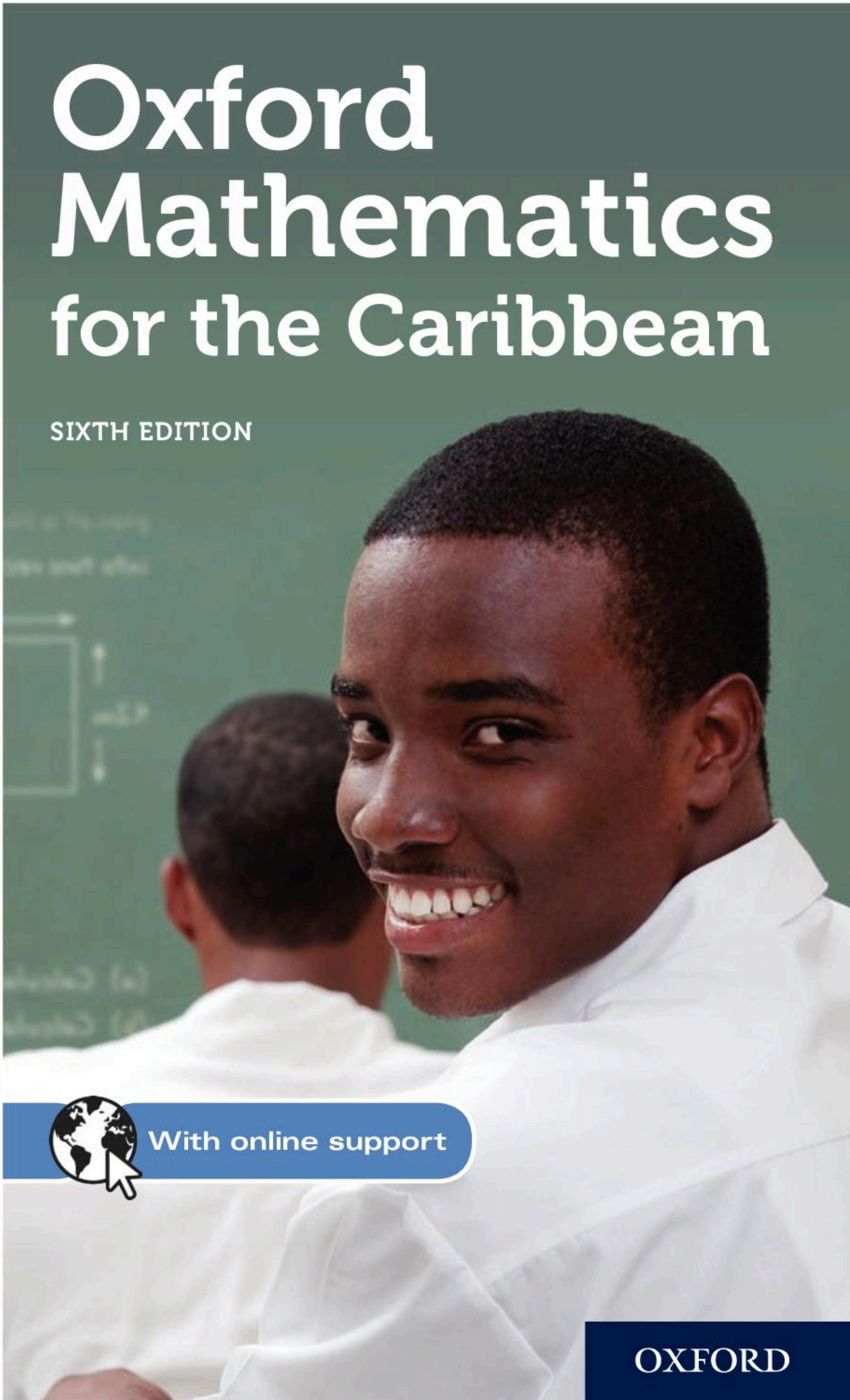
Oxford excellence for the Caribbean

Book 3

Oxford Mathematics for the Caribbean

SIXTH EDITION



With online support

Nicholas Goldberg

with Neva Cameron-Edwards

OXFORD

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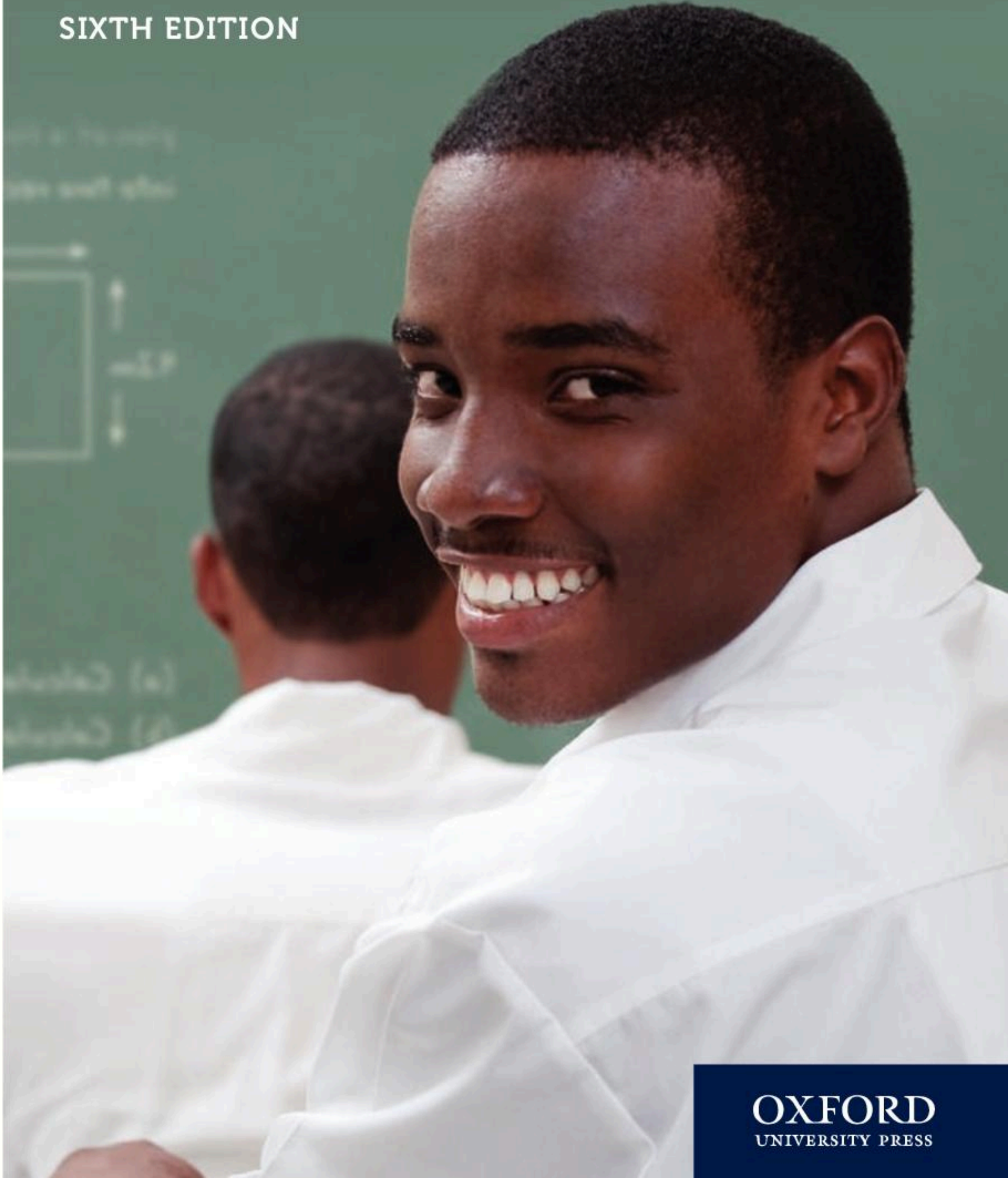

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Contents

About this book

This new edition of Oxford Mathematics for the Caribbean Student Book 3 has been revised to encompass recent changes to the mathematics curriculum throughout the region. Careful attention has been given to the major changes. As in the last editions, there is some content overlap across the three books in the series. This will allow schools to choose when certain topics are introduced but without any loss of continuity should they desire to begin a topic in a subsequent year.

In this edition, new activity boxes have been included that provide for more 'hands-on' experience and to offer a more student-centered approach. Some boxes provide links with other subject areas that teachers may wish to reinforce, while others include suggestions for group and project work.

To encourage greater use of technology in schools, the technology boxes and suggested websites have been revised. Many of these sites provide topic review and enhancement material – and many also showcase games to motivate learning. Support material is now available online at www.oxfordsecondary.com/9780198425793

The major features of previous editions have been retained:

- Check-in boxes at the start of each unit to assess whether a student is ready to begin the unit.
- Worked examples are provided throughout the text.
- Graded exercises, generally easier questions are given in the earlier parts of an exercise with more challenging questions underlined:

  $\underline{1}$ a single underline will challenge the average student

  $\underline{\underline{2}}$ a double underline will challenge the able student.

- Consolidation examples and exercises to provide further practice.
- Summary and checkout: these provide a quick review of the key points within the unit and enable the student to assess progress made.
- Review exercises with both extended response and multiple-choice format provided every four units.
- Units begin with a 'What's the point?' section to address links of the topic with the real world.

The aim of the book is to allow students to experience success and enjoyment in the learning of mathematics in the 21st century.

Nicholas Goldberg

Morne Jaune, Commonwealth of Dominica

December 2018

Computation 1

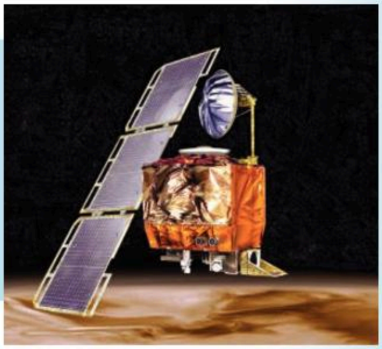
1

Objectives

- ✔ use four rules on fractions to solve problems
- ✔ use four rules on decimals to solve problems
- ✔ convert fractions to decimals to percentages
- ✔ make approximations by rounding numbers
- ✔ Calculate power of numbers
- ✔ Use scientific notation to record numbers

What's the point?

Accurate calculations are critical to make things work. In 1998 a miscalculation when converting one unit to another resulted in the loss of the Mars Climate Orbiter. The cost to the US taxpayer was of the order of US$320 million!



Before you start

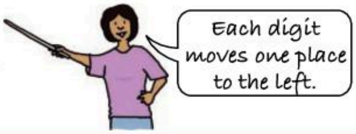
You should know ...

1 How to multiply a decimal by 10 or 100.

For example:

$2.41 \times 10 = 24.1$

$2.41 \times 100 = 241$

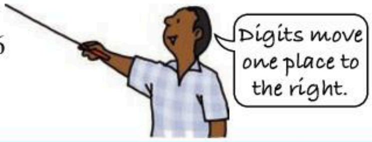


2 How to divide a decimal by 10 or 100.

For example:

$256 \div 10 = 25.6$

$256 \div 100 = 2.56$



Check in

1 Work out

(a) 2.3×10 **(b)** 0.23×10

(c) 2.3×100 **(d)** 0.23×100

(e) 1.61×10 **(f)** 1.61×100

2 Work out

(a) $20 \div 10$ **(b)** $20 \div 100$

(c) $2.4 \div 10$ **(d)** $2.4 \div 100$

(e) $13.1 \div 10$ **(f)** $13.1 \div 100$

1.1 Working with fractions

Fractions with common denominators are easily added or subtracted.

For example:

$\frac{3}{8}+\frac{1}{8}=\frac{4}{8}=\frac{1}{2}$

$\frac{4}{5}-\frac{1}{5}=\frac{3}{5}$

Fractions without common denominators need to be changed to equivalent fractions with the same denominators.

Example 1

Work out: **(a)** $\frac{3}{5}+\frac{1}{4}$ **(b)** $\frac{3}{8}-\frac{2}{7}$

(a) $\frac{3}{5}+\frac{1}{4}=\frac{12}{20}+\frac{5}{20}$ Common denominator is 20

$=\frac{17}{20}$

(b) $\frac{3}{8}-\frac{2}{7}=\frac{21}{56}-\frac{16}{56}=\frac{5}{56}$ Common denominator is 56

Addition and subtraction of mixed numbers is done in a similar manner.

Example 2

Work out: **(a)** $1\frac{3}{4}+2\frac{1}{5}$ **(b)** $3\frac{2}{5}-1\frac{3}{4}$

(a) $1\frac{3}{4}+2\frac{1}{5}=3+\frac{3}{4}+\frac{1}{5}=3+\frac{15}{20}+\frac{4}{20}$

$=3\frac{19}{20}$

(b) $3\frac{2}{5}-1\frac{3}{4}=\frac{17}{5}-\frac{7}{4}$ Change to improper fractions

$=\frac{68}{20}-\frac{35}{20}$ Common denominator is 20

$=\frac{33}{20}$

$=1\frac{13}{20}$ Change to mixed number

Exercise 1A

1 Work out:

(a) $\frac{1}{4}+\frac{1}{4}$
(b) $\frac{2}{5}+\frac{1}{5}$
(c) $\frac{5}{8}-\frac{3}{8}$
(d) $\frac{3}{4}+\frac{1}{4}$
(e) $\frac{4}{9}-\frac{2}{9}$
(f) $\frac{5}{9}-\frac{5}{9}$
(g) $\frac{7}{12}+\frac{5}{12}$
(h) $\frac{7}{20}+\frac{9}{20}$

2 Calculate:

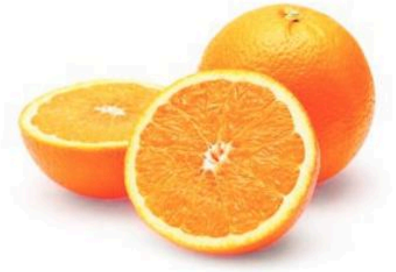
(a) $\frac{1}{2}+\frac{1}{4}$
(b) $\frac{1}{3}+\frac{1}{2}$
(c) $\frac{2}{3}+\frac{1}{5}$
(d) $\frac{2}{7}+\frac{3}{8}$
(e) $\frac{2}{9}+\frac{1}{3}$
(f) $\frac{2}{9}+\frac{1}{6}$
(g) $\frac{4}{7}+\frac{2}{9}$
(h) $\frac{5}{8}+\frac{2}{9}$

3 Calculate:

(a) $\frac{3}{5}-\frac{1}{4}$
(b) $\frac{5}{8}-\frac{1}{4}$
(c) $\frac{3}{7}-\frac{2}{9}$
(d) $\frac{4}{7}-\frac{3}{8}$
(e) $\frac{3}{4}-\frac{1}{2}$
(f) $\frac{11}{12}-\frac{2}{3}$
(g) $\frac{8}{9}-\frac{2}{3}$
(h) $\frac{6}{7}-\frac{3}{4}$

4 Work out:

(a) $3\frac{1}{3}+2\frac{1}{4}$
(b) $3\frac{1}{2}-1\frac{1}{4}$
(c) $7\frac{2}{3}+1\frac{3}{4}$
(d) $4\frac{3}{5}-2\frac{2}{7}$
(e) $8\frac{1}{2}-3\frac{2}{3}$
(f) $6\frac{3}{5}+4\frac{1}{2}$
(g) $5\frac{2}{3}-2\frac{3}{4}$
(h) $5\frac{1}{4}-1\frac{4}{5}$

5 

Jackson bought $3\frac{1}{4}$ kg of oranges. He gave his sister $1\frac{2}{3}$ kg.
What weight of oranges did he have left?

6 A plank of wood is 3 m in length. How long will it be if I cut $\frac{5}{8}$ m of wood from it?

7 Karen spent $\frac{4}{7}$ of her money on Monday. She spent $\frac{1}{3}$ of her money on Tuesday. What fraction of money did she
 - **(a)** spend on Monday and Tuesday?
 - **(b)** have left?

8 A container holds $4\frac{1}{2}$ litres of water.
 - **(a)** How much water is left in the container if Jerry drinks $\frac{7}{8}$ litre?
 - **(b)** What fraction of the water did Jerry drink?

Multiplying and dividing fractions

To multiply fractions you just need to multiply the numerators and the denominators of the two fractions.

For example:

$$\frac{2}{3}\times\frac{2}{5}=\frac{2\times2}{3\times5}=\frac{4}{15}$$

$$1\frac{1}{2}\times\frac{3}{4}=\frac{3}{2}\times\frac{3}{4}=\frac{3\times3}{2\times4}=\frac{9}{8}=1\frac{1}{8}$$

Often you can simplify the fractions before doing the multiplication.

For example:

$$\frac{2}{\cancel{3}_1}\times\frac{\cancel{3}^1}{8}=\frac{\cancel{2}^1}{\cancel{3}_1}\times\frac{\cancel{3}^1}{\cancel{8}_4}=\frac{1\times1}{1\times4}=\frac{1}{4}$$

Division of fractions is a little more tricky.
Recall that:

$$9\div3=9\times\frac{1}{3}=3$$

$$8\div2=8\times\frac{1}{2}=4$$

Notice that the operation

$\div\,3$ is the same as $\times\frac{1}{3}$

$\div\,2$ is the same as $\times\frac{1}{2}$

In general, division by a number is the same as multiplying by the number's **inverse**.

Under multiplication, the inverse of

3 is $\frac{1}{3}$ $(3\times\frac{1}{3}=1)$

$\frac{1}{2}$ is 2 $(\frac{1}{2}\times2=1)$

$\frac{3}{4}$ is $\frac{4}{3}$ $(\frac{3}{4}\times\frac{4}{3}=1)$

Using this idea, all divisions of fractions can be turned into multiplications.

Example 3

Calculate:

(a) $\frac{3}{5}\div\frac{2}{3}$ **(b)** $2\frac{1}{4}\div1\frac{3}{8}$

(a) $\frac{3}{5}\div\frac{2}{3}=\frac{3}{5}\times\frac{3}{2}$

$\div\frac{2}{3}$ is the same as $\times\frac{3}{2}$

$=\frac{9}{10}$

(b) $2\frac{1}{4}\div1\frac{3}{8}=\frac{9}{4}\div\frac{11}{8}$

Change to improper fractions

$=\frac{9}{4}\times\frac{8}{11}$

$\div\frac{11}{8}$ is the same as $\times\frac{8}{11}$

$=\frac{9}{{}_1\cancel{4}}\times\frac{\cancel{8}^2}{11}$

Simplify

$=\frac{18}{11}=1\frac{7}{11}$

Exercise 1B

1 Work out:

(a) $3\times\frac{1}{2}$ **(b)** $\frac{1}{2}\times\frac{1}{2}$

(c) $\frac{3}{4}\times\frac{1}{2}$ **(d)** $\frac{3}{4}\times\frac{2}{3}$

(e) $\frac{7}{8}\times\frac{2}{5}$ **(f)** $\frac{3}{4}\times\frac{4}{7}$

(g) $\frac{6}{11}\times\frac{4}{9}$ **(h)** $\frac{5}{8}\times\frac{4}{5}$

2 Calculate:

(a) $1\frac{1}{2} \times 3$ **(b)** $1\frac{1}{2} \times 1\frac{1}{2}$

(c) $2\frac{1}{4} \times 1\frac{1}{4}$ **(d)** $3\frac{2}{3} \times 2\frac{1}{2}$

(e) $2\frac{1}{2} \times 2\frac{1}{2}$ **(f)** $4\frac{3}{4} \times 1\frac{7}{8}$

(g) $3\frac{1}{3} \times 4\frac{3}{4}$ **(h)** $5\frac{2}{5} \times 3\frac{7}{11}$

3 Calculate:

(a) $\frac{1}{2} \div \frac{1}{4}$ **(b)** $\frac{1}{2} \div \frac{1}{3}$

(c) $\frac{3}{4} \div \frac{1}{4}$ **(d)** $\frac{7}{8} \div \frac{3}{4}$

(e) $\frac{7}{8} \div \frac{2}{5}$ **(f)** $\frac{3}{4} \div \frac{4}{5}$

(g) $\frac{6}{11} \div \frac{4}{9}$ **(h)** $\frac{3}{14} \div \frac{4}{7}$

4 Work out:

(a) $1\frac{1}{4} \div 2\frac{1}{2}$ **(b)** $2\frac{1}{2} \div 1\frac{1}{4}$

(c) $3\frac{1}{2} \div 1\frac{1}{3}$ **(d)** $2\frac{2}{3} \div 1\frac{3}{4}$

(e) $4\frac{2}{5} \div 1\frac{2}{3}$ **(f)** $4\frac{3}{7} \div 1\frac{6}{7}$

(g) $5\frac{4}{5} \div 2\frac{4}{7}$ **(h)** $3\frac{3}{4} \div 6\frac{2}{3}$

5 Calculate:

(a) $\dfrac{2\frac{1}{2} + 1\frac{1}{4}}{2\frac{1}{2} \times 1\frac{1}{4}}$ **(b)** $\dfrac{(2\frac{1}{2})^2}{4\frac{1}{2} \times 1\frac{3}{4}}$

(c) $\dfrac{3\frac{1}{2} - 1\frac{7}{8}}{4\frac{1}{4} + 2\frac{1}{2}}$ **(d)** $\dfrac{\frac{7}{8} - \frac{3}{4}}{2\frac{2}{5} \times 1\frac{1}{4}}$

6

A packet of biscuits has a mass of $\frac{1}{3}$ kg.
Jason eats $\frac{2}{3}$ of the packet.
What is the mass of biscuits left?

7 The area of a rectangular field is $130\frac{2}{3}$ m^2.
What is the field's length if its width is $10\frac{1}{2}$ m?

8

$\frac{3}{4}$ litre bottles of washing-up liquid are taken from a barrel holding $164\frac{1}{2}$ litres of liquid.

(a) How many bottles can be filled from the container?

(b) When 75 bottles have been removed from the container, what fraction of liquid remains in the container?

Technology

Need help with fractions?
Visit the website

www.coolmath.com

There is a complete course on fractions in the prealgebra section. Choose the lessons you need further help on.
Don't forget to do the questions!

1.2 Working with decimals

Decimal numbers are usually easier to use than fractions.

Decimals are really special kinds of fractions called decimal fractions.

Decimal fractions are fractions whose denominator is a multiple of ten.

For example:

$$\frac{7}{10} = 0.7 \qquad \frac{4}{100} = 0.04$$

$$\frac{56}{100} = 0.56 \qquad \frac{378}{100} = 3.78$$

Adding and subtracting decimals is just like adding and subtracting whole numbers. You just need to keep the digits in their correct columns.

Example 4

Calculate:

(a) 1.7 + 0.06 + 53.845

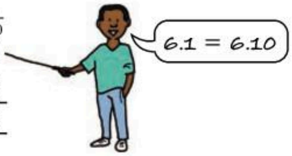
(b) 6.1 − 2.45

(a) 1.7 + 0.06 + 53.845

T	O	.	$\frac{1}{10}$	$\frac{1}{100}$	$\frac{1}{1000}$
	1	.	7		
	0	.	0	6	
5	3	.	8	4	5
5	5	.	6	0	5

(b) 6.1 − 2.45

T	O	.	$\frac{1}{10}$	$\frac{1}{100}$
	6	.	1	0
	2	.	4	5
	3	.	6	5



Exercise 1C

1 Work out:

(a) 0.3 + 0.2 **(b)** 7.6 + 1.9

(c) 5.3 − 1.7 **(d)** 48.2 − 17.6

(e) 14 − 0.3 **(f)** 6.1 + 1.2 + 0.8

(g) 8.3 − 0.6 **(h)** 8 + 0.2 + 3.45

2 Find:

(a) 13.3 − 1.45 **(b)** 100 − 6.8

(c) 18 + 0.2 + 1.85 **(d)** 53.1 − 0.07

(e) 17.8 − 12.35 **(f)** 2 − 0.017

(g) 63 + 0.2 + 15.61 **(h)** 77.84 − 34.95

3 Calculate:

(a) 13.3 + 0.68 + 0.7 + 4

(b) 15.8 − 1.45 + 6

(c) 68.435 − 1.8 + 6.2

(d) 17 − 6.23 + 0.8 − 0.91

(e) 53.2 − 13.86 + 4.5 + 6

4

The results in the final of the men's 200 m at the 2009 World Athletics Championships in Berlin were:

Usain Bolt	19.19 s
Alonso Edward	19.81 s
Wallace Spearman	19.85 s
Shawn Crawford	19.89 s
Steve Mullings	19.98 s
Charles Clark	20.39 s
Ramil Guliyev	20.61 s
David Alerte	20.68 s

(a) What was the difference between Bolt's and Edward's times?

(b) By how much did

(i) Bolt

(ii) Mullings

beat the 20 s barrier?

5 The rainfall on five days in August, on a certain island, was:

Day 1	2 mm
Day 2	6.3 mm
Day 3	1.7 mm
Day 4	0 mm
Day 5	0.8 mm

(a) How much rain fell over the 5-day period?

(b) How much more rain fell on day 1 than on day 5?

6 Explain the mistakes in these calculations.

(a) 4 − 1.27 = 3.27

(b) 6.1 + 0.8 + 8 = 7.7

(c) 0.6 − 0.031 = 0.631

Work out the correct answer in each case.

Multiplying and dividing decimals

Multiplying decimals is the same as multiplying whole numbers. You need to be careful where you put the decimal point.

One way to do this is to make an estimate of the answer.

Example 5

Work out 13.4×6.2

Estimate $13.4 \times 6.2 \simeq 13 \times 6 = 78$

Multiplying the numbers without decimal points:

$$\begin{array}{r} 134 \\ \times\, 62 \\ \hline 268 \\ 8040 \\ \hline 8308 \end{array}$$

The estimate shows that the answer to 13.4×6.2 is around 80, so the decimal point must be placed to give 83.08.
That is $13.4 \times 6.2 = 83.08$

Another way is simply to count the number of digits after the decimal point.

Example 6

Work out 1.73×1.9

$173 \times 19 = 3287$

$$\begin{array}{lrl} \text{so} & 1.\underline{7}\,\underline{3} & \text{2 decimal places} \\ & \times\, 1.\underline{9} & \text{1 decimal place} \\ \cline{2-2} & 3.2\,8\,7 & \text{3 decimal places} \end{array}$$

There are 2 digits after the decimal point in 1.73 and 1 digit after the decimal point in 1.9.
Hence, there should be $2 + 1 = 3$ digits after the decimal point in the answer.

Exercise 1D

1 Given that $5.6 \times 23 = 1288$, work out:
(a) 5.6×2.3 **(b)** 56×2.3
(c) 0.56×2.3 **(d)** 0.56×23
(e) 0.056×0.23 **(f)** 5.6×0.23
(g) 0.56×0.023 **(h)** 560×0.0023

2 Calculate:
(a) 0.3×6 **(b)** 0.13×6
(c) 1.41×5 **(d)** 0.38×7
(e) 6.18×9 **(f)** 7.3×12
(g) 13×0.13 **(h)** 29×0.814

3 Calculate:
(a) 0.4×0.3 **(b)** 1.6×0.4
(c) 6.2×0.9 **(d)** 0.31×0.3
(e) 0.82×0.7 **(f)** 0.34×7.2
(g) 16.1×0.31 **(h)** 4.84×0.62

4 What is the area of a rectangle with
(a) length 2.3 m, width 6.8 m
(b) length 19.3 cm, width 8.3 cm
(c) length 14.32 m, width 6.14 m?

5 Cloth costs $43.15 per metre. What is the cost of
(a) 12 metres
(b) 6.1 metres
(c) 0.4 metres?

6 Patsy walks 4.2 km in one hour. How far does she travel in
(a) 5 hours
(b) 1.3 hours
(c) 0.6 hours?

7 Raymond worked the multiplication 1.6×0.4 as follows:

$$\begin{aligned} 1.6 \times 0.4 &= 1\frac{6}{10} \times \frac{4}{10} \\ &= \frac{16}{10} \times \frac{4}{10} \\ &= \frac{64}{100} = 0.64 \end{aligned}$$

Work out these multiplications using Raymond's method.
(a) 1.5×0.5 **(b)** 6.2×0.8
(c) 0.8×0.3 **(d)** 0.84×0.6
(e) 1.3×2.4 **(f)** 0.94×3.1
(g) 6.45×2.14 **(h)** 17.2×0.68

8 **(a)** Rework Question 3 using Raymond's method of Question 7.
(b) Which of the three methods shown in Examples 5 and 6 and Question 7 do you prefer? Explain.
(c) Which method would you use to explain the multiplication of decimals to a new student? Why?

9 Raymond's friend Rani worked 1.6×0.4 like this:

$$\begin{aligned}1.6 \times 0.4 &= 16 \text{ tenths} \times 4 \text{ tenths}\\ &= 64 \text{ hundredths}\\ &= 0.64\end{aligned}$$

(a) Rework Question 3 using Rani's method.
(b) Which method do you prefer, Raymond's or Rani's? Why?

To divide decimals by decimals, it is usually easier to change the division so that the divisor is a whole number.

Example 7

Calculate:

(a) $8.4 \div 0.4$ **(b)** $3 \div 0.8$

(a) $$\begin{aligned}8.4 \div 0.4 &= \frac{8.4}{0.4}\\ &= \frac{8.4 \times 10}{0.4 \times 10}\\ &= \frac{84}{4} = 21\end{aligned}$$

Change 0.4 to whole number

(b) $$\begin{aligned}3 \div 0.8 &= \frac{3}{0.8}\\ &= \frac{3 \times 10}{0.8 \times 10}\\ &= \frac{30}{8} = 3.75\end{aligned}$$

Exercise 1E

1 Calculate:
- **(a)** $8 \div 0.2$
- **(b)** $6 \div 0.3$
- **(c)** $4.2 \div 2$
- **(d)** $17.4 \div 5$
- **(e)** $12 \div 0.4$
- **(f)** $12 \div 0.04$
- **(g)** $9 \div 0.3$
- **(h)** $16 \div 0.08$

2 Calculate:
- **(a)** $8.5 \div 0.5$
- **(b)** $6.4 \div 0.4$
- **(c)** $6.4 \div 0.04$
- **(d)** $0.64 \div 0.4$
- **(e)** $13.4 \div 0.5$
- **(f)** $23.68 \div 0.4$
- **(g)** $5.472 \div 1.2$
- **(h)** $12.488 \div 2.23$

3

A ball of string measures 21.6 m. How many 0.16 m lengths of string can be cut from it?

4 How many 0.3 litre bottles of juice can be filled from an urn containing 45.9 litres?

5 The area of a rectangular field is 100.8 m^2. What is the width of the field if its length is
- **(a)** 14.4 m
- **(b)** 11.2 m
- **(c)** 6.3 m
- **(d)** 0.35 m?

6 What is the cost of beef per kilogram, if 6.12 kg of beef sells for $94.86?

7 Fatima works the division $6.24 \div 0.4$ as follows:

$$\begin{aligned}6.24 \div 0.4 &= 6\frac{24}{100} \div \frac{4}{10}\\ &= \frac{624}{100} \div \frac{4}{10}\\ &= \frac{624}{10\cancel{0}} \times \frac{1\cancel{0}}{4}\\ &= \frac{156}{10}\\ &= 15\frac{6}{10} = 15.6\end{aligned}$$

Work out these divisions using Fatima's method.
- **(a)** $8 \div 0.2$
- **(b)** $3.4 \div 0.2$
- **(c)** $8.5 \div 1.7$
- **(d)** $4.5 \div 0.15$
- **(e)** $1.28 \div 3.2$
- **(f)** $0.576 \div 1.2$
- **(g)** $3.61 \div 0.019$
- **(h)** $14.4 \div 0.24$

8 **(a)** Which method of dividing decimals do you prefer, Fatima's way in Question 7 or that of Example 7? Explain.
(b) Try and find other ways of doing decimal division. Which one do you like the best?

Technology

You can get lots of extra practice using the four rules on decimals by visiting the website

www.mathsisfun.com

and following the links to Worksheets, Decimals.

There's enough practice even for a workaholic!!
Great lessons to complement the worksheet practice can be found at

www.mathsisfun.com

(follow the links to Numbers, Decimals Menu)

or *www.coolmath.com*

(follow the links to Prealgebra, Decimals).

1.3 Fractions, decimals and percentages

To change any fraction to a decimal you divide the numerator by the denominator.

For example, $\frac{3}{4} = 3 \div 4 = 3.00 \div 4$

$$\begin{array}{r} 0.75 \\ 4\overline{)\,3.00} \\ \underline{2\,8} \\ 20 \\ \underline{20} \\ \underline{0} \end{array}$$

Check the answer this way:

$$0.75 = \frac{7}{10} + \frac{5}{100} = \frac{75}{100} = \frac{15}{20} = \frac{3}{4}$$

Exercise 1F

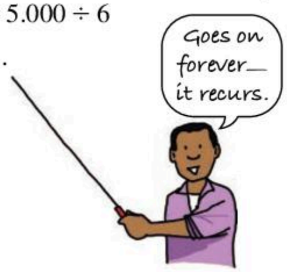
1 Change these fractions to decimals:

(a) $\frac{1}{2}$ **(b)** $\frac{3}{5}$ **(c)** $\frac{13}{20}$ **(d)** $\frac{3}{8}$
(e) $\frac{6}{25}$ **(f)** $\frac{17}{25}$ **(g)** $\frac{7}{8}$ **(h)** $\frac{3}{16}$
(i) $\frac{13}{16}$ **(j)** $\frac{5}{12}$

2 Check your answers to Question 1 by writing your decimal answers as fractions.

3 Some fractions do not turn into simple decimals:
$\frac{5}{6} = 5 \div 6 = 5.000 \div 6$

$$\begin{array}{r} 0.833\,\ldots \\ 6\overline{)\,5.000} \\ \underline{4\,8} \\ 20 \\ \underline{18} \\ 20 \\ \underline{18} \\ \underline{2} \end{array}$$



That is $\frac{5}{6} = 0.833\ldots$. Using this method write these fractions as decimals.

(a) $\frac{2}{3}$ **(b)** $\frac{1}{6}$ **(c)** $\frac{4}{9}$ **(d)** $\frac{3}{11}$
(e) $\frac{5}{7}$ **(f)** $\frac{6}{19}$ **(g)** $\frac{12}{13}$ **(h)** $\frac{14}{17}$
(i) $\frac{8}{33}$ **(j)** $\frac{37}{43}$

Investigation

Fractions like $\frac{2}{3} = 0.66\ldots$
and $\frac{1}{9} = 0.11\ldots$
recur after one decimal place.

(a) What other fractions recur after one decimal place?
(b) What fractions recur after two decimal places?
For example, $\frac{5}{33} = 0.1515\ldots$

Fractions and percentages

Percentages are really fractions with denominators of 100.

For example, $15\% = \frac{15}{100}$

Percentages with fractions can be dealt with in the same way.

Example 8

Write $37\frac{3}{4}\%$ as a fraction.

$$37\tfrac{3}{4}\% = \frac{37\frac{3}{4}}{100} = \frac{\frac{151}{4}}{100}$$

$$= \frac{\frac{151}{4} \times 4}{100 \times 4}$$

$$= \frac{151}{400}$$

The quickest way to change a fraction to a percentage is to multiply the fraction by 100%.

Example 9

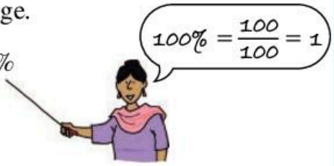
Write $\frac{3}{8}$ as a percentage.

$$\tfrac{3}{8} = \tfrac{3}{8} \times 100\%$$

$$= \tfrac{300}{8}\%$$

$$= 37\tfrac{4}{8}\%$$

$$= 37\tfrac{1}{2}\%$$



Exercise 1G

1 Write these percentages as fractions in their simplest form.

(a) 25% **(b)** 30% **(c)** 5% **(d)** 85%
(e) 48% **(f)** 3% **(g)** 91% **(h)** 83%

2 Write each of these percentages as fractions in their simplest form.

(a) $12\frac{1}{2}\%$ **(b)** $87\frac{1}{2}\%$ **(c)** $6\frac{1}{4}\%$
(d) $6\frac{3}{4}\%$ **(e)** $66\frac{2}{3}\%$ **(f)** $11\frac{1}{9}\%$
(g) $48\frac{2}{5}\%$ **(h)** $21\frac{5}{7}\%$ **(i)** $2\frac{2}{7}\%$
(j) $99\frac{9}{11}\%$

3 Write these fractions as percentages.

(a) $\frac{1}{2}$ **(b)** $\frac{3}{4}$ **(c)** $\frac{4}{5}$
(d) $\frac{6}{20}$ **(e)** $\frac{17}{25}$ **(f)** $\frac{7}{8}$
(g) $\frac{2}{3}$ **(h)** $\frac{5}{6}$ **(i)** $\frac{3}{11}$
(j) $\frac{14}{15}$ **(k)** $\frac{13}{17}$ **(l)** $\frac{15}{23}$
(m) $\frac{3}{29}$ **(n)** $\frac{52}{63}$ **(o)** $\frac{5}{101}$

You should now be able to write any number as a fraction, decimal or percentage.

Example 10

Write the decimal 0.625 as:

(a) a fraction
(b) a percentage.

(a) $0.625 = \frac{625}{1000} = \frac{125}{200} = \frac{25}{40} = \frac{5}{8}$
(b) $0.625 = 0.625 \times 100\% = 62.5\%$

Example 11

Write 48.5% as:

(a) a fraction
(b) a decimal.

(a) $$48.5\% = \frac{48.5}{100} = \frac{48.5 \times 10}{100 \times 10} = \frac{485}{1000}$$

$$= \frac{97}{200}$$

(b) $$48.5\% = \frac{48.5}{100} = 48.5 \div 100$$

$$= 0.485$$

Exercise 1H

1 Write the decimal 0.65 as:

(a) a fraction **(b)** a percentage.

2 Write 15% as:

(a) a fraction **(b)** a decimal.

3 Write each of these percentages as decimals.

(a) 25% **(b)** 30%
(c) 78% **(d)** 85%
(e) $2\frac{1}{2}\%$ **(f)** $42\frac{3}{4}\%$

4 Write these decimals as percentages.

(a) 0.5 **(b)** 0.65
(c) 0.37 **(d)** 0.475
(e) 0.395 **(f)** 0.8625

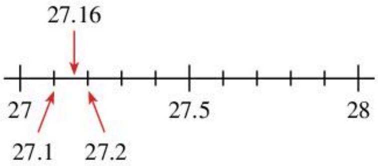
Technology

Want further practice or help on these types of conversions?
Visit the Prealgebra section in
www.coolmath.com

You will find lessons on converting fractions to decimals to percentages, under the topics fractions, decimals and percentages.
If you want to review operations with decimals just click on the decimal lessons.

1.4 Rounding numbers

You can round decimal numbers to the nearest tenth, hundredth or thousandth. A number line shows how this is done.



27.16 when rounded to the nearest tenth is 27.2 because 27.16 is closer to 27.2 than to 27.1.
You can write

27.16 = 27.2 correct to 1 decimal place (1 d.p.)

On the other hand

27.14 = 27.1 (1 d.p.)

since 27.14 is closer to 27.1 than 27.2.

In general if the next placed number is 0, 1, 2, 3 or 4 round down. If it is 5, 6, 7, 8 or 9 round up.

Example 12

Round 4.655 and 0.8723 to
(a) 2 decimal places
(b) 1 decimal place.

(a) The second decimal place is underlined.

4.6$\underline{5}$⑤ $\xrightarrow{\text{round up}}$ 4.66 (2 d.p.)

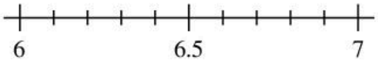
0.8$\underline{7}$②3 $\xrightarrow{\text{round down}}$ 0.87 (2 d.p.)

(b) The first decimal place is underlined

4.$\underline{6}$⑤5 $\xrightarrow{\text{round up}}$ 4.7 (1 d.p.)

0.$\underline{8}$⑦23 $\xrightarrow{\text{round up}}$ 0.9 (1 d.p.)

Exercise 1I



1 Use the number line to write these numbers correct to one decimal place.

(a) 6.14	**(b)** 6.25	**(c)** 6.71
(d) 6.38	**(e)** 6.91	**(f)** 6.99
(g) 6.41	**(h)** 6.75	**(i)** 6.08

2 Write correct to one decimal place.

(a) 3.22	**(b)** 4.36	**(c)** 0.82
(d) 0.76	**(e)** 0.65	**(f)** 4.98
(g) 6.02	**(h)** 7.07	**(i)** 1.19

3 Write correct to two decimal places.

(a) 6.231	**(b)** 0.782	**(c)** 0.965
(d) 4.025	**(e)** 4.204	**(f)** 6.107
(g) 4.003	**(h)** 0.028	**(i)** 0.205

4 Write these numbers

(a) 6.4183	**(b)** 0.7062	**(c)** 0.0215
(d) 9.6008	**(e)** 7.1808	**(f)** 0.3144
(g) 6.2999	**(h)** 4.0818	

correct to:
(i) 3 decimal places
(ii) 2 decimal places
(iii) 1 decimal place.

5 Copy and complete, rounding each number to the given number of decimal places.

Number	1 d.p.	2 d.p.	3 d.p.
3.1674			
0.0396			
17.8074			
6.0782			
105.1648			
94.0718			
1.2309			
6.7071			

6 **(a)** What is the perimeter of a rectangle with length 6.12 cm and width 4.77 cm correct to
(i) 2 d.p. **(ii)** 1 d.p.?

(b) What is its area correct to
(i) 2 d.p. **(ii)** 1 d.p.?

Significant figures

Rounding to one or two places of decimals does not always give a simple approximation for a number.

For example:

6 381 278 . 68 when rounded to 1 d.p. is

6 381 278 . 7

Which is not much simpler!

In such cases it is more useful to round off to a given number of **significant figures**.

Significant figures show the relative importance of the digits in a number, the first non-zero digit being the most important.

For example, to one significant figure (1 s.f.), 2914 becomes 3000.

Notice the column heads are:

Th	H	T	O
2	9	1	4

The first non-zero digit is the 2, which is thousands. In this case rounding to 1 s.f. is the same as rounding to the nearest thousand.

Example 13

Write 83 562 to

(a) 2 s.f.

(b) 1 s.f.

The column heads are

TTh	Th	H	T	O
8	3	5	6	2

(a) The second-digit head is thousands so round to the nearest thousand.
83 562 = 84 000 (2 s.f.)

(b) The first-digit head is tens of thousands so round to the nearest ten thousand.
83 562 = 80 000 (1 s.f.)

Here are some other examples

Number	67 341	23.478	346.12
3 s.f.	67 300	23.5	364
2 s.f.	67 000	23	350
1 s.f.	70 000	20	300

Exercise 1J

1 Write correct to one significant figure.

(a) 420 **(b)** 8314 **(c)** 396
(d) 48 **(e)** 4.81 **(f)** 91 265
(g) 10 034 **(h)** 24.68 **(i)** 7.0036

2 Write to two significant figures:

(a) 962 **(b)** 499 **(c)** 6183
(d) 17 638 **(e)** 17.34 **(f)** 29.92
(g) 4976.2 **(h)** 133.68 **(i)** 8.035

3 Copy and complete.

Number	**613 752**	**1.6831**	**8769.2**
3 s.f.			
2 s.f.			
1 s.f.			

4 Ramsingh wrote the following.

(i) 6384 = 6000 (1 s.f.)
(ii) 6384 = 6400 (2 s.f.)
(iii) 816 952 = 8 (1 s.f.)
(iv) 74.6 = 7 (1 s.f.)
(v) 947.82 = 95 (2 s.f.)

(a) Which items are correct?
(b) Write down the correct answers for any items he got wrong.

5 Round 8 134 767 to

(a) 4 s.f. **(b)** 3 s.f. **(c)** 2 s.f. **(d)** 1 s.f.

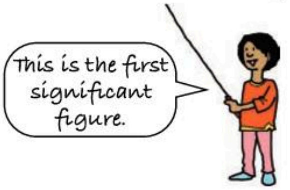
6 A number when rounded to 1 s.f. is 5000.
What is the
(a) largest number it could have been
(b) smallest number it could have been?

You can round numbers that are smaller than one to a given number of significant figures in the same way.

However, you must remember that the first **non-zero** digit is the first significant figure.

For example

0.0415 = 0.04 (to 1 s.f.)



Notice that the zeros before the 4 do not count as significant figures.

Example 14

Write 0.007092 to

(a) 3 s.f. **(b)** 2 s.f. **(c)** 1 s.f.

(a) The first three significant figures are 709 so 0.007092 = 0.00709 (3 s.f.)

(b) The first two significant figures are 70 so 0.007092 = 0.0071 (2 s.f)
(the zero is rounded up to 1 as the third significant figure is a 9)

(c) The first significant figure is 7 so 0.007092 = 0.007 (1 s.f.)

Exercise 1K

1 Write correct to one significant figure

(a) 0.67 **(b)** 0.92 **(c)** 0.07
(d) 0.069 **(e)** 0.123 **(f)** 0.00894
(g) 0.00612 **(h)** 0.000201 **(i)** 0.0009046

2 Write to two significant figures

(a) 0.672 **(b)** 0.915 **(c)** 0.777
(d) 0.0942 **(e)** 0.0106 **(f)** 0.00374
(g) 0.00655 **(h)** 0.000999 **(i)** 0.0316

3 Write to three significant figures

(a) 0.1411 **(b)** 0.3894 **(c)** 0.9625
(d) 0.03975 **(e)** 0.046661 **(f)** 0.02118
(g) 0.0007256 **(h)** 0.006082 **(i)** 0.090909

4 Copy and complete

Number	1 s.f.	2 s.f.	3 s.f.
0.42846			
0.09626			
0.07207			
0.009966			
0.0007109			

5 Daylana wrote

(i) 0.062 = 0.6 (1 s.f.)
(ii) 0.062 = 0.06 (2 s.f.)
(iii) 0.00716 = 0.007 (1 s.f.)
(iv) 0.000888 = 0.00088 (2 s.f.)
(v) 0.040067 = 0.040 (2 s.f.)

(a) Which items are correct?
(b) Write down the correct answers for the items she got wrong.

6 A number when rounded to 2 s.f. is 0.69.
What is the
(a) largest number it could have been
(b) smallest number it would have been?

7 The number 0.08471 when rounded to three significant figures is 0.0847. Explain why the answer is not 0.08470.

8 The number of persons attending a cricket match was 8327.
If you were reporting on the cricket match, what approximation would you use for the crowd site? Explain why.

9

(a) Find the distances in km of the planets from the sun.
(b) Write the distances
  (i) correct to two significant figure
  (ii) correct to two significant figures

10 One inch = 0.0254 metres.
Write
(a) 1 inch in metres to 1 s.f.
(b) 0.6 inches in metres to 3 s.f.
(c) 1.3 inches in metres to 2 s.f.
(d) 4 metres in inches to 2 s.f.
(e) 3.18 metres in inches to 1 s.f.

1.5 Estimation

When you have to measure something, it is a good habit to estimate the measurement, as a way to check your answer.

When you have to do a calculation, it is also a good habit to estimate the answer as a way to check your working out.

Making an estimate is important when you are calculating decimals, because it is easy to put a decimal point in the wrong place during your calculation.

The simplest way to make an estimate of a calculation is to write each number in the calculation correct to one significant figure.

Example 15

Estimate 3.9×5.1

$3.9 = 4$ (1 s.f.)
$5.1 = 5$ (1 s.f.)

So $3.9 \times 5.1 \simeq 4 \times 5 = 20$.

Exercise 1L

1 Estimate the answer in your head:
- **(a)** 1.9×3.2
- **(b)** 4.8×12.1
- **(c)** 2.36×11.01
- **(d)** 10.33×4.98
- **(e)** 6.87×6.21
- **(f)** $(4.31)^2$

2 Estimate the answer:
- **(a)** $(3.84)^3$
- **(b)** $2.7 \times 3.1 \times 6.2$
- **(c)** $3.8 \times 10.1 \times 1.9$
- **(d)** $2.8 \times 0.7 \times 6.9$
- **(e)** $(0.11)^2$
- **(f)** 0.107×0.091
- **(g)** 0.062×0.048
- **(h)** 0.067×0.081

3 Check the accuracy of your estimates in Questions 1 and 2 using a calculator.

The same idea can be used in more complex estimation of calculations. You can also improve your estimates by approximating numbers to two rather than one significant figure.

Example 16

Estimate **(a)** $47.83 \div 2.99$
(b) $0.78 \div 42$

(a) $47.83 \approx 48$ (2 s.f.)
$2.99 \approx 3$ (1 s.f.)

$$48 \div 3 = \tfrac{48}{3} = 16$$

So the answer for $47.83 \div 2.99$ is about 16.

(b) $0.78 \approx 0.8$ (1 s.f.)
$42 \approx 40$ (1 s.f.)

$$\begin{aligned} 0.8 \div 40 &= \tfrac{0.8}{40} \\ &= \tfrac{8}{400} \\ &= \tfrac{1}{50} \\ &= 0.02 \end{aligned}$$

So the answer for $0.78 \div 42$ is about 0.02.

Exercise 1M

1 Estimate the answer in your head:
- **(a)** $11.7 \div 2.8$
- **(b)** $13.8 \div 6.8$
- **(c)** 4.1×0.9
- **(d)** $6.32 \div 1.87$
- **(e)** $8.24 \div 0.96$
- **(f)** $15.72 \div 2.13$

2 Estimate the answer to:
- **(a)** $5.42 \div 2.81$
- **(b)** $1.82 \div 2.6$
- **(c)** $14.08 \div 32.2$
- **(d)** $3.256 \div 3.1$
- **(e)** $123.69 \div 1.9$
- **(f)** $6.72 \div 320$

3 Check the accuracy of your estimates in Questions 1 and 2 using a calculator.

4 A list of calculations is given. The correct answers are included in the set at the end. Without doing the calculations, pick out the correct answer for each one.
- **(a)** $\dfrac{39 \times 71}{54}$
- **(b)** $\dfrac{28 \times 18}{589}$
- **(c)** $\dfrac{31 \times 7.9}{3021}$
- **(d)** $\dfrac{501 \times 81.2}{30.9}$

{137.24, 1316.54, 4.72, 0.86, 51.28, 12.46, 0.08}

5 St. Lucia has a population of 178 000 people. Its land area is 617 km². Estimate its population density.

6 **(a)** Find the current populations and land area of nations shown in the table below.

Nation	Population	Land area (km²)	Population density
Antigua			
Barbados			
Belize			
Dominica			
Jamaica			
St. Kitts			
St. Vincent			
Trinidad			

(b) Copy and complete the table estimating the population density in each nation.

7 **(a)** Estimate the value of 31.68×0.294
(b) Calculate the value of 31.68×0.294 writing your answer
- **(i)** correct to 2 decimal places
- **(ii)** correct to 2 significant figures

1.6 Indices

Recall that

$$5^3 = 5 \times 5 \times 5$$

and that

$$2^4 = 2 \times 2 \times 2 \times 2.$$

You can read 5^3 as '5 to the **power** of 3' and 2^4 as '2 to the **power** of 4'.

The number 3 in 5^3 is sometimes called the **index**.

Power or indices have their own rules. For example

$$\begin{aligned} 5^3 \times 5^4 &= 5 \times 5 \times 5 \quad \times \quad 5 \times 5 \times 5 \times 5 \\ &= 5^7 \end{aligned}$$

Notice that

$$5^3 \times 5^4 = 5^{3+4} = 5^7$$

In general

$$\boxed{a^m \times a^n = a^{m+n}}$$

In the same way

$$\begin{aligned} 5^8 \div 5^2 &= \frac{5 \times 5 \times 5 \times 5 \times 5 \times 5 \times 5 \times 5}{5 \times 2} \\ &= 5 \times 5 \times 5 \times 5 \times 5 \times 5 \\ &= 5^6 \end{aligned}$$

Notice that

$$5^8 \div 5^2 = 5^{8-2} = 5^6$$

In general

$$\boxed{a^m \div a^n = a^{m-n}}$$

You can use these **laws of indices** to calculate powers quickly.

Example 17

Calculate
(a) $3^8 \times 3^6$ **(b)** $15^8 \div 15^3$

(a) $3^8 \times 3^6 = 3^{8+6} = 3^{14}$
(b) $15^8 \div 15^3 = 15^{8-3} = 15^5$

Exercise 1N

1 Copy and complete
(a) $2^5 = 2 \times 2 \times 2 \times 2 \times 2 =$ ____
(b) $3^4 = 3 \times 3 \times$ ...... $=$ ____
(c) $4^3 =$ .... $= 64$
(d) $16 = 2 \times$ .... $= 2^{-}$
(e) $1000 = 10 \times$ .... $= 10^{-}$

2 Calculate
(a) 3^2 **(b)** 6^3 **(c)** 5^3
(d) 16^2 **(e)** 4^4 **(f)** 6^4

3 Work out
(a) $3^2 \times 3^2$ **(b)** 4×4^2
(c) $5^2 \times 5^2$ **(d)** $7^3 \times 7^2$
(e) $8^2 \times 8$ **(f)** $9^2 \times 9^2$

4 Work out
(a) $10^4 \div 10$ **(b)** $6^5 \div 6^3$
(c) $4^5 \div 4^4$ **(d)** $17^4 \div 17^3$
(e) $6^9 \div 6^6$ **(f)** $2^6 \div 2^{10}$

What was your answer to question 4(f)? You should have found that

$$\begin{aligned} 2^6 \div 2^{10} &= \frac{2 \times 2 \times 2 \times 2 \times 2 \times 2}{2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2} \\ &= \frac{1}{2 \times 2 \times 2 \times 2} \\ &= \frac{1}{2^4} \end{aligned}$$

But $2^6 \div 2^{10} = 2^{6-10} = 2^{-4}$

So $2^{-4} = \frac{1}{2^4}$.

This means that indices can be negative

$$2^{-1} = \frac{1}{2} = 0.5$$

$$2^{-2} = \frac{1}{2^2} = \frac{1}{4} = 0.25$$

$$2^{-3} = \frac{1}{2^3} = \frac{1}{8} = 0.125 \text{ etc.}$$

In general $a^{-n} = \frac{1}{a^n}$.

Example 18

Calculate

(a) $3^{-4} \times 3^2$ **(b)** $3^{-5} \div 3^2$

(a) $3^{-4} \times 3^2 = 3^{-4+2} = 3^{-2} = \frac{1}{3^2}$

(b) $3^{-5} \div 3^2 = 3^{-5-2} = 3^{-7} = \frac{1}{3^7}$

Exercise 1O

1 Calculate

(a) $3^2 \times 3^{-1}$ **(b)** $4^5 \times 4^{-3}$
(c) $6^2 \times 6^{-3}$ **(d)** $2^5 \times 2^{-8}$
(e) $4 \times 4^2 \times 4^{-4}$ **(f)** $7^2 \times 7^{-1} \times 7^{-2}$

2 Calculate

(a) $2^5 \div 2^{-2}$ **(b)** $2^3 \div 2^{-3}$
(c) $3^3 \div 3^{-1}$ **(d)** $3^{-2} \div 3^4$
(e) $6^{-3} \div 6^{-4}$ **(f)** $7^{-4} \div 7^{-3}$

3 **(a)** What is $5^3 \div 5^3$?
(b) use indices to show that
$5^0 = 1$

4 Write down the value of

(a) 3^0 **(b)** 3^{-1} **(c)** 3^{-2} **(d)** 7^{-3}

5 Write in index form

(a) $\frac{1}{5}$ **(b)** $\frac{1}{25}$ **(c)** $\frac{1}{8}$ **(d)** $\frac{1}{1000}$

Fractional indices

Indices can not only be both positive and negative whole numbers, they can also be fractions.

For example, what meaning could be attached to $16^{\frac{1}{2}}$?

Well

$16^{\frac{1}{2}} \times 16^{\frac{1}{2}} = 16^{\frac{1}{2}+\frac{1}{2}} = 16^1$

That is

$16^{\frac{1}{2}} \times 16^{\frac{1}{2}} = 16$

But $4 \times 4 = 16$

So $16^{\frac{1}{2}} = 4$

$16^{\frac{1}{2}}$ is simply the square root of 16 or $\sqrt{16}$. In the same way

$9^{\frac{1}{2}} = \sqrt{9} = 3$

$36^{\frac{1}{2}} = \sqrt{36} = 6$

$49^{\frac{1}{2}} = \sqrt{49} = 7$ etc.

What about $8^{\frac{1}{3}}$?

Well

$8^{\frac{1}{3}} \times 8^{\frac{1}{3}} \times 8^{\frac{1}{3}} = 8^{\frac{1}{3}+\frac{1}{3}+\frac{1}{3}} = 8$

But $2 \times 2 \times 2 = 8$

So $8^{\frac{1}{3}} = 2$

That is, $8^{\frac{1}{3}}$ is the **cube root** of 8 or $\sqrt[3]{8}$.

You can use these ideas to find the value of fractional powers of numbers.

Example 19

Work out:

(a) $64^{\frac{1}{2}}$ **(b)** $27^{\frac{1}{3}}$ **(c)** $27^{-\frac{2}{3}}$

(a) $64^{\frac{1}{2}} = \sqrt{64} = 8$

(b) $27^{\frac{1}{3}} = \sqrt[3]{27} = 3$

(c) $27^{-\frac{2}{3}} = \frac{1}{27^{\frac{2}{3}}} = \frac{1}{27^{\frac{1}{3}} \times 27^{\frac{1}{3}}}$
$= \frac{1}{3 \times 3} = \frac{1}{9}$

Exercise 1P

1 Calculate the value of

(a) $4^{\frac{1}{2}}$ **(b)** $64^{\frac{1}{3}}$ **(c)** $81^{\frac{1}{2}}$
(d) $4^{-\frac{1}{2}}$ **(e)** $64^{-\frac{1}{3}}$ **(f)** $81^{-\frac{1}{2}}$

2 Calculate

(a) $4^{\frac{3}{2}}$ **(b)** $25^{\frac{3}{2}}$ **(c)** $8^{\frac{2}{3}}$
(d) $4^{-\frac{3}{2}}$ **(e)** $25^{-\frac{3}{2}}$ **(f)** $8^{-\frac{2}{3}}$

3 Work out

(a) $125^{-\frac{1}{3}}$ **(b)** $81^{\frac{3}{2}}$ **(c)** $100^{-\frac{3}{2}}$
(d) $16^{-\frac{3}{2}}$ **(e)** $4^{-\frac{5}{2}}$ **(f)** $8^{\frac{5}{3}}$

1.7 Standard form

Approximating very large or very small numbers to one or two significant figures is one way of making them easier to work with. However, even approximations of huge and tiny numbers are troublesome to write down.

For example the mass of the moon is about 73 500 000 000 000 000 000 000 kg. It takes quite some time to write this out.

Very small numbers are just as awkward; a proton's mass is about

0.000 000 000 000 000 000 000 001 675 000 kg!

Scientists use **standard form** or **scientific notation** to write such numbers.

In standard form, the number is written as:

Number between 1 and 10 × ten to a power.

For example:

$$8000 = 8 \times 1000 = 8 \times 10^3$$

$$7\,000\,000 = 7 \times 1\,000\,000 = 7 \times 10^6$$

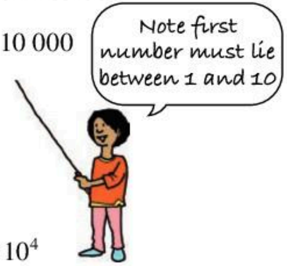
The idea is the same whatever the number.

Example 20

Write in standard form:

(a) 81 432 **(b)** 91 285 000

(a) $81\,432 = 8.1432 \times 10\,000$



$= 8.1432 \times 10^4$

(b) $91\,285\,000 = 9.1285 \times 10\,000\,000$

$= 9.1285 \times 10^7$

In standard form the mass of the moon becomes

7.35×10^{22} kg

which is much easier to write than

73 500 000 000 000 000 000 000 kg.

Exercise 1Q

1 Copy and complete

(a) $8000 = 8 \times \_\_\_\_\_ = 8 \times 10^2$
(b) $16000 = 1.6 \times \_\_\_\_\_ = 1.6 \times 10^{-}$
(c) $5600 = 5.6 \times \_\_\_\_\_ = 5.6 \times 10^{-}$
(d) $200\,000 = 2 \times \_\_\_\_\_ = 2 \times 10^{-}$
(e) $531\,000 = 5.31 \times \_\_\_\_ = \_\_\_ \times 10^{-}$

2 Write in standard form:

(a) 2000 **(b)** 400 **(c)** 80
(d) 90 000 **(e)** 4000 **(f)** 700 000
(g) 3 000 000 **(h)** 40 000 000 **(i)** 100 000

3 Write in full:

(a) 3×10^2 **(b)** 5×10^3 **(c)** 6×10^5
(d) 2×10^7 **(e)** 3×10^6 **(f)** 4×10^9

4 Write in standard form:

(a) 420 **(b)** 6300 **(c)** 170 000
(d) 23 000 **(e)** 61 300 **(f)** 9230
(g) 416 **(h)** 98 185 **(i)** 6 310 040

5 Write in full:

(a) 1.6×10^3 **(b)** 2.8×10^2 **(c)** 3.81×10^2
(d) 4.75×10^5 **(e)** 3.01×10^8 **(f)** 1.6×10^9

You can write very small numbers such as the mass of a proton in standard form also. To do so you need to use negative powers of 10.

Recall

$$10^{-1} = \frac{1}{10} = 0.1$$

$$10^{-2} = \frac{1}{100} = 0.01$$

$$10^{-3} = \frac{1}{1000} = 0.001$$

$$10^{-4} = \frac{1}{10\,000} = 0.0001 \text{ etc.}$$

Using this idea, you can write small numbers in standard form.

For example:

$$0.003 = 3 \div 1000 = 3 \times \frac{1}{1000}$$
$$= 3 \times 10^{-3}$$

$$0.000\,05 = 5 \div 100\,000 = 5 \times \frac{1}{100\,000}$$
$$= 5 \times 10^{-5}$$

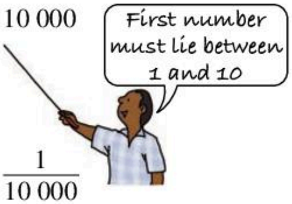
More complex numbers can be written in standard form in the same way. Just remember that the first number must lie between 1 and 10.

Example 21

Write in standard form.

(a) 0.0526 **(b)** 0.000 078 45

(a) $0.0526 = 5.26 \div 10\,000$



$$= 5.26 \times \frac{1}{10\,000}$$
$$= 5.26 \times 10^{-4}$$

(b) $0.000\,078\,45 = 7.845 \div 100\,000\,000$
$$= 7.845 \times \frac{1}{100\,000\,000}$$
$$= 7.845 \times \frac{1}{10^8}$$
$$= 7.845 \times 10^{-8}$$

In standard form the mass of a proton is

1.675×10^{-27} kg

which is much easier to write than 0.000 000 000 000 000 000 000 001 675 g.

Exercise 1R

1 Work out:

(a) 6×10^{-2} **(b)** 4×10^{-1}
(c) 8×10^{-3} **(d)** 5×10^{-4}
(e) 2×10^{-6} **(f)** 7×10^{-5}

2 Copy and complete

(a) $0.04 = 4 \div 100 = 4 \times \frac{1}{100} = 4 \times 10^{-}$
(b) $0.005 = 5 \div$ ___ $= 5 \times$ ___ $= 5 \times 10^{-}$
(c) $0.0009 = 9 \div$ ___ $= 9 \times$ ___ $= 9 \times 10^{-}$
(d) $0.031 = 3.1 \div$ ___ $= 3.1 \times$ ___ $= 3.1 \times 10^{-}$
(e) $0.00835 = 8.35 \div$ _ $= 8.35 \times$ _ $= 8.35 \times 10^{-}$

3 Write in standard form:

(a) 0.63 **(b)** 0.0074
(c) 0.028 **(d)** 0.000 13
(e) 0.023 56 **(f)** 0.000 82
(g) 0.000 003 91 **(h)** 0.0016
(i) 0.003 83

4 Write in standard form:

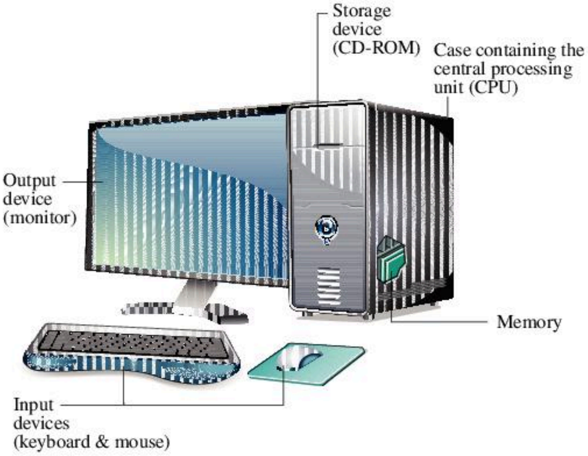
(a) 0.000 030 6 **(b)** 0.000 000 492 5
(c) 0.004 283 1 **(d)** 0.000 901
(e) 0.000 000 000 25 **(f)** 0.000 000 084 6
(g) 0.000 000 086
(h) 0.000 000 000 003 675

5 What is the smallest number in each group?

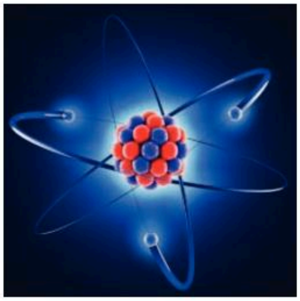
(a) 0.003, 10^{-2}, 1.7×10^{-4}, 0.071
(b) 10^{-4}, 0.0003, 0.001 48, 1.7×10^{-5}
(c) 0.000 000 38, 3.2×10^{-9}, 10^{-10}, 0.000 06
(d) 3.15×10^{-7}, 0.000 000 81, 6.9×10^{-8}, 0.001 340 2

Scientists have given names to powers of 10 in common use.

Number	Power of 10	Prefix	Symbol
1000 000 000 000	10^{12}	tera	T
1000 000 000	10^{9}	giga	G
1000 000	10^{6}	mega	M
1000	10^{3}	kilo	K
0.001	10^{-3}	milli	m
0.000 001	10^{-6}	micro	μ
0.000 000 001	10^{-9}	nano	n
0.000 000 000 001	10^{-12}	piko	p



A computer hard drive with capacity 2 terabytes holds $2 \times 10^{12} = 2\,000\,000\,000\,000$ bytes of data.



The distance across an atom is about 50 nanometre (50 nm).

$$50 \text{ nm} = 50 \times 10^{-9} \text{ m} = 5 \times 10^{-8} \text{ m}$$
$$= 0.000\,000\,05 \text{ m}$$

Activity

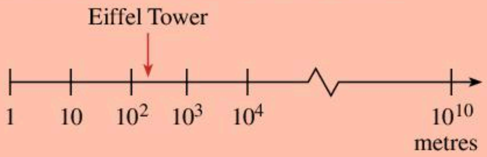
(a) Give two examples of items that have a mass of
  (a) 5 kilograms
  (b) 5 milligrams
  (c) 5 micrograms

(b) Give two examples of things that have a length of
  (a) 5 kilometres
  (b) 5 millimetres
  (c) 5 nanometres

Activity

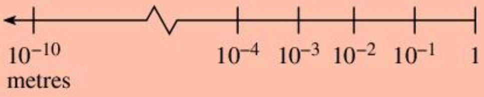
The Eiffel Tower is about 300 m tall. That is 3×10^2 m.

You can show this on a number line.



(a) Copy and complete the number line for length/distance, inserting other objects or distances with lengths/distances up to 10^{10} metres.

(b) Repeat this number line for lengths/distances which are very small



Technology

You can review how to round off a number by visiting the website

www.mathsisfun.com

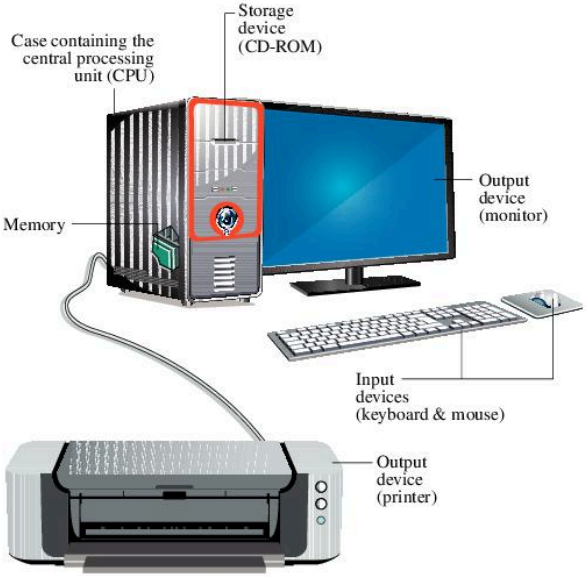
(follow the links to Numbers, Decimals Menu, Rounding Numbers).

Check out the significant digit calculator. It will solve all your significant figure problems!

1.8 Calculators and computers

Calculators are electronic devices that can be used to perform most numerical computations.

A computer differs from a calculator in that it can be **programmed** to perform specific tasks. Calculators can only do pre-designed tasks such as multiplication, division etc. Computers, on the other hand, can be used to write letters, house databases and surf the internet.



In the computer, data is usually fed in through the keyboard or an external memory device. This data is processed in the central processing unit (CPU) and the output is displayed on the monitor or printed.

Exercise 1S

1 **(a)** What types of calculator can be purchased today?
(b) How do these calculators differ from one another?

2 **(a)** What is the difference between a desktop computer and a laptop?
(b) Which would you prefer to have? Explain.

3 **(a)** Find out more about hand-held computers or palmtops.
(b) Are these really computers or advanced calculators?

4 **(a)** Find out more about computer programmes or software used by computers.
(b) How do spreadsheets differ from database software?

Technology

Have a look at different types of calculators that are available. Visit

www.coolmath.com

(and follow the links to Other stuff, Online calculators) to give yourself an idea of what's available.
You can also work with a graphing calculator on the same website.

- Which of these calculators do you like? Why?
- Which ones would you recommend to your friends? Why?
- Should teachers allow students to use calculators in all maths lessons? Give reasons. What about computers?
- What do you think maths lessons will be like in 30 years' time?

Using your calculator

Investigation

(a) Use your calculator to write $\frac{1}{33}, \frac{2}{33}, \frac{3}{33}, \ldots$ as decimals.
(b) What do you notice about your answers?
(c) What about fractions of the form: $\frac{1}{55}, \frac{2}{55}, \frac{3}{55}, \ldots$?
Can you find other fractions which give decimals like these?

Converting one set of units to another is simple with a calculator.

Example 22

1 mile is roughly 1.607 km.
How many miles is 25 km?

25 km = 25 ÷ 1.607 miles
On the calculator press:

$\boxed{2}\ \boxed{5}\ \boxed{\div}\ \boxed{1}\ \boxed{\cdot}\ \boxed{6}\ \boxed{0}\ \boxed{7}\ \boxed{=}$

25 ÷ 1.607 = 15.557

So 25km is 15.56 miles.

Exercise 1T

1 1 inch is roughly 2.54 centimetres
1 foot is roughly 30.48 centimetres
1 pound is roughly 0.454 kilograms
1 pint is roughly 0.568 litres

Using the conversions find the number of:

(a) cm in 23 inches
(b) feet in 3.5 cm
(c) kilograms in $2\frac{3}{4}$ lb
(d) pints in 21.68 litres
(e) litres in $16\frac{1}{2}$ pints

2 Which is the better buy:

(a) $2\frac{1}{2}$ kg of chicken wings costing $13.95 or 3 kg of chicken wings costing $16.40
(b) 0.454 kg of flour costing $2.22 or 1.38 kg of flour costing $6.04
(c) 1150 g of milk powder costing $13.28 or 925 g of milk powder costing $9.75
(d) 115 ml of toothpaste at $7.95 or 60 ml of toothpaste at $3.67?

3 How many lengths of rope 6.3 m long can be cut from 58 m?

4

Thirteen students hire a minibus. The trip costs $95. How much should each student pay?

5 A rectangular room is 6.5 m long and 3.61 m wide.

(a) What is the area (length × width) of the room?
(b) If carpet is sold for $29.75 per square metre, find the cost of carpet for the room.

6 A bottle holds 2 litres of a chemical; 1.638 litres are removed.

(a) What is the volume of chemical remaining?
(b) What is the cost of the remainder if a litre of the chemical costs $322?

7 Albert Munroe visits his supermarket. He buys 6 tins of milk at $1.43 each, 7 kg of rice at $1.95 per kg, 3 bars of washing soap at $0.92 each, $\frac{3}{4}$ kg of cheese at $13.44 per kilogram and 4 packets of biscuits at $2.73 a packet. What is his total bill?

8

Radio waves travel at a speed of about 186 000 miles/second.

(a) What is this speed in km/h? (1.609 km = 1 mile)
(b) How far do radio waves travel in one year?
(c) A creature from outer space sends a radio message to earth. The message takes 17 years to reach the earth. How far from the earth is the creature?

9 A tin of peas weighs 0.45 kg. The tin itself weighs 0.06 kg.

(a) What fraction of the total weight is peas?
(b) What is this as a percentage?

10 **(a)** A doctor tries a new vaccine on 95 patients with typhoid fever. If 68 of his patients recover, what percentage do not?
(b) To test its effectiveness he gives a sugar pill to 58 other typhoid patients. Only 24 of these recover. What percentage do not?

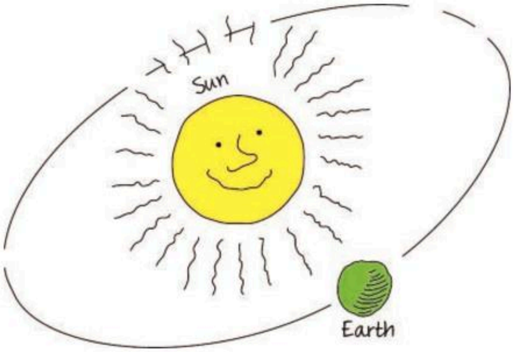
11 A man spends $\frac{2}{5}$ of his income on food, $\frac{1}{7}$ on clothes and $\frac{1}{3}$ on housing.

- **(a)** What fraction of his income does he have left?
- **(b)** What percentage is this?

12 Without tax a shirt costs \$35.60. After the tax is added it sells for \$43.15.

- **(a)** What fraction of the cost price is tax?
- **(b)** What is the rate of taxation?

13



The earth is 93 million miles from the sun. It travels around the sun in a nearly circular path once every $365\frac{1}{4}$ days.

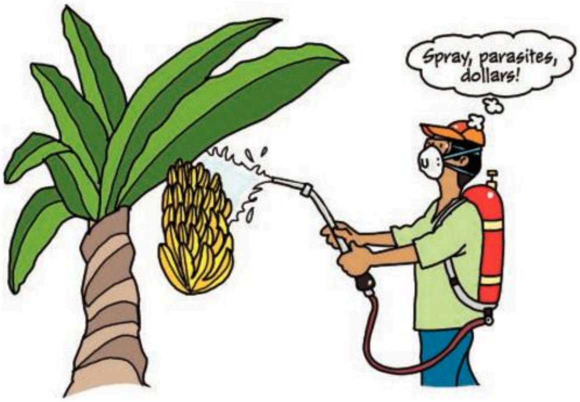
- **(a)** How far is the earth from the sun in kilometres? (1.609 km = 1 mile)
- **(b)** How many kilometres does the earth travel each year?
  (Remember, the circumference of a circle is given by the formula $C = 2\pi r$)
- **(c)** How many kilometres does the earth travel each day?
- **(d)** What is the earth's speed in km/h?
- **(e)** What is the earth's speed in km/s?

14 In a certain island, the government spent \$8 300 000 on education. \$7.7 million was spent on teachers' salaries, the remainder for supplies and materials. The island's 70 schools employ 634 teachers who teach 24 218 children.

- **(a)** How much money was spent on supplies and materials for schools?
- **(b)** What percentage of the education budget was spent on teachers' salaries?
- **(c)** What is the average (mean) salary of a teacher?
- **(d)** How much money does the government spend, on average, for each child?
- **(e)** How many teachers, on average, are attached to each school?
- **(f)** What is the average size of a class?

15 A farmer plants bananas on $4\frac{1}{2}$ acres of his land.

- **(a)** What is the area of his land in square metres?
  (4840 square yards = 1 acre,
  $0.836\ \text{m}^2 = 1$ squre yard)
- **(b)** He plants one banana tree per $4.8\ \text{m}^2$.
  How many trees does he plant?
- **(c)** Each tree bears an average of 12.7 kg of bananas annually. How many kilograms of bananas does his land produce?
- **(d)** What is his total annual income if bananas sell for \$0.47 per kilogram?
- **(e)** He has to spend \$5600 of his annual income on fertilisers and insecticides. Express this as a percentage of his income.



1 Consolidation

Example 1

Work out:

(a) $3\frac{1}{4}+2\frac{2}{5}$

$=5+\frac{1}{4}+\frac{2}{5}$

$=5+\frac{5}{20}+\frac{8}{20}$

$=5+\frac{13}{20}=5\frac{13}{20}$

(b) $4\frac{1}{2}\times 2\frac{2}{3}$

$=\frac{9}{2}\times\frac{8}{3}$

$=\frac{\cancel{9}^3}{\cancel{2}_1}\times\frac{\cancel{8}^4}{\cancel{3}_1}$

$=\frac{3\times 4}{1\times 1}=12$

(c) $2\frac{1}{2}\div 1\frac{3}{4}$

$=\frac{5}{2}\div\frac{7}{4}$

$=\frac{5}{2}\times\frac{4}{7}$

$\div\frac{7}{4}$ is the same as $\times\frac{4}{7}$

$=\frac{20}{14}=1\frac{6}{14}=1\frac{3}{7}$

Example 2

Work out:

(a) 0.68×0.045

An estimate is $1\times 0.04=0.04$

```
   0.68
 × 0.045
 -------
    340
   2720
 -------
 0.03060
```

So $0.68\times 0.045=0.0306$

(b) $0.035\div 0.5$

$\frac{0.035}{0.5}=\frac{0.035\times 10}{0.5\times 10}=\frac{0.35}{5}$

$=0.07$

Example 3

Write:

(a) 42% as a decimal.

$42\%=\frac{42}{100}$

$=0.42$

(b) $\frac{13}{24}$ as a decimal.

$\frac{13}{24}=13\div 24$

```
       0 . 5 4 ...
24) 1 3 . 0 0 0 0
    1 2   0
    ---------
        1  0 0
           9 6
           ---
             4
```

So $\frac{13}{24}=0.54\ldots$

(c) $\frac{7}{12}$ as a percentage.

$\frac{7}{12}=\frac{7}{12}\times 100\%=\frac{700}{12}\%$

$=58.3\%$

Example 4

Write

(a) 4.378 to 1 d.p.

= 4.4 (1 d.p.)

(b) 8947.62 to 2 s.f.

= 8900 (2 s.f.)

Example 5

Write in standard form.

(a) 63 100

$=6.31\times 10\,000=6.31\times 10^4$

(b) 0.000 38

$=3.8\div 10\,000=3.8\times 10^{-4}$

Exercise 1

1 Work out:

(a) $\frac{1}{2}+\frac{2}{3}$ **(b)** $\frac{3}{8}+\frac{4}{7}$ **(c)** $2\frac{3}{5}+1\frac{2}{7}$

(d) $\frac{6}{7}-\frac{2}{3}$ **(e)** $6\frac{1}{2}-2\frac{5}{8}$

2 Work out:

(a) $\frac{3}{5}\times\frac{2}{3}$ **(b)** $2\frac{1}{2}\times 3\frac{1}{3}$ **(c)** $\frac{6}{7}\div\frac{2}{3}$

(d) $\frac{4}{5}\div 1\frac{2}{3}$ **(e)** $4\frac{3}{4}\div 2\frac{2}{5}$

3 Calculate:

(a) 3×0.5 **(b)** 2.3×1.5
(c) 14.3×0.06 **(d)** 16.8×0.19
(e) 0.061×0.342

4 Calculate:

(a) $12 \div 0.5$ **(b)** $1.7 \div 0.02$
(c) $36.1 \div 0.0019$ **(d)** $69.3 \div 0.3$
(e) $2.34 \div 8.6$

5 **(a)** Write these fractions as decimals.

(i) $\frac{13}{25}$ **(ii)** $\frac{29}{40}$ **(iii)** $\frac{23}{37}$
(iv) $\frac{19}{47}$ **(v)** $\frac{14}{31}$

(b) Write the fractions in part (a) as percentages.

6 Copy and complete:

(a)

	5.613	**0.0138**	**1.0824**
Correct to 2 d.p.			
Correct to 1 d.p.			

(b)

	83 145	**16.382**	**0.002 59**
Correct to 2 s.f.			
Correct to 1 s.f.			

7 Write in standard form:

(a) 316 000 **(b)** 0.0251
(c) 1 394 000 **(d)** 0.000 416
(e) 8 100 000 000

Application

8 A piece of cheese at a supermarket weighs 0.38 kg and is priced at $7.35.

(a) What is the cost per kg of cheese?
(b) Adam buys $1\frac{1}{4}$ kg of cheese, how much does he pay?
(c) The supermarket gives a 7% discount on cheese purchases of 5 kg or more. How much will 6.14 kg of cheese cost?

