Global Financial Management

Valuation of Stocks

3.0 Introduction

This lecture provides an overview of equity securities (stocks or shares). These securities provide an ownership interest in the firm whereas debt securities (loans, bonds or other fixed-interest securities) establish a creditor relationship with the firm. After a brief overview of some of the institutional details of these securities, this module focuses on valuing equity securities by making some simplifying assumptions. This leads us to a discussion of financial ratios that are widely used in practice, in particular, dividend yields and price/earnings multiples. After completing this module, you should be able to:

- Understand basic transactions involving stocks

- Demonstrate why stocks can always be valued as the present value of future dividends.

- Determine the value of a stock that pays a constant dividend

- Determine the value of a stock that pays a dividend that grows at a constant rate.

- Use the dividend growth model to infer the expected return on equity if you know the expected growth rate of a company.

- Use the dividend growth model to infer the expected growth rate of future dividends for a company where you know the expected rate of return on equity.

- Value a company using appropriate P/E-multiples and understand the limitations of this methodology.

- Show how the value of a company can be decomposed into the value of growth options and value of a constant earnings stream.

1

3.1 Introduction to Stocks

Stocks represent an ownership interest in a company and confer three rights on the owner of a share:

- Vote at company meetings: Shareholders vote on meetings on issues ranging from merger proposals to changes in the corporate charter to the election of corporate directors.
- Collect periodic dividend payments. Unlike interest payments dividends are not contractually fixed and can vary. Omission of dividends does not trigger bankruptcy.
- Sell the share at his or her discretion. In some countries this right can be limited.

In this lecture we focus on the valuation of stocks. Therefore, we are mainly concerned with the second and third point. However, the first point is important for understanding the market for corporate control and corporate governance.

Stocks are first issued to investors through what is known as the *primary* or new issues market. Typically, companies are founded by one or few entrepreneurs and initially held by a small number of investors. At some point the company decides to raise capital by offering shares to the general public. This is known as an initial public offering (IPO). The company may decide to raise more capital through selling shares in the future. These subsequent offerings are called seasoned equity offerings (SEO). IPOs and SEOs together form the equity primary market. In most cases companies enlist the help of an investment bank for conducting these offerings. The bank handles the distribution of shares to investors. Sometimes they also provide companies with a guarantee to sell a certain number of shares in exchange for a fee.

Investors purchase stocks for their returns. These returns come in the form of:

- capital gains - the appreciation in value over time, and

- dividends - most companies pay periodic dividends.

Investors will be reluctant to purchase a stock unless there is a mechanism available for the speedy resale of these stocks. This allows them to realize capital gains and to obtain liquidity independently of the payout policy of the company. Provision of a resale mechanism is the function of the stock exchange (also known as the *secondary* market). Investors are able to buy and sell stocks through the stock exchange. Investors trade between themselves on these exchanges. The company is not a party to the transaction and receives no funds as a result of these transactions. Conversely, investors can liquidate their investments for consumption purchases without forcing the company to liquidate investments. This feature of a secondary market is crucial for economic development: companies can plan their investment policies independently of the consumption patterns of their investors.

Various stock indexes are also maintained and are closely watched by investors. When we think of how the stock market performed in a particular period, we invariably refer to one of these indexes. The following tables give the major stock market indices and their values on November 24, 1997.

Index	Value 11/24/1997, 12:56pm EST
Dow Jones Industrial Average	7800.50
S&P 500	953.57
NASDAQ Combined Composite Index	1600.36
Toronto Stock Exchange 300 Index	6746.70
Mexico Bolsa Index	4721.97

Index	Value 11/24/1997, 12:56pm EST
FT-SE 100 Index	4898.60
CAC 40 Index	2802.48
DAX Index	3830.63
IBEX 35 Index	6670.25
Milan MIB30 Index	22916.00
BEL20 Index	2357.44
Amsterdam Exchanges Index	875.46
Swiss Market Index	5645.70

Index	Value 11/24/1997, 12:56pm EST
Nikkei 225 Index	16721.58
Hang Seng Stock Index	10586.36
ASX All Ordinaries Index	2482.10

These indices give some kind of average return for a particular market. A major difference between stock indices is between equally weighted and value-weighted indices. Equally weighted indices give the same weight to all stocks, independently of the size of a particular company. Value-weighted indices use the market capitalization (the total value of all shares outstanding) of each company.

3.2 Stock Transactions

There are three ways of transacting in stocks:

Buy - we believe that the stock will appreciate in value over time, or require the stock for its risk characteristics as part of our portfolio (We are expecting a **bullish** market for the stock). It is also said that we are **long** in the stock.

4

Sell - we believe that the stock will depreciate in value over time or we require funds for another purpose (liquidity selling).

Short Sell - here we do not own the stock, but we borrow it from another investor, sell it to a third party, and, in theory, receive the proceeds. We are obligated to pass on to the lender of the stock any dividends declared on the stock and also to pay to the lender the market price of the stock if he himself should decide to sell. When we short sell, we believe that the stock will decline in value thus enabling us to buy it back at a low price later on to make up our obligations to the lender. We are expecting a **bearish** market for the stock. It is also said that we are **short** in the stock.

When a short sale is executed, the brokerage firm must borrow the shorted security from its own inventory or that of another institution. The borrowed security is then delivered to the purchaser on the other side of the short-sale. The purchaser then receives dividends paid out by the corporation. The short-seller must pay out any dividends declared by the firm to the original owner from which the security was borrowed during the period in which the short-sale is outstanding. To close out the short sale, the short seller must buy the stock in order to return the security originally borrowed. Note that borrowing fees can be significant for "hard-to-borrow" securities because these securities are in high demand due to a high level of short-selling (e.g., *Netscape* immediately after it went public).

