of course can not belong top the content or fall within it" (PL 13) But this statement is withdrawn, "We have not here a case of mere Yes or No" he says. (PL39 n17).

ously calls a 'special' or 'limited' or 'selected' subject — the point at which reality as a whole presents itself to us in perception, the point at which we feel ourselves called upon to qualify it as we do.[11] "In judgment the Reality to which in fact we refer is always something distinguished.... It is a limited aspect and portion of the Universe... some special and emphasised feature in the total mass" he says, yet because "[what] we have distinguished remains also inseparably in one with our whole Universe," they also qualify that wider whole. (PL 629, cf. 662)

To say that the mind abstracts from what is given it in perception some universal content which it then applies to Reality makes it seem as though there were two tasks which we must carry out in order to judge, as though the mind's coming to have ideas and its subsequent use of them were distinct and separable functions. In the first edition of the *Principles* Bradley certainly talks in this way as though one and the same content could be affirmed, hoped, questioned, etc, even that an idea might be "held before the mind without any judgment." (PL 76–7) But soon after the book appeared [12] he changed his position and argued that abstraction and reference were indistinguishable aspects of a single process, and that nowhere can we find any such 'floating ideas'.

> "Now a thought only 'in my head', or a bare idea separated from all relation to the real world, is a false abstraction. For we have seen that to hold a thought is, more or less vaguely, to refer it to Reality. And hence an idea, wholly un-referred, would be a self-contradiction." (AR 350)

> "There is no and there can not be any such thing as a *mere* idea, an idea outside any judgment and standing or floating by itself. We have here again not an actual fact but an unreal abstraction. The essence of an idea consists always in the loosening of 'what' from 'that'. But, apart from some transference, some reference elsewhere of the 'what', no such loosening is possible." (PL 640) (cf AR 324, ETR 28–64)

Bradley's retraction here could be viewed as simply his coming to a fuller recognition of his basic thesis that ideas are signs or symbols. For as such it is their essence to refer beyond themselves; assertion is built into them from the start. To put the point in another way, ideas can only be ideas in the context of a judgment, but all judgments in their nature make a claim.

But there is more to the point that just this. Ideas do not simply *try* to refer, thinks Bradley, they *succeed* in doing so. But an idea can only refer if there is something in the world to which it does refer, and so in a sense it must be conceded that all ideas are true. A comparison is helpful here. Frege, since he holds that a name could have sense but no reference, would allow a singular thought even where there existed no corresponding object. Bradley, in rejecting all such floating ideas,

[11] See Sprigge [1993, p. 301]; Ferreira [1999, p. 26].

[12] The change is clear by the time of *Appearance and Reality*, ten years later, but seems to have occurred even earlier.

ideas un-referred to any object, holds that singular thought requires the presence of the actual object. *Reality itself* enters into the judgement. The objects of our thinking are not intentional objects, but reality itself; in modern parlance 'meanings ain't in the head.'

> "A judgment, we assume naturally, says something about some fact or reality. If we asserted or denied about anything else, our judgment would seem to be a frivolous pretence. We must not only say something, but it must also be about something actual that we say it." (PL 41)

But floating ideas were rejected not simply because they ran counter to his theory of reference and judgment. Bradley came to see that, introducing a kind of dualism between pure thought and reality, they were incompatible with the kind of idealism he wished to develop. As Bradley sees it, both thought and its object originate in a single experience and however distinct they may appear to become, there remains a continuity between thought and its perceptual ground or context. Floating ideas were rejected also because they were incompatible with Bradley's coherentism. For if we may start from ideas floating free of any reference to reality, we might construct around them systems as extensive and coherent as those we currently endorse, yet these would not thereby be true.[13] This was Russell's famous objection to the coherence theory of truth.[14] An insistence that all ideas first be anchored in or referred to reality effectively blocks this objection to coherentism.

Of course, the thought that ideas may be simply held before the mind without judging is itself plausible, so if Bradley is to reject it, he will have some explaining to do. In this regard, he adopts a two-fold strategy.

In the first place he holds that surface grammar often misleads us as to the true logical form of a judgment. Although the thought must be referred to reality, "[the] ideal content may be applied subject to more or less transformation; its struggling and conditional character may escape our notice, or may again be realised with more or less transformation." (AR 324) We saw above how a single word — 'Wolf!' 'Fire!' etc. — might really be an implicit judgment. And in a similar way, though I might take myself simply to imagine, say, a unicorn or a tree in winter, there will always be involved some judgment or other, perhaps, that unicorns are white or that trees in winter look sad. Bare entertaining is not possible; thought is always for some purpose, which involves judgment and, with analysis, that may be uncovered.

A central role here is given to the notion of what Bradley calls 'my real world,' the universe of things continuous in space with my body, and in time with the states and actions of that body. (ETR 460) This notion provides him with a tool whereby many thoughts, which seem at first to float are, on analysis found to

[13] [Holdcroft, 1998, p. 171 ff].
[14] [Russell, 1906–7, p. 33].

refer implicitly to my real world. For example, a question seeks a truth which we take to be already there (ETR 36), while something is called 'imaginary' only by exclusion from this real world. (ETR 47)

The second part of the response to the apparent possibility of floating ideas is to expand the realm of possible reference. My real world is not the only one. In addition to this narrow sphere, he says, there are the worlds of duty, of religion, of hope and desire, of dreams, madness and drunkenness, of politics or commerce, of art and imagination, "all counting as elements in the total of reality." (ETR 31) Reality is the sum total of all these different realities. Ideas which seem to float may be true of some other reality. "Because there are many worlds, the idea which floats suspended above one of them is attached to another." (ETR 32) Though it comes into its own here as a device to assist in his rejection of floating ideas, this (it should be noted) was not a new idea but something Bradley had already used in the first edition of the *Principles.* For considering there how best to analyse the claim that 'The wrath of the Homeric Gods is fearful' in view of his thesis that judgment must affirm something of some reality, he responds that "In Homer it *is* so; and surely a poem, surely any imagination, surely dreams and delusions, and surely much more our words and our names are all of them facts of a certain kind." (PL 42)

In general, it can be said that the doctrine of floating ideas was an unfortunate intrusion into the system, whose removal improved consistency. But it did necessitate one or two changes elsewhere in the system, for example, as will be discussed below, in his view of Negation.

2.2 The dialectic between categorical and hypothetical judgments

It is natural to make a distinction between categorical and hypothetical judgments, between those which assert something unconditionally and those which make a claim subject to other factors. Bradley argues that at a deeper level, however, this distinction breaks down. Rather than mutually exclusive classes, all judgments are seen to be somehow *both* conditional *and* categorical. The phrase here 'at a deeper level,' is interesting and worth stopping to note. It reflects, in part, Bradley's sense that the surface grammar of what we say may mask the true logical grammar. But it is an indication also of his belief in a distinction — or perhaps rather a continuum — between what we might term pragmatic and metaphysical levels of discourse. Thought aims at ultimate truth and in the end this must take it into deep metaphysical waters, but at a more pragmatic level many of these complication may be relatively disregarded.

Bradley maintains both that all judgments are conditional and that all are categorical. We can take each of these theses in turn. We can begin with the first. Although at one level it makes sense to think of categorical judgements, Bradley admits, in the end this breaks down, and we must regard all judgments as hypothetical.[15] In order to appreciate this it is best to take the cases of universal

[15] Following Bradley in the *Principles*, I use here the terms categorical and hypothetical. But

and singular judgments separately.

Universal categorical judgments are conditionals because they have same truth conditions as conditionals. For example, in asserting that 'All animals are mortal' it is not simply animals *now* that we are talking about but future animals too, indeed, all possible animals. "We *mean*, 'Whatever is an animal will die,' but that is the same as *If* anything is an animal *then* it is mortal. The assertion really is about a mere hypothesis: it is not about fact." (PL 47) Similarly Bradley suggests that the assertion 'Equilaterial triangles are equiangular' simply affirms that with one quality you inevitably get the other; it says nothing about where, when or if such things are to be found. (PL 82) Rather than a categorical assertion about a determinate set of referents, what the assertion offers is simply a connection between ideas. This analysis of universal judgments will of course seem both familiar and natural to any one acquainted with modern logic. Indeed a case could made that Russell, who is the principal source for this analysis in modern times, derived it from Bradley. It should be noted, however, that Bradley's position is not exactly the same as that adopted today. For (as we shall see below) he draws a sharp distinction between true universal statements and mere collective statements, restricting the hypothetical analysis to the former. Modern logic by contrast makes no such restriction, but uses this analysis for all types of universal statement.

Turning to s*ingular categorical judgments,* Bradley offers two main arguments for the conclusion that singular judgments are conditional. His first argument is one from what we might call the distorting incompleteness of perceptual selection. In this argument Bradley takes what might be considered the paradigm example of a singular categorical judgement, one in which we ascribe some character to presently given sensation, and argues that it is quite unable to sustain its claim to be truly categorical. Such judgments Bradley terms *Analytic judgments of sense*; 'analytic' not in any Kantian sense here, but because "in these we simply analyse the given" (PL 49)

The problem with such analytic judgments of sense is that they take up only a fragment of the reality given in ordinary experience; they latch upon one part of what we perceive and characterise that ignoring the rest. Because of this they suffer from the twin defects of *incompleteness* and *distortion*. It is incomplete because it leaves out the context, but nothing can exist without context, and so the judgment

> "must always presuppose a further content which falls outside the fraction it offers. What it says is true, if true at all, because of something else. The fact it states is really fact only in relation to the rest of the context. It is not true except under that condition." (PL 97)

But worse than this, by selecting something out of its context we distort it. Just

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following the distinction that Bosanquet [1888] makes between them in his *Logic* (Bk.I, ch.VI. p.234) Bradley changed his mind: "It is... better, I am now persuaded, not to say that every judgment is hypothetical." (AR 320).

as a colour considered on its own in isolation can look quite different from how it does in context.

> "It was in the fact and we have taken it out. It was of the fact and we have given it independence. We have separated, divided, abridged, dissected, we have mutilated the given. And we have done this arbitrarily; we have selected what we choose. But if this is so, and if every analytic judgment must inevitably so alter the fact, how can it any longer lay claim to truth?" (PL 94)

Clearly what lies behind both of these arguments is a committed holism, such that everything is connected to everything else and everything depends on everything else. We assume the context is unimportant and can be pushed to the back without cost, but that is wrong. Generally the *Principles* tries to avoid metaphysics, and it was not until afterwards, in *Appearance and Reality*, that he went on to really demonstrate this. But his reasoning here, and elsewhere, can leave us in no doubt that his basic holism was already in place at this stage.[16] Bradley concludes that "analytic judgments of sense are all false" (PL 93) What he means is that taken as they stand they are false, but that properly they are to be read as subject to conditions, and thus really hypothetical. Treating singular judgments in this way transforms them into universal conditionals; rather than picking out individuals, they state entirely general relationships between grounds and consequents.

Even if what Bradley has said here is correct, it might be objected that he has hardly shown that all categorical judgments are hypothetical, for not all categorical judgments are direct reports of sensory experience. Bradley is sensitive to this charge and considers two further candidates, but neither he finds any more able to defend their claim.

Synthetic judgments of sense state facts of time or space which go beyond what is here and now directly perceived; they extend what is given in sensation. For example, 'This road goes to London,' or 'There is a garden on the other side of that wall.' (PL 49, 62). In this they are doubly hypothetical. In the first place, in going beyond the given, they explicitly involve inference. (PL 49, 55, 62, 73, 75) Finding a link or point of identity between what is presented or given ("This road..." "...that wall") and its ideal extension ("...goes to London" "There is a garden on the other side of...") they synthetically extend our knowledge. However, for Bradley inference is something essentially universal and hypothetical, so the synthetic judgment can be no more categorical than the analytic upon which it is based. (93). But these, of course, we have already seen to be conditional, and so it is doubly so.

Another class of putative singular categorical judgments which Bradley consider are *non-sensory judgements*, e.g. 'God is a spirit,' 'the soul is a substance.' (PL 49) Despite himself mentioning them, Bradley says very little about such judgments.

[16] See, for example, PL 71.

(79, 107, 113n66) His final view is that they cannot, but he is not explicit about the reasons. But certainly the argument that we are next to consider would have that result.

Bradley's second main argument for the conclusion that singular judgments are conditional is quite different and works from the alleged impossibility of singular reference. The argument, in fact, is one which he derives from Herbart,[17] and can be set out as follows: If all ideas are universal, and judgment consists in the union of ideas, then all judgment is hypothetical. "A sentence like 'All mules are sterile' links universals, and can only mean 'anything with the property of being a mule has the property of being sterile', making no reference to anything specific. It gives no information about the actual world unless we assume at the same time that these general properties are in fact instantiated, and so in itself it would be better read as saying, 'If anything has the property of being a mules then it will also have the property of being sterile.' In other words, it gives us, not specific details about how things are, but merely generalized information about how things would be if certain other things were the case. A mere union of universals cannot be true or correspond to individual reality itself.