9 The world record for the 100 m dash is held by Usain Bolt of Jamaica at 9.58 s.

(a) What is Bolt's speed in km/hr to 1 d.p.?
(b) What is Bolt's speed in miles/hr, given 1 mile = 1.609 km to 3 s.f.?
(c) A cheetah is timed to run 217 m in 6.93 s. What is the cheetah's speed in
  (i) km/hr to 1 d.p.
  (ii) miles/hr to 2 s.f.?
(d) How much faster is the cheetah than Bolt?

Support Website

Additional material to support this topic can be found at www.oxfordsecondary.com/9780198425793

Summary

You should know ...	Check out
1 How to add, subtract, multiply and divide fractions. *For example:* $\frac{4}{7} \div 1\frac{2}{3} = \frac{4}{7} \div \frac{5}{3} = \frac{4}{7} \times \frac{3}{5} = \frac{12}{35}$	**1** Calculate: **(a)** $\frac{3}{5} + \frac{3}{8}$ **(b)** $2\frac{3}{4} - 1\frac{7}{8}$ **(c)** $2\frac{4}{5} \times \frac{5}{6}$ **(d)** $1\frac{2}{3} \div 3\frac{3}{4}$
2 How to add, subtract, multiply and divide decimals. *For example:* $3.8 \div 0.04 = \frac{3.8}{0.04} = \frac{3.8 \times 100}{0.04 \times 100} = \frac{380}{4} = 95$	**2** Work out: **(a)** 3.5×0.16 **(b)** $8 \div 0.02$ **(c)** 1.42×2.7 **(d)** $0.86 \div 0.004$

3 **(a)** How to change fractions to decimals.

For example:

$\frac{3}{8} = 3 \div 8 = 3.000 \div 8 = 0.375$

$$\begin{array}{r} 0.375 \\ 8\overline{)\,3.000} \end{array}$$

$\longleftarrow$ decimal points line up

(b) How to change decimals to fractions.

For example: $0.375 = \frac{375}{1000} = \frac{15}{40} = \frac{3}{8}$

(c) How to change fractions to percentages.

For example: $\dfrac{3}{8} = \dfrac{3}{8} \times 100\% = \dfrac{300}{8}\% = 37\frac{1}{2}\%$

4 How to round decimals to a given number of significant figures.

For example:

46 891.47 = 46 900 (3 s.f.)

46 891.47 = 47 000 (2 s.f.)

46 891.47 = 50 000 (1 s.f.)

5 How to work with powers of numbers.

For example:

(a) $3^5 \times 3^{-4} = 3^{5-4} = 3^1 = 3$

(b) $5^2 \div 5^5 = 5^{2-5} = 5^{-3} = \dfrac{1}{5^3} = \dfrac{1}{125}$

(c) $64^{\frac{2}{3}} = (\sqrt[3]{64})^2 = 4^2 = 16$

6 How to write a number in standard form.

For example:

$629\,300 = 6.293 \times 100\,000 = 6.293 \times 10^5$

$0.0049 = 4.9 \div 1000 = 4.9 \times 10^{-3}$

7 How to solve fraction and decimal problems using a calculator.

For example:

To convert 4 feet to centimetres, press

$\boxed{4}\ \boxed{\times}\ \boxed{3}\ \boxed{0}\ \boxed{\cdot}\ \boxed{4}\ \boxed{8}\ \boxed{=}$

to get 123.36 cm.

(1 foot is roughly 30.48 centimetres.)

3 **(a)** Write as decimals:

(i) $\frac{3}{4}$ **(ii)** $\frac{5}{8}$

(iii) $\frac{4}{5}$ **(iv)** $\frac{7}{16}$

(b) Write as fractions in their lowest form:

(i) 0.85 **(ii)** 0.315

(iii) 0.92 **(iv)** 0.625

(c) Change these fractions to percentages

(i) $\frac{1}{4}$ **(ii)** $\frac{3}{5}$

(iii) $\frac{17}{20}$ **(iv)** $\frac{7}{8}$

4 **(a)** Write correct to 3 s.f.

(i) 8712 **(ii)** 0.004 864

(b) Write correct to 2 s.f.

(i) 48 487 **(ii)** 0.0195

(c) Write correct to 1 s.f.

(i) 58 **(ii)** 0.000 783

5 Calculate

(a) $4^3 \div 4^5$ **(b)** $2^7 \times 2^{-4}$

(c) $36^{-\frac{1}{2}}$ **(d)** $81^{\frac{3}{2}}$

6 Write in standard form:

(a) 8000 **(b)** 7236

(c) 0.038 **(d)** 183 000

(e) 0.000 062 **(f)** 0.03

7 Convert:

(a) 5 feet to centimetres

(b) 22 feet to metres

(c) 13.2 feet to metres

(d) 23.6 metres to feet

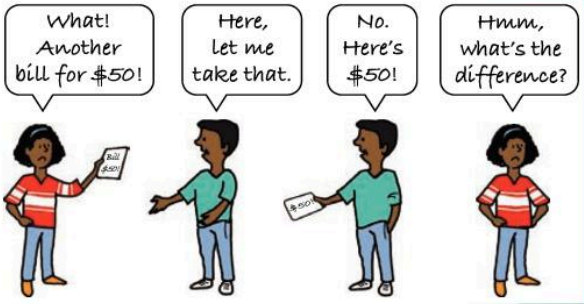
(e) $\frac{3}{4}$ metre to feet.

Computation 2

2

Objectives

- ✔ perform the four operations on integers
- ✔ solve problems involving integers
- ✔ solve problems involving sets of numbers
- ✔ identify a matrix as an array of numbers
- ✔ add and subtract matrices



What's the point?

When you earn money from a job or are given money, you often will put that money in a bank account.

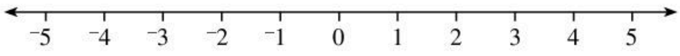
The money you earn is called credit. Money you spend is a debit. Banks show debits on their statements as negative numbers. If you spend more than you earn your balance will show a negative amount.

Name: Tyrone Johnson		**Account 3647128**		**Code 21-22-71**	
Date	**Details**	**From**	**Withdrawn**	**Paid In**	**Balance**
12-12-17	Credit	Bank		$300	$300
12-19-17	Debit	Max Trading	–$200		$100
03-01-18	Debit	Water bill	–$50		$50
07-01-18	Debit	J.J. Giltes	–$200		–$150

Before you start

You should know …

1 Integers are positive and negative whole numbers



Negative numbers are always smaller than positive numbers.
For example:
$^-3 < 2$

Larger negative numbers are always smaller than smaller negative numbers.
For example:
$^-5 < {}^-3$

Check in

1 Insert >, < for each of these number pairs

(a) 5   2   **(b)** $^-5$ $^-2$

(c) $^-5$ 2   **(d)** 5 $^-2$

2 Write these numbers in ascending order

(a) 3, $^-2$, 6, $^-5$, 0

(b) $^-7$, $^-11$, $^-3$, 4, $^-2$, 1

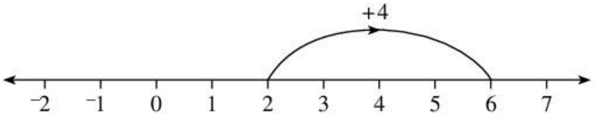
(c) $^-1$, 1, $^-2$, 4, $^-3$, 2

2.1 Operations on integers

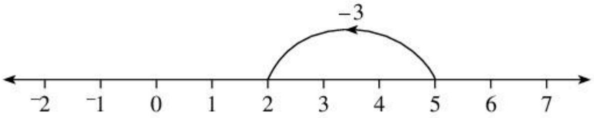
You can use the number line to show simple addition and subtraction of numbers.

For example

$2 + 4 = 6$



Start on 2, add 4 by moving 4 places to the right. Subtraction involves moving to the left, for example



shows $5 - 3 = 2$

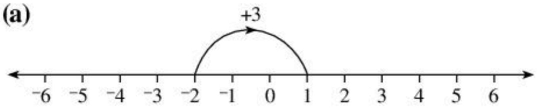
This method also works when adding and subtracting integers.

Example 1

Use a number line to find

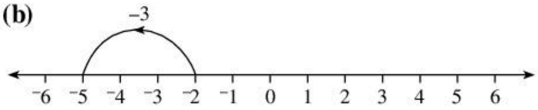
(a) ${}^-2 + 3$ **(b)** ${}^-2 - 3$

(a)



So ${}^-2 + 3 = 1$

(b)



So ${}^-2 - 3 = {}^-5$

In Example 1(a) you are adding the number 3 to negative 2, while in (b) you are subtracting 3 from negative 2.

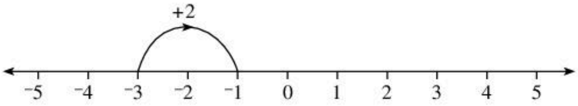
What happens if you want to add a negative number to another?

For example

$2 + ({}^-3)$

Well, the order you add two numbers does not matter, so

$2 + ({}^-3) = {}^-3 + 2$



Thus $2 + ({}^-3) = {}^-3 + 2 = {}^-1$

Notice that

$2 + ({}^-3) = 2 - 3 = {}^-1$

That is,

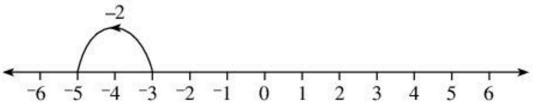
Adding a negative number is the same as subtracting a positive number.

Example 2

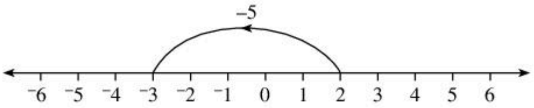
Work out

(a) ${}^-3 + ({}^-2)$ **(b)** $2 + ({}^-5)$

(a) ${}^-3 + ({}^-2) = {}^-3 - 2 = {}^-5$



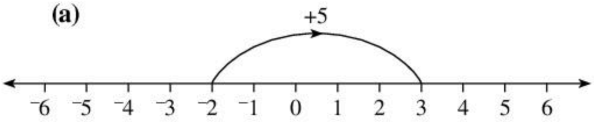
(b) $2 + ({}^-5) = 2 - 5 = {}^-3$



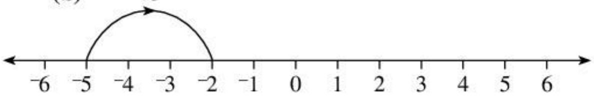
Exercise 2A

1 Write down the additions shown on these number lines

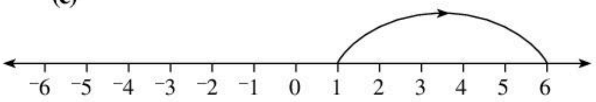
(a)



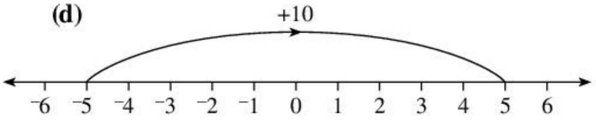
(b)



(c)

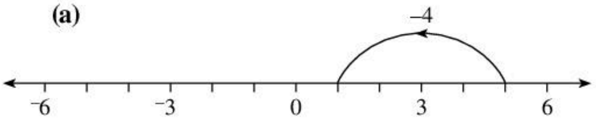


(d)

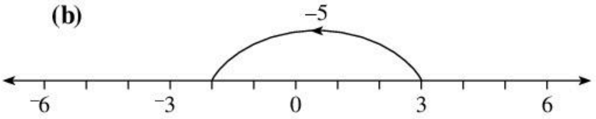


2 Write down the subtractions shown on these number lines

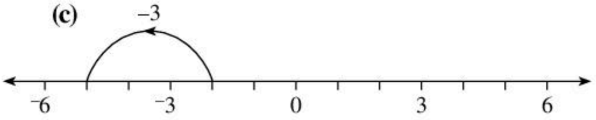
(a)



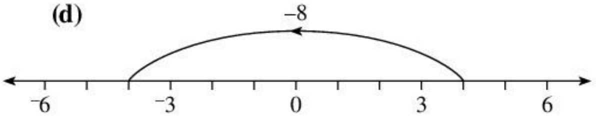
(b)



(c)



(d)



3 Draw number lines to calculate

(a) $3 + 2$ **(b)** $^{-}5 + 2$
(c) $^{-}2 + 4$ **(d)** $^{-}6 + 1$

4 Draw number lines to calculate

(a) $5 - 2$ **(b)** $3 - 4$
(c) $0 - 3$ **(d)** $^{-}1 - 3$

5 Work out

(a) $^{-}2 - 4$ **(b)** $5 - 6$
(c) $^{-}1 + 3$ **(d)** $^{-}5 + 2$
(e) $6 - 7$ **(f)** $^{-}7 + 6$
(g) $^{-}5 + 3$ **(h)** $^{-}5 - 3$

6 Copy and complete

$^{-}2 + 3 =$ _____
$^{-}2 + 2 =$ _____
$^{-}2 + 1 =$ _____
$^{-}2 + 0 =$ _____
$^{-}2 + (^{-}1) =$ _____
$^{-}2 + (^{-}2) =$ _____
$^{-}2 + (^{-}3) =$ _____

7 Copy and complete

(a) $4 + (^{-}1) = 4 - 1 =$ ____
(b) $6 + (^{-}2) =$ ____ $=$ ____
(c) $2 + (^{-}1) =$ ____ $=$ ____
(d) $3 + (^{-}5) =$ ____ $=$ ____
(e) $1 + (^{-}4) =$ ____ $=$ ____

8 Work out

(a) $^{-}3 + (^{-}2)$ **(b)** $^{-}1 + (^{-}3)$
(c) $4 + (^{-}5)$ **(d)** $^{-}4 + (^{-}6)$
(e) $^{-}2 + (^{-}7)$ **(f)** $^{-}7 + (^{-}2)$
(g) $^{-}3 + (^{-}3)$ **(h)** $^{-}9 + (^{-}11)$

9 Copy and complete the addition table

+	$^{-}4$	$^{-}2$	0	2	4
$^{-}4$					
$^{-}2$		$^{-}4$			
0					
2	$^{-}2$				
4				6	8

10 Calculate

(a) $^{-}2 + 3 - 2$ **(b)** $^{-}2 - 1 - 1$
(c) $3 - 6 - 4$ **(d)** $^{-}3 + (^{-}2) + (^{-}3)$
(e) $4 - 7 + (^{-}1)$ **(f)** $3 - 1 + (^{-}6)$
(g) $12 - 4 + (^{-}11)$ **(h)** $^{-}14 - 8 + (^{-}9)$

Subtracting integers

Look at the pattern

$4 - 3 = 1$
$4 - 2 = 2$
$4 - 1 = 3$
$4 - 0 = 4$
$4 - (^{-}1) = ?$

It seems clear that $4 - (^{-}1) = 5$.

That is,

Subtracting a negative number is the same as adding a positive number.

Example 3

Work out

(a) $6 - (^{-}2)$ **(b)** $^{-}3 - {^{-}2}$

(a) $6 - (^{-}2) = 6 + 2 = 8$

(b) $^{-}3 - {^{-}2} = {^{-}3} + 2 = {^{-}1}$

Exercise 2B

1 Work out

(a) $2 - {^{-}3}$ **(b)** $4 - {^{-}1}$
(c) $5 - {^{-}2}$ **(d)** $6 - {^{-}4}$
(e) $^{-}2 - {^{-}3}$ **(f)** $^{-}3 - {^{-}4}$
(g) $^{-}5 - {^{-}4}$ **(h)** $^{-}6 - {^{-}1}$

2 Calculate

(a) $3 + {^{-}2}$ **(b)** $3 - {^{-}2}$
(c) $^{-}3 + {^{-}2}$ **(d)** $^{-}3 - {^{-}2}$
(e) $^{-}3 - 5$ **(f)** $4 - {^{-}5}$
(g) $^{-}6 + {^{-}7}$ **(h)** $^{-}8 - {^{-}2}$

3 Copy and complete the subtraction table.

Second number

First number

−	$^{-}5$	$^{-}3$	$^{-}1$	2	3	5
$^{-}5$		$^{-}2$				
$^{-}3$			$^{-}2$			
$^{-}1$						
1						
3			4			
5					2	

4 Calculate

(a) $3 - {^{-}2} + 4$ **(b)** $6 - 7 + {^{-}2}$
(c) $^{-}4 - {^{-}3} - 4$ **(d)** $^{-}2 + 4 + {^{-}3}$
(e) $3 - 4 - {^{-}2}$ **(f)** $^{-}7 + {^{-}3} + {^{-}2}$
(g) $^{-}4 + {^{-}2} - {^{-}2}$ **(h)** $4 - 2 + {^{-}2}$

5 Copy and complete

(a) $3 - \square = 1$ **(b)** $3 - \square = 4$
(c) $\square - 2 = {^{-}1}$ **(d)** $\square + 3 = {^{-}5}$
(e) $^{-}2 + \square = {^{-}5}$ **(f)** $^{-}6 + \square = {^{-}2}$
(g) $^{-}2 - \square = 0$ **(h)** $^{-}5 - \square = {^{-}2}$

6 The temperature in Moscow is − 5°C. What is the current temperature if it

(a) rose 4°C **(b)** rose 6°C
(c) fell 4°C **(d)** fell 8°C

7 Adam has $350 in his bank account.

(a) Construct his bank statement if he makes the following transactions

12-02-18	$60 payment to Megastore
15-02-18	$130 payment to Pricemart
21-02-18	$210 payment to Savemore

(b) What is his balance?

Multiplying and dividing integers

Multiplication is really repeated addition so

$$4 \times {^{-}3} = {^{-}3} + {^{-}3} + {^{-}3} + {^{-}3} = {^{-}12}$$

and

$$5 \times {^{-}2} = {^{-}2} + {^{-}2} + {^{-}2} + {^{-}2} + {^{-}2} = {^{-}10},$$

That is,

$$4 \times {^{-}3} = {^{-}12}$$

and $5 \times {^{-}2} = {^{-}10}$

The order in which you multiply does not matter either, so

$$^{-}3 \times 4 = {^{-}12}$$

and $^{-}2 \times 5 = {^{-}10}$.

Hence

The product of a positive number and a negative number is negative.

What about the product of a negative number and a negative number?

Look at the pattern

$$4 \times {}^-2 = {}^-8$$
$$3 \times {}^-2 = {}^-6$$
$$2 \times {}^-2 = {}^-4$$
$$1 \times {}^-2 = {}^-2$$
$$0 \times {}^-2 = 0$$
$${}^-1 \times {}^-2 = ?$$

Again, it is clear that

$${}^-1 \times {}^-2 = 2$$

If the pattern is to continue.

Hence,

the product of two negative numbers is positive.

Example 4

Calculate

(a) ${}^-5 \times 4$ **(b)** ${}^-6 \times ({}^-3)$

(a) ${}^-5 \times 4 = {}^-20$

(b) ${}^-6 \times ({}^-3) = 18$

The two rules for multiplying integers give rise to two similar rules for the division of integers.

Since $4 \times {}^-3 = {}^-12$

then ${}^-3 = {}^-12 \div 4$

Thus

a negative number divided by a positive number gives a negative number

Since $4 \times {}^-3 = {}^-12$

Then $4 = {}^-12 \div {}^-3$

Thus,

a negative number divided by a negative number gives a positive number

You can summarise these rules as follows

Multiply(×) Divide(÷)	**Positive number**	**Negative number**
Positive number	+	–
Negative number	–	+

Example 5

Calculate

(a) ${}^-18 \div 2$ **(b)** ${}^-24 \div {}^-6$

(a) ${}^-18 \div 2 = {}^-9$

(b) ${}^-24 \div {}^-6 = 4$

Exercise 2C

1 Work out

(a) $4 \times {}^-2$ **(b)** $6 \times {}^-5$
(c) ${}^-3 \times 6$ **(d)** ${}^-5 \times {}^-2$
(e) ${}^-3 \times {}^-9$ **(f)** $5 \times {}^-4$
(g) ${}^-6 \times 8$ **(h)** ${}^-8 \times 6$
(i) ${}^-3 \times {}^-11$ **(j)** ${}^-12 \times {}^-8$

2 Calculate

(a) $4 \div {}^-2$ **(b)** $25 \div {}^-5$
(c) ${}^-8 \div 2$ **(d)** ${}^-16 \div {}^-4$
(e) $32 \div {}^-4$ **(f)** ${}^-36 \div 9$
(g) $48 \div {}^-4$ **(h)** ${}^-16 \div 2$
(i) ${}^-72 \div 3$ **(j)** ${}^-42 \div {}^-7$

3 Copy and complete

(a) $3 \times \square = {}^-6$ **(b)** ${}^-4 \times \square = {}^-12$
(c) ${}^-4 \times \square = 24$ **(d)** $6 \times \square = {}^-12$
(e) $\square \times 3 = {}^-18$ **(f)** $\square \times {}^-4 = 48$

4 Copy and complete

(a) $18 \div \square = {}^-3$ **(b)** ${}^-12 \div \square = 3$
(c) ${}^-27 \div \square = 9$ **(d)** $\square \div {}^-9 = 5$
(e) $\square \div 12 = {}^-3$ **(f)** $\square \div {}^-8 = 12$

5 Copy and complete

×	$^-6$	$^-4$	$^-2$	0	2	4	6
$^-6$							
$^-4$				0			
$^-2$							
0							
2							
4		$^-16$			8		
6							36

6 Calculate

(a) $({}^-1)^2$ **(b)** $({}^-3)^2$ **(c)** $({}^-2)^3$ **(d)** $({}^-4)^3$
(e) $({}^-1)^9$ **(f)** $({}^-1)^{10}$ **(g)** $({}^-2)^5$ **(h)** $({}^-2)^8$

7 Copy and complete the multiplication tables

(a)

×	2	
	4	$^-6$
	$^-6$	9

(b)

×		
$^-1$	3	$^-4$
	15	$^-20$

8 Copy and complete these division tables

(a)

Second number

First number

÷		
	$^-4$	3
	12	$^-9$

(b)

Second number

First number

÷			
	5	$^-10$	15
	2	$^-4$	6
	$^-1$	2	$^-3$

Technology

Having negative thoughts about integers?
Visit

www.mathsisfun.com/positive-negative-integers.html

and

www.mathsisfun.com/multiplying-negatives.html.

Technology

Having trouble with all these signs? Check out a complete course on integer operations at

www.coolmath.com

(follow the links to Prealgebra, Signed Numbers (Integers))

or

www.purplemaths.com

(follow the links to Negative Numbers in Preliminary Topics).

These should give you the help you need!

2.2 Using integers

Substitution and negative numbers

Substituting negative values into expressions is just like substituting positive values.

Example 6

If $x = {}^-3$, find the value of

(a) x^2 **(b)** $5x^2$

(a) $x^2 = ({}^-3)^2 = {}^-3 \times {}^-3 = 9$

(b) $5x^2 = 5 \times ({}^-3)^2 = 5 \times 9 = 45$

You can also work out more complex expressions.

Example 7

If $x = {}^-2$, find the value of $2x^3 - x^2 + 10x + 6$

$$= 2({}^-2)^3 - ({}^-2)^2 + 10({}^-2) + 6$$
$$= 2({}^-8) - (4) + 10({}^-2) + 6$$
$$= {}^-16 - 4 + {}^-20 + 6$$
$$= {}^-40 + 6$$
$$= {}^-34$$

Exercise 2D

1 Write each of these in product form. The first has been done for you.

(a) $5^3 = 5 \times 5 \times 5$ **(b)** 2^3 **(c)** 3^2

(d) x^6 **(e)** y^4 **(f)** $2a^4$

(g) $3x^2$ **(h)** ab^3 **(i)** p^2q^3

2 Write each of these products using index notation. The first has been done for you.

(a) $3 \times 3 \times 3 \times 3 = 3^4$

(b) $2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2$

(c) $x \times x \times x \times x \times x \times x$

(d) $6 \times p \times p \times p \times p$

(e) $4 \times p \times p \times q \times q \times q$

3 Find the value of each of the following. The first has been worked out for you.

(a) $3x^4$ when $x = {}^-2$

$$\begin{aligned}3x^4 &= 3 \times ({}^-2)^4\\ &= 3 \times ({}^-2) \times ({}^-2) \times ({}^-2) \times ({}^-2)\\ &= 48\end{aligned}$$

(b) x^3 when $x = {}^-1$ **(c)** $4x^3$ when $x = {}^-1$

(d) $3x^2$ when $x = {}^-2$ **(e)** $4t^4$ when $t = {}^-3$

(f) $2p^3$ when $p = {}^-5$

4 **(a)** Find the value of each of the following:

(i) $({}^-1)^3$ **(ii)** $({}^-1)^6$

(iii) $({}^-1)^4$ **(iv)** $({}^-1)^7$

(b) Can you guess the value of $({}^-1)^{100}$ and $({}^-1)^{101}$?

(c) What is the value of $({}^-1)^n$ if n is an odd number?

5 Find the value of each expression when $x = {}^-2$:

(a) $3x^2 - x$ **(b)** $2x^3 + x - 10$

(c) $x^4 + 2x^2 + 7$ **(d)** $4x^3 + 3x^2 + 2x + 1$

(e) $x^3 - 5x^2 - 4x - 15$

6 Find the value of each expression:

(a) $x^2 - x$ when $x = {}^-3$

(b) $x + x^2 - x^3$ when $x = {}^-4$

(c) $2x^2 - x - 1$ when $x = {}^-5$

(d) $x^3 + 4x^2 - 16x + 19$ when $x = {}^-1$

(e) $x^3 - 3x$ when $x = {}^-3$

7 Find the value of each expression when

(i) $x = 2$ **(ii)** $x = {}^-2$

(a) $5x^2$ **(b)** $x^4 + x^2$

(c) $3x^4 - 2x^2$ **(d)** ${}^-5x^2 + x^4$

What do you notice about the answers?

8 Find the value of each expression when:

(i) $x = 2$ **(ii)** $x = {}^-2$

(a) $7x^3$ **(b)** $x^3 - x$

(c) $2x - 3x^3$ **(d)** $x^5 - x^3 + x$

What do you notice about the answers?

9 Look at the expressions in Questions 7 and 8. Can you see a pattern in the answers? Make up an expression in terms of x so that:

(a) it has the same value for $x = 3$ and $x = {}^-3$

(b) it has the same value but opposite sign for $x = 3$ and $x = {}^-3$.

Negative numbers and square roots

The square of a positive or negative number is always positive.

For example,

$$({}^-5)^2 = {}^-5 \times {}^-5 = 25$$
$$(5)^2 = 5 \times 5 \quad = 25$$

That is both ${}^-5$ and 5 have the same square, 25.

On the other hand any positive number has two square roots, one positive, the other negative.

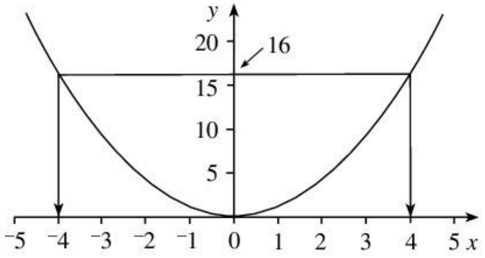
For example

$\sqrt{25} = 5$ or ${}^-5$

If you do not have a calculator a graph can help you find the square or square root of a number.

Example 8

Use the graph of $y = x^2$ to find $\sqrt{16}$.



Notice that when $y = 16$ or $x^2 = 16$ there are two values of x, 4 and ${}^-4$, so $\sqrt{16} = 4$ or ${}^-4$.

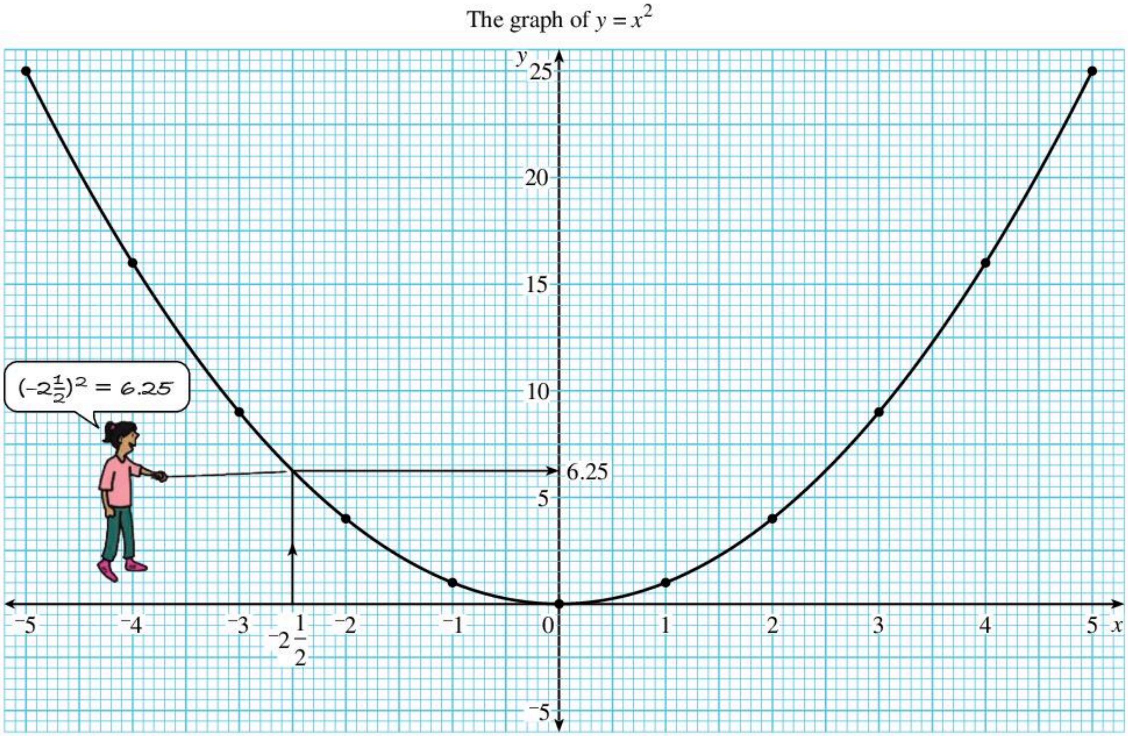
Exercise 2E

1 The graph below shows x^2 for the whole numbers from ${}^-5$ to 5.

(a) Calculate the square of the numbers ${}^-4\frac{1}{2}, {}^-3\frac{1}{2}, {}^-2\frac{1}{2}, {}^-1\frac{1}{2}, {}^-\frac{1}{2}, \frac{1}{2}, 1\frac{1}{2}, 2\frac{1}{2}, 3\frac{1}{2}, 4\frac{1}{2}$.

(b) Can you find your answers on the graph? Check each one.

(c) Do you think the curved line passes through the squares of *all* the numbers between ${}^-5$ and 5?



2 Use the graph below to find as accurately as you can:

(a) 1.7^2 **(b)** 3.2^2 **(c)** $(^-3.8)^2$

(d) $(^-4.7)^2$ **(e)** 4.7^2 **(f)** $(^-2.3)^2$

3 **(a)** Does the graph show that there are two square roots of 25? What are they?

(b) Are there two square roots for every number between 0 and 25?

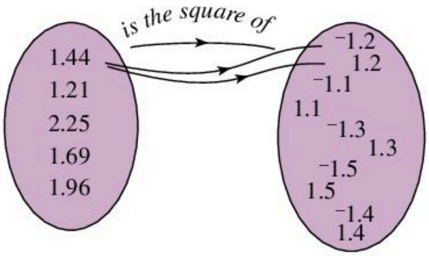
4 What are the two square roots of:

(a) 4 **(b)** 1 **(c)** 9 **(d)** 6.25 **(e)** 20.25?

5 Use the graph to find the two square roots of:

(a) 5 **(b)** 15 **(c)** 0 **(d)** 23

6 The arrow in this diagram shows *is the square of*. Copy and complete it.



7 **(a)** If $A = \{^-4, ^-2, ^-1, 0, 1, 2, 4\}$ and $B = \{^-2, ^-1, 0, 1, 2\}$, draw an arrow graph from A to B to show *is the square of*.

(b) Why can you *not* draw an arrow from some of the numbers in set A? How is this shown on the graph of $y = x^2$ above?

2.3 Sets and numbers

The language of sets

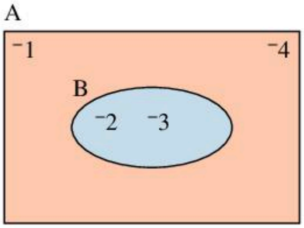
Recall that you can describe a set by listing its members.

For example,

If A is the set of negative whole numbers from $^-1$ to $^-4$

Then $A = \{^-4, ^-3, ^-2, ^-1\}$

If $B = \{^-3, ^-2\}$ then you can show both sets A and B using a Venn diagram.



Notice that B is a subset of A, that is, $B \subseteq A$. The set B is totally contained in the set A.

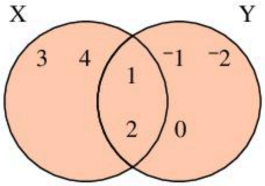
You can form the **union** of two sets by joining them together. The symbol for union is $\cup$.

For example,

If $X = \{1, 2, 3, 4\}$ and $Y = \{^-2, ^-1, 0, 1, 2\}$ then

$X \cup Y = \{^-2, ^-1, 0, 1, 2, 3, 4\}$

On a Venn diagram, this is



Notice that the numbers 1 and 2 are common to both sets. This common part is called the **intersection** of the two sets. The symbol for intersection is $\cap$.

In this case $X \cap Y = \{1, 2\}$

Example 9

Given $A = \{^-3, ^-2, ^-1, 0\}$, $B = \{0\}$ and $C = \{^-1, 0, 1\}$

(a) Find $A \cup C$ **(b)** Find $A \cap C$

(c) Is $B \subseteq C$?

(a) $A \cup C = \{^-3, ^-2, ^-1, 0, 1\}$

(b) $A \cap C = \{^-1, 0\}$

(c) B is a subset of C since all the members of B are contained in C.

Exercise 2F

1 Using {negative whole numbers from $^-1$ to $^-10$} as the universal set, draw separate Venn diagrams to show the subset of:

(a) negative multiples of two
(b) negative odd numbers
(c) negative primes
(d) negative multiples of 5.

2 Given the set $A = \{^-3, ^-2, ^-1)$, write down all the possible subsets of this set with just two members.

3 Copy and complete

(a) $\{^-3, ^-2, ^-1\} \cup \{^-2, ^-1, 0\}$
(b) $\{2, 4, 6, 8, 10\} \cup \{^-8, ^-6, ^-4, ^-2\}$
(c) {factors of 10} $\cup$ {factors of 6}.

4 Copy and complete

(a) $\{^-6, ^-3, ^-2\} \cap \{^-10, ^-2\}$
(b) $\{3, 5, 8\} \cap \{^-7, ^-5, ^-3, ^-1, 3, 5\}$
(c) $\{^-3\frac{1}{2}, ^-2\frac{1}{2}, ^-1\frac{1}{2}\} \cap \{^-3\frac{1}{2}, ^-1\frac{1}{2}, 0\}$

5 Given $A = \{^-3, ^-2, ^-1\}$, $B = \{^-4, ^-2, 0, 2\}$, $C = \{^-5, ^-2, 1, 3, 5\}$.

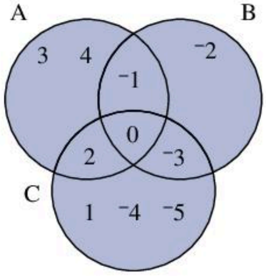
Find the members of the sets

(a) $A \cap B$ **(b)** $B \cup A$
(c) $A \cup C$ **(d)** $C \cap A$
(e) $C \cup B$ **(f)** $B \cap C$

6 Using {negative whole numbers from $^-15$ to $^-1$} as the universal set and $A = \{^-6, ^-4, ^-2\}$, $B = \{^-9, ^-6, ^-4\}$ and $C = \{^-9, ^-6, ^-3, ^-1\}$

(a) Draw Venn diagrams to show
  (i) $A \cap B$ **(ii)** $B \cap C$ **(iii)** $C \cup A$
(b) Find the members of the sets
  (i) $B \cup C$ **(ii)** $B \cup A$ **(iii)** $A \cap C$

7 Look at the Venn diagram below



(a) List the members of
  (i) A **(ii)** B **(iii)** C
(b) Find
  (i) $A \cap B$ **(ii)** $A \cap C$ **(iii)** $B \cap C$
  (iv) $A \cup B$ **(v)** $A \cup C$ **(vi)** $B \cup C$

Finite and infinite sets

A finite set has a limited number of members.

For example,

$A = \{^-2, ^-1, 0, 1, 2\}$ is a finite set. It has five members. You can say $n(A) = 5$

$B = \{^-17, ^-7, 3, 13\}$ is a finite set. It has four members, $n(B) = 4$.

An infinite set is one whose membership has no limit.

For example,

The set of even numbers is $\{2, 4, 6, 8, \ldots\}$.
The dots ... show that the set is endless. You simply cannot list all the members of this set!

There are a number of important infinite sets in our number system.

The set of natural or counting numbers, $\mathbb{N}$.

$\mathbb{N} = \{1, 2, 3, 4, \ldots\}$

The set of whole numbers, $\mathbb{W}$.

$\mathbb{W} = \{0, 1, 2, 3, \ldots\}$

Notice $\mathbb{W}$ is the same as $\mathbb{N}$ except $\mathbb{W}$ includes the number zero but $\mathbb{N}$ does not.

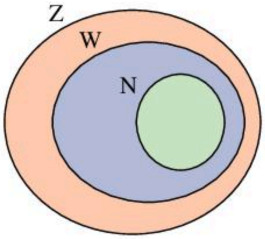
The set of integers, $\mathbb{Z}$.

$\mathbb{Z} = \{\ldots, {}^{-}3, {}^{-}2, {}^{-}1, 0, 1, 2, 3, \ldots\}$

The set of integers contains all the positive and negative whole numbers.

Did you observe that $\mathbb{N} \subseteq \mathbb{W}$ and $\mathbb{W} \subseteq \mathbb{Z}$?

You can represent these sets on a Venn diagram.



Exercise 2G

1 $\mathbb{N} = \{1, 2, 3, 4, \ldots\}$
$A = \{\text{factors of } 12\}$
$B = \{\text{factors of } 8\}$
- **(a)** List the elements of A and B.
- **(b)** What is **(i)** $n(A)$, **(ii)** $n(B)$?
- **(c)** Which of the three sets is an infinite set?
- **(d)** Draw a Venn diagram to show these three sets.
- **(e)** Use your diagram to find
  - **(i)** $A \cap B$ **(ii)** $A \cup B$?
- **(f)** What is $n(A \cup B)$?

2 $\mathbb{W} = \{0, 1, 2, 3, \ldots\}$
$A = \{\text{factors of } 9\}$
$B = \{\text{multiples of } 3\}$
- **(a)** Which of these sets are infinite sets?
- **(b)** Draw a Venn diagram to represent these three sets.
- **(c)** List the members of $A \cap B$
- **(d)** What is $n(A \cap B)$?

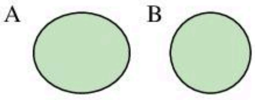
3 $\mathbb{Z} = \{\ldots, {}^{-}2, {}^{-}1, 0, 1, 2, \ldots\}$
$A = \{{}^{-}8, {}^{-}4, 0, 4, 8\}$
$B = \{2, 4, 6, 8, 10, 12, 14\}$
- **(a)** What is **(i)** $n(A)$, **(ii)** $n(B)$?
- **(b)** Which of the three sets is an infinite set?
- **(c)** Draw a Venn diagram to show these three sets.
- **(d)** Use your diagram to find
  - **(i)** $A \cap B$ **(ii)** $A \cup B$
- **(e)** What is $n(A \cup B)$?

4 $M = \{\ldots {}^{-}3, {}^{-}1, 0, 1, 3, \ldots\}$
$A = \{{}^{-}5, {}^{-}1, 3, 7, 11\}$
$B = \{{}^{-}10, {}^{-}8, {}^{-}6, {}^{-}4, {}^{-}2\}$
- **(a)** Describe the set M in your own words.
- **(b)** What is **(i)** $n(A)$, **(ii)** $n(B)$?
- **(c)** Which of the three sets is an infinite set?
- **(d)** Draw a Venn diagram to show these three sets.
- **(e)** Use your diagram to find
  - **(i)** $A \cap B$ **(ii)** $A \cup B$
  - **(iii)** What is $n(A \cup B)$?

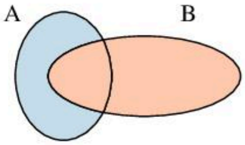
5 $J = \{\ldots {}^{-}9, {}^{-}6, {}^{-}3, 0, 3, 6, 9 \ldots\}$
$K = \{2, 4, 6, 8, \ldots\}$
$L = \{{}^{-}3, {}^{-}2, {}^{-}1, 0, 1, 2, 3\}$
- **(a)** Describe the set J in your own words.
- **(b)** Which of these three sets are an infinite set?
- **(c)** Draw a Venn diagram to show these three sets.
- **(d)** Use your diagram to find
  - **(i)** $J \cap L$ **(ii)** $J \cap K$
- **(e)** What is $n(J \cap L)$?

6 In the Venn diagrams below the sets A and B are finite.

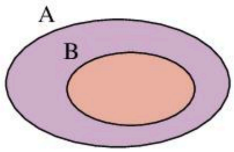
(a)



(b)



(c)



List some possible members of A and B that satisfy each diagram.

7 Repeat question 6 but this time assume sets A and B are infinite.

8 $A = \{{}^-5, {}^-2, 1, 8, 15, 21\}$,
$B = \{{}^-2, {}^-1, 0, 1\}$

(a) List the members of
(i) $A \cap B$ **(ii)** $A \cup B$

(b) Write down
(i) $n(A)$ **(ii)** $n(B)$
(iii) $n(A \cap B)$ **(iv)** $n(A \cup B)$

(c) Is it true that
$n(A \cup B) = n(A) + n(B) - n(A \cap B)$?

9 Make two sets A and B of your own. Repeat question 8. What do you notice?

Technology

Learn a great deal more about sets by visiting the web site *http://www.onlinemathlearning.com/math-sets.html*

Look especially at the page on finite and infinite sets.
Watch the video presentation which will give you much interesting information on many other infinite sets and their importance.

2.4 Matrices

Tailortown has two high schools, the Valley School and Tailortown Secondary School. The number of boys and girls in the two schools is given in the tables below.

Valley School

	Form 1	Form 2	Form 3
Boys	50	48	45
Girls	55	52	48

Tailortown Secondary

	Form 1	Form 2	Form 3
Boys	60	58	55
Girls	65	57	53

Any table can, instead, be written as a **matrix**.

A **matrix** is an array of numbers arranged in rows and columns.

The matrices that refer to the populations of the Valley School, V, and Tailortown Secondary, T, can be written as

$$V = \begin{pmatrix} 50 & 48 & 45 \\ 55 & 52 & 48 \end{pmatrix}$$

$$\text{and } T = \begin{pmatrix} 60 & 58 & 55 \\ 65 & 57 & 53 \end{pmatrix}$$

The matrix V has 2 rows and 3 columns

$$\begin{array}{cl} \begin{pmatrix} \boxed{50 \quad 48 \quad 45} \\ \boxed{55 \quad 52 \quad 48} \end{pmatrix} & \begin{array}{l} \text{Row 1} \\ \text{Row 2} \end{array} \end{array}$$

$$\begin{array}{ccc} \boxed{\begin{matrix} 50 \\ 55 \end{matrix}} & \boxed{\begin{matrix} 48 \\ 52 \end{matrix}} & \boxed{\begin{matrix} 45 \\ 48 \end{matrix}} \\ \text{Column 1} & \text{Column 2} & \text{Column 3} \end{array}$$

The matrix, T, also has 2 rows and 3 columns.

The matrices V and T are known as 2 × 3 matrices as they are made up of 2 rows and 3 columns.

Matrices of the same size can be added.

For example,

$$V + T = \begin{pmatrix} 50 & 48 & 45 \\ 55 & 52 & 48 \end{pmatrix} + \begin{pmatrix} 60 & 58 & 55 \\ 65 & 57 & 53 \end{pmatrix}$$

$$= \begin{pmatrix} 50+60 & 48+58 & 45+55 \\ 55+65 & 52+57 & 48+53 \end{pmatrix}$$

$$= \begin{pmatrix} 110 & 106 & 100 \\ 110 & 109 & 101 \end{pmatrix}$$

Notice the matrix sum V + T represents the number of boys and girls in Forms 1, 2 and 3 in **both** schools.

Example 10

Given $A = \begin{pmatrix} 3 & 1 \\ -4 & 6 \end{pmatrix}$, $B = \begin{pmatrix} 2 & 2 \\ 3 & -4 \end{pmatrix}$, $C = \begin{pmatrix} 5 \\ 7 \end{pmatrix}$

Find **(a)** A + B **(b)** C + A

(a) $A + B = \begin{pmatrix} 3 & 1 \\ -4 & 6 \end{pmatrix} + \begin{pmatrix} 2 & 2 \\ 3 & -4 \end{pmatrix}$

$$= \begin{pmatrix} 3+2 & 1+2 \\ -4+3 & 6-4 \end{pmatrix} = \begin{pmatrix} 5 & 3 \\ -1 & 2 \end{pmatrix}$$

(b) $C + A = \begin{pmatrix} 5 \\ 7 \end{pmatrix} + \begin{pmatrix} 3 & 1 \\ -4 & 6 \end{pmatrix}$

This cannot be done as A is a 2×2 matrix while C is 2×1 matrix. They have different sizes.

Exercise 2H

1 Write down the numbers of rows and columns of each of these matrices

(a) $\begin{pmatrix} 2 & 1 \\ -6 & 4 \end{pmatrix}$ **(b)** $\begin{pmatrix} -3 \\ 0 \end{pmatrix}$

(c) $(-1, 0)$ **(d)** $\begin{pmatrix} 2 \\ 1 \\ 5 \end{pmatrix}$

(e) $\begin{pmatrix} 2 & 4 \\ 5 & -6 \\ 1 & -2 \end{pmatrix}$ **(f)** $\begin{pmatrix} 2 & 3 & 5 \\ 1 & 4 & 6 \\ -2 & 1 & 7 \end{pmatrix}$

2 Given

$A = \begin{pmatrix} 3 \\ 4 \end{pmatrix}$, $B = \begin{pmatrix} 2 \\ -1 \end{pmatrix}$, $C = \begin{pmatrix} 4 \\ 0 \end{pmatrix}$

Find

(a) A + B **(b)** B + A **(c)** A + C
(d) C + A **(e)** B + C **(f)** C + B

3 In Question 2, is it true that A + B = B + A?

4 Given

$A = \begin{pmatrix} 3 & 1 \\ 0 & 4 \end{pmatrix}$, $B = \begin{pmatrix} 1 & 1 \\ 1 & 1 \end{pmatrix}$, $C = \begin{pmatrix} 2 & 1 \\ -1 & 0 \end{pmatrix}$

$D = \begin{pmatrix} -3 & 1 \\ 0 & -4 \end{pmatrix}$, $E = (4, 2)$, $F = \begin{pmatrix} 3 \\ 1 \\ 0 \end{pmatrix}$

Work out, where possible

(a) A + B **(b)** A + C **(c)** A + D
(d) C + D **(e)** E + F **(f)** F + A
(g) C + B **(h)** D + E **(i)** D + B

5 **(a)** Using the matrices in Question 4, calculate
(i) A + (B + C) **(ii)** (A + B) + C
(b) Is matrix addition associative?

6 The scores of three boys on a maths test are shown in the table below.

Name	Score
Andy	6
Brian	2
Chad	5

The boys did a retest and scored as follows

Name	Score
Andy	8
Brian	5
Chad	10

(a) Write their scores on the first test as a matrix, T
(b) Write their scores on the retest as a matrix, R.
(c) Calculate R − T.
(d) What does your answer to (c) tell you?

7 If $A = \begin{pmatrix} 2 & 1 \\ 1 & 2 \end{pmatrix}$, $B = \begin{pmatrix} -2 & 1 \\ 1 & -2 \end{pmatrix}$ $C = \begin{pmatrix} 3 & 0 \\ 0 & 3 \end{pmatrix}$,

$D = \begin{pmatrix} -3 & 0 \\ 0 & -3 \end{pmatrix}$

Find

(a) A + B **(b)** A − B
(c) C + D **(d)** C − D
(e) B − C **(f)** D − C

8 Tiny's Fast Food corner has the following price list

Food	Price
hamburger	$12
cheese burger	$14
roti	$10

(a) Write the price list as a matrix, P.
(b) What would be the cost of ordering 4 of each of these items?

(c) Find the matrix representing
(i) 2P **(ii)** 4P **(iii)** 5P

(d) What meaning would you attach to 5P?

9 Write as a single matrix

(a) $2\begin{pmatrix}2\\1\end{pmatrix}$ **(b)** $4\begin{pmatrix}3\\0\\1\end{pmatrix}$

(c) $-2\begin{pmatrix}1\\0\end{pmatrix}$ **(d)** $5\begin{pmatrix}2 & 0\\0 & 2\end{pmatrix}$

10 Find the value of the letters

(a) $3\begin{pmatrix}4\\2\end{pmatrix}=\begin{pmatrix}a\\6\end{pmatrix}$

(b) $b\begin{pmatrix}-1\\3\end{pmatrix}=\begin{pmatrix}-6\\18\end{pmatrix}$

(c) $4\begin{pmatrix}2 & c\\d & 5\end{pmatrix}=\begin{pmatrix}8 & 16\\-12 & 20\end{pmatrix}$

(d) $e\begin{pmatrix}3 & -2\\4 & 1\end{pmatrix}-\begin{pmatrix}1 & 0\\0 & 1\end{pmatrix}=\begin{pmatrix}5 & -2e\\4e & e-1\end{pmatrix}$

Activity

Find out more about matrices.

- Give at least three examples of the use of matrices in real life.
- Illustrate each example with actual matrices.
- Make a short presentation of your findings to your teacher.

Exercise 2I – mixed questions

1 Work out:
(a) $4+{}^{-}5$ **(b)** ${}^{-}4+5$
(c) $6+{}^{-}8$ **(d)** ${}^{-}6+8$
(e) ${}^{-}5+{}^{-}9$ **(f)** ${}^{-}1+{}^{-}3$
(g) ${}^{-}12+8$ **(h)** $14+{}^{-}18$

2 Calculate:
(a) ${}^{-}3-2$ **(b)** $6-8$
(c) $4-{}^{-}3$ **(d)** ${}^{-}4-{}^{-}3$
(e) $2-{}^{-}9$ **(f)** ${}^{-}9-{}^{-}2$
(g) $12-{}^{-}1$ **(h)** ${}^{-}12-{}^{-}1$

3 Work out
(a) $3+2-6$ **(b)** ${}^{-}4+2-1$
(c) $4+{}^{-}2-6$ **(d)** ${}^{-}6+3+{}^{-}2$
(e) $4-{}^{-}3-2$ **(f)** ${}^{-}3-{}^{-}4+2$
(g) ${}^{-}1-{}^{-}2-3$ **(h)** $2+{}^{-}3-{}^{-}1$

4 Calculate
(a) $3\times{}^{-}2$ **(b)** $4\times{}^{-}5$
(c) ${}^{-}3\times{}^{-}3$ **(d)** ${}^{-}8\times4$
(e) $16\div{}^{-}2$ **(f)** ${}^{-}32\div8$
(g) ${}^{-}15\div{}^{-}3$ **(h)** ${}^{-}48\div6$

5 Find the value of $3x^2-x$
When
(a) $x=1$ **(b)** $x={}^{-}1$
(c) $x=0$ **(d)** $x={}^{-}4$

6 If N = {1, 2, 3, …}
Z = {… $^{-}3$, $^{-}2$, $^{-}1$, 0, 1, 2, 3, …}
W = {0, 1, 2, 3, …}

(a) Describe the set N, Z and W.
(b) Draw a venn diagram to show N, Z and W
(c) What is
(i) N ∩ Z **(ii)** W ∩ Z ?
(d) Is it true that
W ⊂ Z ⊂ Z?

7 On Monday I had $23.50 in my bank account. On Tuesday I wrote cheques totalling $39. I paid $22 into my account on Wednesday. How much did I have in my account on Thursday?

8 Find the value of each expression when $x={}^{-}1$.
(a) x^2+4
(b) x^3-2x+3
(c) x^4+3x^2-9

9 $A=\begin{pmatrix}3 & 1\\-4 & 6\end{pmatrix}$, $B=\begin{pmatrix}2 & -1\\3 & -2\end{pmatrix}$ $C=\begin{pmatrix}1 & 0\\0 & 1\end{pmatrix}$.

Evaluate
(a) A + B **(b)** B + C **(c)** A + C
(d) 3A **(e)** −2C **(f)** 3A − 2C
(g) 4B **(h)** 3B − C **(i)** 4A − 3B

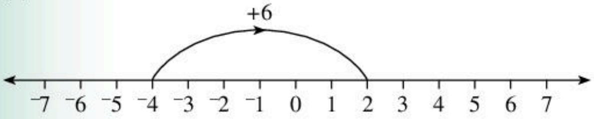
10 Find the value of a, b, c and d if

$$3\begin{pmatrix}a & b\\c & d\end{pmatrix}=\begin{pmatrix}15 & -5\\12 & 21\end{pmatrix}$$

2 Consolidation

Example 1

(a) Work out $^{-}4 + 6$



$^{-}4 + 6 = 2$

(b) Work out $^{-}2 - 3$

$^{-}2 - 3 = {}^{-}5$

Example 2

Work out:

(a) $^{-}3 + {}^{-}5$

$$^{-}3 + {}^{-}5 = {}^{-}3 - 5$$
$$= {}^{-}8$$

(b) $^{-}3 - {}^{-}5$

$$^{-}3 - {}^{-}5 = {}^{-}3 + 5$$
$$= 2$$

Example 3

Work out:

(a) $4 \times {}^{-}5$

$4 \times {}^{-}5 = {}^{-}20$

(b) $^{-}18 \div {}^{-}3$

$^{-}18 \div {}^{-}3 = 6$

Example 4

Find the value of $6 - 2x - 3x^2$ when:

(a) $x = {}^{-}2$

$$6 - 2x - 3x^2$$
$$= 6 - 2({}^{-}2) - 3({}^{-}2)^2$$
$$= 6 + 4 - 3 \times 4$$
$$= 10 - 12 = {}^{-}2$$

(b) $x = {}^{-}5$

$$6 - 2x - 3x^2$$
$$= 6 - 2({}^{-}5) - 3({}^{-}5)^2$$
$$= 6 + 10 - 3 \times 25$$
$$= 16 - 75$$
$$= {}^{-}59$$

Example 5

Work out:

(a) $\begin{pmatrix} 2 & -4 \\ -3 & 2 \end{pmatrix} + \begin{pmatrix} 3 & -4 \\ 2 & -1 \end{pmatrix}$ **(b)** $3\begin{pmatrix} -2 & 1 \\ 3 & 4 \end{pmatrix}$

(a) $$\begin{pmatrix} 2 & -4 \\ -3 & 2 \end{pmatrix} + \begin{pmatrix} 3 & -4 \\ 2 & -1 \end{pmatrix} = \begin{pmatrix} 2+3 & {}^{-}4-4 \\ {}^{-}3+2 & 2-1 \end{pmatrix}$$
$$= \begin{pmatrix} 5 & -8 \\ -1 & 1 \end{pmatrix}$$

(b) $$3\begin{pmatrix} -2 & 1 \\ 3 & 4 \end{pmatrix} = \begin{pmatrix} -6 & 3 \\ 9 & 12 \end{pmatrix}$$

Exercise 2

1 Use a number line to work out

(a) $^{-}4 - 5$ **(b)** $2 - 6$

(c) $^{-}4 + 2$ **(d)** $^{-}5 + 7$

2 Calculate:

(a) $7 + {}^{-}2$ **(b)** $^{-}7 + {}^{-}2$

(c) $^{-}6 + {}^{-}3$ **(d)** $6 + {}^{-}3$

3 Calculate:

(a) $3 - {}^{-}2$ **(b)** $^{-}7 - {}^{-}4$

(c) $^{-}4 - {}^{-}1$ **(d)** $^{-}13 - {}^{-}12$

4 Work out:

(a) $15 \div {}^{-}5$ **(b)** $6 \times {}^{-}4$

(c) $\dfrac{-12}{-4}$ **(d)** $\dfrac{20}{-5}$

5 Work out:

(a) $3 \times {}^{-}5 + 2$ **(b)** $\dfrac{12}{-2} - 3$

(c) $(4 - {}^{-}2)^2 - 1$ **(d)** $({}^{-}1)^3 - 3 \times 2$

6 Find the value of these expressions when

(i) $x = {}^{-}3$ **(ii)** $x = 3$.

(a) $x^2 - 3x + 9$ **(b)** $9 - 3x - x^2$

(c) $(1 - 2x)^2$ **(d)** $5 - 2x - 3x^3$

(e) $4 - x - x^2 - x^3$ **(f)** $3x^4 - x^3 + 2x - 5$

Application

7 The temperature in Calgary is 5°C.

(a) What is the temperature in:

- **(i)** Helsinki if its is 9°C colder than Calgary
- **(ii)** Moscow if it is twice as cold as Helsinki
- **(iii)** Anchorage if it is 2°C warmer than Moscow?

(b) Write the cities in descending order of temperature.

(c) How much colder is

- **(i)** Moscow than Calgary
- **(ii)** Anchorage than Helsinki?

8 **(a)** Copy and complete the table for $y = 7 - x^2$.

x	$^{-}3$	$^{-}2$	$^{-}1$	0	1	2	3
y		3					$^{-}2$

(b) Draw the graph of $y = 7 - x^2$.

(c) Use your graph to find the value of x when $y = 0$.
Hence find $\sqrt{7}$.

9 The number of people living in a house can be shown as a matrix, for example,

$$\begin{array}{rc} & \text{Males} \quad \text{Females} \\ \begin{array}{r}\text{Adults}\\ \text{Children (under 18)}\end{array} & \begin{pmatrix} 2 & 1 \\ 1 & 2 \end{pmatrix} \end{array}$$

(a) Write a matrix to show the number in

- **(i)** your household
- **(ii)** your neighbours' household

(b) Write a matrix to show the total number of person's in your own and your neighbours' house.

Summary

You should know …

1 Adding a negative number is the same as subtracting a positive number.
For example:
$^{-}3 + {^{-}4} = {^{-}3} - 4 = {^{-}7}$

2 Subtracting a negative number is the same as adding a positive number.
For example:
$^{-}4 - {^{-}3} = {^{-}4} + 3 = {^{-}1}$

3 How to multiply and divide integers.
For example:
$^{-}2 \times {^{-}3} = {^{-}6}$
$^{-}6 \div 2 = {^{-}3}$

Check out

1 Work out:
(a) $2 + {^{-}1}$ **(b)** $6 + {^{-}15}$
(c) $^{-}3 + {^{-}3}$ **(d)** $^{-}8 + {^{-}12}$
(e) $^{-}6 + {^{-}9}$ **(f)** $1 + {^{-}13}$

2 Calculate:
(a) $3 - {^{-}6}$ **(b)** $1 - {^{-}2}$
(c) $^{-}3 - {^{-}6}$ **(d)** $^{-}1 - {^{-}2}$
(e) $^{-}13 - {^{-}4}$ **(f)** $26 - {^{-}18}$

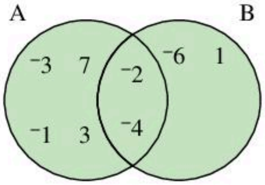
3 Calculate:
(a) $3 \times {^{-}7}$ **(b)** $28 \div {^{-}4}$
(c) $^{-}32 \div {^{-}4}$ **(d)** $^{-}7 \times 13$
(e) $\frac{-52}{13}$ **(f)** $13 \times {^{-}6}$

4 How to substitute negative numbers in expressions.
For example:
Find $2x^2 - 3x + 5$ if $x = {}^{-}4$.

$$\begin{aligned} 2x^2 - 3x + 5 &= 2 \times ({}^{-}4)^2 - 3 \times ({}^{-}4) + 5 \\ &= 2 \times {}^{-}4 \times {}^{-}4 - {}^{-}12 + 5 \\ &= 32 - {}^{-}12 + 5 \\ &= 32 + 12 + 5 \\ &= 49 \end{aligned}$$

5 Any positive number has two square roots; one is positive and one is negative.
For example: The square roots of 16 are 4 and $^{-}4$.

6 You can show negative numbers on a Venn diagram
For example:
$A = \{{}^{-}4, {}^{-}3, {}^{-}2, {}^{-}1, 3, 7\}$, $B = \{{}^{-}6, {}^{-}4, {}^{-}2, 1\}$



The sets of natural numbers, whole numbers and integers are all infinite sets.

4 Find the value of each expression when $x = {}^{-}3$.
(a) $2x^2$ **(b)** $x - 3$
(c) $(x - 4)^2$ **(d)** $3x^2 - x$
(e) $6 - x^2$
(f) $4x^3 - 2x^2 - x + 2$

5 Work out:
(a) $({}^{-}3)^2$ **(b)** 6^2
(c) $\sqrt{36}$ **(d)** $\sqrt{49}$

Measurement 1

3

Objectives

- ✔ calculate the perimeter of shapes
- ✔ estimate the area of shapes by counting squares
- ✔ calculate the area of parallelograms, trapeziums and circles
- ✔ find the area of composite shapes
- ✔ calculate the area of a sector of a circle
- ✔ calculate the length of arc of a circle

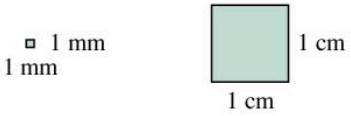
What's the point?

Real estate agents deal with the sale of properties. The key criteria for fixing the price of a piece of land is its location and its area. The larger its area the more valuable the land. Precise measurements of land area are therefore most important!

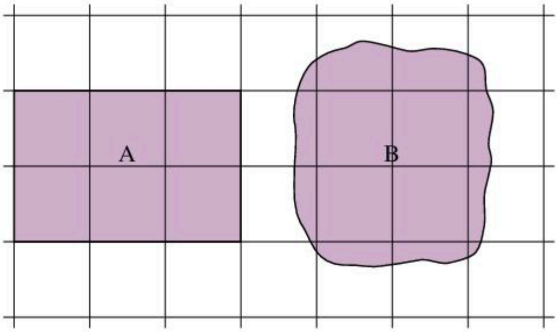
Before you start

You should know ...

1 Area can be measured in mm^2 and cm^2.



2 You can find the area of a shape by counting squares.
For example:



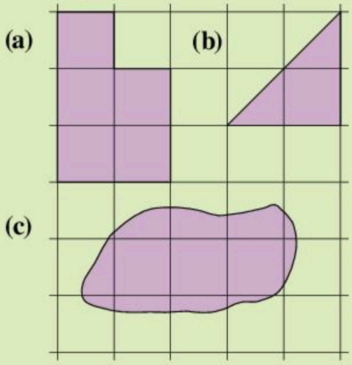
Area A is 6 cm^2 Area B is roughly 6 cm^2

Check in

1 Estimate the area of:

(a) a 20 cent coin
(b) a postage stamp
(c) your thumb nail
(d) a page of your exercise book.

2 Find the areas of these shapes.
Each square is 1 cm^2.



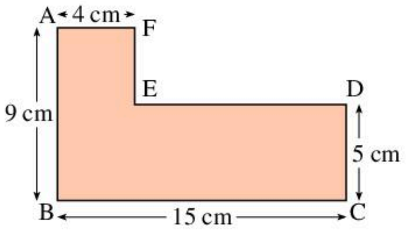
3.1 Perimeter

You will recall that the distance around a shape is called the **perimeter**. Generally, it is easy to find the perimeter of a shape by simply adding up the lengths of its sides.

In some cases you have to calculate some of the lengths before you can find the perimeter.

Example 1

Find the perimeter of the shape below.



The lengths of all the sides of the shape ABCDEF are given except for EF and DE.

To find EF:

Notice EF + 5 cm = 9 cm

so EF = 4 cm

To find DE:

Notice DE + 4 cm = 15 cm

so DE = 11 cm

The perimeter, P, is given by

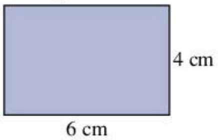
P = 9 cm + 15 cm + 5 cm + 11 cm + 4 cm + 4 cm

= 48 cm

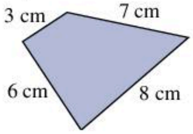
Exercise 3A

1 Find the perimeter of these shapes.

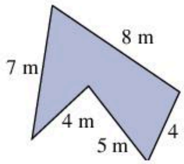
(a)



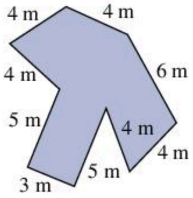
(b)



(c)

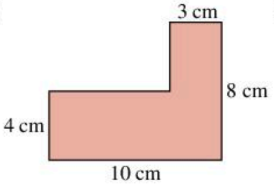


(d)

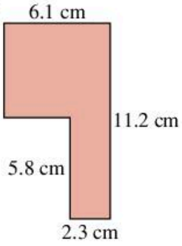


2 Calculate the perimeter of these shapes. You will first have to find the missing lengths.

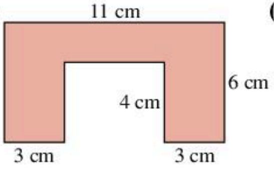
(a)



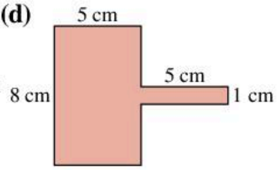
(b)



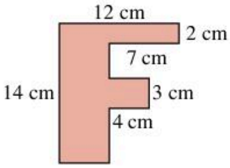
(c)



(d)

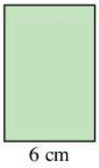


(e)

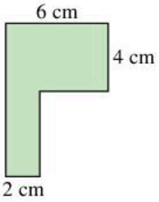


3 The perimeter of each shape is 30 cm. Find the lengths of the missing sides.

(a)

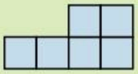


(b)



Investigation

The shape below is made up of six small squares.



Its perimeter is 12 units.

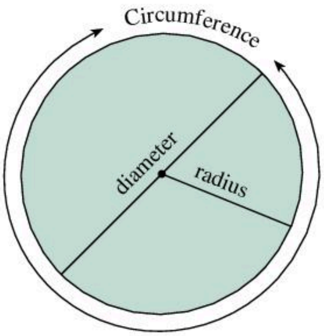
Rearrange the six squares to make a shape with a smaller perimeter.

Which shape made up of six squares has the smallest perimeter? Which has the largest?

What if you used five squares? seven squares?

Circumference of a circle

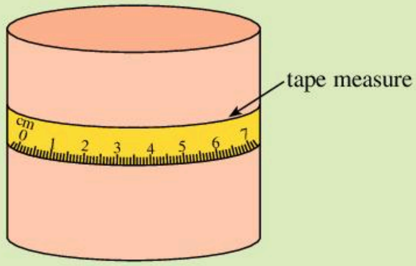
The perimeter of a circle is called its **circumference**.



The circumference of a circle is about 3 times the distance across it or its diameter.

Try the activity below to get a better idea of the relationship between the diameter and circumference of a circle.

Activity



(a) Using a tape measure, measure the diameter and circumference of five tins with circular bases.

(b) Copy and complete the table.

Object	Diameter (D) cm	Circum-ference (C) (cm)	$C \div D$
1			
2			
3			
4			
5			

(c) What values did you get for the last column, $C \div D$?

In the activity you should have found that

$$C \div D \simeq 3.1$$

More exactly the circumference C of a circle is given by the formula

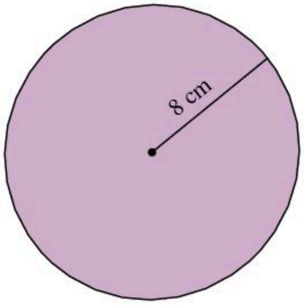
$$\boxed{\begin{array}{r} \mathbf{C = 2\pi r} \\ \textbf{or } \mathbf{C = \pi d} \end{array}}$$

where r is the radius and d is the diameter of the circle. The value of π (pi) is approximately $3\frac{1}{7}$ or 3.14. A scientific calculator has a π button which gives a very accurate value.

To find the circumference of a circle, you just need to use the formula.

Example 2

What is the circumference of a circle with radius 8 cm? Take $\pi = 3.14$.

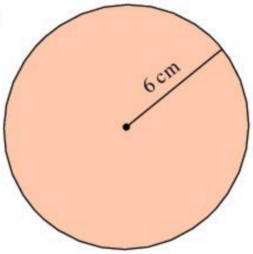


$$\begin{aligned} \text{Circumference} &= 2\pi r \\ &= 2 \times 3.14 \times 8 \text{ cm} \\ &= 50.24 \text{ cm} \end{aligned}$$

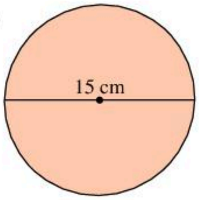
Exercise 3B

1 Find the circumference of these shapes. Take $\pi = 3.14$.

(a)



(b)



2 Using $\pi = \frac{22}{7}$, find the circumference of a circle with radius

(a) 7 cm **(b)** 21 cm

3 Using $\pi = 3.14$, find the circumference of a circle with diameter

(a) 6.2 cm **(b)** 11.7 cm

4

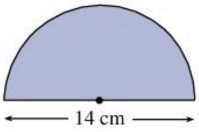
A bicycle wheel has diameter 75 cm.

(a) What is the circumference of the wheel?

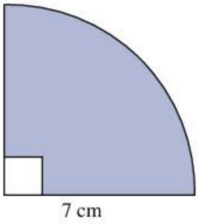
(b) A cyclist travels 100 m on this bicycle. Through how many revolutions does the wheel turn?

5 Find the perimeter of these shapes. Take $\pi = 3\frac{1}{7}$

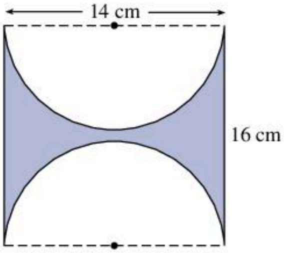
(a)



(b)



(c)



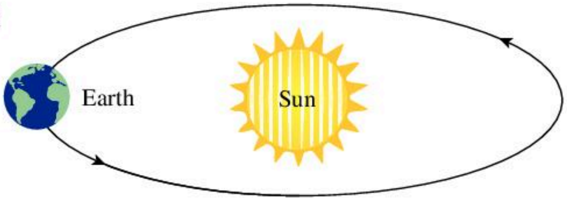
6 What is the radius of a circle that has circumference

(a) 31.4 cm **(b)** 10 cm?

(Take $\pi = 3.14$)

7 The Earth's circumference is 40 000 km. What is its diameter?

8



The Earth is about 150 000 000 km from the Sun.

(a) How far does the Earth travel in one year?

(b) How far does the Earth travel in one day?

(c) What is the Earth's speed in km/h?

9 A bicycle wheel has diameter 60 cm. How far has a cyclist travelled when the wheel has made 250 revolutions? (Take $\pi = 3.14$)

10 A satellite is placed in orbit 10 km above the earth's surface. Scientists want to send a second satellite in orbit 10.1 km above the earth's surface. For every complete revolution, how much further will the second satellite travel? (Take $\pi = 3.14$)

11 The second hand of a watch is 15 mm long. How far does the pointer travel in

(a) 15 seconds

(b) 20 seconds

(c) $5\frac{1}{2}$ minutes?

(Take $\pi = 3.14$)

Investigation

Car tyres have codes to distinguish them, for example, P225/60R17 98T

- Find out what the letters and numbers mean.
- Which numbers are related to the diameter of the wheel's rim or circumference?
- What codes are used on bicycle tyres?
- On motorbikes?

Technology

Use the online calculator at

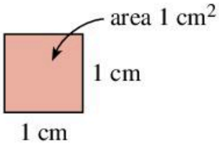
www.onlineconversion.com/circlessolve.htm

To calculate diameters and circumferences of circles.

Use the calculator to check your answer to Exercise 3B.

3.2 Basics of area

The area of a shape is a measure of the size of its surface. Area is measured in square units. In the metric system these may be square centimetres (cm^2)

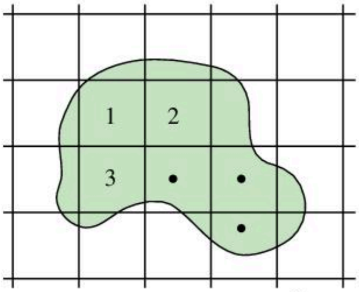


or square metres (m^2) or square kilometres (km^2).

To find the area of an irregular shape, you will need to find the number of square centimetres (or square metres) that will cover it.

Example 3

What is the area of the shaded shape?

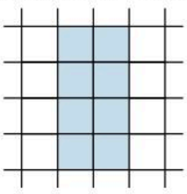


The area of the shape is about 6 cm^2.

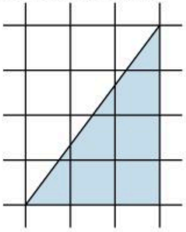
Exercise 3C

1 Estimate the area in squares of these shapes.

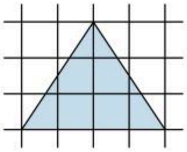
(a)



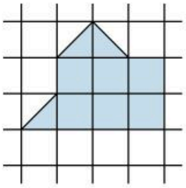
(b)



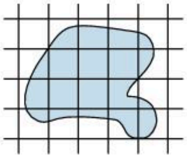
(c)



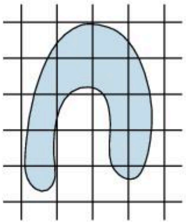
(d)



(e)



(f)



2 Draw a shape with an area of exactly
(a) 2 cm^2 **(b)** 3 cm^2 **(c)** 10 cm^2

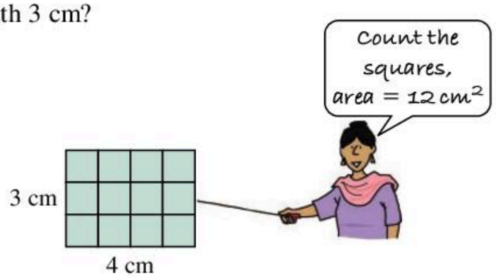
3 On centimetre squared paper, draw an outline of your hand.
- **(a)** Count squares to estimate its area.
- **(b)** Repeat for your foot.
- **(c)** Which has the larger area?

Areas of rectangles and triangles

There are simple formulae that can be used to find the area of common shapes.

Area of a rectangle = length × width

For example:
What is the area of a rectangle with length 4 cm and width 3 cm?



$$\text{Area of rectangle} = \text{length} \times \text{width}$$
$$= 4\text{ cm} \times 3\text{ cm}$$
$$= 12\text{ cm}^2$$

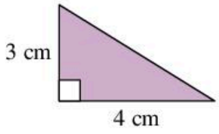
The formula for the area of a triangle is

$$\boxed{\textbf{Area of triangle} = \tfrac{1}{2} \times \textbf{base} \times \textbf{height}}$$

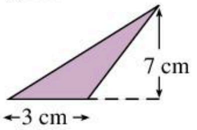
Example 4

What is the area of these triangles?

(a)



(b)



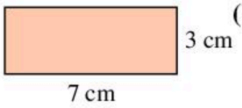
(a) Area triangle $= \frac{1}{2} \times \text{base} \times \text{height}$
$= \frac{1}{2} \times 4 \text{ cm} \times 3 \text{ cm}$
$= 6 \text{ cm}^2$

(b) Area triangle $= \frac{1}{2} \times \text{base} \times \text{height}$
$= \frac{1}{2} \times 3 \text{ cm} \times 7 \text{ cm}$
$= 10.5 \text{ cm}^2$

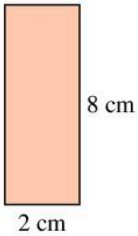
Exercise 3D

1 Find the area of these rectangles.

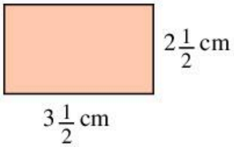
(a)



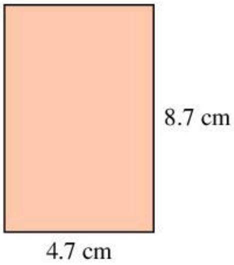
(b)



(c)

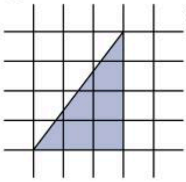


(d)

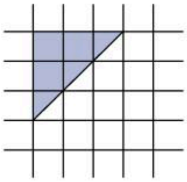


2 Find, by counting squares, the areas of these triangles.

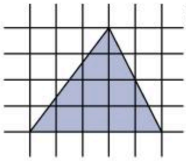
(a)



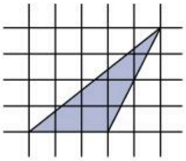
(b)



(c)



(d)



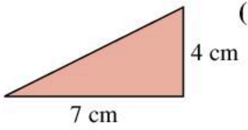
3 **(a)** Copy and complete the table for the triangles in Question 2.

Triangle	Base	Height	$\frac{1}{2} \times$ base $\times$ height
(a)	3		
(b)			
(c)			
(d)		4	

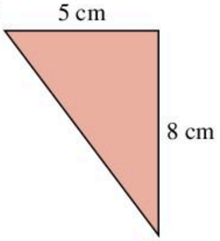
(b) Do you agree that the formula area of a triangle $= \frac{1}{2} \times$ base $\times$ height is correct?

4 Find the area of these triangles.

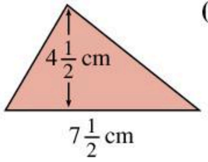
(a)



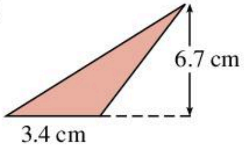
(b)



(c)



(d)



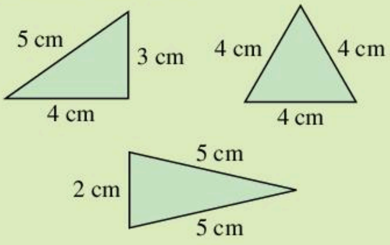
Technology

Watch the video on finding the areas of triangles at the website

www.mathplayground.com

Investigation

Look at these triangles:



They each have perimeter 12 cm.

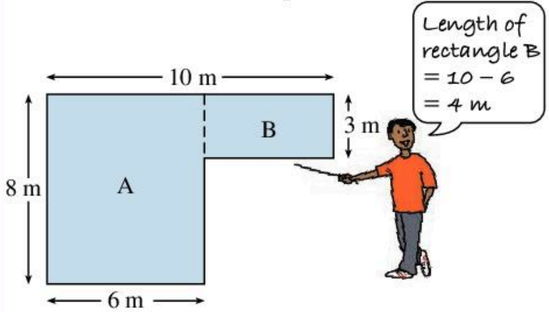
(a) Draw five other triangles with perimeter 12 cm.
(b) Find the area of the triangles you drew.
(c) Which triangle has the largest area?

3.3 Area of composite shapes

The areas of most shapes with straight edges can be found by dividing them up into rectangles or triangles.

Example 5

Find the area of the L-shaped room.



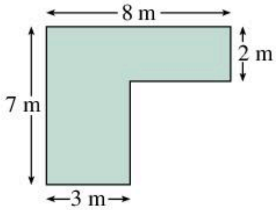
The shape can be divided into two rectangles A and B.

Area of rectangle A $= 8\text{ m} \times 6\text{ m}$
$= 48\text{ m}^2$
Area of rectangle B $= 4\text{ m} \times 3\text{ m}$
$= 12\text{ m}^2$
Area of room $= A + B = 48\text{ m}^2 + 12\text{ m}^2$
$= 60\text{ m}^2$

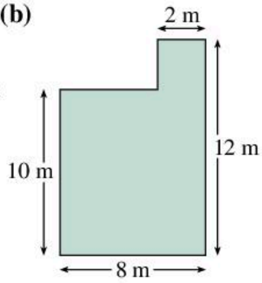
Exercise 3E

1 Find the areas of the shapes by dividing them into rectangles:

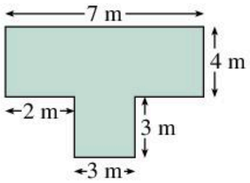
(a)



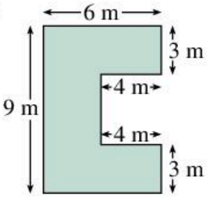
(b)



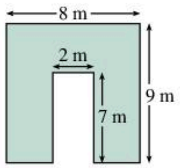
(c)



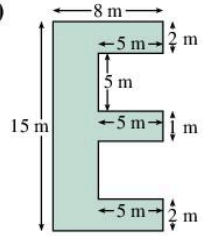
(d)



(e)

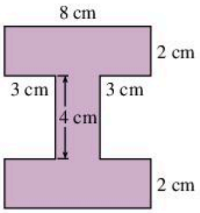


(f)

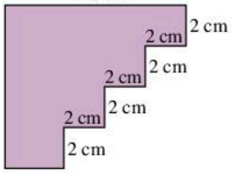


2 Find the area of each of these shapes.

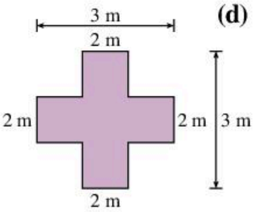
(a)



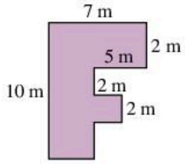
(b)



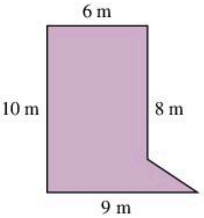
(c)



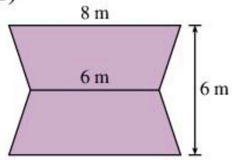
(d)



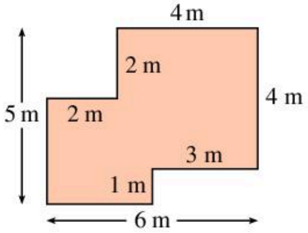
(e)



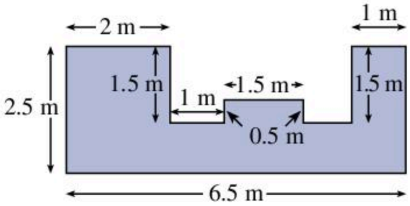
(f)



3 Calculate the area of this shape:

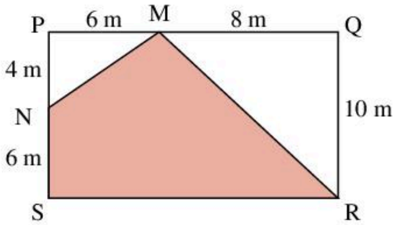


4 This is the plan of the paved patio outside Mr Ramchand's house. Find its area.



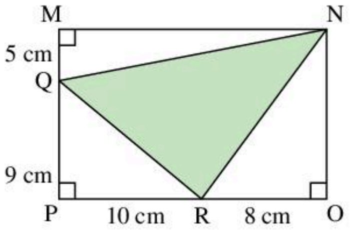
5 The front of a brick house is 10 m wide and 6 m high. There are four windows, each 2.5 m by 1.5 m, and one door 2 m by 0.8 m. What area of the front is brick?