In modeling finance problems we often assume that the investor receives the full proceeds of a short sale. There are a number of practical mechanics, which limit the investors' ability to access these funds. The proceeds from a short sale are usually held by an investor's brokerage firm as

collateral. The investor usually does not receive the interest from the short sale proceeds, and will likely have to meet a margin requirement. In practice, short sales require a cash outlay. They do not provide a cash inflow.

3.3 Valuation of Stocks

In this section, we determine the value of a typical stock. Assume that a stock has just paid a dividend so that the series of future periodic dividends (D_t) can be represented as:

Period	0	1	2	...	t	…
Dividend		D_1	D_2	...	D_t	…

We start by looking at a typical share traded on the stock exchange and bought and sold once a year. The original buyer at t=0 buys the share with a view to sell it at the end of the first year at an expected price of P_1. This entitles the investor to receive the first year's dividend D_1. Assume the discount rate (= required rate of return) for this stock is constant and equal to r_e. Then the buyer values the share as:

$$P_0 = \frac{D_1 + P_1}{1 + r_e} \tag{1}$$

But what determines P_1? Simply assume the buyer in one year's time determines the price in just the same way, and uses the same discount rate:[1]

$$P_1 = \frac{D_2 + P_2}{1 + r_e} \tag{2}$$

---

[1] The important assumption here is that the hypothetical investors concerned here use the same discount rate. This is not a strong assumption. The assumption that discount rates are identical across periods simplifies the analysis, but is not essential.

Or, generally, for period T:

$$P_{T-1} = \frac{D_T + P_T}{1 + r_e} \tag{3}$$

Substituting equation (2) into equation (1) gives:

$$P_0 = \frac{D_1}{1 + r_e} + \frac{D_2 + P_2}{(1 + r_e)^2} \tag{4}$$

Continuing the same process:

$$P_0 = \frac{D_1}{1 + r_e} + \frac{D_2}{(1 + r_e)^2} + \dots + \frac{D_T + P_T}{(1 + r_e)^T} \tag{5}$$

Since $(1 + r_e)^T$ becomes very large as T becomes very large, the expression $\frac{P_T}{(1 + r_e)^T}$ can be neglected for a large time horizon.[2] Hence:

$$P_0 = \frac{D_1}{1 + r_e} + \frac{D_2}{(1 + r_e)^2} + \frac{D_3}{(1 + r_e)^3} + \dots \tag{6}$$

This shows the first important result:

> The share price equals the present value of dividends.

---

[2] Mathematically, this requires that P_T does not grow "too fast" in some appropriate sense as T becomes large.

This formula is interesting in its own right because it shows that even though investors may turn over their portfolios very frequently, this does not have any impact on the value of the stock: *short term investment horizons do not translate into a short termist valuation of shares.* However, in order to make use of expression (6), we have to make some assumptions about future dividends. Before we turn to this topic, it is useful to turn to equation (1) once more and express it in terms of returns. We solve for r_e to find:

$$r_e = \frac{D_1}{P_0} + \frac{P_1 - P_0}{P_0} \qquad (7)$$

The first part on the right hand side is commonly known as the dividend yield. This is a financial ratio widely used by practitioners. However, note that in practice we do not know D_1 since it is an expected value about a future dividend payment. Practitioners commonly refer to the dividend yield as D_0/P_0. This difference is important and we shall therefore refer to D_0/P_0 as the *historic* or trailing dividend yield, and to D_1/P_0 as the *prospective* dividend yield. The second part on the right hand side of (7) is the capital gain, expressed as a percentage of the current stock price. Then we can express (7) as:

$$\boxed{\begin{array}{c} \text{Return on equity} \\ = \text{Prospective Dividend Yield} + \text{Expected Capital Gain} \end{array}}$$

3.4 The "Constant Growth" Formula

The simplest assumption about dividends is that they stay constant over time, so that $D_1 = D_2 = D_3 = .. = \overline{D}$. Then expression (6) simplifies to:

8

$$P_0 \; = \; \frac{\overline{D}}{r_e} \quad \Rightarrow \quad r_e \; = \; \frac{\overline{D}}{P_0} \; = \; DY \qquad\qquad (8)$$

where *DY* denotes the dividend yield. Hence, we have two important conclusions:

1. **If the dividend is expected to stay constant over time, shares can be valued like perpetual bonds *as $P_0=D/r_e$.***

2. **If the dividend is expected to stay constant, the expected return on equity is equal to the dividend yield.**

Unfortunately, constancy of dividends is a very specific assumption with little realism, and therefore few applications. A more general assumption is that dividends grow at a constant rate. Hence, assume that dividends grow at a constant rate *g* forever:

$$D_2 \; = \; D_1(1 \, + \, g)$$

$$D_3 \; = \; D_2(1 \, + \, g) = \; D_1(1 \, + \, g\,)^2$$

$$D_4 \; = \; D_3(1 \, + \, g) = \; D_1(1 \, + \, g\,)^3$$

$$......$$

$$D_T \; = \; D_{T-1}(1 \, + \, g) = \; D_1(1 \, + \, g\,)^{T-1}$$

Substituting these expressions into (6) gives:

$$P_0 \; = \; \frac{D_1}{1 \, + \, r_e} \; + \; \frac{D_1(1 \, + \, g)}{(1 \, + \, r_e\,)^2} \; + \; ... \; + \; \frac{D_1(1 \, + \, g\,)^{T-1}}{(1 \, + \, r_e\,)^T} \; + \; ... \qquad\qquad (9)$$

Assume that g is smaller than r_e.[3] Then the general formula for adding this series is (see the appendix for a derivation):

$$P_0 = \frac{D_1}{r_e - g} \qquad (10)$$

Note that (10) reduces to (8) if $g=0$, hence the constant dividend case is covered as a special case. From this we can see immediately:

$$r_e = \frac{D_1}{P_0} + g \qquad (11)$$

This gives the third important result:

Expected Return on Equity
= Prospective Dividend Yield + Growth Rate

Using (7) together with (11) gives also:

$$g = \frac{P_1 - P_0}{P_0} \Leftrightarrow P_1 = (1+g)P_0 \qquad (12)$$

---

[3]   It turns out that $g<r_e$ is precisely the condition noted above to conclude that $\dfrac{P_T}{(1+r_e)^T}$ becomes small as T becomes large. If $g<r_e$, then $\dfrac{P_T}{(1+r_e)^T}$ would become infinitely large, hence we would have to conclude that P_0 is infinitely large, hardly a plausible conclusion.