But this contradicts our intuition that judgment is about reality. As such it is not something Bradley can simply take on board. So his response to this argument is somewhat equivocal. At the deepest level he rejects it by denying that judgment is the union of ideas. It thus forms part of his case against subject-predicate structure of judgment. But at a lower level he finds it irresistible. At that level it tells us that in so far as we take judgments to be composed of many ideas, it must be regarded as merely hypothetical.

The engine that drives Herbart's argument, and that so impresses Bradley, is the notion that there is a gap between ideas and facts, that facts are singular and ideas universal. Bradley spends a lot of time establishing this. To take the first half of this dichotomy, it is an axiomatic intuition for him that reality is singular. "The fact given us is singular, it is quite unique; but our terms are all general, and state a fact which may apply as well to many other cases" e.g. I have toothache (PL 49–50) Again, "A fact is individual," (PL 43) "The real is what is individual" (PL 45)

That facts are singular is not perhaps so controversial. Nor is it usually felt to be so worrying. Because most people have held that we have the power of singular thought. But this is something Bradley denies. For him all ideas are universal. "Nothing in the world that you can do to ideas, no possible torture will get out of them an assertion that is not universal" (PL 63) Of course, no one, would deny that many ideas are general, but it would be thought that some are particular.

Bradley considers four classes of putatively singular expression, none of which are able to secure genuinely singular reference. Definite descriptions, such as 'the man wearing a pork-pie hat,' or 'the third turning on the left', might seem to pick out unique individuals. But on reflection we see that they always could apply to other possible individuals as well. "The event you describe is a single occurrence,

[17] [Herbart, 1850, pp. 91–4].

but what you say of it will do just as well for any number of events, imaginary or real." (PL 63)

We might think that time and space could come to our aid here. If 'the man in the hat' could pick out more than one such individual, surely a further specification of the form, 'at place x and time t' can make good this gap. Bradley disagrees. He says "We must get rid of the erroneous notion ... that space and time are 'principles of individuation,' in the sense that a temporal or spatial exclusion will confer uniqueness on any content." (PL 63) For in themselves points of space and time are utterly indistinguishable. It is only when "the series is taken as one continuous whole, and the relations between its members are... fixed by the unity of the series" (PL 63–4) that exclusion makes its appearance. A given item may be located *in* or *by* its context but cannot be picked out *from* its context.

Mill, along with many others thought that proper names were mere marks, that they had denotation but no connotation. By contrast Bradley thinks proper names are universals (PL 59–63) A person would not get a name unless he were recognised as distinct, continuing though change, but this is a matter of his attributes. (PL 61)

But can we not at least pick out unique individuals using demonstratives, like 'this' or 'that'? Bradley disagrees. Rather than being specific demonstratives such as 'this' or 'that' are really the most general words there are, for anything whatever may be called 'this' or 'that.' He draws a distinction between what he calls 'This' and 'thisness.' The former is the felt unique encounter with reality — "unique, not because it has a certain character, but because it is *given*" (PL 64) — the latter what we get when we try to think or express this, the generic property of being a unique individual which applies to everything. A thing's "stamp of uniqueness and singularity comes to it from the former and not from the latter" (PL 65) but all we can think is the latter.

These two main arguments, then, from the conditioned nature of perceptual abstraction and the impossibility of singular reference, together with his preferred analysis of general statements, Bradley takes to establish that *all* judgment is really hypothetical. What this means for him is that all judgment is limited and false, that all is inadequate to what is given in feeling. For if judgment is conditional reality most certainly is not.

We have spent quite some time showing how and why, for Bradley, all judgments are hypothetical. However it would be wrong to suppose that that represents the whole story for him, for he insists too that there is an important sense in which all are categorical. "All judgments are categorical, for they do all affirm about the reality, and assert the existence of a quality in that" (PL 106) Each takes reality as a whole as its subject and asserts a quality of it. Even hypothetical judgments are in a sense categorical. They assert a quality of reality which makes us link ideas in a certain way. (PL 86–9)

2.3 The dialectic between identity and difference

In addition to that between the hypothetical and the categorical, there is a second dialectic at work in Bradley's theory of judgment. He argues that every judgement expresses a union of identity and difference. 'A union of identity and difference *between what*?' it will be asked and immediately we turn to consider the doctrine it must be confessed that he is somewhat equivocal in identifying the terms of these relationships. Sometimes he seems to be talking about the relation between special subject and the predicate we attribute to it, other times he seems to be talking about a relation between the judgment as a whole and the reality to which it is referred. But while confusing, this ambiguity is of little moment here, for in truth he believes that the superficial and the ultimate form of judgment must display the relationship which, along with many other British Idealists, comes to call identity-in-difference. Bradley arrives at the doctrine that every judgment expresses an identity-in-difference via the rejection of two extremes. It is the only possible middle ground between the two poles that judgments may not occupy: complete identity and complete difference. We may look at these two prohibitions in turn.

Bradley rejects as impossible the notion that there might be no difference whatsoever between the terms of a given judgment, a relation which he variously calls 'simple identity,' (PL374) 'abstract identity' (PL146) or 'mere identity' (PL373) and symbolises with an equals sign. (PL 25) The idea here comes from Hegel, and Bradley is quite explicit in acknowledging the debt, [18] but it is probably more familiar to us today from Wittgenstein. In his *Tractatus* Wittgenstein says that a perfect language would have no use for the sign of identity, for an identity statement does not tell us anything either about the world or about our language. It could be debated to what extent this understanding of a judgment is one actually adopted by philosophers or just a theoretical extreme to be exploited for the purpose of debate, but it is not without its real advocates. And certainly this is how Bradley understood the equational logic of his philosophical contemporary Jevons.[19]

The comparisons with Wittgenstein and Jevons should not mislead us into thinking that Bradley's claim is simply one about the sign of *identity*. Rather it is a point about the difference and/or identity between the elements of a judg-

[18]See Hegel [1812–16, Vol. II, pp. 37–43] and Hegel [1817, pp. 165–8]. "It is no judgement at all. As Hegel tells us, it sins against the very form of judgement; for, while professing to say something, it really says nothing" (PL 141). "Thought most certainly does not demand mere sameness, which to it would be nothing. A bare tautology (Hegel has taught us this, and I wish we could all learn it) is not even so much as a poor truth or a thin truth. It is not a truth in any way, in any sense, or at all" (AR 501 also 508).

[19]According to Jevons [1874] all propositions should be read as asserting an identity between their subject and their predicate. Bradley takes issue with this. (PL 370–88, 644ff) He does not deny that judgments involve an identity, but he can not agree that that is *all* they assert. Another target in Bradley's sights is Henry Mansel. It can be argued that both Mansel and Jevons derive this idea that Subject-predicate statements could be thought of as one of identity from a common source, William Hamilton.

ment, however the judgment itself may choose to join them. For if '$A = A$' is vacuous, the same holds of such judgments as 'A and A,' 'A or A', 'If A then A.' 'A only if A', or 'A is a-ish.'

At times Bradley seems to imply that such abstract identities are false, and this is how Wollheim takes him.[20] But his deeper meaning seems to be that they are simply nonsense. $A = A$, he says, "sins against the very form of judgment." (PL 141) In any judgment there must be a movement of thought or else nothing is *said*. Bare identity, he think, could never satisfy the intellect's demand that something at least must be asserted.[21]

The most common response to the suggestion that statements of identity have no place in logic is, of course, that taken by Frege in his distinction between the sense and reference of a term: a common reference does not make identity trivial if there exists a difference in sense. At one level Bradley's response would be to agree with Frege, for it is Frege's point precisely that identity makes no sense except where there is also difference. But at a deeper level he would take issue with Frege's analysis, for that analysis implies that read in intension or read in extension a judgement may be found to be saying quite different things. Read in extension (reference) it asserts identity, read in intension (sense) it asserts difference. However, for Bradley, whether we read it in intension or extension a judgment always says the same; it always combines identity and diversity. He admits that our mode of reading it affects the face which it shows to — in extension the judgment seems to assert the identity of a subject via an indirect reference to its different attributes, while in intension it seems to assert a connection between distinct attributes via an indirect reference to an identical subject — but in substantial content it remains unaffected. (PL 174) He disagrees with Frege's alternative analysis because he does not believe that in the end it makes any sense to speak of 'mere extension' or 'mere intension,' either without the other (PL 642–3)

But not only does Bradley reject judgments of abstract identity, he equally — and much more controversially — rejects any judgment of what he calls 'mere difference' (PL 373). Any attempt to combine in thought elements with nothing in common is ruled out as impossible. Why? This doctrine might be thought to come from Hegel also, for certainly Hegel says something very much like it (as the antithesis for which his discussion of identity forms the thesis).[22] But unlike the case of abstract identity, Bradley never acknowledges such a debt nor, I suggest, is it especially helpful to see Hegel as his source. Instead I think we can isolate a number of different lines of argument that lead Bradley to make this stipulation.

[20] [Wollheim, 1959, pp. 81–5].

[21] "Thought most certainly does not demand mere sameness, which to it would be nothing. . . if the law of contradiction forbade diversity, it would forbid thinking altogether." (PL 501) "That abstract identity should satisfy the intellect... is wholly impossible" (AR 508, cf. PL 25, 371–2). "what then do we assert by AB = AB? It seems we must own that we do not assert anything. The judgement has been gutted and finally vanishes... In removing the difference of subject and predicate we have removed the whole judgement" (PL 26). "whenever we write '=' there must be a difference, or we should be unable to distinguish the terms we deal with." (PL 27).

[22] See [Hegel, 1812–16, Vol. II, pp. 43–58] and [Hegel, 1817, pp. 168–75].

One crucial element for this view revolves around his theory of negative judgment (which will be considered below) but there seem to be others as well. For example, he offers a rather different line of argument about what it is to *think* a conjunction. [23] The argument is not fully clear, but I reconstruct as follows

As T. H. Green argued that a succession of perceptions is not a perception of succession,[24] so it is true that conjoined thoughts of different things are not a thought of those different things conjoined. Thinking of 'A' and (then) thinking of 'B' is not the same as thinking of 'A and B'. Simply to think them both is not to actually combine them in thought, for I can do that with 'square' and 'circle' but I do not thereby manage to think their union, to imagine a square circle. The point is that when we think 'A and B' the 'and' gets predicated of reality as well, [25] but in that case it is necessary to give an account of what this means, to assert how *in reality* they are conjoined.

Sometimes Bradley puts this point in terms of the *powers* of thought. Thought itself (he says) does not make the conjunction — it has not the resources to do so — rather it describes the conjunction it finds. Matters would be fine, he suggests, "[if] thought in its own nature possessed a 'together,' a 'between,' and an 'all at once."' (AR 504) but "Thought can of itself supply no internal bond by which to hold them together," (AR 504) "The intellect has in its nature no principle of mere togetherness." (AR 511).

But where we are faced with a mere conjunction of complete differences, a purely external connection there seems no scope in reality to think such a union. All we have is a brute combination. The problem with a wholly external link is that there is no explanation. An external 'and' says Bradley "in the end does but conjoin aliens inexplicably. . . there is in the end neither self-evidence nor any 'because' except that brutally thing come so." (502) He compares it to the mechanical view of the world. He objects that such a union is "groundless" or "without reason" (501) "I understand by an external relation to mean a mere conjunction for which in the quality there exists no reason whatever." [26]

At times this looks like some sort of appeal to sufficient reason, and certainly the accusation has been made, [27] but Bradley also presents it as some sort of law of thought. Unless there is some ground or point of union, he seems to be saying, thought can not bring them together. Thought cannot be *satisfied.* That, he seems to be saying, is just how thought works. "you are left in short with brute conjunctions where you seek for connexions, and where this need for connexions seems part of your nature" (ETR 115) "Thought demands to go proprio motu, or, what is the same thing with a ground and reason." (501) "to be satisfied my intellect must understand. . . my intellect can not simply unite a diversity, nor has

[23]See especially a 1897 paper, 'Contradiction and the Contrary' (included in the second edition of AR, pp. 500–11)

[24][Green, 1883, Chapter I].

[25]"Upon the view which I advocate when you say R is a, and R is b, and R is c', the 'and' qualifies a higher reality with includes $RaRbRc$ together with 'and'." (ETR 230).

[26][Bradley, 1999, p. 216].