6 Ali's garden is the shaded region.



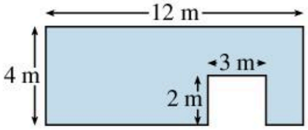
Find the area of Ali's garden.

7 Andy wants to cut the shaded triangle from a piece of cardboard.



Find the area of the shaded triangle.

8 Here is the plan of a room.



(a) What is the area of the room?

(b) If the room is to be paved with rectangular tiles 20 cm by 30 cm, what is the area of a tile in **(i)** cm^2 **(ii)** m^2?

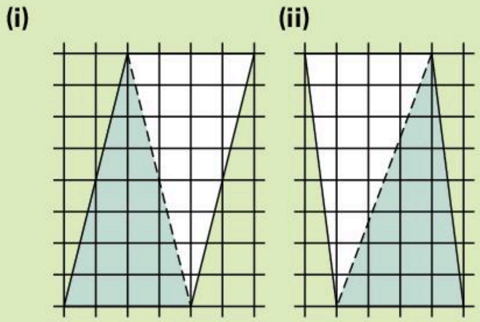
(c) How many tiles are needed to pave the room?

(d) What would be the cost of paving the whole room if each tile cost $1.30?

3.4 Areas of parallelograms and trapeziums

Area of a parallelogram

Activity


(i) (ii)

(a) Make a tracing of the shaded triangle in the first parallelogram above.

(b) Does your tracing fit exactly on to the white triangle in the first parallelogram?

(c) What can you say about the white triangle and the shaded triangle?

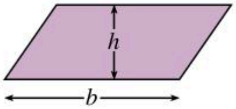
(d) Repeat parts (a) to (c) for drawing (ii).

(e) What is the connection between the area of each parallelogram above and the area of its shaded triangle?

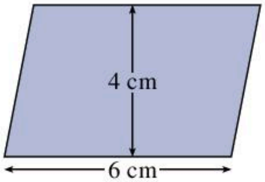
(f) Can you suggest a quick way to find the area of parallelogram?

A parallelogram is made up of two identical triangles. The area of each triangle is $\frac{1}{2}(b \times h)$. The area of the parallelogram is twice this.

- The area of a parallelogram is $\boldsymbol{A} = \boldsymbol{b} \times \boldsymbol{h}$.
  (b is base length; h is vertical height)



Example 6



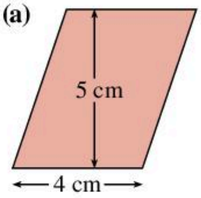
What is the area of the parallelogram?

Area of the parallelogram = base × height
= 6 cm × 4 cm
= 24 cm^2

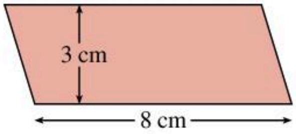
Exercise 3F

1 Find the area of each parallelogram.

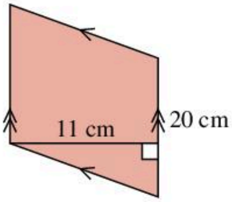
(a)



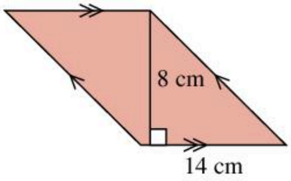
(b)



(c)

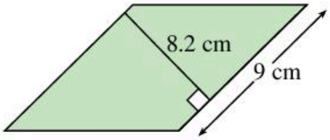


(d)

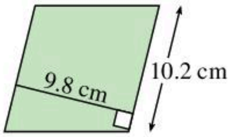


2 Find the area of these parallelograms.

(a)



(b)



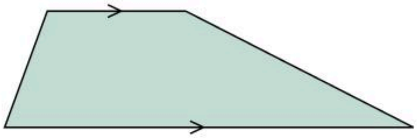
3 By measuring carefully, find the area of this parallelogram.

4 Copy and complete the table for parallelograms.

	Base (cm)	Height (cm)	Area (cm^2)
(a)	3.5	8	
(b)		16	144
(c)	6.5		52
(d)	2.3	3.2	
(e)		7.1	26.27

Area of a trapezium

This shape has only one pair of parallel sides. It is called a **trapezium**.

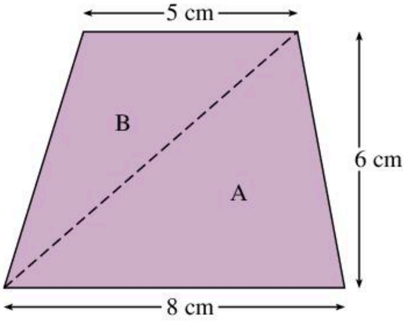


- A trapezium is a quadrilateral with one pair of sides parallel.

The area of a trapezium can be found by dividing it into two triangles.

Example 7

Find the area of this trapezium.

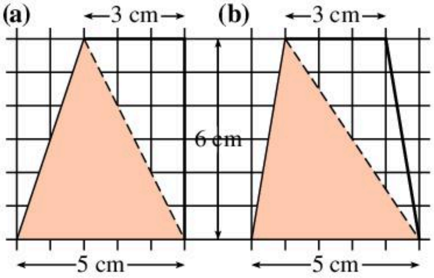


Divide the trapezium into two triangles A and B.

Area of triangle A $= \frac{1}{2} \times 8 \text{ cm} \times 6 \text{ cm}$
$= 24 \text{ cm}^2$
Area of triangle B $= \frac{1}{2} \times 5 \text{ cm} \times 6 \text{ cm}$
$= 15 \text{ cm}^2$
Area of trapezium = A + B
$= 24 \text{ cm}^2 + 15 \text{ cm}^2$
$= 39 \text{ cm}^2$

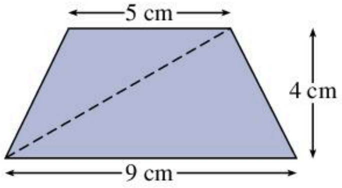
Exercise 3G

1 For each diagram, find the area of:
  - **(i)** the shaded triangle
  - **(ii)** the unshaded triangle
  - **(iii)** the trapezium.

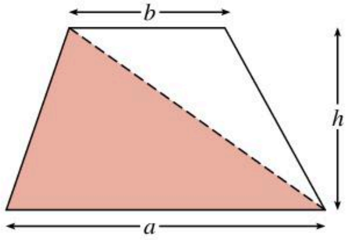


2 Draw a set of trapeziums each with a base of 5 cm, a height of 6 cm and a top edge of 3 cm. Do they all have the same area?

3 Find the area of the trapezium:



Look at this trapezium.

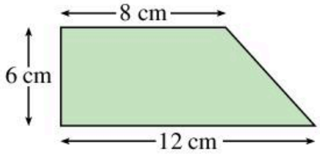


The area of the shaded triangle $= \frac{1}{2}a \times h$
The area of the white triangle $= \frac{1}{2}b \times h$
The area of the trapezium $= \frac{1}{2}a \times h + \frac{1}{2}b \times h$
$= \frac{1}{2}(a + b) \times h$

- The area of a trapezium with two parallel sides of length *a* and *b*, a perpendicular distance *h* apart, is $\boldsymbol{A = \frac{1}{2}(a + b) \times h}$

Example 8

Find the area of the trapezium.

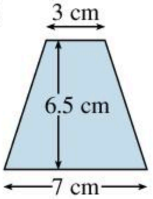


Area of trapezium $= \frac{1}{2}(a + b) \times h$
$= \frac{1}{2}(12 + 8) \times 6$
$= \frac{1}{2} \times 20 \times 6$
$= 60 \text{ cm}^2$

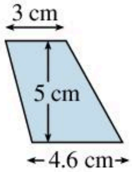
Exercise 3H

1 Find the area of each trapezium:

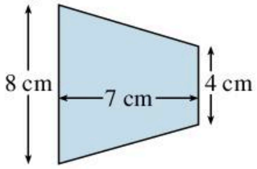
(a)



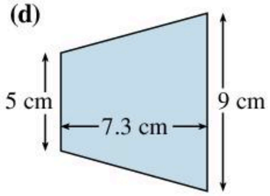
(b)



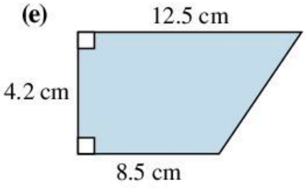
(c)



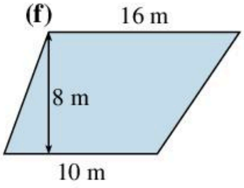
(d)



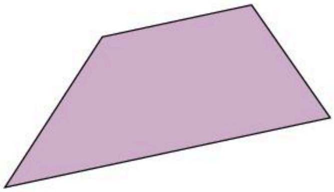
(e)



(f)



2 By measuring, find the area of the trapezium.



3 Using the formula $A = \frac{1}{2}(a + b) \times h$ find the area of a trapezium when:

- **(i)** $a = 7$ cm, $b = 11$ cm, and $h = 10$ cm
- **(ii)** $a = 4$ cm, $b = 3$ cm, and $h = 8$ cm
- **(iii)** $a = 2.6$ cm, $b = 7.4$ cm, and $h = 5.6$ cm
- **(iv)** $a = 9.3$ m, $b = 6.3$ m, and $h = 12.2$ m

4 Find the area of a trapezium with parallel sides of length 24 cm and 16 cm and perpendicular height of 18 cm.

5 The area of a trapezium is 80 cm^2. Its parallel sides are 32 cm and 16 cm in length.
Find the perpendicular height.

6 Copy and complete the table for trapeziums.

	Length of parallel sides		Perpendicular distance between PQ and RS	Area of trapezium
	PQ	RS		
(a)	9 m	15 m	7 m	
(b)	16.8 m	12.5 m	8.4 m	
(c)	23 cm	37 cm	40 cm	
(d)	12.4 m	6.8 m	4.5 m	
(e)	24 cm	16 cm	15.5 cm	

Technology

Learn more about finding areas of shapes. Visit

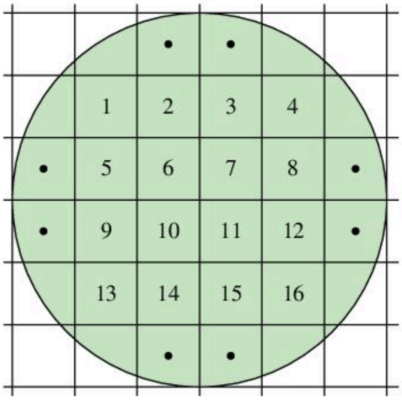
www.bbc.co.uk/education/guides/z2mtyrd/revision/1

Check all seven pages of the website.

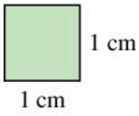
Do not forget the activities and tests!

3.5 Circles, arcs and sectors

One way of finding the area of a circle is to draw the circle on squared paper and count the number of squares, as you did in Exercise 3C.



Each small square



has area 1 cm^2.
Area of circle $\simeq 28$ cm^2

Exercise 3I

1 **(a)** On centimetre squared paper draw circles with radius

- **(i)** 2 cm
- **(ii)** 3 cm
- **(iii)** 4 cm
- **(iv)** 5 cm

(b) By counting the number of square centimetres, find the area of each circle.

(c) Copy and complete the table

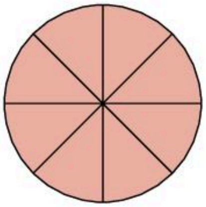
Radius r, of circle (cm)	r^2	Area (cm^2)
2		
3	9	
4		
5	25	

2 In Question 1 find the value of Area $\div r^2$ for each circle. What do you notice?

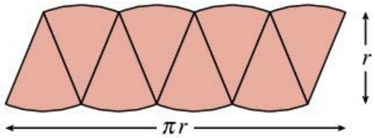
3 Using the result you obtained in Question 2, estimate the area of a circle with radius

- **(a)** 6 cm
- **(b)** 8 cm

Another way of finding the area of a circle is to divide the circle into eight equal parts.



and fit the parts together to make a 'parallelogram'



The 'height' of the 'parallelogram' is r

The base length of the 'parallelogram' is πr, can you see why?

$$\begin{aligned} \text{so the area } &= \pi r \times r \\ &= \pi r^2 \end{aligned}$$

This shows that the area of a circle, A, has area given by the formula

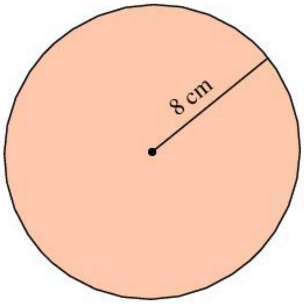
$$A = \pi r^2$$

where r is the radius of the circle and π is the same constant that you met while finding the circumference of a circle.

That is, $\pi \simeq 3.14$ or $3\frac{1}{7}$

You can use the formula, $A = \pi r^2$, to find the area of circles very easily.

Example 9



Find the area of a circle with radius, 8 cm.

$$\begin{aligned} \text{Area of circle } &= \pi r^2 \\ &= 3.14 \times 8^2 \\ &= 3.14 \times 64 \\ &= 200.96 \text{ cm}^2 \end{aligned}$$

Exercise 3J

1 Using $\pi = 3.14$, find the area of a circle with radius

(a) 4 cm **(b)** 5 cm

2 Compare your answers to Question 1 to those in Exercise ____ Question 1(c).

How accurate was your square counting?

3 A circle has diameter 12 cm.

(a) What is its radius?

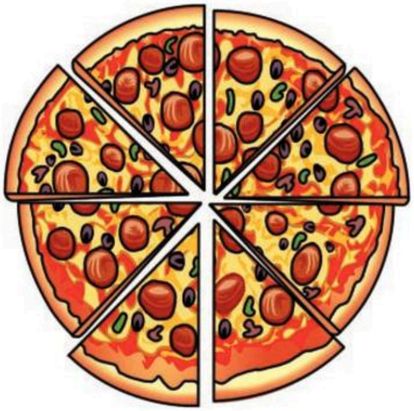
(b) Find the area of the circle.

4 Find the area of circles with diameter

(a) 7 cm **(b)** 15 cm

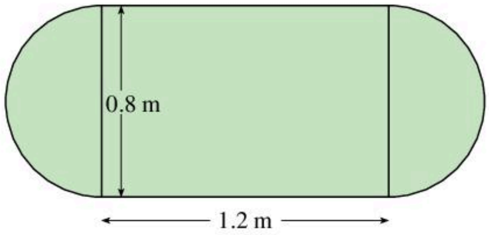
5 A circle has area 60 cm^2. Find its radius.

6



A pizza has a diameter of 30 cm. Find the area of one portion of pizza if the pizza is cut into eight equal pieces.

7 The diagram shows the dimensions of a table with semicircular ends.



Find the area of the table.

Technology

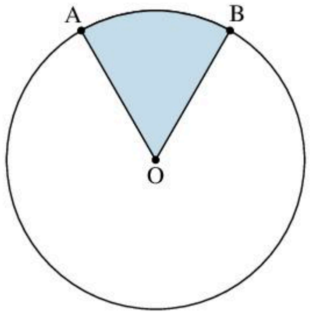
Visit the site

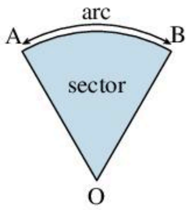
www.calculatorsoup.com/calculators/geometry-plane/circle.php

- Use the circle area calculator to find the area of a circle with
  - (a) radius 9 cm
  - (b) diameter 6.2 cm
- Use the calculator to check your answers to Questions 1–5 of Exercise 3J.

Arcs and sectors

The diagram shows a circle with centre O and points A and B on the circumference





The distance along the circumference from A to B is called the **arc** AB.

The shaded portion of the circle is called a **sector**.

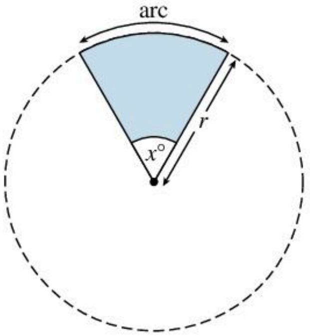
Activity

Research the meanings of the following terms

- minor arc
- major arc
- minor sector
- major sector
- segment
- chord

Draw diagrams to illustrate each term.

Look at the sector below.



The fraction of the circle that represents the sector is $\frac{x}{360°}$. Can you see why?

You can use this fraction to help you find the length of an arc and the area of a sector of a circle.

An **arc** is any piece of the circumference of a circle.

The arc length is a fraction of the circumference.

In the diagram

$$\text{Arc length} = \frac{x°}{360°} \times 2\pi r$$

A **sector** of a circle is the shape enclosed by an arc and two radii of the circle.

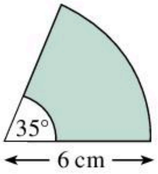
The area of a sector is a fraction of the circle's area.

In the diagram

$$\text{Sector area} = \frac{x°}{360°} \times \pi r^2$$

Example 10

Find the arc length and sector area of the shape below.



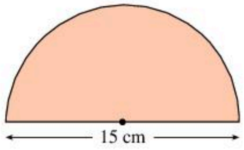
$$\text{Length of arc} = \frac{x^\circ}{360^\circ} \times 2\pi r$$
$$= \frac{35^\circ}{360^\circ} \times 2 \times 3.14 \times 6 \text{ cm}$$
$$= \frac{35^\circ}{360^\circ} \times 37.68 \text{ cm}$$
$$= 3.66 \text{ cm}$$

$$\text{Sector area} = \frac{x^\circ}{360^\circ} \times \pi r^2$$
$$= \frac{35^\circ}{360^\circ} \times 3.14 \times 6 \text{ cm} \times 6 \text{ cm}$$
$$= \frac{35^\circ}{360^\circ} \times 113.04 \text{ cm}^2$$
$$= 10.99 \text{ cm}^2$$

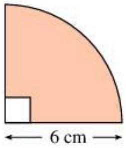
Exercise 3K

1 Find the arc length and sector area of these shapes. Use $\pi = 3.14$.

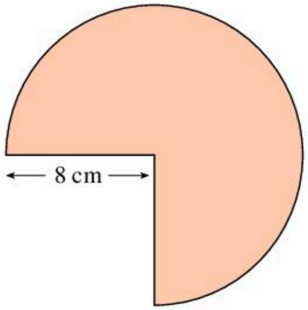
(a)



(b)

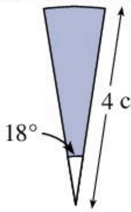


(c)

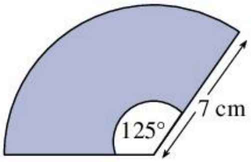


2 Find the areas of these sectors.

(a)



(b)



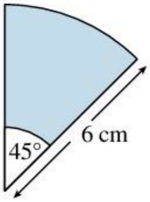
3 A circular watch has a minute hand that is 2.3 cm long.

(a) What distance does the tip of the hand move through in 20 minutes?

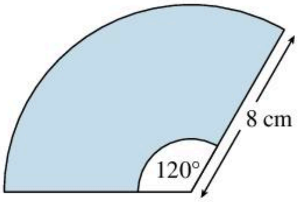
(b) What area of the watch face is traced out by the minute hand in 25 minutes?

4 Using $\pi = 3.14$, find the area of each sector

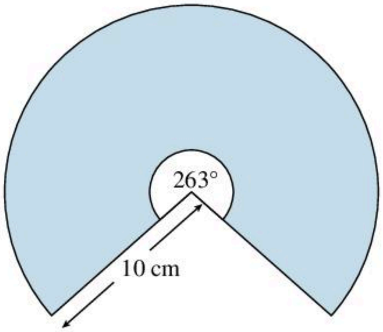
(a)



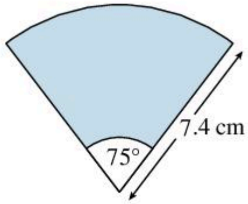
(b)



(c)



(d)



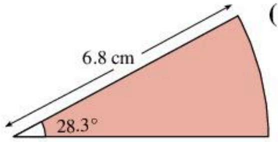
5 Find the arc length of each of the sectors in Question 4.

6

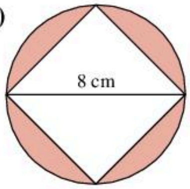
A windscreen wiper is 30 cm in length. Find the area of the windscreen it wipes if it turns through an angle of 140°.

7 Find the area of the shaded parts of these shapes.

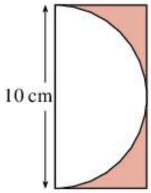
(a)



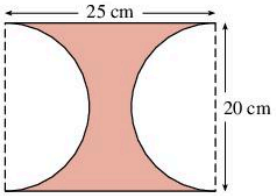
(b)



(c)



(d)

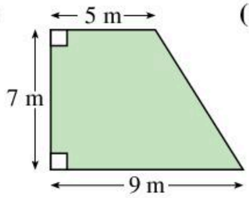


8 A sector of a circle has area 45 cm^2. What is the angle of the sector if the radius is 14.2 cm?

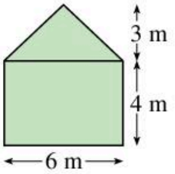
Exercise 3L – mixed questions

1 Find the area of each of these shapes.

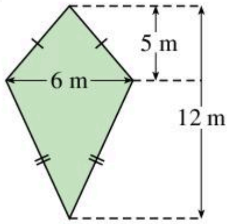
(a)



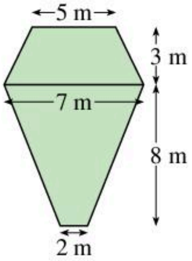
(b)



(c)



(d)

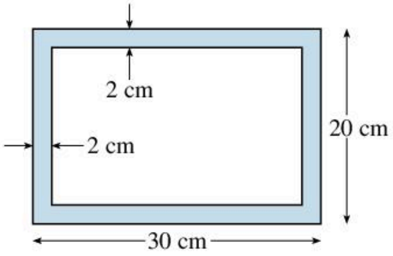


2 Write down the formula for the area of a circle with diameter *d*. Find the area of a circle of diameter:

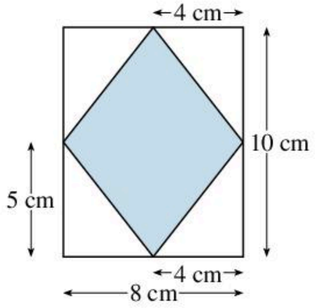
(a) 4 cm **(b)** 12 cm **(c)** 5.6 cm

3 Find the area of the shaded parts in these figures:

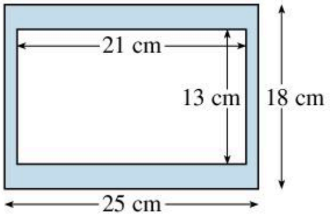
(a)



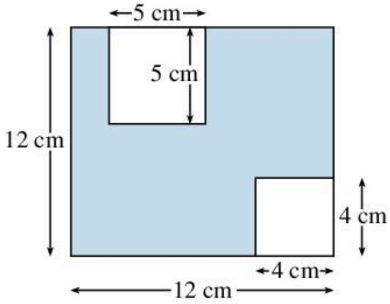
(b)



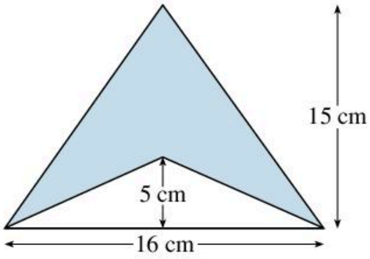
(c)



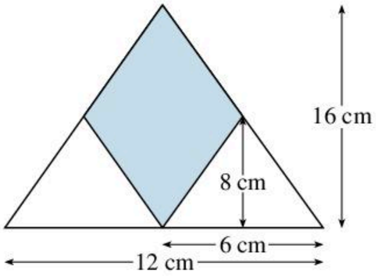
(d)



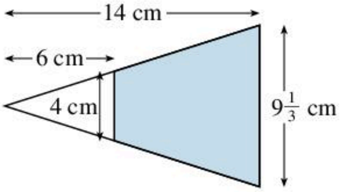
(e)



(f)

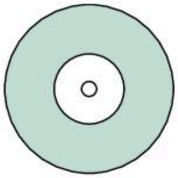


(g)



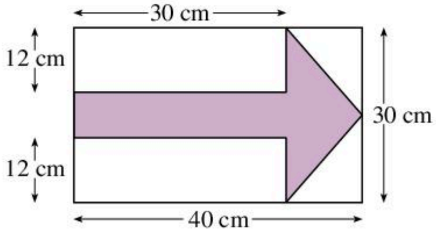
4 At metalwork class, Michael cut a square of tin, of edge 3.7 cm, from a larger square of edge 6.3 cm. What area of tin was left?

5 The diagram shows a gramophone record of diameter 29 cm.



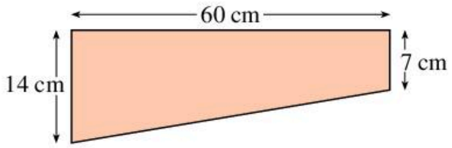
The diameter of the label is 7 cm. Find the area of the playing surface of the record.
Take $\pi = 3.14$

6 An arrow for a sign post has to be cut from a rectangular metal sheet measuring 30 cm by 40 cm.



Find the area of the arrow.

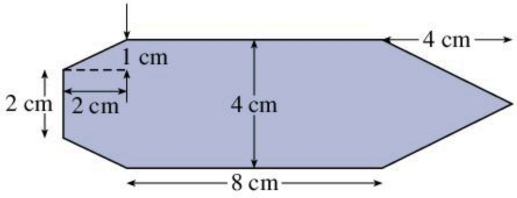
7 The diagram shows a piece of metal which is to be used to make the blade of a saw.



(a) Find the area of the blade.

(b) Metal costs \$78 per square metre.
What is the cost of metal contained in the blade?

8 The diagram shows a piece of wood which has been cut to make the deck of a toy boat. Find its area.



9 Give the base and height of as many triangles with area 24 cm^2 as you can. (use only whole number lengths).

10 A rectangular floor measuring 18 metres by 15 metres is to be covered with carpet, leaving a border 1.6 m wide round the room.
Find the area of the carpet required.

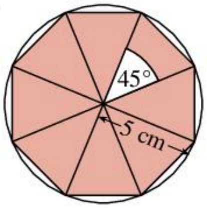
11 **(a)** Calculate the area of a square with side 5 cm. What is the area of a square whose sides are twice as long? How are the two areas connected?

(b) What happens if the new square has sides three times the original?

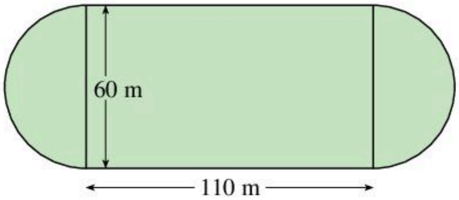
12 Use a protractor and compasses to draw an equilateral triangle with side of 4 cm. Find its area.

13 Use your answer for Question 11 to calculate the area of a regular hexagon inscribed in a circle of radius 4 cm.

14 Draw a circle of radius 5 cm.
Construct a regular octagon within it.
Find the area of:
(a) one triangle
(b) the octagon



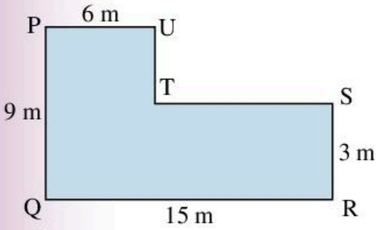
15 The inside of an athletics track needs to be re-seeded with grass. The area that needs seeding is sketched below. Find the total amount of seed required if 0.25 kg of seed is needed for 1 m^2 of ground.



3 Consolidation

Example 1

What is the perimeter of this shape?



Perimeter = Distance around

$= PQ + QR + RS + ST + TU + UP$

$TS = 15\text{ m} - 6\text{ m} = 9\text{ m}$

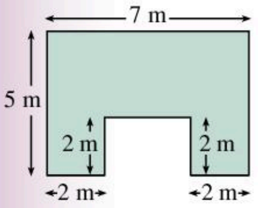
$UT = 9\text{ m} - 3\text{ m} = 6\text{ m}$

So Perimeter $= 9\text{ m} + 15\text{ m} + 3\text{ m} + 9\text{ m} + 6\text{ m} + 6\text{ m}$

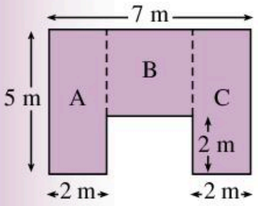
$= 48\text{ m}$

Example 2

Find the area of this shape.



First divide the shape into rectangles.



Notice that the length of rectangle

$B = 7\text{ m} - 2\text{ m} - 2\text{ m} = 3\text{ m}$

and width of rectangle $B = 5\text{ m} - 2\text{ m} = 3\text{ m}$

Area of rectangle $A = 5\text{ m} \times 2\text{ m} = 10\text{ m}^2$

Area of rectangle $B = 3\text{ m} \times 3\text{ m} = 9\text{ m}^2$

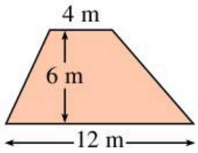
Area of rectangle $C = 5\text{ m} \times 2\text{ m} = 10\text{ m}^2$

Area of shape $= 10\text{ m}^2 + 9\text{ m}^2 + 10\text{ m}^2 = 29\text{ m}^2$

Example 3

Find the area of:

(a)

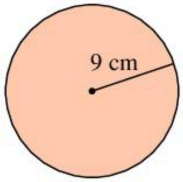


Area of trapezium $= \frac{1}{2}(a + b) \times h$

$= \frac{1}{2}(4\text{ m} + 12\text{ m}) \times 6\text{ m}$

$= 48\text{ m}^2$

(b)



Area of circle $= \pi r^2$

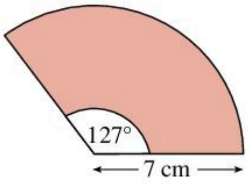
$= 3.14 \times 9^2\text{ cm}^2$

$= 3.14 \times 81\text{ cm}^2$

$= 254.34\text{ cm}^2$

Example 4

Find the area of the sector and the length of the arc of this shape. Take $\pi = 3.14$.



Area of sector $= \frac{x^\circ}{360^\circ} \times \pi r^2$

$= \frac{127^\circ}{360^\circ} \times 3.14 \times 7\text{ cm} \times 7\text{ cm}$

$= \frac{127^\circ}{360^\circ} \times 153.86\text{ cm}^2$

$= 54.28\text{ cm}^2$

Length of arc $= \frac{x^\circ}{360^\circ} \times 2\pi r$

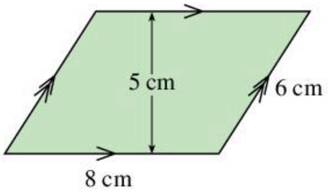
$= \frac{127^\circ}{360^\circ} \times 2 \times 3.14 \times 7\text{ cm}$

$= \frac{127^\circ}{360^\circ} \times 43.96\text{ cm} = 15.5\text{ cm}$

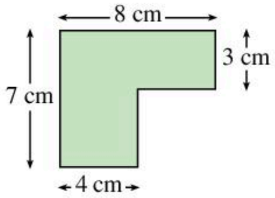
Exercise 3

1 Find the area of these shapes.

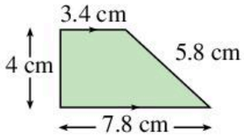
(a)



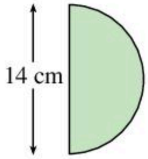
(b)



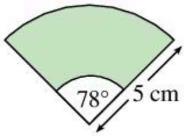
(c)



(d)



(e)



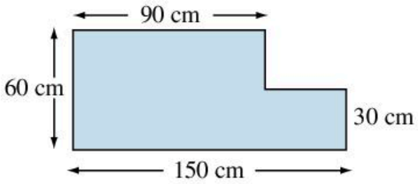
2 Find the perimeter of the shapes in Question 1.

Application

3 A store room has a rectangular base 3.2 m by 4.1 m with walls 2.4 m tall. On one wall there is a door 2.1 m high and 0.9 m wide.

- **(a)** What is the area of the door?
- **(b)** What is the total surface area of the four walls?
- **(c)** If paint costs $26.25 per litre
  - **(i)** how many litres of paint are required to paint the walls if 1 litre can cover 24 m^2?
  - **(ii)** What will be the cost of painting two coats on the walls?

4 Ashton wishes to tile the area above his kitchen sink.
The area is shown in the diagram.



- **(a)** What is the area that he has to tile?
- **(b)** Kitchen tiles are square with side 15 cm.
  - **(i)** What is the area of a kitchen tile?
  - **(ii)** How many tiles are needed to tile Ashton's kitchen?

5 Part of an athletics field is set apart for the throwing of a discus. The throwing area is in the shape of a sector of a circle of angle 40°.
What area of the field should be set aside for discus throwing if the maximum throw is not likely to exceed 75 m?

Support Website

Additional material to support this topic can be found at
www.oxfordsecondary.com/9780198425793

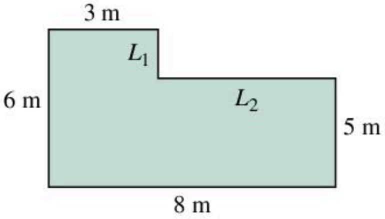
Summary

You should know …

1 How to find the perimeter of a shape.

For example:

The perimeter of an L-shaped room.



Length $L_1 = 6\text{ m} - 5\text{ m} = 1\text{ m}$

Length $L_2 = 8\text{ m} - 3\text{ m} = 5\text{ m}$

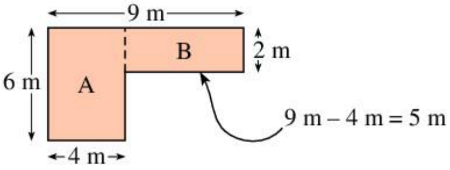
Perimeter $= 6\text{ m} + 8\text{ m} + 5\text{ m} + 5\text{ m} + 1\text{ m} + 3\text{ m}$

$= 28\text{ m}$

2 To find the area of a composite shape, you need to divide it into simple shapes.

For example:

The area of the L-shaped room.



Area rectangle A $= 6\text{ cm} \times 4\text{ cm} = 24\text{ m}^2$

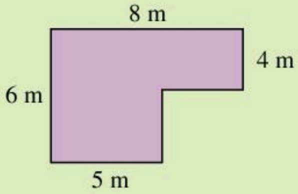
Area rectangle B $= 5\text{ cm} \times 2\text{ cm} = 10\text{ m}^2$

Area of room $= 24\text{ m}^2 + 10\text{ m}^2 = 34\text{ m}^2$

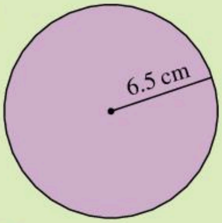
Check out

1 What is the perimeter of these shapes?

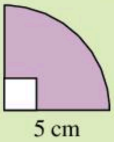
(a)



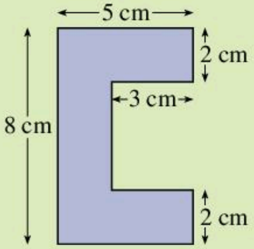
(b)



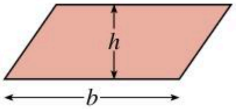
(c)



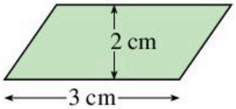
2 Find the area of this shape.



3 The area of a parallelogram, A, is given by the formula
$A = b \times h$

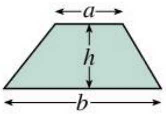


For example:

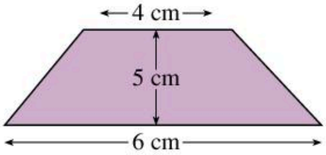


$A = 3\text{ cm} \times 2\text{ cm} = 6\text{ cm}^2$

4 The area of a trapezium, A is given by the formula
$A = \frac{1}{2}(a + b) \times h$

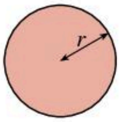


For example:



$A = \frac{1}{2}(4\text{ cm} + 6\text{ cm}) \times 5\text{ cm}$
$= 25\text{ cm}^2$

5

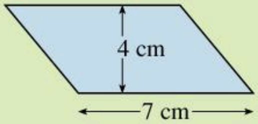


The area of a circle, radius r, is πr^2.

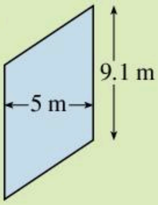
For example:
The area of a circle, radius 5 cm
$= \pi \times r^2$
$= 3.14 \times 5^2$
$= 3.14 \times 25$
$= 78.5\text{ cm}^2$

3 Find the area of these parallelograms.

(a)

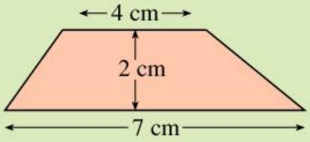


(b)

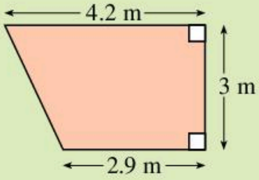


4 Find the area of these trapeziums.

(a)



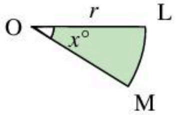
(b)



5 Find the area of a circle with radius:
(a) 3 cm
(b) 6 cm
(c) 8.2 m

6 **(a)** Part of the circumference of a circle is called an arc.
(b) The area bounded by two radii and an arc is called a sector.

For example:



LM is an arc.
LOM is a sector.

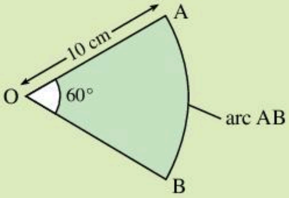
Length of arc LM $= \frac{x}{360} \times 2\pi r$

Area of sector LOM $= \frac{x}{360} \times \pi r^2$

6 Find

(a) the length of arc AB

(b) the area of sector AOB.



Computation 3

4

Objectives

- ✔ solve problems involving ratio and proportion
- ✔ use scales to find distances, areas and volumes
- ✔ add and subtract number in different number bases

What's the point?

Earthquakes and hurricanes are common in the Caribbean. To withstand such forces of nature, buildings have to be constructed with solid foundations. Builders need to use suitable ratios of sand to cement to ensure structures are solid.

Before you start

You should know ...	Check in
1 Equivalent fractions are different ways of showing the same fraction. *For example*: $\frac{2}{3} = \frac{4}{6} = \frac{6}{9} = \frac{8}{12}$	**1** Write down two fractions equivalent to **(a)** $\frac{3}{4}$ **(b)** $\frac{4}{5}$
2 Fractions can be simplified by dividing the numerator and the denominator by the same number. *For example*: $\frac{18}{24} \underset{\div 3}{\overset{\div 3}{=}} \frac{6}{8} \underset{\div 2}{\overset{\div 2}{=}} \frac{3}{4}$	**2** Simplify **(a)** $\frac{12}{36}$ **(b)** $\frac{20}{25}$ **(c)** $\frac{85}{100}$ **(d)** $\frac{16}{48}$
3 A ratio compares the size of two or more quantities. The ratios 5 : 8 and 10 : 16 are equivalent since $\frac{5}{8} = \frac{10}{16}$	**3** Copy and complete these ratios **(a)** $3:4 = 6:?$ **(b)** $4:5 = 12:?$ **(c)** $?:7 = 15:21$ **(d)** $2:9 = ?:63$

4.1 Ratio and proportion

Ratios are commonly used to compare one item with another.

Look at the signs

Buy 3 Ties
Get One FREE!

Buy 5 Ties
Get Two FREE!

In the first sign you must buy 3 ties to get 1 free tie, that is, a ratio of 3 : 1

In the second sign you must buy 5 ties to get 2 free ties, that is, a ratio of 5 : 2

Which is the better bargain?

You can simplify the ratio

$$5:2=2.5:1$$

so the second deal may be better.

In this case you were able to simplify a ratio by writing it in the form $1:n$. The rules for this are similar to those for equivalent fractions.

Example 1

Write the ratios

(a) 3 : 12 **(b)** 5 : 8

in the ratio $1:n$

(a) $3:12=\frac{3}{12}=\frac{1}{4}=1:4$

(b) $5:8=\frac{5}{8}=\frac{5\div 5}{8\div 5}=\frac{1}{1.6}=1:1.6$

Exercise 4A

1 Write the ratios in the form $1:n$

(a) 3 : 6 **(b)** 4 : 12 **(c)** 5 : 10

(d) 2 : 3 **(e)** 5 : 12 **(f)** 7 : 11

2 In a cricket match, Garvin scored 50 runs from 100 balls, while Hilroy scored 40 runs from 90 balls.

- **(a)** Write in the simplest form
  - **(i)** Garvin's strike ratio
  - **(ii)** Hilroy's strike ratio
- **(b)** Writing the ratios in the form $1:n$ find who had the better strike ratio.

3 The ratio of rayon to cotton in two shirts is 4 : 5 and 7 : 9. Which shirt has the greater amount of cotton?

4 In cricket the strike rate of a batsman is the ratio of the number of runs scored to the number of balls faced by the batsman × 100.

(a) Copy and complete the table to find the strike rates of these well-known cricketers in one-day internationals.

Years	Cricketer	Runs Scored	Balls faced	Strike rate
2004–2015	Darren Sammy	1871	1870	
2011–2015	Andre Russell	985	753	
2007–2016	Kieron Pollard	2289	2466	
1975–1991	Viv Richards	6721	7451	

(b) Who had the best strike rate?

(c) Who was the best batsman?

5

In basketball the ratio of 'assists' to 'turnovers' is one method coaches use to find how efficient players are.

(a) Copy and complete the table from the 2016–2017 NBA season.

Player	Number of assists	Number of turnovers	Assist/ turnover ratio
James Hardin	318	154	
Stephen Curry	637	297	
LeBron James	787	376	
Jeremy Lin	184	86	

(b) Who had the best assist: turnover ratio?

(c) Who was the best basketballer?

Activity

- Find out who uses ratios.
- Give examples of ratios used in
  - **(a)** construction trades
  - **(b)** the kitchen
  - **(c)** sports
  - **(d)** health.
- Make a presentation of your findings.

Technology

Use a sports calculator at

www.captaincalculator.com/sports/cricket/

to find batting strike rates. Research data from any cricket statistics site and pick your ODI cricket team.

Working with ratios

Ratios can be used to divide quantities into two or more parts.

Example 2

\$200 is shared between Amy, Brenda and Celia in the ratio $7:2:1$
How much does each get?

$$\begin{aligned} \text{Number of equal parts} &= 7+2+1 = 10 \\ \text{Each part} &= \$200 \div 10 \\ &= \$20 \\ \text{Amy gets } 7 \times \$20 &= \$140 \\ \text{Brenda gets } 2 \times \$20 &= \$40 \\ \text{Celia gets } 1 \times \$20 &= \$20 \end{aligned}$$

Notice the three shares, \$140, \$40 and \$20 sum to \$200.

You can also calculate quantities from given ratios when one of the shares is known.

Example 3

A sum of money is shared between Andy and Brian in the ratio $3:4$. Brian receives \$20. How much does Andy get?

Andy gets 3 equal parts
Brian gets 4 equal parts
4 equal parts $= \$20$
so 1 equal part $= \$20 \div 4 = \$5$
Hence Andy gets $3 \times \$5 = \$15$

Exercise 4B

1 Share \$10 between Bernard and Beverly in the ratio

(a) $3:2$ **(b)** $3:7$ **(c)** $1:4$
(d) $13:7$ **(e)** $17:3$ **(f)** $21:4$

2 Share these quantities in the ratios given

(a) 60 oranges in the ratio $12:3$
(b) 40 mangos in the ratio $3:17$
(c) 12 books in the ratio $1:5$
(d) 28 fish in the ratio $2:5$

3 Share \$900 among Roselyn, Sandra and Tessa in the ratio

(a) $1:2:6$ **(b)** $2:3:4$
(c) $2:2:5$ **(d)** $4:5:9$

4 

The ratio of sand to cement to make mortar is 3 : 1. How many buckets of sand should be mixed with

(a) 4 buckets of cement
(b) 7 buckets of cement
(c) 12 buckets of cement

5 In Question 4, how many buckets of cement are needed to make

(a) 12 buckets of mortar
(b) 20 buckets of mortar
(c) 14 buckets of mortar

6 The ratio of boys to girls at Baytown Secondary School is 5 : 4. How many boys are at the school if there are

(a) 200 girls
(b) 300 girls
(c) 480 girls
(d) 420 girls

7 The ratio of students to teacher at The Grange School is 15 : 1. What is the total population of the school, staff and students if there are

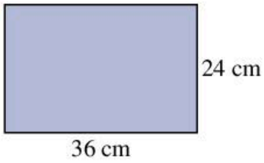
(a) 23 teachers
(b) 40 teachers
(c) 300 students
(d) 1350 students

8 Concrete for the foundation of a house is made from cement, sand and gravel mixed in the ratio 1 : 2 : 3. Find the number of wheelbarrows of

(a) sand to mix with 60 barrows of cement
(b) gravel to make 87 barrows of concrete
(c) gravel to mix with 12 barrows of cement
(d) cement to mix with 21 barrows of gravel.

9 A fruit juice is made from oranges and pineapple juice mixed in the ratio 7 : 2.

(a) How many litres of pineapple juice are needed to make 63 litres of fruit juice?
(b) How many litres of orange juice are needed to make 45 litres of juice?
(c) How much fruit juice can be made from 28 litres of pineapple juice?

10 

The diagram shows the plan of a rectangular house, drawn using a scale of 1 : 50. Find

(a) the length of the house in metres
(b) the width of the house in metres
(c) the length of a 4m long wall on the plan.

More complex ratio problems can be solved using the same ideas.

Example 4

A sum of money was shared among Jim, Joseph and Jon in the ratio 2 : 3 : 5. If Jon received $60 more than Joseph, find

(a) how much Jim received
(b) the total sum of money.

(a) Joseph got 3 equal parts
Jon got 5 equal parts
Jon got $5 - 3 = 2$ more parts than Joseph
Hence 2 parts = \$60
and 1 part = \$60 ÷ 2
= \$30
Jim got 2 parts = 2 × \$30
= \$60

(b) There are $2 + 3 + 5 = 10$ equal parts
Total sum = 10 × \$30
= \$300

Exercise 4C

1 A sum of money is divided among Lester, Lily and Mara in the ratio 2 : 3 : 7. Find Mara's amount if

(a) Lester receives \$20
(b) Lily receives \$21
(c) Lily gets \$30 more than Lester
(d) Mara gets \$80 more than Lily
(e) Mara gets \$325 more than Lester.

2 A length of ribbon is divided in the ratio 3 : 5 : 8. Find the length of the ribbon if

(a) the shortest length is 18 cm
(b) the longest length is 72 cm

(c) the difference between the longest and shortest lengths is 35 cm.

3 A bag of sweets is shared in the ratio 4 : 5 : 7 among Clara, Claribel and Cletus.
- **(a)** If the bag had 48 sweets, how many sweets did Cletus get?
- **(b)** If Cletus got 15 more sweets than Clara, how many sweets
  - **(i)** did Claribel get
  - **(ii)** were there altogether?

4 The sum of $120 was shared among Marelle, Nadia and Oscar. Marelle received $20, while Nadia got one third of the money.
- **(a)** How much did Oscar receive?
- **(b)** Calculate the ratio in which the $120 was divided among the three.
- **(c)** What fraction of the sum did Nadia receive?

5 A sum of money is shared among three brothers Finlay, Gary and Hodge so that Gary gets $\frac{2}{7}$ of the money and Hodge gets $\frac{1}{3}$ of the money.
- **(a)** Calculate the ratio in which the money was divided among the brothers
- **(b)** If Hodge got $70, how much money
  - **(i)** did Finlay receive
  - **(ii)** was there altogether?

The golden ratio

A

B

C

D

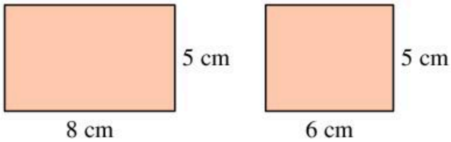
Look at the pictures above.

Which one do you prefer? Why?

Many people will say C because they like its proportions. Its not too long and not too short.

One way to find the 'best looking' rectangle (think of it as a picture frame) is to calculate the ratio of the rectangles length to its width.

For example
Look at the two rectangles



In rectangle A

$$\begin{aligned} \text{length} : \text{width} &= 8 \text{ cm} : 5 \text{ cm} \\ &= 1.6 : 1 \end{aligned}$$

In rectangle B

$$\begin{aligned} \text{length} : \text{width} &= 6 \text{ cm} : 5 \text{ cm} \\ &= 1.2 : 1 \end{aligned}$$

Some 500 years ago, artists felt that the 'best looking' rectangle or the rectangle with the best proportions was one with a length to width ratio of 1.618 : 1. They called this ratio the golden ratio.

In fact the golden ratio appears to occur a great deal in nature. Go through Exercise 4D to discover more!

Exercise 4D

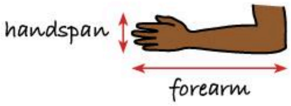
1 **(a)** Measure length and width of your maths book in centimetres.
- **(b)** What is the ratio length to width?
- **(c)** Write this ratio in the form $n : 1$

2 Measure a number of rectangles in your classroom.
- **(a)** Copy and complete the table to learn more about the ratios length to width.

Rectangle	Length	Width	Length/Width
Exercise book			
Door			
Window frame			

- **(b)** What do you notice?

3



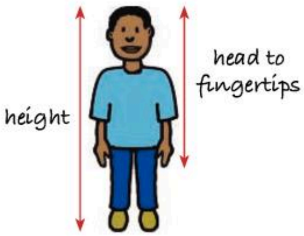
Work with five friends.

(a) Measure the lengths of your forearm (elbow to fingertips) and your hand span (distance from thumb to little finger when your hand is spread out as far as possible). Copy and complete the table

Student	Forearm length (cm)	Hand span (cm)	Forearm/ hand span
1			
2			
3			
4			
5			
6			

(b) What do you notice?

4

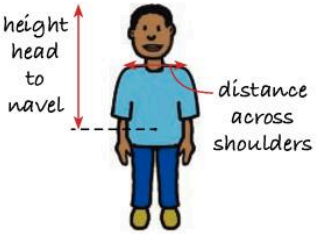


(a) Work in a small group. Measure your height and the height from your head to your fingertips. Repeat for each group member. Copy and complete the table.

Student	Height (cm)	Head to fingertips (cm)	Height/Head to fingertips
1			
2			

(b) What do you notice?

5



(a) Work in a small group.

Measure the distance from your head to your navel and the distance across your shoulders. Repeat for each group member. Copy and complete the table.

Student	Height head to navel (cm)	Distance across shoulders (cm)	Height/ distance
1			
2			

(b) What do you notice?

6 **(a)** Look at other body measurement ratios. For example
- **(i)** length of leg: distance to knee.
- **(ii)** length of head: distance from ear to ear

(b) Are there any patterns?

7 The sequence 1, 1, 2, 3, 5, 8, 13, - - -. is known as the Fibonacci Sequence.

(a) Can you see how it is formed?

(b) What do you think are the next two terms?

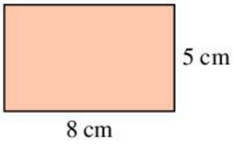
(c) Find the ratio of successive terms of the sequence by completing the table.

N	Term $(n + 1)$	Term (n)	Ratio term $n + 1$ to term n
1	1	1	1 : 1
2	2	1	2 : 1
3	3	2	3 : 2 = 1.5 : 1
4	5	3	
5	8	5	
5			
6			
10			

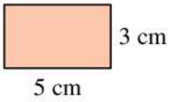
(d) What do you notice about the ratio?

8 **(a)** Draw some rectangles with sides having the width and length of successive numbers in the Fibonnaci sequence. For example,

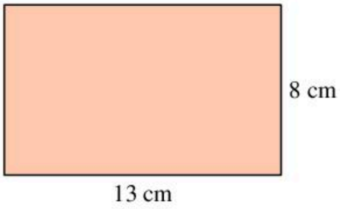
(i)



(ii)



(iii)



(b) Write down the ratio length to width for each 'golden' rectangle drawn.

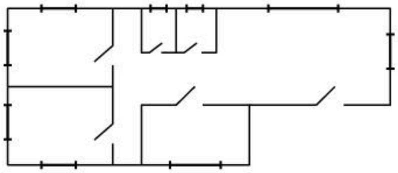
(c) Do the rectangles become successively more pleasing?

Activity

Learn more about the golden ratio and geometry by visiting *www.mathsisfun/numbers/nature-golden-ratio-fibonacci.html* and Wikipedia.

Use these sites to help you write a report on the Fibonacci sequence and the golden ratio.

4.2 Scale drawings



Scale: 1 cm represents 2 m

Ratios are also used in maps and building plans as scales.

Scales can be written using

- units – such as 1 cm to 2 m
- no units – such as 1 : 200

Both forms are the same.

Example 5

Write the scale 1 cm to 2 m in ratio form

$$\begin{aligned} 1 \text{ cm to } 2 \text{ m} &= 1 \text{ cm} : 2 \text{ m} \\ &= 1 \text{ cm} : 200 \text{ cm} \\ &= 1 : 200 \end{aligned}$$

Exercise 4E

1 Write these scales in ratio form
  (a) 1 cm to 1 m
  (b) 1 cm to 5 m
  (c) 1 cm to 20 m
  (d) 1 cm to 500 m
  (e) 1 cm to 1 km
  (f) 1 cm to 5 km

2 What does a distance of 1 cm represent on a map with scale
  (a) 1 : 20 000 **(b)** 1 : 50 000
  (c) 1 : 10 000 **(d)** 1 : 25 000
  (e) 1 : 500 000 **(f)** 1 : 100 000?

3 **(a)** Measure the length and width of your classroom.
  (b) Draw a plan of your classroom using a scale of 1 cm to represent 1 m.
  (c) Draw a plan of your classroom using a scale of 1 cm to represent 2 m.
  (d) Write the scales you used in **(c)** and **(d)** in ratio form.

On a scale drawing you can easily calculate actual distances.

Example 6

A map has a scale of 1 : 50 000.
What is the actual distance between two towns that are 2 cm apart on the map?

The map's scale is 1 : 50 000.
The actual distance is 50 000 times greater than that on the map.

$$\begin{aligned} \text{Actual distance} &= 2 \text{ cm} \times 50\,000 \\ &= 100\,000 \text{ cm} \\ &= 1000 \text{ m} \\ &= 1 \text{ km} \end{aligned}$$

You can also find the plan or map distance if you know the actual distance.

Example 7

A plan has a scale of 1 : 800. What distance would a wall of length 4 m be represented on a plan?

Scale is 1 : 800
The plan distance is 800 times smaller than the actual distance.

Plan distance = 4 m ÷ 800
= 400 cm ÷ 800
= 0.5 cm

Exercise 4F

1 A map has scale 1 : 50 000. What is the actual distance when the distance on the map is

(a) 1 cm **(b)** 5 cm
(c) 0.5 cm **(d)** 3.2 cm ?

2 Two villages are 4 km apart. What would be the distance represented on a map if the map has a scale

(a) 1 : 10 000 **(b)** 1 : 25 000
(c) 1 : 50 000 **(d)** 1 : 100 000?

3 A plan uses a scale of 2 cm to represent 7 m.

(a) Write this scale as a ratio in the form 1 : n

(b) What actual distance would 3 cm on the plan be?

(c) What would a distance of 4 m on the ground be represented on the plan?

4 Find the scale in the form of a ratio (1 : n) of a map in which

(a) 3 cm represents 60 m
(b) 5 cm represents 20 km
(c) 4 cm represent 200 km
(d) 3.2 cm represent 80 m

5 **(a)** Measure the width and length of your desk in centimetres.

(b) Make a plan of the top of your desk using a scale of

(i) 1 : 100 **(ii)** 1 : 50
(iii) 1 : 25 **(iv)** 1 : 20

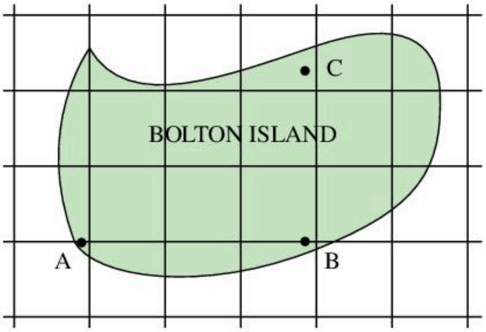
6

(a) Using the map of Dominica with a scale of 1 : 500 000, measure with your ruler the map distances

(i) Roscau to Portsmouth
(ii) Grand Bay to Marigot
(iii) Castle Bruce to St. Joseph
(iv) La Plaine to Roseau

(b) Find the actual distances between the towns in **(a)**.

7 The diagram shows the map of Bolton Island

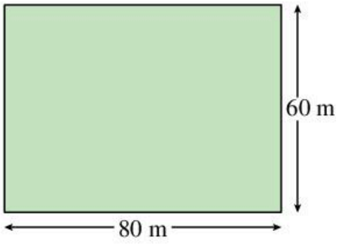


Scale 1:50000

What is

(a) the map distance AB
(b) the actual distance AB
(c) the map distance BC
(d) the actual distance BC

8



The diagram shows a rectangular field 80 m long and 60 m wide.

(a) Draw a scale drawing of the field using a scale of 1 : 2000

(b) What is the area of your scale drawing in cm^2?

(c) What is the area of the field in

(i) m^2 **(ii)** cm^2

(d) What is the ratio

area of field: area of field on scale drawing?

9 Jemason used a scale of 1 : 100 to make a plan of his rectangular bedroom floor with length 4 m and width 3 m. Find the

(a) dimensions of the floor on the plan

(b) area of the floor as drawn on the plan

(c) area of the actual floor in

(i) m^2 **(ii)** cm^2

(d) ratio area floor : area floor plan.

10 A water tank is in the form of a cube with side 2 m. Alan makes a scale model of the tank using a scale of 5 cm to represent 1 m.

(a) Write the scale as a ratio.

(b) What is the volume of the model tank in cm^3?

(c) What is the volume of the actual tank in

(i) m^3 **(ii)** cm^3

(d) Find the ratio

volume of tank : volume of model

4.3 Number bases

Look at the number 736.
It is really made up of

7 hundreds + 3 tens + 6 ones

or $7 \times 100 + 3 \times 10 + 6 \times 1$ when written in expanded form. Notice that the column headings of the number are powers of ten.

H	T	O		10^2	10	1
7	3	6	or	7	3	6

Our system of numbers is known as a denary or base ten system.

You can, in fact, make number systems using different bases.

Computers operate on the presence or absence of electrical current to show, for example, a light on or a light off. Computers use a base 2 or binary system to operate.

Binary system

In the binary or base 2 system there are only two digits, 0 and 1. The column headings are powers of 2 instead of powers of 10 in the base 10 system.

In the base 2 system, the number 1101 has column headings

2^3	2^2	2^1	2^0
1	1	0	1

It is really

$$\begin{aligned} & 1 \times 2^3 + 1 \times 2^2 + 0 \times 2^1 + 1 \times 2^0 \\ = & 1 \times 8 + 1 \times 4 + 1 \times 1 \\ = & 13 \end{aligned}$$

That is,

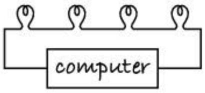
1101 (base 2) = 13 (base 10).

you can shorten this to

$$1101_2 = 13_{10}$$

Where the subscripts indicate which base is being used.

Notice that a very simple computer would show 13_{10} as



or 1101_2 with 1 being indicated by a glowing electric bulbs and 0 by a bulb with no current passing through it.

To change binary numbers to denary, all you need to do is to write down the column headings as powers of 2.

Example 8

Write **(a)** 101 **(b)** 101011 as denary numbers

(a) The column headings are

2^2	2^1	2^0
1	0	1

$$\begin{aligned} \text{so } 101_2 &= 1 \times 2^2 + 0 \times 2^1 + 1 \times 2^0 \\ &= 1 \times 4 + 0 + 1 \times 1 \\ &= 5 \end{aligned}$$

(b) with column headings 101011 is

2^5	2^4	2^3	2^2	2^1	2^0
1	0	1	0	1	1

$$\begin{aligned} 101011 &= 1 \times 2^5 + 0 \times 2^4 + 1 \times 2^3 + 0 \times 2^2 \\ &\quad + 1 \times 2^1 + 1 \times 2^0 \\ &= 1 \times 32 + 1 \times 8 + 1 \times 2 + 1 \times 1 \\ &= 43 \end{aligned}$$

To convert a denary number to a binary, you can use a repeated division method.

Example 9

Convert **(a)** 12 **(b)** 57 to binary

(a)

$$\begin{array}{r|ll} 2 & 12 & \\ \hline 2 & 6 & \text{r } 0 \\ \hline 2 & 3 & \text{r } 0 \\ \hline 2 & 1 & \text{r } 1 \\ \hline & 0 & \text{r } 1 \end{array} \quad \Big\uparrow \text{ read up}$$

$12 = 1100_2$

(b)

$$\begin{array}{r|ll} 2 & 57 & \\ \hline 2 & 29 & \text{r } 1 \\ \hline 2 & 14 & \text{r } 1 \\ \hline 2 & 7 & \text{r } 0 \\ \hline 2 & 3 & \text{r } 1 \\ \hline 2 & 1 & \text{r } 1 \\ \hline & 0 & \text{r } 1 \end{array} \quad \Big\uparrow \text{ read up}$$

$57 = 111011_2$

Exercise 4G

1 Write these binary numbers as decimals

(a) 11 **(b)** 110 **(c)** 111
(d) 1001 **(e)** 1101

2 Write the numbers 1 to 8 as binary numbers.

3 Convert the binary number to base 10

(a) 1000 **(b)** 10000
(c) 10001 **(d)** 100001
(e) 1111 **(f)** 11111
(g) 11101 **(h)** 1101101

4 Convert the base 10 numbers to binary.

(a) 9 **(b)** 11
(c) 15 **(d)** 20
(e) 31 **(f)** 32
(g) 59 **(h)** 65

5 Match the pairs correctly

Base 10	**Base 2**
14	110010
63	11011
50	111111
130	1110
27	10000010

Base 3

Numbers can be written in other number bases. For example, in base 3 the column headings are power of 3, that is

$$\dots.\ 3^4, 3^3, 3^2, 3^1, 3^0$$

The number 2201 (base 3) is

3^3	3^2	3^1	3^0
2	2	0	1

or

$$\begin{aligned} & 2 \times 3^3 + 2 \times 3^2 + 0 \times 3^1 + 1 \times 3^0 \\ &= 2 \times 27 + 2 \times 9 + 0 \times 3 + 1 \times 1 \\ &= 54 + 18 + 0 + 1 \\ &= 73 \end{aligned}$$

Hence $2201_3 = 73_{10}$

Conversion of a base 10 number to a base 3 is again repeated division, but this time dividing by 3 repeatedly.

Example 10

Convert 95 to base 3

3	95	
3	31	r 2
3	10	r 1
3	3	r 1
3	1	r 0
	0	r 1

↑ read up!

Hence $95_{10} = 10112_3$

Exercise 4H

1 Write these base 3 numbers in base 10
- **(a)** 10 **(b)** 11 **(c)** 21
- **(d)** 22 **(e)** 212 **(f)** 1021

2 Write the numbers 1 to 10 in base 3.

3 Convert the base 3 numbers to base 10
- **(a)** 2000 **(b)** 1010
- **(c)** 2210 **(d)** 2020
- **(e)** 12120 **(f)** 11111
- **(g)** 22222 **(h)** 21212

4 Convert the base 10 numbers to base 3.
- **(a)** 4 **(b)** 12
- **(c)** 15 **(d)** 31
- **(e)** 55 **(f)** 82
- **(g)** 242 **(h)** 244

5 Match the pairs correctly.

Base 10	**Base 3**
18	22212
45	2111
239	200
110	1200
67	11002

Adding and subtracting in different number bases

In base 10 you exchange ones into tens, and tens into hundreds when you add, for example,

$$\begin{array}{r} 2^13^14 \\ +\ 386 \\ \hline 620 \end{array}$$

4 + 6 = 10 or 1 ten 0 ones
4 tens + 8 tens = 12 tens or 1 hundred 2 tens

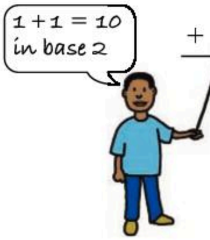
Addition in base 2 or base 3 involves, instead, the exchange of 2^1s for 2^2s or the exchange of 3^1s for 3^2s.

Example 11

Work out
- **(a)** $110_2 + 101_2$
- **(b)** $202_3 + 121_3$

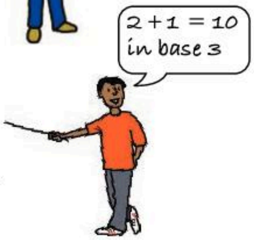
(a)

$$\begin{array}{rcccc} & 2^3 & 2^2 & 2^1 & 2^0 \\ & & 1 & 1 & 0 \\ + & & 1 & 0 & 1 \\ \hline & 1 & 0 & 0 & 1 \end{array}$$



(b)

$$\begin{array}{rcccc} & 3^3 & 3^2 & 3^1 & 3^0 \\ & & 2 & 0 & 2 \\ + & & 1^1 & 2 & 1 \\ \hline & 1 & 0 & 0 & 0 \end{array}$$



Exercise 4I

1 Add these binary numbers
- **(a)** 11 + 10 **(b)** 11 + 1
- **(c)** 101 + 11 **(d)** 111 + 11

2 Work out these binary additions

(a) $\begin{array}{r} 1101 \\ +\ 1101 \\ \hline \end{array}$ **(b)** $\begin{array}{r} 1111 \\ +\ 1111 \\ \hline \end{array}$

3 Add these base 3 numbers
- **(a)** 21 + 10 **(b)** 21 + 20
- **(c)** 20 + 12 **(d)** 112 + 21

4 Work out these base 3 additions

(a) $\begin{array}{r} 2102 \\ +\ 1212 \\ \hline \end{array}$ **(b)** $\begin{array}{r} 2211 \\ +\ 222 \\ \hline \end{array}$

5 Subtract these binary numbers
- **(a)** 11 − 1 **(b)** 10 − 1
- **(c)** 110 − 11 **(d)** 1100 − 10

6 Work out these binary subtractions

(a) $$\begin{array}{r} 1001 \\ -\ \ 111 \\ \hline \end{array}$$

(b) $$\begin{array}{r} 11001 \\ -\ \ 1101 \\ \hline \end{array}$$

7 Subtract these base 3 numbers

(a) 21 − 2 **(b)** 12 − 1

(c) 121 − 22 **(d)** 1200 − 21

8 Work out these base 3 subtractions

(a) $$\begin{array}{r} 12111 \\ -\ \ 1221 \\ \hline \end{array}$$

(b) $$\begin{array}{r} 10211 \\ -\ \ 2020 \\ \hline \end{array}$$

Exercise 4J – mixed questions

1 Share $120 in the ratio

(a) 5 : 1 **(b)** 7 : 5

(c) 19 : 1 **(d)** 13 : 7

2 The ratio of cement, sand and gravel to make concrete is 1 : 2 : 3. How many

(a) buckets of sand are needed to mix with 15 buckets of gravel

(b) buckets of gravel are needed to mix with 8 buckets of cement.

(c) buckets of concrete are made from 3 buckets of cement, 6 buckets of sand and 9 buckets of gravel?

3 A box of chocolates is shared in the ratio 3 : 5 : 7 among Daniel, Doris and Diana. If Doris got 8 more chocolates than Daniel, how many chocolates

(a) did Diana get

(b) were there altogether?

4 A ‘six-pack’ of soft drinks cost $13.50. What is the cost of

(a) 4 soft drinks **(b)** 7 soft drinks?

5 Write the denary numbers in binary

(a) 56 **(b)** 42

(c) 89 **(d)** 112

6 Work out these base 3 problems

(a) $$\begin{array}{r} 202 \\ -\ \ 12 \\ \hline \end{array}$$

(b) $$\begin{array}{r} 200 \\ 22 \\ +\ 121 \\ \hline \end{array}$$

7 Write these base 3 numbers in base 10

(a) 2221 **(b)** 1121

(c) 10001 **(d)** 21121

8 Write these scales in ratio form (1 : *n*)

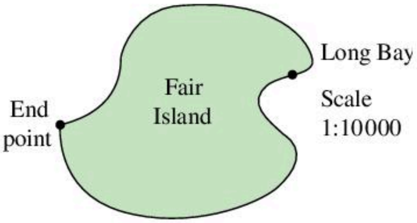
(a) 1 m to 5 km

(b) 1 mm to 10 m

(c) 1 cm to 400 m

(d) 1 cm to 250 km

9



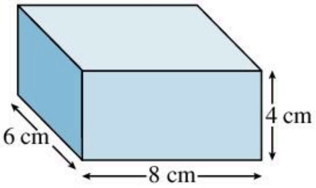
The diagram shows a map of Fair Island drawn on a 1 : 10 000 scale.

(a) Measure the map distance from End Point to Long Bay.

(b) What is the actual distance from End Point to Long Bay in km?

(c) High land is 400 m from End Point. How far is this on the map?

10



A scale model of a tank has dimensions 8 cm × 4 cm × 6 cm when made using a scale of 1 : 50.

(a) What are the dimensions of the actual tank?

(b) What is the volume of the model tank?

(c) What is the volume of the actual tank?

(d) Find the ratio model tank volume to actual tank volume.

11 A box of 13 floor tiles costs $58.50. Find the cost of

(a) 26 tiles **(b)** 20 tiles.

12 At Weston School there are 375 boys, 325 girls and 35 teachers.

(a) What is ratio of boys to girls?

(b) What is the student–teacher ratio?

(c) If three teachers left and only two were replaced, what would be the new student–teacher ratio?

13 The scale on a plan is 1 : 2500.

(a) How long is a wall which is 3 cm in length on the plan?

(b) What length would be represented on the plan for a window whose actual length is 2 m?

14 The recipe for pumpkin soup is

For two people use:	
500 ml	water
400 g	pumpkin
30 g	butter
10 g	seasoning

(a) How much butter is needed if five people are to be served?

(b) A cook uses 35 g of seasoning to make the soup. How much pumpkin will he need?

4 Consolidation

Example 1

Share \$20 in the ratio:

(a) 4 : 1

There are $4 + 1 = 5$ parts.
Each part $= \$20 \div 5 = \$4$
So 4 parts $= 4 \times \$4 = \$16$
That is, $4 : 1 = \$16 : \$4$

(b) 7 : 3

There are $7 + 3 = 10$ parts.
Each part $= \$20 \div 10 = \$2$
So 7 parts $= 7 \times \$2 = \$14$
3 parts $= 3 \times \$2 = \$6$
That is, $7 : 3 = \$14 : \$6$

Example 2

Share 80 marbles among Alan, Brian and Charles in the ratio 2 : 3 : 5.

There are $2 + 3 + 5 = 10$ parts.
Each part = 80 marbles ÷ 10 = 8 marbles
Alan gets $2 \times 8 = 16$ marbles
Brian gets $3 \times 8 = 24$ marbles
Charles gets $5 \times 8 = 40$ marbles

Example 3

A map has a scale of 1 : 5000. Find the

(a) actual distance if the map distance is 6 cm
(b) map distance if the actual distance is 1 km

(a) Actual distance $= 6\text{ cm} \times 5000$
$= 30\,000\text{ cm}$
$= 300\text{ m}$

(b) Map distance $= 1\text{ km} \div 5000$
$= 1000\text{ m} \div 5000$
$= 0.2\text{ m}$
$= 20\text{ cm}$

Example 4

Write 45_{10} in base 3

$$\begin{array}{r|ll} 3 & 45 & \\ \hline 3 & 15 & r\ 0 \\ \hline 3 & 5 & r\ 0 \\ \hline 3 & 1 & r\ 2 \\ \hline 3 & 0 & r\ 1 \\ \hline \end{array} \quad \Big\uparrow \quad 45_{10} = 1200_3$$

Exercise 4

1 Share the following between Anton and Dannisha:

(a) 20 marbles in the ratio 3 : 2
(b) 36 biscuits in the ratio 4 : 5
(c) 9 pencils in the ratio 1 : 2
(d) \$72 in the ratio 7 : 5
(e) 400 oranges in the ratio 3 : 17

2 The ratio of boys to girls at a football match is 4 : 1.

(a) How many boys are at the match if the number of girls is:
(i) 15 **(ii)** 25 **(iii)** 62
(iv) 100 **(v)** 214?

(b) How many girls are at the match if the total number of children there is:
(i) 200 **(ii)** 125 **(iii)** 460
(iv) 1025 **(v)** 985?

3 Share the following among Linda, Maryann and Neisha.

(a) \$300 in the ratio 1 : 2 : 2
(b) \$825 in the ratio 8 : 8 : 9
(c) 25 oranges in the ratio 5 : 2 : 13
(d) 84 cakes in the ratio 2 : 3 : 2

4 Write these numbers in base 2

(a) 54 **(b)** 68
(c) 102 **(d)** 248

5 Ellen, Fanny and Gwen shared a sum of money in the ratio 2 : 7 : 9 among them. Fanny received \$60 more than Ellen.

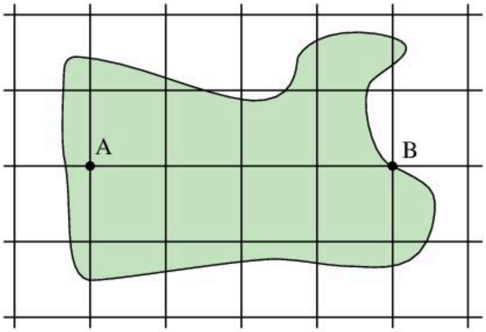
(a) How much did Ellen get?
(b) How much money was there altogether?

Application

6 The student–teacher ratio at the Mixed School is 15 : 1.

(a) How many students are there if there is a total of 320 students and staff?
(b) What would be the new student–teacher ratio if two more teachers joined the school?

7



The diagram shows the map of a region with scale 1 : 20 000.

Find the

(a) map distance AB

(b) actual distance AB in km

(c) approximate area of the map in cm^2

(d) actual area of the region in km^2.

8 A fruit juice is made of 2 parts pineapple to 3 parts orange to 4 parts passion fruit.

(a) Diane wishes to make 18 litres of fruit juice. How much

- **(i)** pineapple
- **(ii)** orange
- **(iii)** passion fruit

juice should she make?

(b) If Diane has 5 litres of pineapple juice, how many litres of

- **(i)** orange
- **(ii)** passion fruit

will she need to make the fruit juice?

Summary

You should know …

1 How to share something in a given ratio.

For example:

Share $30 in the ratio 2 : 3

There are $2 + 3 = 5$ parts

Each part is worth $\$30 \div 5 = \$6$

so 2 parts are worth $2 \times \$6 = \$12$

and 3 parts are worth $3 \times \$6 = \$18$

2 How to solve more complex ratio problems.

For example:

A sum of money is shared in the ratio 2 : 6 : 9 among A, B and C. If C got $14 more than A how much money was there altogether?

C got $9 - 2 = 7$ parts more than A

so 1 part $= \$14 \div 7 = \$2$

Total amount $= (2 + 6 + 9) \times \$2$

$= \$34$

Check out

1 Share $50 in the ratio

(a) 4 : 1 **(b)** 7 : 3

(c) 1 : 9 **(d)** 17 : 3

(e) 6 : 19 **(f)** 21 : 4

2 A pile of oranges was shared among Amanda, Ann and Amy in the ratio 1 : 4 : 7.

How many oranges did Ann get if Amanda had

(a) 30 less than Ann

(b) 12 less than Ann?

3 How to use scale in maps.

For example:

A map has scale 1 : 25000.

What is the distance on the map between two towns 3 km apart.

Map distance = 3 km ÷ 25000
= 3000 m ÷ 25000
= 300 000 cm ÷ 25000
= 12 cm

3 The scale on a map is 1 : 40 000.

- **(a)** Find the actual distance if the distance on the map is
  - **(i)** 2 cm **(ii)** 5.3 cm
- **(b)** Find the map distance if the actual distance is
  - **(i)** 2 km **(ii)** 0.4 km

4 How to write numbers in different bases.

For example:

(a) $1101_2 = 2 \times 2^3 + 2 \times 2^2 + 0 \times 2^1 + 1 \times 2^0$
$= 2 \times 8 + 2 \times 4 + 0 + 1$
$= 25$ (base 10)

(b) change 42 to base 2

$$\begin{array}{r|ll} 2 & 42 & \\ \hline 2 & 21 & \text{r } 0 \\ \hline 2 & 10 & \text{r } 1 \\ \hline 2 & 5 & \text{r } 0 \\ \hline 2 & 2 & \text{r } 1 \\ \hline 2 & 1 & \text{r } 0 \\ \hline & 0 & \text{r } 1 \end{array} \quad \uparrow$$

42 (base 10) = 101010

4

- **(a)** Change to base 10
  - **(i)** 101_2 **(ii)** 1110_2
  - **(iii)** 210_3 **(iv)** 2121_3
- **(b)** Change to base 2
  - **(i)** 14 **(ii)** 43
- **(c)** Change to base 3
  - **(i)** 14 **(ii)** 43

Revision exercise 1

Computation 1

1 Find:

(a) 2.3×4 **(b)** 3.2×4.1
(c) 5.7×12 **(d)** 7.8×0.3
(e) 0.64×7 **(f)** 0.5×0.9
(g) 1.36×8 **(h)** 1.71×0.7
(i) 0.43×0.06 **(j)** 6.3×0.47

2 Given that $51 \times 43 = 2193$, find:

(a) 5.1×43 **(b)** 510×4.3
(c) 510×430 **(d)** 5.1×4.3
(e) 0.51×4.3 **(f)** 0.51×0.43
(g) 510×0.043 **(h)** 0.051×4300
(i) 0.0051×4.3 **(j)** 0.51×0.0043

3 Find:

(a) $4.6 \div 4$ **(b)** $7.5 \div 8$
(c) $71 \div 0.2$ **(d)** $56 \div 1.6$
(e) $51.4 \div 0.25$ **(f)** $0.84 \div 0.014$

4 Given that $1541 \div 67 = 23$, find:

(a) $15.41 \div 67$ **(b)** $15.41 \div 0.67$
(c) $1.541 \div 670$ **(d)** $1.541 \div 6.7$
(e) $15.41 \div 23$ **(f)** $1.541 \div 0.23$

5 Mrs Cloth bought 3.5 metres of dress material to make some sashes. Each sash requires 0.25 metres. How many sashes can she make?

6 Change the fraction to **(i)** a decimal and **(ii)** a per cent:

(a) $\frac{3}{4}$ **(b)** $\frac{1}{8}$ **(c)** $\frac{1}{5}$
(d) $\frac{3}{40}$ **(e)** $\frac{17}{80}$ **(f)** $\frac{123}{200}$

7 Change the per cent to **(i)** a decimal and **(ii)** a fraction in its lowest form:

(a) 25% **(b)** 65% **(c)** 16%
(d) 37.5% **(e)** 1.5% **(f)** $66\frac{2}{3}\%$

8 Write in order of size, smallest first:

(a) 30%, $\frac{3}{8}$, 0.35, $\frac{4}{13}$, 0.333
(b) 0.163, 16%, $\frac{7}{40}$, 163%, $\frac{9}{55}$

9 Copy and complete:

Number	42 819	0.038 45	1888.3	79 000 061
3 s.f.	42 800			
2 s.f.				
1 s.f.				