Hence, if we assume that the company is in a steady state where dividends are expected to grow at a constant rate g, we also expect that the stock price grows at the same rate constant rate g.

The strongest assumption we made in deriving (11) is the constancy of the growth rate, that is, we assume the firm is in a "steady state". This is a strong assumption for any firm, but if we view
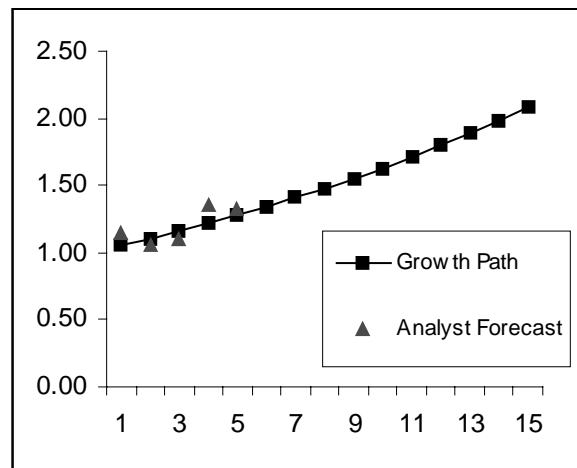
g as some kind of average we can sacrifice some generality for simplicity. However, for firms which are clearly not in a steady state (consider firms where the current dividend and is zero, so in the first year in which they pay a dividend the dividend growth will be infinity!), this procedure is entirely inappropriate. In this case we have to extend the constant growth model and define subperiods with different growth rates. Alternatively, we could formulate a model where the dividend growth model holds for all periods after 3-5 years, and we use analysts' dividend forecasts for the first few years. This is illustrated in the following graph:

The graph illustrates exponential dividend growth, starting at a dividend of $1.00 in year 0. The square-shaped points illustrate exponential growth (i. e., growth at a constant rate). The triangle shaped points illustrate analyst's forecast based on detailed projections fore the first 5 years.

11

3.5. Valuation of General Motors: an example

In order to see how these formulae may be applied, consider the case of General Motors. The trailing (historic) dividend of GM in December 96 was $1.60 per share. Other data are:

Number of shares outstanding:	856,695,000
Market capitalization:	$ 46.31bn

The market capitalization of a company is always defined as:

$$MCAP = Number\ of\ shares\ outstanding * Share\ price$$

Hence, we can use the apparatus we have built so far either on a per share basis (divide total earnings, dividends and MCAP by the number of shares), or for the company as a whole. Suppose you forecast that until the end of 1997 GM's dividend will be $1.75 per share, and then grow at a constant rate forever after. What valuation for GM do you obtain for alternative combinations of the growth rate and the discount rate? The following table shows the type of results you obtain:

Table 1

Return/ Growth	3%	4%	4.50%	5%	6%	7%
7%	37.48	49.97	59.97	74.96	149.92	-
8%	29.98	37.48	42.83	49.97	74.96	149.92
9%	24.99	29.98	33.32	37.48	49.97	74.96
10%	21.42	24.99	27.26	29.98	37.48	49.97
11%	18.74	21.42	23.06	24.99	29.98	37.48
12%	16.66	18.74	19.99	21.42	24.99	29.98

In order to see how you obtain these results, consider the case of a 5% annual growth rate and 9% return. (the boxed entry in the table). Our dividend per share forecast was $1.75. Multiplying this with the number of shares outstanding gives a total expected dividend for GM for 1998 of $1.499bn, or a prospective dividend yield of 3.78%. Then we have:

$$MCAP_{GM} = \frac{D_{1998}}{r_{GM} - g_{GM}} = \frac{\$1.499bn}{0.09 - 0.05} = \$37.48bn \qquad (13)$$

Hence, we can use the dividend growth model in order to value the equity of a company by using the following steps:

1. Forecast the end of year dividend of the company

2. Estimate the growth rate of dividends and the required rate of return on capital

3. Use formula (10)

Conversely, we can also use the formula in the other versions discussed above in order to:

- Infer the growth rate of dividends: If you know the expected return on equity and the current value, you can infer the growth rate (rearrange (10) or (11)) expected by the market. One way of estimating expected returns is using another model for predicting required returns. We will discuss one such model, the Capital Asset Pricing Model, in a subsequent lecture.

- Infer expected returns. If you know the growth rate of dividends (e. g., from industry forecasts), you can infer the cost of equity capital used by the market.