[27]For example by Russell [1910, p. 374].

it in itself any form or way of togetherness, and you gain nothing if beside A and B you offer me their conjunction in fact." (509) Bradley seems to be saying that the only way we can significantly think 'and,' the only way we can give that thought any content, is to find a point in common and that this is why "An identity must underlie every judgment" (PL 28)

It might be objected that we can *think* it because we can *experience* it, for is not bare conjunction something we are *given* in experience? But that is something which Bradley denies. He rejects the notion that bare conjunction ever is *given*. What is given, he argues, is given always against a background context. And this background Bradley insists we cannot simply discount as unnecessary. (AR 503)

A quite separate line of argument against judgments which attempt to combine mere differences that we find in Bradley's thinking revolves around his puzzles about predication. [28] We could represent his thought as follows. Conjunctive judgements may be reduced to or are at least equivalent with predicative ones, but with predication the unacceptable choice between identity and difference is more clear. We can take these in reverse order.

Taking a basic subject-predicate statement 'R is a' we can not hold that R and a are literally identical, for that would automatically exclude the possibility of our also holding that 'R is b'. But neither can we say that R is something wholly other than a, for what then would be left of the claim that R is qualified by a? The only possible way out, thinks Bradley, is read the judgment as saying that they are different *in some sense*, or that *under certain conditions* they are identical. In this sense predication must be a middle-ground, a matter of identity-in-difference.

'But predication is not identity' it will be objected. No indeed. However, that is not our point. The question we ask here is, whatever it's nature, does predication connect identical or different things. And neither answer, unless qualified by the other, seems acceptable.

It is relatively easy to see how this works for predication, but why does it follow from this that two predicates, or even a simple conjunction need a point of union? But if these cases can (in substance at least) be reduced to subject-predicate judgements, then they are not really so different. And is this not possible? We may say 'R is a and b' (it is red and square) but this can be converted into 'Ra is b' (the red thing is square) and again we can ask if the predicate is something different from the subject or not. Even 'A and B' (it is raining and it is cold) can with paraphrase be rendered 'R is a and b' (the conditions are rainy and cold) or even 'Ra is b' (the cold conditions are rainy ones) facing us with the old dilemma.

Combining, then, the impossibility of abstract identity with the impossibility of mere difference, Bradley takes himself to have shown that all judgment must be an identity-in-difference. But what does it really *mean* to say that all judgments must occupy a middle ground between identity and diversity, that all must assert a *point* of identity between differences? It is possible to distinguish between a

[28] This is perhaps best seen in a 1909 paper 'Coherence and contradiction' (reprinted in ETR pp. 226–9) which repeats a case broadly similar to that he had made earlier in ch..II of *Appearance and Reality*.

number of different senses in which this thesis might be taken.

1. It can be read as the harmless enough point that a thing cannot be self-identical without being different from other things; being oneself and being distinct from others are but two sides of the same thing.

2. Another way to read Bradley's thesis here is to see it as one about the unity of a judgment. There must exist something which binds many ideas into one thought. There can be no thought without parts or multiple contents, but they only function as contents at all in the space of one judgment

3. Much of what Bradley says seems best understood in a third sense: the coming together of things into union or association with each other. (e.g. the union that is the United Kingdom.) Thus he says, "Every judgement makes a double affirmation, or a single affirmation which has two sides. It asserts a connection of different attributes, with an indirect reference to an identical subject; or it directly asserts the identity of the subject, with an implication of the difference of its attributes" (PL 174). For example the statement 'Dogs are mammals' he thinks should be read as saying that any subject which has the property of being a dog has also the property of being a mammal (PL178)

4. Another reading would distinguish between qualitative and numerical identity. The thesis tells us that qualitative identity is compatible with numerical diversity (i.e. many different things can all be red) or numerical identity is compatible with qualitative diversity, (i.e. one and the same thing with different attributes).

5. These four interpretations Bradley's pronouncements of identity-in-diversity each break down the point somehow into an ambiguity — identical in one respect, but different in another. However, it is unclear in the end that this is correct. For we can identify a fifth sense — things which are in the same respect at the same time, both identical and diverse — which however puzzling and seemingly contradictory, has textual support. For in his *Collected Essays* Bradley insists that "Identity and difference... are inseparable aspects of one complex whole. They are not even 'discernible'. If this means you can separate them in idea, so as to treat one as remaining itself when the other is excluded. And the whole is emphatically not a 'synthesis', if that means it can be mentally divided." (CE 295–6) Statement such as this or the puzzling claim that "It takes two to make the same" (PL 141), it is tempting to understand in this fifth sense. Indeed, it might be argued that this is the essence of the doctrine of immediate experience. Bradley thinks that we have immediate experience, but thought breaks this up. Thought's fault is precisely its exclusive 'either-or.' The only way it can combine identity and difference between things is to break them up into

separate aspects, such that they are the same in one respect, but different in another respect.

If the doctrine of identity-in-difference is hard to interpret precisely, it is hard also to locate in proper sphere of application. For although we have considered it in connection with *judgment*, in Bradley's mind the doctrine of identity-in-difference can be easily be extended to other fields. For example, the result can be extended outwards to *inference*, for Bradley sees no sharp distinction between judgment and inference. Inference requires a point of identity, but also a genuine difference. (PL 288, 460) It can also be contracted inwards to units smaller than whole judgments, viz. individual *ideas*. Ideas too can be identity-in-difference. As we saw, there is no simple answer to question whether 'wolf-eating-lamb' is one idea or several. Thirdly the doctrine spreads into the issue of universals. A true universal — what Bradley calls a 'concrete universal' — is also, he argues, a species of identity-in-difference. This I consider in the next section.

2.4 Extension and Intension

It is common to distinguish between intension and extension. And in his chapter on Quantity in judgment Bradley makes two points about this. He argues that any judgment may be taken either extensionally or intensionally (§§11–29). This is obscure point, to an extent withdrawn in Terminal Essay III. But far more significantly, we are introduced to the crucial notion of the concrete universal. This is done via a consideration of the traditional doctrine of the inverse variance of extension and intension of terms. According to this doctrine, the wider the extension the narrower the extension. For example, as we move from dog to mammal to animal we pick out larger and larger classes, but say something less and less specific. Bradley is wholly opposed to this view. He describes it as either "false or frivolous" (PL 170)

It is frivolous in so far as it describes merely a psychological accident about how we may visualise things. As a matter of psychological fact about human mental images, it may be that the more universal they become the more detail we find that we have to drop from them, or, as he puts it elsewhere, the more 'schematic' they become.[29] But this paucity, belonging as it does to psychology, does not make them logically any the thinner. Really this is beside the point, for our concern is not with images, but with logical function

The doctrine is frivolous too, in so far as it reports the uninteresting fact that *if* you arrange ideas pyramidically, subtracting from each layer below to form the layer above, then they will have those properties. (PL 172–4). The more important question is whether our ideas really *are* arranged in this way. And Bradley is quite certain they are not. The traditional pyramid system treats each level of classification as separate, but real qualities can't come apart in that fashion;

[29] Indeed Bradley admits that "There are psychological difficulties as to abstract ideas." (PL 7note).

they determine one another. Each quality is what it is only in its context, not just horizontally, but vertically also; not just in relation to the qualities that surround it, but in relation to the higher level genus they belong to. The sub-classifications of the species are chosen precisely because of their unique and special relation to the character of the genus; they are *ways* of realizing that genus. For example, humans are rational animals. But this distinctive feature of human animals, their rationality, means that they are animals in a different way to the that in which, say, lions or tigers are animals. Their animality is of a modified type. Similarly isosceles, scalene and equilateral are all different ways of being a triangle; blue, red and green different ways of being coloured; loud and soft different ways of being a noise. Were universals formed by mere subtraction, the inverse relation would be correct. But in fact true universals are *concrete*; for content increases rather than decreases the more universal our ideas become.

Bradley's paradigm example of a concrete universal is the individual. "A particular object, and especially a living individual, is universal, thinks Bradley, in the sense that it is spread out in the world, that it brings together under one label a host of elements that from another point of view seem to be different or distinct. The contrast is that rather than, as with the traditional view, bringing together many instances of the same property, we are thinking here of a union of many different properties. The union could be either of diversity at a time (what unites all an individual's properties together as his or hers) or of diversity over a time (the reason why properties held or acts performed at different times are still properties or acts of the very same person), but either way it is a bringing of many diverse elements together into one unified whole. Bradley argued that,

> "[What] exists must be individual, and the individual is no atom. It has an internal diversity of content. It has a change of appearance in time, and this change brings with it a plurality of attributes. But amid its manyness it still remains one. It is the identity of differences, and therefore universal ... So far as it is the same throughout its diversity, it is universal ... [a man] is universal because he is one throughout all his different attributes". (PL 187–8)

Such a union of diverse predicates is to be contrasted with one formed by abstraction. For in the latter case we pull out what is held in common by many different individuals, bringing these shared features under one label. We end up with a union of similar predications rather than of differing ones.

The idea that universals are abstract, no doubt gains much of its support from contrasting them with particulars. And so Bradley is keen to attack these also. Experience is not, as the empiricists think, an encounter with bare particulars which we then overlay with concepts. Atomic particulars are an unreal abstraction from surrounding context as are universal — that without what is as impossible as what without that. On Bradley's account both particulars and universals arises simultaneously through the process of thought on immediate experience, an inevitable pair neither ultimate true not capable of working alone.

2.5 Other Types of Judgment

All that has been said thus far concerns judgment in general, but there are a great many points that can be made about specific varieties of judgment; and after his initial more general discussion Bradley turns, in the remainder of Book I, to consider various different types of judgment.

2.5.1 Negative judgment

One of the most important types (important in the sense that that what he says here affects many other issues) is that which he begins, namely negative judgment.[30] There are two main components to his view.

First of all he argues that negative judgments stand "at a different level of reflection" (PL 114) from affirmative ones. In affirmative judgment we attribute content to reality directly, but for negation there must first be "the suggestion of an affirmative relation" (PL114) which we go on to deny. If I maintain that the tree is not yellow, this must be understood as rejecting the suggestion that it is. What is repelled is "the suggested synthesis, not the real judgment" (PL116) so Bradley is not maintaining that there must be or have been some actual belief which is then denied; his point is a logical rather than a psychological or historical one.[31] In this thesis Bradley is quite at odds with most modern logicians who see positive and negative propositions as standing at the same level. A negation sign reverses the truth value of any proposition, but since not-not-P is equivalent to P, any negative may be rewritten as positive or any positive as a negative.

Secondly, he argues that negative judgment presuppose a positive ground. (PL 114) For there exist no negative facts and, in consequence, there can be no 'mere' or 'bare' denial.

> "If not-A were solely the negation of A, it would be an assertion without a quality, and would be a denial without anything positive to serve as its ground. A something that is only not something else, is a relation that terminates in an impalpable void, a reflection thrown upon empty space. It is a mere nonentity which cannot be real" (PL 123)

Instead negation must be understood as working on the basis of something positive. If a negative judgement is true of some reality it must be because that reality has some positive feature incompatible with whatever it is that is being denied. The tree is not yellow because it is green instead. Although this ground is not made explicit — it is "undetermined" (PL 110), "unknown" to us (PL 117), even "occult" (PL 120) — negation makes sense only if we think of it as in this way grounded in the assertion of some such positive contrary. In this way Bradley reconciles negative judgment with his overall conception of judgment in general as always referring some content to reality. The thought that there could be no

[30] For an excellent discussion of Bradley's view of negation see Stock [1985].

[31] A.J.Ayer [1952, p. 39] misinterprets Bradley in precisely this way.

negative facts has met with wider approval in modern logical thinking, though fewer have followed precisely Bradley's way of securing this.

It might be thought here that Bradley has simply confused the contradictory and the contrary. But that would be unfair. The distinction remains. Insofar as logic needs to distinguish between the contrary and the contradictory, Bradley describes the contradictory as "the general idea of the contrary. Not-A for example is any and every possible contrary of A." (PL 146) Not-A "is a general name for any quality which, when you make it a predicate of A, or joint predicate with A, removes A from existence. The contradictory idea is the universal idea of the discrepant or contrary." (PL 123) But although possible in this way to distinguish them for the most part Bradley does not bother to do so, finding the contrast of little moment. (AR 500)

The reduction of negation to positive contrariety is of great significance in Bradley's thinking. It is, for example, a crucial plank in explaining the rejection of bare conjunction that we encountered above. For Bradley urges that the bringing together in judgment of mere difference is something that may be dismissed as contradictory. Indeed, it is not simply as a species or example of contradiction but, Bradley suggests, the very essence of contradiction itself. As Bradley sees it, contradiction is precisely the attempt to bring together two things with nothing in common. [32]

The suggestion that difference just is contradiction is a strange one — for *prima facie* they do not seem to be the same at all — and in order to properly understand it is necessary to understand that Bradley is exploiting a distinction between appearance and reality.