10 Write in scientific notation:

(a) 6834 **(b)** 794 300 **(c)** 245 000 000
(d) 943 560 **(e)** 0.0042 **(f)** 0.000 000 78
(g) 67.6 **(h)** 0.41 **(i)** 438 000 000 000

11 Work out

(a) $3^3 \times 3^2$ **(b)** $4^5 \div 4^2$
(c) $4^2 \div 4^{-2}$ **(d)** $5^3 \times 5^{-2}$
(e) $81^{\frac{1}{2}}$ **(f)** $64^{\frac{1}{3}}$
(g) $4^{-\frac{3}{2}}$ **(h)** $9^{-\frac{1}{2}}$

12 Carpet costs \$15.34 per square metre. Find the cost of carpeting a rectangular room 3.2 metres long and 2.6 metres wide.

13 A school party of 36 has to hire a coach for a day trip. If the coach costs \$63 for the day, how much will each person have to pay?

14 What fraction of a day is 1 second? Use your calculator to show this as a decimal.

15 1 kilogram is approximately 2.2 pounds. How many grams are there in 1 pound?

16 1 inch is approximately 2.53 centimetres. Convert 1 km to inches.

17 Find the cost of $\frac{3}{4}$ kg of rice at \$1.80 per kilogram and 25 g of curry powder at \$4.32 per kilogram.

18 A jet travels 34.78 m in $\frac{1}{5}$ second. Find its speed in kilometres per hour.

19 A piece of lead weighing 5.36 kg is melted down to make lead shot. How many lead shot each weighing 0.175 g can be made?

20 Find the weight in kg of 1000 lead shot in Question 18.

Computation 2

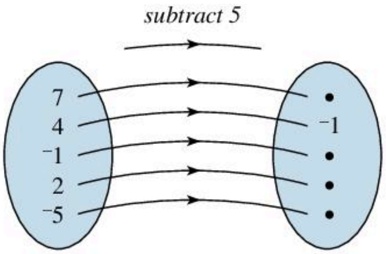
21 Simplify:

(a) $3 + {}^{-}7$ **(b)** $7 + {}^{-}3$ **(c)** ${}^{-}3 + {}^{-}7$
(d) ${}^{-}2 + 7$ **(e)** $17 + {}^{-}8$ **(f)** $7 + {}^{-}18$

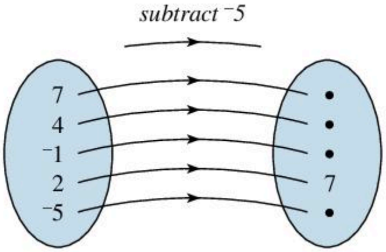
22 Simplify:

(a) $2 + {}^{-}1 + 2$ **(b)** ${}^{-}3 - 3 + 2$
(c) $4 - 5 + {}^{-}2$ **(d)** ${}^{-}1 + 5 + {}^{-}4$
(e) ${}^{-}4 + 5 - 1$ **(f)** ${}^{-}5 + 4 + {}^{-}1$

23 **(a)** Copy and complete this arrow diagram:



(b) Copy and complete this arrow diagram:



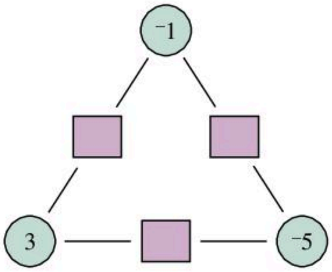
24 Copy and complete this table.

		Second number			
	Subtract	2	$^-3$	$^-1$	4
First number	3	1		4	
	$^-1$				
	4				
	$^-2$				

25 Work out

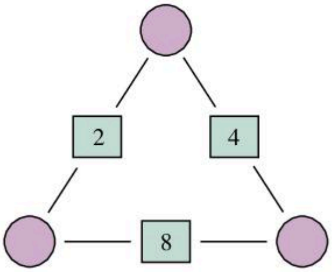
(a) $^-3 \times 5$ **(b)** $^-3 \times {}^-6$
(c) $^-3 \times {}^-4$ **(d)** $^-15 \div 5$
(e) $35 \div {}^-7$ **(f)** $^-45 \div {}^-9$

26 **(a)** In this diagram, the number in the square on each side of the triangle is the sum of the numbers in the circles.



Find the numbers in the squares.

(b) This time, find the numbers in the circles so that their sum on each side of the triangle is the number in the square:



27 Given $\mathbb{N}$ = {natural numbers}
$\mathbb{Z}$ = {integers}
$\mathbb{Q}$ = {rational numbers}

(a) Draw a Venn diagram to show $\mathbb{N}$, $\mathbb{Z}$ and $\mathbb{Q}$.
(b) What is
(i) $\mathbb{N} \cap \mathbb{Z}$ **(ii)** $\mathbb{N} \cup \mathbb{Z}$ **(iii)** $\mathbb{Z} \cap \mathbb{Q}$
(iv) $\mathbb{Z} \cup \mathbb{Q}$ **(v)** $\mathbb{N} \cap \mathbb{Q}$ **(iv)** $\mathbb{Q} \cap \mathbb{N}$?

28 Given $a = 2$, $b = {}^-1$, $c = 3$, evaluate:

(a) $a + 2b$ **(b)** $ab - c$
(c) $ac - 3b$ **(d)** $ab + bc$
(e) $\frac{a}{b} - \frac{c}{b}$ **(f)** $\frac{abc}{^-6}$

29 Find the value of $x^2 - 3x + 2$ when x equals

(a) $^-1$ **(b)** 4 **(c)** $^-5$

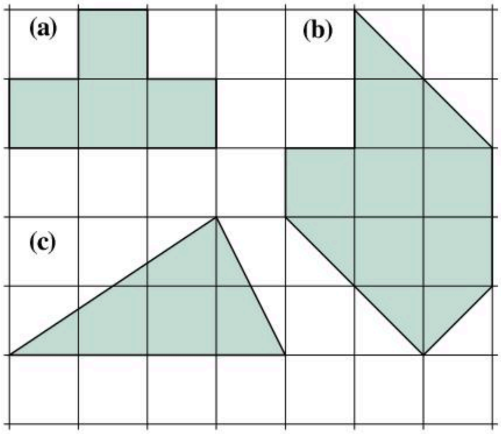
30 Given $A = \begin{pmatrix} 2 & 1 \\ 4 & ^-5 \end{pmatrix}$, $B = \begin{pmatrix} 3 & ^-1 \\ 2 & ^-4 \end{pmatrix}$

Find

(a) $A + B$ **(b)** $A - B$ **(c)** $3A - 2B$

Measurement 1

31 Find the area and perimeter of each shape drawn below on a 1 cm grid.

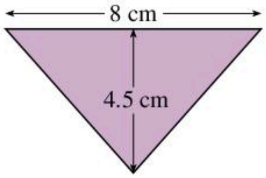


32 **(a)** Find the area of a rectangle which is 8 cm wide and 15 cm long.

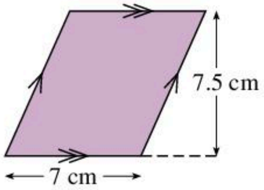
(b) A rectangle has an area of 96 cm^2 and a length of 16 cm. What is the width of the rectangle?

33 Find the area of each shape:

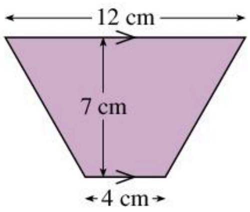
(a)



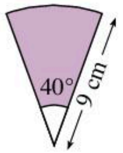
(b)



(c)



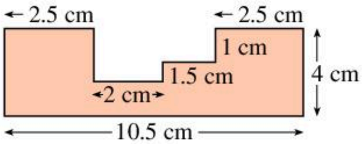
(d)



34 **(a)** Find the area of a triangle which has a height of 7.2 cm and a base of 2.5 cm.

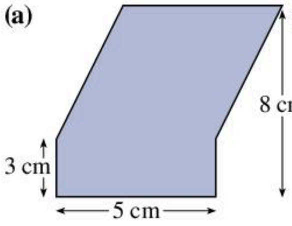
(b) The area of a triangle is 40 cm^2. If the base of the triangle is 2.5 cm find the height of the triangle.

35 The cross-section of a metal casting is shown below. Find: **(a)** its perimeter **(b)** its area.

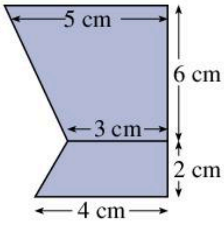


36 Find the area of each shape:

(a)



(b)



37 A circle has diameter 8.4 cm. A sector with angle 75° is cut from the circle. (Take π as 3.14) Find the sector's:

(a) perimeter **(b)** area.

38 The area of a circle is 43.2 cm^2. Find:

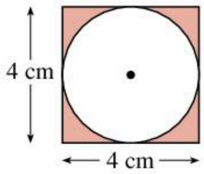
(a) the radius of the circle

(b) the circumference of the circle.

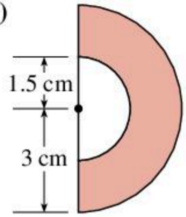
(Take π as 3.14)

39 Find the area of the shaded part of each shape: (Take π as 3.14).

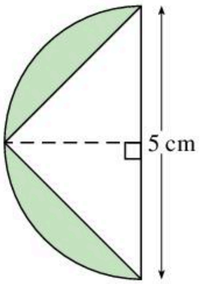
(a)



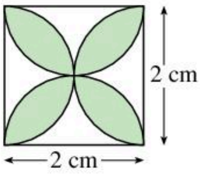
(b)



(c)



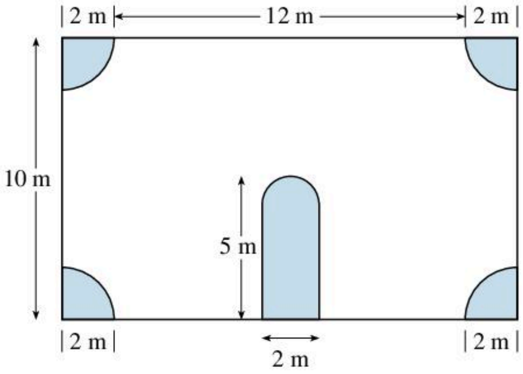
(d)



40 The diagram below is the plan of a courtyard with the shaded regions representing flower beds.

(a) Find the total area of the flower beds.

(b) The unshaded area is to be concreted. If concrete costs \$5 per m^2, find the cost of doing this.



Computation 3

41 Use a ratio to compare the quantities:

(a) 1 m; 20 cm
(b) 20 mm; 1 cm
(c) 3 minutes; 30 seconds
(d) 45 minutes; 2 hours
(e) 25 g; 0.75 kg
(f) 3.6 kg; 90 g

42 Find n:

(a) $3:4=1:n$ **(b)** $4:5=n:1$
(c) $6:3=1:n$ **(d)** $2:8=n:1$
(e) $5:9=2:n$ **(f)** $7:3=n:6$
(g) $n:5=4:3$ **(h)** $2:n=8:4$

43 The ratio of boys to girls in Class 4W is 5 : 3.

(a) Find how many girls there are, if there are 15 boys.
(b) Find how many boys there are, if there are 18 girls.
(c) Find how many boys and girls there are, if there are altogether
  (i) 32 students **(ii)** 48 students

44 360 sheep are to be shared between Jo and Jim in a given ratio. How many sheep will each get if the ratio is:

(a) 7 : 3 **(b)** 16 : 20 **(c)** 1 : 8
(d) 7 : 5 **(e)** 35 : 37 **(f)** 5 : 13?

45 Share the following between Ann and Jane in the given ratio:

(a) 12 apples, in the ratio 1 : 5
(b) 18 oranges, in the ratio 8 : 1
(c) 42 bananas, in the ratio 4 : 3
(d) 45 figs, in the ratio 4 : 5
(e) 16 records, in the ratio 5 : 3
(f) 84 tapes, in the ratio 13 : 15
(g) 16.8 metres of cloth, in the ratio 3 : 1
(h) 25.6 kg of rice, in the ratio 7 : 9
(i) 0.63 litres of juice, in the ratio 5 : 4
(j) $3328, in the ratio 6 : 7

46 A farm has cows, goats and sheep in the proportion 1 : 2 : 5.

(a) Find how many goats and sheep there are, if there are 8 cows.
(b) Find how many cows and goats there are, if there are 35 sheep.
(c) Find how many cows and sheep there are, if there are 12 goats.
(d) Find how many of each type of animal there are, if there are altogether:
  (i) 16 animals **(ii)** 48 animals
  (iii) 72 animals **(iv)** 120 animals.

47 Share the following between Adam, Barney and Carol in the given proportion.

(a) $400, in the proportion 1 : 7 : 8
(b) $250, in the proportion 3 : 1 : 1
(c) 84 cokes, in the proportion 4 : 2 : 1
(d) 96 nuts, in the proportion 3 : 4 : 5
(e) 180 records, in the proportion 7 : 6 : 5
(f) 169 tapes, in the proportion 6 : 4 : 3
(g) 128 sheep, in the proportion 5 : 5 : 6
(h) 94 camels, in the proportion 19 : 17 : 11
(i) 52.2 kg of ground nuts in the proportion 1 : 5 : 12
(j) 4.76 m of dress material in the proportion 2 : 3 : 12

48 Aldy, Anand and Amos share a sum of money in the ratio 2 : 5 : 9. Find the amount of money if

(a) Anand gets $27 more than Aldy
(b) Amos gets $60 more than Anand
(c) Aldy has $343 less than Amos

49 A map has a scale of 1 : 40 000. Find the

(a) actual distance between two towns that are 3.7 cm apart on the map
(b) map distance between two village that are 4.3 km apart.

50 A model sailing boat is built using a scale of 1 : 20.

(a) What is the length of the actual boat if the model has length 1.2 m?
(b) The area of a sail on the model is 0.5 m^2. What is the area of the sail on the actual boat?

51 Write as base 10 numbers

(a) 222 (base 3) **(b)** 1011 (base 2)
(c) 12121 (base 3) **(d)** 1101011 (base 2)

52 Work out these base 2 calculations.

(a)
$$\begin{array}{r} 11011 \\ 110 \\ +\ 10111 \\ \hline \end{array}$$

(b)
$$\begin{array}{r} 101110 \\ -\ \ 10010 \\ \hline \end{array}$$

Mixed questions 1

1 Work out $3 \div 0.02$
A 0.15 **B** 1.5 **C** 15 **D** 150

2 The fraction $\frac{3}{16}$ written as a decimal is
A 5.33 **B** 3.16 **C** 0.533 **D** 0.1875

3 The number 3.15×10^{-4} is
A 3.150 000 **B** 0.003 15 **C** 0.000 315
D 0.000 031 5

4 How many inches are there in 25 cm?
(1 inch = 2.54 cm)
A 6.35 **B** 9.84 **C** 63.5 **D** 98.4

5 What is 6% of 0.3?
A 18 **B** 1.8 **C** 0.18 **D** 0.018

6 2.3 kg of fish cost $37.40. How much would 1.6 kg of fish cost?
A $16.26 **B** $26.02
C $26.18 **D** $53.76

7 What is 791 436 to one significant figure?
A 7 **B** 8 **C** 700000 **D** 800 000

8 What is the decimal equivalent of 6.3%?
A 63 **B** 6.3 **C** 0.63 **D** 0.063

9 Sound travels at 760 miles per hour.
What is this speed in metres per second?
(1 mile = 1607 metres)
A 127 m/s **B** 339 m/s
C 7612 m/s **D** 20355 m/s

10 Which is the best buy?
A 1.6 kg of sugar for $2.16
B 0.75 kg of sugar for $1.00
C 2.3 kg of sugar for $3.75
D 3.4 kg of sugar for $5.75

11 What is $^{-}3 - {}^{-}2$?
A $^{-}5$ **B** $^{-}1$ **C** 1 **D** 5

12 $(^{-}1)^5$ is
A $^{-}5$ **B** $^{-}1$ **C** 1 **D** 5

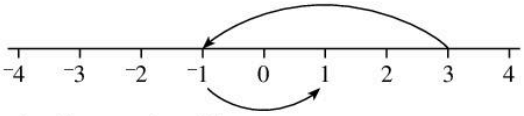
13 $^{-}3 - \square = 8$. What is $\square$?
A $^{-}11$ **B** $^{-}5$ **C** 5 **D** 11

14 What is the value of $x^2 - 3x + 1$ when $x = {}^{-}2$?
A $^{-}1$ **B** $^{-}3$ **C** 10 **D** 11

15 On Sunday night the temperature in Alaska fell by 5°C. What was the temperature earlier that day if the night temperature was $^{-}4$°C?
A $^{-}9$°C **B** $^{-}1$°C **C** 1°C **D** 9°C

16 $(^{-}2\frac{1}{2})^2$ is
A $^{-}6\frac{1}{4}$ **B** $^{-}5$ **C** 5 **D** $6\frac{1}{4}$

17



The diagram best illustrates
A $^{-}1 + 2 = 3$ **B** $3 - 4 + 2 = 1$
C $3 - 1 = 1$ **D** $3 + 4 - 2 = 1$

18 What is the value of $\frac{-3}{3} + \frac{-15}{-3}$
A $^{-}4$ **B** $^{-}2$ **C** 2 **D** 4

19 When I subtract $^{-}3$ from a certain number and then add 2 to my answer I get 4. What is this number?
A $^{-}1$ **B** 1 **C** 4 **D** 5

20 If $A = \begin{pmatrix} 1 & 0 \\ 0 & 2 \end{pmatrix}$ and $B = \begin{pmatrix} 3 & ^{-}1 \\ ^{-}5 & 3 \end{pmatrix}$

What is 3A − B?

A $\begin{pmatrix} 0 & 1 \\ 5 & 3 \end{pmatrix}$ **B** $\begin{pmatrix} 0 & ^{-}1 \\ ^{-}5 & 3 \end{pmatrix}$

C $\begin{pmatrix} ^{-}2 & 1 \\ 5 & ^{-}1 \end{pmatrix}$ **D** $\begin{pmatrix} 3 & 0 \\ 0 & 6 \end{pmatrix}$

21 Two opposite sides of a parallelogram each measure 8 cm. The distance between them if the area of the parallelogram is 72 cm^2 is
A 8 cm **B** 9 cm **C** 12 cm **D** 16 cm

22 The area of a circle is 25 cm^2. The radius in centimetres is approximately
A 1.8 **B** 2.8 **C** 3.2 **D** 5.6

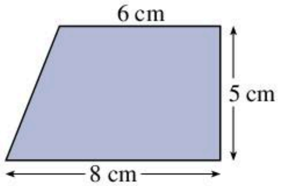
23 A rectangle 24 cm long and 16 cm wide is divided into as many rectangles as possible, each 3 cm by 2 cm. The number of small rectangles is
A 64 **B** 60 **C** 48 **D** 16

24 A triangle has base 6 cm and perpendicular height 5 cm. What is its area?
A 30 cm^2 **B** 11 cm^2 **C** 15 cm^2 **D** 60 cm^2

25 A triangle has area 30 cm^2 and base 3 cm. What is its height?
A 20 cm **B** 10 cm **C** 5 cm **D** 90 cm

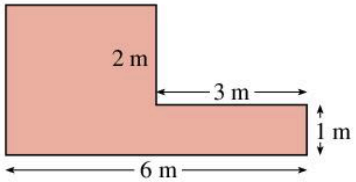
26 The area of a square is 36 cm^2. What is the length of its base?
A 6 cm **B** 18 cm **C** 12 cm **D** 9 cm

27 Find the area of this trapezium:



A 30 cm^2 **B** 40 cm^2
C 35 cm^2 **D** 70 cm^2

28 Find the area of this shape:



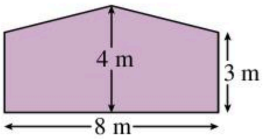
A 6 m^2 **B** 12 m^2
C 18 m^2 **D** 24 m^2

29 The minute hand of a clock is 28 cm long. How far does the tip of the hand move in 15 minutes? Take $\pi = \frac{22}{7}$.
A 11 cm **B** 22 cm **C** 44 cm **D** 88 cm

30

A pizza is cut into six equal slices. What is the area of a slice if the pizza has diameter 20 cm? Take $\pi = 3.14$
A 10.46 cm^2 **B** 20.93 cm^2
C 52.33 cm^2 **D** 209.33 cm^2

31



The diagram shows the side end of a house. What is its area?
A 14 m^2 **B** 24 m^2 **C** 28 m^2 **D** 32 m^2

32 What is the denary number 26 in base 2?
A 1101 **B** 10110 **C** 11010 **D** 11000

33 What is 108 : 16 as a ratio in its simplest form?
A 27 : 4 **B** 4 : 27 **C** 54 : 8 **D** 8 : 54

34 A mechanical digger can dig a trench in 3 hours. It takes a man two days to dig the same trench. What is the ratio of digger to the man's times?
A 2 : 3 **B** 1 : 6 **C** 1 : 8 **D** 1 : 16

35 On a map a length of 3 cm represents 6 m. What is the scale of the map?
A 1 : 2 **B** 1 : 18 **C** 1 : 50 **D** 1 : 200

36 The ratio of boys to girls in a school is 2 : 3. How many boys are there if the school has 500 students?
A 100 **B** 120 **C** 180 **D** 200

37 240 g of flour are needed to make a cake for four persons. How much flour is required for a cake suitable for five persons
A 192 **B** 200 **C** 280 **D** 300

38 Alan, Brian and Cedric share $10 in the ratio 1 : 2 : 2. How much does Alan receive?
A $1 **B** $2 **C** $10 **D** $20

39 In Priscilla's class the ratio of tall students to short students is 3 : 4. What is the size of her class if there are 12 tall students in it?
A 9 **B** 16 **C** 21 **D** 28

40 A model of a tower is built on a scale of 1 : 6000. What is the actual height of the tower if the tower in the model is 2 cm tall?
A 3 m **B** 12 m **C** 30 m **D** 120 m

41 What is □ if 9 : 12 = 15 : □?
A 18 **B** 20 **C** 21 **D** 24

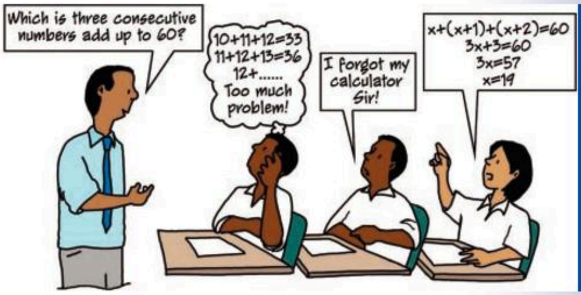
42 A dozen eggs are sold for $5.04. What is the cost of 14 eggs?
A $0.36 **B** $4.32 **C** $5.88 **D** $70.56

Algebra

5

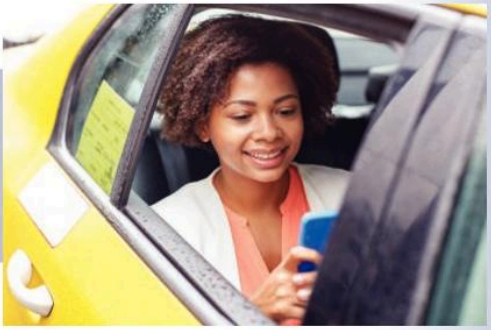
Objectives

- ✔ simplify basic algebraic expressions
- ✔ simplify algebraic fractions
- ✔ use distributive law to factorise or expand algebraic expressions
- ✔ solve linear equations
- ✔ use linear equations to solve problems
- ✔ solve pairs of simultaneous linear equations
- ✔ solve factorisable quadratic equations



What's the point?

Basic formulas are part of your life. When you want to find out how long a journey will take you will be using an algebraic formula: Journey time = distance ÷ speed

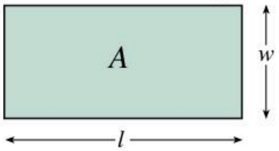


Before you start

You should know ...

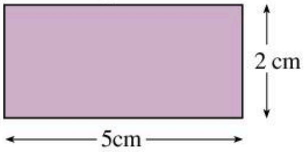
1 $>$ is the symbol for 'is greater than'
$<$ is the symbol for 'is less than'.
For example:
$9 > 7$
$^{-}2 < 2$

2 The area of a rectangle, A, is



$A = l \times w$

For example:



$A = 2\text{ cm} \times 5\text{ cm} = 10\text{ cm}^2$

Check in

1 Fill in the correct symbol $>$ or $<$ between the two numbers
(a) $3 \quad ^{-}10$
(b) $^{-}2 \quad 6$
(c) $^{-}2 \quad ^{-}5$

2 Work out the area of the rectangle with:
(a) length 2 cm, width 3 cm
(b) length 6 cm, width 6.5 cm
(c) length 18 m, width 3.25 m.

5.1 Basic algebra

Algebra is really generalised arithmetic. It follows the same rules of arithmetic but uses letters or symbols instead of numbers.

In arithmetic you have:

$$4 + 4 + 4 + 4 + 4 = 5 \times 4$$

while in algebra you have:

$$x + x + x + x + x = 5 \times x = 5x$$

In the same way

$$3 \times 3 \times 3 \times 3 = 3^4$$

while

$$x \times x \times x \times x = x^4$$

You see basic algebraic expressions in many settings. For example

Area of a rectangle = lw
where l is length and w is width of the rectangle.

Circumference of a circle = $2\pi r$
where r is the radius of the circle.

To be able to work with algebraic expressions, you need to be able to simplify them.

The basic rule is that you can only add or subtract **like terms**.

In the expression

$$7 + 6x + 4y - 2x$$

$6x$ and $-2x$ are like terms. The number 7 and the term $6x$ are **unlike terms** and cannot be combined.

To simplify expressions you have to combine like terms.

Example 1

Simplify:
(a) $7 + 4x + 6y + 2x + y$
(b) $xy - y + 2xy + 3y$

(a) $7 + 4x + 6y + 2x + y$
$= 7 + (4x + 2x) + (6y + y)$
$= 7 + 6x + 7y$

(b) $xy - y + 2xy + 3y$
$= (xy + 2xy) + (3y - y)$
$= 3xy + 2y$

The method is the same even with more complex expressions.

Example 2

Simplify:
(a) $4x^2y - 3xy^2 + xy + 2x^2y$
(b) $3abc + abc^2 - 2abc - a^2bc$

(a) $4x^2y - 3xy^2 + xy + 2x^2y$
$= (4x^2y + 2x^2y) - 3xy^2 + xy$
$= 6x^2y - 3xy^2 + xy$

(b) $3abc + abc^2 - 2abc - a^2bc$
$= (3abc - 2abc) + abc^2 - a^2bc$
$= abc + abc^2 - a^2bc$

Exercise 5A

1 Simplify:
(a) $3x + 4x$
(b) $5y - 2y$
(c) $3x + 4y$
(d) $5y - 2x + y$
(e) $4x - 3y + 2y$
(f) $ab - 2ab + 3ab$
(g) $3x - 4y + 2x$
(h) $4y - 2x - 2y + x$
(i) $3xy - 2x + 2xy$
(j) $2y - 3x + 3y - 2x$

2 Simplify:
(a) $2x^2 - x^2 + 3x^2$
(b) $4y - y + 6$
(c) $3y - 2y + 4y$
(d) $3x^2 - 2x^2 - x^2$
(e) $4 + 3y - 2 + 4y$
(f) $3 - 3x + 6 - 6x$
(g) $3a^2 - 3a - a^2$
(h) $4a^2 - 3a^2 + a^2$
(i) $3xy - y + 2xy + y$
(j) $4a^3 - 2a^2 + 3a^2$

3 Find the value of each expression when $a = 4$ and $b = 3$.
(a) $3a + b$
(b) $b - 3a$
(c) $a + b$
(d) $4b - 3a + b$
(e) $3 + 3a + 3b$
(f) $6 - 3a + 2b$
(g) $a^2 + b$
(h) $a^2 + b^2$
(i) $9 - a^2$
(j) $3ab + 2a - b$

Technology

Learn more about simplifying algebraic expressions by visiting the website

www.onlinemathlearning.com

and following the links to Algebra, Simplifying Expressions.

Make sure you watch the videos.

Working with powers

Recall that

$x \times x \times x \times x = x^4$

and

$$\begin{aligned} x^2 \times x^4 &= x \times x \times x \times x \times x \times x \\ &= x^6 \end{aligned}$$

In general

$$\boxed{x^a \times x^b = x^{a+b}}$$

The rule for division of powers is similar, for example

$$\begin{aligned} x^5 \div x^2 &= \frac{x \times x \times x \times \cancel{x} \times \cancel{x}}{\cancel{x} \times \cancel{x}} \\ &= x \times x \times x \\ &= x^3 \end{aligned}$$

Notice that

$x^5 \div x^2 = x^{5-2} = x^3$

In general

$$\boxed{x^a \div x^b = x^{a-b}}$$

What about $(x^2)^3$?

$$\begin{aligned} (x^2)^3 &= x^2 \times x^2 \times x^2 \\ &= x^{2+2+2} = x^{2\times 3} \\ &= x^6 \end{aligned}$$

In general

$$\boxed{(x^a)^b = x^{ab}}$$

Example 3

Simplify:

(a) $x^4 \times x^5$ **(b)** $a^6 \div a^4$ **(c)** $(y^3)^5$

(a) $x^4 \times x^5 = x^{4+5} = x^9$

(b) $a^6 \div a^4 = a^{6-4} = a^2$

(c) $(y^3)^5 = y^{3\times 5} = y^{15}$

Exercise 5B

1 Simplify:

(a) $x \times x \times x \times y$

(b) $x \times x \times y \times y$

(c) $x \times x \times x \times x \times y \times y$

(d) $a \times a \times b \times b \times a \times b$

2 Simplify:

(a) $x^2 \times x^3$ **(b)** $y^2 \times y^6$ **(c)** $2x^3 \times x^4$

(d) $3x^4 \times x^5$ **(e)** $x^3 \times x^4 \times x^2$ **(f)** $y^2 \times y^3 \times y^6$

3 Simplify:

(a) $x^5 \div x^4$ **(b)** $y^5 \div y^5$ **(c)** $x^7 \div x^3$

(d) $y^8 \div y^3$ **(e)** $x^2 \times x^7 \div x^4$ **(f)** $y^3 \times y^4 \div y^5$

4 Simplify:

(a) $(x^2)^4$ **(b)** $(y^3)^2$ **(c)** $(x^3)^4$

(d) $(y^2)^5$ **(e)** $(ab^2)^3$ **(f)** $(2a^3b^2)^2$

5 Simplify:

(a) $\dfrac{2x^2 \times x^3}{x}$ **(b)** $\dfrac{4y^3 \times y \times y^2}{y^3}$

(c) $\dfrac{x^2 \times y^2 \times x^2}{y^3 \times x}$ **(d)** $\dfrac{4x^4 \times x^5}{y^2 \times x^3}$

6 Simplify:

(a) $(x^2y)^2 \div xy$ **(b)** $\dfrac{x^2 \times y^4 \times x^3}{(2x^2)^2}$

(c) $(3x^2y^2)^2 \times x^2y^2$ **(d)** $(2x^3y)^3 \div x^2y$

You can use these rules to simplify more complex expressions.

Example 4

Simplify:

(a) $25x^2y^3z \div -5xy^2$

(b) $(-3x^2) \times (4y^2) \times (2x)^3$

You can do this two ways either by multiplying everything out

(a)
$$\begin{aligned} \frac{25x^2y^3z}{-5xy^2} &= \frac{\overset{5}{\cancel{25}} \times x \times \cancel{x} \times y \times \cancel{y} \times \cancel{y} \times z}{-\cancel{5} \times \cancel{x} \times \cancel{y} \times \cancel{y}} \\ &= -5xyz \end{aligned}$$

or you can work it out using the rules for powers

$$\begin{aligned} \frac{25x^2y^3z}{-5xy^2} &= -5x^{2-1}y^{3-2}z \\ &= -5xyz \end{aligned}$$

(b)
$$\begin{aligned} &(-3x^2) \times (4y^2) \times (2x)^3 \\ &= -3 \times x \times x \times 4 \times y \times y \times 2x \times 2x \times 2x \\ &= -96x^5y^2 \end{aligned}$$

or using the rules for powers

$$\begin{aligned} &(-3x^2) \times (4y^2) \times (2x)^3 \\ &= -3x^2 \times 4y^2 \times 8x^3 \\ &= -24x^{2+3} \times 4y^2 \\ &= -96x^5y^2 \end{aligned}$$

Exercise 5C

1 Simplify by multiplying everything out

- **(a)** $3x^2 \times 4x^3 \times 2y^2$
- **(b)** $3y^2 \times (-4y) \times (-2y)$
- **(c)** $4x^2y \times 3xy^2 \times 2x^3y$
- **(d)** $(-3x^2y^2) \times (-2x^3y) \times (-3y^2)$
- **(e)** $(-5xz^3) \times (2x^3yz) \times z^4$
- **(f)** $3y^2z \times (-4y^2x) \times (x^2y)^3$
- **(g)** $4x^2y^3z^2 \times x^3y^2z^3 \times (-2xz^3)$
- **(h)** $(-3yz)^2 \times (2x^2)^3 \times (x^2y^3)^2$
- **(i)** $3xyz^2 \times x^3y^2 \times 2xy^4$
- **(j)** $6x^2z^2 \times 2x^3z \times (x^2y)^3$

2 Check your answers to Question 1 by using the rules for powers.

3 Simplify:

- **(a)** $3x^2y^3 \div x^3y^4$
- **(b)** $5x^2y^3z^2 \div 2xy^3z$
- **(c)** $14x^3y^2z^5 \div 7x^2yz^4$
- **(d)** $(-6xy^3) \div 3x^2z^3$
- **(e)** $(-8x^3y^2) \div (-2x)^4$
- **(f)** $(-12x^2y^3) \div (3xyz)^3$
- **(g)** $28x^3z^2 \div (-4x^3y^2z^2)$
- **(h)** $(-3x^2y^2)^3 \div (2xy^3z)^3$
- **(i)** $(2x^4y^3z^4)^4 \div (x^2y^2z^3)^3$
- **(j)** $(-x^3y^2)^5 \div (-2x^2y^3z)^4$

Using the distributive law

You can work out the multiplication

$$6 \times 74$$

using the distributive law:

$$\begin{aligned} 6 \times 74 &= 6 \times (70 + 4) \\ &= 6 \times 70 + 6 \times 4 \\ &= 420 + 24 \\ &= 444 \end{aligned}$$

In algebraic terms the distributive law is

$$a \times (b + c) = a \times b + a \times c$$

That is, everything inside the bracket is multiplied by what is outside.

Example 5

Expand the brackets.

(a) $3(x + 2y)$ **(b)** $x(x + 1)$

(a) $$\begin{aligned} 3(x + 2y) &= 3 \times x + 3 \times 2y \\ &= 3x + 6y \end{aligned}$$

(b) $$\begin{aligned} x(x + 1) &= x \times x + x \times 1 \\ &= x^2 + x \end{aligned}$$

You can simplify more complex expressions using the same ideas.

Example 6

Simplify:

(a) $3(a - 2b) + a(4 - 2b)$

(b) $2a(3 - 2b) - a(4b - 2)$

(a) $$\begin{aligned} &3(a - 2b) + a(4 - 2b) \\ &= 3 \times a - 3 \times 2b + a \times 4 - a \times 2b \\ &= 3a - 6b + 4a - 2ab \\ &= 3a + 4a - 6b - 2ab \\ &= 7a - 6b - 2ab \end{aligned}$$

(b) $$\begin{aligned} &2a(3 - 2b) - a(4b - 2) \\ &= 2a \times 3 - 2a \times 2b - a \times 4b + a \times 2 \\ &= 6a - 4ab - 4ab + 2a \\ &= 6a + 2a - 4ab - 4ab \\ &= 8a - 8ab \end{aligned}$$

Exercise 5D

1 Expand the brackets.

- **(a)** $3(x + 2)$
- **(b)** $4(2x - 6)$
- **(c)** $5(3x - 3)$
- **(d)** $6(4 - 3x)$

2 Expand the brackets.

- **(a)** $x(x + 2)$
- **(b)** $x^2(x + x^2)$
- **(c)** $3x(x - y)$
- **(d)** $4x^2(2 - 3x)$
- **(e)** $x^3(1 + 2x - x^2)$
- **(f)** $4x(2y - x + y^2)$

3 Expand the brackets and simplify.

- **(a)** $2(x + 1) + 3(x + 2)$
- **(b)** $4(y - 1) + 2(y - 2)$
- **(c)** $3(2x + 1) + 4(3x - 4)$

(d) $4(1-2x)+3(2-3x)$
(e) $5(x-3)-2(x+2)$
(f) $3(4x-2y)+3(2x-3y)$
(g) $2(x-4y)-2(x+y)$
(h) $3(x-y)-3(2x+3y)$
(i) $4(2x-3y)-2(2x-y)$
(j) $5(3y-x)-4(x-3y)$

4 Simplify:
(a) $3x+4x(1+x+x^2)$
(b) $5y^2-y(1+2y)$
(c) $x(3+2x)+x^2(1+2x)$
(d) $3x(1-2x)+x(x-1)$
(e) $4x(2x+1)+3(2x+1)$
(f) $5y(1-y)-y(y+3)$
(g) $4y(2-3y)-2y(1-y)$
(h) $3y(3+2y)-3y(1-2y)$
(i) $3y(1+y-y^2)-y^2(2-3y)$
(j) $5y^2(1-2x)+xy^2(3-2y)$

5 Remove brackets and simplify:
(a) $3(x-2y)+2(x+y)-3(x+2y)$
(b) $4x(1+y)+3y(2-x)-2(xy+3y)$
(c) $x(x^2-3)+x^2(1-x)-3x(3+x)$

Factorisation

Instead of removing brackets it is sometimes useful to insert brackets. This process is called **factorisation**.

For example, the expression

$4x+6y$

can be rewritten as

$2(2x+3y)$

Notice that 2 is a factor of both $4x$ and $6y$:

$2 \times 2x = 4x$
$2 \times 3y = 6y$

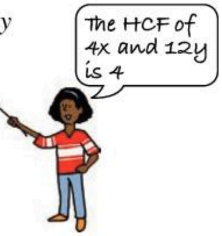
To factorise an expression you need to find the common factors of each term in the expression.

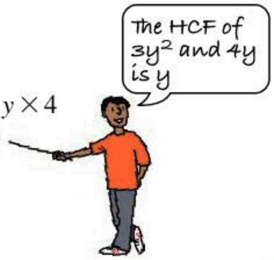
In fact, you are finding the highest common factor, HCF, of the terms.

Example 7

Factorise:
(a) $4x-12y$
(b) $3y^2-4y$

(a) $4x-12y = 4 \times x - 4 \times 3y$
$= 4(x-3y)$





(b) $3y^2-4y = y \times 3y - y \times 4$
$= y(3y-4)$

In Questions 1–3 of Exercise 5E the HCF has been found for you!

Exercise 5E

1 Copy and complete:
(a) $2x+2y=2(\qquad)$
(b) $3x-6y=3(\qquad)$
(c) $5x-15y=5(\qquad)$
(d) $6x-15y=3(\qquad)$
(e) $7x+14y=7(\qquad)$
(f) $35x-15y=5(\qquad)$
(g) $6x+3y-9z=3(\qquad)$
(h) $16x-24y+32z=8(\qquad)$

2 Copy and complete:
(a) $bx+by=b(\qquad)$
(b) $ax^2+ay^2=a(\qquad)$
(c) $3ax+4ay=a(\qquad)$
(d) $6px-3py=3p(\qquad)$
(e) $8rx+4ry=4r(\qquad)$
(f) $xy+3x^2=x(\qquad)$
(g) $5x^2y+4y=y(\qquad)$
(h) $6xy+4y-3x^2y=y(\qquad)$

3 Copy and complete:

(a) $4x^2 + 3xy - yx^2 = x\ (\qquad)$
(b) $6ax + 4by + 12cz = 2\ (\qquad)$
(c) $4xy - 6x^2y = 2xy\ (\qquad)$
(d) $3x^3y - 2x^2y = x^2y(\qquad)$
(e) $a^2b^2 - ab^2 + ab = ab\ (\qquad)$
(f) $3ab^2 + 4a^2b + a^2b^2 = ab(\qquad)$
(g) $3x^2y - 2xy^2 + 4x^2yz = xy(\qquad)$
(h) $4xy - 3x^2y^2 + 5xyz = xy(\qquad)$

4 Find the HCF of the terms and hence factorise

(a) $3x + 9y$ **(b)** $2a - 4b$
(c) $6x - 12y$ **(d)** $14x - 7y$
(e) $15x + 18y$ **(f)** $6x - 24y$
(g) $6x + 72y$ **(h)** $ax + ay$

5 Factorise:

(a) $ax + 3ay$ **(b)** $an - 3am$
(c) $6rx + 2ry$ **(d)** $3ax - 18ay$
(e) $5mn - 15mp$ **(f)** $4pq - 3pr$
(g) $6r + 4pr - 2qr$ **(h)** $3ab - 3a + ac$

6 Factorise:

(a) $3x^2 + x$ **(b)** $x^2y + 3xy$
(c) $4m^2 - m$ **(d)** $a^2b - 3ab^2$
(e) $\pi r^2 + \pi r^2h$ **(f)** $\pi rh + h^2$
(g) $3x^2y - xy + 3xy^2$ **(h)** $pq + 3p^2q - 2q^2$

7 Factorise:

(a) $4a^2b - 6ab^2 + 2ab$
(b) $x^2y^2 - 3x^2y + 4xy^2$
(c) $4xy^3 - 2xy^2 + 3y^2$
(d) $g^2h^3 - g^2h^2 + g^3h^2$
(e) $14abc - 7ab^2c + 21abc^2$
(f) $6ab^2c - 3a^2bc + 9abc^2$

5.2 Algebraic fractions

Working with algebraic fractions is similar to working with numerical fractions. Recall how you would add two numerical fractions, for example

$$\frac{2}{5} + \frac{3}{7} = \frac{2 \times 7}{5 \times 7} + \frac{3 \times 5}{7 \times 5}$$
$$= \frac{14}{35} + \frac{15}{35} = \frac{29}{35}$$

You could not add 2 fifths to 3 sevenths, you first had to turn the fractions into equivalent fractions with the same common denominator, 5 × 7 = 35.

The same idea is used for algebraic fractions

$$\frac{2x}{5} + \frac{3x}{7} = \frac{2x \times 7}{5 \times 7} + \frac{3x \times 5}{7 \times 5}$$
$$= \frac{14x}{35} + \frac{15x}{35} = \frac{29x}{35}$$

The first step is to find a common denominator or least common multiple, LCM.

Example 8

Simplify:

(a) $4x + \frac{2y}{3}$ **(b)** $\frac{x}{3} - \frac{4}{y}$

(a) $4x$ is the fraction $\frac{4x}{1}$, the LCM of 1 and 3 is 3, so

$$4x + \frac{2y}{3} = \frac{4x \times 3}{3} + \frac{2y}{3}$$
$$= \frac{12x + 2y}{3}$$

(b) The LCM of 3 and y is $3y$, so

$$\frac{x}{3} + \frac{4}{y} = \frac{x \times y}{3 \times y} - \frac{4 \times 3}{y \times 3}$$
$$= \frac{xy - 12}{3y}$$

Exercise 5F

1 Simplify:

(a) $\frac{3}{5} + \frac{1}{5}$ **(b)** $\frac{2}{5} + \frac{1}{3}$
(c) $\frac{5}{8} + \frac{2}{5}$ **(d)** $\frac{3}{16} + \frac{4}{7}$

2 Simplify:

(a) $\frac{3x}{5} + \frac{x}{5}$ **(b)** $\frac{2x}{5} + \frac{x}{3}$
(c) $\frac{5x}{8} + \frac{2x}{5}$ **(d)** $\frac{3x}{16} + \frac{4x}{7}$

3 Simplify:

(a) $\frac{4x}{3} + \frac{3x}{4}$ **(b)** $\frac{5x}{3} - \frac{2x}{5}$
(c) $3x + \frac{2}{7}$ **(d)** $\frac{3}{4} - 2x$

4 Simplify:

(a) $\frac{3}{x}+\frac{2}{3}$ **(b)** $\frac{2}{y}-\frac{3}{5}$

(c) $\frac{4}{x}+\frac{5}{y}$ **(d)** $\frac{3}{y}-2$

5 Simplify:

(a) $\frac{2}{x}+\frac{x}{2}$ **(b)** $\frac{3}{x}-\frac{x}{5}$

(c) $\frac{2x}{y}+\frac{3x}{4}$ **(d)** $\frac{4x}{3y}-\frac{2y}{5x}$

6 **(a)** $\frac{2a}{5b}+\frac{3b}{a}$ **(b)** $\frac{2a}{5c}-\frac{2c}{3b}$

(c) $\frac{2}{ab}-\frac{c}{3}$ **(d)** $\frac{2}{ab}+\frac{3}{c}$

7 **(a)** $\frac{2}{3}+\frac{2}{a}+\frac{1}{b}$ **(b)** $\frac{a}{3}+\frac{2}{b}+\frac{3}{c}$

(c) $\frac{2a}{3}-\frac{4b}{c}-\frac{3}{4}$ **(d)** $\frac{2a}{b^2}+\frac{b}{a}+\frac{2}{3}$

More complex algebraic fractions can be dealt with in the same way. That is, in each case find the LCM of the terms and use the LCM as the common denominator.

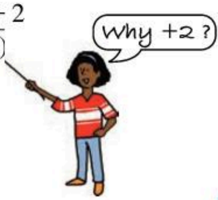
Example 9

Simplify:

$$\frac{3}{x-2}-\frac{5}{x+4}$$

The LCM of denominators is the product $(x-2)(x+4)$. Hence

$$\begin{aligned}\frac{3}{x-2}-\frac{5}{x+4}&=\frac{3(x+4)}{(x-2)(x+4)}-\frac{5(x-2)}{(x+4)(x-2)}\\&=\frac{3x+12}{(x-2)(x+4)}-\frac{5x-2}{(x+4)(x-2)}\\&=\frac{3x+12-5x+2}{(x-2)(x+4)}\\&=\frac{-2x+14}{(x-2)(x+4)}\end{aligned}$$



Exercise 5G

1 Copy and complete

(a) $\frac{2}{x}+\frac{3}{x+1}=\frac{2(x+1)}{x(x+1)}+\frac{3x}{(x+1)x}=$ _______

(b) $\frac{6}{x+1}-\frac{2}{x}=\frac{\square}{(x+1)x}-\frac{\square}{(x+1)x}=$ _______

(c) $\frac{3}{x+3}+\frac{2}{x+1}=\frac{\square}{(x+3)(x+1)}+\frac{\square}{(x+3)(x+1)}$

$=$ _______

2 Simplify:

(a) $\frac{5}{x}+\frac{2}{x+2}$ **(b)** $\frac{4}{x-3}+\frac{3}{x}$

(c) $\frac{5}{x-1}+\frac{3}{x-5}$ **(d)** $\frac{2}{x+4}-\frac{3}{x}$

3 Simplify:

(a) $\frac{2x-1}{3}+\frac{x+1}{4}$ **(b)** $\frac{3x-5}{2}+\frac{5x-7}{3}$

(c) $\frac{2x+3}{5}-\frac{4x-1}{3}$ **(d)** $\frac{2x-7}{3}-\frac{3x-2}{4}$

4 Simplify:

(a) $\frac{3}{2x-5}-\frac{2}{x-4}$ **(b)** $\frac{4}{x-3}-\frac{6}{2x-4}$

(c) $\frac{3}{x-2}-\frac{4x}{x-1}$ **(d)** $\frac{3x}{x-5}-\frac{4}{3x-7}$

5 The cost m mangoes is \$5, while the cost of p pineapples is \$23.

(a) What is the cost of one mango?

(b) What is the cost of one pineapple?

(c) What is the cost of one mango and one pineapple?

(d) Write in its simplest form the cost of a mangos and b pineapples.

Multiply and dividing algebraic fractions

You can multiply and divide algebraic fractions just as you can multiply and divide numerical fractions.

Recall

$$\frac{2}{3}\times\frac{5}{7}=\frac{2\times5}{3\times7}=\frac{10}{21}$$

and

$$\frac{2}{3}\div\frac{5}{7}=\frac{2}{3}\times\frac{7}{5}=\frac{14}{15}$$

Algebraic fractions follow the same rules.

Example 10

Simplify:

(a) $\dfrac{a^2}{3bc^2} \times \dfrac{c^3}{ab^2}$ **(b)** $\dfrac{4a^2}{3bc^2} \div \dfrac{2ac^2}{b^3}$

(a) $$\begin{aligned}\frac{a^2}{3bc^2} \times \frac{c^3}{ab^2} &= \frac{a^2 \times c^3}{3bc^2 \times ab}\\ &= \frac{a^{\cancel{2}}c^{\cancel{3}}}{3\cancel{a}b^2\cancel{c^2}}\\ &= \frac{ac}{3b^2}\end{aligned}$$

(b) $$\begin{aligned}\frac{4a^2}{3bc^2} \div \frac{2ac^2}{b^3} &= \frac{4a^2}{3bc^2} \times \frac{b^3}{2ac^2}\\ &= \frac{4a^{\cancel{2}}b^{\cancel{3}}}{6\cancel{a}\cancel{b}c^4}\\ &= \frac{2b^2}{3c^4}\end{aligned}$$

Exercise 5H

1 Simplify:

(a) $\dfrac{3ac}{b^2} \times \dfrac{4bc}{a}$ **(b)** $\dfrac{4a^2c}{b^3} \times \dfrac{2a^2}{c^3}$

(c) $\dfrac{2ab^2c}{5} \times \dfrac{10b^2c}{a^3}$ **(d)** $\dfrac{4a^3}{b^2c^4} \times \dfrac{a^2b^3}{c}$

2 Simplify:

(a) $\dfrac{a^2b^2}{c} \div \dfrac{c^2}{a}$ **(b)** $\dfrac{2a}{3c} \div \dfrac{4a}{3c}$

(c) $\dfrac{5a^2b^3}{c^2} \div \dfrac{a^2b}{c^2}$ **(d)** $\dfrac{a^3b}{3c^3} \div \dfrac{ab^2}{2c^2}$

3 Simplify:

(a) $\dfrac{3a^2b}{4c^2} \times \dfrac{2ab^2}{c} \times \dfrac{ac^3}{6b^2}$

(b) $\dfrac{5a}{6b^2c^3} \times \dfrac{a^2bc^2}{4} \times \dfrac{c^2}{b^3a^2}$

(c) $\dfrac{3ac^2}{b^3} \times \dfrac{4b^2}{a^2c^3} \times \dfrac{c^2}{a^3b}$

(d) $\dfrac{ab^2}{(b^2)^2} \times \dfrac{c^3a}{5bc^3} \times \dfrac{b^4}{2c^2}$

4 Simplify:

(a) $\dfrac{a^3}{b^2} \times \dfrac{2c^3}{3ba} \div \dfrac{b^2}{c^3}$

(b) $\dfrac{2a^2b}{c^3} \times \dfrac{ab^2}{c} \div \dfrac{4b^2a^3}{c^2}$

(c) $\dfrac{12ac^3}{b^2} \times \dfrac{4c^2}{a^3b} \div \dfrac{24a^2c^5}{b^3}$

(d) $\dfrac{-2ac^2}{3b} \times \dfrac{6a^3c^2}{7b^3} \div \dfrac{3a^3c}{14b^3}$

5.3 Products of binomial expressions

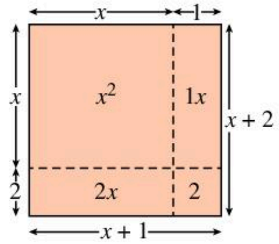
An expression such as $3x + 4$ or $7 - 2x$ are known as binomial expressions as they are both made up of two terms.

In mathematics you often have to deal with the product of two such expressions.

For example, what is

$$(x + 1)(x + 2)?$$

One way of thinking about this product is that it can represent the area of a rectangle with length $x + 1$ and width $x + 2$:



The area of the rectangle

$$\begin{aligned}&= \text{sum of the four smaller rectangles}\\ &= x^2 + 1x + 2x + 2\\ &= x^2 + 3x + 2\end{aligned}$$

Hence $(x + 1)(x + 2) = x^2 + 3x + 2$

Another way to calculate the product is to use the **distributive law**.

$$\begin{aligned}(x + 1)(x + 2) &= x(x + 2) + 1(x + 2)\\ &= x^2 + 2x + 1x + 2\\ &= x^2 + 3x + 2\end{aligned}$$

Notice, you multiply the $x + 2$ first by x then by 1. Each term in the second bracket is multiplied **each** term in the first bracket.

You can use the mnemonic FOIL to make sure you do not forget any terms!

$$(x + 1)(x + 2)$$

First terms $x \times x = x^2$
Outer terms $x \times 2 = 2x$
Inner terms $1 \times x = 1x$
Last terms $1 \times 2 = 2$

That is, $(x + 1)(x + 2) = x^2 + 2x + 1x + 2$
$= x^2 + 3x + 2$

This works for more complex expressions too.

Example 11

Expand and simplify

$(3x + 2y)(5x - 3y)$

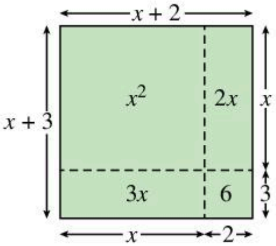
First terms $3x \times 5x = 15x^2$
Outer terms $3x \times -3y = -9xy$
Inner terms $2y \times 5x = 10xy$
Last terms $2y \times -3y = -6y^2$

so $(3x + 2y)(5x - 3y) = 15x^2 - 9xy + 10xy - 6y^2$
$= 15x^2 + xy - 6y^2$

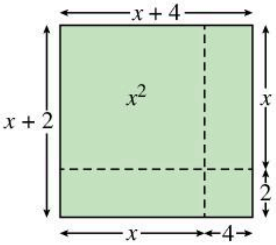
Exercise 5I

1 Find the area of the rectangles to help you simplify:

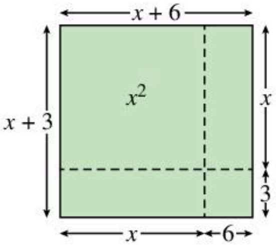
(a) $(x + 2)(x + 3)$



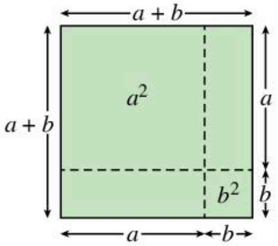
(b) $(x + 4)(x + 2)$



(c) $(x + 6)(x + 3)$



(d) $(a + b)(a + b)$



2 Expand and simplify:
(a) $(x + 1)(x + 3)$ **(b)** $(x + 4)(x + 3)$
(c) $(x + 3)(x + 3)$ **(d)** $(x + 3)(x + 7)$
(e) $(x + 5)(x + 6)$ **(f)** $(x + 7)(x + 8)$

3 Expand and simplify:
(a) $(x - 1)(x + 1)$ **(b)** $(x - 2)(x + 2)$
(c) $(x - 5)(x + 5)$ **(d)** $(x + 6)(x - 6)$
(e) $(x + 8)(x - 8)$ **(f)** $(x - 9)(x + 9)$

4 What did you notice about your answers to Question 3?

5 Expand and simplify:
(a) $(x + 2)(x - 3)$ **(b)** $(x - 3)(x + 4)$
(c) $(x - 3)(x + 6)$ **(d)** $(x + 4)(x - 6)$
(e) $(x - 5)(x + 6)$ **(f)** $(x + 5)(x - 7)$

6 Expand and simplify:
(a) $(x - 2)(x - 3)$ **(b)** $(x - 1)^2$
(c) $(x - 5)^2$ **(d)** $(x - 3)(x - 6)$
(e) $(x - 4)(x - 5)$ **(f)** $(x - 6)(x - 9)$

7 Expand and simplify:
(a) $(a + b)(c + d)$ **(b)** $(a + b)(c - d)$
(c) $(a - b)(c + d)$ **(d)** $(a - b)(c - d)$

8 Expand and simplify:
(a) $(2x + 1)(2x + 3)$ **(b)** $(2x + 3)(x + 2)$
(c) $(4x + 3)(3x + 1)$ **(d)** $(3x + 2)(2x + 5)$
(e) $(5x + 1)(3x + 3)$ **(f)** $(4x + 1)(2x + 3)$

9 Expand and simplify:
(a) $(2x - 1)^2$ **(b)** $(3x + 5)^2$
(c) $(2x - 1)(3x + 5)$ **(d)** $(2x + 1)(3x - 5)$
(e) $(2x - 1)(3x - 5)$ **(f)** $(4x - 1)(3x - 8)$

10 Expand and simplify:
(a) $(a - b)^2$ **(b)** $(a - b)(a + b)$
(c) $(3a - 2b)(3a + 2b)$ **(d)** $(4a - 7b)(4a + 7b)$
(e) $(2a - 3b)^2$ **(f)** $(2a + 3b)^2$

Technology

Visit

www.mathsisfun.com/algebra/special-binomial-products.html

or check www.coolmath.com/prealgebra and look at polynomials to get further assistance and practice

5.4 Solving linear equations

An **equation** is a statement showing the equality of two expressions.

For example:

$4x + 3 = 4$

or $3x - 7 = 6 - 2x$

are both equations.

To solve linear equations you can use the **balance** method. This means that what you do to one side of an equation you must do to the other to maintain equality.

For example:

In the equation

$$x + 3 = 8$$

adding 2 to both sides

$$x + 3 + 2 = 8 + 2$$

maintains equality.

You can use this powerful idea to solve all linear equations.

Example 12

Solve:

(a) $x - 7 = 8$

(b) $3x + 2 = 14$

(a) $x - 7 = 8$

(add 7 to both sides) $x - 7 + 7 = 8 + 7$

$x = 15$

(b) $3x + 2 = 14$

(subtract 2 from both sides) $3x + 2 - 2 = 14 - 2$

$3x = 12$

(divide both sides by 3) $\frac{3x}{3} = \frac{12}{3}$

$x = 4$

Notice that you add or subtract numbers to simplify the expressions on each side of the equation. The aim is to put the unknown terms on one side of the equation and the constant terms (numbers) on the other side.

Exercise 5J

1 Solve for x.

(a) $x - 3 = 5$ **(b)** $x + 3 = 5$

(c) $x + 4 = 11$ **(d)** $x - 15 = 16$

(e) $x + 1 = 0$ **(f)** $x - 8 = 6$

2 Solve for x.

(a) $3x = 9$ **(b)** $5x = 15$

(c) $4x = 12$ **(d)** $6x = 36$

(e) $7x = 21$ **(f)** $11x = 33$

3 Solve for x.

(a) $\frac{x}{3} = 2$ **(b)** $\frac{x}{4} = 5$

(c) $\frac{x}{4} = 6$ **(d)** $\frac{x}{9} = {}^{-}3$

(e) $\frac{x}{16} = 1$ **(f)** $\frac{x}{11} = {}^{-}2$

4 Solve:

(a) $3x + 1 = 4$ **(b)** $2x - 1 = 1$

(c) $3x - 3 = 12$ **(d)** $4x + 5 = 25$

(e) $6x - 7 = 47$ **(f)** $3x - 8 = 12$

5 Solve:

(a) $\frac{x}{2} + 3 = 4$ **(b)** $\frac{x}{3} + 4 = 5$

(c) $\frac{x}{4} - 1 = 1$ **(d)** $\frac{x}{7} + 2 = 5$

(e) $\frac{x}{5} - 4 = 0$ **(f)** $\frac{x}{6} - 7 = 5$

You can solve more complex equations using the same idea. In each case try to put the like terms together.

Example 13

Solve:

(a) $5x - 2 = 2x + 5$
(b) $7 - 3x = 6x - 15$

(a) $5x - 2 = 2x + 5$

First, collect the x terms on the left-hand side of the equation by subtracting $2x$ from both sides.

$$(-2x) \qquad 5x - 2 - 2x = 2x + 5 - 2x$$
$$3x - 2 = 5$$

Then, collect the constant terms on the right-hand side by adding 2 to both sides.

$$(+2) \qquad 3x - 2 + 2 = 5 + 2$$
$$3x = 7$$
$$(\div 3) \qquad \frac{3x}{3} = \frac{7}{3}$$
$$x = 2\frac{1}{3}$$

(b) $7 - 3x = 6x - 15$

To collect x-terms, add $3x$ to both sides.

$$(+3x) \qquad 7 - 3x + 3x = 6x - 15 + 3x$$
$$7 = 9x - 15$$

To collect constant terms, add 15 to both sides.

$$(+15) \qquad 7 + 15 = 9x - 15 + 15$$
$$22 = 9x$$
$$(\div 9) \qquad \frac{22}{9} = \frac{9x}{9}$$

So $$x = 2\frac{4}{9}$$

Exercise 5K

1 Solve:
- **(a)** $3x + 2 = 2x + 6$
- **(b)** $5x - 2 = x + 6$
- **(c)** $4x + 3 = 3x + 7$
- **(d)** $2x - 3 = x - 5$
- **(e)** $3x - 4 = 2x + 5$
- **(f)** $6x - 9 = 3x + 4$

2 Solve these equations:
- **(a)** $6 - x = x$
- **(b)** $6 - 2x = x$
- **(c)** $4 - 3x = 2 + x$
- **(d)** $4x - 3 = 6 - 3x$
- **(e)** $6x - 13 = 7 - 4x$
- **(f)** $13 - 5x = {}^{-}11 - 3x$

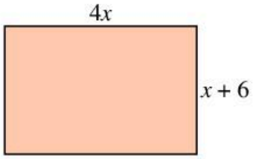
3 Solve:
- **(a)** $3(x + 1) = 4x - 2$
- **(b)** $2(x - 2) = 3x - 2$
- **(c)** $7(x - 1) = 2 - 2x$
- **(d)** $3(2x - 1) = 4x - 5$
- **(e)** $2(x + 5) = 3(x + 1)$
- **(f)** $4(x - 1) = 5(2 - x)$

4 Solve:
- **(a)** $3(x + 1) + 3 = 2x - 4$
- **(b)** $4(2x - 1) - 3 = 3 - 2x$
- **(c)** $7(1 - x) + x = 2 + x$
- **(d)** $6x + 3(1 - 4x) = 3(x + 2)$
- **(e)** $3 + 4(2x - 1) = 6x - 2$
- **(f)** $4 - 2(x + 2) = 3(x + 1)$

5 When 3 is added to a number and then multiplied by 4 the result is 32.
- **(a)** Denote the number by x and write down an equation.
- **(b)** Solve the equation to find the value of x.

6 A triangle has perimeter $6x$ cm. The lengths of the sides of the triangle are x, $x + 2$, $x + 4$ cm.
- **(a)** Write down an equation relating the perimeter to the lengths of the sides.
- **(b)** Solve the equation and hence find the lengths of each side of the triangle.

7



The perimeter of the rectangle above is 48 cm.
- **(a)** Form an equation for the perimeter of the rectangle.
- **(b)** Solve the equation to find the length and width of the rectangle.

8 The sum of two consecutive even numbers is 226. What are the numbers?

Technology

Use the equation calculator at

www.algebrahelp.com

to check your solutions to the equations in Exercise 5K.

How successful were you?

Technology

Need more assistance with equations?
Look at the sites

www.algebrahelp.com

or

www.coolmath.com

Both sites give a complete course on solving simple linear equations.
Both sites give lots of practice questions.
Try them!
Check your answers.

5.5 Forming and solving equations

Many problems can be solved by first forming equations.

Example 14

The sum of three consecutive numbers is 60.
What are the numbers?

Let x be the first number.

Then $x + 1$ is the next consecutive number and $x + 2$ is the next consecutive number

The numbers add up to 60 so,

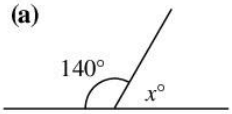
$$\begin{aligned} & x + (x + 1) + (x + 2) = 60 \\ & 3x + 3 = 60 \\ (-3)\quad & 3x = 57 \\ (\div 3)\quad & x = \tfrac{57}{3} = 19 \end{aligned}$$

If $x = 19$, then $x + 1 = 20$, and $x + 2 = 21$ so the numbers are 19, 20 and 21

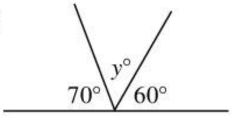
Exercise 5L

1 The sum of the angles on a straight line is180. Write an equation for each diagram, then find the value of the unknown letter.

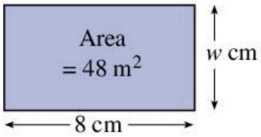
(a)



(b)

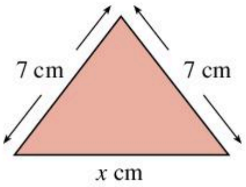


2 **(a)** Write down an equation in terms of w for the area of the rectangle.



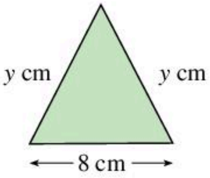
(b) Solve the equation to find w and write down the width of the rectangle.

3 **(a)** Write down an expression in terms of x for the perimeter of the isosceles triangle.



(b) If the perimeter is 20 cm, find the value of x.

4 **(a)** The perimeter of this isosceles triangle is 32 cm. Write down an equation in terms of y for the perimeter.



(b) Solve this equation to find y and write down the lengths of the two equal sides of the triangle.

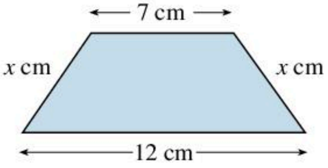
5 In an exam Curtis scored x marks and Alfred scored 20 marks more than Curtis.

(a) What was Alfred's score?

(b) What was the total score of Curtis and Alfred?

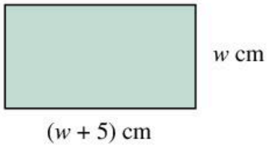
(c) If their total score was 130 what was Curtis's actual score?

6 **(a)** Write down an expression for the perimeter of the trapezium.



(b) If the perimeter is 30 cm, find the value of x.

7 **(a)** The width of the rectangle shown is w cm.



If its length is 5 cm longer than its width write down an expression for the perimeter of the rectangle.

(b) If the perimeter is 50 cm find the dimensions of the rectangle.

8 One side of a rectangle is 4 cm longer than the other. The perimeter of the rectangle is 28 cm. Find the length of each side.

9 Peter is three years older than his sister Paula. If their combined age is 21 find their actual ages.

10 Two consecutive whole numbers add up to 31. What are they?

11 Three consecutive whole numbers add up to 72. What are they?

12 The sum of three consecutive odd numbers is 69. Find them.

13 An adult's ticket to a cricket match costs twice as much as a child's ticket. The cost for Mr and Mrs Bun and their five children is \$3.15.

(a) If a child's ticket costs c cents, write an equation to show this information.

(b) Solve your equation to find the cost of each ticket.

14

At the Windham Cinema, 'stalls' tickets cost \$3 more than 'house' tickets. Ram bought four 'stalls' tickets and three 'house' tickets for the film 'Creature from the dark lagoon'. Writing n as the cost of a house ticket, write an expression for the cost of Ram's tickets. If Ram paid \$82 in total find the cost of a 'house' ticket.

15 A number is multiplied by 3 and then 7 is added. The result is doubled, giving the answer 80. Using n to represent the original number, write an equation to show this information.
Solve the equation to find n.

5.6 Simultaneous equations

The equations you have looked at so far involve just one unknown. When there are two unknowns you need two equations in order to find the values of both unknowns. Two such equations are known as **simultaneous** equations.

For example:

$$3x + 2y = 8$$
$$2x - 3y = {}^{-}1$$

are a pair of simultaneous equations.

The easiest way to solve a pair of simultaneous equations is to eliminate either the x or y terms and solve the remaining linear equation.

Look at the simultaneous linear equations:

$$3x + 2y = 8 \qquad [1]$$
$$2x + 2y = 6 \qquad [2]$$

Subtract equation [2] from equation [1].

$$\begin{array}{rl} 3x + 2y = 8 & \quad [1] \\ \underline{2x + 2y = 6} & \quad [2] \\ x = 2 & \end{array}$$

Substituting $x = 2$ into equation [1]

$$\begin{array}{lr} & 3 \times 2 + 2y = 8 \\ & 6 + 2y = 8 \\ (-6) & 2y = 2 \\ (\div 2) & y = 1 \end{array}$$

Hence, $x = 2$, $y = 1$ is the solution to the equations.

Sometimes you need to multiply each equation by an appropriate number to make the coefficients of x or y the same before adding or subtracting the equations.

Example 15

Solve:

$3x + 4y = 13 \quad \ldots \quad [1]$
$2x + 3y = 9 \quad \ldots \quad [2]$

To make the coefficients of x the same, multiply [1] by 2 and [2] by 3.

[1] × 2: $6x + 8y = 26 \;\ldots\; [3]$
[2] × 3: $6x + 9y = 27 \;\ldots\; [4]$

To eliminate the x term, subtract equation [3] from equation [4].

[4] – [3]: $y = 1$

Substituting $y = 1$ into equation [1]

$$3x + 4 \times 1 = 13$$
$$3x + 4 = 13$$
(-4) $\quad 3x = 9$
$(\div 3)$ $\quad x = 3$

Hence, $x = 3$, $y = 1$ is the solution to the equations.

You can check your answers by substitution. For example 11 the solution is $x = 3$, $y = 1$. To check this answer look at each equation in turn:

[1] is $3x + 4y = 3 \times 3 + 4 \times 1 = 9 + 4 = 13$
[2] is $2x + 3y = 2 \times 3 + 3 \times 1 = 6 + 3 = 9$

Exercise 5M

1 Solve these simultaneous equations by subtracting them.

(a) $3x + 2y = 5$, $2x + 2y = 4$ **(b)** $6x + y = 13$, $4x + y = 9$
(c) $4x - 3y = 1$, $3x - 3y = 0$ **(d)** $5x - 4y = 11$, $3x - 4y = 5$

2 Solve these simultaneous equations by adding them.

(a) $4x - y = 3$, $6x + y = 7$ **(b)** $3x - 5y = {}^{-}2$, $4x + 5y = 9$
(c) $3x - 2y = {}^{-}3$, $3x + 2y = 9$ **(d)** $4x - 7y = {}^{-}3$, $3x + 7y = 10$

3 Solve these simultaneous equations by multiplying one equation by an appropriate number.

(a) $3x - 2y = 10$, $4x + y = 17$ **(b)** $6x - 5y = 1$, $3x + 2y = 5$
(c) $x - 3y = {}^{-}1$, $4x - 2y = 12$ **(d)** $3x + 4y = 14$, $5x - 8y = {}^{-}6$

4 Solve these simultaneous equations.

(a) $2x + 3y = 5$, $5x + 2y = 7$ **(b)** $6x - 2y = 4$, $4x + 3y = 7$
(c) $3x - 2y = 11$, $2x + 3y = 16$ **(d)** $5x - 3y = {}^{-}1$, $2x + 4y = 10$
(e) $7x + 3y = 4$, $3x + 4y = {}^{-}1$ **(f)** $9x - 7y = 4$, $6x - 5y = 2$

Another way of solving a pair of simultaneous equations is by substitution.

Example 16

Solve:

$x - 2y = 7 \quad \ldots \quad [1]$
$3x - 4y = 22 \quad \ldots \quad [2]$

Equation [1] is

$$x - 2y = 7$$
$(+2y)$ $\quad x = 7 + 2y \quad \ldots \quad [3]$

Substituting [3] into [2]

$$3(7 + 2y) - 4y = 22$$
$$21 + 6y - 4y = 22$$
$$21 + 2y = 22$$
(-21) $\quad 2y = 1$
$(\div 2)$ $\quad y = \frac{1}{2}$

Substituting in [3]

$$x = 7 + 2 \times \frac{1}{2}$$
$$= 8$$

Hence the solution is $x = 8$, $y = \frac{1}{2}$.

Notice that in this approach the two equations in two unknowns are reduced to one equation in one unknown.

Exercise 5N

1 Solve these equations using the substitution method.

(a) $x - 3y = 1$, $2x - 5y = 3$ **(b)** $x + 3y = 4$, $3x - 2y = 1$

(c) $x - 5y = 6$, $2x - 13y = 15$ **(d)** $x + 3y = 10$, $5x - 2y = {}^{-}1$

2 Solve these simultaneous equations using the substitution method.

(a) $2x - 3y = 1$, $3x + 2y = 8$ **(b)** $2x + y = 5$, $5x - 2y = 8$

(c) $3x - y = 6$, $2x + 3y = 15$ **(d)** $4x + 3y = 10$, $5x - 2y = 1$

3 **(a)** Use the substitution method to solve the equations in Exercise 5M Questions 3 and 4.

(b) Which method do you prefer?

Technology

Do an internet search to find some sites on simultaneous equations.

Which ones help you?

5.7 Quadratic equations

A **quadratic equation** contains a term in x^2, but no higher power.

For example,

$$x^2 + 4x - 3 = 0$$
$$2x^2 = 7$$
$$4 - 3x^2 = 5x$$

are all quadratic equations.

To solve some of these types of equations, you first need to be able to factorise a quadratic expression.

Factorising quadratic expressions

Some, but not all, quadratic expressions can be written as the product of two binomial expressions (see Section 5.3).

For example

$$x^2 + 3x + 2 = (x + 1)(x + 2)$$

The expression

$$x^2 + 3x + 2$$

is said to be factorised with factors $(x + 1)$ and $(x + 2)$.

Factorisation is, of course, the reverse of expanding a product of two factors.

A quadratic expression such as

$$x^2 + 5x + 6$$

will factorise, if it can be written as a product of factors

$$x^2 + 5x + 6 = (x + \triangle)(x + \square)$$

where the numbers $\triangle$ and $\square$ have to be found.

Now

$$\begin{aligned}(x + \triangle)(x + \square) &= x^2 + \triangle \times x + \square \times x + \triangle \times \square \\ &= x^2 + x(\triangle + \square) + \triangle \times \square\end{aligned}$$

Equating the coefficients

$$5 = \triangle + \square$$

and $6 = \triangle + \square$

In other words, $\triangle$ and $\square$ are the factors of 6 that sum to 5.

2 and 3 meet this requirement

so $x^2 + 5x + 6 = (x + 2)(x + 3)$

You can check this is correct by expansion

$$\begin{aligned}(x + 2)(x + 3) &= x(x + 3) + 2(x + 3) \\ &= x^2 + 3x + 2x + 6 \\ &= x^2 + 5x + 6\end{aligned}$$

In general, you need to look at the factors of the whole number term in the quadratic expression and find which factor pair when added gives the value of the coefficient of x.

Example 17

Factorise $x^2 + 8x + 12$

The factors of 12 are

1 and 12 or –1 and –12
2 and 6 or –2 and –6
3 and 4 or –3 and –4

When added the pair that gives 8 are 2 and 6.

Hence

$$x^2 + 8x + 12 = (x + 2)(x + 6)$$

Check this is correct!

This method works even for more complex quadratic expressions.

Example 18

Factorise $x^2 - 2x - 8$.

The factors of –8 are

−8 and 1 or 8 and −1
−4 and 2 or 4 and −2

The only pair which sum to the coefficient of x, −2, is −4 and 2.

Hence

$$x^2 - 2x - 8 = (x - 4)(x + 2).$$

Exercise 5O

1 Copy and complete

(a) $x^2 + 5x + 4 = (x + 4)(x + \triangle)$
(b) $x^2 + 8x + 7 = (x + 1)(x + \triangle)$
(c) $x^2 + 9x + 8 = (x + \quad)(x + \triangle)$
(d) $x^2 + 7x + 6 = (x + \quad)(x + \triangle)$

2 Copy and complete

(a) $x^2 + 13x + 12 = (x + \square)(x + \triangle)$
(b) $x^2 + 11x + 18 = (x + \square)(x + \triangle)$
(c) $x^2 + 8x + 15 = (x + \square)(x + \triangle)$
(d) $x^2 + 12x + 20 = (x + \square)(x + \triangle)$

3 Copy and complete

(a) $x^2 - 7x + 12 = (x - 3)(x \quad \square)$
(b) $x^2 - 8x + 15 = (x - 5)(x - \square)$
(c) $x^2 - 12x + 20 = (x - 2)(x - \square)$
(d) $x^2 - 10x + 25 = (x - 5)(x - \square)$

4 Copy and complete

(a) $x^2 - 6x + 8 = (x - \triangle)(x - \square)$
(b) $x^2 - 8x + 15 = (x - \triangle)(x - \square)$
(c) $x^2 - 10x + 24 = (x - \triangle)(x - \square)$
(d) $x^2 - 13x + 30 = (x - \triangle)(x - \square)$

5 Copy and complete

(a) $x^2 + 2x - 8 = 0 = (x + 4)(x - \square)$
(b) $x^2 + x - 12 = 0 = (x - 3)(x + \square)$
(c) $x^2 + 7x - 18 = 0 = (x - 2)(x + \square)$
(d) $x^2 - 9x - 36 = 0 = (x + 3)(x - \square)$

6 Copy and complete

(a) $x^2 + 8x - 9 = (x + \triangle)(x - \square)$
(b) $x^2 - 6x - 16 = (x + \triangle)(x - \square)$
(c) $x^2 - 4x - 21 = (x + \triangle)(x - \square)$
(d) $x^2 - 3x - 40 = (x + \triangle)(x - \square)$

7 Factorise

(a) $x^2 + 18x + 17$ **(b)** $x^2 - 5x - 14$
(c) $x^2 - 16x + 28$ **(d)** $x^2 + x - 42$

8 Go through Questions 1 – 7. check by expanding your factors that your answers are correct.

Solving quadratic equations

Once you are able to factorise a quadratic expression, it is relatively easy to solve a corresponding quadratic equation.

For example
Solve $x^2 - 7x + 12 = 0$

$$x^2 - 7x + 12 = 0$$
$$\Rightarrow (x - 3)(x - 4) = 0$$

For the product $(x - 3)(x - 4)$ to be zero, either

$x - 3 = 0$ so $x = 3$
or $x - 4 = 0$ so $x = 4$

Notice a quadratic equation has two solutions, $x = 3$ and $x = 4$.

You can check your answer are correct.
When $x = 3$

$$\begin{aligned} x^2 - 7x + 12 &= 3^2 - 7 \times 3 + 12 \\ &= 9 - 21 + 12 \\ &= 0 \end{aligned}$$

When $x = 4$

$$\begin{aligned} x^2 - 7x + 12 &= 4^2 - 7 \times 4 + 12 \\ &= 16 - 28 + 12 \\ &= 0 \end{aligned}$$

Unfortunately, not all quadratics equations can be factorised.

Example 19

Solve

(a) $x^2 + x - 12 = 0$
$(x + 4)(x - 3) = 0$
Either $x + 4 = 0 \quad \Rightarrow x = -4$
or $x - 3 = 0 \quad \Rightarrow x = 3$

(b) $x^2 + x + 12 = 0$

$x^2 + x + 12$ cannot be factorized so you cannot solve this equation by factorisation

Exercise 5P

1 Solve by factorisation.
- **(a)** $x^2 + 4x + 3 = 0$
- **(b)** $x^2 + 7x + 12 = 0$
- **(c)** $x^2 + 8x + 15 = 0$
- **(d)** $x^2 + 23x + 22 = 0$

2 Solve:
- **(a)** $x^2 - 7x + 12 = 0$
- **(b)** $x^2 - 8x + 15 = 0$
- **(c)** $x^2 - 7x + 10 = 0$
- **(d)** $x^2 - 14x + 13 = 0$

3 Solve:
- **(a)** $x^2 + 2x - 8 = 0$
- **(b)** $x^2 + 15x - 16 = 0$
- **(c)** $x^2 + 22x - 23 = 0$
- **(d)** $x^2 + x - 6 = 0$

4 Solve:
- **(a)** $x^2 - 4x - 21 = 0$
- **(b)** $x^2 - 10x - 11 = 0$
- **(c)** $x^2 - 4x - 32 = 0$
- **(d)** $x^2 - 2x - 35 = 0$

Exercise 5Q – mixed questions

1 Simplify:
- **(a)** $3x + 7y + 10 + 2x - 2y - 3$
- **(b)** $9x + 3y - 5z + 8y - 4x + 12z$
- **(c)** $9(5x + 3y) - 5(4x + 5y)$
- **(d)** $5x + 7y + 3(x - y + 1)$

2 If $x = 2$, $y = 3$, $z = 4$ find the value of:
- **(a)** $4x - 3y + 2z$
- **(b)** $5x + 2y - 3z$
- **(c)** $\dfrac{3x + 2y + z}{x + y + z}$
- **(d)** $3x^2y$
- **(e)** $2xy^2 + 3z$
- **(f)** $(3x + y)^2$
- **(g)** $3x^2 - 2y + 5$

3 Simplify:
- **(a)** $2p^3 \times 3p^4$
- **(b)** $2q^5 \times 4q^2$
- **(c)** $18f^5 \div 6f^5$
- **(d)** $(2y^2)^3$
- **(e)** $\dfrac{(9x^2)(2x^7)}{3x^5}$
- **(f)** $\dfrac{36x^5y^3 \times 5x^2}{9x^3y^2}$

4 Factorise:
- **(a)** $3x^2y - 2xy$
- **(b)** $6xy^2 - 3y$
- **(c)** $8x^3y^2 - 4x^2y^2$
- **(d)** $12x^3 - 4x^2y$
- **(e)** $3xy + 4x^2y - y^2$
- **(f)** $8x^2y - 2xy^2 - xy$

5 Simplify:
- **(a)** $\dfrac{3}{x} + \dfrac{5}{x + 1}$
- **(b)** $\dfrac{5}{x} - \dfrac{2}{y}$
- **(c)** $4 + \dfrac{3}{x}$
- **(d)** $4 - \dfrac{2}{3y}$
- **(e)** $\dfrac{3}{x - 1} + \dfrac{5}{x + 1}$
- **(f)** $\dfrac{8}{x - 2} - \dfrac{2}{x - 1}$

6 Expand and simplify:
- **(a)** $(x + 3)(x + 1)$
- **(b)** $(x + 2)(x - 2)$
- **(c)** $(x + 4)(x - 3)$
- **(d)** $(x - 3)^2$
- **(e)** $(2x - 3)(3x + 11)$
- **(f)** $(3x - 1)(2x - 1)$

7 Solve each equation.
- **(a)** $5x + 7 = 19 - x$
- **(b)** $8(2d - 3) = 3(4d - 7)$
- **(c)** $3(6 + 7y) + 2(1 - 5y) = 42$
- **(d)** $\dfrac{x}{4} - 9 = 16$
- **(e)** $\dfrac{x + 5}{4} = 11$
- **(f)** $4(7x + 6) - 8 = 72$

8 Mr Johnson needs 320 m of fencing to enclose his rectangular field. If the width of his field is w m and the length of it is 90 m, find the value of w.