3.6 Earnings yields and P/E ratios

The most widely used ratio are price earnings multiples, or short P/E multiples. Denote earnings per share by E_1. Then the earnings yield is defined as E_1 / P_0. It is therefore the reciprocal of the P/E-ratio defined as P_0 / E_1. Note that these are *prospective* P/E-ratios and earnings yields,

and that financial analysts refer often to *historic* or *trailing* values, defined as E_0/P_0 and P_0/E_0 respectively. Dividends and earnings are related via the company's payout policy. This can be summarized in the payout ratio d defined as the ratio of dividends per share and earnings per share:

$$d = \frac{D_1}{E_1} \tag{14}$$

Then the dividend can be written as $D_1 = d\,E_1$ which can be substituted into (11) to give:

$$r_e = \frac{E_1}{P_0} * d + g \tag{15}$$

which relates to required return on equity to the earnings yield. Rearranging once more gives:

$$\frac{P_0}{E_1} = \frac{d}{r_e - g} \tag{16}$$

This shows the result that:

If two companies have the same payout policy, the same cost of equity capital and the same growth rate, then they should also have the same P/E ratio.

The problem with using the above measures is that they refer to prospective dividend and earnings yields, whereas the financial press often reports historic yields. However, it is easy to see that they can be related in a similar way by assuming that dividends and earnings grow at the constant rate g from now on, i. e. that $D_1 = (1+g)D_0$. If d is constant over time, this implies also that $E_1 = (1+g)E_0$. Then (11) and (15) and (16) become:

$$r_e = g + (1+g)\frac{D_0}{P_0}$$

$$\frac{P_0}{E_0} = (1+g)\frac{P_0}{E_1} = \frac{d(1+g)}{r_e - g}$$

(17)

We shall work through one example on inferring the growth rate from publicly available data using (17). Consider the big three American car manufacturers. The main data are given in the following table:

Table 2

	Chrysler	Ford	GM
MCAP ($billion)	30.92	40.71	46.31
No. of shares (in '000s)	702,500	1,187,000	856,695
Share Price	$44.01	$34.30	$54.06
Dividend per share	$1.35	$1.43	$1.60
Dividend Yield	3.07%	4.17%	2.96%
EPS	$5.03	$3.72	$6.06
P/E ratio	8.75	9.22	8.92

Then rearranging (17) gives:

$$g = \frac{r_e - D_0/P_0}{1 + D_0/P_0}$$

(18)

where D_0/P_0 is the historical dividend yield referred to in table II. Then we obtain the following implied growth rates (depending on the discount rates in the left-hand column).

Returns	Implied growth rates		
	Chrysler	Ford	GM
9%	5.76%	4.64%	5.87%
10%	6.73%	5.60%	6.84%
11%	7.70%	6.56%	7.81%
12%	8.67%	7.52%	8.78%
13%	9.64%	8.48%	9.75%
14%	10.61%	9.44%	10.72%
15%	11.58%	10.40%	11.69%

So, if we know (e. g. from analysis deriving from the capital asset pricing model, see the lecture on the capital asset pricing model) that the required rate of return on equity is 12%, then we can derive the expected growth rate for Ford (historic dividend yield 4.17%) as:

$$g_{Ford} = \frac{0.12 - 0.0417}{1.0417} = 0.0752$$

which gives the result of 7.52% stated in the table.

3.7 (How) Should you use P/E ratios?

Analysts often refer to companies with a low P/E multiple as being undervalued, or as overvalued if the P/E ratio is high. (e.g. if they say that Philip Morris "has a modest P/E"). The P/E ratio becomes then like a price tag in a supermarket: the industry average says that $1m of retail earnings or earnings from computer manufacturing etc. "sell" for a certain price or "multiple", say 28. If you can then buy $1m of earnings in computer manufacturing for 14, you strike a bargain, because you are entitled to the same earnings, hence dividend stream, for a lower price.

Our analysis has two implications for this type of argument. On the positive side, we have shown that a simple dividend discount model can rationalize the P/E ratio. Then a P/E ratio can be used for company valuation using the following steps:

1. Forecast the company's end of period earnings (e. g. use forecasts of sales and margins etc.).

2. Estimate growth and the required rate of return (use industry forecasts, asset-pricing theory).

3. Estimate which proportion of earnings needs to be retained so that investment is sufficient to generate the growth we have assumed in step 2. The retention ratio of earnings is then 1-d in our notation.

4. Use formula (16) to value the share as $P_0 = E_1 \dfrac{d}{r_e - g}$.

However, P/E ratios are almost never used this way. The whole point of using financial ratios is to avoid the estimations involved in steps 1-3. Instead, practitioners use the following two-step approach:

1. Find a sample of companies in the same industry, which are "similar" to the company you wish to value and determine the average (historical or trailing) P/E ratio of this sample.

2. Value the company by using the approximation:

$$V_{Company} = P/E_{Average} * E_{Company} \qquad (19)$$

This advantage of the second procedure is that it uses market estimates of the ratio $\dfrac{d}{r_e - g}$. Since measuring and estimating each of the components is fraught with errors deriving from the respective forecasting models for required returns, growth rates etc., using a market measurement may avoid this. The disadvantage of this procedure is that it makes it easy to overlook the assumptions that go into the analysis:

1. Constant growth, the company is in a steady state.

2. The company is comparable to the industry sample with respect to expected returns on equity(r_e), growth prospects (g), and retention ratio(1-d).

Assumption 2 is the more restrictive one. Note in particular that two companies can have the same technological structure and markets, and therefore the same growth prospects, but still differ with respect to expected returns on equity, e. g. because they have different leverage and different debt policies. We discuss this at a later stage when we analyze capital structure and borrowing. Many times practitioners feel uneasy about the constant growth assumption. This is a much less serious problem. In most cases, even a fully specified discounted cash flow analysis (see the lectures on project appraisal) will proceed in two steps: (1) estimate the PV of net cash flows for 5-10 years into the future, and (2) estimate a "horizon value" or "terminal value" by using some variant of a constant growth scenario. Typically, more than 75% of a company's value comes from the horizon value, and the (implicit) assumptions about future growth. Hence, even though the constant growth assumption may be a strong one, it will be part of most valuation procedures in one way or another.