For Bradley, at the level of ultimate reality there are no genuinely contradictory, that is, genuinely different, predicates. Contradiction is a theoretically limiting case that never in fact happens. [33] This might seem obvious, something we could all agree on, nothing more than the principle of non-contradiction. But we need to see that Bradley's understanding of the matter is more radical than our own. Our view would be that while there are many predicates which *might* contradict, nowhere are they instantiated in any way such that they *do* so. In other words there exist contradictory predicates, but nowhere are they co-instantiated. Bradley's view by contrast is that there simply are no contradictory predicates. Nowhere, even in possibility, can we find two utterly different, or contrary, predicates. This then is a far more radical reading of the principle on non-contradiction: faced with apparent opposites, rather than take the milder course of denying that they really

[32] "If you merely conjoin it with something outside that is different and not itself, this in principle is contradiction." (ETR 227 note) "The 'And', if you take it *simply* as *mere* 'And', is itself contradiction." (ETR 228 note). "Things are self-contrary when, and just so far as, they appear as bare conjunctions, when in order to think them you would have to predicate differences without an internal ground of connection and distinction, when, in other words, you would have to unite diversities simply, and that means in the same point. This is what contradiction means, or I at least have been able to find no other meaning." (AR 505).

[33] "Nothing in itself is opposite and refuses to unite. . . There are no native contraries." (AR 510–11) "In the end nothing is contrary nor is there any insoluble contradiction" (505).

are united, Bradley pursues the more radical option of denying that they really are opposite. [34]

But. of course, there certainly *seem* to be such predicates, a fact which Bradley must explain. He suggests that the appearance of contradiction arises where we take an incomplete or insufficiently developed view of the predicates or the subject in which they are supposed to be united. We take an unduly narrow view of the subject, or exclude from it all internal diversity. In general, predicates merely seem to be contrary when "we have abstracted from them and from the subject every condition of union." (ETR 271)

We seem faced with an insoluble contradiction because the things seem incapable of further analysis. (AR 505) But in fact, suggests Bradley, on spelling out the conditions, apparent contraries or discrepants may turn out to be only differences. "[If] one arrangement has made them opposite, a wider arrangement may perhaps unmake their opposition, and may include them all at once and harmoniously." AR 500–1) The so-called 'opposites' turn out not to be really opposite at all, but simply different.[35, 36]

Some examples may help us here. Bradley himself observes that if we narrow the existence of a thing down to one moment in time then it cannot be in two places, or that if we regard the soul as some sort of indivisible unity then it cannot affirm and deny at the same time. But why insist on such limitations? (AR 506) Alternatively we might consider the fact that a painting can be beautiful (in daylight) and ugly (in artificial light), or the fact that dynamite is combustible (when dry) and non-combustible (when wet).[37] Similarly on a simple (timeless) conception of a thing colour predicates are incompatible, but once we widen our perspective to include duration they become compatible.[38]

Apparent contrariety is thus resolved into difference, but this should not be thought the end of the matter. For (to Bradley) complete difference is no more acceptable, indeed no different from, contradiction. And so Bradley argues that where predicates seem to be quite different — whether they strike as explicitly contradictory or not — this too must be regarded as but an appearance; something which occurs only because we have ignored some wider context or point of union and which on closer examination can be removed. Bradley insists that wherever things seem completely different there will always be in fact a point in common.

[34] "'Opposites will not unite, and their apparent union is mere appearance.' But the mere appearance really perhaps only lies in their intrinsic opposition." (AR 5001).

[35] They are "moments which would be incompatible if they really were separate, but, conjoined together, have been subdued into something within the character of the whole." (PL 149) .

[36] Bosanquet's view on this matter is identical to Bradley's. He says "no predicates are intrinsically contrary to one another. They only become so by the conditions under which they are drawn together" [Bosanquet, 1912, p. 223] "There are places for all predicates; and when all predicates are in their places, none of them is contrary to any other. It is the bringing them together on an inadequate basis of distinction, which is the essence of contradiction and contrariety. . . Contradiction. . . is not a dead fact about certain predicates; it is an imperfection in the organisation of systems." [Bosanquet, 1912, p. 225].

[37] [Ferreira, 1999, pp. 102–4].

[38] [Wollheim, 1959, p. 147].

Again, an example may help: black and cold are different, but it may well be that a thing's blackness and coldness are found to come from a common source.

It is in the merely apparent nature of contrariety and difference that we find the key to understanding Bradley's view of Hegel's dialectic. Dialectic is not in Bradley's understanding,[39] as some have taken it to be, some sort of denial of the law of contradiction, it is rather an exploitation of the point that elements which seem at first irreconcilable may in a wider perspective in fact be combined. (PL 149, 410)

Thus armed with this understanding of the distinction between appearance and reality, Bradley is able to hold that while, at the level of ultimate reality, contradiction is the same as complete difference, at the level of appearance, we may yet distinguish between apparent contradiction and apparent difference. Which of these they appear to us as is a function of the kind and degree of abstraction we make. It is at this level of appearance that Bradley insists that "differents and discrepants should never be confused." (PL 146) Roughly, the narrower our view of the subject the more pairs will seem contrary, but as we widen our view the more they will seem to be different. At the level of ultimate reality, however, these two relations coincide.

But why, it will be asked, even if it can be reconciled with appearances, would anyone *want* to identify contradiction and difference in the first place? The move needs to be understood in terms of Bradley's view of negation. As we have seen Bradley understands negation — the contradictory — in terms of what is discrepant or contrary. Negative judgments stand "at a different level of reflection" (PL 114) from affirmative ones; when we deny something we always do so on the basis of something positive, for there can be no 'mere' or 'bare' denial, no negative facts.

This understanding of negation renders much clearer his denial of genuinely contrary predicates. For in a world were all properties are affirmative, where all reality is positive, there is no intrinsic opposition.[40] It also helps us understand why he wishes to hold that mere difference is the same as contradiction. If the negation of A is taken as not-A, something inherently negative in its being, then its opposition to A seems obvious. But if we remember that it is in reality just some other positive quality B that excludes A, then its discrepant nature becomes much less obvious. It is something different from A which has the further capacity to exclude A. But why, we can not see. (We should note here Bradley's claim that "There is no logical principle which will tell us what qualities are really discrepant." (PL 146) Indeed thinking further on the matter we would seem to be in a strange situation. For holding on to the traditional thought that contradiction and difference are to be distinguished, there would seem to be differences which exclude one another and differences which can be united. But insofar as both were simple differences there would be nothing more that could be said on the

[39] I leave open the issue of how *Hegel* understood it.

[40] In a similar way Leibniz argues that the concept of God can contain no contradictions because God's nature contains only positiove properties.

point. 'Some differences can be united and others can't and that's all you can say.' Yet this is completely unacceptable. There surely needs to be some basis for this contrast, for otherwise there would be no rational basis for the contrast between difference and contrariety, just an arbitrary fiat stating which things may and which may not be combined. Once we have got rid of pure negation, once there is no longer anything intrinsically negative, the contrast between differences that exclude one another and those that may be combined is totally ungrounded. They are just brute relations, and the difference between them equally brute. Bradley's view of negation leaves him with no way of distinguishing between difference and contrariety. His response is two-fold. At the level of appearance a relative distinction may be drawn. But in the last analysis no distinction can be drawn. So he equates them. He takes his theory of negation at face value and concludes that in the end there is no difference between exclusion and difference. He lumps these together as one class.

Bradley's account of negative judgment was one of the aspects of his logic that came under the most sustained attack by Bernard Bosanquet, in Chapter V of his *Logic and Knowledge,* and later on for the second edition of the *Principles*, Bradley modified his views accepting many of Bosanquet's criticisms (PL p.125 note 1; Terminal Essay VI)

First of all the rejection of floating ideas necessitates a slight change. For in the first edition Bradley describes negation is something 'subjective' in so far as what is rejected is merely a 'suggestion'. He says that "[the] process takes place in the unsubstantial region of ideal experiment. And the steps of that experiment are not even asserted to exist in the world outside our heads." (PL120) However, if there are no floating ideas, ideas can not be simply 'suggested' or 'entertained,' all we can do with them is to affirm them.

Negation is, perhaps, more 'reflective,' in the sense that we tend to make assertions before we make denials but such prior awareness is (Bradley acknowledges) irrelevant to the logical point at issue. (PL 665) Instead he seeks to solve the problem by appeal to the distinction which his further reflections on judgment in general had already encouraged him to draw (see above) between reality in general and the 'special subject' of a judgment. He suggests that negation does involve a rejection or denial, but "[the] content which it denies is never excluded absolutely. Far from falling nowhere, that content qualifies elsewhere the Universe." (PL 665) What he had earlier called the 'suggested synthesis' does in fact apply to some reality, and hence does not float, but nothing is actually asserted of the special subject.

Another problem with Bradley's first edition account comes out of his treatment of double negation. It is a generally accepted axiom that double negation is equivalent to affirmation, that, not-not-A is equivalent to A. However, it might be objected that Bradley's understanding of negation precludes him from this principle for the contrary of a contrary need not be the original. (Think of colours: a contrary of red might be green, but a contrary of green might be yellow.)

In the first edition of the *Principles* Bradley attempts to preserve the axiom of double negation by insisting that, although we might use any positive ground, y, to deny that A is b, if we then choose to deny our denial the choice of ground is limited to b itself, since any other ground, z, might be just as exclusive of b as y, leaving us no further on. (PL 159)

However, in the second edition he withdraws this solution deferring to Bosanquet's alternative response. (PL 167 n.25) He now suggests that introducing a special subject into the judgment in the manner just considered above in effect implies a dichotomy — a 'this' as opposed to a 'that' — and thus he urges, "disjunction within a whole is the one way in and by which in the end negation becomes intelligible." (PL 662)[41] The negation is under-laid by an exclusive disjunction (The tree is either x or yellow), such that when we implicitly assert the positive ground (The tree is x) we can then conclude our negative assertion (The tree is not yellow). Viewed in this way negation (like many other forms of judgment) is seen to involve an aspect of inference.

This seems a retraction of his earlier view that negative judgment has a wholly positive basis, for while something that is merely contrary to yellow (green) can be positive, the exclusive disjunction of yellow (not yellow) has a negativity about it. For even if we read it as itself a disjunction of positive options (blue or red or orange, etc.) the assumption of completeness (not anything else) seems negative. Bradley's view is more nuanced. He suggests that perhaps it is better to say that all judgments are both positive and negative at the same time; that there is no sharp line between positive and negative. "Negation everywhere has a ground, not on one side merely but on both sides." (PL 664)

2.5.2 Conditional judgment

Some judgments are openly conditional in form, But further than this Bradley also thinks that some (e.g. the universal categorical) although not openly so are best analysed as conditional, and that there is a sense in which, or a degree to which, all judgments are conditional. But just what is a conditional? How is it to be analysed?

For Bradley a conditional or hypothetical judgment is an ideal experiment. "A supposal is in short, an ideal experiment. It is the application of a content to the real, with a view to see what the consequence is, and with a tacit reservation that no actual judgment has taken place." (PL 86) The judgment says that if you entertain one thought you must go on and entertain a second, it asserts a relation between two possible judgments without in fact making either of them. It is in this sense "a subjective operation" (PL 86) something taking place in thought only.

But how can this be squared with the claim that Bradley has already made so much of, that all judgement asserts something of reality? The answer to this is that conditionals assert that reality has a certain 'latent' or 'occult' quality in virtue of which antecedent and consequent are connected. (PL 87, 88) We imply that the world is so arranged, the laws and conditions so set up, that if the one

[41] "Negation. . . implies at its base a disjunction which is real." (PL 666).

were to happen the second would follow. This is not stated explicitly — "The fact that is affirmed as an adjective of the real, and on which depends the truth and falsehood, does not explicitly appear in the [hypothetical] judgment." (PL 87) — but it is what grounds our assertion.