9 Solve the simultaneous linear equations.

(a) $3x - 4y = {}^{-}1$
$6x + 7y = 13$

(b) $4x - 2y = 3$
$5x + 6y = 8$

(c) $7x - 3y = 1$
$2x - 3y = {}^{-}4$

10 The sum of two consecutive even numbers is 38. Find the two numbers.

11 12 years from now, Ali will be three times older than he was 18 years ago. If Ali is a years old today, write down:

(a) his age 12 years from now

(b) his age 18 years ago

(c) an equation which shows the above information.
Solve the equation to find Ali's age now.

12 Factorise:

(a) $x^2 + 13x + 42$

(b) $x^2 - 24x - 81$

(c) $x^2 - 7x - 60$

(d) $x^2 - 13x - 30$

13 Solve the equations:

(a) $(x - 3)(x + 4) = 0$

(b) $x^2 - 16 = 0$

(c) $x^2 + 3x - 4 = 0$

(d) $x^2 - 4x - 45 = 0$

14 A box has width 1 cm longer than its height and length 2 cm longer than its width. If the height, width and length add up to 52 cm, find the dimensions of the box.

5 Consolidation

Example 1

Simplify:

(a)
$$\begin{aligned} &3x(2y - z)\\ &= 3x \times 2y - 3x \times z\\ &= 6xy - 3xz \end{aligned}$$

(b)
$$\begin{aligned} &2x(1 - x) + 6(x - 3)\\ &= 2x \times 1 - 2x \times x + 6 \times x - 6 \times 3\\ &= 2x - 2x^2 + 6x - 18\\ &= 8x - 2x^2 - 18 \end{aligned}$$

Example 2

Simplify:

(a)
$$\begin{aligned} &\frac{3}{2x-1} - \frac{4}{2-3x}\\ &= \frac{3(2-3x)}{(2x-1)(2-3x)} - \frac{4(2x-1)}{(2x-1)(2-3x)}\\ &= \frac{6-9x-8x+1}{(2x-1)(2-3x)} - \frac{7-17x}{(2x-1)(2-3x)} \end{aligned}$$

Example 3

Solve:

(a)
$$\begin{aligned} &3(2x - 5) = 9\\ &- 3 \times 2x - 3 \times 5 = 9\\ &\qquad 6x - 15 = 9\\ (+15)\quad &\qquad 6x = 24\\ (\div 6)\quad &\qquad x = 4 \end{aligned}$$

(b)
$$\begin{aligned} &\frac{2x+3}{5} = x - 4\\ (\times 5)\quad &2x + 3 = 5(x - 4)\\ &2x + 3 = 5x - 20\\ (+20)\quad &2x + 23 = 5x\\ (-2x)\quad &23 = 3x\\ (\div 3)\quad &x = \frac{23}{3} = 7\frac{2}{3} \end{aligned}$$

Example 4

Expand:

$$\begin{aligned} &(3x - 2)(4x + 3)\\ &= 3x(4x + 3) - 2(4x + 3)\\ &= 12x^2 + 9x - 8x - 6\\ &= 12x^2 + x - 6 \end{aligned}$$

Example 5

Solve the simultaneous equations.

(a)
$$\begin{aligned} &2x - 3y = 5 \quad \ldots \quad [1]\\ &3x + 2y = 14 \quad \ldots \quad [2]\\ 2 \times [1]:\quad &4x - 6y = 10\\ 3 \times [2]:\quad &\underline{9x + 6y = 42}\\ \text{Add}\quad &13x = 52\\ &x = 4 \end{aligned}$$

Substitute in [1]

$$\begin{aligned} &2 \times 4 - 3y = 5\\ &8 - 3y = 5\\ (-8)\quad &{}^{-}3y = {}^{-}3\\ (\div {}^{-}3)\quad &y = 1 \end{aligned}$$

(b)
$$\begin{aligned} &3x + y = 4 \quad \ldots \quad [1]\\ &5x - 2y = 3 \quad \ldots \quad [2] \end{aligned}$$

Equation [1] is

$$\begin{aligned} &3x + y = 4\\ (-3x)\quad &y = 4 - 3x \quad \ldots \; [3] \end{aligned}$$

Substitute [3] into [2]

$$\begin{aligned} &5x - 2(4 - 3x) = 3\\ &5x - 8 + 6x = 3\\ &11x - 8 = 3\\ (+8)\quad &11x = 11\\ (\div 11)\quad &x = 1 \end{aligned}$$

Substitute in [1]

$$\begin{aligned} &3 \times 1 + y = 4\\ &3 + y = 4\\ (-3)\quad &y = 1 \end{aligned}$$

Exercise 5

1 Simplify:

(a) $3 \times s \times t \times t$ **(b)** $p^3 \times p^3$

(c) $3p^2 \times 2p^3$ **(d)** $r^2 \times 3r^3 \times r^2$

(e) $p^4 \div p^2$ **(f)** $3p^4 \div p^3$

(g) $\dfrac{6p^2 \times 3r^2}{2pr}$ **(h)** $\dfrac{2s \times t^3 \times u^4}{8tu^2}$

2 Simplify:

(a) $3(2 + 3p)$ **(b)** $6(x - 2y)$

(c) $m(2m - 3n)$ **(d)** $3s(2r - 5s)$

(e) $4(x - y) + 3(x + y)$ **(f)** $g(2h + f) + f(h + 2f)$

3 Simplify:

(a) $\frac{7}{x} - \frac{x}{3}$ **(b)** $5 - \frac{x}{4}$

(c) $12 - \frac{x^2}{y}$ **(d)** $\frac{2}{x+1} + \frac{3}{x+4}$

(e) $\frac{5}{x-2} + \frac{3}{2x-1}$ **(f)** $\frac{3}{3x-4} - \frac{2}{2x-5}$

4 Solve these equations.

(a) $3x + 2 = 8$ **(b)** $3x - 2 = 10$

(c) $2 - x = 1$ **(d)** $\frac{x}{3} = 5$

(e) $\frac{x}{3} + 5 = 9$ **(f)** $4x = 5 - x$

(g) $2x - 3 = x - 4$ **(h)** $3(x - 7) + 4 = 20$

5 Expand and simplify.

(a) $(3x - 1)(x + 1)$ **(b)** $(x - 7)(x + 7)$

(c) $(x + 5)^2$ **(d)** $(3x - 5)^2$

(e) $(2x + 1)(2x - 1)$ **(f)** $(2 - x)(3 - x)$

6 Solve these simultaneous equations.

(a) $3x - 2y = 1$ **(b)** $6x + 2y = 7$
$4x - 3y = 1$ $5x - 6y = 2$

(c) $7x - 2y = 9$
$3x + 4y = {}^{-}1$

Application

7 The labour cost, C, in dollars charged by a firm for electrical repairs is given by

$$C = 30 + 75h$$

where h is the number of hours taken for the repair job.

(a) Find the repair cost for a job taking 6 hours.

(b) Alice gets a repair bill for \$205.
How long did the repair take?

8 Delia has \$945 in twenty dollar and five dollar bills. She finds she has five times as many twenty dollar bills as five dollar bills.

(a) Writing x as the number of \$20 bills, form an equation to show the information.

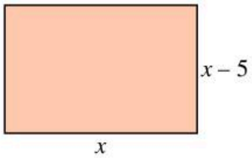
(b) Solve this equation to find the number of bills of each type she has.

9 The cost for 5 adults and 3 children to attend a show is \$210. For 2 adults and 5 children the cost is \$160. Form a pair of simultaneous linear equations and solve them to find the cost of

(a) an adult ticket

(b) a child's ticket.

10



A rectangular room has width 5 m shorter than its length, x.

(a) If the area of the room is 24 m^2, write down an equation in terms of x to show this information.

(b) Solve this quadratic equation by factorisation to find the length and width of the room.

Summary

You should know ...

1 How to simplify an expression.

For example:

$$\begin{aligned} &3x^2 + 4y^2 - 2x^2 - y^2 \\ &= 3x^2 - 2x^2 + 4y^2 - y^2 \\ &= x^2 + 3y^2 \end{aligned}$$

2 How to use the distributive law to simplify an expression.

For example:

$$\begin{aligned} &6(3x - 2y) + 2(x - y) \\ &= 6 \times 3x - 6 \times 2y + 2 \times x - 2 \times y \\ &= 18x - 12y + 2x - 2y \\ &= 20x - 14y \end{aligned}$$

3 How to factorise an expression.

For example:

$3ax + 4ay = a(3x - 4y)$

4 How to simplify algebraic fractions.

For example:

$$\begin{aligned} \frac{4}{x-1} - 2 &= \frac{4}{x-1} - \frac{2(x-1)}{x-1} \\ &= \frac{4 - 2(x-1)}{x-1} \\ &= \frac{4 - 2x + 2}{x-1} \\ &= \frac{6 - 2x}{x-1} \end{aligned}$$

5 How to simplify binary product.

For example:

$$\begin{aligned} (3x + 7)(2x - 5) &= 3x(2x - 5) + 7(2x - 5) \\ &= 6x^2 - 15x + 14x - 35 \\ &= 6x^2 - x - 35 \end{aligned}$$

6 How to solve a linear equation.

For example:

$$\begin{aligned} & & 4x - 2 &= x + 9 \\ &(-x) & 3x - 2 &= 9 \\ &(+2) & 3x &= 11 \\ &(\div 3) & x &= \frac{11}{3} = 3\frac{2}{3} \end{aligned}$$

Check out

1 Simplify:

(a) $3x + 2y - 4x + 6y$

(b) $6xy - 2y + 2xy - 3y$

2 Expand the brackets and simplify:

(a) $4(2x - 3)$ **(b)** $6(2x + 7y)$

(c) $3x(2 - 4y)$ **(d)** $6x^2(3 - 4x)$

(e) $3x(1 + 2y) - 2y(1 - 2x)$

(f) $4y(1 - y) + 3y(2 - y)$

3 Factorise:

(a) $3x - x^2$ **(b)** $4xy - x^2y$

(c) $6x + 72$

(d) $a^2bc - abc + ab^2c$

4 Simplify:

(a) $\frac{3}{x+1} + 4$

(b) $\frac{2}{a} - \frac{b}{4}$

(c) $\frac{6}{2x-1} + \frac{3}{1-x}$

(d) $\frac{a}{b-1} - \frac{b}{a-1}$

5 Simplify:

(a) $(x + 3)(x + 11)$

(b) $(x - 9)(x + 9)$

(c) $(2x - 7)^2$

(d) $(3 - x)(5 - 2x)$

6 Solve:

(a) $4x - 3 = 1$

(b) $3x - 2 = 7$

(c) $\frac{x}{3} + 2 = 5$

(d) $6x - 3 = 4 + 2x$

(e) $16x - 2 = 11 + 3x$

7 How to form equations to solve problems.
For example:
In four years' time James will be five times as old as he is now. What is his present age?

Let his present age be x years.
In four years' time his age will be $x + 4$
Five times as old as his present age is $5x$

$$
\begin{array}{ll}
\text{so} & x + 4 = 5x \\
(-x) & 4 = 4x \\
(\div 4) & 1 = x
\end{array}
$$

James is 1 year old now.

7 The width of a rectangle is w cm. Its length is 4 cm more than its width. If the perimeter of the rectangle is 28 cm, find the length and width.

8 How to solve simultaneous equations.
For example:

$$
\begin{array}{lrl}
 & 5x - 2y = 8 & \dots \ [1] \\
 & 3x + 3y = 9 & \dots \ [2] \\
3 \times [1]: & 15x - 6y = 24 & \\
2 \times [2]: & \underline{6x + 6y = 18} & \\
\text{Add} & 21x = 42 & \\
(\div 21) & x = 2 &
\end{array}
$$

Substitute into [1]

$$
\begin{array}{lr}
 & 5 \times 2 \ - 2y = 8 \\
 & 10 \ - 2y = 8 \\
(-10) & {}^{-}2y = {}^{-}2 \\
(\div {}^{-}2) & y = 1
\end{array}
$$

8 Solve:
(a) $3x - 5y = 1$
$2x + 3y = 7$
(b) $6x + 3y = 18$
$7x - 2y = 10$
(c) $4x - 3y = 10$
$3x - 4y = 11$

9 How to solve a quadratic equation by factorisation.
For example:

$$x^2 - 11x - 26 = 0$$

The factors of -26 are
-26 and 1 or 26 and -1
-13 and 2 or 13 and -2
Only -13 and 2 add to -11

$$
\begin{array}{lll}
\text{So} & x^2 - 11x - 26 = 0 & \\
 & \Rightarrow (x - 13)(x + 2) = 0 & \\
 & \Rightarrow x - 13 = 0 \quad \Rightarrow & x = 13 \\
\text{or} & x + 2 = 0 \quad \Rightarrow & x = -2
\end{array}
$$

9 Solve by factorisation:
(a) $x^2 + 4x + 3 = 0$
(b) $x^2 - 6x - 7 = 0$
(c) $x^2 - 2x - 8 = 0$

Geometry

6

Objectives

- ✔ use basic angle facts to calculate angles in shapes
- ✔ use geometric properties to classify triangles, quadrilaterals and other polygons
- ✔ identify similar and congruent triangles
- ✔ use a ruler and compasses to construct angles and shapes

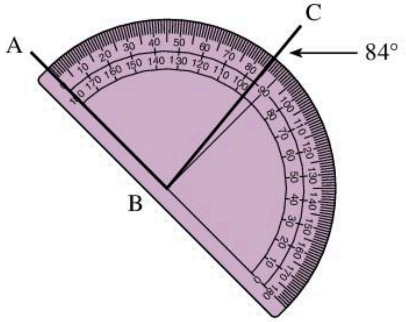
What's the point?

Cycling is a popular sport in the Caribbean. Top cyclists, like Jamaica's Ricardo Lynch, practise on special cycle tracks called velodromes. The velodromes are usually banked at an angle of 42° to prevent cyclists from falling off at the bends.

Before you start

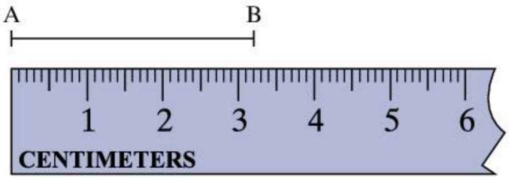
You should know ...

1 How to use a protractor to draw and measure angles.



$A\hat{B}C = 84°$

2 How to draw and measure lines.



AB = 3.2 cm

Check in

1 Use your protractor to draw angles of:

(a) 30° **(b)** 45°
(c) 72° **(d)** 143°

2 **(a)** Measure these lines:

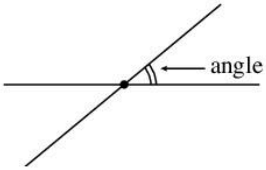
(i) __________
(ii) __________________

(b) Draw lines of length:

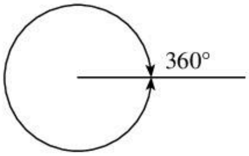
(i) 6.1 cm **(ii)** 4.3 cm

6.1 All about angles

An angle is formed when two straight lines meet at a point.

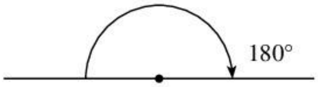


Angles are measured in degrees.
A complete turn is 360°.



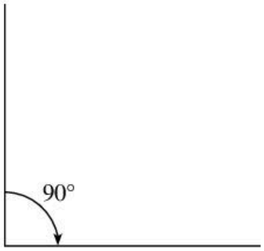
That is, angles at a point add up to 360°.

A half turn is 180°.



That is, angles on a straight line add up to 180°.

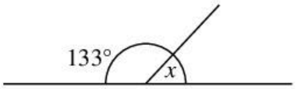
A quarter turn is 90°.



You can use these facts to calculate angles.

Example 1

Find the angle x.



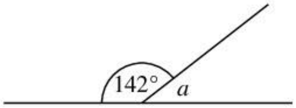
The angle on a straight line is 180°, so

$$\begin{aligned} 133^\circ + x &= 180^\circ \\ x &= 180^\circ - 133^\circ \\ &= 47^\circ \end{aligned}$$

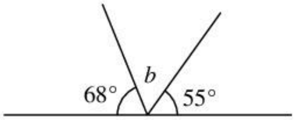
Exercise 6A

1 Calculate the missing angles a, b, c, d.

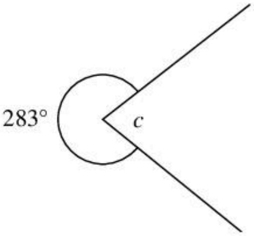
(a)



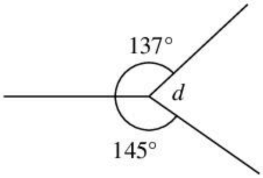
(b)



(c)

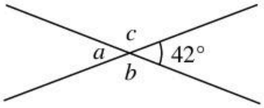


(d)

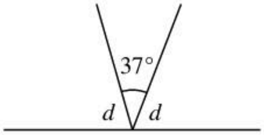


2 Find the missing angles a, b, c, d.

(a)

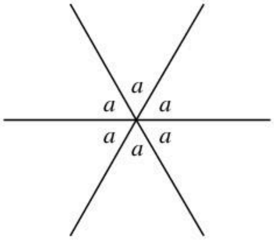


(b)

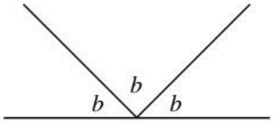


3 Work out the missing angles a, b, c, d, e.

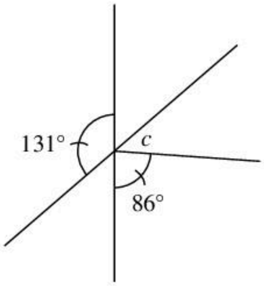
(a)



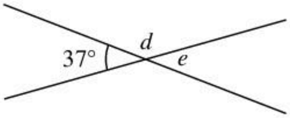
(b)



(c)



(d)

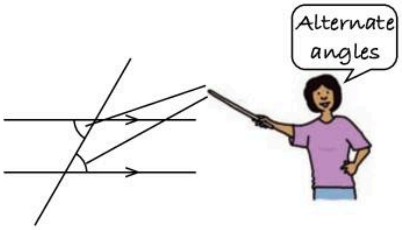


Parallel lines

When a line crosses two parallel lines you get other properties.

Corresponding angles are equal

and

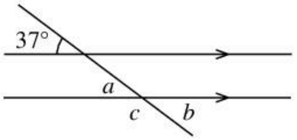


alternate angles are equal.

You can use the ideas of corresponding and alternate angles to work more complex problems involving missing angles.

Example 2

Find the missing angles a, b, c.

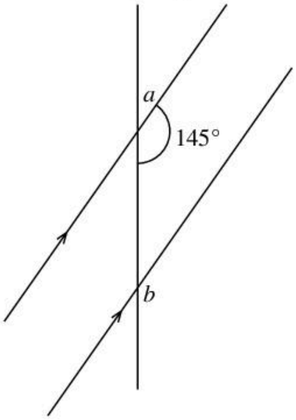


$a = 37°$ (corresponding angles)
$a = b = 37°$ (vertically opposite angles)
$b + c = 180°$ (angles on a straight line)
So $37° + c = 180°$
hence $c = 180° - 37° = 143°$

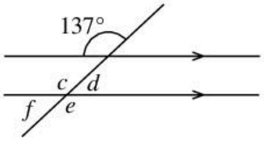
Exercise 6B

1 Find the missing angles a–i.

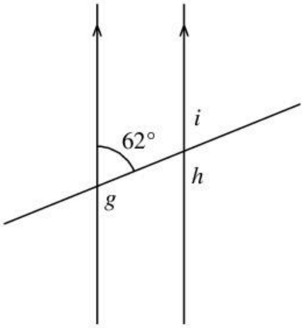
(a)



(b)

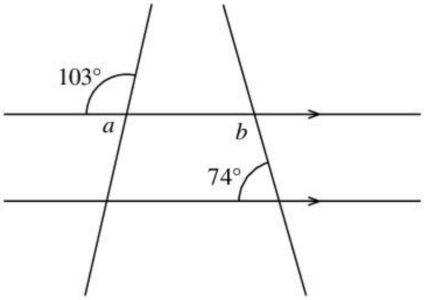


(c)

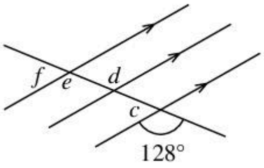


2 Calculate the angles *a*–*l*.

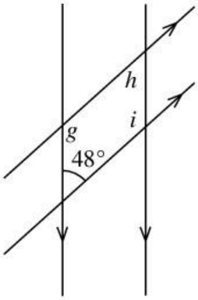
(a)



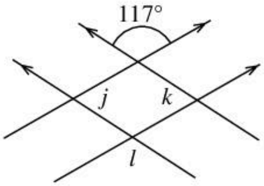
(b)



(c)

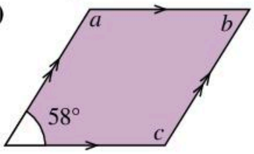


(d)

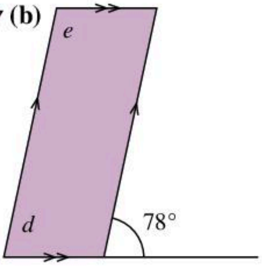


3 Find the marked angles *a*–*i* in these parallelograms.

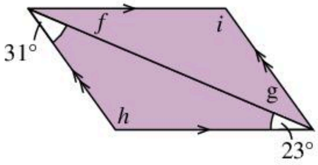
(a)



(b)



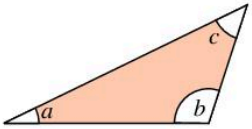
(c)



4 One interior angle of a parallelogram is 142°. What are the other angles?

Angles in common shapes

The sum of the angles in a triangle is 180°.

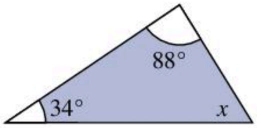


$$a + b + c = 180°$$

Given two angles in a triangle you can find the third.

Example 3

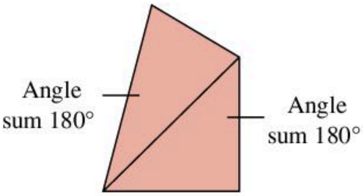
Find the missing angle in the triangle.



As the angle sum is 180°

$$\begin{aligned} 34° + 88° + x &= 180° \\ 122° + x &= 180° \\ x &= 180° - 122° \\ &= 58° \end{aligned}$$

Quadrilaterals are made up of two triangles.

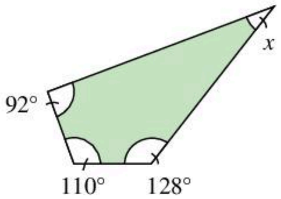


Hence the angle sum in a quadrilateral is $180° + 180° = 360°$

You can use this fact to calculate missing angles in a quadrilateral.

Example 4

Find the missing angle in the quadrilateral.



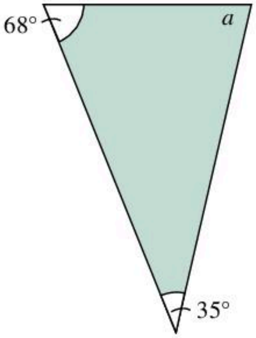
The angle sum is 360°.

$$\begin{aligned} \text{Hence, } \quad 92° + 110° + 128° + x &= 360° \\ 330° + x &= 360° \\ x &= 30° \end{aligned}$$

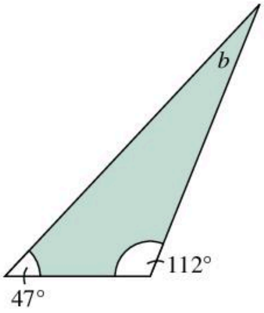
Exercise 6C

1 Find the missing angles in these shapes.

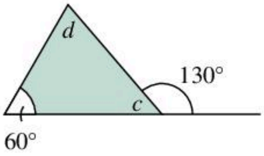
(a)



(b)

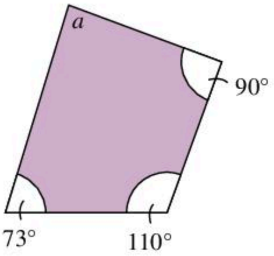


(c)

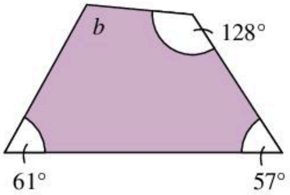


2 Calculate the missing angles.

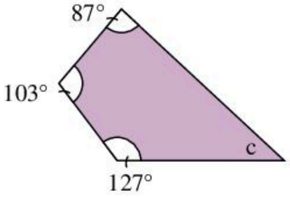
(a)



(b)

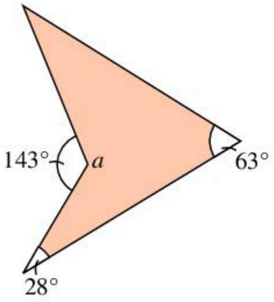


(c)

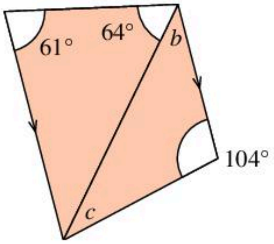


3 Find the missing angles.

(a)



(b)



Technology

For further revision on angles including videos, visit

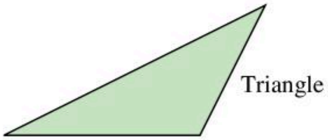
www.onlinemathlearning.com

Click on the relevant sections under angles. It has lots of information!

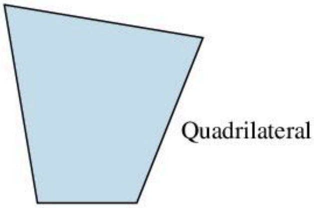
6.2 Classifying polygons

A **polygon** is a closed shape with 3 or more sides.

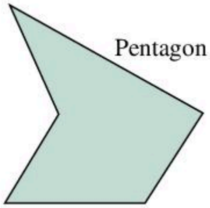
A three-sided polygon is called a triangle.



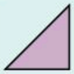
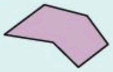
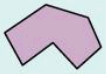
A four-sided polygon is called a quadrilateral.



A five-sided polygon is called a pentagon.



The easiest way to sort polygons is to check how many sides they have. See the table below.

Shape	Number of sides	Name of shape
	3	Triangle
	4	Quadrilateral
	5	Pentagon
	6	Hexagon
	7	Heptagon
	8	Octagon
	9	Nonagon
	10	Decagon

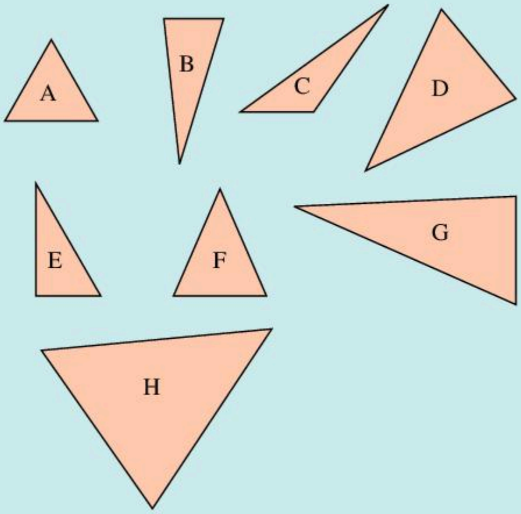
It's easy to find the properties of simpler polygons such as triangles and quadrilaterals.

Properties of triangles

You can discover all the major properties of triangles by sorting triangles into different groups.

Activity

Copy and cut out these triangles below.



Work in a small group.

- Divide the triangles into two distinct groups. Explain to your group how you divided them.
- Repeat, but sort the triangles into other groups.
- What criteria did you use for sorting?

Exercise 6D

1 **(a)** Draw three different triangles in your exercise book.
(b) With your protractor measure the interior angles of each triangle?
(c) What is the sum of the angles in each triangle?

2 **(a)** Using the triangles you cut out in the activity, measure with your ruler the length of each side.

(b) Copy and complete the table.

Triangle	Side 1 length (cm)	Side 2 length (cm)	Side 3 length (cm)
A			
B			
C			
D			
E			
F			
G			
H			

(c) **(i)** Which triangles have three equal sides (equilateral triangles)?
(ii) Which triangles have just two equal sides (isosceles triangles)?
(iii) Which triangles have no equal sides (scalene triangles)?

3 **(a)** Repeat Question 2, but this time measure the interior angle of each of the triangles.
(b) Copy and complete the table.

Triangle	Angle 1	Angle 2	Angle 3
A			
B			
C			
D			
E			
F			
G			
H			

(c) **(i)** Which triangles have three equal angles?
(ii) Which triangles have just two equal angles?
(iii) Which triangles have one 90° angle?
(iv) Which triangles have no equal angles?

4 Compare your answers to Question 2 with Question 4. What do you notice?

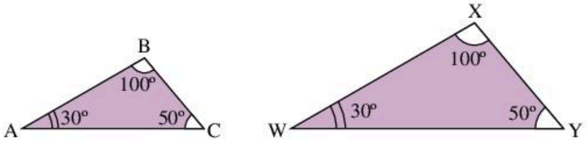
5 Copy and complete the table.

Triangle	Number of equal sides	Number of equal angles
Equilateral		
Isosceles		
Scalene		

6 **(a)** Draw an isosceles, right-angled triangle.
(b) Can you draw an equilateral right-angled triangle? Explain.

Similar and congruent triangles

Look at triangles ABC and XYZ.



Are these triangles identical?

Clearly, they are not even though their angles are the same.

They have the **same shape** but **different sizes**.

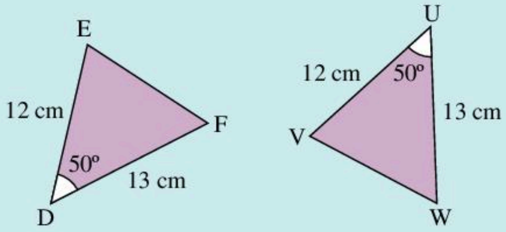
Such triangles are called **similar triangles**.

You will learn more about the properties of similar triangles in Unit 10, section 10.4

So when are two triangles identical?

Activity

Look at triangles DEF and UVW



- Using a protractor and ruler, make copies of triangles DEF and UVW.
- Cut them out.
- Are they identical?

In the activity you should have found the triangles are identical.

You can say two triangles are **congruent** if they have the **same shape** and the **same size**.

That is, if all three angles are equal and all three sides are equal. However, you just need to know *three* things about two triangles that are equal to say the two triangles are congruent.

Activity

Learn about the conditions for triangles to be congruent.

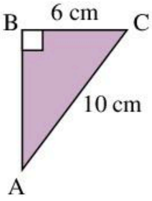
Visit

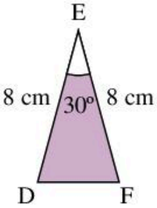
www.mathsisfun.com/geometry/triangles-congruent-finding.html

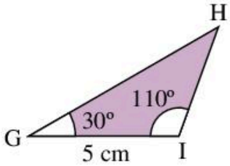
- Make a list of the different conditions.
- Make a presentation to your class.

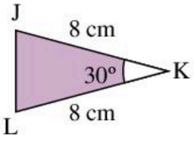
Exercise 6E

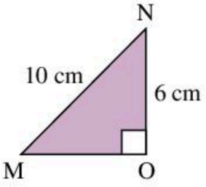
1

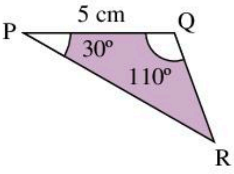






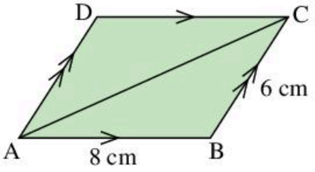






(a) Which pairs of triangles are congruent?
(b) Give reasons for each of your answers.

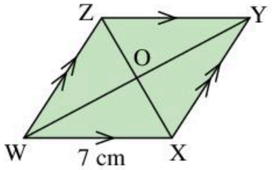
2 In the parallelogram ABCD, AB = 8 cm and BC = 6 cm.



The diagonal AC splits the parallelogram into two triangles.

(a) Are triangles ADC and ABC congruent?
(b) Give reasons for your answer.

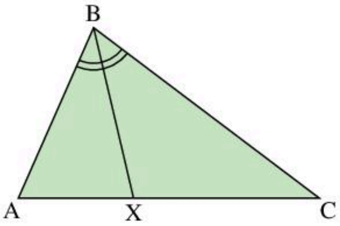
3



WXYZ is a rhombus with side 7 cm. The diagonals WY and XZ meet at O.

(a) Is triangle ZWX congruent to triangle ZYX? Give reasons for your answer.
(b) Write down **two** other pairs of congruent triangles. Give reasons why they are congruent.

4



In triangle ABC, the angle B is bisected, with the bisector cutting AC at X.

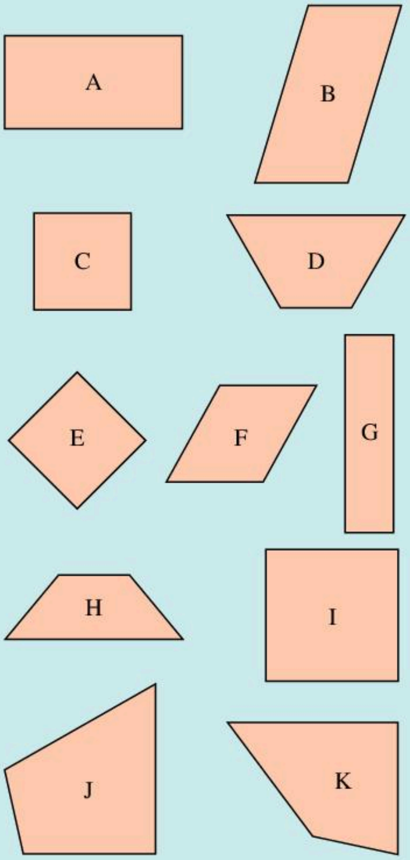
(a) Is triangle ABX congruent to triangle CBX? Give reasons.
(b) If AB = BC, would the triangles in (a) be congruent? Give reasons.

Properties of quadrilaterals

You can classify quadrilaterals based on their sides and angles.

Activity

Copy and cut out these quadrilaterals



Work in a small group:

- Divide the quadrilaterals into two or more groups. Explain how you divided them.
- Repeat, but sort the quadrilaterals into other groups.
- What rules did you use to sort them?
- Sort the quadrilaterals again. This time get other members of your group to guess the sorting rule you used.

Exercise 6F

1 **(a)** Draw three different quadrilaterals in your exercise book.
(b) With your protractor measure the interior angles of each quadrilateral.
(c) Did you find that the angle sum of the interior angles is 360°?

2 **(a)** Using the quadrilaterals you cut out in the Activity on page 110, measure with a ruler, the length of each side.
(b) Copy and complete the table.

Quadrilateral	Side 1 (cm)	Side 2 (cm)	Side 3 (cm)	Side 4 (cm)
A				
B				
C				
⋮				
K				

(c) Which quadrilaterals have
- **(i)** all four sides equal in length
- **(ii)** just two sides equal in length
- **(iii)** two pairs of sides equal in length
- **(iv)** no sides equal in length?

3 **(a)** Repeat Question 2, but this time measure the interior angle of the triangles with your protractor.
(b) Copy and complete the table.

Quadrilateral	Angle 1	Angle 2	Angle 3	Angle 4
A				
B				
C				
⋮				
K				

(c) Which quadrilaterals have
- **(i)** all four angles equal
- **(ii)** just two angles equal
- **(iii)** two pairs of angles equal
- **(iv)** no equal angles?

4 **(a)** Using the quadrilaterals you cut out in the Activity on page 110, identify those with pairs of parallel sides.

Copy and complete the table.

Quadrilateral	Two pairs of parallel sides	One pair of parallel sides	No parallel sides
A			
B			
C			
⋮			
K			

(b) A parallelogram has two pairs of parallel sides. Which of the shapes are parallelograms?

(c) How does quadrilateral F differ from I?

(d) How does quadrilateral B differ from G?

(e) A quadrilateral with four equal sides is called a rhombus. Which of the shapes are rhombi?

5 Copy and complete the table, by putting a ✓ or a ✗ in the cells.

Name of shape	Opposite sides equal	Opposite angles equal	Opposite sides parallel	All sides equal	All angles equal
square	✓				
rectangle					
parallelogram					
rhombus					
trapezium					
kite			✗		

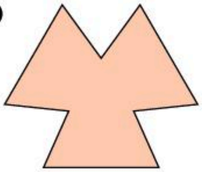
Sorting polygons

Another way to sort polygons is to find how many lines of symmetry the shapes have.

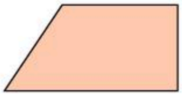
Example 5

How many lines of symmetry do these shapes have?

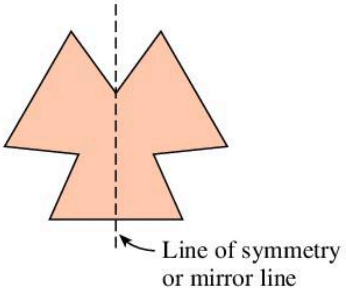
(a)



(b)



(a) The shape has one line of symmetry.



When folded along the mirror line the two halves match exactly.

(b) The shape has no lines of symmetry.

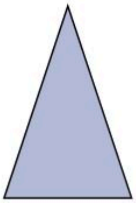
The best way to find if a shape has a line of symmetry is to cut out the shape and fold along a possible line of symmetry. If the halves match then it *is* a line of symmetry.

Exercise 6G

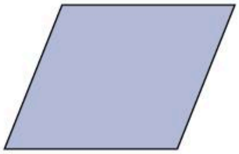
1 Make copies of these shapes.

(a)

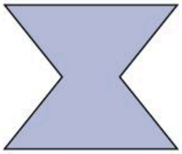
(b)



(c)



(d)



By folding them, find out the number of lines of symmetry they have.

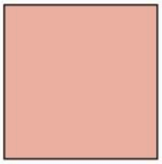
2 Draw a triangle with:

- **(a)** one line of symmetry
- **(b)** three lines of symmetry
- **(c)** no lines of symmetry.

3 Look at these quadrilaterals.

(a) Which ones have just one line of symmetry?

(i)



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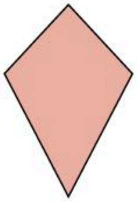

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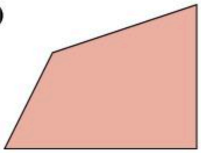

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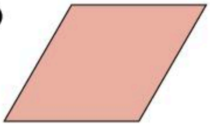

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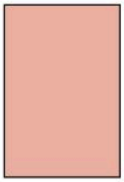

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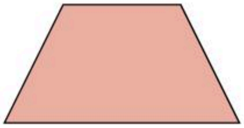


- **(b)** Which have two lines of symmetry?
- **(c)** Which have three lines of symmetry?
- **(d)** Which have no lines of symmetry?
- **(e)** What are the names of these shapes?

4 **(a)** Draw a pentagon with 5 lines of symmetry.
(b) Draw a six-sided shape with 6 lines of symmetry.

Investigation

Stringy polygons

Visit

www.nrich.maths.org/2913

Do the investigation on symmetry lines in a quadrilateral using string!

Repeat the investigation, this time for a pentagon.

What about a hexagon?

The number of pairs of parallel sides is another way of sorting polygons.

Exercise 6H

1 Draw a quadrilateral with:

- **(a)** one pair of parallel sides
- **(b)** two pairs of parallel sides.

2 What do you call the shapes drawn in Question 1?

3 Draw a shape with:

- **(a)** six sides and three pairs of parallel sides
- **(b)** eight sides and four pairs of parallel sides
- **(c)** six sides and one pair of parallel sides.

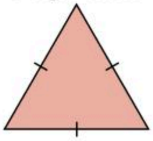
4 **(a)** Draw a quadrilateral with one line of symmetry and one pair of parallel sides.
(b) Which quadrilaterals have two lines of symmetry and two pairs of parallel sides?
(c) Which quadrilaterals have three lines of symmetry and two pairs of parallel sides?

Other ways of sorting polygons include:

- – length of side
- – number of right angles.

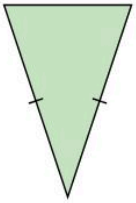
Triangles are often sorted by side length.

3 equal sides



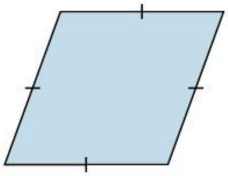
Equilateral triangle

Two equal sides

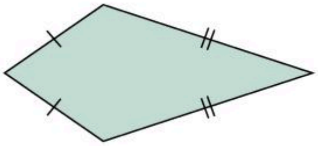


Isosceles triangle

A quadrilateral with four equal sides is a rhombus.



A quadrilateral with two pairs of equal sides is a kite.



With these ideas you should now be able to sort and classify most simple shapes.

Exercise 6I

1 Copy and complete the table.

Name of triangle	Number of equal sides
	0
	2
	3

2 Copy and complete the table.

Name of quadrilateral	Number of lines of symmetry
Square	
	2
	1
Parallelogram	
Rhombus	

3 Copy and complete the table.

Shape	Pairs of equal angles	Pairs of parallel sides
Kite		0
Rectangle		
Rhombus		
Square		

4 Draw a pentagon with
- **(a)** one pair of parallel sides
- **(b)** three equal sides
- **(c)** three right angles
- **(d)** two right angles
- **(e)** two right angles and one pair of parallel sides
- **(f)** two pairs of equal sides and two right angles

5 Write down all the major properties of a regular hexagon.

6 **(a)** Draw a Venn diagram to show:
Q = {quadrilaterals}
R = {rhombuses}
T = {rectangles}
(b) What can you say about R ∩ T?

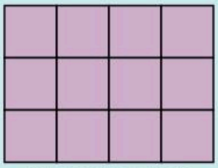
7 **(a)** Draw a Venn diagram to show:
T = {triangles}
E = {equilateral triangles}
I = {isosceles triangles}
(b) What is E ∩ I?

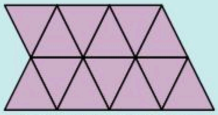
8 **(a)** Draw a Venn diagram to show:
Q = {quadrilaterals}
R = {rectangles}
S = {squares}
P = {parallelograms}
(b) Is it true to say that all rectangles are parallelograms?
(c) Is it true that all squares are rectangles?

Investigation

A shape is said to tessellate if when fitted together with identical shapes it leaves no gaps.

For example,


squares tessellate


isosceles triangles tessellate

By making identical copies of a shape, find out which of these shapes tessellate
- **(a)** scalene triangle
- **(b)** trapezium
- **(c)** kite
- **(d)** any quadrilateral
- **(e)** regular pentagon
- **(f)** regular hexagon

6.3 Construction geometry

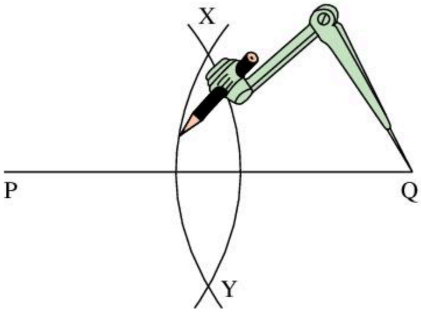
Architects, draftsmen and carpenters rely on accurate drawings. You can use a ruler and a protractor to construct shapes. However, to be more accurate a ruler and compasses are all you need.

In this section you will learn about a number of different constructions that will help you to draw shapes accurately.

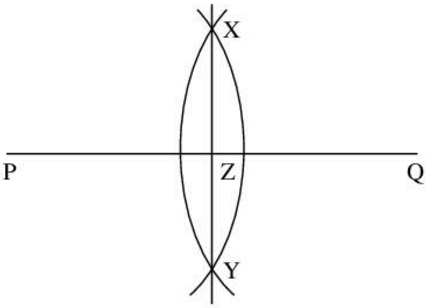
Bisecting a straight line

You can bisect the line PQ with a pair of compasses and a ruler.

Draw an arc with centre P and radius at least $\frac{1}{2}$ PQ. Draw another arc, centre Q, with the same radius.



The arcs meet at X and Y. Join XY.



XY is perpendicular to PQ and bisects PQ at Z.
PZ = QZ

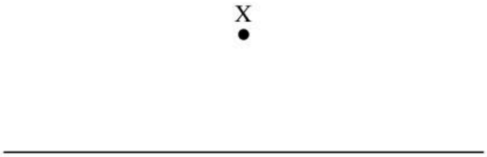
Exercise 6J

1 Draw a line PQ = 8 cm.
  (a) Bisect the line.
  (b) How long is each part?

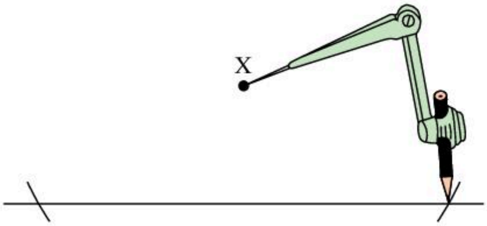
2 **(a)** Draw four different triangles.
  (b) Bisect the sides of each triangle.
  (c) What do you notice?

Constructing a perpendicular

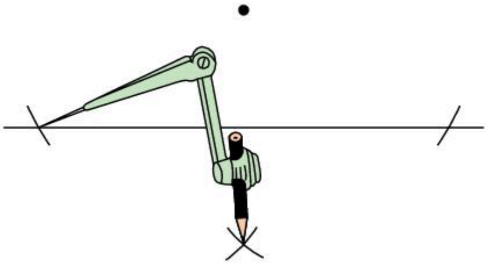
Sometimes you want to draw a perpendicular line from a point, X, to a line.



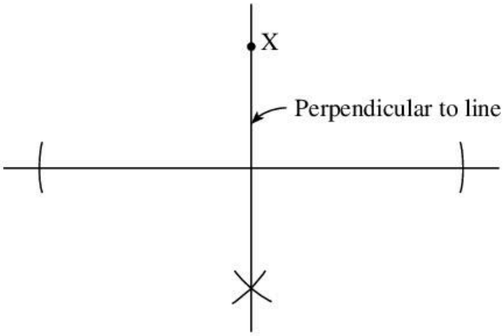
To do this, with your compasses draw arcs centred at X that cut the line.



Then draw arcs centred where the previous arcs cut the line.



Finally, with your ruler join the point X to where the arcs intersect.



Exercise 6K

1 Draw a line AB and a point C 6 cm above the line. Construct a perpendicular from C to the line AB.

2 **(a)** Draw a triangle.
  (b) Construct a perpendicular from each corner of the triangle to its opposite side.
  (c) What do you notice?

3 Repeat Question 2 for four other triangles.

Constructing angles

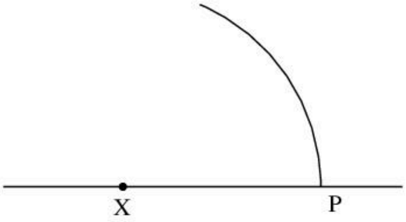
Important angles such as 90°, 60°, 30° and 45° can be constructed with just a ruler, pencil and compass.

There are three basic angle constructions to learn.
(i) 60° angle
(ii) 90° angle
(iii) angle bisection

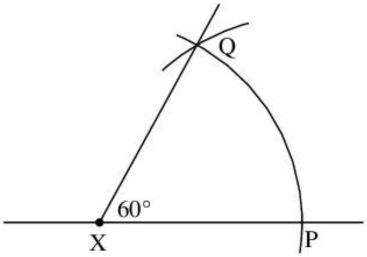
Most of the other angles can be made from these three.

(i) 60° angle

To construct a 60° at X on a line, first draw an arc with your compass with centre X to cut the line at P.



Using the **same** radius draw an arc, with centre P, to cut the first arc at Q.
Join XQ.



The angle QXP = 60°.

Exercise 6L

1 Construct a 60° angle.

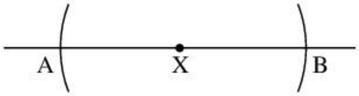
2 Construct the equilateral triangle XYZ with side XY = 5.8 cm.

3 Construct an angle of 120° by constructing two angles of 60° next to each other.

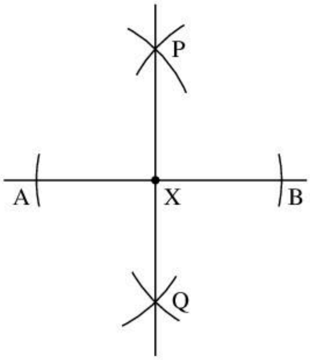
4 Construct a regular hexagon with side 6 cm.

(ii) 90° angle

To draw a 90° angle at a point X on a line, first draw two arcs centred at X to cut the line at A and B.



Increase the radius of your compasses. Draw arcs centred at A and B to intersect each other at P and Q. Join PQ.



Angle AXP = 90°

Exercise 6M

1 Construct an angle of 90°.

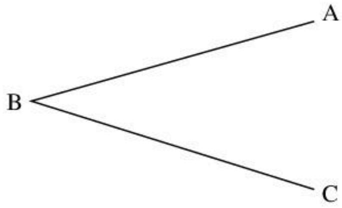
2 Construct a square with side 7 cm.

3 Construct the rectangle ABCD with side AB = 10 cm and side BC = 6 cm.

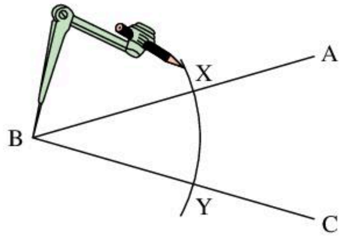
4 Construct the triangle PQR with $P\hat{Q}R = 60°$, $P\hat{R}Q = 90°$ and PR = 8 cm.

(iii) Bisecting an angle

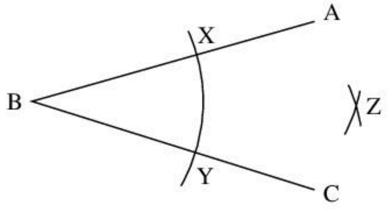
You can bisect the angle ABC with a pair of compasses and a ruler.



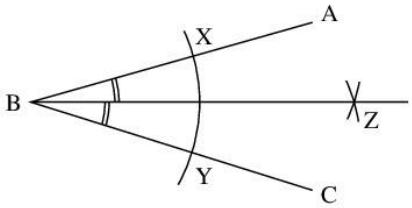
Draw an arc with centre B to cut AB at X and BC at Y.



Draw two more arcs, with the same radius, centred at X and Y. Label the point where they meet Z.



Join BZ. This line bisects $A\hat{B}C$.



Exercise 6N

1 Construct an angle of 60°. Bisect it to get angles of 30°.

2 Construct an angle of 45° by first constructing a 90° angle and then bisecting it.

3 Construct angles of

(a) 15° **(b)** $22\frac{1}{2}°$

(c) 120° **(d)** 150°

4 **(a)** Construct the triangle ABC with AB = 9 cm, $C\hat{A}B = 60°$ and $A\hat{B}C = 30°$.

(b) Measure the angle $A\hat{C}B$.

5 Write detailed instructions on how to construct an angle of 75°.

6 Construct an angle of 135°.

7 Construct triangle XYZ, where XY = 7.5 cm, $Z\hat{X}Y = 75°$ and $X\hat{Y}Z = 22\frac{1}{2}°$.

Technology

Need more practice? Look at the construction videos at

www.onlinemathlearning.com/angle-construction.html

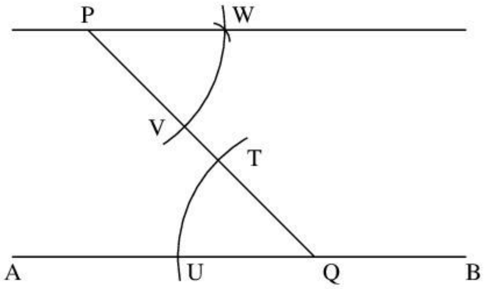
or view

http://www.mathopenref.com/tocs/constructionstoc.html

Videos are often the easiest way to learn constructions!

Parallel lines

You can construct a straight line, through a point P, parallel to another straight line, AB.



[1] Join P to any point Q on AB.

[2] With Q as the centre and using any radius, draw an arc cutting PQ and AB. Label the point T and U.

[3] With centre P and the *same* radius, draw an arc cutting PQ. Label the point V. With centre V and radius equal to the distance TU, draw another arc. Label the point where the two arcs meet W.

[4] Draw a line through P and W. This line is parallel to AB.

Exercise 6O

1 Draw two parallel lines using your ruler and compasses.

2 **(a)** Draw the line AB = 7 cm.
  (b) Construct the angle $B\hat{A}C = 60°$.
  (c) Mark the point C so that AC = 6 cm.
  (d) Draw a line through C parallel to AB.
  (e) Mark the point X on this parallel line so that CX = 7 cm.
  (f) Join the points B and X.
  (g) What is the shape ABXC?

3 Use the method in Question 2 to construct a parallelogram WXYZ with WX = 9 cm, $W\hat{X}Y = 45°$ and XY = 7 cm.

4 Construct a rhombus ABCD with side AB = 8 cm and $A\hat{B}C = 30°$.

Constructing triangles

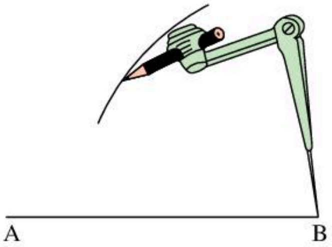
The construction of a triangle given the lengths of its sides is particularly simple.

Example 7

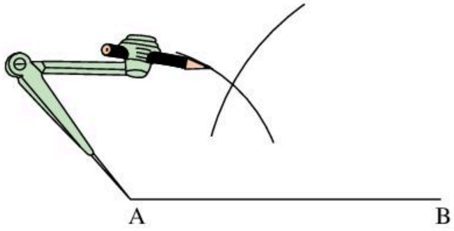
Construct a triangle ABC with AB = 8 cm, BC = 6 cm and AC = 4 cm, using a ruler and compasses only.

(a) Draw AB.

(b) With your compasses, draw an arc of radius 6 cm centred at B.

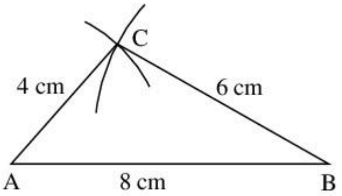


(c) Draw an arc of radius 4 cm centred at A.



(d) Label the point where the two arcs intersect as C.

(e) Join AC and BC.

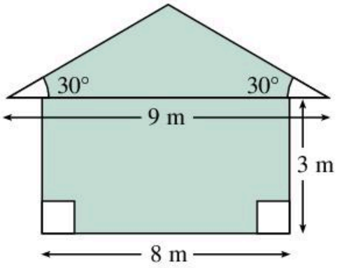


Exercise 6P

1 Construct triangles with sides:
  (a) 3 cm, 4 cm and 5 cm
  (b) 6 cm, 8 cm and 10 cm
  (c) 7 cm, 6 cm and 9 cm
  (d) 6.3 cm, 4.2 cm and 8.7 cm
  (e) 11.3 cm, 7.9 cm and 6.3 cm.

2 Draw the triangle ABC such that:
  (a) AB = 8 cm, $C\hat{A}B = 60°$ and AC = 7 cm
  (b) AB = 9 cm, $C\hat{A}B = 45°$ and AC = 6.8 cm
  (c) AB = 4.8 cm, $C\hat{A}B = 30°$ and AC = 7.3 cm.

3



The diagram shows the side view of a house.

Using a scale of 1 cm to represent 1 m, draw an accurate drawing of the side view.

Technology

Get a complete review of all these geometric constructions and more by visiting

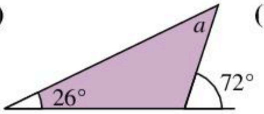
www.onlinemathlearning.com

Click on the constructions you are interested in and watch the video demonstrations. It's a great way to learn!

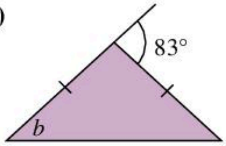
Exercise 6Q – mixed questions

1 Calculate the lettered angles.

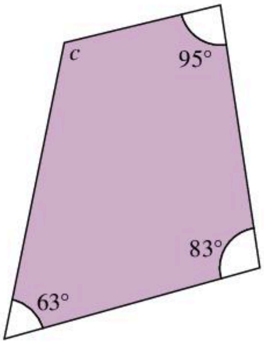
(a)



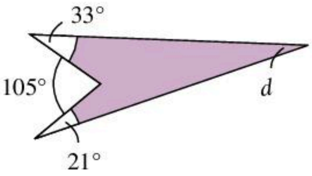
(b)



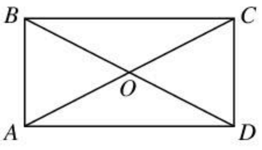
(c)



(d)



2



ABCD is a rectangle with diagonals AC and BD meeting at O.

(a) Explain why $C\hat{A}D = B\hat{C}A$

(b) Is triangle ABC congruent to triangle CDA? Give reasons.

(c) Identify three other pairs of triangles that are congruent to one another. Say why they are congruent to each other.

3 Draw a triangle with:

(a) one line of symmetry

(b) three lines of symmetry.

4 Construct with ruler and compasses only:

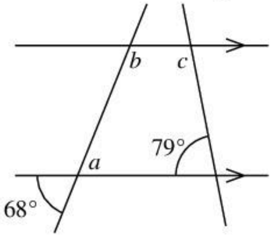
(a) a square with side 6 cm

(b) a rectangle with sides 6 cm and 8 cm

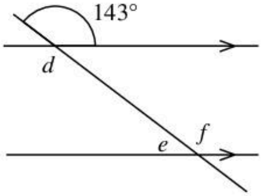
(c) a rhombus with side 8 cm and interior angles 60° and 120°.

5 Calculate the lettered angles.

(a)



(b)



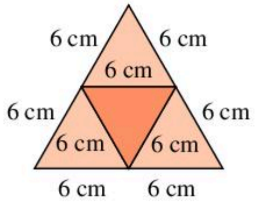
6 Using ruler and compasses only, construct the triangle XYZ with:

(a) XY = 7 cm, YZ = 8 cm, XZ = 12 cm

(b) XY = 8 cm, $Z\hat{X}Y = 60^\circ$, $X\hat{Y}Z = 90^\circ$

(c) XY = 8.5 cm, $Z\hat{X}Y = 30^\circ$, $X\hat{Y}Z = 30^\circ$

7



(a) Construct the four equilateral triangles with side 6 cm as shown above.

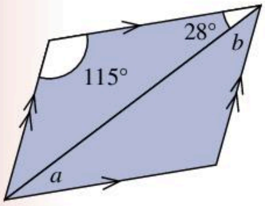
(b) Cut out the large triangle and fold along the inside edges. What three-dimensional shape have you made?

6 Consolidation

Example 1

Find the sizes of the lettered angles.

(a)



$a = 28°$ (alternate angles)

$b + 28° + 115° = 180°$ (supplementary angles)

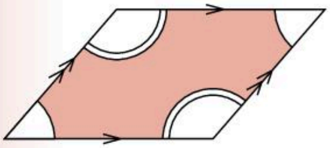
Hence $b = 37°$

Example 2

Write down four properties of:

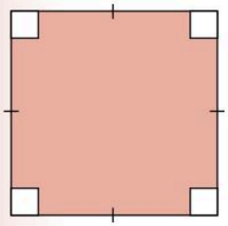
- **(a)** parallelogram
- **(b)** square.

(a) Parallelogram



Two pairs of parallel sides
Two pairs of equal sides
Two pairs of equal angles
Diagonals bisect each other

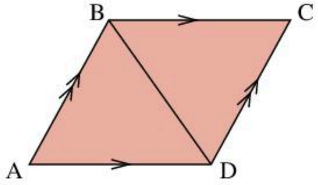
(b) Square



Two pairs of parallel sides
Four sides equal in length
Four right angles
Four axes of symmetry

Example 3

ABCD is a parallelogram with diagonal BD. Identify with reasons one pair of congruent triangles.



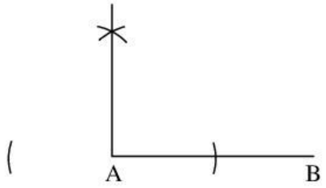
Angle ADB = angle DBC (alternate angles)
AD = BC and BD is common to both triangles.
Hence, △ABD is congruent to △CDB – side, angle, side (SAS).

Example 4

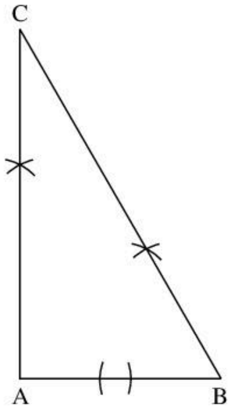
Construct the triangle ABC with $A\hat{B}C = 90°$, $C\hat{A}B = 60°$ and AB = 7 cm.

First draw the line AB.

Then construct a perpendicular at A.



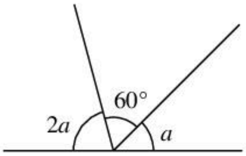
Construct 60° angle at B and join line to meet perpendicular of A at C.



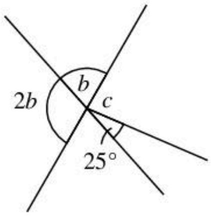
Exercise 6

1 Find the lettered angles.

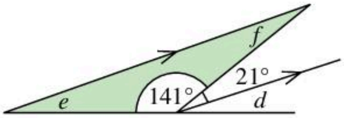
(a)



(b)

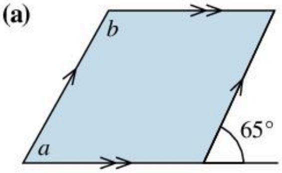


(c)

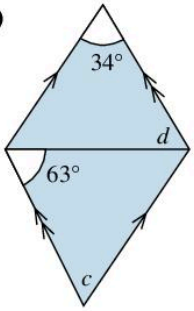


2 Calculate the sizes of the lettered angles.

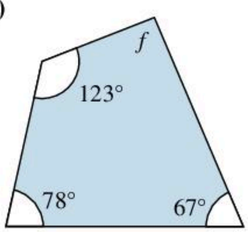
(a)



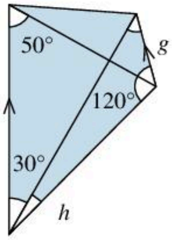
(b)



(c)



(d)



3 Copy and complete the tables.

(a)

Triangle	Lines of symmetry	Number of equal sides
Equilateral		
Isosceles		
Right-angled		

(b)

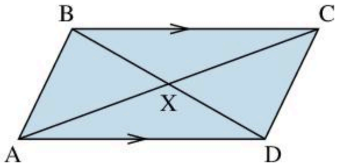
Quadrilateral	Lines of symmetry	Number of right angles	Number of parallel sides
Square			
Parallelogram			
Kite			
Trapezium			

4 Using a ruler and compasses only, construct:

(a) triangle ABC with $A\hat{B}C = 30^\circ$, $C\hat{A}B = 60^\circ$ and AB = 8 cm

(b) parallelogram ABCD with $D\hat{A}B = 45^\circ$, AB = 7 cm and AC = 6 cm.

5



ABCD is a parallelogram.
The diagonals AC and BD meet at X.

(a) Show that triangle ABC is similar to triangle AXD.

(b) Is triangle ABX congruent to triangle CDX? Give reasons.

Application

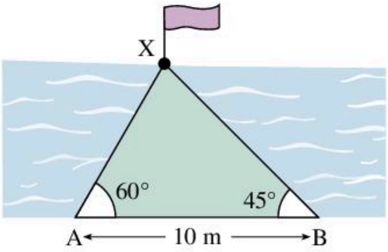
6 A big wheel in a fairground is in the shape of a regular decagon whose 'spokes' form ten interior triangles.

(a) Draw one of these interior triangles.

(b) What type of triangle is it?

(c) Determine the interior angles of the triangle.

7



A surveyor stands at point A on a riverbank and measures the angle between the riverbank and a flag on the opposite bank at a point X to be 60°. When the surveyor walks 10 m downstream to point B the angle measured to X is 40°.

(a) Using a scale of 1 cm = 1 m and a ruler and compasses only, draw a scale drawing of triangle ABX.

(b) How wide is the river?

Support Website

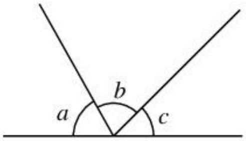
Additional material to support this topic can be found at www.oxfordsecondary.com/9780198425793

Summary

You should know ...

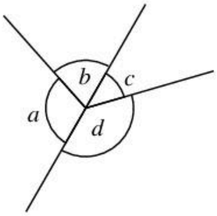
1 **(a)** Angles on a straight line add up to 180°.

$$a + b + c = 180°$$

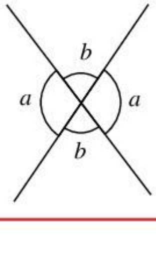


(b) Angles at a point add up to 360°.

$$a + b + c + d = 360°$$



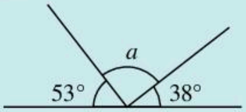
(c) Vertically opposite angles are equal.



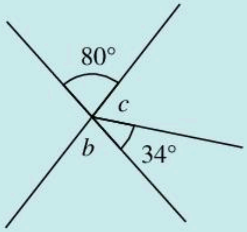
Check out

1 Calculate the size of the lettered angles.

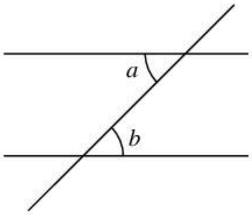
(a)



(b)

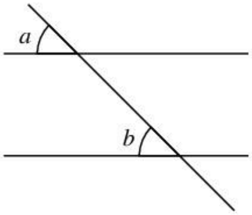


2 **(a)** Alternate and corresponding angles are equal.



a and *b* are alternate angles so $a = b$

(b)



a and *b* are corresponding angles so $a = b$

3 How to classify shapes in terms of:

- number of sides
- number of lines of symmetry
- number of right angles
- number of equal sides
- number of parallel sides.

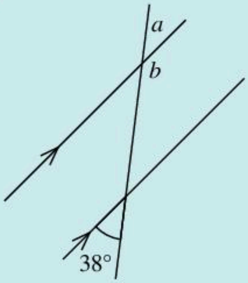
4 What a

(a) similar triangle is

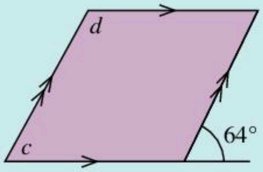
(b) congruent triangle is.

2 Find the lettered angles.

(a)



(b)



3 Write down three properties of:

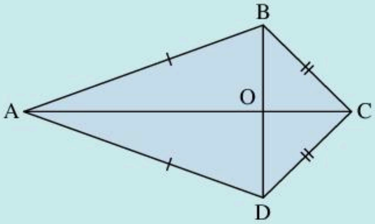
(a) isosceles triangle

(b) rectangle

(c) kite

(d) parallelogram.

4



ABCD is a kite with AB = AD and BC = DC. The diagonals AC and BD meet at O.

Identify with reasons

(a) one pair of similar triangles

(b) one pair of congruent triangles.

5 How to use a ruler and compasses to:
- **(a)** bisect a straight line
- **(b)** construct angles of 90° and 60°
- **(c)** bisect an angle
- **(d)** construct a triangle.

5 **(a)** Construct triangle ABC with AB = 7 cm, $C\hat{A}B = 60°$ and $A\hat{B}C = 90°$.

(b) Construct a parallelogram ABCD with AB = 8 cm, AD = 5 cm and $B\hat{A}D = 30°$.

Trigonometry

7

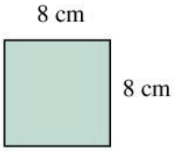
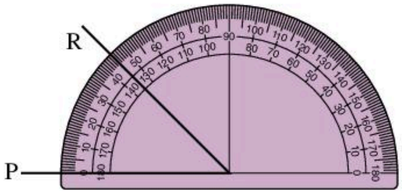
Objectives

- ✔ use Pythagoras' theorem to solve problems involving right-angled triangles
- ✔ use trigonometrical ratios to solve problems involving right-angled triangles
- ✔ use trigonometrical ratios to solve problems involving angles of elevation and depression

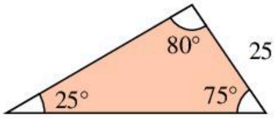
What's the point?

The study of trigonometry allows you to answer questions such as: How wide is this river? How tall is this mountain?

Before you start

You should know ...	Check in
1 The area of a square is given by the formula Area = length × length 	**1** **(a)** What is the area of this square? **(b)** Find the area of a square with sides x cm.
2 How to use a protractor to measure and draw angles. *For example*:  $P\hat{Q}R = 45°$	**2** Draw these angles with a protractor. **(a)** 30° **(b)** 70° **(c)** 120° **(d)** 150°

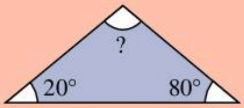
3 The angle sum in a triangle is 180°.
For example:



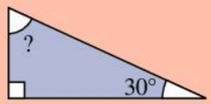
25° + 80° + 75° = 180°

3 Work out the missing angles.

(a)



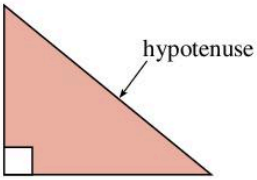
(b)



7.1 Right-angled triangles

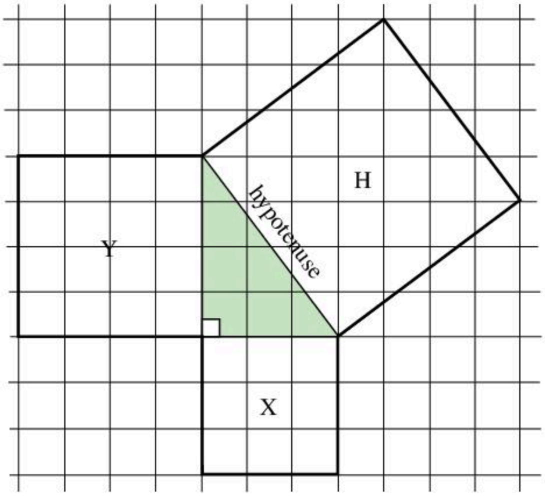
You will need a set square and centimetre squared paper.

The longest side of a right-angled triangle is called the **hypotenuse**.



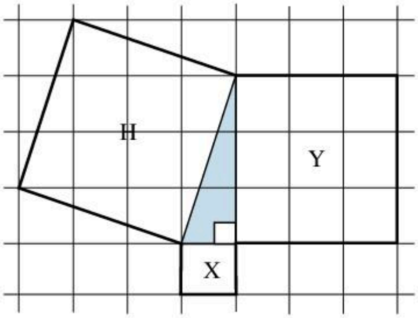
Exercise 7A

1 This shape has been drawn on squared paper.



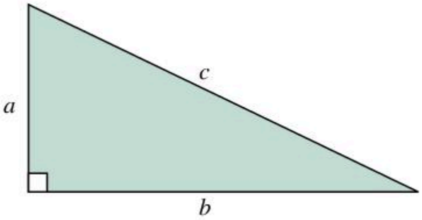
(a) Find the areas of the squares X and Y.
(b) Find the area of the square H.
(c) Can you find a relationship between the areas of squares X and Y and the square H?

2 Repeat Question 1 for this shape.



3 **(a)** Draw a right-angled triangle on centimetre squared paper. Make the two shorter sides an exact number of centimetres.
(b) Draw a square on each side. Draw the square on the hypotenuse very carefully, using a set square.
(c) Find the area of each square.
(d) What is the relationship between the three areas?

4



Draw accurately right-angled triangles with the following lengths:

(a) $a = 12$ mm, $b = 16$ mm
(b) $a = 30$ mm, $b = 40$ mm
(c) $a = 5$ mm, $b = 12$ mm
(d) $a = 20$ mm, $b = 21$ mm
(e) $a = 12$ mm, $b = 35$ mm

5 Measure the hypotenuse, c of each of the triangles you drew in Question 4.

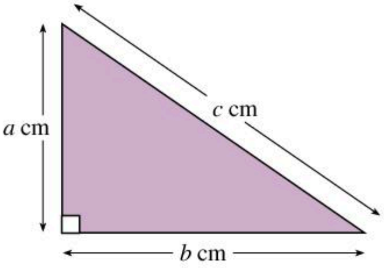
6 Use your results from Questions 4 and 5 to copy and complete the table:

	a	b	c	a^2	b^2	a^2+b^2	c^2
(a)	12	16		144	256	400	
(b)	30	40					
(c)	5	12			144		
(d)	20	21					
(e)	12	35					

What do you notice about the last two columns in the table?

7.2 Pythagoras' theorem

You will need a calculator.



In a right-angled triangle

$$c^2 = a^2 + b^2$$

This relationship was written down over 2000 years ago by a Greek mathematician called Pythagoras.

This result is known as Pythagoras' theorem.

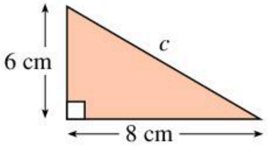
It is sometimes stated as:

- **The square on the hypotenuse of a right-angled triangle equals the sum of the squares on the other two sides.**

You can use Pythagoras' theorem to find the length of the third side of a right-angled triangle, if you know the other two sides.

Example 1

Find the length, c, of the hypotenuse of the triangle.



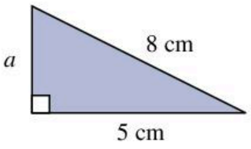
Using Pythagoras' theorem

$$\begin{aligned} & c^2 = a^2 + b^2 \\ \text{so} \quad & c^2 = 6^2 + 8^2 \\ & = 36 + 64 \\ & = 100 \\ \text{so} \quad & c = \sqrt{100} \\ & = 10 \text{ cm} \end{aligned}$$

The length of the hypotenuse is 10 cm.

Example 2

In the triangle, find the length a.

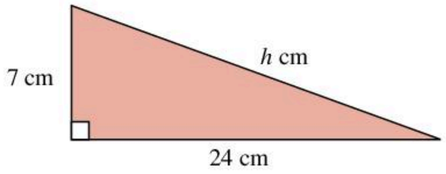


By Pythagoras' theorem:

$$\begin{aligned} & a^2 + 5^2 = 8^2 \\ & a^2 + 25 = 64 \\ (-25) \quad & a^2 = 39 \\ & a = \sqrt{39} \\ \text{so} \quad & a = 6.24 \text{ cm} \end{aligned}$$

Exercise 7B

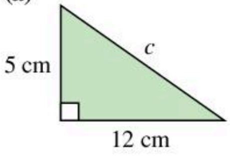
1 Look at the triangle, then copy and complete the statements which follow.



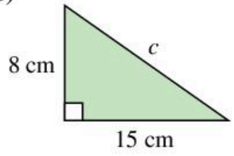
$$\begin{aligned} & h^2 = \square^2 + 24^2 \\ & h^2 = \square + \square \\ & h^2 = \square \\ \text{so} \quad & h = \sqrt{\square} = \square \end{aligned}$$

2 Find the length of c in these right-angled triangles.

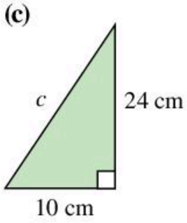
(a)



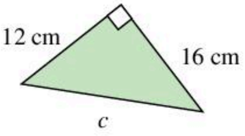
(b)



(c)

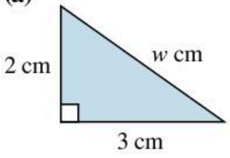


(d)

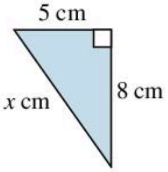


3 Use your calculator to find the unknown length represented by the letter.

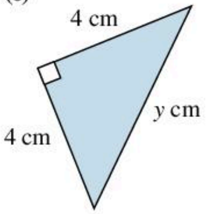
(a)



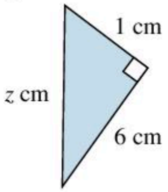
(b)



(c)



(d)



4 Copy and complete the statements for this triangle.

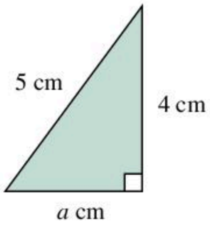
$a^2 + \square^2 = 5^2$

$a^2 + \square = \square$

$a^2 = \square$

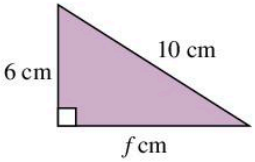
$a = \sqrt{\square}$

so $a = \square$

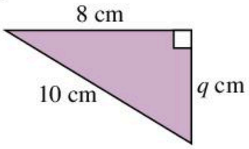


5 For each triangle, find the unknown length represented by the letter.

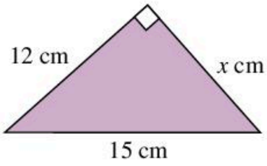
(a)



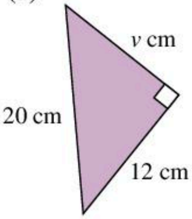
(b)



(c)

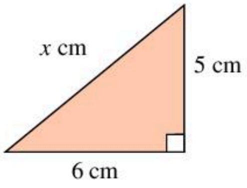


(d)

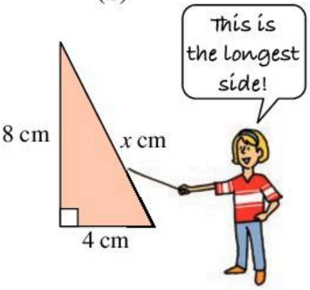


6 Find the value of x.

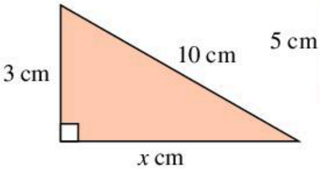
(a)



(b)



(c)



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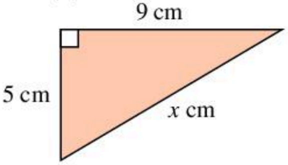

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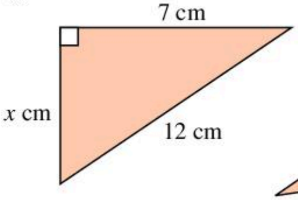

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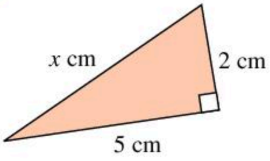

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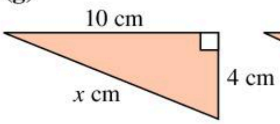

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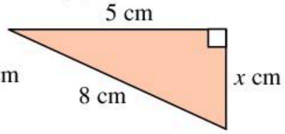
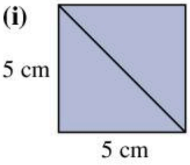
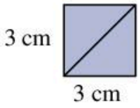


7 **(a)** Calculate the length of the diagonal in each square.

(i)

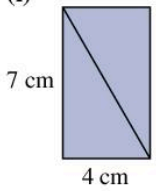


(ii)

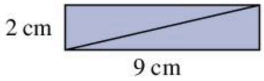


(b) Calculate the length of the diagonal in each rectangle.

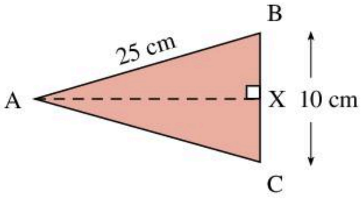
(i)



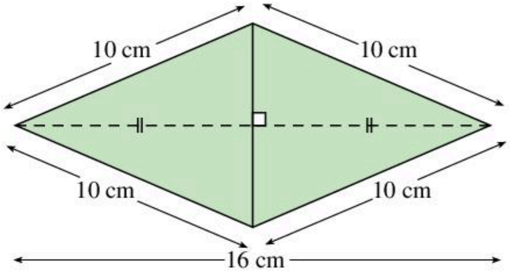
(ii)



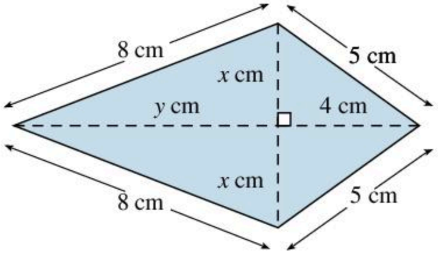
8 Triangle ABC is isosceles, with AB = AC = 25 cm and BC = 10 cm
Calculate its height AX.