Hence, P/E ratios can provide useful information for company valuation, but they need to be handled with care. The apparent simplicity of just using one financial ratio can be problematic if the strong assumptions behind it are overlooked. However, if you use it, make sure that the company you wish to value is always comparable with respect to the coefficients that determine the multiple. This requires, among other things, to compare companies with broadly similar leverage and operating characteristics.

3.8 Financial ratios in practice

This section lists a number of typical applications, where financial commentators refer to price/ earnings or other multiples for valuation purposes:

(A) Company valuation

On December 30 1996 Linda Sandler, Staff Reporter of the WSJ wrote in the "Heard on the Street" column:

> Investors pay close attention to Mr. Buffett's oracular pronouncements about investments, particularly his notion of real or "intrinsic," value. (…) The easy part is putting a multiple on Berkshire's earnings or cash flow from its wholly owned insurance and manufacturing businesses. (…) The hard part is deciding on a fair value for Berkshire's investment portfolio (…).

> Berkshire's big stake in Coca-Cola cost around $1.3 billion. (…) Coke stock is up 45% this year; it now sells for nearly 40 times earnings in the past 12 months. Gillette is Berkshire's second biggest holding, valued at $3.5 billion at September 30. That stock is up 37% this year and its trailing price-earnings multiple is 35.

Hence, the article argues that a portfolio (Buffet's Berkshire Hathaway) may be fully valued (or even overvalued), because some of the major individual assets of this portfolio (1) trade at high price earnings multiples and (2) the stocks have risen a lot recently. We see from (16) that P/E-

ratios depend directly on the growth rate, hence growth stocks have higher P/E-ratios. Hence, the argument above could be right in arguing that 40 times earnings is too high a price for Coke if Coke is in a mature market were the growth potential is limited.

(B) Mergers and Acquisitions

Steven Lipin, Greg Jaffe and Benjamin A. Holden, wrote in the WSJ on December 19 on in an article entitled "Western Resources Bids To Acquire Rest of ADT"

> The bid for ADT as a multiple of revenue appears to be valued at less than the $368 million Western agreed to pay for Westinghouse Electric Corp.'s alarm business. Western paid a price equivalent to about 4.8 times the Westinghouse security systems estimated annual recurring revenue for 1996, says Edward Wojaechowski, an analyst with Strong Capital Management, which owns about 620,000 ADT shares. The Western bid is about 4.5 times ADT's projected annual recurring revenue for 1996, he said. "Because ADT's dominant position in the market, it should be able to get at least five times annual recurring revenue" (…)

Mergers and acquisitions always require a careful valuation of the asset to be bought. The article shows how financial analysts use a multiple - here revenue multiple - in order to compare this transaction with a similar transaction in the same industry and assess whether the price is fair. Note that revenues and earnings are related (earnings before interest and taxes divided by revenues equals the sales margin). Here the analyst argues that a low multiple is indicative of a low transaction price. The merit of this argument depends critically on whether the "dominant position" translates into more growth than that of other competitors: you can only grow at a higher rate than your competitors if you increase your market share.

(C) Public stock offerings

On December 20 1996 Baker Li (AP - Dow Jones News Service) wrote the following in the Dow Jones Business News in an article on "Asian IPO Focus/Taiwan: Ultima Electronics Seen A Bargain" (quoted according to *Wall Street Journal Interactive Edition*):

> Half of the planned IPO shares were sold through a public offering completed December 17 with a winning price of $36.0 (Taiwan) per share (…) Ultima is posed to reach around NT$ 44.1 in the first three days upon listing, forecasts Hsieh Chih-mo, an electronics analyst with Top Soon Portfolio Corporation. (…).

> Kevin Chan, an electronics analyst with Core Pacific Securities Corp., is more cautious, saying NT$38 is a reasonable expectation for the stock's debut. He takes into account his company's forecast 1997 earnings per share (EPS) of NT$3.6 for Ultimate which gives a conservative price/earnings ratio (P/E) of 14 times, well below the industry's average of 28 times.

> 'Annual growth rate of global scanner output is expected…(at) 40% in the following two years' says Sancy Wang, company chairman and chief executive officer.[4]

Public stock offerings (IPOs) are generally offered at prices to investors that are below the prices they trade subsequently on the stock exchange. In some sense, IPOs are offered at a discount. This phenomenon has been widely observed and is referred to as IPO underpricing. IPOs are notoriously difficult to price, since it is not always clear what a "comparable" company is.

---

[4] There may be an error here, either in the source or in the copy. EPS of NT$3.6 multiplied by P/E of 14 gives NT$50.4, not $38.

3.9 Present Value of Growth Opportunities

You often hear the buzz words *growth firm*. We will explore what this means in the context of our stock valuation model. From the formulas already developed, we can separate the price of a company's stock into two components: a no growth component and a growth component.

Before we work out the details, we could write the price of the share as:

$$P_0 = \frac{E_1}{r_e} + PVGO \tag{20}$$

where PVGO equals the present value of growth opportunities. The first term is the present value of an earnings flow if all earnings in the future were constant and equal to E_1. If the firm were not expected to grow, all earnings would be paid in dividends ($D_t = E_t$) and the value of the stock would be the present value of the "dividend annuity" given above.

It is unlikely that the firm will not have any growth opportunities. The more realistic scenario is that some of the earnings are plowed back into the firm's operations and there is an additional term that represents the present value of growth opportunities. In terms of the formula above, you can consider the PVGO to be a remainder term. We can determine the value of the no growth portion of the stock price. If the stock price is available, then the difference must be the present value of the growth opportunities.