Understood in this way hypothetical judgments also may be regarded as involving inference, as in effect condensed arguments. Indeed, thinks Bradley, it is in hypothetical judgment that inference first definitely emerges. Supposition of the truth of the antecedent, together with all the relevant circumstances and laws of nature, is supposed to entail the truth of the consequent. A conditional is true if the argument which it abbreviates is sound.[42]

2.5.3 Disjunctive judgment

Turning from hypothetical to disjunctive judgment, to which he does not give a very great prominence, the first point to note is that, unlike modern logicians, Bradley insists on taking 'or' exclusively. A disjunctive judgment is true if and only if one of its disjuncts is true and all the others are false. He says "I confess I should despair of human language, if such distinctions as separate 'and' from 'or' could be broken down." (PL 134) [43]

Disjunctive judgment presents something of a dilemma for Bradley, for neither of the most obvious ways of taking it will work. It is thinks Bradley intuitively obvious that there are no such things as disjunctive facts; nothing can exist in reality in the form b or c. He says, "[This] mode of speech can not possibly answer to real fact. No real fact can be 'either — or.' It is both or one, and between the two there is nothing actual" (PL 129; cf. 46–7) But, if not straightforwardly categorical, neither thinks Bradley can disjunctive judgments be understood as straightforwardly hypothetical. They cannot simply be reduced to a combination of four conditionals — if b then not-c, if not-b then c, if c then not-b, and if not-c then b — for that leaves out of account on what basis they are thus combined; why they are thought together to exhaust the field. (PL 128–9)

Disjunction says Bradley "if not quite categorical, is certainly not quite hypothetical. It involves both these elements." (PL 137) The way this works is as follows. The judgment 'A is b or c' asserts that A has a quality x shared by both b and c and not by anything else. (PL 130) This is the 'ground' of the disjunction. Asserting quality x assumes a sort of omniscience (PL 137). For quality x tells us that each excludes the other, and that these are the only possibilities, allowing us to erect hypotheticals on its basis On its basis the judgment can then be read as asserting two pairs of hypotheticals: (i) if A is b it is not c, and if A is c it is not b, (ii) if A is not b it is c, and if A is not c it is b. (PL 136) Disjunction is thus "the union of hypotheticals on a categoric basis." (PL 131). It is worth noting the similarity with between this account of disjunction and Bradley analysis of

[42] [Allard, 2005, p. 87–8]; [Griffin, 1996, p. 198].

[43] Bosanquet makes the same assumption. For detailed discussions of this point see [Allard, 2005, pp. 114–9]; [Crossley, 1878, pp. 115–23].

conditionals considered above. Both types of judgment are read as asserting a hidden or latent quality on the basis of which a hypothetical judgment is erected.

2.5.4 Collective judgment

Unlike modern logicians, Bradley distinguishes what he calls collective judgments (such as 'Everyone in this room is wearing shoes' or 'All of the sheep have been dosed') from universal categorical judgments (such as 'All animals are mortal' or 'Equilaterial triangles are equiangular.') Where the latter uses 'all' in a wholly unrestricted sense, in the former case 'all' refers to "a real collection of actual cases" (PL 82, cf.355–7) He analyses them as conjunctions: 'This sheep has been dosed,' 'This sheep has been dosed' and so on.

By the time of the second edition, Bradley had become unhappy with this account and withdrew it, deferring to Bosanquet.[44] The problem he says is "that all counting presupposes and depends on a qualitative Whole, and that Collective Judgment asserts a generic connection within its group. Hence no mere particulars can be counted" (368 note 2) For example, we never count just people, books, or cars, but rather people *in the boat*, book *in the pile*, or cars *in the queue*, but in doing so we make them all instances of a type or kind. However since universal judgments also pick out members of a type or kind (e.g. all members of the type: animal) this is to blur the difference between collective and universal judgment.

2.5.5 Modal judgment

In Chapter VII of the *Principles* Bradley turns to discuss the modality of judgments, using the terminology 'assertoric,' 'apodictic,' and 'problematic' to refer respectively to judgments of actuality, necessity and possibility judgments. For Bradley, possibility and necessity as such have no existence in the real world. All that exists is the actual. "Reality in itself is neither necessary, nor possible, nor again impossible. These predicates (we must suppose in logic) are not found as such outside our reflection." (PL 212)

Such a view is not uncommon in modern times, and many have concluded from it that necessity is matter solely to do with words and definitions. This is the view of necessity as analytic (in the Kantian sense.) And in saying that they "are not found as such outside our reflection" Bradley might seem to be endorsing that line. But in fact he rejects this account on the grounds that meanings vary between people and over times. Specifically, as knowledge grows, our meanings become enlarged and what was synthetic becomes analytic — "[what] is added today is implied tomorrow." (PL 185). Analyticity is an inadequate basis to explain modality.

So how, then, are modal judgments to be analysed? He begins by rejecting the idea that modality affects only the mode of an assertion and not its content, for

[44] PL 110 note 37, 368 note 2 referring to Bosanquet [1885, p. 76ff] and Bosanquet [1888, 152ff, 209ff].

on his scheme there is only one way of asserting and that is the assertoric. (PL 197) So instead of one content $S — P$ declared in three different ways, we assert of reality itself ('It is true that. . . ') one or other of three slightly different contents, 'actual $S — P$', 'possible $S — P$' or 'necessary $S — P$.' (PL 198).

As a class modal judgments present nothing fundamentally new. Bradley argues that assertoric judgements are just categorical ones, while both apodictic and problematic judgments are species of conditionals. (PL 198) "What may be and what must be involve a supposition." (PL 199) he says.

To begin with necessity, Bradley suggests that a thing is necessary when it is, or is said to be, *because of* something else. "Necessity carries with it the idea of mediation, of dependency, of inadequacy to maintain an isolated position." (PL 199) "Where the necessary [judgment] asserts strongly, it borrows its strength from a concealed assertorical." (PL 202) For Bradley necessity is a relation of consequence between judgments, not a feature of judgments taken singly. The judgment of necessity is to be read as asserting or implying a relation of logical consequence between the facts which it states and some further unstated conditions on which they depend. So 'S must be P' really says something like 'because R is Q then S must be P.' One difference which may be admitted between a more usual conditional judgment and one of necessity is that we say not simply that *if* something holds then something else must, but we further imply that those conditions *do* in fact hold. However, thinks Bradley, this is a difference of *knowledge* and the *necessity* in each case remains the same. (PL 201)

Judgments of possibility are a sub-species of judgments of necessity, like them they say that something would exist if certain other things were the case. But where judgments of necessity imply that *all* of their conditions are met, judgments of possibility imply only that *some* of them are. (PL 202) We are stating that under certain conditions a things must occur and that at least some of those conditions obtain, but we can not say that they all do.

As stated, there are no modal facts and so, with respect to both possibility and necessity, the implied relations are ideal rather than factual, ones holding between ideas rather than facts. As proof of this Bradley draws attention to the point that there exist hypotheticals with impossible antecedents and consequents. 'If two were three then four must be six' is the example he gives (PL 201). Nevertheless, he is keen to demonstrate that both have "a basis in fact and depend[s] upon experience" (PL 207) and thus that they are in accordance with his general theory of judgment. "All necessity affirms a real ground explicit or implicit" (PL 207) he insists. Likewise, he thinks, possibility must always be 'real' possibility — "possibility apart from or antecedent to the real world is utter nonsense." (PL 203)

In the last analysis, as we move from logic to metaphysics, in so far as necessity is a matter of one thing holding *because* of another, Bradley's view that everything is connected to everything else, that there are no mere conjunctions, amounts in effect to the view that ultimately everything is necessary. No judgment is, at the end of the day, ultimately or irreducibly contingent; these are rather indicators of a still incomplete analysis

2.5.6 Probabilistic judgment

In the final section of chapter VII Bradley deals briefly with judgments of probability. As with modality, these can not be taken as directly descriptive of reality — "[no] statement that we make about probabilities can, as such, be true of the actual facts." (PL 217) — nevertheless they do make an affirmation about reality. He is able to maintain this because he bases statements of probability on disjunction for which, as we have already see, he believes it is possible to find a categorical basis.

For instance, in considering the throw of a dice, we make a number of categorical assumptions about the world (about the dice and how it will fall) to the effect that there must occur one of only six possible outcomes: 'Reality is such that there will occur 1 *or* 2 *or* 3 *or* 4 *or* 5 *or* 6.' From this point Bradley follows the classical theory of probability and, assuming each outcome to be equally likely (PL 218), shows how we may calculate chances as the ratio of favourable cases to the total number of outcomes.

3 INFERENCE

3.1 Context-setting

We have now seen how Bradley deals with judgment, both in general and in its various different species. Moving on from this topic, which occupies Book I of the *Principles*, Bradley proceeds, in Books II and III, to consider the question of inference.

However it must be noted from the start that Bradley finds no great gulf between judgment and inference. For judgment itself is an inferential process. Within every judgment there are deeper levels of meaning and implication which are inferential in structure. Sometimes (as we have already seen in the cases of synthetic judgments of sense, hypothetical judgments or negative judgments) the analyses which Bradley offers make this latent structure quite explicit, but it is something that holds true, in some measure, for all judgments; since all judgments are made subject to conditions.

Bradley locates two activities, analysis and synthesis, both of which he holds take place simultaneously in both judgments and inferences. This observation might lead us to wonder if he is not simply removing the distinction between judgment and inference; reducing them to a single class. But in truth his position is more subtle than that. In their explicit forms Bradley is clear that there remains a distinction between judgment and inference. In a judgment of perception there is no datum, its starting point is *for the intellect* nothing. (PL 479) But it belongs to the essence of inference to get or draw its product from some cognitive premise or datum (PL 437). In this sense "[explicit] judgment comes before explicit inference." But if we consider their earlier stages, he continues, they "emerge together" as "two sides of one act," process and product together. "The earliest judgment will imply an operation, which, though it is not inference is something like it; and the earliest reasoning will begin with a *datum*, which though kin to judgment, is not intellectual" he says. (PL 481)

This understanding of the relationship between judgment and inference might seem to raise as many questions as it answers. For what is meant by 'earlier' or 'develops into' here? At times Bradley appears to suggest that the difference is just a question of psychological history, but more fundamentally his point is a logical one. To think is to employ or assert ideas. But there are only two ways to use ideas, either directly (judgment) or indirectly (inference). These modes are different, but neither is possible without the other, and if one appears to take priority over the other this is just a matter of their differing degrees of logical articulation.

Perhaps nowhere more than in his discussion of inference does it become manifest that Bradley is not a formal logician. It is common to distinguish between formal and material inferences — between inferences which do not depend on the particular subject matter of what their propositions describe and those which do — and to hold that logic considers only the former; or at least that a separate branch of the subject, inductive logic, is needed for the latter. However, Bradley places no such restriction on himself. He includes under the heading of inference any extension beyond what is simply given to us in perception, whether formal or material in character. Inferences such as 'Charles I was a king, he was beheaded, so a king may be beheaded' or 'Today is Monday so tomorrow is Tuesday' or scientific inferences cannot in any straightforward way be rendered formal but are not for that reason excluded from consideration; for the subject matter of logic is simply *true thought* and therefore includes any systematic movement in thought from one truth to another. Of course we may not go *outside* of our premises to establish any conclusion, for the activity then would simply not be *inference* — which is essentially the derivation of a result from a source. (PL 521) But inference is a matter of developing connections and relations to ideas beyond that source. And these, though internal to our initial content, may lie hidden or implicit within it; outside our explicit cognitive apprehension. For this reason valid inferences can not be captured in merely formal structures.

This is not to exclude absolutely considerations of form. Bradley freely admits that inferences fall into *types*; we cannot reason from mere particulars, and so any inference must in principle be applicable in other imaginable cases. (PL 522) But no distinction which we might draw between form and matter could ever be anything other than relative, such that on a different framework what was material could be presented as formal. (PL 532) Moreover, thinks Bradley, such distinctions are in the end rather useless for, even if we were to make and stick with them, they can never provide us with a fixed set of forms or models adequate to cover all valid inferences.

> "It is impossible that there should be fixed models for reasoning; you can not draw out exhaustive schemata of valid inference" (PL 268)

> "...no possible logic can supply us with schemes of inference. You may have classes and kinds and examples of reasoning, but you can not have a set of exhaustive types. The conclusion refuses simply to fill up the blanks you have supplied...the attempt to provide for these endless varieties is...irrational and hopeless." (PL 521)

That there could be such a fixed schedule is, he says, simply not possible because in the end we must always be defeated by "the endlessness of the field." (PL 268) Taking a general conception of inference as the bringing together of various data such that some special relationship is observed between them which we then draw as a conclusion, Bradley points out that "the number of special relations has no end," (PL 268) and hence that there would be need for an infinite number of inference schemes to parallel them. In other words, no collection of models could ever be complete. Bradley does admit that we may find some "principles which are tests of the general possibility of making a construction," (PL 268) but these only ever hold within certain limited categories of inference and only give us general guidance — they can not, for example, identify specific new relations. The way in which an inference develops has a fluidity which defies regimentation. On the one hand it leaves room for individual judgment, inspiration and insight. (PL 268; 273 n.8) To notice connections we need a good eye. Inference is an art, not a science. And on the other hand there is also the capriciousness of the world itself. Bradley makes this point by comparing real and ideal experiments.

> "It is clear, I think, that when trying experiments in the actual world by combining and dividing things, or by drawing upon paper, we may be surprised by qualities which we did not anticipate. And the same must be true of ideal experiment." (PL 397)

At bottom, the truth and reality of our reasoning stems not from classifying it under some general authorised pattern, but rather consists "in the development of an unbroken individual identity to a result which is its own and which meets its particular requirement." (PL 618) In other words, although inference is always general because it deals with ideas, what it aims to capture and what drives its inner working is the concrete particularity of the world itself.