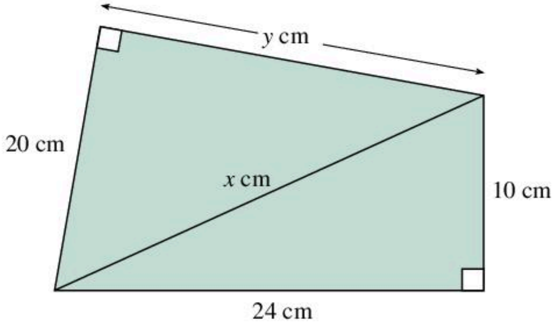
9 The diagonals of a rhombus intersect at right angles. Find the length of the second diagonal in the rhombus shown.



10 The diagonals of a kite intersect at right angles. Find the length of each diagonal using the measurements given below.

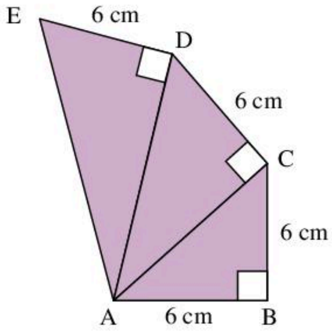


11 From the diagram find the values of x and y.



12 PQRS is a kite and QS is its line of symmetry. Angle P = angle R = 90°, QR = 36 cm and RS = 16 cm. Find the length QS.

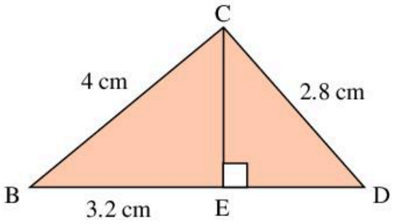
13 **(a)** Draw accurately the diagram, starting with triangle ABC, then adding ACD and ADE.



(b) Measure AC, AD and AE.

(c) Calculate AC, AD and AE and check the accuracy of your drawing.

14 In the diagram $B\hat{E}C = 90°$, BC = 4 cm, BE = 3.2 cm, CD = 2.8 cm.



Find: **(a)** CE **(b)** BD

Is $B\hat{C}D$ a right angle?
Give a reason for your answer.

Investigation

The whole numbers 3, 4, and 5 form a Pythagorean triple because:

$$3^2 + 4^2 = 5^2$$

{5, 12, 13} is another Pythagorean triple since

$$5^2 + 12^2 = 13^2$$

What other Pythagorean triples can you find?

Technology

- Review what you have learnt by visiting the website

  www.mathsisfun.com

  Have a look at the two interactive proofs of Pythagoras' theorem.
- How many, different, ways can Pythagoras' theorem be proved? Study the webpage

  www.cut-the-knot.org/pythagoras

  It contains 81 different proofs! Select your favourite three and share them with your class.

7.3 Naming sides

You use Pythagoras' theorem to find unknown lengths in right-angled triangles.

To find unknown angles in right-angled triangles, you need to use **trigonometry**.

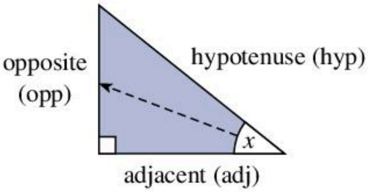
Trigonometry relates the sides and angles of triangles.

There are three trigonometric ratios:

sine, cosine, tangent

First you need to be able to name the sides of a triangle.

You name the sides of a right-angled triangle in relation to the angle of interest:



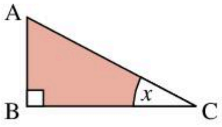
The hypotenuse is the longest side, the opposite is the side opposite the angle, x, and the adjacent is the side next to the angle, x.

Exercise 7C

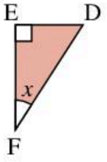
You will need a set square and a protractor.

1 Identify the hypotenuse, opposite and adjacent sides in relation to angle x.

(a)



(b)



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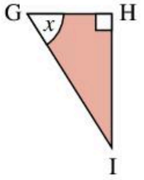

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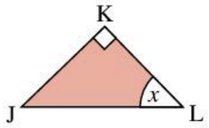
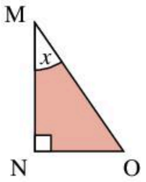
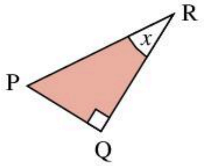


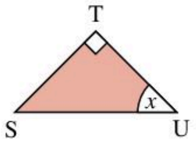
(e)



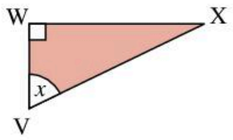
(f)



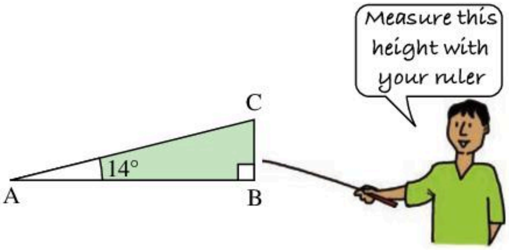
(g)



(h)



2



(a) Draw accurately six triangles ABC with $\hat{A} = 14°$, $\hat{B} = 90°$ and base length:

(i) AB = 1 cm **(ii)** AB = 2 cm

(iii) AB = 4 cm **(iv)** AB = 5 cm

(v) AB = 8 cm **(vi)** AB = 10 cm

(b) In each case measure accurately the vertical height, BC.

(c) Calculate the ratio $\frac{BC}{AB} = \frac{\text{opposite}}{\text{adjacent}}$.

(d) Copy and complete the table.

	Angle	**AB (adj)**	**BC (opp)**	$\frac{BC}{AB}\left(=\frac{opp}{adj}\right)$
(i)	14°	1 cm		
(ii)	14°	2 cm		
(iii)	14°	4 cm	1 cm	1 ÷ 4 = 0.25
(iv)	14°	5 cm		
(v)	14°	8 cm		
(vi)	14°	10 cm		

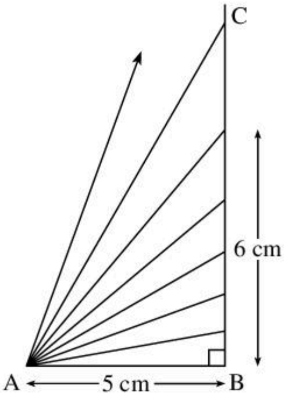
3 Did you find that each ratio in Question 2 was 0.25? This means that the ratio opp : adj is always 0.25 : 1, for an angle of 14°. Do you agree?

4 Repeat Question 2 but make the angle 45° instead of 14°. Make a new table.

What do you notice about the ratios?

5 **(a)** Draw a line AB 5 cm long, at the bottom of a sheet of paper. Draw BC about 15 cm long, at 90° to AB.

(b) Mark angles at A at 10° intervals, as shown, up to 70°.



(c) The 50° angle cuts off a height of 6 cm on BC, as shown on the diagram above. Measure the height cut off on BC by each of the other angles. Write them in the table.

Angle	**adj**	**opp**	**Ratio** $\left(\frac{opp}{adj}\right)$
10°	5 cm		
20°	5 cm		
30°	5 cm		
40°	5 cm		
50°	5 cm	6 cm	1.2
60°	5 cm		
70°	5 cm		

(d) Fill in the last column of the table.

6 **(a)** Repeat Question 5, but this time make AB 8 cm, and BC about 24 cm.

(b) Compare the last column in the two tables. Does the ratio for a given angle stay the same?

7 Suppose AB is 10 cm, in Question 5. Without drawing the diagram, write down the height when the angle is:

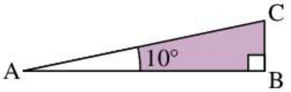
(a) 20° **(b)** 30° **(c)** 50° **(d)** 70°

7.4 The tangent of an angle

In the last section you should have found:

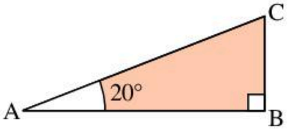
For a given angle the $\frac{\text{opp}}{\text{adj}}$ ratio is always the same.

For example, in Question 5 of Exercise 7C, you should have found:



$$\frac{\text{opp}}{\text{adj}} = \frac{\text{BC}}{\text{AB}} = 0.18$$

For any angle of 10°, the ratio $\frac{\text{opp}}{\text{adj}} = 0.18$



$$\frac{\text{opp}}{\text{adj}} = \frac{\text{BC}}{\text{AB}} = 0.36$$

For any angle of 20°, the ratio $\frac{\text{opp}}{\text{adj}} = 0.36$

The $\frac{\text{opp}}{\text{adj}}$ ratio associated with an angle is called the **tangent** of that angle.

- **tangent** $= \frac{\textbf{opp}}{\textbf{adj}}$

You can write **tan** instead of 'tangent of':

tangent of 10° = tan 10° = 0.18
tangent of 20° = tan 20° = 0.36

If you have a scientific calculator you can easily find the tangent of an angle. For example, to find tan 20° press:

$\boxed{2}\ \boxed{0}\ \boxed{\tan}$ and $\boxed{0.36397023}$

or $\boxed{\tan}\boxed{(}\boxed{2}\boxed{0}\boxed{)}\boxed{=}$ (depending on your calculator) will appear on the screen.
That is, tan 20° = 0.36397023

You can write
tan 20° = 0.364 (3 sig.fig.)

Exercise 7D

You will need cm/mm graph paper, a set square, protractor and a calculator.

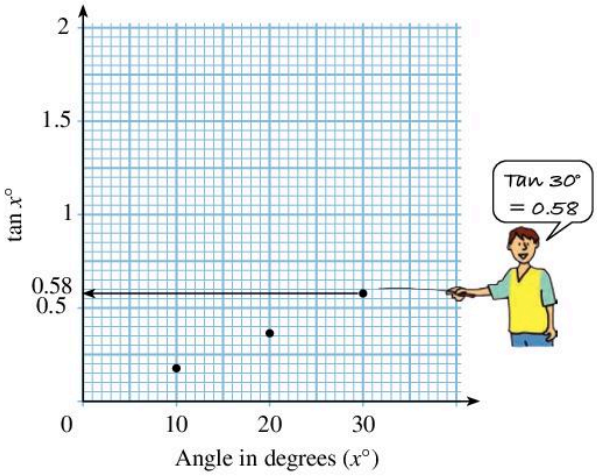
1 Use your calculator to find:

(a) tan 10° **(b)** tan 30° **(c)** tan 50°
(d) tan 45° **(e)** tan 58° **(f)** tan 16°
(g) tan 70° **(h)** tan 84° **(i)** tan 41.6°

2 Copy and complete the table.

x°	10	20	30	40	50	60	70	80
tan x°			0.58			1.73		

3 **(a)** Using the table in Question 2, copy and complete the graph of tan x°.



(b) Join the points on your graph with a smooth curve.

4 Using your graph for Question 3, find the value of

(a) tan 15° **(b)** tan 25° **(c)** tan 35°
(d) tan 49° **(e)** tan 37° **(f)** tan 64°
(g) tan 62° **(h)** tan 74° **(i)** tan 76°

5 From your graph for Question 3 find the angle whose tangent is

(a) 0.4 **(b)** 0.6 **(c)** 0.8
(d) 1 **(e)** 1.5 **(f)** 2
(g) 1.3 **(h)** 2.4 **(i)** 1.75

6 Using your calculator:

(a) What is the difference between the tangents of angles less than 45° and angles greater than 45°?
(b) What happens to the tangent as the angle gets smaller and smaller?
(c) What happens to the tangent as the angle gets near to 90°?

7 Do your answers for Question 6 (b) and (c) agree with the shape of your graph for Question 3?

7.5 Working with tangents

You can use your calculator to find an angle when you know its tangent. To do this you must use the inverse function button $\boxed{\text{INV}}$.

Note: On some calculators the inverse button is marked

$\boxed{\text{2nd}}$ or $\boxed{\text{SHIFT}}$

WARNING!

Not all calculators are the same!

Check the instructions for use on trig functions!

Example 3

Find x, when $\tan x = 0.745$

To find x, you must find the inverse, that is

$$x = \tan^{-1}(0.745)$$

On your calculator press:

$\boxed{\cdot}$ $\boxed{7}$ $\boxed{4}$ $\boxed{5}$ $\boxed{\text{INV}}$ $\boxed{\tan}$

or $\boxed{\text{2nd}}$ $\boxed{\tan}$ $\boxed{\cdot}$ $\boxed{7}$ $\boxed{4}$ $\boxed{5}$

or $\boxed{\tan^{-1}(}$ $\boxed{.}$ $\boxed{7}$ $\boxed{4}$ $\boxed{5}$ $\boxed{)}$ $\boxed{=}$

$\boxed{36.6861107}$ should appear on the screen, so

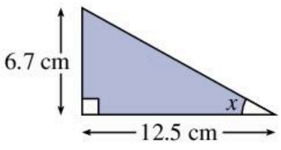
$x = 36.6861107°$

That is, $x = 36.7°$ (3 sig.fig.)

Given the opposite and adjacent sides of a right-angled triangle you can now calculate the angle.

Example 4

Calculate the angle x in the diagram



$$\tan x = \frac{\text{opp}}{\text{adj}} = \frac{6.7}{12.5}$$

so $\tan x = 0.536$

Use the inverse tan function on your calculator to get

$x = 28.2°$

Exercise 7E

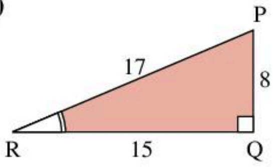
Give your answers to 3 significant figures where appropriate.

1 Use your calculator to find the value of x (to 3 sig fig) when $\tan x$ is

(a) 0.649 **(b)** 0.781 **(c)** 0.933
(d) 1.11 **(e)** 1.38 **(f)** 2.9
(g) 0.400 **(h)** 0.888 **(i)** 4.59

2 Write down the tangent of the marked angle as a fraction.

(a)



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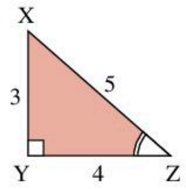

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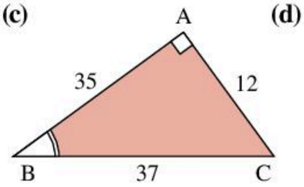

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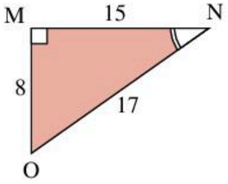
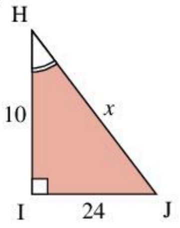


(e)



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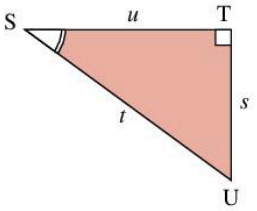
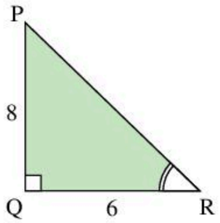
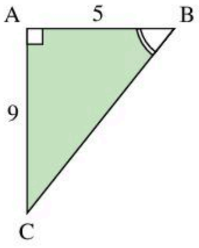


3 Find the marked angle in each diagram.

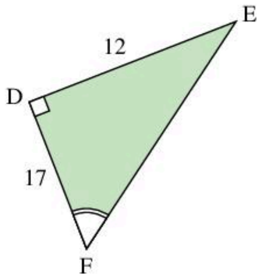
(a)



(b)



(c)



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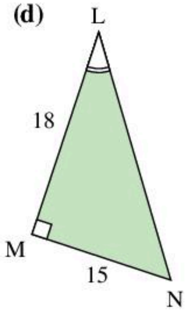

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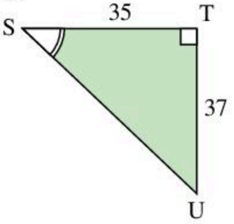

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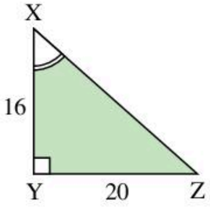
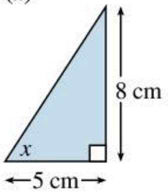
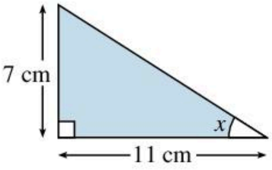


4 Find the tangent of x. Then use your calculator to find the angle x:

(a)



(b)



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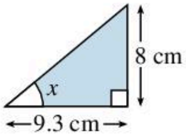

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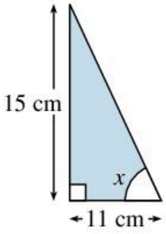
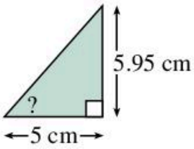
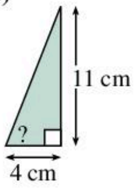


5 Calculate the angle marked in each triangle:

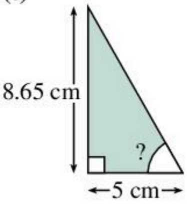
(a)



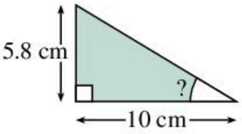
(b)



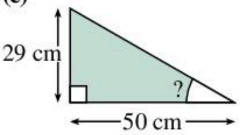
(c)



(d)

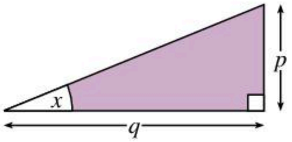


(e)



6 Copy and complete this statement for the diagram:

$\tan x = \frac{\square}{\square}$



Use the statement to find tan x and x when

(a) $p = 8$ cm and $q = 8$ cm

(b) $p = 7$ cm and $q = 10$ cm

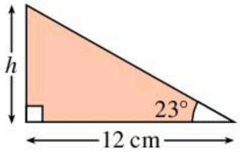
(c) $p = 2$ cm and $q = 5$ cm

(d) $p = 16$ cm and $q = 8$ cm.

If you are given an angle and the adjacent side of a right-angled triangle you can find the opposite side.

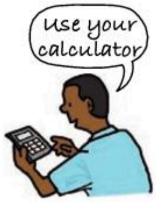
Example 5

Find the height, h in the figure.



$$\frac{\text{opposite}}{\text{adjacent}} = \tan 23°$$

$$\frac{h}{12} = 0.424$$



Now solve the equation. You have to multiply both sides by 12.

$h = 0.424 \times 12$

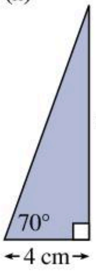
$h = 5.088$ cm

Exercise 7F

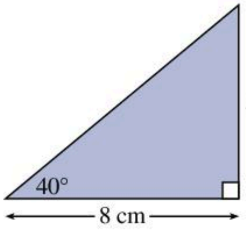
Give your answers to three significant figures where appropriate.

1 Use the method of Example 5 to find the length, h, in these triangles.

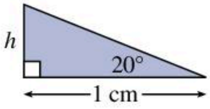
(a)



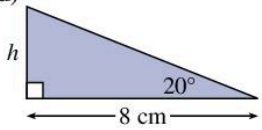
(b)



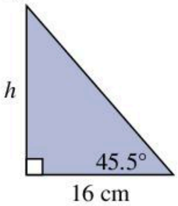
(c)



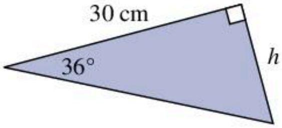
(d)



(e)

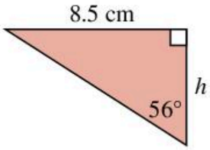


(f)

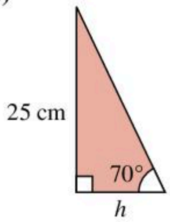


2 Find *h*:

(a)



(b)



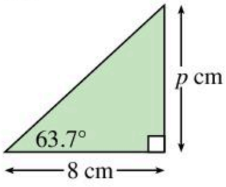
3 In triangle ABC, $\hat{C} = 90°$, $\hat{B} = 10°$ and BC = 24 cm. Find AC.

4 In triangle XYZ, $\hat{Z} = 90°$, $\hat{Y} = 58°$ and ZY = 32 cm. Find XZ.

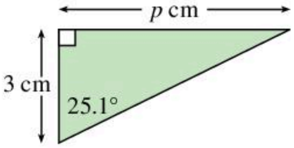
5 In triangle PQR, $\hat{Q} = 64.5°$, $\hat{P} = 90°$ and PR = 28 m. Find PQ.

6 Find *p* in each of these triangles:

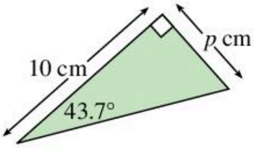
(a)



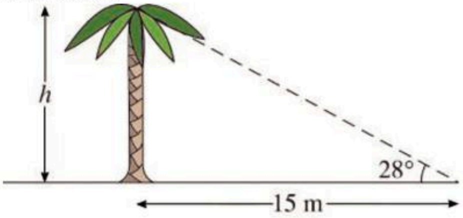
(b)



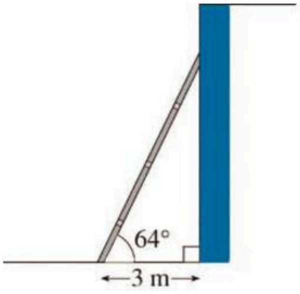
(c)



7 The top of a tree makes an angle of 28° with a point 15 m from its base. Calculate the height *h* of the tree.



8 A ladder resting against a wall makes an angle of 64° with the ground. If the end of the ladder is 3 m from the foot of the wall find how far the ladder reaches up the wall.

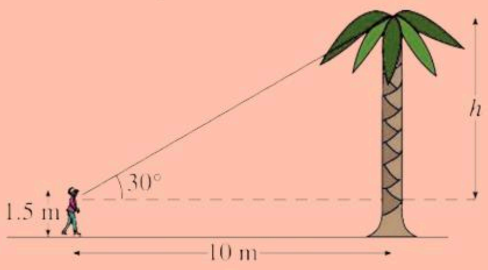


9 A man sitting at a window with his eyes 20 m above the ground can just see the sun over the top of a roof 45 m high which is 30 m from him horizontally. Find the elevation of the sun.

Activity

Making and using a clinometer
You will need a piece of card, a protractor, thread, a weight, scissors and glue.

You can now calculate the heights of buildings and trees around your school without having to climb them!
Look at the diagram below.



In the diagram a boy is standing 10 m from the foot of a tree. The angle made by the boy's line of sight to the tree top from the horizontal is 30°.

So $\frac{h}{10} = \tan 30°$

$\frac{h}{10} = 0.577$

$h = 0.577 \times 10 = 5.77$ m

Since the boy is 1.5 m tall,

height of the tree $= 5.77 + 1.50$

$= 7.27$ m

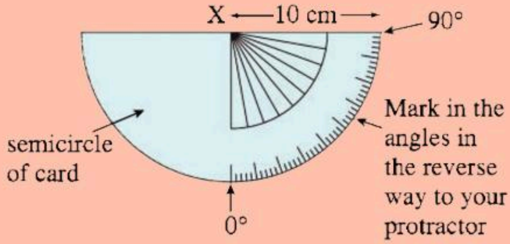
For you to do the same sort of calculation you need to be able to measure the angle made by the line of sight and the horizontal. To do this you will have to make an 'angle finder' or clinometer.

Making a clinometer

Step 1 Draw a semicircle, radius 10 cm, on your card.

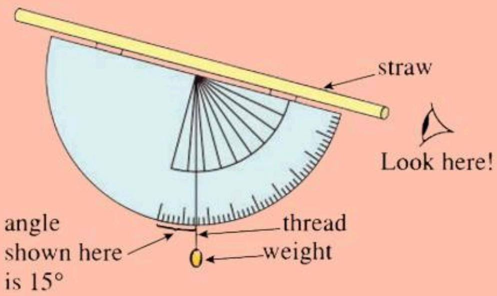
Step 2 Cut out your semicircle.

Step 3 With your protractor, mark carefully angles from 90° to 0° on your semicircle as shown below.



Step 4 Join one end of your thread to a weight and attach the other end of the thread to the upper centre of your card at X.

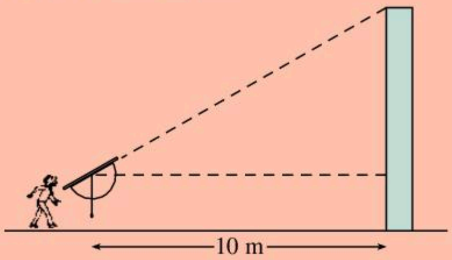
Step 5 Attach a straw as a sight along the top edge of the card.



Research other designs for a clinometer on the internet. Which one do you prefer?

Using a clinometer

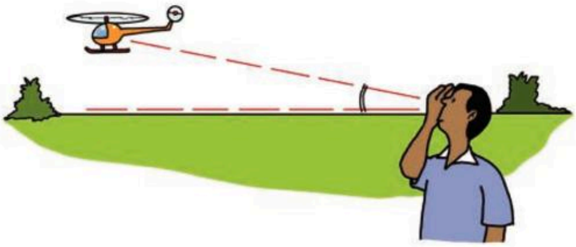
To use your clinometer measure 10 m from the foot of the building whose height you want to find. Line up the straw sight with the top of the building and read off the angle indicated by the thread.



Use your 'angle finder' to help you to find the heights of six buildings or trees near your school. (Do not forget to include your height in the answer!)

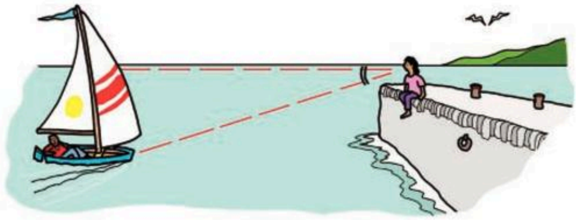
Angles of elevation and depression

When you look up to view an object you **elevate** or raise your eyes.



The **angle of elevation** of an object is the angle between the horizontal and the observer's line of sight.

When you look down or **depress** your eyes the angle between your line of vision and the horizontal is called the **angle of depression**.

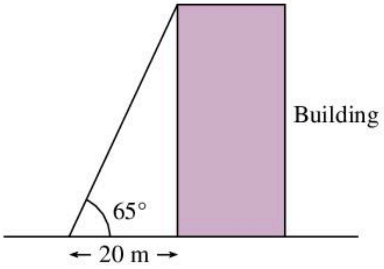


You can use the idea of angle of elevation or depression to find the heights of towers or buildings as you did in the previous activity.

Example 6

A building has an angle of elevation of 65° when measured from a point 20 m from its base. What is the height of the building?

First, draw a diagram:



Using the tangent ratio

$$\frac{\text{height}}{20} = \tan 65^\circ = 2.145$$

$$\text{Height} = 2.145 \times 20 \text{ m}$$

$$= 42.9 \text{ m}$$

Technology

Visit the website *www.onlinemathlearning.com/basic-trigonometry.html* and click on Angle of Elevation and Depression to learn more!

Exercise 7G

Give your answers to three significant figures where appropriate.

1 The angle of elevation of an electricity pole from a point 8 m from its base is 52.7°. What is the height of the pole?

2 A boat is 60 m from the foot of a cliff. A lookout at the top of the cliff finds the angle of depression from him to the boat to be 34°. What is the height of the cliff?

3 A tree is 16 m tall. How far is a student from the foot of the tree if the angle of elevation is:
(a) 45° **(b)** 25° **(c)** 10°?

4 A surveyor standing on a hill known to be 258 m in height measures the angle of depression to another hill, 164 m in height, to be 13°. What is the horizontal distance between the two hills?

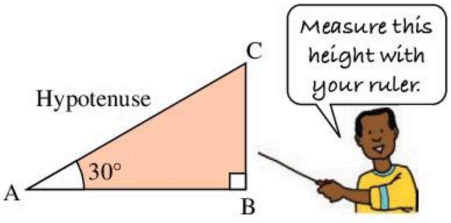
7.6 The sine of an angle

You will need graph paper, a set square, a protractor and a calculator.

Exercise 7H

1 **(a)** Draw accurately six triangles ABC with $\hat{A} = 30^\circ$, $\hat{B} = 90^\circ$ and hypotenuse:
  (i) AC = 1 cm **(ii)** AC = 2 cm
  (iii) AC = 4 cm **(iv)** AC = 5 cm
  (v) AC = 8 cm **(vi)** AC = 10 cm

(b) In each case measure accurately, with your ruler, the vertical height BC.



(c) Calculate the ratio $\frac{BC}{AC} = \frac{\text{opposite}}{\text{hypotenuse}}$ for each triangle.

(d) Copy and complete this table:

	Angle	AC (hypotenuse)	BC (opposite)	$\frac{BC}{AC} = \left(\frac{\text{opposite}}{\text{hypotenuse}}\right)$
(i)	30°	1 cm		
(ii)	30°	2 cm		
(iii)	30°	4 cm	2 cm	0.5
(iv)	30°	5 cm		
(v)	30°	8 cm		
(vi)	30°	10 cm		

(e) What do you notice about the last column?

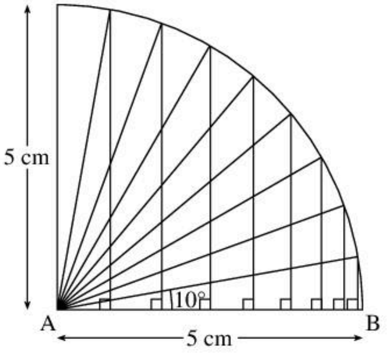
2 Repeat Question 1 for A = 60° instead of 30°.

3 **(a)** Draw a line AB 5 cm long in the middle of a sheet of paper.

(b) Use compasses to draw an arc of a circle with centre A and radius 5 cm.

(c) Mark angles at A at 10° intervals. Continue the lines until they meet the arc, as shown in the drawing. They touch the arc at a set of points.

(d) Through each of these points draw a line at right angles to AB, as shown.



(e) Now copy the table.

Angle	Radius	Height	Ratio $\left(\frac{\text{height}}{\text{radius}}\right)$
10°	5 cm		
20°	5 cm		
30°	5 cm	2.5 cm	0.5
40°	5 cm		
50°	5 cm		
60°	5 cm	4.3 cm	0.86
70°	5 cm		
80°	5 cm		

(f) Measure each of the vertical heights. Record your measurements in the table.

(g) Complete the last column of the table.

4 **(a)** Repeat Question 3, but use a circle of radius 8 cm.

(b) Compare the last column of your table with the one for Question 3. What do you notice?

5 If the radius in Question 3 was 10 cm, what would the height be for an angle of

(a) 30°

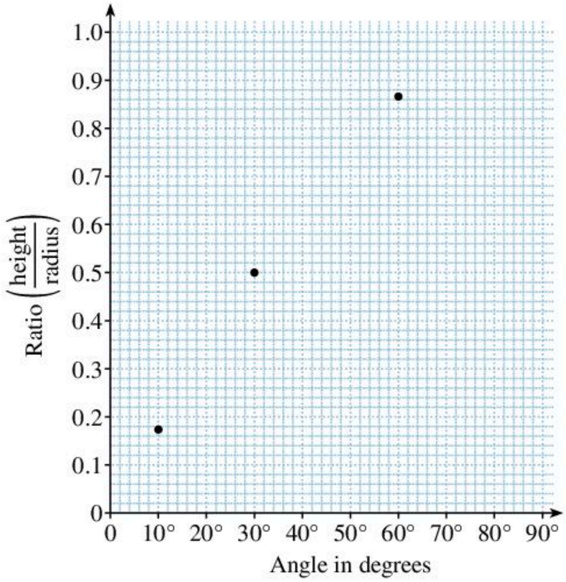
(b) 60°?

6 **(a)** Copy and complete, for the $\frac{\text{opp}}{\text{hyp}}$ ratio:

- **(i)** It is always . . . , for a given angle.
- **(ii)** It gets . . . as the angle gets closer to 90°.

(b) Are these properties the same as for the tangent ratio?

7 Copy and complete the graph, using your results from Question 3. Join the points with a smooth curve.



8 From your graph in Question 7 find the ratio which corresponds to an angle of:

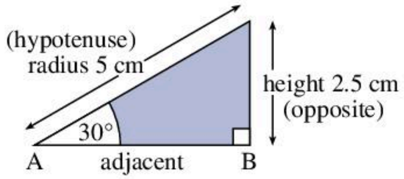
(a) 25° **(b)** 35° **(c)** 45° **(d)** 55°

9 From your graph in Question 7 find the angle which corresponds to the ratio:

(a) 0.3 **(b)** 0.4 **(c)** 0.6 **(d)** 0.7

The sine

Here is a triangle from the diagram for Question 3 of Exercise 7H.



The ratio $= \dfrac{\text{height}}{\text{radius}} = \dfrac{\text{opp}}{\text{hyp}}$

This ratio is called the **sine**.

- $\textbf{sine} = \dfrac{\textbf{opp}}{\textbf{hyp}}$

$$\begin{aligned}\text{The sine of } 30^\circ &= \frac{\text{opp}}{\text{hyp}}\\ &= \frac{2.5}{5}\\ &= 0.5\end{aligned}$$

'sine of' is often shortened to **sin**:

sine of $30^\circ = \sin 30^\circ = 0.5$
sine of $70^\circ = \sin 70^\circ = 0.94$

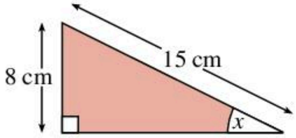
You can use your calculator to find the sine of any angle.

For example, to find sin 30° press

$\boxed{3}\ \boxed{0}\ \boxed{\sin}$

Given the opposite and hypotenuse of a right-angled triangle you can now find its angle.

Example 7



Find the angle x.

$$\begin{aligned}\sin x &= \frac{\text{opposite}}{\text{hypotenuse}}\\ &= \frac{8}{15}\\ &= 0.533\ldots\\ x &= \sin^{-1}(0.533)\end{aligned}$$

On your calculator press

$\boxed{\cdot}\ \boxed{5}\ \boxed{3}\ \boxed{3}\ \boxed{\text{INV}}\ \boxed{\sin}$

which gives $x = 32.2^\circ$

The sine of a given angle is always the same. As the angle increases, the sine increases. As the angle gets closer to 90° the sine gets closer to 1.

Exercise 7I

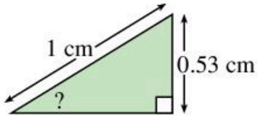
Give your answers to three significant figures where appropriate.

1 Use your calculator to find
(a) sin 32° **(b)** sin 45° **(c)** sin 76°
(d) sin 65° **(e)** sin 30° **(f)** sin 60°
(g) sin 14.5° **(h)** sin 23.6° **(i)** sin 65.5°
(j) sin 70.5° **(k)** sin 90° **(l)** sin 87.2°

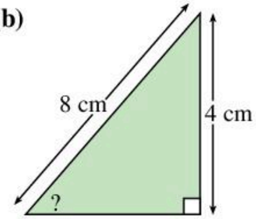
2 Use your calculator to find the angle whose sine is
(a) 0.559 **(b)** 0.809 **(c)** 0.891 **(d)** 0.900
(e) 0.331 **(f)** 0.639 **(g)** 0.866 **(h)** 0.800
(i) 0.500 **(j)** 0.707 **(k)** 0.445 **(l)** 0.254

3 Find the missing angle.

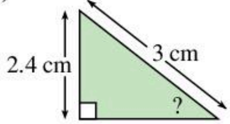
(a)



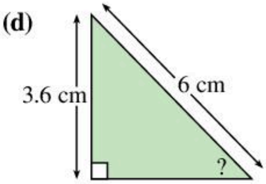
(b)



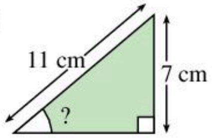
(c)



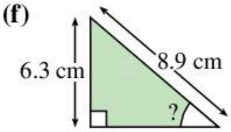
(d)



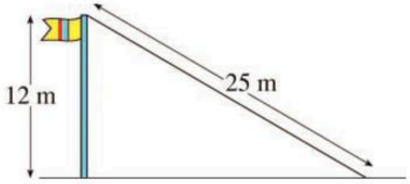
(e)



(f)



4 A flagpole, 12 m high is supported by a guy rope 25 m long. Find the angle the rope makes with the ground.



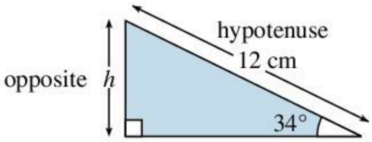
5 In triangle ABC, $\hat{C} = 90^\circ$, BC = 12 cm and AB = 15 cm. Find angle A.

6 A triangle PQR, has $\hat{Q} = 90°$, PQ = 60 cm and PR = 90 cm. Find angle R.

7 An aeroplane flies a distance of 700 m from its take off point climbing steadily. If it is now 300 m above the ground, what is its angle of climb?

If you are given an angle and the hypotenuse you can find the opposite side.

Example 8



Find the height h in the triangle above.

$$\frac{\text{opp}}{\text{hyp}} = \sin 34°$$

$$\frac{h}{12} = \sin 34°$$

Use your calculator:

$$\frac{h}{12} = 0.559$$

Solve the equation:

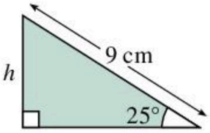
$$(\times 12) \qquad h = 0.559 \times 12$$
$$= 6.708 \text{ cm}$$

Exercise 7J

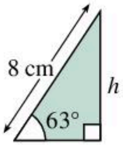
Give your answers to three significant figures where appropriate.

1 Use the method in Example 8 to find the height, h, in the triangles.

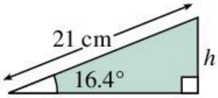
(a)



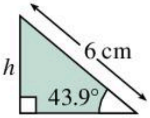
(b)



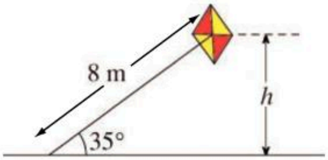
(c)



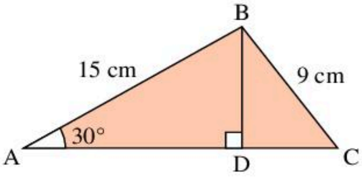
(d)



2 A kite makes an angle of elevation of 35° with the ground. Find the vertical height of the kite above the ground.



3 **(a)** Calculate the length BD in the figure.



(b) Use your answer to (a) to find the angle BCD.

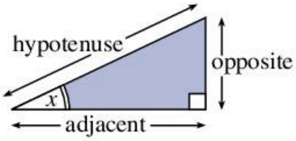
4 An aeroplane flies for 850 m at a constant angle of elevation of 23° after take off. Find the vertical height through which it rises.

5 A woman walks 300 m up a road with a slope of 18°. How far vertically is she from her starting point?

7.7 The cosine of an angle

You will need graph paper, a set square, a protractor and a calculator.

So far we have discovered two ratios for a right-angled triangle, **tan** and **sin**.



$$\tan x = \frac{\textbf{opposite}}{\textbf{adjacent}} \qquad \sin x = \frac{\textbf{opposite}}{\textbf{hypotenuse}}$$

There is another important ratio for a right-angled triangle. It is called the **cosine**.

- $$\textbf{cosine } x = \frac{\textbf{adjacent}}{\textbf{hypotenuse}}$$

'cosine of' is usually shortened to **cos**.

Exercise 7K

1 **(a)** Copy the table.

Angle x	Hypotenuse AC	Adjacent AB	cos x = $\left(\dfrac{\text{adjacent}}{\text{hypotenuse}}\right)$
30°	1 cm		
30°	2 cm		
30°	4 cm		
30°	5 cm	4.3 cm	0.86
30°	8 cm		
30°	10 cm		

(b) Now turn back to the diagram you drew for Question 1 of Exercise 7H. Measure the base (along AB) of each right-angled triangle in your diagram. Write the measurements in your table.

(c) Complete the last column of the table. What do you notice?

2 **(a)** Copy the table, and continue it for all angles up to 80°.

Angle x	Hypotenuse AC	Adjacent AB	cos x = $\left(\dfrac{\text{adjacent}}{\text{hypotenuse}}\right)$
10°	5 cm		
20°	5 cm		
30°	5 cm	4.3 cm	0.86

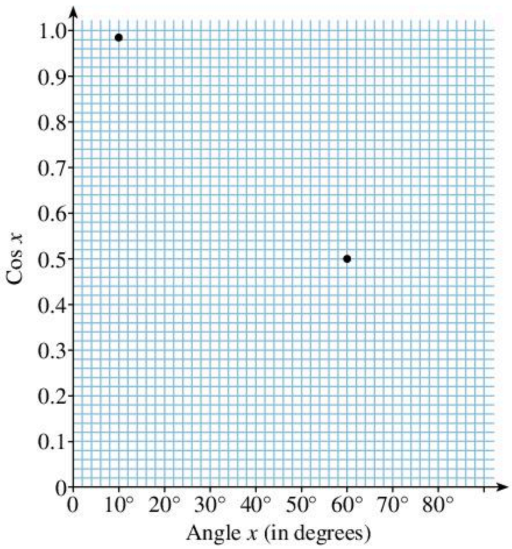
(b) Now turn to the diagram you drew for Question 3 of Exercise 7H. Measure the base of each right-angled triangle in your diagram. Record the measurements in your table.

(c) Complete the last column of the table. What do you notice?

3 **(a)** What happens to the cosines in Question 2 as the angles get larger?

(b) Compare these cosine ratios with the sine ratios you found in Question 3 of Exercise 7H. What do you notice?

4 Copy and complete the graph, using your results from Question 2. Join the points with a smooth curve.



5 Using your graph from Question 4 find the cosine of:

(a) 35° **(b)** 45° **(c)** 55° **(d)** 65°
(e) 60° **(f)** 30° **(g)** 80° **(h)** 24°
(i) 9° **(j)** 15° **(k)** 12° **(l)** 89°

Check your results using your calculator. Is your graph accurate?

6 Using your graph from Question 4, find the angle which has the cosine:

(a) 0.3 **(b)** 0.4 **(c)** 0.6 **(d)** 0.7
(e) 0.25 **(f)** 0.65 **(g)** 0.55 **(h)** 0.38

Check your results on your calculator.

7 Use your calculator to find to three significant figures:

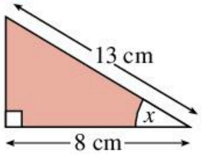
(a) cos 42° **(b)** cos 65°
(c) cos 28.6° **(d)** cos 7.1°
(e) cos 84.8° **(f)** cos 10.5°
(g) cos 65.5° **(h)** cos 28.8°

8 Use your calculator to find, to three significant figures, which angle has the cosine:

(a) 0.599 **(b)** 0.809
(c) 0.891 **(d)** 0.331
(e) 0.639 **(f)** 0.800
(g) 0.866 **(h)** 0.707

If you are given the adjacent and hypotenuse of a right-angled triangle you can find its angle.

Example 9



Calculate the angle x.

$$\cos x = \frac{\text{adj}}{\text{hyp}}$$

$$\cos x = \frac{8}{13}$$

$$\cos x = 0.615$$

So $x = \cos^{-1}(0.615)$

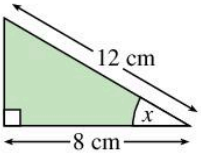
Using your calculator:
$x = 52.0°$

Exercise 7L

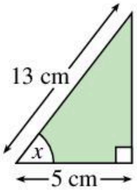
Give your answers to three significant figures where appropriate.

1 Calculate the angle x in each triangle:

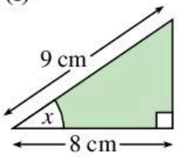
(a)



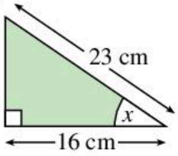
(b)



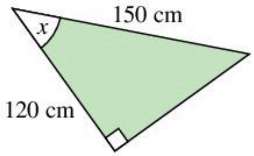
(c)



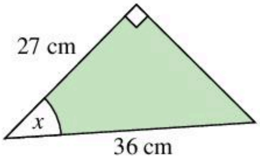
(d)



(e)



(f)



2 In triangle ABC angle ABC = 90°, AB = 32 cm and AC = 50 cm. Find angle BAC.

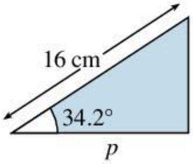
3 In triangle XYZ angle XZY = 90°, XY = 28 cm and YZ = 22 cm. Find angle XYZ.

4 A ladder 3 m long rests against a vertical wall. If the foot of the ladder is 1 m from the wall, calculate the angle the ladder makes with the ground.

5 What is the angle of slope of a road if a man has walked 100 m up the road and has travelled 30 m horizontally?

You can also use cosine to find the adjacent if you know the hypotenuse and the angle.

Example 10



Find the base length, p in the triangle.

$$\frac{\text{adj}}{\text{hyp}} = \cos 34.2°$$

$$\frac{p}{16} = \cos 34.2°$$

Using your calculator:

$$\frac{p}{16} = 0.827$$

Solve the equation:

$(\times 16)$ $\quad p = 0.827 \times 16$

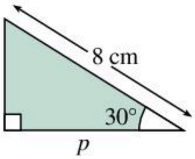
$= 13.232$ cm

Exercise 7M

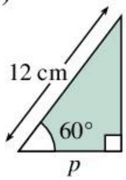
Give your answers to three significant figures where appropriate.

1 Find p in each of these triangles:

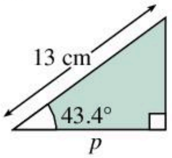
(a)



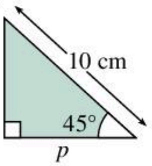
(b)



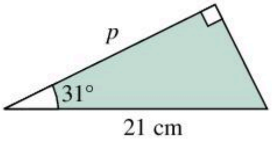
(c)



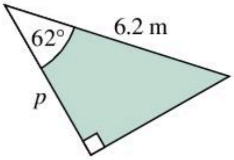
(d)



(e)



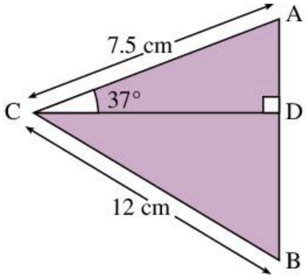
(f)



2 In triangle PQR angle PQR = 90°, angle QRP = 41° and length PR = 30 cm. Find QR.

3 In triangle ABC angle BAC = 90°, BC = 11 cm and angle ABC = 68°. Find AB.

4 From the diagram, calculate:
- **(a)** the length CD
- **(b)** the angle DCB.



5 A road 70 m long rises at an angle of 5°.
- **(a)** What horizontal distance does the road cover?
- **(b)** Through what vertical distance does the road rise?

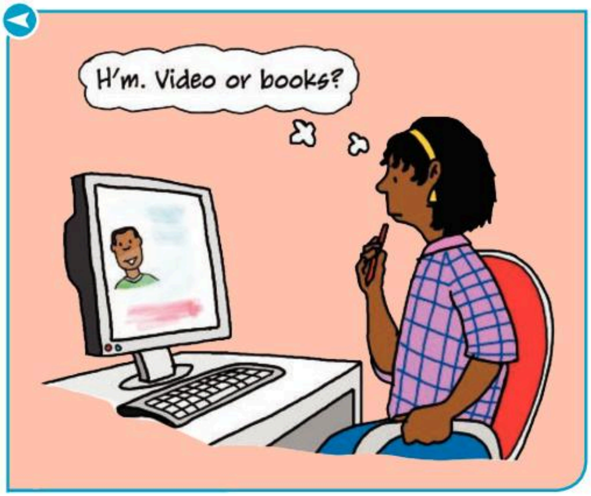
6 A man walks 100 m up a slope of 22° and then 50 m up a slope of 18°. How far horizontally is he from his starting point?

Technology

A complete course on right-angled triangles and trigonometric ratios can be found at

www.onlinemathlearning.com/basic-trigonometry.html

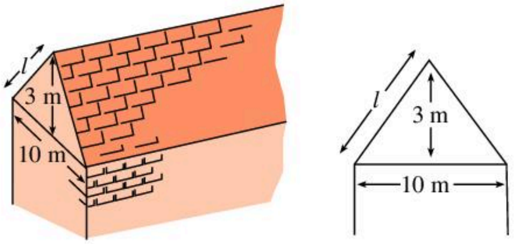
The course comes with lots of instructive videos.



Exercise 7N – mixed questions

Give your answers to three significant figures where appropriate.

1 The diagram shows part of a roof.

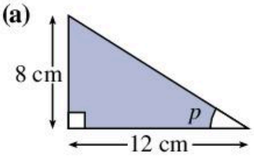


The width of the roof is 10 metres, and the highest part of the roof is 3 metres above the top of the front wall.

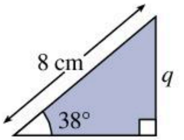
Find the length, *l*, of the sloping part of the roof.

2 Find the unknown value represented by the letter in each triangle.

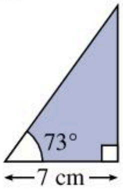
(a)



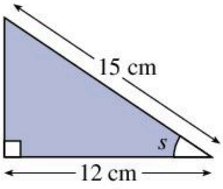
(b)



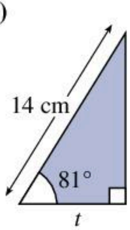
(c)



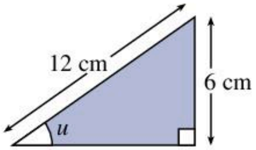
(d)



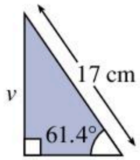
(e)



(f)

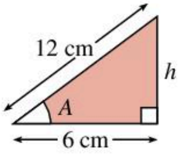


(g)

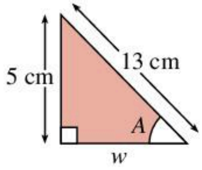


3 Find the sides and angles marked by the letters.

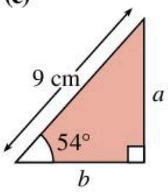
(a)



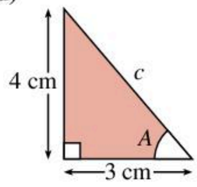
(b)



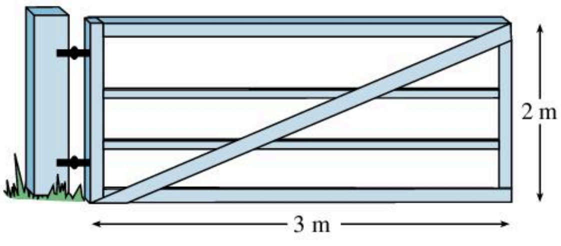
(c)



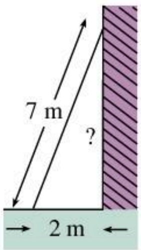
(d)



4 Mr Hassan's gate has a diagonal bracing strut to strengthen it.
What is the length of this bracing strut?



5 The diagram shows a ladder leaning against Mr Hassan's house. The ladder is 7 metres long. The foot of the ladder is 2 m from the wall. How high up the wall does the ladder reach?



6 If the ladder in Question 5 was 1 metre longer, and its foot in the same position, how much higher up the wall would it reach?

7 Suppose in Question 5, Mr Hassan moves the foot of the ladder in 1 m closer to the wall. How high up the wall does it reach now?

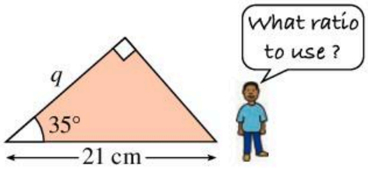
8 Mike and Ali start walking from the same point. Mike walks due North for 2.4 km. Ali walks due East for 3.2 km.

(a) Make a sketch to show this information.

(b) How far is Ali from Mike when they stop?

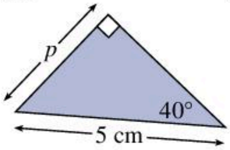
(c) If Ali now stands still, how much further North must Mike walk to get 5 km away from him?

9 Look at the triangle below. Can you find the value of q?

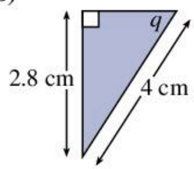


10 Find the unknown values represented by the letters.

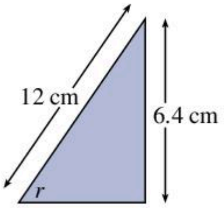
(a)



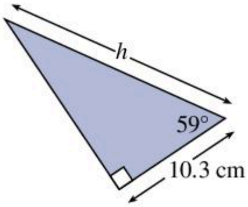
(b)



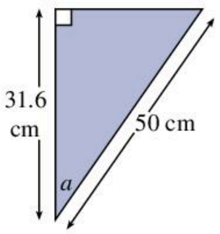
(c)



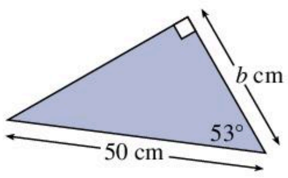
(d)



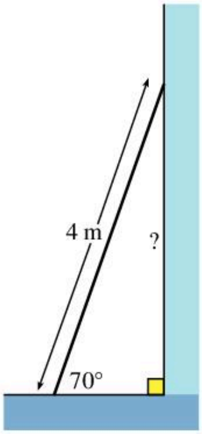
(e)



(f)

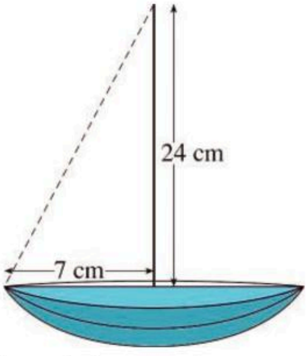


11 A ladder 4 m long rests against a wall and makes an angle of 70° with the ground.



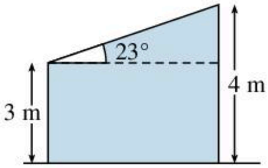
(a) How far up the wall is the top of the ladder?
(b) How much further up the wall is the top of the ladder when the ladder is moved to make an angle of 76° with the ground?

12 Sam's model boat has a mast 24 cm tall. The rigging, shown by the dotted line, is fixed 7 cm from the foot of the mast.

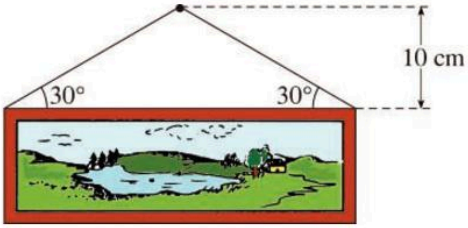


(a) Find the angle between the mast and the rigging.
(b) Find the length of the rigging.

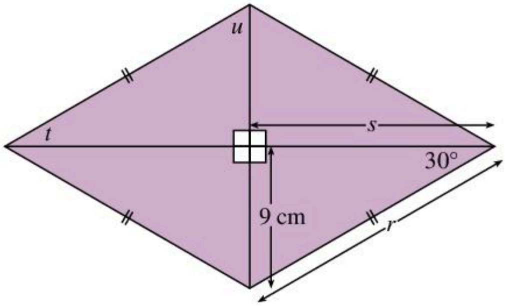
13 The diagram below shows the side elevation of a small house.
Find the width of the side of the house.



14 A picture is 10 cm vertically below a nail from which it is hung. If the cord makes an angle of 30° with the picture frame, find the width of the frame.

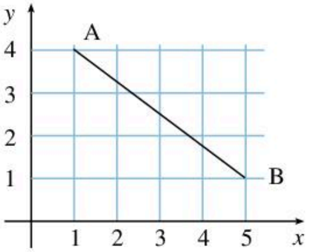


15 Find the unknown values represented by the letters.



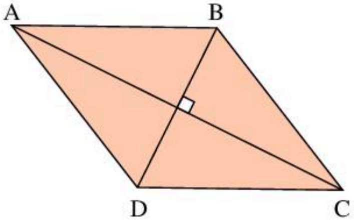
16 The angle of elevation of the top of a church tower, from a point on level ground 300 m away is 24°. Find the height of the tower.

17 **(a)** A is the point (1, 4) and B the point (5, 1). Calculate the distance AB.



(b) Calculate the distance between the points P(2, 3) and Q(5, 4).

18 The rhombus ABCD has diagonals AC = 32 cm and BD = 24 cm. Find the lengths of its sides:



19 The rectangle ABCD has AB = 3.6 m and BC = 4.8 m. Find the angle between the diagonal AC and AB.

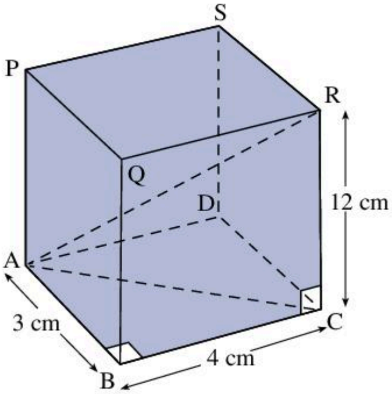
20 PQRS is a rhombus of side 25 cm. The diagonal PR is 30 cm. Find the angle between PR and RS.

21 Look at the cuboid shown.

(a) Find the distance from A to C.

(b) Using triangle ACR, find the distance from A to R.

(c) How far is it from B to S?



22 For the cuboid in Question 21, find the distance from:

(a) A to the midpoint of RC

(b) P to the midpoint of BC.

23 The base of a cardboard box measures 20 cm by 30 cm. Its height is 10 cm.

(a) Sketch it.

(b) How long is the longest rod which will lie flat on the bottom of the box?

(c) How long is the longest rod which will fit into the box (not lying flat)?

24 A triangle with sides of length 3, 4 and 5 is right-angled because $3^2 + 4^2 = 5^2$.

The lengths of the sides of some triangles are given. Which of them are right-angled?

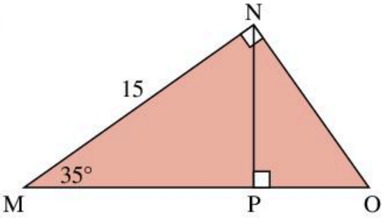
(a) 10, 15, 20 **(b)** 5, 12, 13

(c) 9, 40, 41 **(d)** 9, 12, 15

(e) 12, 16, 20 **(f)** 20, 21, 29

(g) 8, 12, 16 **(h)** 7, 8, 10

25



In the figure MN = 15, angle PMN = 35° and angles MNO and NPO are 90° each. Copy the figure and mark in the sizes of the remaining angles.

Find **(a)** NP **(b)** NO **(c)** MO

26 **(a)** Use your calculator to find:

(i) sin 30° and cos 60°

(ii) sin 53° and cos 37°

(iii) sin 21° and cos 69°

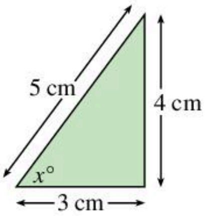
(iv) sin 45° and cos 45°

(b) What do you notice about each pair of results?

Can you explain this?

(*Hint*: draw a right-angled triangle. Call the two acute angles x° and $(90 - x)$°. Call the sides f, g and h. Now write down the sine and cosine of each angle.)

27 **(a)** Use Pythagoras' theorem to show that this triangle has a right angle.



(b) Find the unknown angle using:

(i) tan x° **(ii)** sin x° **(iii)** cos x°

Are your three answers the same?

28 Look again at the triangle in Question 27.

(a) Do you agree that:
$\sin x° = \frac{4}{5}$ and $\cos x° = \frac{3}{5}$?

(b) Find $(\frac{4}{5})^2$ and $(\frac{3}{5})^2$ and show that:
$(\frac{4}{5})^2 + (\frac{3}{5})^2 = 1$

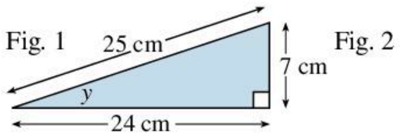
(c) Is $(\sin x°)^2 + (\cos x°)^2 = 1$?

29 For the triangle below, find:

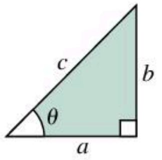
(a) $\sin y$ **(b)** $\cos y$

(c) $(\sin y)^2$ **(d)** $(\cos y)^2$

(e) $(\sin y)^2 + (\cos y)^2$ in its simplest form.

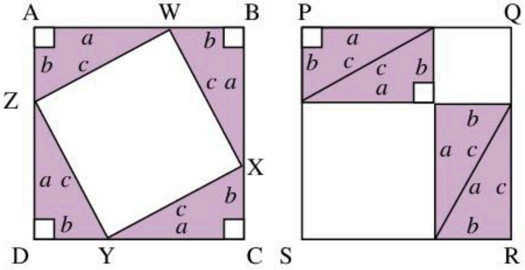


30 **(a)** For the triangle below, write $\sin \theta$ and $\cos \theta$ in terms of a, b and c.



(b) According to Pythagoras' theorem
$c^2 = a^2 + b^2$.
Show that $(\sin \theta)^2 + (\cos \theta)^2 = 1$

31 Fig. 1 shows the square ABCD made up of four identical right-angled triangles and the quadrilateral WXYZ.



(a) Show that $W\hat{X}Y = 90°$ and that WXYZ is a square.

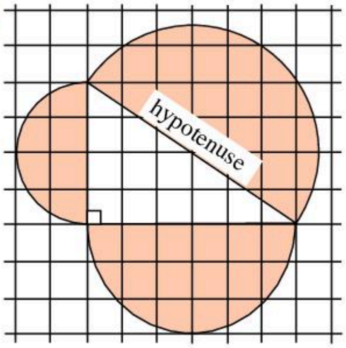
(b) What is the area of WXYZ?

(c) What is the area of the two white squares in Fig. 2?

(d) Use Fig. 1 and Fig. 2 to show that

$$c^2 = a^2 + b^2$$

32



(a) Draw a right-angled triangle on squared paper. On each side carefully draw a semicircle.

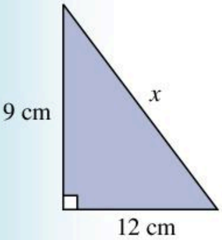
(b) Find the area of each semicircle.
What do you notice?
(*Note*: the area of a circle is πr^2)

(c) Repeat this process, but this time draw an equilateral triangle on each side.
What do you notice about the areas?

7 Consolidation

Example 1

Find the missing length x in this right triangle.



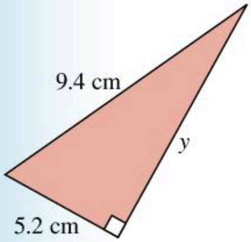
By Pythagoras' theorem:

$$9^2 + 12^2 = x^2$$
$$81 + 144 = x^2$$
$$x^2 = 225$$
$$x = \sqrt{225}$$

So $x = 15$ cm

Example 2

Find the length marked y in this right triangle:



By Pythagoras' theorem:

$$y^2 + 5.2^2 = 9.4^2$$
$$y^2 + 27.04 = 88.36$$
$$(-27.04) \qquad y^2 = 61.32$$
$$y = \sqrt{61.32}$$
$$\approx 7.83 \text{ cm}$$

Example 3

Find the value of the angle x when:

(a) $\tan x = 1.86$

so $x = \tan^{-1}(1.86)$

On your calculator press

$\boxed{1}\ \boxed{\cdot}\ \boxed{8}\ \boxed{6}\ \boxed{\text{INV}}\ \boxed{\text{tan}}$

or $\boxed{\text{2nd}}\ \boxed{\text{tan}}\ \boxed{1}\ \boxed{\cdot}\ \boxed{8}\ \boxed{6}$

$\boxed{61.73599585}$ should appear on the screen, so $x = 61.7°$ (3 sig. fig.)

(b) $\cos x = 0.254$

On your calculator press

$\boxed{\cdot}\ \boxed{2}\ \boxed{5}\ \boxed{4}\ \boxed{\text{INV}}\ \boxed{\text{cos}}$

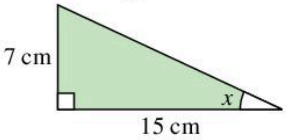
or $\boxed{\text{2nd}}\ \boxed{\text{cos}}\ \boxed{\cdot}\ \boxed{2}\ \boxed{5}\ \boxed{4}$

$\boxed{75.28566146}$ should appear on the screen, so $x = 75.3°$ (3 sig. fig.)

Example 4

Calculate the angle x.

(a)



$$\tan x = \frac{\text{opp}}{\text{adj}} = \frac{7}{15}$$

so $\tan x = 0.466$

$$x = \tan^{-1}(0.466)$$

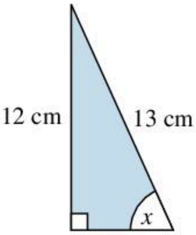
On your calculator press

$\boxed{\cdot}\ \boxed{4}\ \boxed{6}\ \boxed{6}\ \boxed{\text{INV}}\ \boxed{\text{tan}}$

or $\boxed{\text{2nd}}\ \boxed{\text{tan}}\ \boxed{\cdot}\ \boxed{4}\ \boxed{6}\ \boxed{6}$

to get $x = 25.0°$ (3 sig. fig.)

(b)



$$\sin x = \frac{\text{opp}}{\text{hyp}} = \frac{12}{13}$$

so $\sin x = 0.923$

On your calculator press

$\boxed{\cdot}\ \boxed{9}\ \boxed{2}\ \boxed{3}\ \boxed{\text{INV}}\ \boxed{\text{sin}}$

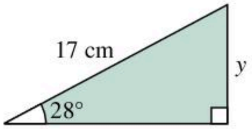
or $\boxed{\text{2nd}}\ \boxed{\text{sin}}\ \boxed{\cdot}\ \boxed{9}\ \boxed{2}\ \boxed{3}$

to get $x = 67.4°$

Example 5

Find the length marked y in these triangles.

(a)

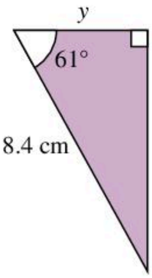


$$\frac{\text{opp}}{\text{hyp}} = \frac{y}{17} = \sin 28°$$

so $$\frac{y}{17} = 0.469\ldots$$

$$(\times 17) \quad y = 0.469\ldots \times 17 = 7.98 \text{ cm}$$

(b)



$$\frac{\text{adj}}{\text{hyp}} = \frac{y}{8.4} = \cos 61°$$

$$\text{so} \quad \frac{y}{8.4} = 0.485$$

$$y = 0.485 \times 8.4 \text{ cm}$$

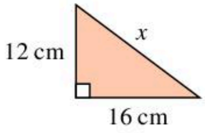
$$= 4.07 \text{ cm}$$

Exercise 7

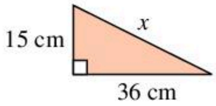
Give your answers to three significant figures where appropriate.

1 Find the missing lengths in these right angled triangles.

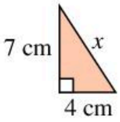
(a)



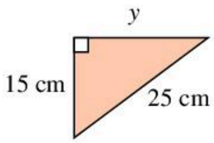
(b)



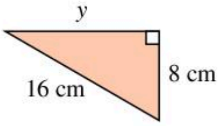
(c)



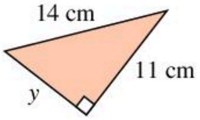
(d)



(e)



(f)

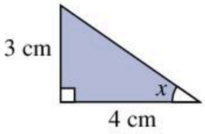


2 Use your calculator to find x when:

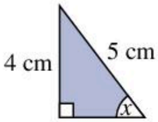
(a) $\tan x = 1.73$ **(b)** $\cos x = 0.707$
(c) $\tan x = 0.577$ **(d)** $\sin x = 0.8$
(e) $\cos x = 0.05$ **(f)** $\sin x = 0.124$
(g) $\tan x = 13.6$ **(h)** $\cos x = 0.615$

3 Calculate the angle x in each of these triangles.

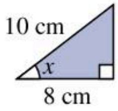
(a)



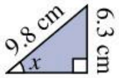
(b)



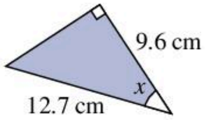
(c)



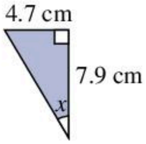
(d)



(e)

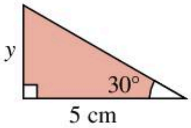


(f)

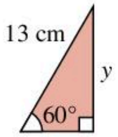


4 Find the side marked y in each of the triangles.

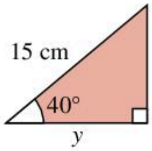
(a)



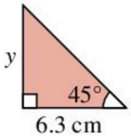
(b)



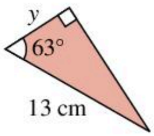
(c)



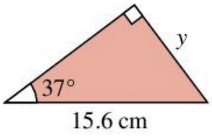
(d)



(e)



(f)



Application

5 A man stands 30 m from the base of a radio mast. He measures the angle of elevation to the top of the mast to be 58.3°.

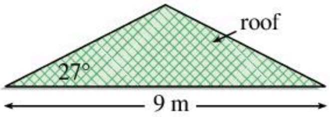
(a) Draw a diagram to show the man and mast.

(b) Find the height of the mast, assuming the man to be 1.7 m tall.

6 A boat sails due south from English Harbour, Antigua for 8 km and then sails due west for 6 km.

- **(a)** How far is the boat from English Harbour?
- **(b)** The boat then sails a further 4 km due south. How far is the boat now from English Harbour?

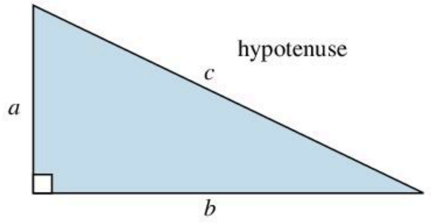
7 Some authorities recommend a roof slope of 27° to be better protect homes from the effect of hurricanes. John's house is 9 m wide and has roof angle 27°.
What is the length of the sloping side of his roof?



Summary

You should know ...

1 Pythagoras' theorem.

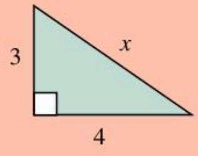


For a right-angled triangle
$a^2 + b^2 = c^2$

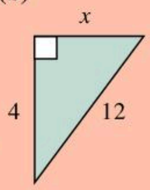
Check out

1 Find the missing lengths in these right-angled triangles.

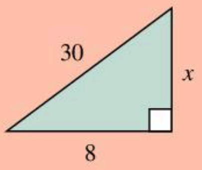
(a)



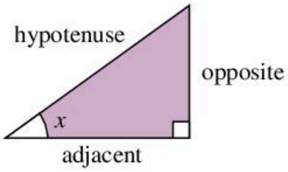
(b)



(c)



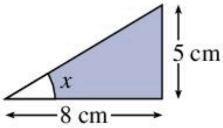
2 The sides of a right-angled triangle are named in relation to the angle of interest:



3 The tangent of $x^\circ = \tan x^\circ$

$$= \frac{\text{opposite}}{\text{adjacent}}$$

For example:



$\tan x = \frac{5}{8} = 0.625$

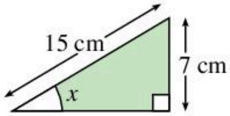
$x = \tan^{-1}(0.625)$

so $x = 32.0^\circ$

4 The sine of $x^\circ = \sin x^\circ$

$$= \frac{\text{opposite}}{\text{hypotenuse}}$$

For example:



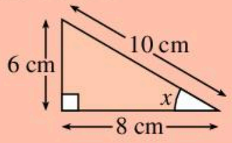
$\sin x = \frac{7}{15} = 0.466\ldots$

$x = \sin^{-1}(0.466)$

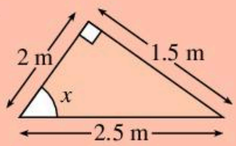
so $x = 27.8^\circ$

2 For each triangle write down the length of the opposite and adjacent sides.

(a)

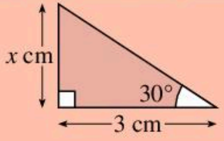


(b)

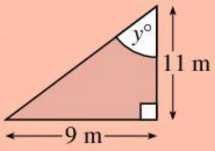


3 Using tangents find x and y.

(a)

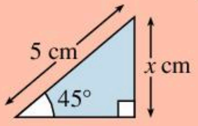


(b)

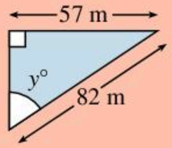


4 Use sine to find x and y.

(a)



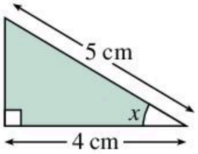
(b)



5 The cosine of $x° = \cos x°$

$$= \frac{\text{adjacent}}{\text{hypotenuse}}$$

For example:



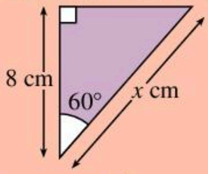
$\cos x = \frac{4}{5} = 0.8$

$x = \cos^{-1}(0.8)$

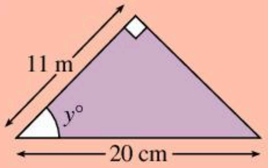
so $x = 36.9°$

5 Use cosine to find x and y.

(a)



(b)

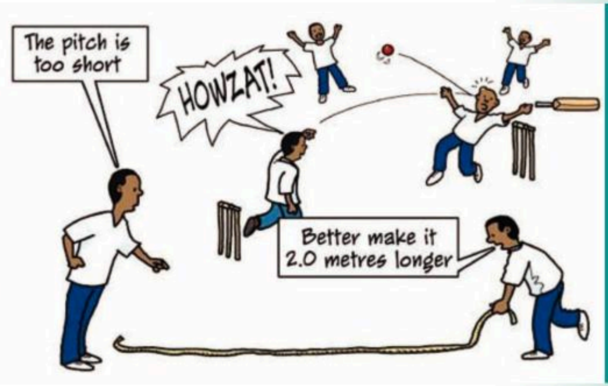


Measurement 2

8

Objectives

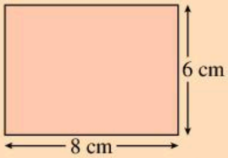
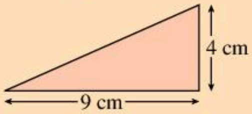
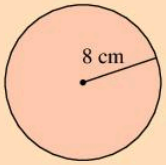
- ✔ calculate the volume of cuboids, prisms, cylinders, pyramids and cones
- ✔ calculate the surface area of cuboids, prisms, cylinders and cones
- ✔ solve problems involving volumes and surface areas of solids



What's the point?

To move heavy loads you need vehicles with powerful engines. The larger the engine's volume the larger load it can carry. A sports utility vehicle (SUV) will generally have an engine with a volume of at least 2000 cm^3.

Before you start

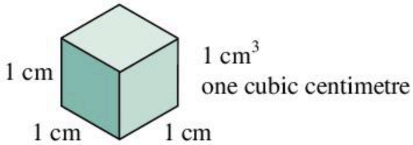
You should know ...	Check in
1 The units used to measure area cm^2, m^2 and km^2.	**1** **(a)** Draw a shape with area 4 cm^2. **(b)** Estimate the area of **(i)** your classroom **(ii)** your hand.
2 How to find the area of simple shapes.	**2** Find the area of **(a)**  **(b)**  **(c)**  (take π = 3.14)

3 How to calculate the volume of a cuboid.
For example:
the volume of a cuboid with sides of length 4 m, 3 m and 2 m
is $4 \times 3 \times 2 \text{ m}^3$
$= 24 \text{ m}^3$

3 **(a)** Find the volume of a cuboid which is 7 m long, 5 m wide and 3.5 m high.
(b) A cuboid of length 10 cm and width 5.5 cm has a volume of 110 cm^3. Find its height.

8.1 Volume basics

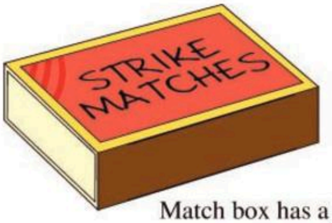
Volume is always measured in cubic units – mm^3, cm^3, m^3.



Small objects are measured in mm^3:

Match head has a volume ≈ 50 mm^3

Medium-sized objects are measured in cm^3:



Match box has a volume ≈ 40 cm^3

Larger objects are measured in m^3:

A refrigerator has a volume ≈ 1 m^3

Exercise 8A

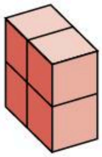
1 Which unit is most suitable to measure the volume of:
- **(a)** your classroom
- **(b)** a pencil case
- **(c)** an orange
- **(d)** a car
- **(e)** a grain of rice
- **(f)** a cricket ball
- **(g)** a swimming pool
- **(h)** a hen's egg
- **(i)** a shoe box
- **(j)** an oil drum?

2 Write down five objects which have a volume:
- **(a)** more than 1 m^3
- **(b)** less than 1 cm^3.

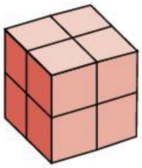
3 **(a)** How many cm make a m?
(b) How many cm^2 make a m^2?
(c) How many cm^3 make a m^3?

4 Find the volume of these solids if each cube is 1 cm^3.

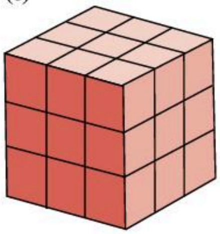
(a)



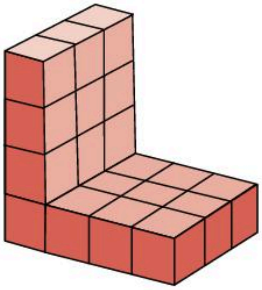
(b)



(c)

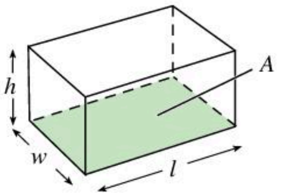


(d)



Cuboids

A cuboid has length l, width w, and height h, as shown.



- Its volume is:

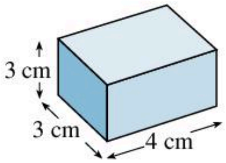
  $V = l \times w \times h$ or $\boldsymbol{V = lwh}$

  Since $A = l \times w$ is the area of the shaded face of the cuboid, we can also write: $\boldsymbol{V = Ah}$

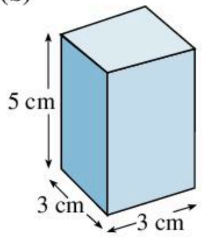
Exercise 8B

1 Find the volume of these cuboids:

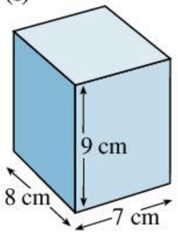
(a)



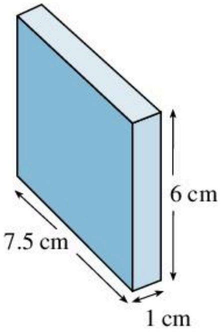
(b)



(c)



(d)

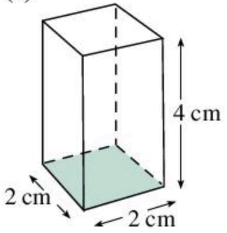


2 Copy and complete the table for cuboids.

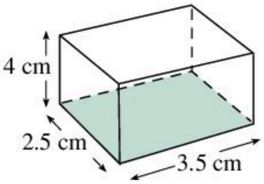
	l **cm**	w **cm**	h **cm**	V **cm^3**
(i)	2	6	12	
(ii)	8		4	64
(iii)		0.5	8.2	82
(iv)	2.4	6.7		48.24

3 Find the area of the shaded face of the cuboid, then find its volume:

(a)



(b)



4 Copy and complete the table for cuboids.

	A **cm^2**	h **cm**	V **cm^3**
(i)	4		64
(ii)	4	10.4	
(iii)	5.2	2.5	
(iv)		3.55	28.40

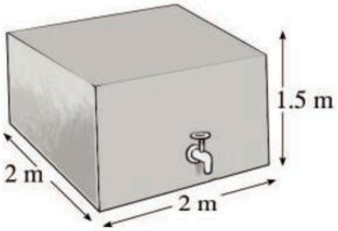
5 What is the height of a room which is 8 m long, 6 m wide and contains 144 m^3 of air.

6 How many cubes of side 2 cm can be fitted into a box 12 cm long, 8 cm wide and 4 cm high?

7 The store sells cereal packets that measure 20 cm × 30 cm × 8 cm.
They are delivered to stores in larger boxes measuring 90 cm × 40 cm × 40 cm. Draw a diagram to show how the packets may be packed to fit inside the larger box. How many packets would fill the larger box?

8 What is the capacity in litres of a metal box 20 cm wide, 50 cm long and 30 cm high?
(1 litre = 1000 cm^3)

9



(a) What is the volume of the water tank
(i) m^3 **(ii)** cm^3?

(b) How many litres of water does it hold?

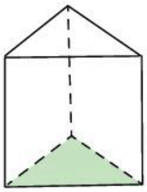
10 A water tank holds 3000 litres of water when full. The tank has a rectangular base with area 3 m^2. Find the height of the tank.

8.2 Volume of prisms and cylinders

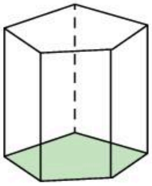
Volume of a prism

A **prism** is a solid with two identical and parallel faces in the form of a polygon.
A cuboid is one example of a **prism**.

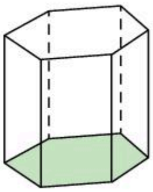
Here are some more:



Triangular prism



Pentagonal prism



Hexagonal prism

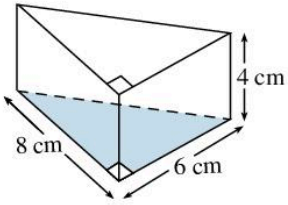
- Volume of a prism = area of cross-section × height

$V = A \times h$

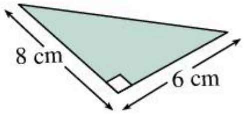
To find the volume of a prism you first need to the area of its base or cross-sectional area.

Example 1

Find the volume of this prism.



The base area is a triangle.