The splitting of the value of the stock into a growth and a no growth component is an insightful exercise. It is now time to reconcile our dividend growth formula with this split. Recall that d is the fraction of earnings distributed as dividends, hence *(1-d)*, represents that fraction of earnings that are plowed back into the firm.

Rearranging (15) we obtain:

$$P_0 = \frac{E_1}{r_e} + \frac{gP_0 - (1-d)E_1}{r_e}$$

$$\Rightarrow PVGO = \frac{gP_0 - (1-d)E_1}{r_e}$$

(21)

Note that the constant stream of earnings, E_1, is pulled out of the PVGO to avoid double counting. Equation (21) has a simple interpretation: gP_0 is the expected appreciation in the stock price over the next period ($gP_0 = P_1 - P_0$). The second term in the numerator ($(1-d)E_1$) is simply the amount of investment necessary to generate this appreciation in price. Hence, the numerator is the net increase in shareholder value per period: shareholders "buy" the appreciation in stock price gP_0 through a reduced distribution of dividends ($(1-d)E_1$). The present value of growth opportunities is the capitalized value of all future increments of shareholder value.

To illustrate some of these points, it is useful to go through a few examples. Suppose that TECHCO, Inc. will have earnings per share next year of US $4. The company will plow back 60% of its earnings into continuing operations. The required rate of return for the firm, r_e, is 16%, the expected growth rate of future earnings is 12%. Calculate: 1) the stock price, 2) the PE ratio, 3) the PVGO, and 4) the amounts of investment that produce the PVGO.

The first step of this exercise is to carefully map these data into the parameters of our model:

g	12%
r_e	16%
E_1	$4.00
1-d	60%

The stock price calculation is straightforward. We know the next period's earnings, so the dividend will be:

$$D_1 = dE_1 = (0.4)(4.00) = \$1.60$$

We can deduce what the growth in earnings is going to be. The company is going to plow back 60% of its earnings to be used in projects that are expected to earn 20% per year. Now we can plug directly into the constant dividend growth model:

$$P_0 = \frac{D_1}{r_e - g} = \frac{\$1.60}{0.16 - 0.12} = \$40.00$$

The PE ratio is also fairly elementary. We know that:

$$\frac{P_0}{E_1} = \frac{\$40}{\$4} = 10$$

Since earnings will be growing at 12%, we can calculate the trailing PE-ratio:

$$\frac{P_0}{E_0} = \frac{P_0}{E_1 / (1 + g)} = \frac{\$40}{\$4.00 / 1.12} = 11.2$$

We can use our formula for the PVGO to calculate this quantity:

$$PVGO = P_0 - \frac{E_1}{r_e} = \$40 - \frac{\$4.00}{0.16} = \$15.00$$

So, $25 of the stock price is related to the present value of earnings without growth opportunities, and $15 of the stock price is related to the growth opportunities of the firm.

Now the hardest part of the problem is to show the amounts of investment that produce the PVGO. Let's take it in steps. In the first year, the firm plows back (0.6)(4.00), or $2.40 of their earnings back into the firms operations. Hence, shareholders' distribution of dividends is reduced by exactly this amount. This generates an increase in stock price of $40.00*0.12=$4.80, i. e., we expect the stock price to increase from $40.00 to $44.80. Hence, shareholders' net gain is $4.80-$2.40=$2.40. The value of this in perpetuity is:

PVGO=$2.40 / 0.16 = $15.00

A PVGO of $15 is the same result that we arrived at earlier.

3.10 The Dividend Discount Model - A User's Guide

The number of different equations of the form $P_0 = \&\#) * ?$ we have derived and discussed above is sometimes bewildering and confusing. This section attempts to provide some guidance as to how these equations are related and when to use which one. The fundamental and general equation that is at the basis of all others is:[5]

$$P_0 = \frac{D_1}{1 + r_e} + \frac{D_2}{(1 + r_e)^2} + \frac{D_3}{(1 + r_e)^3} + \dots \tag{6}$$

---

[5] See the formula on slide 11 of the class notes.

which is repeated here for convenience. This expresses the current price as a function of *all* future dividends. Hence, in order to make use of (6) you need a complete forecasting model that extends into all periods into the future. If you have such a forecast, simply use the forecast values for $D_1, D_2,...,D_\infty$ and obtain a valuation.

You are rarely - if ever - in a privileged situation where you have such a forecast, so you need to generate a forecast. All other equations for the dividend discount model are about generating forecasts. Consider the case is where you only have a dividend forecast for a finite time horizon T, say 10 years, so you know $D_1, D_2,...,D_T$. Then you can use a finite horizon version of the model, where you use the explicit forecasts for the first T periods, and the constant growth formula for the horizon value P_T. This gives us:[6]

$$P_0 = \sum_{i=1}^{i=T} \frac{D_i}{(1+r_e)^i} + \frac{P_T}{(1+r_e)^T} = \sum_{i=1}^{T} \frac{D_i}{(1+r_e)^i} + \frac{(1+g)D_T}{(1+r_e)^T (r_e - g)} \tag{22}$$

Here we have substituted $P_T = \dfrac{D_{T+1}}{r_e - g} = \dfrac{(1+g)D_T}{r_e - g}$ for the dividends *after* period T, for which we have no other forecast, so we use constant growth as a first approximation to generate such a forecast.

If we can only forecast dividends one period into the future, then we use a special version of (22) for T=1, and in this case we obtain the standard equation for the dividend growth model:[7]

---

[6] See slide 22 of the class notes.