Rather than distinguish different types of inference, or separate out correct from incorrect inferences, as many logicians have sought to do, Bradley's real concern in the *Principles* is to establish the general nature of inference.

With the scene thus set, as we turn to look at Bradley's discussion of inference itself, perhaps the first thing to strike us is its length. Bradley devotes at least twice the amount of space to inference as he did to judgment. It has been suggested that it is because he finds inference more common than do other logicians that he devotes so much space to it,[45] but the length of his discussion is also a function of the way in which he proceeds. Bradley's discussion covers two Books, each itself with two parts. First he outlines a basic theory, then he attacks rival theories. Next, identifying weaknesses in the initial account, he moves on to a better version of the theory, before finally considering the question of the validity of inference. For clarity of presentation, however, it is perhaps easier to the first and third parts of his discussion together.

3.2 The basic account of inference

Bradley begins his discussion of inference with three adequacy conditions on any acceptable theory, although perhaps rather than three distinct points thee may better be regarded as three aspects of a single idea. First of all he notes that there is a difference between reasoning and mere observation; reasoning is something active that we *do,* not something passive that happen to us. If a truth is inferred, it is more than simply seen. (PL 245) Inference is more than one thought prompted or followed by another, with the conclusion coming to us irresistibly from without, as associationism or Platonism (in their different ways) would have it. But although we *make* inferences, neither is this some arbitrary or capricious act on our part, rather we are lead or constrained in doing so by reality itself; we *discover* rather than invent the result of our inference. Secondly, he notes that in inference we pass from one truth already possessed to a further truth. The conclusion is thus not self-existent but in an important sense dependent on its premise or premises. (PL 245) Thirdly, inference must convey some new piece of information. (PL 246) It must do more than repeat a part or the whole of the premises. Thus 'A therefore A' or 'A and B therefore A' do not count as inferences. In making this requirement Bradley differs from modern logic which would regard patterns such as these as valid inferences, albeit dull and pointless ones.

Bradley then gives eight examples of inferences which any adequate theory must accommodate. (PL 246) Both here (and again as he later develops the theory) Bradley uses such intuitions about what counts as an inference to act as a litmus test for the adequacy of any theory since, he argues, we are more certain that these are indeed inferences than we could ever be of any theory that might say otherwise.

On the basis of these conditions and examples, Bradley then proposes a general model of inference.

> "Every inference combines two elements; it is in the first place a process, and in the second a result. The process is an operation of synthesis; it takes its data and by ideal construction combines them into a whole. The result is a perception of a new relation within that unity." (PL 256)

[45][Manser, 1983, p. 152].

By way of illustration Bradley shows how a number of typical inferences may be placed under this scheme. For example, we may infer 'This man is a logician, this man is a fool, therefore a logician may be a fool.' (PL 265). Here two attributes are brought together in a single individual, and a new relation observed to hold between them. Again we infer, 'A is to the north of B and B west of C, therefore C south-east of A' (PL 266). Here the inference consists in bringing together two spatial relations and thereby allowing us to uncover a third. The examples are listed under five general types (PL 264) but that taxonomy does not pretend to completeness for, as we have already seen he thinks such exhaustive enumeration impossible. (PL 266)

Although there can be produced no list that would exhaust every possible inference, there are, Bradley holds, two general conditions which may be laid down to hold on them all. First there is the necessity of an identical point. He says, "It is impossible to reason except on the basis of an identity." (PL 285) Bradley's idea here is that premises can only be combined where there is a 'common point" (PL 285) or 'centre of identity' (PL 389) between them. And he means here an actual identity, not merely some similarity or likeness. (PL 286) There must, of course, exist also difference between them — mere repetition would be pointless — but were they wholly different it would simply not be possible to think them together in one construction. (PL 288) In large part Bradley is repeating here thoughts we considered above when we looked at the impossibility of judgments of bare conjunction; what holds of two elements in a judgment holds equally of two premises in an inference.

Bradley's second condition — the necessity of at least one universal premise (PL 285) is, when one reflects upon its basis, really not much more than a reapplication of the first. For he defends it on the grounds that if there must be a term common to the premises, a single content in multiple different context, that is universal, and thus so will be at least one of the premises. (PL 294) Taking the premises 'A precedes B' and 'B precedes C' we might doubt that this is so, but we should note that Bradley is using the term 'universal' in a rather unusual way here, a use already encountered in his notion of the concrete universal. He holds that a "universal judgment is one that holds of any subject which is a synthesis of differences. It is a proposition the truth of which is not confined to any single this." (PL 295)

Bradley first attempt provides him with a good initial account of inference, but when he returns to the subject in Book III of the *Logic* he finds a number of deficiencies with this basic model. He notes that inference does not always result in a new relation, that sometimes what we perceive as the conclusion is instead a new quality. In this connection he uses the example of sailing around some land and, realising that one has arrived back at one's starting point, concluding that the land is in fact an island. (PL 396) He worries too about mathematical inferences. Arithmetical operation "produce new results; they are ideal operations which give conclusions, and justify what they give". So "they are palpable inferences." (PL 401) But a sum like $5 + 5 = 10$ "establishes no relation between the terms of the premises. On the contrary the relation, which appears in the conclusion, has one terminal point which never appeared in the data at all." (PL 404) A third type of inference to slip the net is, he suggests, inference with only one premise. (PL 407) For example, we may infer 'If A then C.' This might be shorthand for 'if anything is B it is C, but here A is B, and therefore it is C,' but no such premises need ever come before the mind. Dialectical inference too seems to escape the model. (PL 408f) This he describes as a passage of thought in which one idea is felt insufficient and supplemented by a contrary one. In allowing both sides of dialectical scheme here to be positive, Bradley disagrees with more standard interpretations of dialectic, which emphasise negation and call for a unity of opposites. (PL 410) But, however one takes it, it is a pattern of inference that cannot be accommodated on the original model. Last of all Bradley also rethinks the crucial role of identity in the operation of inference. Where before he had insisted there must always be a common centre, his account is modified to accommodate inferences with no explicit centre. He continues to hold that a centre can be found for every inference, but he now allows that instead of being explicitly stated the centre of identity may be implicit, something we have to extract by a process of analysis or synthesis. For example, in comparison we can only say that A is like B by extracting from A and B some property x which they both share. (PL 461)

Reflecting on these deficiencies in his first account Bradley proposes both a new criterion for inference and a new model of how it works. The criterion is *necessity*. He suggests that "wherever we have necessary truth there is reasoning and inference" (PL 394) [46] The thought behind this criterion is that in inference we are always given some 'because' to our why, some reason why what is the case must be the case. And that, as we saw above in his discussion of modal judgments, is precisely the underlying significance of necessity.

Bradley explains his new model of what happens in inference in terms of what he calls *ideal experiment*. He loosens his initial specification suggesting that in inference,

> "[no] matter what the operation may be, there is always some operation. This operation is an ideal experiment upon something which is given, and the result of this process is invariably ascribed to the original *datum*." (PL 431)

The term 'ideal experiment' conjures up the more modern term 'thought experiment,' with which it has indeed much in common. It is a mental operation in which we take thoughts or ideas and let them evolve under their own logic. It differs from mere imagination in that the result determined not by us but by the way things are. It is even somewhat unpredictable. Yet unlike a concrete experiment in the world that involves particular things, "This process is ideal, in the

[46] "Every inference is necessary, and the necessity of the process can be formulated as an universal truth" (PL531)

sense that it advances on the strength of a connection between universals." (PL 441)

Bosanquet much objected to the notion of 'ideal experiment'[47] and it has to be admitted that the term is perhaps too vague really to tell us anything. Pretty much any sort of operation could count as one. A little more light is shed when in Book III/Part I/Chapter VI — 'The final essence of inference' — Bradley suggests that the operations of such ideal experiment in the end come down to either analysis and synthesis, or some combination of the two. (PL 455) In some inferences synthesis dominates (for example, in construction around an identical centre resulting in our perception of a new relation or quality (PL 451) in some analysis dominates (for example, when we abstract from being burned to the conclusion that it was the fire that burned (PL 452) while in others we find both operations (for example, in the standard inference pattern where we go from $A - B$ and $B - C$, to $A - B - C$, and then to $A - C$, we see first synthesis and then analysis. (PL 450) Although it is useful to draw a distinction between them, Bradley holds that there is no ultimate difference between analysis and synthesis; they are but two sides of a single coin. (PL 470) We cannot break a whole into elements without at the same time relating the elements together, nor yet can we bring things together without recognising their distinctness as elements of one whole; and this fact, he says, explains the apparent paradox that knowledge advances from the abstract to the concrete. (PL 474) The more we leave behind perception's partial and distorted initial encounter with reality, allowing our thought to bring in the its surrounding mass of contextual detail, the more concrete our product becomes.

As with the theory of judgment, so it was with inference that there were certain changes of view between the first and the second editions of the *Principles.* As well as the numerous footnote comments, Bradley wrote a complete new essay on inference for the second edition. Here a new way of describing inference (which had occurred in the first edition (e.g. 393–4) but in a more minor role) comes to the fore. "Every inference" Bradley says "is the ideal self-development of a given object taken as real." (PL 598) The term 'object' here is used in a very loose sense to cover any set of facts or conditions, expressible as premises, which the mind takes together as one object of thought. This presents itself to us in ideal form — that is to say, it comes before the mind — as something real, as one part of the actual universe. But always reality is more than it seems to be; "what in any particular case this object is, and how its limits really are defined, cannot be taken as appearing in those forms of language which serve as its expression." (PL 598) Below its surface it carries with it traces of the wider whole to which it belongs and with which it is continuous. In inference, we penetrate to these deeper levels, or the object reveals its hidden depths to us. (Bradley sees no fundamental difference between these two for, driven by an inner logic which needs must lay bare the structure lying below what is explicitly presented to us, the process is one of 'discovery' rather than 'creation.') But because the object only is what it is insofar as it is an element in a wider connected whole, in thus revealing itself,

[47] [Bosanquet, 1885, p. 288ff].

it takes us beyond our original and explicit starting point to a new insight which forms the conclusion of our inference; even if, in another sense, all the time, it has done no more than develop itself.

3.3 Mistaken views of inference

If in its positive presentation Bradley's theory of inference can begin to seem a bit vague, more teeth are found if we consider those elements in what other writers have said about inference which he *opposes*. Bradley begins this critique with traditional syllogistic logic. He strongly rejects the claim that all valid arguments can be put in syllogistic form, together with the related notion that in all inference we find a major and a minor premise.

Bradley has a twofold strategy. In the first place he simply points out that there exist valid inferences falling outside this scheme. (PL 248) Indeed only one of the eight examples offered is, he thinks, really syllogistic. (PL 247) But he has deeper worries too. He complains that the principle of class inclusion (upon which syllogism is based) contains a *petitio*. For the statement 'All men are mortal,' read as a collection, already contains the conclusion we go on to draw from it, that 'John is mortal.' It therefore yields no new knowledge, violating the third criterion on the adequacy of inference. (PL 248) The problem is avoided if we read the statement as a connection of attributes. (PL 249) But if this preserves the validity of the inference, it can no longer be regarded as syllogistic in form.

Later on Bradley takes the case further. He admits that it is, with some contrivance possible to force, or as he puts it to "torture," most inferences into some sort of syllogistic pattern. (PL 526) But to oblige them to lie in such a "a bed of Procrustes" (PL 267) can only distort how they work producing the most unnatural results. For example, you can try to turn the general principle of any inference, the basic type to which it belongs, into a premise and proceed to run the argument from that point. But this is not really what is going on when we make that inference; for such a major premise will be nothing but an abstracted repetition of the inference itself. (PL 525-6) The rule according to which an inference proceeds is not itself required as a further premise of the inference. (Bradley's point here is similar to that made by Lewis Carroll in his famous paper, 'What the Tortoise said to Achilles.')[48]

Not all of Bradley's readers were wholly impressed by his critique of syllogism and, whatever defects he may have shown it to have, Bosanquet objected that Bradley had not really found a satisfactory alternative theory.[49] But turning from traditional 'deductive' logic to the logic of 'induction' Bradley's arguments became considerably stronger and more influential. Indeed, he himself regarded his differences with the syllogism are minor compared next to those he has with empiricist logicians, such as John Stuart Mill. The *Principles* devotes three chapters to attacking different aspects of Mill's logic: the association of ideas, Mill's method

[48] [Carroll, 1895].
[49] [Bosanquet, 1885, p. 314].

of argument from particulars to particulars, and his inductive cannons. We will consider each of these in turn.