Area of base triangle, $A = \frac{1}{2}bh$

$= \frac{1}{2} \times 6 \text{ cm} \times 8 \text{ cm}$

$= 24 \text{ cm}^2$

Volume of prism $= A \times h$

$= 24 \text{ cm}^2 \times 4 \text{ cm}$

$= 96 \text{ cm}^3$

Technology

Visit the website

www.onlinemathlearning.com

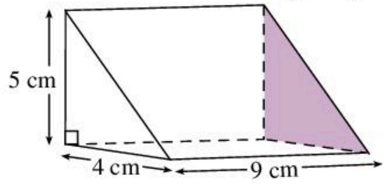
Study what it says about finding volumes of prisms. Watch the videos!

When you are through, you are ready for Exercise 8C!

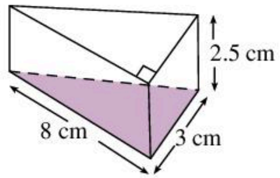
Exercise 8C

1 Find the volume of these triangular prisms:

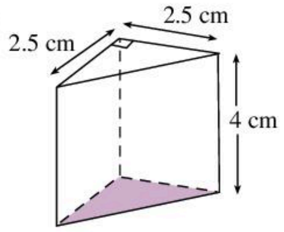
(a)



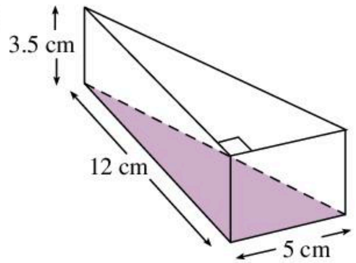
(b)



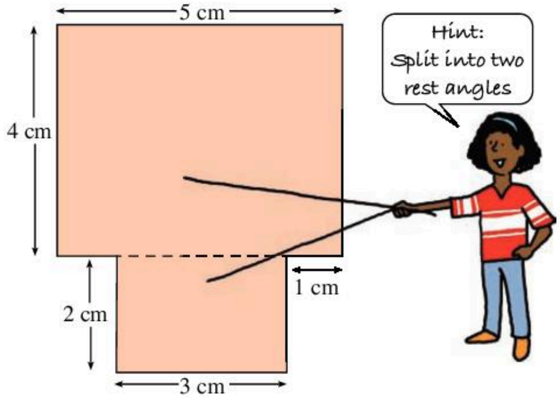
(c)



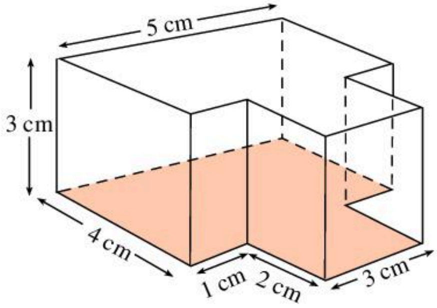
(d)



2 **(a)** Find the area of this shape.

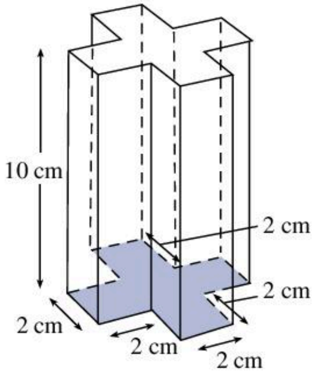


(b) Using your answer to part (a), find the volume of this prism.

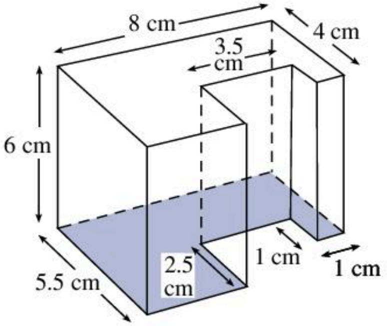


3 Find the volume of the prism, by first finding the area of its base:

(a)

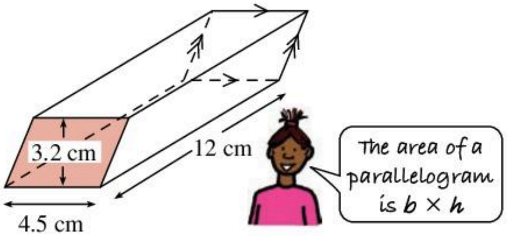


(b)

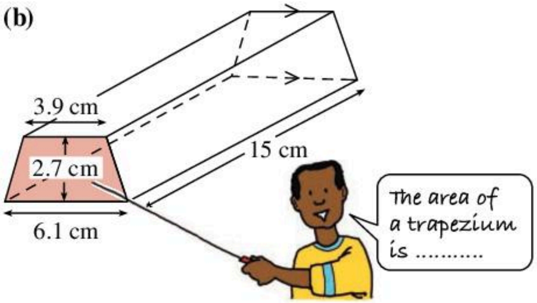


4 Find the volume of each prism:

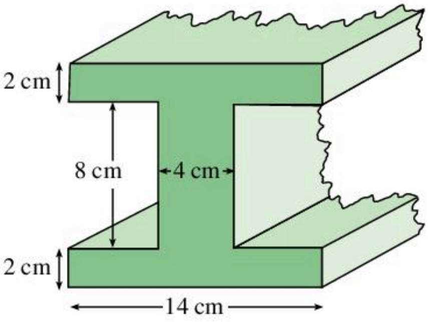
(a)



(b)



5 The drawing shows the end of a steel girder 2 m long.

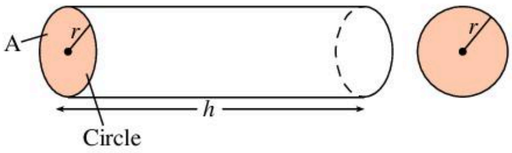


(a) Find the area of the end of the girder.

(b) Find the volume of the girder in cm^3.

Volume of a cylinder

A cylinder is a solid with two identical and parallel circular faces.



The volume of a cylinder uses the same formula as the volume of a prism, that is

$$V = A \times h$$

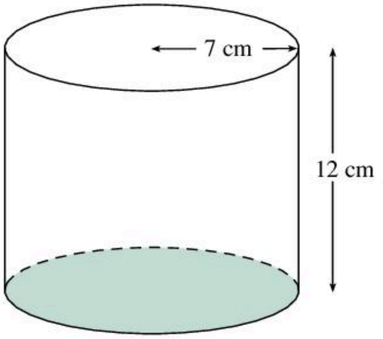
where A is the area of the cross-section or circle

$$\begin{aligned} \text{Area of circle } &= \pi r^2 \\ \text{Volume of cylinder } &= \text{Area of cross section} \times \text{height} \\ &= \pi r^2 \times h \end{aligned}$$

- The volume of a cylinder is **$V = \pi r^2 h$.**

Example 2

Find the volume of this cylinder.
Take $\pi = 3\frac{1}{7}$.



The base is a circle of radius 7 cm.

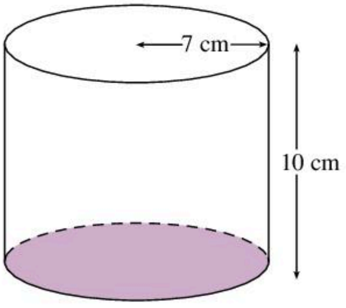
$$\begin{aligned} \text{Area of circle} &= \pi r^2 \\ &= \frac{22}{\cancel{7}} \times \cancel{7} \times 7 \\ &= 154 \text{ cm}^2 \end{aligned}$$

$$\begin{aligned} \text{Volume of cylinder} &= A \times h = \pi r^2 h \\ &= 154 \text{ cm}^2 \times 12 \text{ cm} \\ &= 1848 \text{ cm}^3 \end{aligned}$$

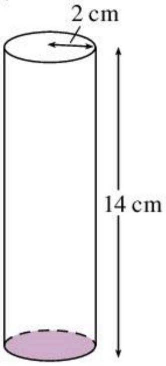
Exercise 8D

1 Find the volume of each cylinder, using $3\frac{1}{7}$ for π:

(a)



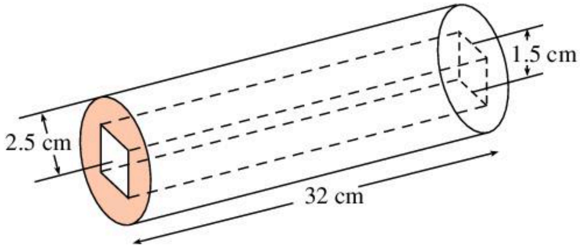
(b)



2 **(a)** Using 3.14 or your calculator's π button, find the volume of a cylinder if:
(i) $r = 3$ cm, $h = 6.4$ cm
(ii) $r = 3.2$ cm, $h = 4$ cm
Give your answer correct to 1 decimal place.
(b) Find the height of a cylinder if:
(i) $V = 37.68$ cm^3, $r = 1$ cm
(ii) $V = 75.36$ cm^3, $r = 2$ cm

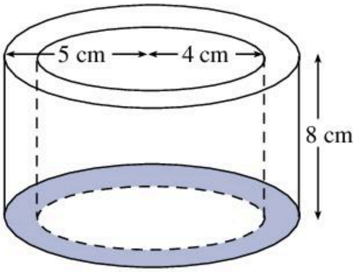
3 A barrel of cask rum has a base radius of 30 cm and is 110 cm high. Find, using 3.14 for π:
(a) its volume in cm^3
(b) its capacity in litres
(c) the number of bottles of rum that can be filled from the barrel if each bottle holds 0.73 litres.

4 A wooden cylinder of radius 2.5 cm and length 32 cm has a square hole of edge 1.5 cm removed from it as shown in the diagram.



Using $\pi = 3.14$:
(a) Find the volume of the original cylinder.
(b) Find the volume of wood removed.
(c) Calculate the volume of the remaining wood.

5 A piece of piping has an inner radius of 4 cm and an outer radius of 5 cm. Use 3.14 for π.

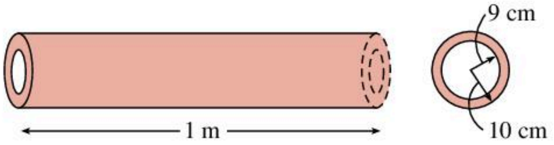


(a) Find the base area of the pipe.
(b) What is the volume of material used to make a pipe of length 8 cm?

6 A cylindrical vase is 15 cm tall and has an inside radius of 4 cm. It is made from clay 1 cm thick. Using 3.14 for π, find:

(a) the height of the *inside* of the vase
(b) the volume of water the vase can hold
(c) the mass of the vase if 1 cm^3 of clay has a mass of 2.8 g.

7 Find the volume of lead in a lead pipe with the dimensions shown. If 1 cm^3 of lead has a mass of 11.3 g, find the total mass of the pipe. Use 3.14 for π.



Technology

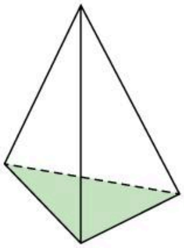
Review what you have learnt about volumes of cylinders by visiting the website

www.onlinemathlearning.com

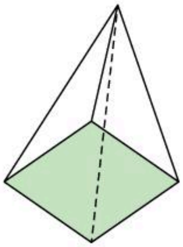
Watch the video examples.

8.3 Volumes of pyramids and cones

A pyramid is a shape whose faces slope upwards from the base to a vertex.



A pyramid with a triangular base is called a **tetrahedron**



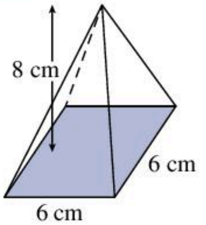
A pyramid with a square base is called a **square-based pyramid**

The volume, V, of any pyramid is given by the formula

$$V = \frac{1}{3} \times \text{base area} \times \text{height}$$

or $\boldsymbol{V = \frac{1}{3} A \times h}$

Example 3



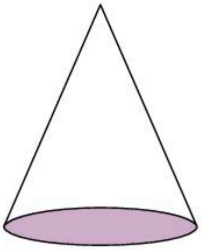
Find the volume of a square-based pyramid with side 6 cm and vertical height 8 cm.

Base area = 6 cm × 6 cm = 36 cm^2

Volume of pyramid

$$= \frac{1}{3} A \times h$$
$$= \frac{1}{3}\, 36\ \text{cm}^2 \times 8\ \text{cm}$$
$$= 96\ \text{cm}^3$$

A cone is a shape whose faces slope upwards from a **circular** base to a vertex.



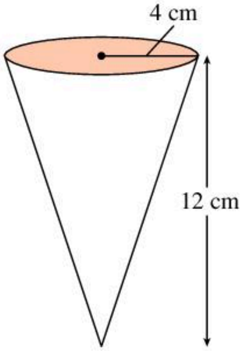
The formula for the volume of a cone is the same for that of a pyramid. That is,

$$\text{Volume of cone } = \frac{1}{3} A \times h$$

where the base area, A, is a circle.

To find the volume of a cone, you first have to calculate the area of the base circle.

Example 4



The radius of an ice-cream cone with vertical height 12 cm is 4 cm. What is it's volume?

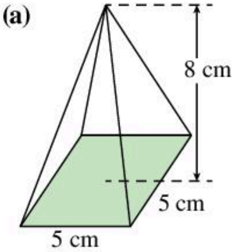
$$\begin{aligned}\text{Base area} &= \pi r^2\\ &= 3.14 \times 4^2\\ &= 50.24\ \text{cm}^2\end{aligned}$$

$$\begin{aligned}\text{Volume of cone} &= \tfrac{1}{3} A \times h\\ &= \tfrac{1}{3}\, 50.24 \times 12\ \text{cm}^3\\ &= 200.96\ \text{cm}^3\end{aligned}$$

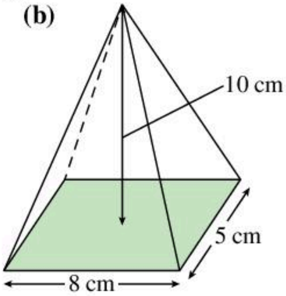
Exercise 8E

1 Find the volume of the pyramids.

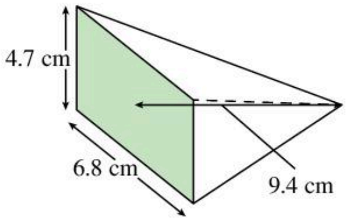
(a)



(b)

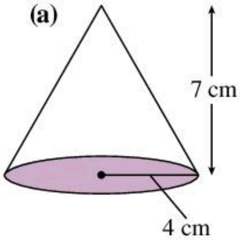


(c)

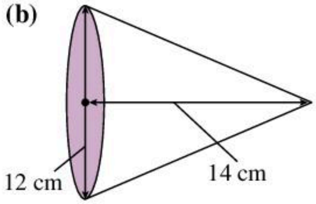


2 Calculate the volume of the cones giving your answers to three significant figures.

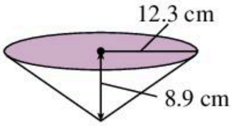
(a)



(b)



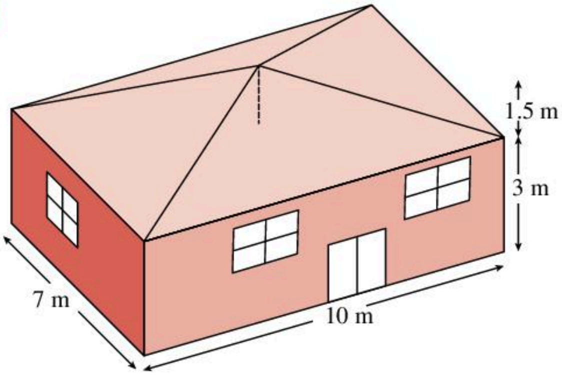
(c)



3 What is the volume of a tetrahedron with base area 12 cm^2 and vertical height 6 cm?

4 What is the vertical height of a square-based pyramid that has base side 6 cm and volume 108 cm^3?

5



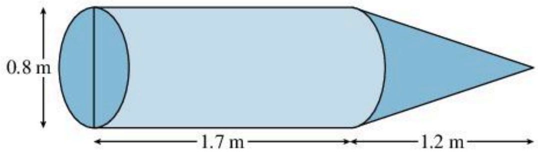
A rectangular based house with width 7 m, length 10 m and height 3 m, has a high roof in the form of a pyramid with vertical height 1.5 m
Find the total volume of the house in m^3.

6

A conical party hat has diameter 15 cm and vertical height 12 cm. Find the volume of the hat.

7 A conical paper cup has top radius 4 cm and vertical height 13 cm.
  (a) What is the volume of the cup in cm^3?
  (b) How much water in litres can the cup hold when filled to the brim?
  (c) The cup is filled with 0.15 l of water. How far up the cup does the water reach?

8



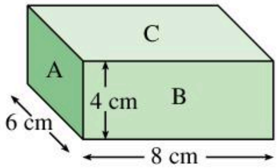
The diagram shows the prototype design for a rocket. What is the total volume of the rocket?

8.4 Surface areas

The area of the total surface of a solid is called its **surface area**. For example, the surface area of a box is the sum of the areas of its six rectangular faces.

Example 5

What is the surface area of a cuboid with dimensions 8 cm by 6 cm by 4 cm?



The cuboid has six faces.

Area side A = length × width
= 6 cm × 4 cm
= 24 cm^2
Area side B = 4 cm × 8 cm = 32 cm^2
Area side C = 8 cm × 6 cm = 48 cm^2

Surface area of cuboid
= 2 × A + 2 × B + 2 × C
= 2 × 24 cm^2 + 2 × 32 cm^2 + 2 × 48 cm^2
= 48 + 64 + 96 cm^2
= 208 cm^2

Exercise 8F

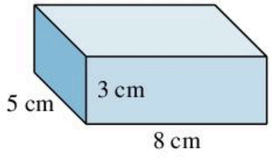
1 (a) What is the surface area of a cube with side 1 cm?
  (b) What is the surface area of a cube with side 3 cm?

2 The face of a cube has area 24 cm^2. What is the surface area of the cube?

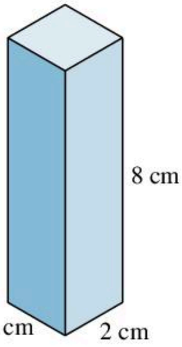
3 (a) Find the total surface area of cubes with sides 2.5 cm and 5 cm.
  (b) What happens to the surface area if the edge length of a cube is doubled?

4 Find the surface area of these cuboids.

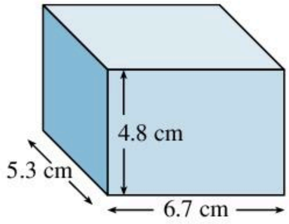
(a)



(b)



(c)



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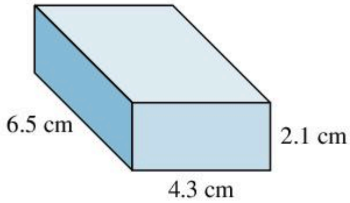

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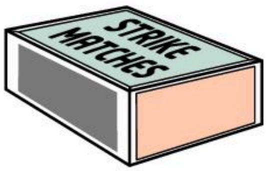
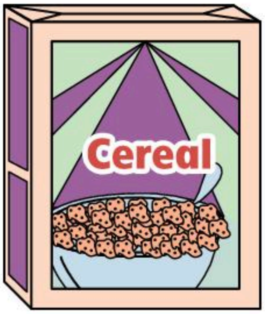


(a) With a ruler measure the dimensions of a matchbox.
(b) Calculate the total surface area of your matchbox.

6



- **(a)** Collect three different cereal boxes.
- **(b)** Measure their dimensions.
- **(c)** Calculate the total surface area of each.
- **(d)** Which box can be made using the least amount of card?

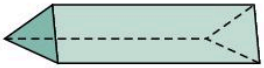
7 A room has a rectangular floor 5.5 m long and 4.5 m wide. The room is 2.5 m in height.

- **(a)** What is the area of wall space in the room?
- **(b)** How much paint is required to paint the walls with two coats if 1 litre of paint can cover 11.25 m^2?

8
- **(a)** Find the total surface area of a cuboid with dimensions 4 cm × 3 cm × 2 cm.
- **(b)** If the length of the cube, 4 cm, is increased by 20%, what is the new surface area of the cuboid?
- **(c)** How much did the surface area increase?
- **(d)** How does the surface area change if both length, 4 cm, and width, 3 cm, are increased by 20%?

Surface areas of prisms and cylinders

The surface area of any prism is also found by adding up the areas of each of the prism's sides.
For example, a triangular wedge has five faces.

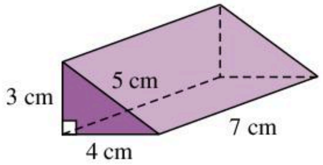


Two of its faces are triangles.
Three of its faces are rectangles.

To find the surface area you have to add the area of the two triangles to that of the three rectangles.

Example 6

Find the surface area of this triangular prism.



$$\begin{aligned}\text{Area of triangular face} &= \tfrac{1}{2}\text{ base} \times \text{height}\\ &= \tfrac{1}{2} \times 4\text{ cm} \times 3\text{ cm}\\ &= 6\text{ cm}^2\end{aligned}$$

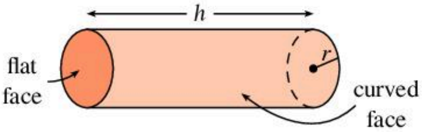
$$\begin{aligned}\text{Area of base rectangle} &= \text{length} \times \text{width}\\ &= 7\text{ cm} \times 4\text{ cm}\\ &= 28\text{ cm}^2\end{aligned}$$

$$\begin{aligned}\text{Area of upright rectangle} &= 7\text{ cm} \times 3\text{ cm}\\ &= 21\text{ cm}^2\end{aligned}$$

$$\begin{aligned}\text{Area of sloping rectangle} &= 7\text{ cm} \times 5\text{ cm}\\ &= 35\text{ cm}^2\end{aligned}$$

$$\begin{aligned}\text{Surface area} &= \text{area triangle} \times 2\\ &\quad + \text{area of rectangles}\\ &= 6\text{ cm}^2 \times 2 + 28\text{ cm}^2\\ &\quad + 21\text{ cm}^2 + 35\text{ cm}^2\\ &= 12 + 28 + 21 + 35\text{ cm}^2\\ &= 96\text{ cm}^2\end{aligned}$$

A cylinder has three faces: two flat circular faces and one curved face.



The area of the curved surface of the cylinder = distance around end × height

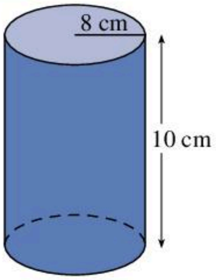
$= 2\pi r \times h$

Area of flat face = area of circle

$= \pi r^2$

Total surface area of cylinder $= 2\pi rh + 2\pi r^2$

Example 7



Find the:

(a) curved surface area

(b) total surface area of the cylinder.

Take $\pi = 3.14$

$$\begin{aligned}\text{Curved surface area} &= 2\pi rh\\ &= 2 \times 3.14 \times 8 \times 10\ \text{cm}^2\\ &= 502.4\ \text{cm}^2\end{aligned}$$

$$\begin{aligned}\text{Area of circle} &= \pi r^2\\ &= 3.14 \times 8 \times 8\ \text{cm}^2\\ &= 200.96\ \text{cm}^2\end{aligned}$$

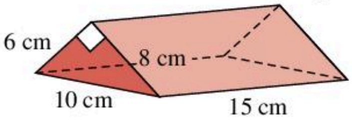
$$\begin{aligned}\text{Total surface area} &= 2 \times 200.96 + 502.4\ \text{cm}^2\\ &= 904.32\ \text{cm}^2\end{aligned}$$

Exercise 8G

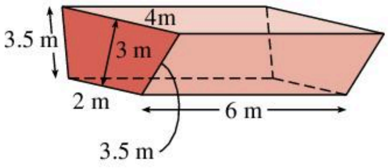
Give your answers to three significant figures where appropriate.

1 Find the surface area of these shapes.

(a)



(b)



2 **(a)** Find the curved surface area of a cylinder with:

- **(i)** $r = 5$ cm, $h = 10$ cm
- **(ii)** $r = 3$ cm, $h = 6$ cm
- **(iii)** $r = 7.5$ cm, $h = 12$ cm.

(b) What is the total surface area of these cylinders?

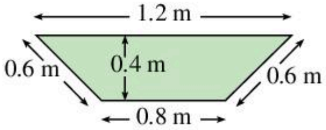
3

(a) Measure the diameter and height of a tin of beans.

(b) What area of paper is needed for the label?

(c) What area of tin is required to make the container?

4 The cross-section of a metal trough is shaped in the form of a trapezium as shown.



If the trough is 3 m in length, what area of metal is required to construct it?

5 A cylindrical tank is 2 m tall and has a cross-sectional area of 3 m^2.

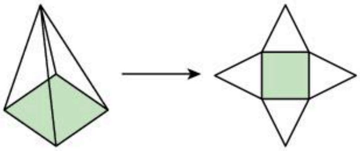
(a) What is the curved surface area of the tank?

(b) How many litres of paint will be needed to paint the curved surface of the tank of one litre of paint covers 11.25 m^2?

Surface area of pyramid and cones

To find the surface area of a pyramid, you need to sum the area of each of its faces.

This is straightforward if you look at its net.



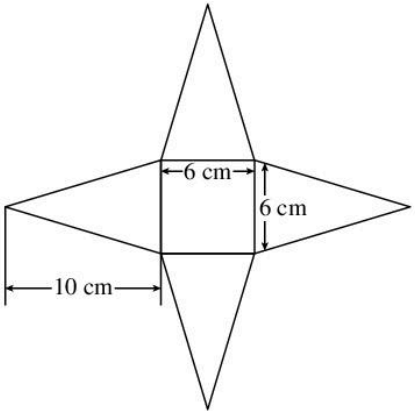
Square-based pyramid

Net of square-based pyramid

A square-base pyramid has four identical triangular faces and one square face.

Example 8

Find the volume of a square-based pyramid whose net is shown.



Area of triangular face $= \frac{1}{2} b \times h$

$= \frac{1}{2} \times 6 \text{ cm} \times 10 \text{ cm}$

$= 30 \text{ cm}^2$

Area of square face $= l \times w$

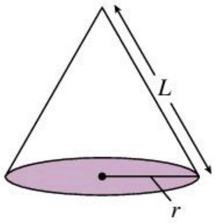
$= 6 \text{ cm} \times 6 \text{ cm}$

$= 36 \text{ cm}^2$

Total surface area $= 4 \times 30 + 36$

$= 156 \text{ cm}^2$

For a cone with base radius, r and start height, l

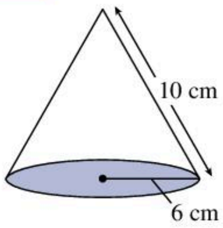


The **curved surface area** $= \pi rl$

The base area $= \pi r^2$

Total surface area $= \pi rl + \pi r^2$

Example 9



Find the

(a) curved surface area

(b) total surface area of a solid cone with base radius 6 cm and slant height 10 cm.

(a) Curved surface area of cone

$= \pi rl$

$= 3.14 \times 6 \text{ cm} \times 10 \text{ cm}$

$= 188.4 \text{ cm}^2$

(b) Base area of cone

$= \pi r^2$

$= 3.14 \times 6^2$

$= 113.04 \text{ cm}^2$

Total surface area of cone

$= 188.4 + 113.04$

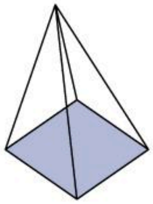
$= 301.44 \text{ cm}^2$

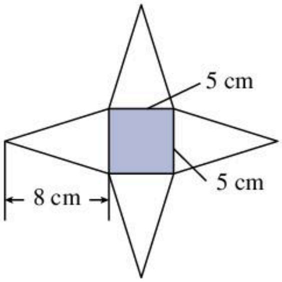
Exercise 8H

Give your answers to three significant figures where appropriate.

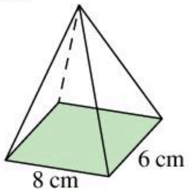
1 Find the surface area of the pyramids made from the nets with given dimensions.

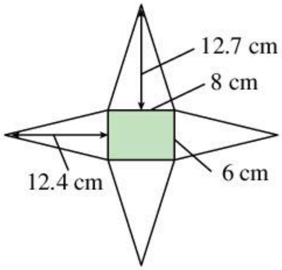
(a)



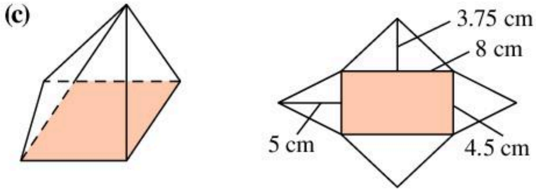


(b)





(c)

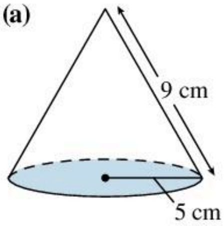


2 Find the surface area of a regular tetrahedron given one of its faces has area 8 cm^2.

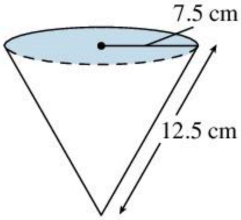
3 Find the curved surface area of a cone with base radius 4.3 cm and slant height 7.6 cm.

4 Find the total surface area of the solid cones shown.

(a)



(b)



Exercise 8I – mixed questions

Give your answers to three significant figures where appropriate.

1 Convert:

- **(a)** 2 m^3 to cm^3
- **(b)** 5 cm^3 to mm^3
- **(c)** 4300 mm^3 to cm^3
- **(d)** 50 000 cm^3 to m^3.

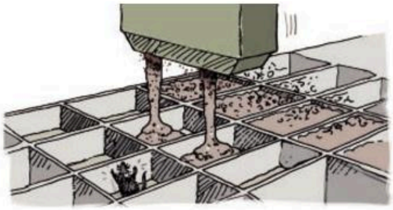
2 A cube with side 6 cm has each of its sides increased in length by 50%.
By what percentage does the surface area of the cube increase?

3
- **(a)** Which has the greater volume, a cube of side 4 cm or a 3 cm by 3 cm by 7 cm cuboid?
- **(b)** How many cm^3 are in a cube of side 3 cm?
- **(c)** How many cubes of side 2 cm can be packed in a box 50 cm × 20 cm × 6 cm?

4
- **(a)** A cube has volume 2.43 cm^3. What is its total surface area?
- **(b)** The surface area of a cube is 128 cm^3. What is its volume?

5
- **(a)** A room is 3 m high and has a volume of 45 m^3.
  Calculate the area of the floor of the room.
- **(b)** A dormitory is 10 m × 6 m × 3 m. How many people can sleep in the dormitory if each person needs 18 m^3 of air space?

6 A concrete block is made by pouring 720 cm^3 of concrete into a 12 cm × 10 cm rectangular tray.



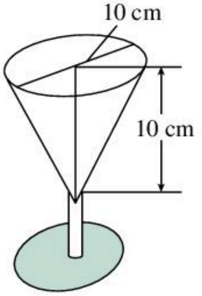
How thick is the block?

7 How many boxes measuring 25 cm × 30 cm × 10 cm can be packed into a large carton measuring 1 m × 60 cm × 50 cm?

8 A container measures 120 cm × 80 cm × 60 cm. The container is $\frac{9}{10}$ full of water.

- **(a)** How many blocks measuring 20 cm × 15 cm × 12 cm should be put in the container to make the water reach the top?
- **(b)** How many litres of water would be needed instead of the blocks to fill the tank?

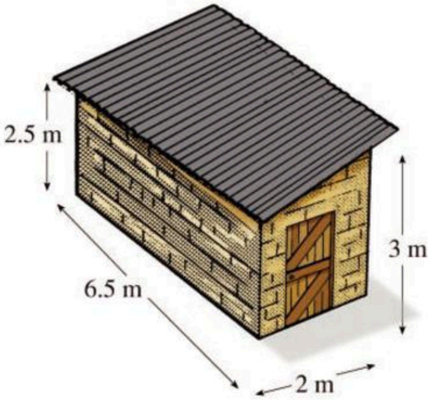
9



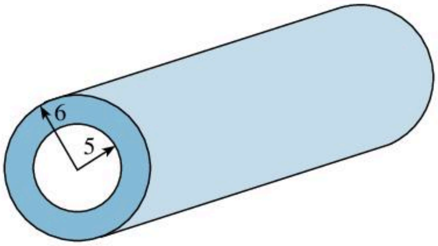
The top of wine glass is in the shape of a cone with diameter 10 cm and vertical height 10 cm.

- **(a)** What is the volume of the glass?
- **(b)** 250 ml of wine is poured into the glass. What height does the wine reach in the glass?

10 A gardener's shed has the dimensions shown in the diagram. Find the volume of the shed.

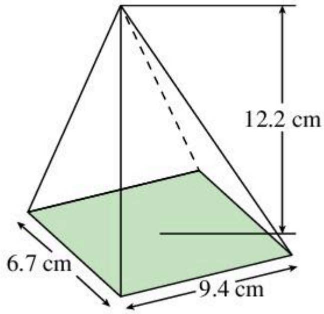


11



Find the volume of plastic needed to make a pipe of length 480 cm whose inner radius is 5 cm and outer radius is 6 cm.

12



The diagram shows a solid rectangular-based pyramid with base dimensions 6.7 cm by 9.4 cm and vertical height 12.2 cm.

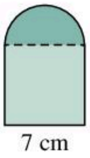
(a) Find the volume of the pyramid.

(b) If the pyramid is made from a substance with density 3.4 g/cm^3, find the mass of the pyramid.

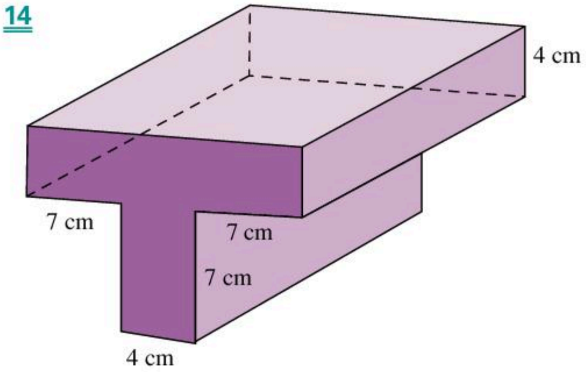
13 The cross-section of a loaf of bread consists of a square of side 7 cm joined to a semicircle.

(a) Calculate the area of the cross section.

(b) Calculate the volume if the length of the loaf is 30 cm.



14



The volume of the steel girder in the diagram is 1200 cm^3.
Find its length.

15 **(a)** A cylindrical glass has a base radius of 3 cm. 200 ml of water is poured into the glass. Find the height to which the water reaches up the side of the glass. Take $\pi = 3.14$.

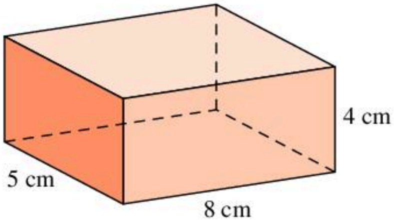
(b) If in part (a) the height of the glass is 12 cm, find:

- **(i)** the volume left in the glass above the water
- **(ii)** the number of ice cubes of side 3 cm which would have to be added to the water to cause it just to overflow. (Assume the ice cubes float beneath the surface of the water.)

8 Consolidation

Example 1

Find the volume of:



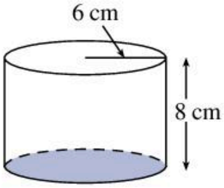
Volume of cuboid $= \text{base area} \times \text{height}$

$= 5 \text{ cm} \times 8 \text{ cm} \times 4 \text{ cm}$

$= 160 \text{ cm}^3$

Example 2

Find the volume of:



Volume of cylinder $= \text{base area} \times \text{height}$

$= \pi r^2 \times h$

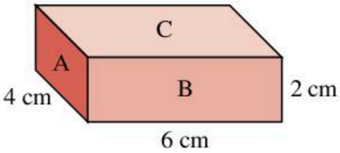
$= 3.14 \times 6^2 \times 8 \text{ cm}^3$

$= 904.32 \text{ cm}^3$

Example 3

Find the surface area of:

(a)



Area of A $= 4 \text{ cm} \times 2 \text{ cm} = 8 \text{ cm}^2$

Area of B $= 6 \text{ cm} \times 2 \text{ cm} = 12 \text{ cm}^2$

Area of C $= 6 \text{ cm} \times 4 \text{ cm} = 24 \text{ cm}^2$

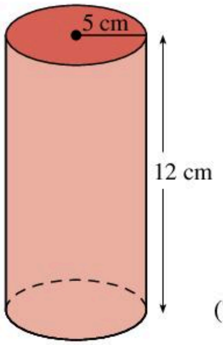
Total surface area $= 2 \times A + 2 \times B + 2 \times C$

$= 2 \times 8 + 2 \times 12 + 2 \times 24 \text{ cm}^2$

$= 16 + 24 + 48 \text{ cm}^2$

$= 88 \text{ cm}^2$

(b)



(Take $\pi = 3.14$

Curved surface area $= 2\pi rh$

$= 2 \times 3.14 \times 5 \times 12 \text{ cm}^2$

$= 376.8 \text{ cm}^2$

Area of circle $= \pi r^2$

$= 3.14 \times 5 \times 5 \text{ cm}^2$

$= 78.5 \text{ cm}^2$

Total surface area $= \text{area circle} \times 2 + \text{curved area}$

$= 2 \times 78.5 + 376.8 \text{ cm}^2$

$= 157 + 376.8 \text{ cm}^2$

$= 533.8 \text{ cm}^2$

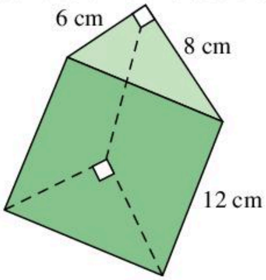
Exercise 8

Give your answers to three significant figures where appropriate.

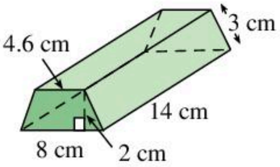
1 Estimate the:
- **(a)** mass of a pen in grams
- **(b)** length of a new pencil in cm
- **(c)** mass of a box of matches
- **(d)** mass of a 5 year old child
- **(e)** length of your stride in m
- **(f)** mass of a bicycle in kilograms
- **(g)** distance from your class to the school gate
- **(h)** distance from your home to the sea.

2 Find the volume of these shapes:

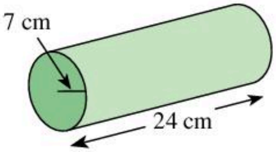
(a)



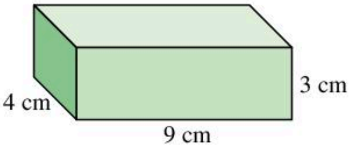
(b)



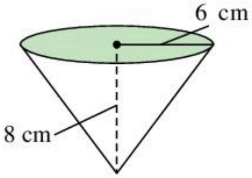
(c)



(d)



(e)



3 Calculate the total surface area of each of the shapes in Question 2.

4

The label on a tin of milk is 15.30 cm long and 13 cm high.

Find

(a) the circumference of the tin of milk

(b) the radius of the tin

(c) the volume of milk in the tin.

5 Antoine wishes to build a store room with a rectangular base 3.8 m wide and 2.7 m long. The room will have walls that are 2.3 m high.

(a) What volume of goods could he store in the room?

(b) What is this volume to one decimal place?

(c) A bag of cement has a volume of about 0.025 m^3. About how many bags of cement could Antoine's store room hold?

6 A cylindrical water tank has base radius 60 cm and height 1.8 m.

(a) What is the volume of the water tank in m^3?

(b) How many litres of water can the tank hold?

(c) How high will water reach up the side of the tank if the tank has only 600 litres of water?

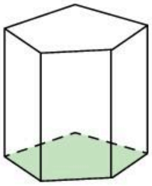
(d) What is the curved surface area of the tank?

Summary

You should know ...

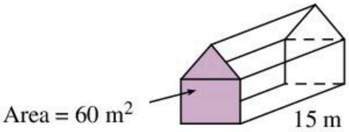
1 A prism is a solid shape with a constant cross section in the form of a polygon.

For example:



2 The volume of a prism or cylinder is $A \times h$, where A is the area of the cross section and h is the height or distance between the flat faces.

For example:



$$\text{Volume} = 60 \times 15 \text{ m}^3$$
$$= 900 \text{ m}^3$$

3 The volume, v, of a pyramid or cone is

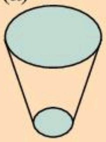
$$V = \tfrac{1}{3} A \times h$$

where A = area of base and h is the perpendicular height.

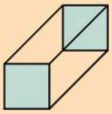
Check out

1 State which of the following is a prism.

(a)

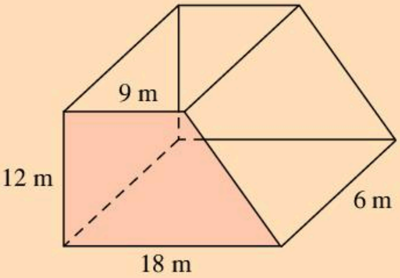


(b)

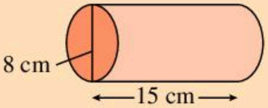


(c)

2 **(a)** Find the volume of this prism.

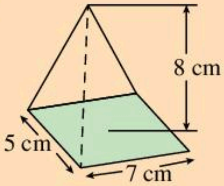


(b) Find the volume of the cylinder

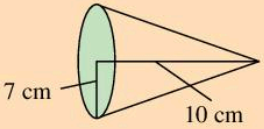


3 Find the volume of

(a)



(b)



4 The area of the total surface of a solid is called its surface area.

4 Find the total surface area of a:
- **(a)** cuboid with length 7 m, width 6 m and height 4 m
- **(b)** cylinder with base radius 7 cm and height 15 cm. (Take $\pi = 3.14$)

Revision exercise 2

Algebra

1 Simplify:

(a) $3x + 2y + 5x - 4y + x$

(b) $7a - b + 3c - 2a - 3b + 5c$

(c) $3x^2 \times x^3 - 7y^4 \times y + z^3 \times z^2 \times z$

(d) $5(2x + y) - 4(x - 5y)$

(e) $3x(y + z) - y(z + x) + 2z(x + y)$

2 Factorise:

(a) $6x + 8y$

(b) $ar - 2br$

(c) $4ac - 3ab + 6ad$

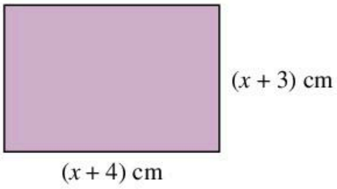
(d) $4a^2bc - 3abc + 5ab^2c$

3 Simplify:

(a) $\dfrac{3}{x} + \dfrac{2}{x + 1}$ **(b)** $\dfrac{4}{x - 1} + \dfrac{3}{x + 1}$

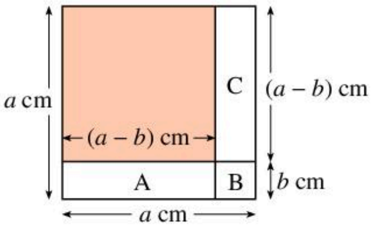
(c) $6 - \dfrac{5}{3 + 2x}$ **(d)** $\dfrac{4x + 2}{5} - \dfrac{3x - 1}{4}$

4 **(a)** Show that the area of the rectangle below can be written as:
$(x^2 + 7x + 12)\ \text{cm}^2$



(b) Find an expression for the area of a rectangle $(x + 5)$ cm long and $(x + 7)$ cm wide.

5 The area of the shaded square below can be written as $(a - b) \times (a - b)\ \text{cm}^2$.



Find the area of the rectangles A, B and C, and then by subtracting these areas from the area of the large square show that the area of the shaded square can also be written as $a^2 - 2ab + b^2$.

6 Simplify:

(a) $(x + 3)(x + 5)$ **(b)** $(x - 3)(x + 5)$

(c) $(x - 3)(x - 5)$ **(d)** $(x + 5)(x - 5)$

7 Solve these pairs of simultaneous equations.

(a) $3x - 4y = {}^{-}1$
$2x + 3y = 5$

(b) $5x + 2y = 11$
$3x - 5y = {}^{-}12$

(c) $4x - 5y = {}^{-}3$
$6x - 7y = {}^{-}4$

(d) $7x + 4y = 9$
$5x + 12y = 11$

8 Use the balance idea to help you to solve:

(a) $3x + 5 = x + 19$ **(b)** $5x - 2 = 4x + 11$

(c) $4 + 3(x - 2) = 10 - x$ **(d)** $7x + 2 < 23$

(e) $8x - 5 > 5x + 13$

9 A number is doubled and then seven is added giving a result of 37. If n is the number, write an equation to show this information. Solve your equation to find the number.

10 Joe Old is 23 years older than his daughter Rita, and 28 years older than his son Clive. The total of their three ages is 51 years.
If Clive's age is c years, write down Rita's age and Joe's age, in terms of c.
Write an equation to show that their total age is 51 years.
Solve your equation to find each of their ages.

11 An adult's bus ticket costs 50 cents more than a child's ticket.
The total cost of 3 adult's tickets and 5 children's tickets is \$7.90.

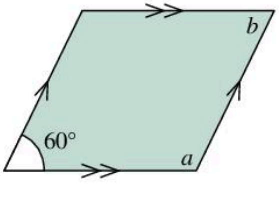
(a) If a child's ticket costs c cents write an equation to show that the total cost is \$7.90.

(b) Solve your equation to find the cost of each ticket.

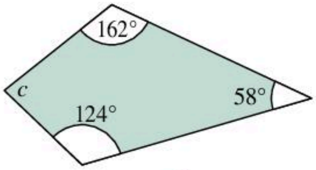
Geometry

12 Find the size of the lettered angles.

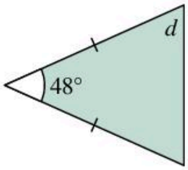
(a)



(b)



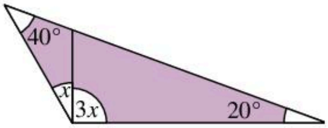
(c)



13 Draw a quadrilateral with:

(a) one line of symmetry
(b) two lines of symmetry
(c) four lines of symmetry.

14 In the diagram, find the value of x.



15 Write down four properties of a:

(a) rhombus **(b)** equilateral triangle.

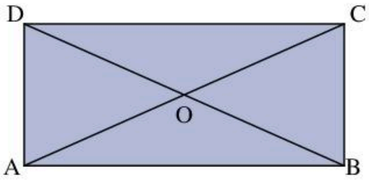
16 Using a ruler and compasses only, construct angles of:

(a) 30° **(b)** 120° **(c)** 150°

17 Using a ruler and compasses only, construct:

(a) an equilateral triangle with side 7 cm
(b) a square with side 6 cm
(c) a rhombus ABCD with $\hat{ABC} = 120°$ and AB = 8 cm.

18



ABCD is a rectangle, with diagonals AC and BD meeting at O.

(a) Is triangle DOA congruent to triangle COB?
(b) Identify two other pairs of congruent triangles.

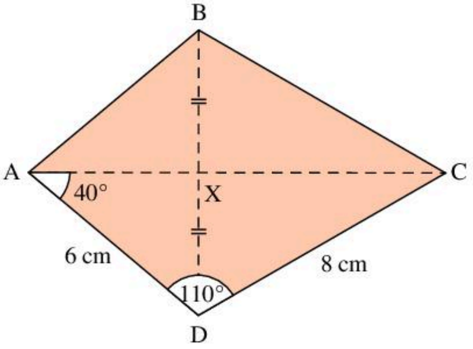
19 Given K = {kites}
R = {rhombuses}
P = {parallelograms}
and Q = {quadrilaterals}

(a) Draw a Venn diagram to show Q, P, R and T.
(b) What is
(i) $Q \cap R$ **(ii)** $P \cup R$ **(iii)** $K \cap Q$?

20 A regular polygon has 30 sides. Find:

(a) the angle sum
(b) the interior angle of the polygon.

21



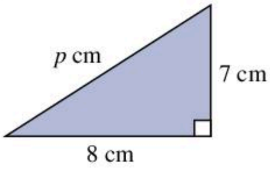
In the kite ABCD,
AD = 6 cm, BD = 10 cm, CD = 8 cm, $A\hat{D}C = 110°$ and $D\hat{A}C = 40°$. Find:

(a) $C\hat{A}B$
(b) $A\hat{B}C$
(c) AB

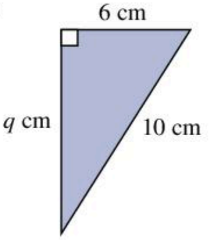
Trigonometry

22 Find the value of the letters in the diagrams.

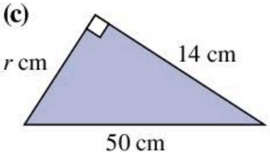
(a)



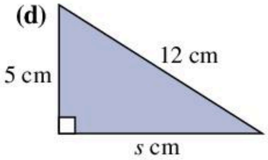
(b)



(c)

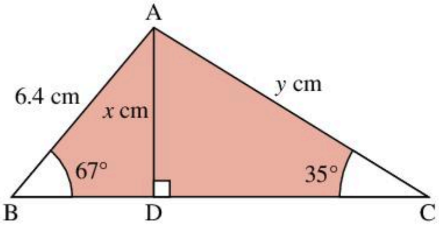


(d)

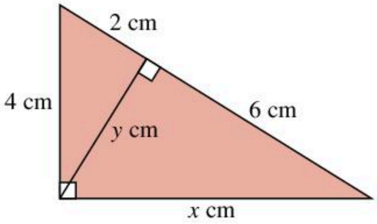


23 Find the value of the letters in the diagrams.

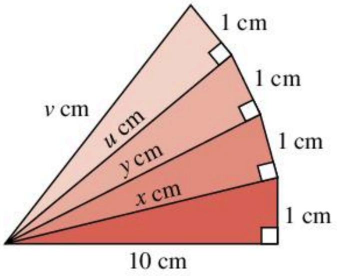
(a)



(b)



(c)



24 A man wishes to reach the top of a tree using a ladder. If the ladder is 6 m long and the tree is 5 m tall, how far will the foot of the ladder be from the tree?

25 A sailor travels 25 km due North and then 32 km due East. How far is he from his starting point? How much further will he have travelled in an Easterly direction when he is 50 km from his starting point?

26 Find the length of the largest rod which will fit inside a cuboid 5 cm by 6 cm by 4 cm.

27 Use a calculator to find:

(a) tan 32° **(b)** sin 49° **(c)** cos 68°

(d) tan 14.3° **(e)** sin 61.4° **(f)** cos 22.6°

28 Use a calculator to find the angle $x°$ when:

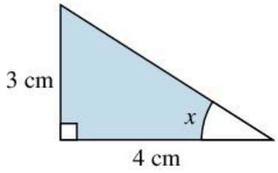
(a) $\tan x° = 1.54$ **(b)** $\tan x° = 0.390$

(c) $\sin x° = 0.515$ **(d)** $\sin x° = 0.953$

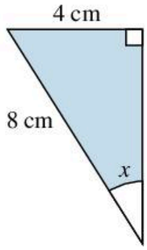
(e) $\cos x° = 0.707$ **(f)** $\cos x° = 0.289$

29 Find the size of angle x in each triangle:

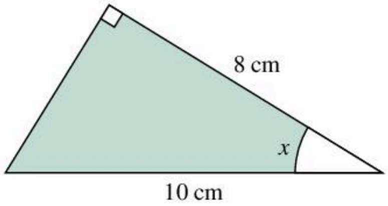
(a)



(b)

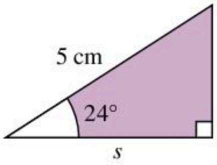


(c)

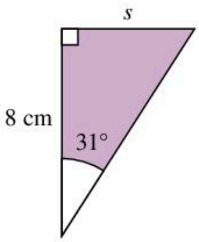


30 Find the length of side s in each triangle:

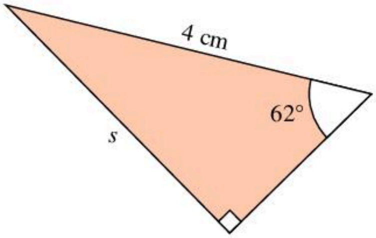
(a)



(b)

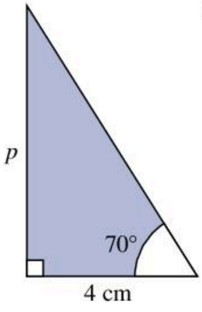


(c)

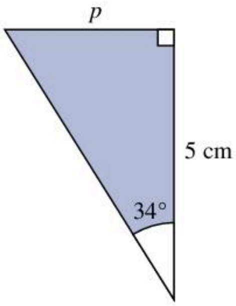


31 Find the length of side p in each triangle:

(a)



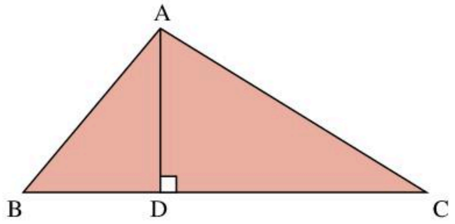
(b)



32 A pilot flies 120 km due North and then 160 km due East. Find the bearing (the angle measured clockwise from the north) of his position from his starting point.

33 A wireless mast is 35 m tall and is secured by a wire from the top to the ground.
If the wire is 40 m long, what angle does it make with the ground?
If the wire were actually fixed to a point 5 m from the top of the mast what angle would it then make with the ground?

34 In the diagram below AB is 6.4 cm and $A\hat{B}D = 67°$ and $A\hat{C}D = 35°$.

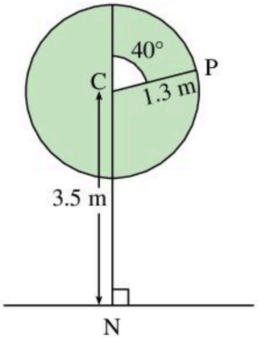


Find:

- **(a)** the length of BD
- **(b)** the length of AD
- **(c)** the length of DC.

35 A boy 1.6 m tall stands 32 m from a building. Using an 'angle finder' like the one on page 70 he finds the angle of elevation to the top of the building is 25°. Find the height of the building.

36 A wheel of radius 1.3 m is fixed to the top of a pole 3.5 m above the ground. A point P lies on the circumference of the wheel and the line joining P to the centre C of the wheel makes an angle of 40° with the vertical. Find the height of P above the ground.



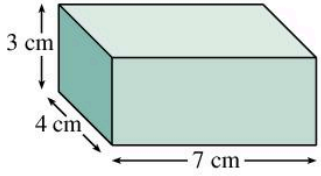
Measurement 2

37 Convert:

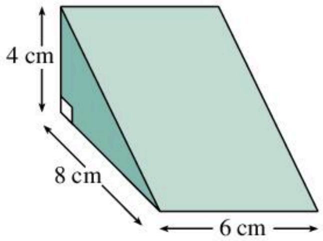
- **(a)** 1 m^3 to cm^3
- **(b)** 0.05 m^3 to cm^3
- **(c)** $200\,000 \text{ cm}^3$ to m^3
- **(d)** 400 mm^3 to cm^3

38 What is the volume of these prisms?

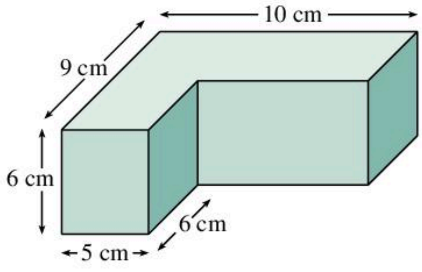
(a)



(b)



(c)



39 What is the total surface area of:

(a) a cuboid with dimensions 5 cm × 6 cm × 7 cm

(b) a cylinder with radius 9 cm and height 12 cm? (Take $\pi = 3.14$)

40

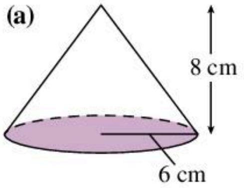
A cylindrical tin holds 500 cm^3 of orange juice.

(a) What is the height of the tin if its radius is 6 cm?

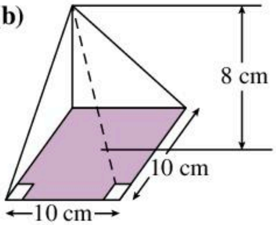
(b) What is the area of the label on the tin? (Take $\pi = 3.14$)

41 Find the volume of these solids

(a)



(b)



42 The total surface area of a cube is 181.5 cm^2. What is the volume of the cube?

43 How many litres of water can be placed in a:

(a) cuboidal tank with base area 2 m^2 and height 1 m

(b) cylindrical tank with base area 2.5 m^2 and height 2 m?

44 How many cereal boxes with dimensions 20 cm × 10 cm × 30 cm can be stored in a crate with dimensions 1 m × 1 m × 0.6 m?

45 400 ml of water are poured into a cylindrical glass with radius 7 cm. How far up the glass will the water reach? (Take $\pi = \frac{22}{7}$)

Mixed questions 2

1 Solve: $3x + 2(x + 1) = 7$

A 1 **B** $1\frac{1}{5}$ **C** $1\frac{3}{5}$ **D** 2

2 Simplify $2a^3 \times 3a^2$

A $5a^6$ **B** $6a^6$ **C** $6a^5$ **D** $5a^5$

3 Solve this equation: $3x + 1 = 25$

A 5 **B** 6 **C** 7 **D** 8

4 Solve: $\frac{x}{4} - 1 = 2$

A 4 **B** 9 **C** 12 **D** 84

5 A rectangular field has length 80 m. What is its width if the perimeter is 240 m?

A 3 m **B** 40 m **C** 80 m **D** 160 m

6 Simplify $\frac{4b^4}{2b^2}$

A $4b^4$ **B** $2b^2$ **C** $2b^6$ **D** $4b^6$

7 $3(2p - 5) =$

A $6p - 5$ **B** $6p - 15$

C $6p + 15$ **D** $2p - 15$

8 $3a + 2(5a - 6) =$

A $7a + 12$ **B** $7a - 12$

C $13a + 12$ **D** $13a - 12$

9 $8 + (p - 3) = 6$, then $p =$

A $^{-}1$ **B** 1 **C** $^{-}5$ **D** 5

10 When factorised $x^2 - 5x + 6$ is

A $(x - 2)(x - 3)$ **B** $(x - 2)(x + 3)$

C $(x + 2)(x - 3)$ **D** $(x + 2)(x + 3)$

11 The values of x and y that satisfy the equations

$4x - 3y = {}^{-}1$

$6x + 5y = 8$

are

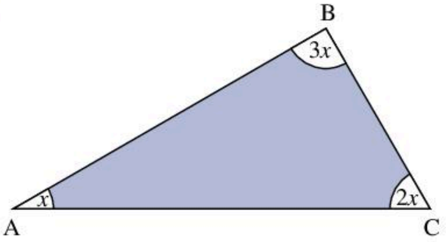
A $x = \frac{1}{2}, y = 1$ **B** $x = 1, y = \frac{1}{2}$

C $x = 1, y = 1$ **D** $x = \frac{1}{2}, y = 2$

12 If $a * b = a^2 - b^2 + 3$ then $4 * 5 =$

A $^{-}6$ **B** 1 **C** 6 **D** 44

13

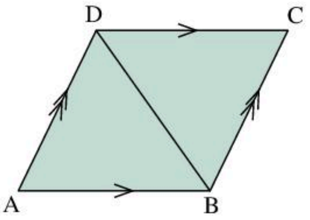


What is the size of $A\hat{B}C$?

A 30° **B** 60° **C** 90° **D** 120°

14 Three angles of a quadrilateral are 38°, 102° and 50°. What is the fourth angle?
A 10° **B** 40° **C** 170° **D** 190°

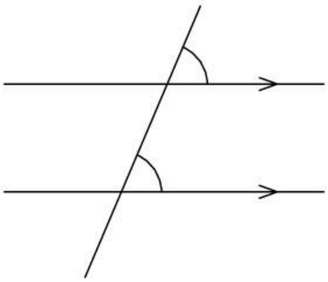
15



ABCD is a parallelogram with diagonal DB. Which statement is true?

A $A\hat{D}C + A\hat{B}C = 180°$
B $A\hat{D}B = D\hat{C}B$
C $A\hat{B}D = B\hat{D}C$
D $B\hat{A}D + D\hat{C}B = 180°$

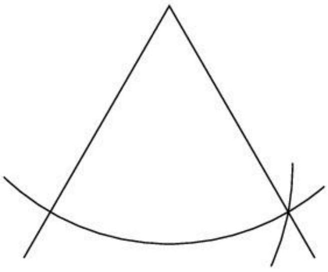
16



The two marked angles are:

A alternate **B** complementary
C corresponding **D** supplementary

17



The diagram shows compass marks for the construction of:

A a 45° angle **B** a 60° angle
C an angle bisector **D** two parallels

18 What is the sum of the interior angles of a hexagon?
A 360° **B** 540° **C** 720° **D** 900°

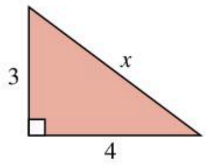
19 Which angle can be constructed with a ruler and compasses only?
A $22\frac{1}{2}°$ **B** $45\frac{1}{2}°$ **C** $66\frac{1}{2}°$ **D** $87\frac{1}{2}°$

20 A four-sided shapes with one line of symmetry and whose diagonals bisect at right angles is a:

A kite **B** rhombus
C rectangle **D** square

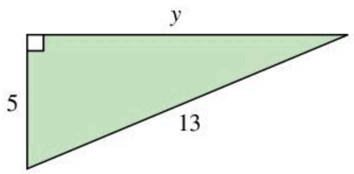
21 How many lines of symmetry does a regular pentagon have?
A 3 **B** 4 **C** 5 **D** 6

22 Find the missing length:



A 7 **B** 6 **C** 5 **D** 25

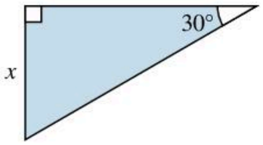
23 Find the missing length:



A 12 **B** 14 **C** 8 **D** 18

24 The two shorter sides of a right-angled triangle are 7 cm and 24 cm. Find the length of the hypotenuse.
A 72 **B** 31 **C** 24 **D** 25

25 What is the name of the side marked x?

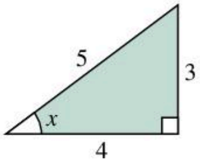


A adjacent **B** opposite
C hypotenuse **D** acute

26 What is the name of the longest side in a right-angled triangle?

A adjacent **B** opposite
C hypotenuse **D** acute

27 What is the sine of the angle x?

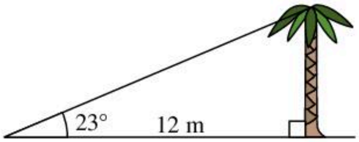


A 3 **B** 4 **C** $\frac{3}{5}$ **D** $\frac{4}{5}$

28 What is the cosine of the angle x in question 27?

A $\frac{5}{3}$ **B** $\frac{5}{4}$ **C** $\frac{3}{5}$ **D** $\frac{4}{5}$

29

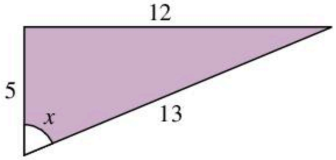


In the diagram the height of the tree is

A 12 cos 23° **B** 12 sin 23°

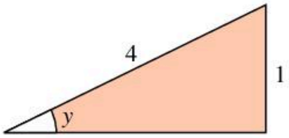
C 12 tan 23° **D** 12 ÷ tan 23°

30 What is angle x?



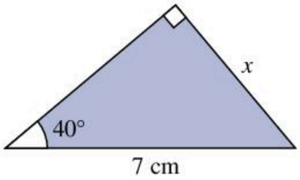
A 23° **B** 90° **C** 43° **D** 67°

31 Find angle y:



A 14° **B** 76° **C** 4° **D** $\frac{1}{4}^{\circ}$

32 Find x:



A 4.5 cm **B** 5.4 cm **C** 7 cm **D** 5.9 cm

33 The surface area of a cube is 150 cm². What is its volume?

A 5 cm³ **B** 25 cm³

C 100 cm³ **D** 125 cm³

34 A refrigerator has a volume of 1.5 m³. How many cm³ is this?

A 15 **B** 15 000

C 150 000 **D** 1 500 000

35 The surface area of a cuboid with dimensions 2 cm × 3 cm × 4 cm is:

A 24 cm² **B** 26 cm²

C 48 cm² **D** 52 cm²

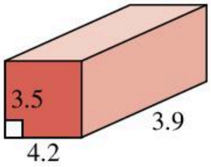
36 A cylindrical tank has base area 2 m². How much water can it hold if its height is 1 m?

A 2 litres **B** 20 litres

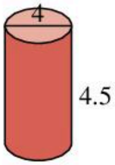
C 200 litres **D** 2000 litres

37 The solid with the smallest volume is (all lengths are in metres)

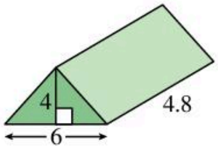
A



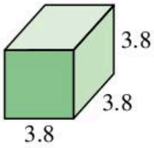
B



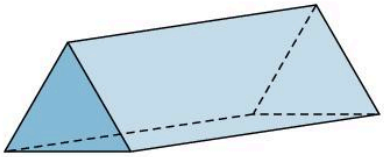
C



D



38 Describe this shape.



A cylinder **B** triangle

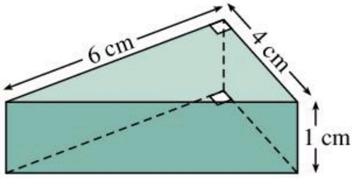
C prism **D** pyramid

39 Find the volume of a cylinder with radius 3 cm and length 8 cm.

A 12 cm³ **B** 24 cm³

C 72 cm³ **D** 226 cm³

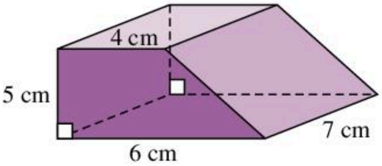
40 Find the volume of this shape.



A 12 cm^3 **B** 24 cm^3

C 6 cm^3 **D** 96 cm^3

41 Find the volume of this shape.



A 210 cm^3 **B** 175 cm^3

C 350 cm^3 **D** 840 cm^3

Functions and graphs

9

Objectives

- ✔ find and plot points on a graph using a variety of scales
- ✔ read information from graphs to solve problems
- ✔ distinguish between relations and functions
- ✔ use function notation
- ✔ draw and use graphs of linear functions
- ✔ find gradient and intercept of straight-line graphs
- ✔ use graphs to solve simultaneous linear equations
- ✔ draw and interpret graphs of quadratic functions

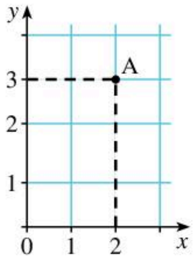
What's the point?

Drawing graphs helps you to identify patterns in numbers more easily. From graphs you can make simple predictions and use them to answer such questions as: How much rainfall is expected in September? Will sales increase in March? How much should my baby weigh next month?

Before you start

You should know ...

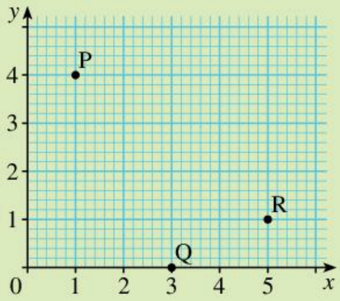
1 How to write down the coordinates of plotted points.
For example:



The coordinates of A are (2, 3).

Check in

1



Write down the coordinates of P, Q and R.

2 How to convert fractions to decimals.
For example:
$\frac{1}{10} = 0.1$

2 Convert these fractions to decimals.

(a) $\frac{1}{8}$ **(b)** $\frac{7}{8}$

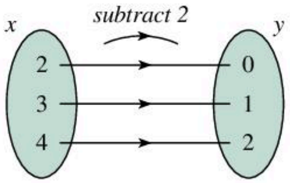
(c) $\frac{9}{11}$ **(d)** $\frac{2}{3}$

3 How to substitute values into a formula.
For example:
If $y = 2x + 3$, then the value of y when $x = 4$ is
$y = 2 \times 4 + 3 = 8 + 3 = 11$

3 Copy and complete the table of values for the formula $y = 3x - 2$

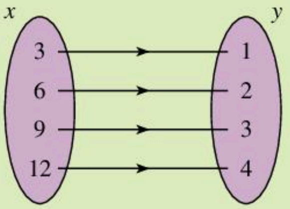
x	$^{-}2$	$^{-}1$	0	1	2	3
y	$^{-}8$				4	

4 Mappings can be represented as ordered pairs.
For example:
The mapping *subtract 2* can be shown on an arrow graph as:



The ordered pairs are: (2, 0), (3, 1) and (4, 2).

4



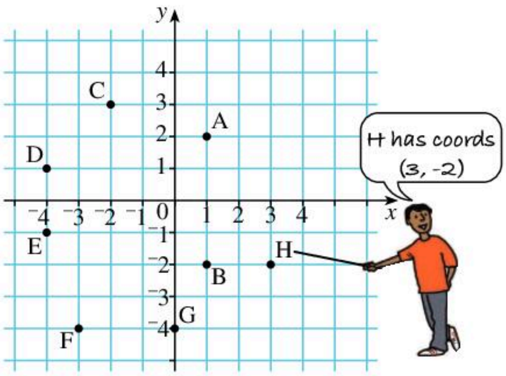
Write down the ordered pairs for the arrow graph.

9.1 Plotting points

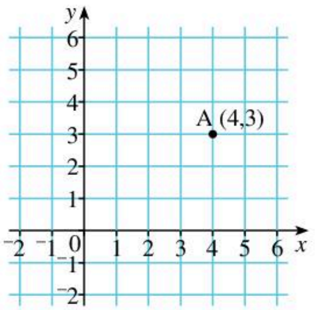
You will need graph paper.

Exercise 9A

1 Write down the coordinates of these lettered points.



2 Make a copy of the graph below. Show the position of the points with coordinates:
B(4, 1), C(4, $^{-}2$), D(4, 6), E(2, 3), F(0, 3), G($^{-}2$, 3)



(a) What can you say about the points A, B, C and D?

(b) What can you say about the points A, E, F and G?

3 **(a)** On graph paper with the x-axis numbered from ${}^-5$ to 5 and the y-axis numbered from ${}^-6$ to 6, plot the following points:
$(2, {}^-1), (2, {}^-5), (3, {}^-4), (5, {}^-4), (4, {}^-5),$
$({}^-3, {}^-5), ({}^-1, {}^-4), ({}^-1, {}^-2), ({}^-3, 0),$
$({}^-2, 1), ({}^-4, 3), ({}^-4, 6), ({}^-2, 5), (0, 6), (0, 3),$
$({}^-1, 2), (2, {}^-1).$

(b) Join the points in order with straight lines.

(c) Suggest a name for your picture.

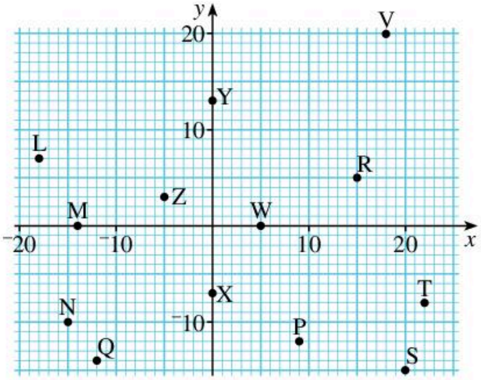
4 Draw x- and y-axes and plot the points:
$A({}^-4, 2)$, $B({}^-1, {}^-2)$, $C(3, {}^-2)$, $D(3, 1)$, $E(0, 5)$, $F({}^-1, 3)$, $G({}^-1, 1)$ and $H({}^-1, 0)$

(a) What type of quadrilaterals are CDFH, ABDE and GBCD?

(b) What sort of triangles are BDF, EFD and AFE?

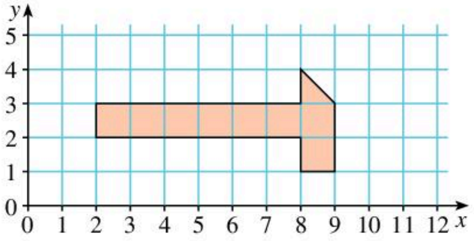
(c) Which two triangles are identical but in different positions?

5



Name the points with the following coordinates:
$(15, 5), (18, 20), (20, {}^-15), (9, {}^-12), (5, 0),$
$(0, 13), ({}^-12, {}^-14), ({}^-14, 0), ({}^-18, 7),$
$({}^-15, {}^-10), (0, {}^-7), ({}^-5, 3), (22, {}^-8)$

6 Write down the coordinates of the eight vertices of this shape:



Activity

A coordinate game for the whole class

Arrange the desks in your class in rows and columns. Number the rows and columns so that each student is sitting at a coordinate (row, column).

The game

A student calls out an ordered pair. The student sitting in that position stands up and calls out another ordered pair. Any student who fails to stand when his or her ordered pair is called is out of the game. Also, any student calling out an ordered pair which is an empty or non-existent seat is out of the game.

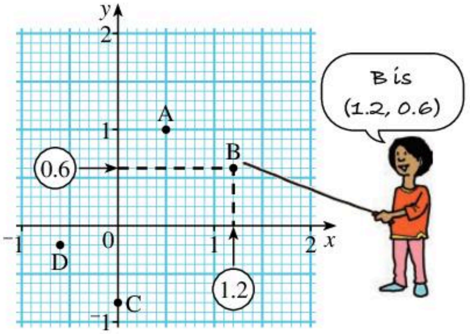
Scales and decimals

- The way in which numbers are marked on an axis is called the **scale**.

The divisions on the scale are not always whole numbers.

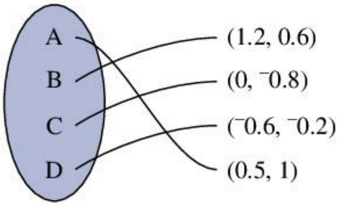
Example 1

What are the coordinates of the points A, B, C and D on the graph below?



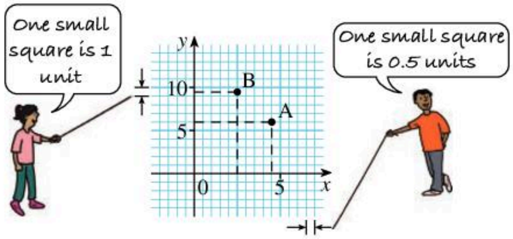
The interval between 0 and 1 is divided into tenths.
Each small division on both axes represents 0.1

Look carefully at the graph to see why B has coordinates (1.2, 0.6) The coordinates for the other three points can then be matched.



Example 2

What are the coordinates of the points A and B?



The interval between 0 and 5 on the *x*-axis is divided into 10 small squares so 2 squares represent 1 unit. The interval between 0 and 5 on the *y*-axis is divided into 5 small squares so 1 square represents 1 unit.

A represents (4.5, 6)

B represents (2.5, 9.5)

Exercise 9B

1 **(a)** On centimetre graph paper as in Example 2 above what does each small division represent if

(i) 1 cm represents 2 units

(ii) 2 cm represents 1 unit

(iii) 1 cm represents 5 units?

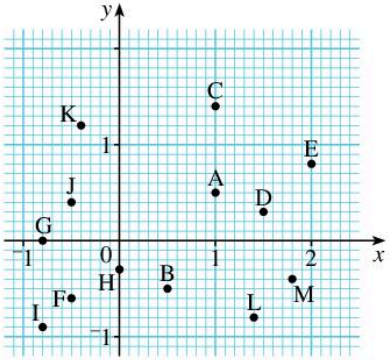
(b) How many small divisions will represent 0.1 if

(i) 4 cm represents 1 unit

(ii) 8 cm represents 1 unit

(iii) 6 cm represents 1 unit?

2 Write down the coordinates of the points shown in the graph.



3 **(a)** Copy the axes in Question 2 on graph paper. Plot the following points:

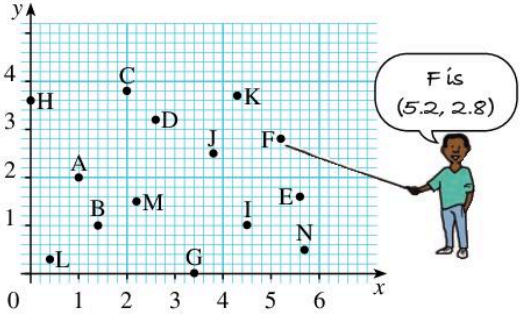
(0.2, 0.8), (0.2, 1.2), (0.6, 1.6), (1, 1.2), (1, 0.8), (1.4, 0.8), (1.4, 1.2), (1.8, 1.4), (2, 1.2), (1.8, 1.2), (1.8, 0.8), (1.6, 0.6), (1.6, 0), (1.4, 0.4), (1, 0), (1, 0.2), (0.6, 0.6), (0.6, 0.4), (0.2, 0.8)

(b) Join the points in order with straight lines.

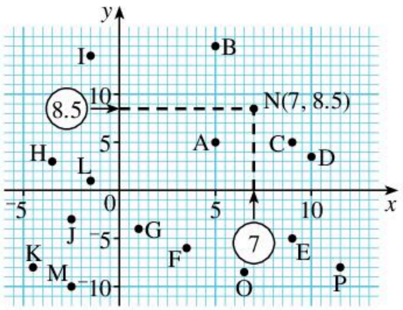
(c) Suggest a name for the picture you have made.

4 **(a)** On the graph how many small divisions are there between 1 and 2? What does each division represent?

(b) Write down the coordinates of the points shown.



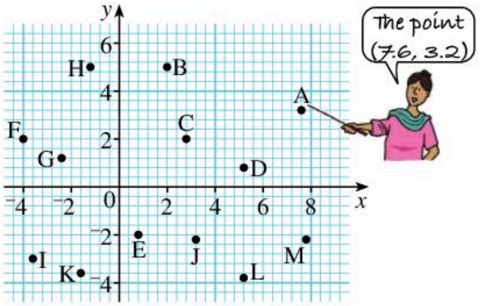
5



(a) What does each small division on the y-axis represent?
(b) What does each small division on the x-axis represent?
(c) Write down the coordinates of the points shown.

6 **(a)** Copy on graph paper the axes in Question 5, using the same scale.
(b) Plot these points:
A(1, 4), B(3, 7), C($^{-}3$, 8), D($^{-}5$, $^{-}5$), E(1.5, 7), F($^{-}4.5$, $^{-}9$), G(7.8, $^{-}2$), H($^{-}2.3$, 6.8), I($^{-}4.9$, $^{-}2.9$), J(6.7, $^{-}3.7$)

7 **(a)** What does each small division on each axis represent in this graph?



(b) Do you agree that the point marked A has coordinates (7.6, 3.2)?
(c) Write down the coordinates of the other points.

8 Use a scale of 4 cm to represent 1 unit on both the x- and y-axes on your graph paper. Number both axes from 0 to 2.
(a) Plot each of the following points and join the points in order with straight lines.
(1.1, 0.9), (1.1, 1), (1, 1.1), (0.8, 1.1), (0.7, 1), (0.7, 0.8), (0.8, 0.7), (0.9, 0.7), (0.9, 0.5), (1, 0.5), (1.4, 0.4), (1.7, 0.3), (1.8, 0.2), (1.8, 0.1), (1.7, 0.2), (1.4, 0.3), (1, 0.4), (0.8, 0.4), (0.4, 0.3), (0.1, 0.2), (0, 0.1), (0, 0.2), (0.1, 0.3), (0.4, 0.4), (0.8, 0.5), (0.9, 0.5)
(b) What have you drawn?

Activity

Each of these graphs uses a different scale, but the figure looks the same.
In each case write down the coordinates of A, B, C and D.
(*Note:* the scale on the x-axis is not necessarily the same as the scale on the y-axis.)

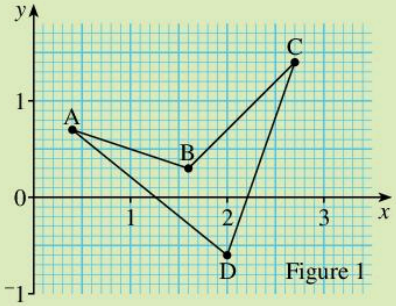


Figure 1

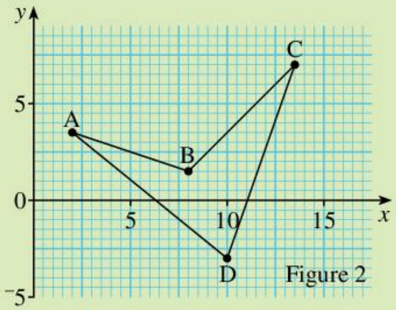


Figure 2

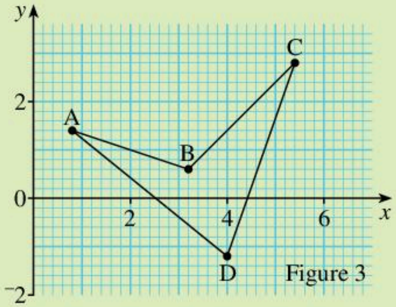


Figure 3

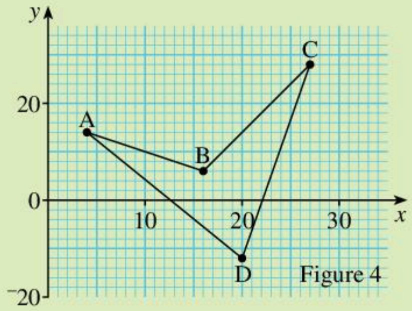


Figure 4

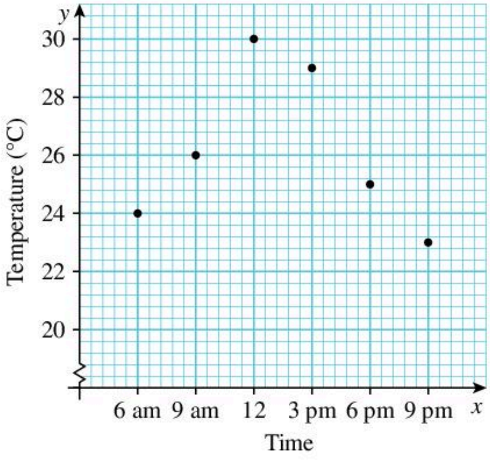
Are the coordinates of A, B, C and D the same in each figure? Explain.