[7] See slide 14 of the class notes.

$$P_0 = \frac{D_1}{r_e - g} \tag{10}$$

Finally, if we do not even obtain a forecast for this year's dividend, then the only value we may have is the historic dividend D_0. Then we use our forecasting model for one period, so that $D_1 = (1 + g)D_0$. As a result the valuation equation becomes:[8]
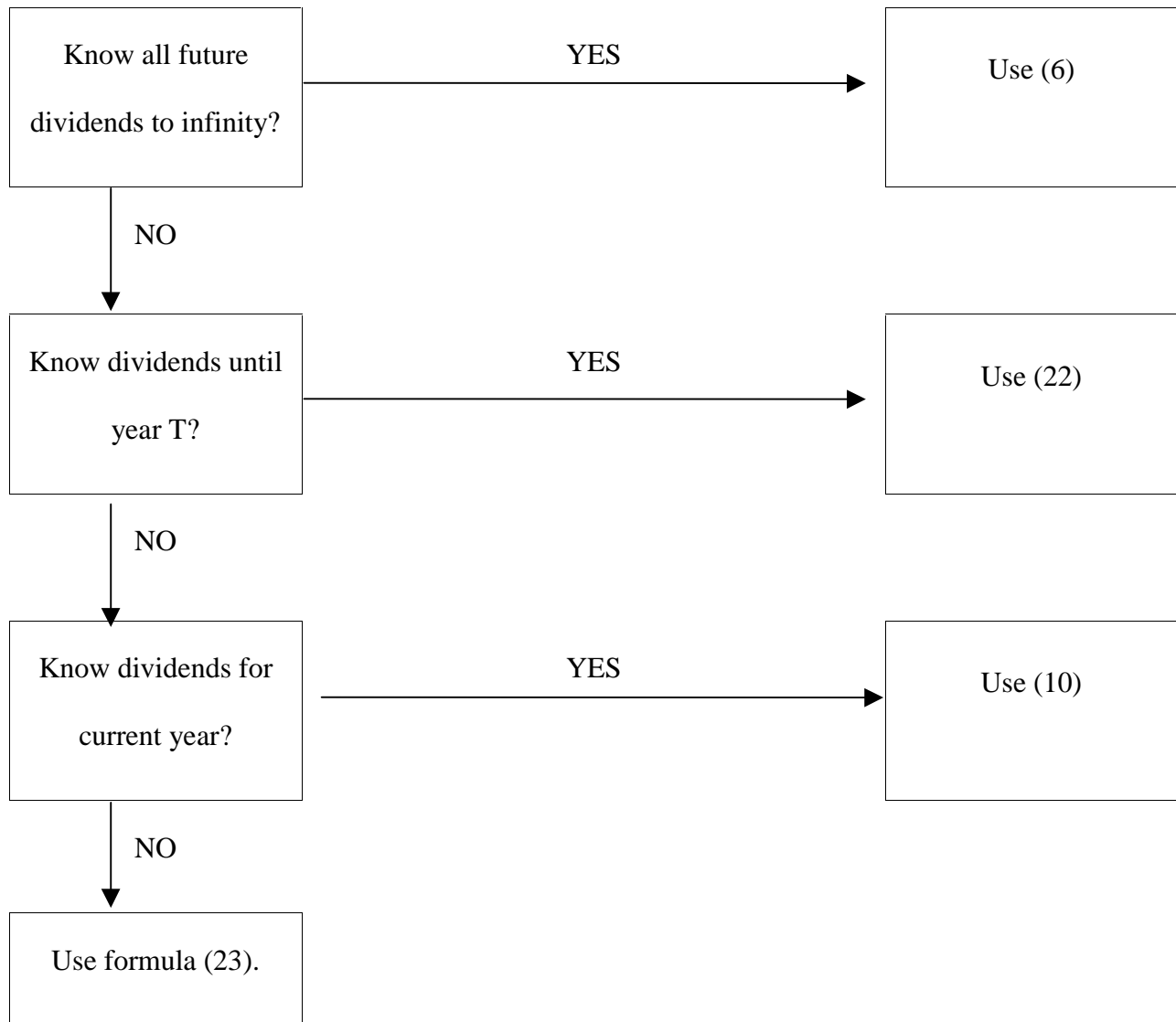
$$P_0 = \frac{(1 + g)D_0}{r_e - g} \tag{23}$$

which only relies on the known historic dividend and our estimates for the constant growth rate and the cost of equity capital. We can summarize our discussion in the following flow chart. In this chart "know.." refers to having a forecast except constant growth.

---

[8] See slides 20, 28 of the class notes.

Figure 1

```
┌─────────────────────┐                                    ┌─────────────────────┐
│  Know all future    │            YES                     │                     │
│                     │ ─────────────────────────────────▶ │      Use (6)        │
│ dividends to infinity?                                   │                     │
└─────────────────────┘                                    └─────────────────────┘
          │
          │ NO
          ▼
┌─────────────────────┐                                    ┌─────────────────────┐
│ Know dividends until│            YES                     │                     │
│                     │ ─────────────────────────────────▶ │      Use (22)       │
│      year T?        │                                    │                     │
└─────────────────────┘                                    └─────────────────────┘
          │
          │ NO
          ▼
┌─────────────────────┐                                    ┌─────────────────────┐
│  Know dividends for │            YES                     │                     │
│                     │ ─────────────────────────────────▶ │      Use (10)       │
│    current year?    │                                    │                     │
└─────────────────────┘                                    └─────────────────────┘
          │
          │ NO
          ▼
┌─────────────────────┐
│                     │
│  Use formula (23).  │
│                     │
└─────────────────────┘
```

3.11 Summary and Preview

The key results of this lecture are:

- Stocks can be valued as present discounted values of future dividends or as dividends plus expected capital gains.

- Projecting growing dividends into the future leads to the dividend growth model. This model implies that the expected return on equity equals the prospective dividends plus expected capital gains.