The doctrine of the *association of ideas* (pioneered by Locke and developed into something of sophisticated orthodoxy by subsequent figures such as Hume, Hartley and Mill) is an empiricist attempt to explain the principles that cause one idea to follow upon another.[50] According to the doctrine all human thought, however complex, may be derived from conditioned connections between simple sensory elements, based on similarity and/or repeated juxtaposition in space and time. Where things A and B, similar or continguous with one another, are repeatedly experienced together this creates in the mind a habit or tendency such that whenever we next experience or think of A, even without B, B nonetheless is recalled in our mind. Perhaps the best known example of this way of thinking is Hume's use of 'constant conjunction' to explain our idea of causation or necessary connection, but for the associationists theirs was an entirely general theory, able to account for all of our thinking, even inference. Mill for instance argues that in syllogism, "the minor premise always affirms a resemblance between a new case and some cases previously known; while the major premise asserts something which, having been found true of those known cases, we consider ourselves warranted in holding true of any other case resembling the former in certain given particulars."[51]

Bradley does not attempt to deny that association *occurs* (PL299) — for whether or not it does is a matter of empirical fact — what he disputes is the empiricists' *explanation* of it.

The fundamental problem is that the images or ideas of which the theory speaks are all particulars, individual dateable private psychological events. But such items can never be associated, for by their nature they are fleeting and can never recur. "There is no Hades where they wait in disconsolate exile, till Associationism announces resurrection and recall." (PL 306) Talk of recalling a connected idea is only plausible to the degree that we forget this, for any new occurrence will always be in fact a different idea. Bradley's point is that where such associative chains of thought occur the connections must be, not between particular ideas, but between *universals*. Far from explaining the origin of our general ideas association would not even be possible unless we first had them. "I maintain that all association is between universals" he insists "and that no other association exists." (PL 307) At least the classical logic recognised this fact, although the debt he explicitly acknowledges for this view is to Hegel. [52]

This general worry is worked in detail for each of the two associationist principles. The principle of contiguity, in the end, amounts to no more than the thesis that where ideas have been contiguous, ideas which are like them tend to excite one another, (PL 316) in other words to reduce to an instance of the principle of

[50] [Ferreira, 1999, pp. 236–9]; [Ferreira, 1996, pp. 298–84].

[51] [Mill, 1843, Book II, Chapter IV].

[52] "Association holds only between universals. This doctrine. . . I owe to Hegel." (346; an attribution repeated at PL 515 n.1)

resemblance. But that principle itself is unable to stand. It claims that current ideas have the power to call up past ones with which resemble them. But resemblance must be perceived and how can something present resemble something absent? “If the relation does not exist until the idea is called up, how can the idea be called up by the relation?” asks Bradley. (PL 321)

Bradley’s general opposition to the association of ideas is, of course, but one further application of his general anti-psychologism. In his view inference depends on logical relations, not causal or psychological one, but these are something which associationism can never capture. In the end it reduces conceptual connections to the arbitrary frequencies of experience, something which wholly bypasses the content of the ideas themselves. Such a view finds its most graphic presentation in Hume’s psychological atomism, where all of our ideas are distinct existences between which the mind never perceives any real connection. Ideas are conjoined “by the agency of chance or fate” rather than any real bond. (PL 302)

It might be asked whether Bradley’s rejection of psychologism as he finds it in Mill is really compatible with his own view of inference as a process of ideal self-development, for does not that amount to saying that there are ‘laws of thought’; laws according to which ideas evolve? But this objection misses its mark for his aim is not to deny that there are discoverable rules determining the way in which our ideas succeed one another; his point is rather to insist that they thus evolve on the basis of *their* content, not on the basis of arbitrary external connections, as the associationists would have it.

That Bradley follows these objections to associationism with an extensive critique of Mill’s account of inductive inference shows that his conception of inference was not one of simple formal deductive inference. But unlike Mill, who saw induction as the only legitimate type in contradistinction to deduction, Bradley does not draw a sharp distinction between the two modes.

There are two aspects of Mill’s theory of inference to which he takes exception. First, there is Mill’s belief that inference is properly the passage in thought from particulars to particulars. Bradley spends but little time on this idea, for its incorrectness follows immediately from what has just been said about the association of ideas. If a universal is always needed to connect our ideas then, “[to] reason directly from particulars to particulars is wholly impossible.” (348) Bradley suggests that the factors that lead Mill into this error were two-fold. In the first place he was mislead by his general conception of inference into making a false dichotomy. He regarded himself as faced with a choice between treating inferences either as question-begging syllogisms which already contain their conclusions, or as movements from one particular idea to another. For reasons we shall consider below, Bradley regards this dichotomy as a false one. But there is a second source to error, in so far as Bradley freely admits that there are some inferences which might *look* as they go from particular to particular. He considers the child who falsely infers from the fact that his dog wags its tail when he is happy, that the cat which swishes its tail is happy also. However, rather than take this at face value suggests that it may be given an alternative analysis, as an argument based on analogy. For what is really happening here is that the child is generalising; taking from the dog certain qualities which he transfers to the cat on the basis that they are analogous — in the case, of course, a false analogy. (PL 351)

Leaving alleged inferences from particulars to particulars, Bradley attacks a second "cognate superstition," (PL 355) the notion that inference may go from particulars to *universal truths;* that is to say, the principle of induction, as most famously formalised in Mill's canons of inductive inference. Bradley has no sympathy at all with these formulations and insists that their methods, although professing to start from mere particulars, in fact imply universals. (PL 359) He further argues that, rather than properly inductive they can all be seen as examples of a single quite different pattern of reasoning, inference by elimination; "they fix a relation between certain wholes, and then, by removal of parts of each, establish this relation between the remaining elements." (PL 363) Running through each of the five cannons in turn in order diagnose their individual faults, Bradley does admit that there is a sense in which all of them work if, that is, you add to them some proviso of the sort, 'in this particular case,' but thus to rescue them is he complains utterly to destroy their generalising power as types of inference.

3.4 The validity of inference

In the final chapter of the *Principles* Bradley addresses the question of the validity of inference. That he should even ask about this shows how different his conception is from that of the modern logician who, for all his efforts to define formally in just what it consists, takes the validity of inference in general as unproblematic. As Bradley sees it, the question can be asked at two levels, logical and metaphysical. (PL 551) We can take these in turn.

At a logical level we ask whether premises really do prove their conclusions. The problem — which he later refers to as "the essential puzzle of inference" (PL 599) — is that inference must yield new knowledge. But in that case it seems, through the mind's operation on its starting material, that we have altered it. If we *make* the conclusion we can not also claim to have *found* it. And so we face a dilemma: "If nothing was altered, then there was no inference; but if we altered aught then the inference is vicious." (PL 554) In other words, inference must yield new knowledge but the requirement of validity seems to prevent that.

Now, this was not a new problem. It had, as already noted above, attracted the attention of John Stuart Mill. Mill argued that syllogistic inference can not go beyond its premises. Its legitimacy derives entirely from the fact that it already asserts its conclusion in its premises. For this reason, he felt all that was left was the direct inference of one particular fact from another. But Bradley was unable to accept this answer, and sought instead to find a way in which inference could indeed yield new knowledge without sacrificing its validity.

He suggests that, in part, the solution we need is "to regard our reasoning as simply a change in our way of knowing." The change is not in the object, but in ourselves. "If, by altering *myself*, I am so able to perceive a connection which before was not visible, then my act conditions, not the consequence itself, but my knowledge of that consequence." (PL 554) Every inference modifies its own starting point. But I play no part in that. "My vision is affected, but the object is left to its own development." (PL 555)

Yet such a breach between the psychological process of inference and the objective validity of the inference can be only half the answer. For the mere correction in our vision, a mere shift in attention, hardly amount to an inference — as Bradley notes in the second edition. (PL 573n7) Even if we play no part it in, and are thus freed from blame, the fact that the premises themselves evolve under an internal logic challenges our belief in the validity of the inference. For the conclusion becomes different from the premises. It is this second aspect of the puzzle which comes to the fore in the extended discussion of inference which Bradley added to the second edition of the *Principles*. He suggests there that the basic solution to our puzzle lies in "the double nature of the object." (PL 599) The inference starts with a special object, but that object is more than itself. An element in a wider whole, it has an identity that points beyond itself, and which develops along its own natural lines through the process of inference. What we uncover in the inference is not something new smuggled in from outside our starting point; it is "nothing beyond the intrinsic development of its proper being." (PL 600). In other words, the conclusion is not asserted in the premises (as Mill had said), but it is thinks Bradley implicit in them. If conclusions are contained in premise it is, to use a phrase from Frege, "as plants are contained in their seeds, not as beams are contained in a house"[53] This is what Bradley means by the term 'self-development'. There must be *development* or else there is no new knowledge, but it must be *self*-development or else the inference would be invalid.

In this way the key to the validity of inference lies in *coherence*. As the premises unpack their latent connect, they connect themselves with the rest our beliefs and thus inferences (like judgments, as we shall see below) are judged by the overall coherence and comprehensiveness they are able to generate in our system of beliefs. The ideal standard here is that of perfected or complete knowledge, where everything connects to everything else. In Bradley's own words, "Our actual criterion is the body of our knowledge, made both as wide and as coherent as is possible... And the measure of the truth and importance of any one judgment or conclusion lies in its contribution to, and its place in, our intelligible system." (PL 620, cf.489) Although this allows us to give an affirmative reply to our first question of the validity of inference, it is a reply that must be qualified. In the second edition, he insists that every inference is fallible (PL 619-21). The reason for this is that each inference is individual and if, from on perspective one given inference may seem to increase coherence and comprehensiveness more than another, from an even wider perspective our judgment may be reversed.

In the second part of the chapter Bradley turns from this concern with the logical validity of inference, to the more metaphysical question of whether what takes place in the ideal realms of inference in fact corresponds to how things are

[53]Quoted by Sievers [1996, p. 248].

in reality. Do our thought processes correspond to actual processes in the world? If there is evolution in our datum, is there a corresponding development *in rerum natura*? (PL 580)

Even as he began his account of inference, stating the requirement for a basis of an identity in all inference, Bradley noted already that the validity of inference depends on the "enormous" metaphysical assumption "[that] what is the same ideally is really the same." (292) So this is a question that has lain in the background throughout the discussion. But now that he comes to address it directly, the implications of an affirmative answer are ones which Bradley finds it hard to accept. He is struck by the overwhelming difference between thought and sensuous reality. Their natures and modes of connection are so fundamentally unlike one another that can we really be sure so that what happens in one is matched by the other. Were we to say, with Hegel, that the Real just is the rational, matters would be easy. But can we really say this? Bradley's doubts come to the fore, and the *Principles* ends, with a famous passage worth quoting in full.

> "Unless thought stands for something that falls beyond mere intelligence, if 'thinking' is not used with some strange implication that never was part of the meaning of the word, a lingering scruple still forbids us to believe that reality can ever be purely rational. It may come from a failure in my metaphysics, or from a weakness of the flesh which continues to blind me, but the notion that existence could be the same as understanding strikes as cold and ghost-like as the dreariest materialism. That the glory of this world in the end is appearance leaves the world more glorious, if we feel it is a show of some fuller splendour; but the sensuous curtain is a deception and a cheat, if it hides some colourless movement of atoms, some spectral woof of impalpable abstractions, or unearthly ballet of bloodless categories. Though dragged to such conclusions, we can not embrace them... They no more *make* that Whole which commands our devotion, than some shredded dissection of human tatters *is* that warm and breathing beauty of flesh which our hearts found delightful." (PL 590–1)

The anti-Hegelian sentiments of these lines and their commitment to the inescapability of some form or other of realism, separated Bradley from a great many of his idealist companions. But rather than recant, what is here but hinted at he went on, in his subject book *Appearance and Reality* (1893), to develop into a full blown metaphysics. Thought, he argues there, is not wholly separate from the real which is given in experience — he is no dualist — but neither is it simply identical with it. Rather it must be regarded as unreal abstraction from presentation pointing, though its own deficiencies, beyond itself to a deeper reality.

4 TRUTH

In so far as the theory of truth is a metaphysical question, concerning the relationship between thought and reality, it perhaps has no place in a discussion of logic. That was certainly Bradley's own attitude in writing the *Principles*, throughout which he tries as far as possible to avoid metaphysical problems. But since for Bradley the very aim and purpose of logic is to assess the truth-bearing character of our thought such questions of metaphysics can never altogether excluded. And, indeed, as we have just seen, in his final discussions of the ultimate validity of inference he finds himself in squarely metaphysical territory. For this reason it seems appropriate to conclude this consideration of Bradley's logical views with a discussion of his conception of truth. In doing so I draw largely on material that came after *Principles*, that is from *Appearance and Reality* and from *Essays on Truth and Reality*. Questions could perhaps be asked about the extent to which these later works are fully compatible with the earlier logical work, but in so far as that was revised in the second edition of 1922 there are, I think, no serious points of discrepancy.

The correspondence theory of truth

Bradley finds himself unable to endorse any of the three traditional theories of the nature of truth, and it is helpful to examine his objections to them. We can begin with the most common theory of truth, the correspondence theory, according to which a judgment is said to be true if it copies, mirrors, or corresponds to the facts.

At time in the *Principles* it might seem as thought Bradley held a correspondence theory of truth. We might take, for example the following statement: "A judgement, we assume naturally, says something about some fact or reality . . . For consider; a judgement must be true or false, and its truth or falsehood can not lie in itself. They involve a reference to a something beyond. And this, about which or of which we judge, if it is not a fact, what else can it be?" (PL 41). And there many other references in that work to copying or corresponding or fitting the facts. [54]

However, it is quite clear that correspondence theory was not in fact one which he endorsed, and in the second edition of the *Principles*, he explains that he employed it there as a simplifying assumption only.

> "The attempt, made at times in this work for the sake of convenience... to identify reality with the series of facts, and truth with copying — was, I think, misjudged. It arose from my wish to limit the subject, and to avoid metaphysics, since, as is stated in the Preface, I was not prepared there to give a final answer" (PL 591 note).

[54] PL 41–2, 579–80, 583, cf. ETR 109 note.

But, in truth, only the most cursory of readings could ever have suggested that this was his view. For even in the first edition, he strongly attacks the theory. "The common-sense view of facts outside us passing over into the form of truth within us, or copying themselves into a faithful mirror, is shaken and perplexed by the simplest enquiries" (PL 46) he complains. His objections to the theory are twofold. In the first place, he believes that thought and reality are so fundamentally different, that any talk of correspondence between them is quite ridiculous. Thought is abstract and general, an "unearthly ballet of bloodless categories" (PL 591), where reality concrete and particular, the felt here and now of sensuous reality. But how can something essentially general and hypothetical ever correspond to something essentially individual and categorical? How can what is thought ever mirror what is felt? Secondly, Bradley attacks the idea of bare unconceptualized facts to which truths are supposed on the account to correspond. For as we saw, perception and judgment emerge together and there can be no intelligible encounter with reality that is not at the same time thought or conceptualised. If we have focused on what is given in perception enough to pick it out and describe it we have already categorised and interpreted it. Nowhere do we encounter bare reality against which to measure our beliefs as the correspondence theory calls upon us to do. "The merely given facts are" Bradley says, "the imaginary creatures of false theory. They are manufactured by a mind which abstracts one aspect of the concrete known whole, and sets this abstracted aspect out by itself as a real thing" (ETR 108).

The pragmatic theory of truth

There are several respects in which Bradley found himself close to the newly developing pragmatic theory of truth. Fellow critics of correspondence, he was able to agree with their insistence that truth be not separated utterly from our procedures for determining it. "Truth indeed must not become transcendent" (ETR 128) he admits. Rather, he says, truth is what satisfies the intellect. In truth, the intellect finds a rest and contentment that is its own good or end. Meaninglessness, contradiction and falsehood, on the other hand, all produce in the intellect a sense of uneasiness or dissatisfaction, in which state it cannot remain. They leave us with a "certain felt need" (ETR 311), that must be meet, and so we search for a state in which the intellect can rest contented (AR 509, ETR 1, 2, 242). Of course, full satisfaction is impossible, and so Bradley admits that in the real world our criteria certainly are pragmatic. For natural science, what this means is that although its theories can never attain ultimate accuracy, they may nonetheless be instrumentally or practically true. This thinks Bradley is their sole aim. "The question is not whether the principles of physical science possess an absolute truth to which they make no claim. The question is whether the abstraction employed by that science, is legitimate and useful" (AR 251) Although ultimately unreal abstractions, the concepts of science are to be thought of as "working ideas" (AR 251), and as such, legitimate in so far as and only in so far as they do work. "I do not object to *anything* that is offered, so long as and so far as it works, and so long as it is offered merely as something which works" (CE 373).

In all of these ways it might be easy to mistake Bradley for a pragmatist. But despite the calls of figures such as F.C.S.Schiller and William James to join their camp, he never felt able to call himself a pragmatist. Part of his problem was that of finding a single clear doctrine to sign up to. And here we must sympathise with his doubts that there every really existed a single pragmatic theory of truth. But he has as well a more fundamental difficulty. For however unattainable it may be to us, Bradley believes in absolute truth. Even if not one which we often enter, or one in which any of us could defend ourselves, there remains a 'supreme court' of metaphysics in which are judgments are assessed for absolute truth, such that however widespread or inevitable it may the use of the pragmatic is always a kind of 'second best.' Elevated to a dogma Bradley sees pragmatism as a threat to objectivity, a slide into relativism. Knowledge aims to reproduce the way things really are, and practical reasoning for all its inevitability remains something different from inquiry into how things really are. And if it is indeed the case that truth satisfies the intellect, this is down to the fundamental nature of *reality* (of which we are, of course, a continuous part) not the fundamental nature of *truth*; it is a practical criterion of truth not its defining essence.[55]

The coherence theory of truth

According to the coherence theory of truth, truth consist in the coherence and comprehensiveness of propositions among each other. By far the greatest number of people have thought that Bradley held a coherence theory of truth. The origin of this belief may be traced to Russell, who attacked the theory severely in a paper which, although explicitly directed against Joachim (who did hold such a view), made it clear that he believed this to be Bradley's view also. [56] This is still the popular opinion. It is supported by the fact that a careless reading might well give this impression. But the real situation is quite otherwise. The fact of the matter is that nowhere does he say that truth *consists* in coherence. For Bradley this is the *criterion*, not the nature, of truth. He clearly states that coherence is a *test* of truth. (ETR 202) On the other hand, it must be admitted that statements to the effect that the more coherent something is, the more true it is, are ambiguous and not hard to misinterpret.

The identity theory of truth

Bradley's actual theory of truth is unusual and rather perplexing. But notwithstanding these facts, there is no excuse for misinterpreting him, for there can be no doubt as to what he actually says.[57] Although it is repeated in numerous other

[55] Candlish (1984) section II.
[56] Russell [1906–7]. The ostensible target was Joachim [1906].
[57] Correct accounts of his theory of truth may be found in [Candlish, 1989] and [Baldwin, 1991].

places, the theory is most clearly expressed in his *Essays on Truth and Reality*, where he says,

> "The division of reality from knowledge and of knowledge from truth must in any form be abandoned. And the only way of exit from the maze is to accept the remaining alternative. Our one hope lies in taking courage to embrace the result that reality is not outside truth. The identity of truth knowledge and reality, whatever difficulty that may bring, must be taken as necessary and fundamental." (ETR 112–3)

The position has been identified as what we might call an identity theory of truth. For a judgment to be true, we must remove all difference between it an its subject, such that in the end it does not simply correspond to reality. But is identical with it. To understand this, more needs to be said about Bradley's understanding of the relationship between thought, feeling and reality. As Bradley sees it, reality comes first in feeling, undifferentiated by thought. From this emerges thought. But though distorts reality, for it treats as hard and fast distinction what is really a fluid identity in diversity. This is the source of all its errors. The process of removing distortion, repairing the damage, is one that in the end must take us beyond thought itself, as Bradley puts it to thought's suicide. Its must be transcended into a higher experience that is Reality itself. The goal of our thinking is an identity with reality that would take us beyond thought itself. We see here how we have transcended logic for metaphysics, for this strictly metaphysical view is of no use for logic; for truth on this understanding is not even a property of judgments.[58]

Degrees of truth

We may return from such far-off metaphysical regions to consider one further important implication of Bradley general conception of judgment for his theory of truth. All judgment is made subject to conditions, facts outside its explicit statement which nonetheless bare upon it. So long as these conditions are ignored the judgment must be regarded as strictly false. Were they all to be included within the body of the judgment it would express a truth. But it is clear that, between these two extremes, there exist a myriad of degrees as more or less of these conditioning factors are explicitly acknowledge in the judgment itself. One of the most notable aspects of Bradley's theory of judgment is his exploitation of this possibility in the form of a doctrine of degrees of truth. [59] Bradley accepts, of course, that for the most part we act as though truth and falsity were absolute but none the less he thinks, when we assess judgments in anything more than a pragmatic way degrees of truth force themselves on us. We see, argues Bradley,

[58] [Walker, 1998, p. 97].

[59] While there can be no doubt that this is his view in *Appearance and Reality*, it is to be noted that he denied it in the first edition of the Principles (PL 197). Allard [1996, p. 141] reads this as saying simply that something can not be more or less affirmed.

that no judgment is wholly true and no judgment wholly false, but rather that all lie somewhere in between.

No judgment is wholly true because any judgment is always subject to unstated conditions, which as long as they are left out vitiate the judgment. But in the end thinks Bradley it would be impossible to wholly correct this, for the presence of these conditions is a consequence of the very action of thought to abstract, separate and divide what is given in experience as a single sensuous whole. So long as we think, we must divide and abstract and our thought must of consequence be incomplete and distorted, but partial truth.

And yet on the other hand, it must always remain partially true, for because there are no floating ideas and because whatever we say is referred to reality as a whole, our judgments can never fail to contain some measure of truth. They may so distort and abstract that putting right their failing would change them almost beyond recognition, but it cannot be that nothing in what they say, not even when "redistributed and dissolved" (AR 323) is able to find a point of reference in reality. For this reason. Says Bradley, "Error *is* truth, it is partial truth, that is false only because partial and incomplete" (AR 169) We abstract a content and attempt to refer it to a reality with which it seems discrepant, but reality owns both of these elements and in a wider view "this jarring character is swallowed up and is dissolved in fuller harmony." (AR 170)

But what, it will be demanded, *are* degrees of truth? The idea is such an alien one to those brought up to believe in only two truth values. Bradley suggests that they may be understood as a measure of *transformation* that would be required to turn something into a complete truth. To add in those factors that have been ignored is to modify the judgment and "the amount of survival in each case" he suggests "gives the degree of reality and truth" of the original judgment. (AR 323) While it is unclear how this could result in any sort of measure, the idea is intuitive.

In an objection addressed to one of Bradley's followers, H. H. Joachim, but aimed as much against Bradley, Bertrand Russell famously argued that the whole doctrine of degrees of truth was just self-contradictory.

> "[If] no partial truth is quite true, it cannot be quite true that no partial truth is quite true; unless indeed the whole of truth is contained in the proposition 'no partial truth is quite true', which is too sceptical a view for the philosophy we are considering. Connected with this is the difficulty that human beings can never know anything quite true, because their knowledge is not the whole truth. Thus the philosophy with which the view in question is bound up cannot quite be true, since, if it were, it could not be known to idealists" [60]

Like many statements of Russell's this is perhaps more memorable than valid. No doubt he wishes us to infer that since the doctrine of partial truth is not itself

[60] [Russell, 1906–7, p. 33].

'quite true' it is false, and thus that if it is true it is false, i.e. it is self-refuting. But not quite true is different from false, and there is no contradiction in regarding the theory that all statements are incomplete as itself incomplete.

But nonetheless there is a puzzle here. For how can we know that it is not incomplete in a way that would radically change it? However, Bradley has within his resources a way to solve this problem. He makes a relative distinction between absolute and finite truth. Both are conditioned and depend for their truth upon factors not included within the judgement, but the former, which include the truths of metaphysics, are as general as judgements can possibly be, and thus unconditioned by anything which it would be possible for us to recognise and add. He says, "Absolute truth is corrected only by passing outside the intellect. It is modified only by taking in the remaining aspects of experience. But in this passage the proper nature of truth is, of course, transformed and perishes." (AR 483) In this way they are as true as any judgements could ever be. If corrigible, they are not *intellectually* corrigible. "There is no intellectual alteration which could possibly, as general truth, bring it nearer ultimate Reality." (AR 483). And so while it might be worrying to think that the doctrine that there is no absolute truth were incomplete if that meant that it might later be completed and modified (perhaps into something very different) if it is as complete as any judgment could be, we need have no such worry.

5 CONCLUSION

Bradley's logic was immensely influential in its day; it became a kind of bible for a generation of subsequent philosophers. And because of his view of the centrality of logic and its relation to other areas of philosophy, this allegiance influenced not just their logical views but their other positions also. However, rapid success of the new logic championed by Frege, Russell, Whitehead and Wittgenstein which made its appearance at the beginning of the twentieth century almost entirely eclipsed its idealist predecessor, rather as it had eclipsed the empiricist logic that came before it. Since that time Bradley's logic has been either derided or ignored. As a result Bradley's logic is still not widely known, which is a great shame. Although it is true the majority of his logical views are ones which modern logicians would still want to reject, the strength of the rejection has led his successors to overlook even those things which he said that are of value. For as we have seen Bradley's system of logic is complex and subtle and there is much in it that rewards careful attention. Fortunately, over the last twenty years or so, this has increasingly come to be recognised as his work has begun once again to attract scholarly attention.

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