- P/E ratios are determined by three key parameters: (1) the growth rate of future dividends, (2) the expected return on equities, and (3) the payout ratio. A valuation using P/E ratio is simple, but requires that comparable companies are carefully selected.

- Stock prices can be decomposed into one component that reflects the present value of future earnings generated by assets in place, and another component that reflects the present value of growth opportunities.

The analysis presented here is simple. This has the advantage of generating powerful results, but implies also some limitations.

- The dividend growth model allows us to determine a relationship between the constant growth rate and the expected return on equity, but we need to know one in order to determine the other.

- Constant growth is a limiting assumption since some companies are not in a steady state.

- Financial ratios are widely used in practice because they are simple, but their applicability depends on a number of assumptions. We need to develop more robust methods to conduct company valuation.

Appendix

Derivation of equation (10): Reconsider equation (9) above. We can rewrite this as:

$$P_0 = \frac{D_1}{1 + r_e}\left(1 + x + x^2 + x^3 + ...\right) \quad where \quad x = \frac{1 + g}{1 + r_e} \tag{A 1}$$

Since $g < r_e$ by assumption, we have $x < 1$. Then we can use a standard algebraic result:

$$\left(1 - x\right)\left(1 + x + x^2 + x^3 + ...\right) = 1 - x + x^2 - x^2 + x^3 - ... = 1$$
$$\Rightarrow 1 + x + x^2 + x^3 + ... \frac{1}{1 - x} = \frac{1 + r_e}{r_e - g} \tag{A 2}$$

Inserting (A 2) into (A 1) we obtain:

$$P_0 = \frac{D_1}{1 + r_e}\frac{1 + r_e}{r_e - g} = \frac{D_1}{r_e - g} \tag{A 3}$$

which reproduces (10).

Derivation of equation (21):

Use (16) and observe the following sequence:

$$P_0 = \frac{dE_1}{r_e - g}$$
$$\Rightarrow r_e P_0 - g P_0 = dE_1 \tag{A 4}$$
$$\Rightarrow r_e P_0 = E_1 + g P_0 - \left(1 - d\right)E_1$$

Then divide the last equation by r_e.

Further Examples

Example 1

A company has just paid a dividend of US $2 per share. The dividend is expected to grow at a rate of 10 percent per year indefinitely. What is the stock price if the required rate of return is 12 percent? Here we apply the constant growth rate formula. We note that since the current dividend $D_0 = 2$, the period 1 dividend, D_1, will be $D_0 (1+g)$. Thus:

$$P_0 = \frac{D_1}{r_e - g} = \frac{D_0(1+g)}{r_e - g} = \frac{2(1.1)}{0.12 - 0.1} = \$110$$

Example 2

Suppose the company in the previous example has a new management. It is now expected that the rate of growth in dividends will be 15 percent per year for 3 years and then 10 percent per year thereafter. What is the new stock price if the required return remains at 12 percent?

Since the stock pays dividends at a constant rate indefinitely after year 3, find the price of the stock at the end of year 3 using the constant growth formula:

$$P_3 = \frac{D_4}{r_e - g} = \frac{3.346}{0.12 - 0.10} = 167.3$$

Finally, compute the current stock price by taking the present value of each of the first three dividends and the present value of the stock price in year 3:

$$P_0 = \frac{2.300}{1.12} + \frac{2.645}{1.12^2} + \frac{3042}{1.12^3} + \frac{167.30}{1.12^3} = \$125.41$$

Example 3

You're interested in buying the stock of company WWWFinance. Analysts expect earnings, and consequently dividends, to increase at a rate of 10% per year. The next dividend payout is expected to be 185 yen. Your required rate of return is 15%. What should the stock price be?

$$P_0 = \frac{185}{0.15 - 0.10} = 3,700$$

b) Assume that you have paid the price from a) for company WWWFinance stock. Suddenly, analysts revise their future earnings estimates for the company upwards to 11%. What is the new stock price? What absolute (percentage) gain have you realized on your investment?

$$P_{divgr=11\%} = \frac{185 * 1.11}{0.15 - 0.11} = 5,133.75 \quad Yen$$
$$Gain_{Absolute} = 5,133.75 - 3700 = 1,433.75$$
$$Gain_{Percent} = \frac{1,433.75}{3,700} = 38.75\%$$

Example 4

A company has a required return on equity of 10% and a prospective dividend yield of 4%. What is the expected growth? How much would the constant growth rate have to decrease so that the stock price falls by 20%? What is the dividend yield after the expected growth rate has fallen?

$$g = 10\% - 4\% = 6\%$$

After the fall the stock price drops from P_0 to P'_0. We have:

$$\frac{P_0^{'}}{P_0} = 0.8 = \frac{D_1/P_0}{D_1/P_0^{'}} = \frac{0.04}{D_1/P_0^{'}} \Rightarrow D_1/P_0^{'} = \frac{0.04}{0.8} = 5\%$$

$$g' = r_e - D_1/P_0^{'} = 10\% - 5\% = 5\%$$

Hence, a 1% drop in the constant growth rate from 6% to 5% would cause a drop in the stock price of 20% and an increase in the dividend yield from 4% to 5%.

Important Terminology

Important Formulae

Equation (6), (10), (16):

$$P_0 = \frac{D_1}{1 + r_e} + \frac{D_2}{(1 + r_e)^2} + \frac{D_3}{(1 + r_e)^3} + \ldots \qquad (6)$$

$$P_0 = \frac{D_1}{r_e - g} \qquad (10)$$

$$\frac{P_0}{E_1} = \frac{d}{r_e - g} \qquad (16)$$

There are many equations in this lecture, but if you remember (10), the definition of the payout ratio d=D/E, the definition of the growth rate g, then you can derive all of them from (10).