9.2 Reading line graphs

Line graphs are a way of displaying data that is connected or related in some way.

For example, a graph may show how the temperature on a particular day is related to the time of day.

Time	6 am	9 am	12 noon	3 pm	6 pm	9 pm
Temperature (°C)	24	26	30	29	25	23



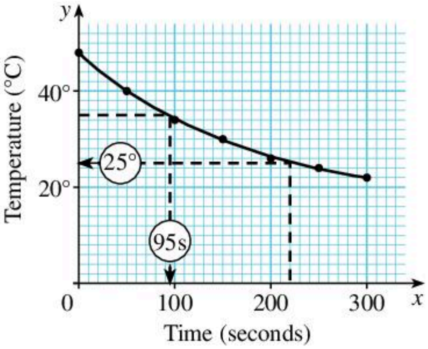
Note that

Temperature (6 am) = 24°C
Temperature (9 am) = 26°C etc.

If it makes sense, you can join the points on a graph to make a curve. Then you can read off values between the points.

Example 3

The temperature of some hot water in a beaker is taken every 50 seconds. The results are shown in the graph



(a) What is the temperature of the water after 220 seconds?

(b) When is the temperature 35 °C?

First look carefully at the scale on both axes. On the time axis, 2 cm represents 100 seconds, so each small division represents 100 ÷ 10 = 10 seconds.

On the temperature axis, 2 cm represents 20 °C, so each small division represents 20 ÷ 10 = 2 °C.

(a) From the graph the temperature after 220 seconds is 25 °C.

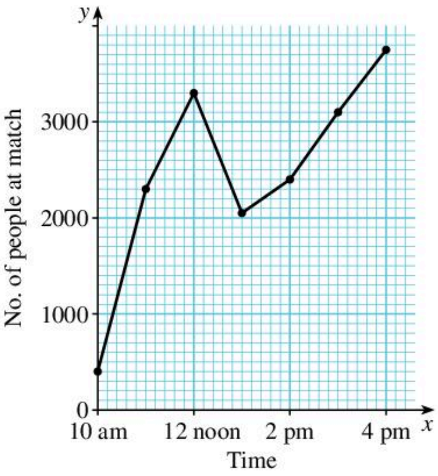
(b) From the graph, the temperature is 35 °C after 95 seconds.

Exercise 9C

1 The graph shows the number of people present at a cricket match at different times during the day.

(a) What does each small division on the vertical axis represent?

(b) How much time does each small division represent on the horizontal axis?



(c) How many people were there at the match at:

(i) 11 am **(ii)** 12.36 pm

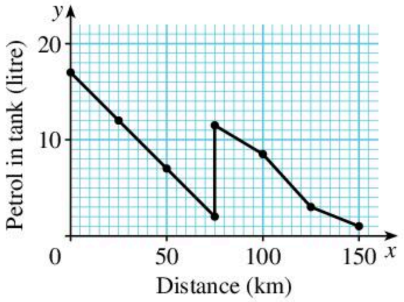
(iii) 2.48 pm?

(d) At what times was the number of people at the match:

(i) 1500 **(ii)** 1900

(iii) 2700?

2 The graph shows the amount of petrol in a car every 25 km.



(a) How much petrol was in the tank after:

(i) 25 km **(ii)** 45 km **(iii)** 108 km?

(b) How far had the car travelled when the amount of petrol in the tank was:

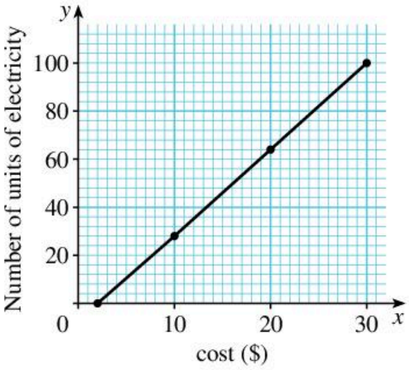
(i) 17 litres

(ii) 14 litres

(iii) 1 litre?

(c) When was the car refilled with petrol? How much petrol was put in the tank?

3 The cost per unit of electricity is shown in the graph.



(a) What is the cost of:

(i) 20 units

(ii) 28 units

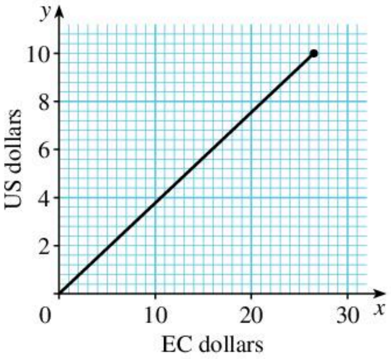
(iii) 76 units?

(b) How many units of electricity are used if the total cost is:

(i) $5 **(ii)** $9.50 **(iii)** $29?

(c) What is the standing charge, that is the charge applied even if no electricity is used?

4 The graph shows the number of US dollars that can be obtained from a given number of EC dollars.



(a) How many US$ can be obtained from:

(i) EC$15 **(ii)** EC$20 **(iii)** EC$8?

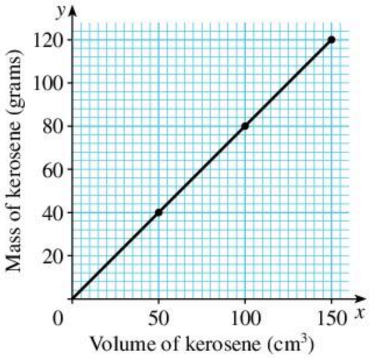
(b) How many EC$ can be obtained from:

(i) US$6

(ii) US$4.40

(iii) US$3.60?

5 The masses of different volumes of kerosene are shown in the graph.

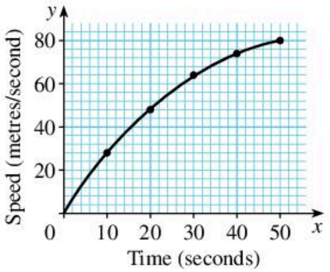


(a) What mass of kerosene has a volume of:
(i) 50 cm^3 **(ii)** 70 cm^3 **(iii)** 106 cm^3?

(b) What volume of kerosene has a mass of:
(i) 20 g **(ii)** 32 g **(iii)** 105 g?

(c) What is the mass of 1 cm^3 of kerosene (its density)?

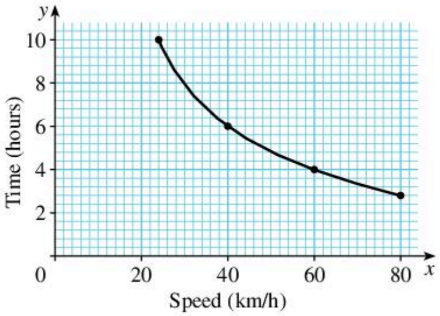
6 A car accelerates from rest. The graph shows the speed of the car during the first 50 seconds of its motion.



(a) What was the speed of the car after:
(i) 10 s **(ii)** 16 s **(iii)** 33 s?

(b) At what time was the speed of the car:
(i) 20 m/s **(ii)** 44 m/s **(iii)** 61 m/s?

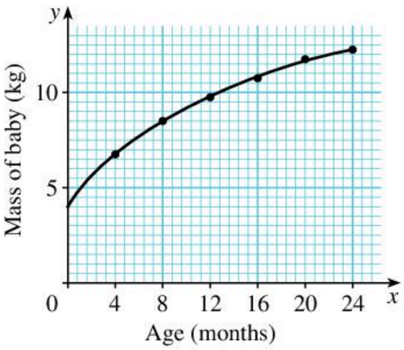
7 The graph shows the time in hours to travel a certain distance at various speeds.



(a) How long did the journey take when the speed was:
(i) 34 km/h
(ii) 65 km/h
(iii) 27 km/h?

(b) What was the speed if the journey time was:
(i) 5 hours
(ii) $7\frac{1}{2}$ hours
(iii) 6 hours 48 min?

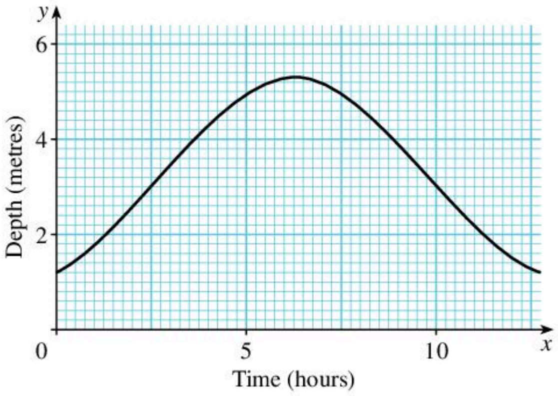
8 The mass of a baby during its first 24 months of life is shown in the graph.



(a) What was the mass of the baby:
(i) at birth
(ii) after 6 months
(iii) after 19 months?

(b) At what age was the baby's mass:
(i) 5 kg **(ii)** 9 kg **(iii)** 7.3 kg?

9 The depth of water in a tidal river over a 12-hour period is shown in the graph below.



(a) What was the river depth after:
(i) $2\frac{1}{2}$ h **(ii)** $6\frac{1}{4}$ h **(iii)** $11\frac{1}{4}$ h?

(b) At what times was the river depth:
(i) 2 m **(ii)** 2.6 m **(iii)** 3.8 m?

9.3 Drawing graphs

When you draw a graph you must pay special attention to the scale.

Example 4

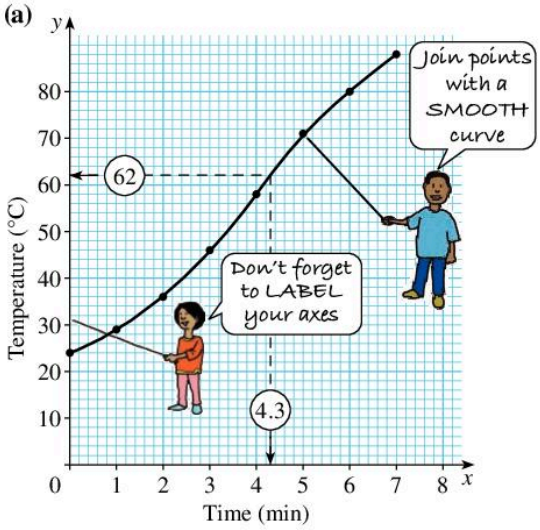
The temperature of water heated in a kettle is taken every minute. The table shows the results.

Time (min)	0	1	2	3	4	5	6	7
Temperature (°C)	24	29	36	46	58	71	80	88

(a) Plot a graph of the information in the table using a scale of 1 cm to 1 minute on the horizontal axis and 1 cm to 10 °C on the vertical axis.

(b) Use your graph to find when the temperature was 62 °C.

(a)



(b) Temperature was 62 °C after 4.3 min.

Exercise 9D

1 The table shows the conversion from metres per second to kilometres per hour.

Speed (m/s)	0	10	20	30	40	50	60
Speed (km/h)	0	36	72	108	144	180	216

(a) Draw a graph to show this information. Use a scale of 2 cm to 10 m/s on the horizontal axis and 1 cm to 10 km/h on the vertical axis.

(b) Use your graph to convert these speeds to km/h:

(i) 15 m/s **(ii)** 45 m/s

(c) Use your graph to convert these speeds to m/s:

(i) 90 km/h **(ii)** 150 km/h

2 A piece of meat is taken out of a freezer. Its temperature rises steadily as shown in the table.

Time (min)	0	10	20	30	40	50
Temp (°C)	$^{-}5$	0	5	10	15	20

Draw a graph, using a scale of 1 cm to represent 5 minutes on the horizontal axis, and 2 cm to represent 5 °C on the vertical axis.

Use the graph to estimate:

(a) the temperature of the meat after 17 minutes

(b) the time taken for the meat to reach a temperature of 12 °C.

3 A baby was 4 kg when he was born. His mass over the next 7 weeks is shown in the table.

Week	0	1	2	3	4	5	6	7
Mass (kg)	4	4.4	4.8	5.2	5.6	6	6.4	6.8

Draw a graph and use this graph to state approximately how many days old the baby was when his mass was 5 kg.

4 The speed of a car is measured in kilometres per hour (km/h).

This table shows the speed of the car starting from rest after a given number of seconds:

Time (s)	0	2	4	6	8	10	12
Speed (km/h)	0	4	16	30	44	56	60

(a) Draw a graph to show this information. Use a scale of 1 cm to 1 s on the horizontal axis and 2 cm to 10 km/h on the vertical axis. Make sure you join the points with a smooth curve.

(b) From your graph find the speed of the car after:

(i) 5 seconds **(ii)** $9\frac{1}{2}$ seconds

(c) From your graph find the time taken for the speed to become:

(i) 20 km/h **(ii)** 35 km/h

5 The volume of water that flows into a water tank over a period of 20 minutes is given by the table.

Time (min)	0	1	3	11	13.5	17	20
Water volume (l)	0	13	39	143	175.5	221	260

(a) Draw a graph to show this information. Use a scale of 2 cm to represent 4 minutes on the horizontal axis and 2 cm to represent 50 litres on the vertical axis.

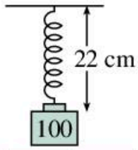
(b) From your graph find how much water is in the tank after:

(i) 10 min **(ii)** 16.4 min

(c) From your graph find when the volume of water in the tank is:

(i) 20 litres **(ii)** 155 litres

6 A spring is stretched by hanging masses from its end. The lengths of the spring for different masses are given in the table:



Mass (g)	0	20	36	52	84	100
Length (cm)	7	10	12.4	14.8	19.6	22

(a) Draw a graph to show this information. Use a scale of 1 cm to represent 10 g on the horizontal axis and 1 cm to represent 2 cm on the vertical axis.

(b) From your graph find the length of the spring when the mass is:

(i) 26 g **(ii)** 63 g

(c) From your graph find the mass which would cause the spring to have a length of:

(i) 16 cm **(ii)** 21.6 cm

7 The cost of petrol for a given number of litres is shown in the table.

No. of litres	2	6	16	22	38	46
cost ($)	3.70	11.10	29.60	40.70	70.30	85.10

(a) Draw a graph of this information. Use a scale of 1 cm to represent 5 litres on the horizontal axis and 1 cm to represent $10 on the vertical axis.

(b) From your graph find the cost of the following quantities of petrol:

(i) 5 litres **(ii)** 33 litres

(c) From your graph find how much petrol can be bought for:

(i) $20 **(ii)** $55

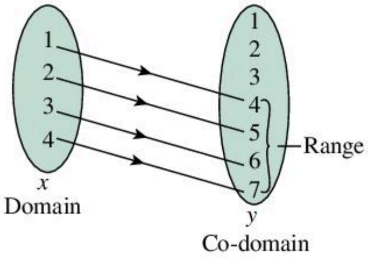
9.4 Relations and functions

Look at the relation 'add 3' defined between the sets

$x = \{1, 2, 3, 4\}$
and $y = \{1, 2, 3, 4, 5, 6, 7\}$

This relation can be shown as

(i) an arrow diagram



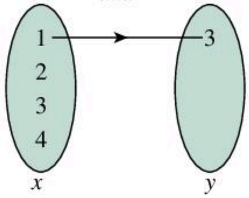
The set x is known as the **domain**.

The set y is known as the co-domain. The image points {4, 5, 6, 7} are known as the **range**.

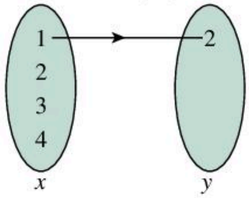
Exercise 9E

1 **(a)** Copy and complete the arrow diagram for these relations

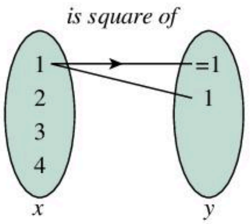
(i) *add 2*



(ii) *is a half of*



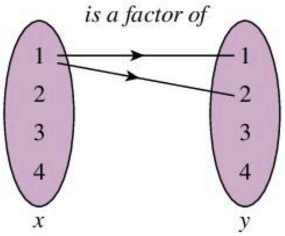
(iii)



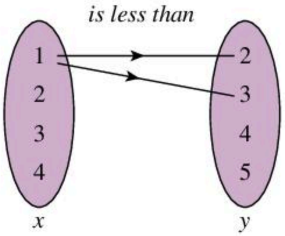
(b) Identify the range in each case.

2 Complete the arrow diagrams for these relations.

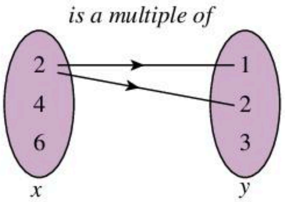
(a)



(b)

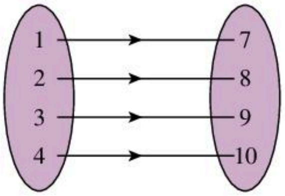


(c)

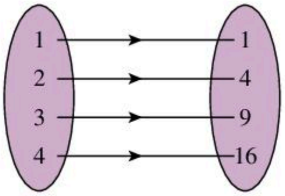


3 Write down the relation for each of these arrow graphs.

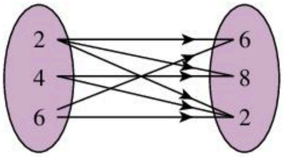
(a)



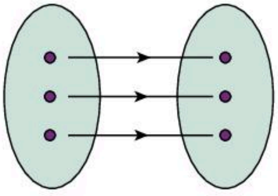
(b)



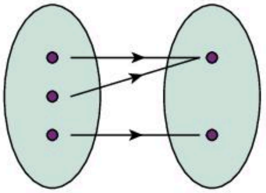
(c)



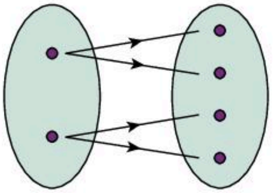
Look at the arrow graphs you draw in Exercise 9E. There are four types of arrow graph for a relation.



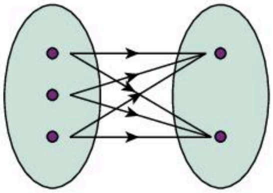
one to one (1–1)



many to one (m–1)



one to many (m–1)

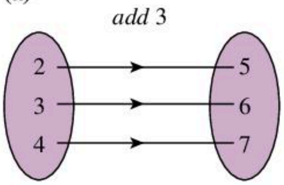


many to many (m–m)

A relation that is one to one or many to one is called a **function**. A function is **well-defined** or **well-behaved** as for each element of the domain there is only *one* image.

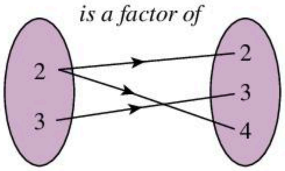
For example:

(a)



This is a function

(b)



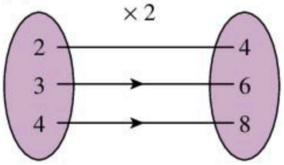
This is NOT a function

In (b) the number 2 can be sent to 2 or 4, as 2 is a factor of both 2 and 4.

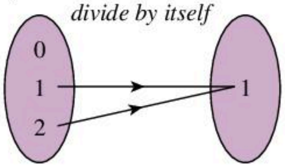
Further, *each* element of the domain must have an image part in the co-domain.

For example,

(c)



(d)

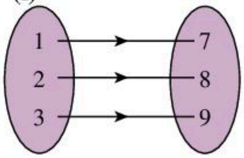


(d) is not a function since 0 in the domain has no image.

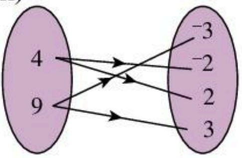
Exercise 9F

1 **(a)** Identify which of these arrow graphs show functions.

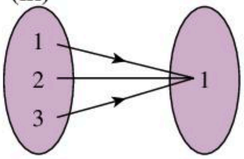
(i)



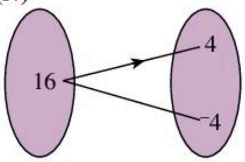
(ii)



(iii)



(iv)



(b) Give reasons for your answers.

(c) State the domain and range for each relation.

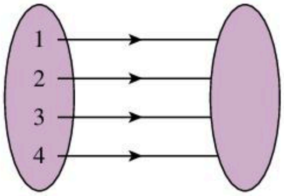
2 **(a)** Draw arrow diagrams for the relations

	Domain
(i) 'add 4'	{0, 1, 2, 3}
(ii) 'subtract 2'	{0, 1, 2, 3}
(iii) 'square'	$\{-2, -1, 0, 1, 2\}$
(iv) 'square root'	{1, 4, 9, 16}
(v) 'is a factor of'	{2, 3}

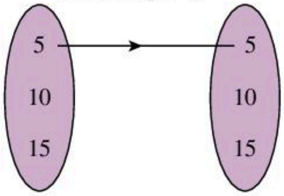
(b) State whether the relation is a function, giving reasons for your answer.

3 **(a)** Copy and complete the arrow graphs.

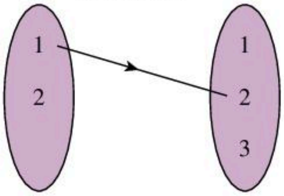
(i) *Subtract 2*



(ii) *is a multiple of*



(iii) *is less then*

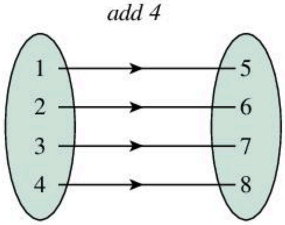


(b) State whether the arrow graph is 1–1, m–1, 1–m or m–m.

(c) Which of the relations is a function?

9.5 Function notation

You can think of a function as an **input**, **output** machine, for example, the relation 'add 4' over the domain {1, 2, 3, 4} is



You can write this as the number machine

Input Relationship Ouput

$$x \rightarrow \boxed{+4} \rightarrow y$$

or $x \rightarrow x + 4$

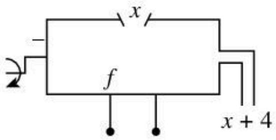
The relation $x \rightarrow x + 4$ over the domain {1, 2, 3, 4} is clearly a function as it is a 1–1 map.

To save writing you can name the function, *f*. That is

$f: x \rightarrow x + 4$

or $f(x) = x + 4$

You can picture this as a function machine



The operation '*f*' on a number simply adds 4, so

$f(3) = 3 + 4 = 7$

$f(2) = 2 + 4 = 6$ etc.

or $f: 3 \rightarrow 3 + 4 = 7$

$f: 2 \rightarrow 2 + 4 = 6$ etc.

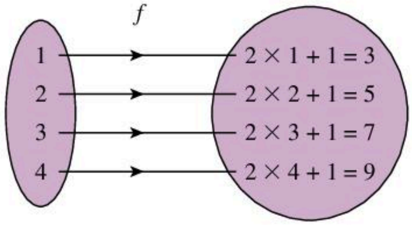
You can use this notation when drawing arrow graphs.

Example 5

Draw an arrow graph to show the function

$$f : x \rightarrow 2x + 1$$

over the domain {1, 2, 3, 4}



The function, *f*, tells you to multiply the number by 2 and then add 1

Note:

$f(1) = 2 \times 1 + 1 = 3$
$f(2) = 2 \times 2 + 1 = 5$
$f(3) = 2 \times 3 + 1 = 7$
$f(4) = 2 \times 4 + 1 = 9$

Exercise 9G

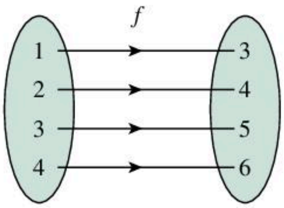
1 Draw an arrow graph for the functions defined by

	Function	Domain
(a)	$f : x \rightarrow x + 1$	{1, 2, 3, 4}
(b)	$f : x \rightarrow x - 3$	{1, 2, 3, 4}
(c)	$f : x \rightarrow x^2$	$\{^-2, ^-1, 0, 1, 2\}$
(d)	$f : x \rightarrow 4 - x$	{1, 2, 3, 4}
(e)	$f : x \rightarrow 3x + 2$	{1, 2, 3, 4}
(f)	$f : x \rightarrow \frac{1}{2}x + 1$	{1, 2, 3, 4}
(g)	$f : x \rightarrow 4 - 2x$	{0, 1, 2, 3}
(h)	$f : x \rightarrow x^2 - 4$	$\{^-2, ^-1, 0, 1, 2\}$
(i)	$f : x \rightarrow 3x - 1$	{0, 1, 2, 3}
(j)	$f : x^n \rightarrow x^2 + x$	{0, 1, 2, 3}

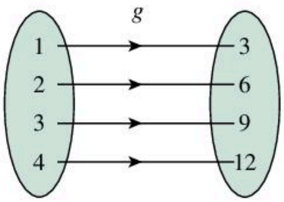
2 In Question 1, identify the range for each function.

3 Describe in function notation the functions shown by the arrow graphs.

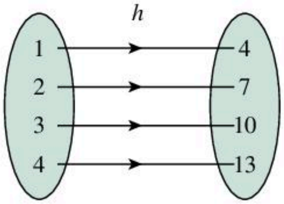
(a)



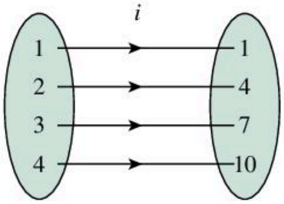
(b)



(c)



(d)



4 Evaluate:

(a) $f(3)$ when $f(x) = 2x + 6$
(b) $f(0)$ when $f(x) = 3x - 1$
(c) $f(4)$ when $f(x) = 6 - x$
(d) $f(^-2)$ when $f(x) = 2x - 1$
(e) $f(^-2)$ when $f(x) = x^2$

5 For each function

(a) $g(x) = 3x + 2$
(b) $g(x) = x^2 + 2$
(c) $g(x) = \frac{1}{x}$
(d) $g(x) = 6 - x^2$

find the value of

(i) $g(3)$ **(ii)** $g(–3)$ **(iii)** $g(1)$

Functions and graphs

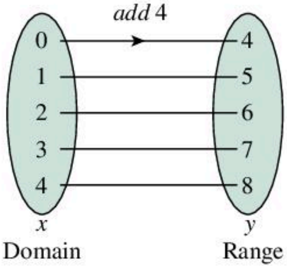
You can show both relations and functions as

- an arrow graph
- an algebraic rule
- a set of ordered pairs
- a coordinate graph.

For example

The relation 'add 4' on the domain {0, 1, 2, 3, 4} can be shown as

- an arrow graph

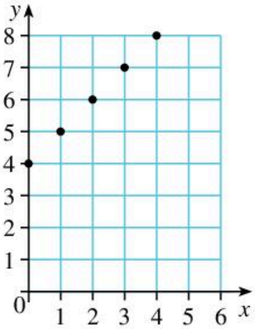


- an algebraic rule

$$x \rightarrow \boxed{+4} \rightarrow x+4$$

or $x \rightarrow x+4$

- a set of ordered pairs (0, 4), (1, 5), (2, 6), (3, 7), (4, 8)
- a coordinate graph



In the above example the relation 'add 4' on the domain {0, 1, 2, 3, 4} is a function. You can write

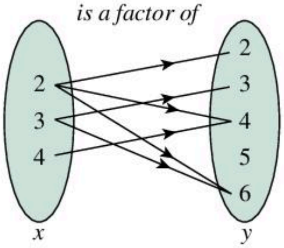
$$f: x \rightarrow x+4$$

as another way of describing it.

You can draw graphs of relations, even if they are not functions.

Example 6

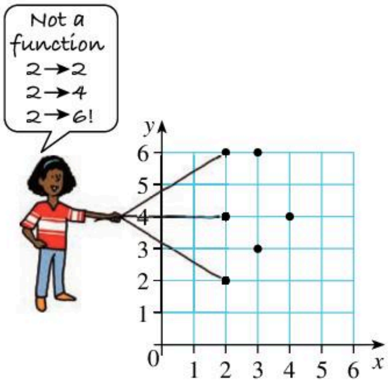
Draw a coordinate graph to show the relation 'is a factor of' as shown on the arrow graph.



The ordered pairs are

(2, 2), (2, 4), (2, 6), (3, 3), (3, 6), (4, 4)

Plotting these on a graph.



In the example, the number 2 has image 2, 4 and 6. As it does not have a **single** image point, the relation is not a function.

Exercise 9H

1 Draw arrow and coordinate graphs for the relations defined by

		Domain
(a)	'subtract 3'	{1, 2, 3, 4, 5}
(b)	'is a multiple of'	{2, 4, 6}
(c)	'is a half of'	{1, 2, 3, 4}
(d)	'square'	$\{^-3, ^-2, ^-1, 0, 1, 2, 3\}$
(e)	'square root of'	{0, 1, 4, 9}

2 In Question 1 which of the relations are functions?

3 Draw arrow and coordinate graphs for the functions defined by

	Domain
(a) $f: x \to 2x + 3$	$\{0, 1, 2, 3, 4\}$
(b) $f: x \to 4 - x$	$\{0, 1, 2, 3, 4\}$
(c) $f: x \to 3x - 2$	$\{{}^-2, {}^-1, 0, 1, 2\}$
(d) $f: x \to x^2 + 3$	$\{{}^-2, {}^-1, 0, 1, 2\}$
(e) $f: x \to \frac{1}{2}x - 1$	$\{{}^-2, {}^-1, 0, 1, 2\}$

4 In Question 3, write down the range for each of the functions.

5 **(a)** Draw coordinate graphs of the ordered pairs
  (i) (0, 3), (1, 4), (2, 5), (3, 6)
  (ii) $({}^-2, 4), ({}^-1, 1), (0, 0), (1, 1), (2, 4)$
  (iii) (3, 1), (3, 2), (3, 0), (2, 1), (2, 0)
  (iv) $(4, {}^-2), (1, {}^-1), (0, 0), (1, 1), (4, 2)$
(b) Which of the graphs show functions?

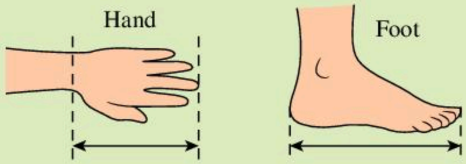
6 **(a)** Over the domain {0, 1, 2, 3, 4, 5} plot the coordinate graph of the function $f: x \to 3x + 4$
(b) Join your points with a straight line.
(c) Use your graph to find the value of $f(2.5)$
(d) For what value of x, is $f(x) = 14$?

7 Angela measures the height, h, of a plant every 5 days. Her results are shown in the table.

Day	5	10	15	20	25
Height (cm)	2	4	6	8	10

(a) Draw a graph to show this information.
(b) Use your graph to find the height after
  (i) 3 days **(ii)** 9 days
(c) When will the plant be 9 cm in height?
(d) Write down an algebraic rule to describe the relationship between height, h, of the plant and the number of days, d.

Activity



(a) In centimetres, measure the length of your hand and your foot.

(b) Measure these lengths for nine other classmates.

(c) Record your results in a table.

Length hand (cm)					
Length foot (cm)					

(d) Plot the graph of hand length against foot length.

(e) Is the relationship between hand and foot length a function? Explain your findings?

Technology

Learn all about relations, functions and graphs by visiting

www.mathsisfun.com/sets/function.html

Make sure you try the questions!

9.6 Linear graphs

A set of points that can be joined by a straight line is called a **linear graph**.

For example, the function

$$f: x \to 2x + 1$$

over the domain {0, 1, 2, 3, 4} is a linear graph.

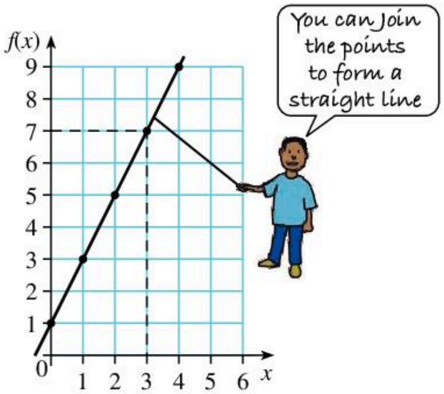
The ordered pairs are:

$0 \to 2 \times 0 + 1 = 1$ (0, 1)
$1 \to 2 \times 1 + 1 = 3$ (1, 3)
$2 \to 2 \times 2 + 1 = 5$ (2, 5)
$3 \to 2 \times 3 + 1 = 7$ (3, 7)
$4 \to 2 \times 4 + 1 = 9$ (4, 9)

You can write these pairs as a table of values

x	0	1	2	3	4
$f(x)$	1	3	5	7	9

and plot them on a graph.



The function

$$f(x) = 2x + 1$$

is a linear function.
This function can also be written as

$$y = 2x + 1 \text{ where } y = f(x)$$

In this case the function has no explicit name, but it is still a function! For every value of x inputted there is a distinct value of y.

Linear functions such as $y = 3x - 1$ can be plotted in the same way. The first step is to draw a table of values which gives the coordinate pairs. From this table the points can be plotted.

Example 7

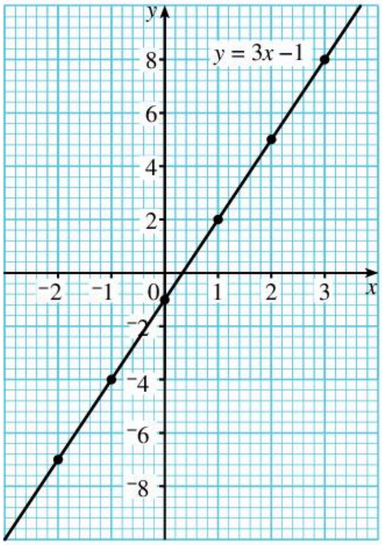
Plot the graph of the relation $y = 3x - 1$ for $x = {}^{-}2$ to $x = 3$.

First, complete the table of values for each value of x.

x	$^{-}2$	$^{-}1$	0	1	2	3
3x	$^{-}6$	$^{-}3$	0	3	6	9
$^{-}$1	$^{-}1$	$^{-}1$	$^{-}1$	$^{-}1$	$^{-}1$	$^{-}1$
y	$^{-}7$	$^{-}4$	$^{-}1$	2	5	8

Notice that the points that satisfy the equation $y = 3x - 1$ are $({}^{-}2, {}^{-}7)$, $({}^{-}1, {}^{-}4)$, $(0, {}^{-}1)$, $(1, 2)$, $(2, 5)$, $(3, 8)$

Plot the points on the graph and join them:



Exercise 9I

1 **(a)** Plot these sets of points on suitable axes.
  - **(i)** $(0, 0)$, $(1, 1)$, $(2, 2)$, $(3, 3)$
  - **(ii)** $({}^{-}1, 0)$, $(0, 1)$, $(1, 2)$, $(2, 4)$
  - **(iii)** $({}^{-}3, {}^{-}2)$, $({}^{-}1, {}^{-}2)$, $(1, 2)$, $(3, 4)$
  - **(iv)** $(0, {}^{-}2)$, $(1, {}^{-}1\frac{1}{2})$, $(2, {}^{-}1)$, $(3, {}^{-}\frac{1}{2})$

(b) Which sets represent a linear relation?

2 **(a)** Copy and complete the table of values for $y = 2x + 1$.

x	$^{-}2$	$^{-}1$	0	1	2	3	4
2x		$^{-}2$				6	
$^{+}$1	$^{+}1$					$^{+}1$	
y			1			7	

(b) Draw a graph of $y = 2x + 1$ using a scale of 1 cm to represent 1 unit on the x-axis and 1 cm to represent 1 unit on the y-axis.

3 **(a)** For each of the following equations, draw tables of values for $x = {}^{-}2$ to $x = 4$.
  - **(i)** $y = 3x$
  - **(ii)** $y = 3x + 1$
  - **(iii)** $y = x - 5$
  - **(iv)** $y = 2x - 5$
  - **(v)** $y = \frac{1}{2}x$
  - **(vi)** $y = 1 - x$

(b) Using a scale of 1 cm to represent 1 unit on the x-axis and 2 cm to represent 1 unit on the y-axis, plot the graphs of the equations in (a).

4 Using suitable scales, plot the graphs of these linear relations for $x = {}^{-}2$ to $x = 4$.

(a) $y = 2x - 3$ **(b)** $y = 2 - x$
(c) $y = 4x + 2$ **(d)** $y = 4x - 2$
(e) $y = 2x - 4$ **(f)** $y = 6 - 2x$

5 **(a)** Copy and complete the table of values for the equation $x + y = 6$.

x	$^{-}3$	$^{-}2$	$^{-}1$	0	1	2	3
y		8		6		4	

(b) Using a suitable scale, plot the graph of $x + y = 6$.

(c) At what point does the graph intercept the y-axis?

6 Which of these lines pass through the point (2, 3)?

(a) $y = 2x + 1$
(b) $y = 2x - 1$
(c) $y = 5x - 7$
(d) $y = \frac{1}{2}x + 2$
(e) $y = 6 - 2x$

7 **(a)** For each equation, complete a table of values for $x = {}^{-}3$ to $x = 3$.

(i) $y = 3x - 2$ **(ii)** $y = 3x - 1$
(iii) $y = 3x$ **(iv)** $y = 3x + 1$
(v) $y = 3x + 2$ **(vi)** $y = 3x + 3$

(b) Using a suitable scale, plot each of these graphs on the same set of axes.

(c) What do you notice about each of these lines?

(d) Write down the point where each equation intercepts the y-axis.

8 **(a)** For each equation, complete a table of values for $x = {}^{-}3$ to $x = 3$.

(i) $y = \frac{1}{2}x + 2$ **(ii)** $y = x + 2$
(iii) $y = 2x + 2$ **(iv)** $y = 3x + 2$

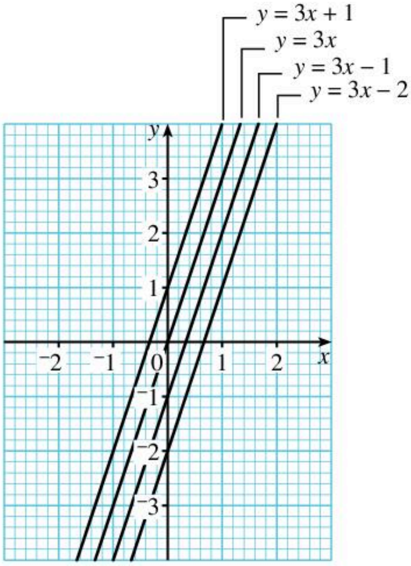
(b) Using a suitable scale, plot each of these graphs on the same set of axes.

(c) What do you notice about the steepness of these lines?

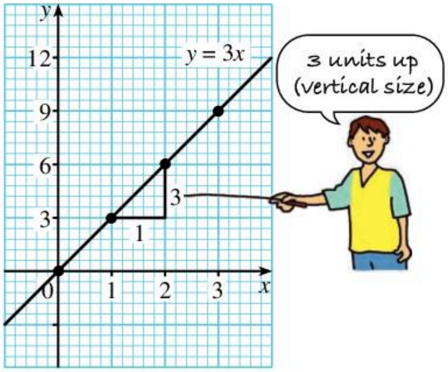
(d) What do you notice about their points of intersection with the y-axis?

Gradients and intercepts

In Question 7 of Exercise 9I, you should have found all the lines were parallel. That is, they all had the same slope.



The slope or gradient of each of these lines is 3.



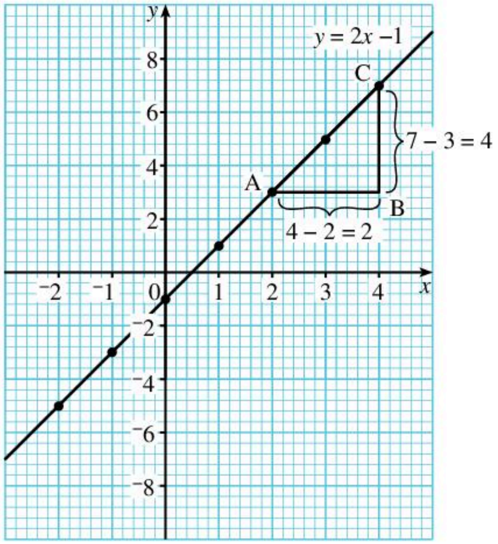
The gradient of $y = 3x$ is 3; for every 1 unit across you move 3 units up.

The **gradient** of a line is sometimes defined as

$$\text{Gradient} = \frac{\text{vertical rise}}{\text{horizontal shift}}$$

Example 8

What is the gradient of the line $y = 2x - 1$?
When plotted the graph of $y = 2x - 1$ is:



$$\text{Gradient} = \frac{\text{vertical rise}}{\text{horizontal shift}}$$

$$= \frac{BC}{AB} = \frac{7-3}{4-2} = \frac{4}{2}$$

$$= 2$$

Notice in the example that the gradient of $y = 2x - 1$ is 2. In general, the gradient of the line $y = mx + c$ is m.

Exercise 9J

1 **(a)** Draw these graphs for $x = {}^{-}1$ to $x = 3$.
 (i) $y = 4x$ **(ii)** $y = \frac{1}{2}x + 1$
 (iii) $y = 3x + 4$ **(iv)** $y = 2x$
 (b) Find the gradient of each line.
 (c) What is their intercept with the y-axis?

2 **(a)** On the same set of axes plot:
 (i) $y = x + 1$ **(ii)** $y = \frac{1}{2}x + 1$
 (iii) $y = 2x + 1$ **(iv)** $y = 3x + 1$
 (b) Which line has the steepest gradient?
 (c) What is the point of interception of each line with the y-axis?

3 Copy and complete the table.

Equation	Gradient	Coordinates of y-intercept
$y = 2x + 1$		(0, 1)
$y = 3x - 1$	3	
$y = \frac{1}{2}x + 1$		
$y = {}^{-}x$	$^{-}1$	
$y = x$		(0, 0)
$y = 4 - x$		
$y = 4 - 2x$		

4 In Question 3,
 (a) which lines have a positive slope
 (b) which lines have a negative slope
 (c) which lines are parallel?

5 Copy and complete the table.

Equation	Gradient	Coordinates of y-intercept
$y =$	2	(0, 3)
	3	(0, $^{-}1$)
	$^{-}2$	(0, 5)
	$^{-}1$	(0, $^{-}2$)

Technology

Learn more about slopes, intercepts and straight-line graphs by visiting
www.onlinemathlearning.com/slope-intercept-method.html

Watch the videos!

Using graphs to solve simultaneous equations

In Chapter 4 you learnt how to solve a pair of simultaneous equations. Graphing the equations offers another way of solving such equations.

Example 9

Plot the graphs

$x + y = 4$
$x - y = 2$

and hence solve this pair of simultaneous equations.

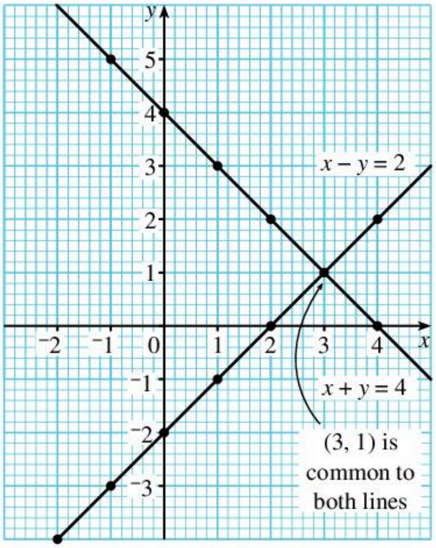
The table of values for $x = {}^{-}2$ to $x = 4$ for $x + y = 4$ is:

x	$^{-}2$	$^{-}1$	0	1	2	3	4
y	6	5	4	3	2	1	0

and for $x - y = 2$ is

x	$^{-}2$	$^{-}1$	0	1	2	3	4
y	$^{-}4$	$^{-}3$	$^{-}2$	$^{-}1$	0	1	2

Plotting the graphs gives:



Points that lie on the line $x + y = 4$ satisfy this equation.
Points that lie on the line $x - y = 2$ satisfy that equation.
The two lines meet at the point where $x = 3$, $y = 1$ (3, 1).
This point lies on both lines and is the solution to the simultaneous equations.

Exercise 9K

1 **(a)** Copy and complete the tables of values for $y = 3x - 1$

x	$^{-}1$	0	1	2	3	4	5
y							

and $y = 2x + 1$.

x	$^{-}1$	0	1	2	3	4	5
y							

(b) Plot the graphs on the same axes.
(c) Where do the two lines intersect?
(d) What is the solution to the equations $y = 3x - 1$, $y = 2x + 1$?

2 Draw graphs for each pair of equations and find their solution.

(a) $y = x + 2$, $y = 4 - x$
(b) $y = 3 - x$, $y = x - 3$
(c) $y = 2x + 3$, $y = 3x - 1$
(d) $y = 2x - 3$, $y = 3x + 4$

3 **(a)** Solve these pairs of simultaneous linear equations graphically.

(i) $x + y = 6$, $x - y = 2$
(ii) $2x = y$, $x + y = 9$
(iii) $y = 4x - 1$, $y = 2x + 1$
(iv) $y = \frac{1}{2}x - 3$, $y = 2x + 3$

(b) Solve the equations algebraically. Do your solutions agree with the graphical ones?

4 **(a)** Plot the equations

$y = 2x - 6$
$y = 2x + 4$

(b) What do you notice about the graphs?
(c) Why is there no solution to the equations in (a)?

Technology

Use the graphing calculator at www.desmos.com

to plot the graphs

$y = 3x - 4$
$y = 4x + 3$

Where do these two lines meet?

Use the website grapher or a graphing calculator to check the solutions to the equations in Exercise 9H.

9.7 Graphs of quadratics

An equation with x^2 as the highest term is known as a **quadratic** equation.

For example

$y = x^2 + 3x - 2$

$x^2 - 4 = 0$

are quadratic equations.

The mapping

$$f : x \rightarrow x^2 - 3x + 2$$

is an example of a **quadratic function**.

To plot the graph of a quadratic equation or function you first need to complete a table of values, to find the coordinate pairs.

Example 10

Plot the graph of $y = x^2 - 1$ for values of x between $^{-}3$ and 3.

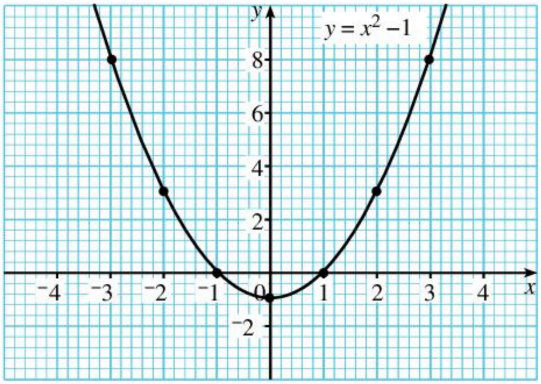
The table of values is:

x	$^{-}3$	$^{-}2$	$^{-}1$	0	1	2	3
x^2	9	4	1	0	1	4	9
-1	-1	-1	-1	-1	-1	-1	-1
y	8	3	0	$^{-}1$	0	3	8

The coordinate pairs are

($^{-}3$, 8), ($^{-}2$, 3), ($^{-}1$, 0), (0, $^{-}1$), (1, 0), (2, 3), (3, 8)

Plot the points.



Notice the graph is symmetrical and the curve is smooth.

For more complex quadratic functions the table of values has more rows.

Example 11

Draw the graph of the function $f(x) = x^2 - 3x + 4$ over the domain $-3 \leq x \leq 3$.

The table of values is:

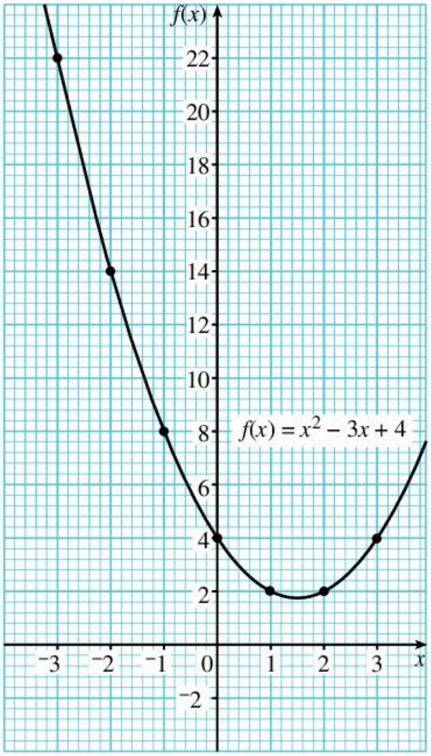
x	$^{-}3$	$^{-}2$	$^{-}1$	0	1	2	3
x^2	9	4	1	0	1	4	9
$^{-}3x$	9	6	3	0	$^{-}3$	$^{-}6$	$^{-}9$
$^{+}4$	4	4	4	4	4	4	4
y	22	14	8	4	2	2	4

The coordinate pairs are:

(–3, 22), (–2, 14), (–1, 8), (0, 4), (1, 2), (2, 2), (3, 4)

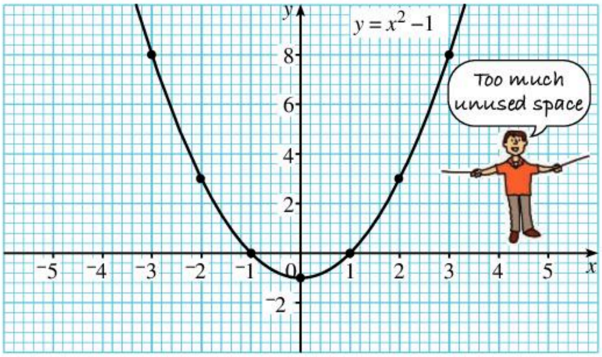
Choose a suitable scale to plot the points.

Plotting the points gives:



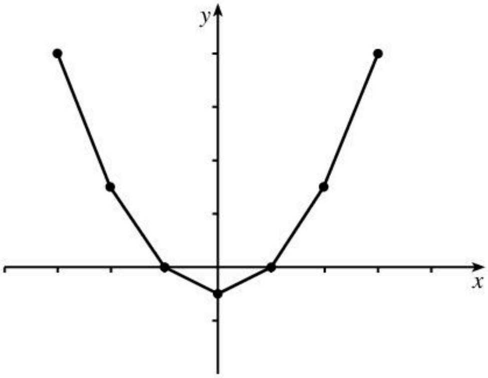
Make sure you draw a smooth curve to connect the points.

Your choice of scale is important. In Example 9 if the scale on the x-axis is too narrow, the graph will look like a hairpin!

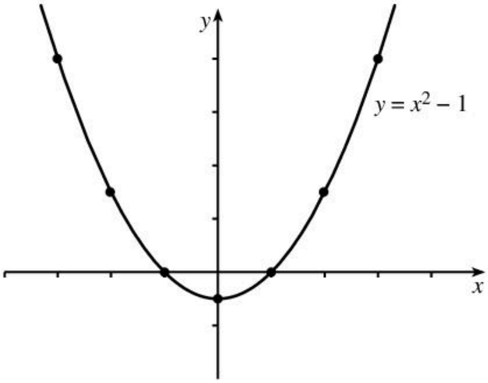


It is also important to join the points with a smooth curve – do not use a ruler!

WRONG



RIGHT



Exercise 9L

1 **(a)** Copy and complete the table of values for the equation $y = x^2 + 3$.

x	$^{-}3$	$^{-}2$	$^{-}1$	0	1	2	3
x^2			1				9
+3		3				3	
y				3			

(b) Plot the values to form a graph.
(c) Join the points with a smooth curve.
(d) Where does the graph cut the y-axis?

2 Copy and complete the table for the equation $y = x^2 + 1$.

x	$^{-}3$	$^{-}2$	$^{-}1$	0	1	2	3
y		5		1			

Draw the graph of $y = x^2 + 1$.

3 Draw a graph of $y = x^2 + 3$ by first making a table of values for x from $^{-}3$ to 3.

4 Draw a graph of $y = x^2 - 4$ by first making a table of values for x from $^{-}3$ to 3.

5 Look at the graphs you drew in Questions 1–4. What do you notice?

6 Draw the graph of the function $f : x \rightarrow 6 - x^2$ over the domain $-4 \leq x \leq 4$.

The table of values is started for you:

x	$^{-}2$	$^{-}1$	0	1	2
x^2	4	1	0	1	4
$f(x) = 6 - x^2$	2	5	6		

In what way is the graph different from the graphs in Questions 2, 3 and 4?

7 **(a)** Copy and complete the table of values for the equation $y = x^2 - 3x$.

x	$^{-}3$	$^{-}2$	$^{-}1$	0	1	2	3
x^2		4			1		
$^{-}3x$	9					$^{-}6$	
y			4				

(b) Using suitable axes, plot the points on a graph. Join the points with a smooth curve.

(c) Where does the curve $y = x^2 - 3x$ cut the x-axis?

8 **(a)** Copy and complete the table of values for the equation $y = x^2 - 2x + 3$.

x	$^{-}3$	$^{-}2$	$^{-}1$	0	1	2	3
x^2		4				4	
$^{-}2x$		4			$^{-}2$		
$^{+}3$			3				3
y	16			3			

(b) Using suitable axes, plot the points on a graph. Join the points with a smooth curve.

(c) Where does the curve $y = x^2 - 2x + 3$ cut the x-axis?

9 **(a)** Draw tables of values for $x = {^{-}3}$ to $x = 3$ for the functions

- **(i)** $f(x) = x^2 - 3x + 4$
- **(ii)** $f(x) = x^2 - 2x - 1$
- **(iii)** $f(x) = 2 + 3x - x^2$
- **(iv)** $f(x) = 4 - 3x - x^2$
- **(v)** $f(x) = 3x^2 - 2x - 1$
- **(vi)** $f(x) = 6 - 5x - 2x^2$

(b) Using suitable axes plot the graph of each quadratic function.

Technology

Plotting graphs of quadratic functions can be hard work.

Download the graphing app at

www.desmos.com

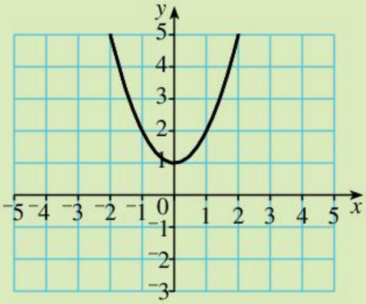
Enter one of the quadratic functions from Question 8 of Exercise 9L. Check its graph. Repeat for the other questions. Did you draw your graphs correctly?

Investigation

Visit

www.demos.com

click on sliders and then on Parabolas : Standard Form. You should see:



$y = ax^2 + bx + c$
$a = 1$
$b = 0$
$c = 1$

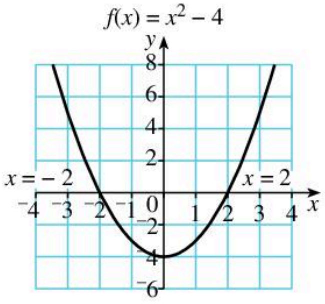
showing the graph of

$y = x^2 + 1$

What happens to the quadratic graphs as you increase/decrease the value of
(i) a **(ii)** b **(iii)** c?

Solving quadratic equations

Look at the graph of the function



where does the graph intersect with the x-axis?

At $x = 2$ and $x = {^{-}2}$

This intersection occurs when $f(x) = 0$ or when

$$x^2 - 4 = 0$$
$$\Rightarrow \quad x^2 = 4$$
$$\Rightarrow \quad x = \sqrt{4}$$
$$= 2 \text{ or } {^-2}$$

That is, the values $x = 2$ and $x = {^-2}$ are the solutions or **roots** of the quadratic equation $x^2 - 4 = 0$.

More complex quadratic equations can be solved in the same way.

Example 12

Draw the graph of

$$y = x^2 - x - 6$$

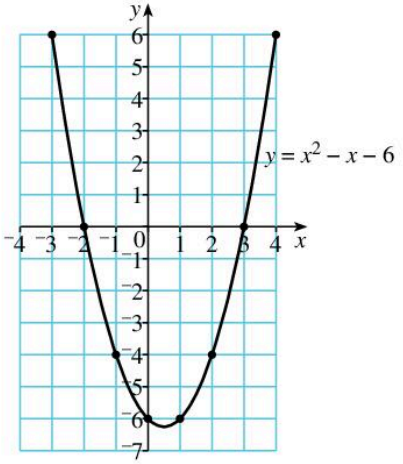
Hence solve the equation

$$x^2 - x - 6 = 0$$

First complete a table of values

x	$^-3$	$^-2$	$^-1$	0	1	2	3	4
x^2	9	4	1	0	1	4	9	16
^-x	3	2	1	0	$^-1$	$^-2$	$^-3$	$^-4$
6	$^-6$	$^-6$	$^-6$	$^-6$	$^-6$	$^-6$	$^-6$	$^-6$
y	6	0	$^-4$	$^-6$	$^-6$	$^-4$	0	6

Then plot the graph



The graph cuts the x-axis at

$$x = {^-2} \text{ and } x = 3$$

Hence the solutions of this equation

$$x^2 - x - 6 = 0$$

are $x = {^-2}$ and $x = 3$

Exercise 9M

1 **(a)** Plot the graph of the function $f(x) = x^2 - 6$

(b) Hence solve the equation $x^2 - 6 = 0$

2 Complete a table of values as shown for the following equations,

x	$^-3$	$^-2$	$^-1$	0	1	2	3
y							

- **(a)** $y = x^2 - 5$
- **(b)** $y = x^2 - x$
- **(c)** $y = x^2 - 2x + 1$
- **(d)** $y = x^2 - 3x - 2$
- **(e)** $y = x^2 - 5x + 6$

3 **(a)** Draw the graph of the equations in Question 2.

(b) Hence find the values of x, when $y = 0$.

4 Solve each quadratic equation by drawing a graph of the associated quadratic function and finding the value for x when $f(x) = 0$. In each case take the domain to be $-4 \leq x \leq 4$.

- **(a)** $f(x) = 8 - x^2$
- **(b)** $f(x) = x^2 + 2x - 3$
- **(c)** $f(x) = x^2 + x - 6$
- **(d)** $f(x) = x^2 - 5x + 4$

Technology

Want to learn more about quadratics and their graphs?

Visit the website

www.purplemath.com

and follow the links to Graphing Quadratic Functions in Intermediate Algebra Topics.

Investigation

Find out about the uses of quadratics.

Search some websites to get information on their use.
Present your findings to your class.

Exercise 9N – mixed questions

1 **(a)** Plot each of the given set of points on the same graph and join them together in order with straight lines.

Set A

$(6, 6), (9, 4), (9, 2), (8, 1), (6, 1), (5, 2), (5, 3), (4, 4), (3, 4), (2, 3), (^-2, 3), (^-3, 4), (^-4, 4), (4, 4), (6, ^-2), (6, ^-5), (^-6, ^-5), (^-6, ^-2), (^-4, 4), (^-5, 3), (^-5, 2), (^-9, 2), (^-9, 4), (^-6, 6), (6, 6)$

Set B

$(1, 1), (1, ^-1), (^-1, ^-1), (^-1, 1), (1, 1)$

Set C

$(1, 2), (2, 1), (2, ^-1), (1, ^-2), (6, ^-2), (^-6, ^-2), (^-1, ^-2), (^-2, ^-1), (^-2, 1), (^-1, 2), (1, 2)$

(b) What have you drawn?

2 Repeat Question 1 for these sets of points.

Set A

$(2, 9), (3, 7), (1, ^-1), (2, ^-5), (4, ^-5), (4, ^-9), (1, ^-9), (1, ^-5), (0, ^-2), (^-1, ^-5), (^-1, ^-9), (^-4, ^-9), (^-4, ^-5), (^-2, ^-5), (^-1, ^-1), (^-3, 7), (^-2, 9), (0, 2), (2, 9)$

Set B

$(0, 1), (1, 0), (0, ^-1), (^-1, 0), (0, 1)$

Set C

$(3, ^-6), (3, ^-8), (2, ^-8), (2, ^-6), (3, ^-6)$

Set D

$(^-2, ^-6), (^-2, ^-8), (^-3, ^-8), (^-3, ^-6), (^-2, ^-6)$

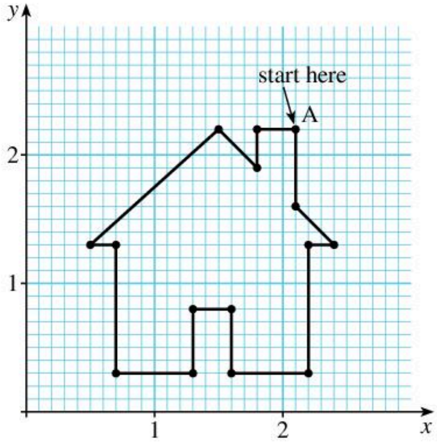
3 Use a scale of 8 cm to represent 1 unit on the x-axis and 4 cm to represent 1 unit on the y-axis.

(a) Plot each of the following points and join the points in order with straight lines.

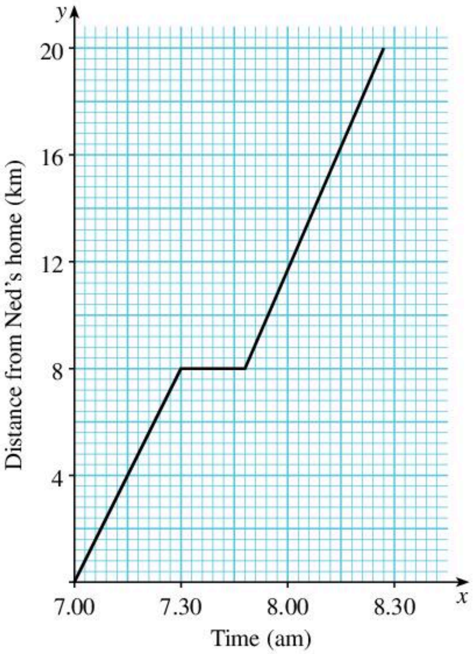
$(0, 1), (0.4, ^-0.1), (0.1, ^-0.1), (0.1, ^-0.2), (0.2, ^-0.2), (0.1, ^-0.6), (^-0.1, ^-0.6), (^-0.2, ^-0.2), (^-0.1, ^-0.2), (^-0.1, ^-0.1), (^-0.4, ^-0.1), (0, 1)$

(b) What have you drawn?

4 Starting at A, and moving in a clockwise direction list, in order, the coordinates of the vertices.



5 The graph shows Ned's journey from home to school. On the way he stops at Michael's house.



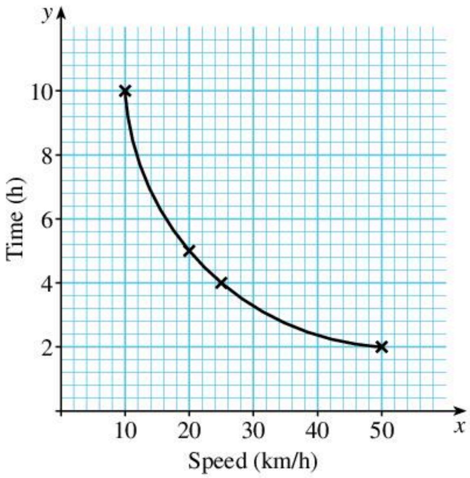
(a) What time did Ned leave home?

(b) How far is Michael's house from Ned's home?

(c) How long does Ned wait for Michael?

(d) At what time do they arrive at school?

6 The graph shows the time it takes a car to complete a journey when travelling at different speeds.



(a) How long does the journey take at a speed of
  (i) 30 km/h **(ii)** 15 km/h?
(b) At what speed is the car travelling when the journey takes
  (i) 5 hours **(ii)** 8 hours?
(c) Approximately how long is the journey?

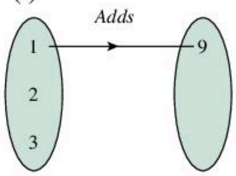
7 The population of a colony of insects increases as follows:

Week	1	2	3	4	5	6
No. of insects	3	9	27	81	243	729

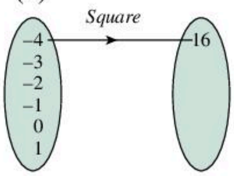
(a) Using a scale of 2 cm to represent 1 week on the horizontal axis and 1 cm to represent 100 insects on the vertical axis, plot the graph of the above information.
(b) From your graph estimate the number of insects after:
  (i) $3\frac{1}{2}$ weeks **(ii)** $5\frac{1}{2}$ weeks
(c) From your graph estimate how long it is before there are:
  (i) 50 insects **(ii)** 620 insects

8 **(a)** Copy and complete the arrow graphs for the relations.

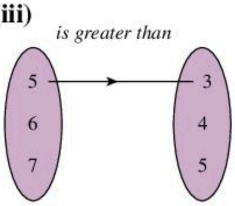
(i)



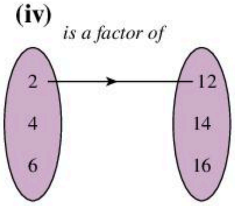
(ii)



(iii)



(iv)



(b) In each case state whether the relationship is 1–1, m–1, 1–m or m–m.
(c) Identify which relations are functions.

9 **(a)** Draw arrow graphs of the functions defined by

	Domain
(i) $f: x \rightarrow 4 - x$	$\{0, 1, 2, 3, 4, 5\}$
(ii) $f: x \rightarrow 3x + 1$	$\{^-2, ^-1, 0, 1, 2\}$
(iii) $f: x \rightarrow x^2 - 1$	$\{^-2, ^-1, 0, 1, 2\}$
(iv) $f: x \rightarrow 3 - 2x$	$\{^-2, ^-1, 0, 1, 2\}$

(b) In each case identify the range.

10 Over the domain $\{^-3, ^-2, ^-1, 0, 1, 2, 3\}$ draw coordinate graphs of the functions.
(a) $f: x \rightarrow 6 - x$
(b) $f: x \rightarrow 2x + 5$
(c) $f: x \rightarrow x^2 + 1$
(d) $f: x \rightarrow 3 - 2x$

11 **(a)** Complete tables of values for $x = {}^-2$ to $x = 4$ for the equations:
  (i) $2x + y = 6$
  (ii) $x - y = 3$
(b) Plot the equations on a graph.
(c) Hence find the solution to the equations.

12 The mass of different volumes of sulphuric acid is given in the table.

Volume (cm^3)	2.6	5.1	15.4	29	47	73.6
Mass (g)	3.4	6.6	20	37.7	61.1	95.7

(a) Draw a graph of the information. Use a scale of 2 cm to represent 10 cm^3 on the horizontal axis and 1 cm to represent 10 g on the vertical axis.
(b) From your graph find the mass of the following volumes of sulphuric acid:
  (i) 20 cm^3 **(ii)** 62 cm^3
(c) From your graph find the volume of sulphuric acid which has a mass of:
  (i) 30 g **(ii)** 74 g

13 By plotting the graphs or otherwise, find the gradient of these lines.

(a) $y = 4x - 3$

(b) $y = 6 - \frac{1}{2}x$

(c) $3x + 2y = 4$

14 The distance fallen by a stone after a certain time is given in the table.

Time (s)	1	2.3	2.9	3.5	4.8	5.9
Distance (m)	5	26.5	42	61.3	115.2	174

(a) Using a scale of 2 cm to represent 1 s on the horizontal axis and 1 cm to represent 20 m on the vertical axis, plot the graph.

(b) From your graph find the distance fallen by the stone after:

(i) 4 s **(ii)** 5.2 s

(c) From your graph find the time taken for the stone to fall:

(i) 50 m **(ii)** 106 m

15 The areas of different circles are shown in the table below:

Radius (cm)	3.2	4.3	9.1	11.6	13
Area (cm^2)	32.2	58.1	260	423	531

(a) Using a scale of 1 cm to represent 1 cm on the horizontal axis and 2 cm to represent 100 cm^2 on the vertical axis. Draw the graph of this information.

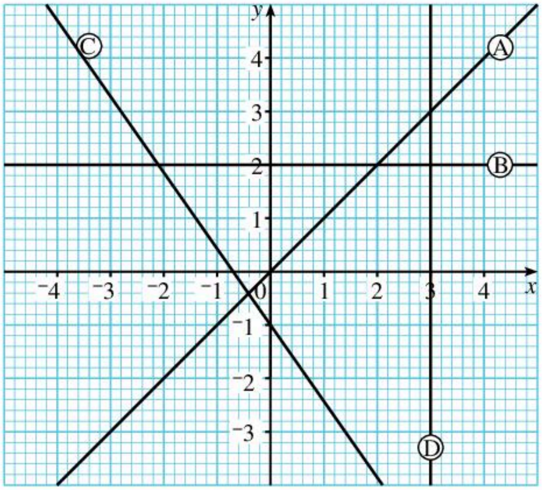
(b) From your graph find the area of a circle with radius:

(i) 6.5 cm **(ii)** 12.3 cm

(c) From your graph find the radius of a circle with area:

(i) 30 cm^2 **(ii)** 330 cm^2

16



Which line in the graph has the equation

(a) $x = 3$ **(b)** $y = 2$

(c) $y - x = 0$ **(d)** $2y + 3x + 2 = 0$?

17 **(a)** Copy and complete the table for the equation $y = 2 - 3x - 2x^2$.

x	$^{-}3$	$^{-}2$	$^{-}1$	0	1	2	3
2	2	2	2	2	2	2	2
$^{-}3x$		6					$^{-}9$
$^{-}2x^2$		$^{-}8$			$^{-}2$		$^{-}18$
y		0					$^{-}25$

(b) Draw the graph of the equation using 1 cm to represent 1 unit on the x-axis and 2 cm for 10 units on the y-axis.

(c) For what values of x does $y = 0$?

(d) What are the solutions to the equation $2 - 3x - 2x = 0$?

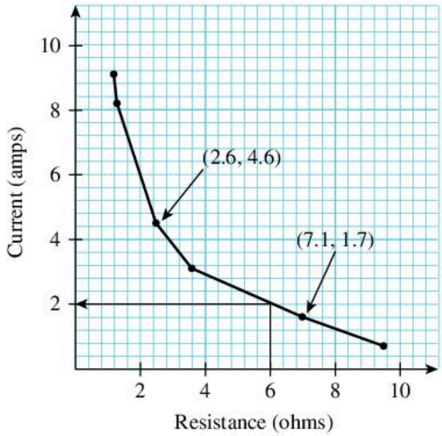
9 Consolidation

Example 1

The current in amps in a circuit for different values of resistance (in ohms) is given in the table.

Resistance (ohms)	9.6	7.1	3.7	2.6	1.4	1.3
Current (amps)	0.8	1.7	3.2	4.6	8.3	9.2

(a) Using a scale of 1 cm to represent 2 amps and 1 cm to represent 2 ohms draw a graph to show the information.



(b) What is the current when the resistance is 6 ohms?
From the graph, when resistance is 6 ohms, current ≃ 2.2 amps.

Example 2

Show the relation

$x \rightarrow 2x - 1$

over the domain $\{^-2, ^-1, 0, 1, 2\}$ as

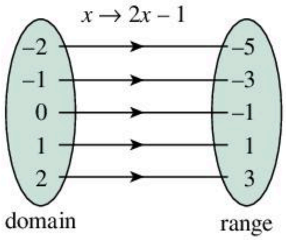
(a) an arrow graph

(b) a set of ordered pairs

(c) a coordinate graph.

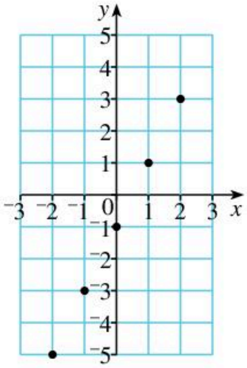
Is the relation a function?

(a)



(b) Coordinate pairs
$(^-2, ^-5), (^-1, ^-3), (0, ^-1), (1, ^-1), (2, 3)$

(c)



The mapping is one to one, hence it is a function.

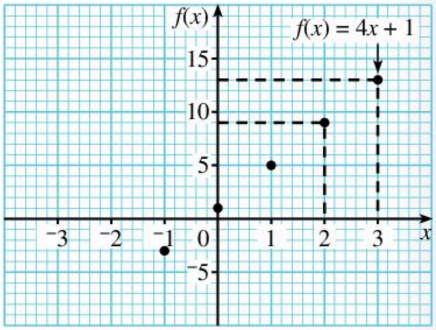
Example 3

For the function $f: x \rightarrow 4x + 1$ over the domain $\{^-1, 0, 1, 2, 3\}$, draw a coordinate graph.

$$f(^-1) = 4 \times ^-1 + 1 = ^-4 + 1 = ^-3$$
$$f(0) = 4 \times 0 + 1 = 0 + 1 = 1$$
$$f(1) = 4 \times 1 + 1 = 4 + 1 = 5$$
$$f(2) = 4 \times 2 + 1 = 8 + 1 = 9$$
$$f(3) = 4 \times 3 + 1 = 12 + 1 = 13$$

The coordinate pairs in the form of a table of values are

x	⁻1	0	1	2	3
***f*(*x*)**	⁻3	1	5	9	13



Example 4

Solve the equations

$x + 2y = 6$

$2x - y = 2$

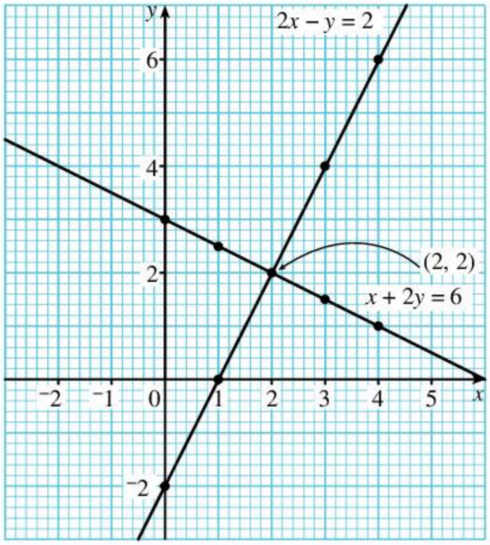
graphically.

The tables of values are $x + 2y = 6$

x	0	1	2	3	4
y	3	$2\frac{1}{2}$	2	$1\frac{1}{2}$	1

and $2x - y = 2$

x	0	1	2	3	4
y	$^{-}2$	0	2	4	6



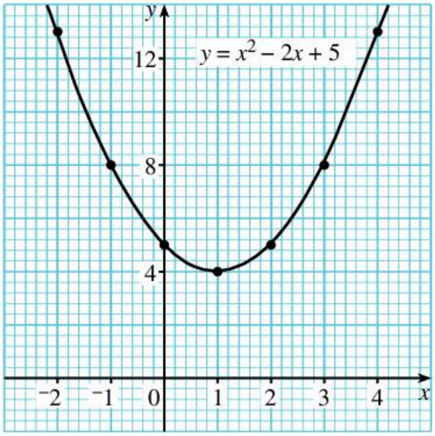
The solution is $x = 2$, $y = 2$.

Example 5

Plot the quadratic equation $y = x^2 - 2x + 5$ for $^{-}2 \leqslant x \leqslant 4$.

Table of values is:

x	$^{-}2$	$^{-}1$	0	1	2	3	4
x^2	4	1	0	1	4	9	16
$^{-}2x$	4	2	0	$^{-}2$	$^{-}4$	$^{-}6$	$^{-}8$
5	5	5	5	5	5	5	5
y	13	8	5	4	5	8	13



Exercise 9

1 **(a)** Complete a table as shown below

x	$^{-}3$	$^{-}2$	$^{-}1$	0	1	2	3	4
$f(x)$								

for the equations:

(i) $f(x) = 2x + 1$ **(ii)** $f(x) = 2x - 3$

(iii) $f(x) = \frac{1}{2}x + 3$ **(iv)** $f(x) = 6 - 2x$

(b) From the table of values, draw graphs of each of the functions.

2 Show the relation x 'is factor of' over the domain {1, 2, 3, 4} as

(a) an arrow graph

(b) a set of ordered pair

(c) a coordinate graph

(d) is the relation a function.

3 Use a graphical method to solve these equations.

(a) $y = 3x - 5$
$y = 2x + 1$

(b) $x - y = 6$
$x + y = 4$

(c) $2x + y = 6$
$x - 2y = 3$

4 Plot these quadratic graphs for $^{-}3 \leqslant x \leqslant 3$.

(a) $y = x^2 - 2x + 5$

(b) $y = 8 - x^2$

(c) $y = 2x^2 - 3x + 4$

Application

5 The distance travelled by a car in metres from its rest position over time is shown in the table.

Time (seconds)	0.5	1.2	2.9	3.2	4.1	5.2	6.1
Distance (metres)	0.4	2.2	12.6	15.4	25.2	40.6	55.8

(a) Using a scale of 1 cm to represent 5 m and 1 cm to represent 1 second, draw a graph to show the data.

(b) Use your graph to find:

  (i) the distance travelled by the car after 4.5 seconds

  (ii) the time it takes for the car to travel 50 metres.

6 The table shows the relationship between centimetres and inches:

Centimetres	0	2.54	5.08	7.62	10.16
Inches	0	1	2	3	4

(a) Plot the graph of this relationship and join the points with a straight line.

(b) Use your graph to convert:

  (i) 2.5 inches to cm

  (ii) 4 cm to inches

  (iii) 6.2 cm to inches.

Support Website

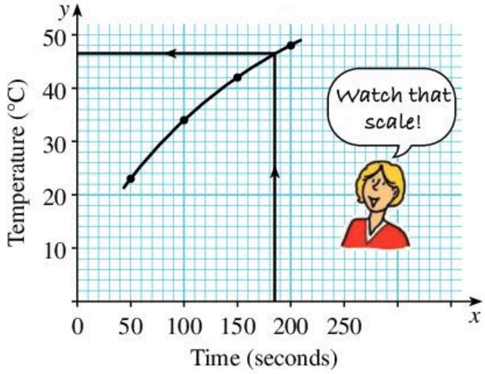
Additional material to support this topic can be found at www.oxfordsecondary.com/9780198425793

Summary

You should know …

1 How to read information from graphs.

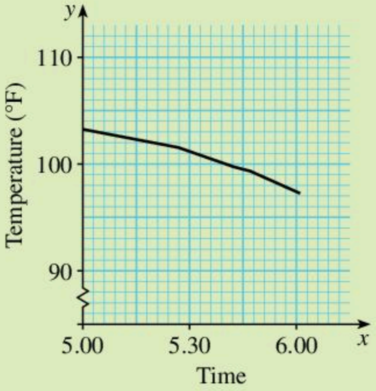
For example:



The temperature after 185 s is 46.5C.

Check out

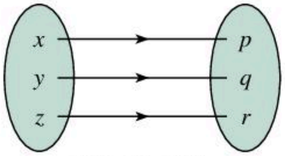
1 A baby's temperature was taken every 30 minutes starting at 5 o'clock.



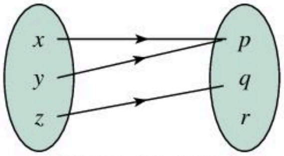
(a) At what time was the temperature 100?

(b) What was the temperature at 5.18?

2 A **function** is a one-to-one or one-to-many mapping.



one - to - one map



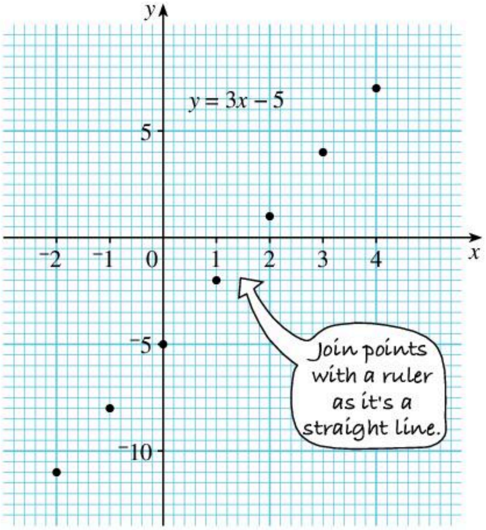
many - to - one map

3 You can show a function as

- an arrow graph
- a set of ordered pairs
- an algebraic rule
- a coordinate graph.

4 How to draw graphs of relations using a table of values.
For example:
$y = 3x - 5$

x	$^-2$	$^-1$	0	1	2	3	4
$3x$	$^-6$	$^-3$	0	3	6	9	12
$^-5$	$^-5$	$^-5$	$^-5$	$^-5$	$^-5$	$^-5$	$^-5$
y	$^-11$	$^-8$	$^-5$	$^-2$	1	4	7



2 Which of these relations with given domains are functions?

(a) $x \rightarrow x + 4$ $\{0, 1, 2, 3, 4\}$

(b) $x \rightarrow 2 - 3x$ $\{0, 1, 2, 3, 4\}$

(c) $x \rightarrow \pm\sqrt{x}$ $\{0, 1, 4, 9\}$

3 Given the function
$f: x \rightarrow 2x - 7$
over the domain $\{0, 1, 2, 3, 4\}$

(a) draw an arrow graph of the function

(b) write down the set of ordered pairs

(c) plot the ordered pairs to make a coordinate graph of two function.

4 **(a)** Draw graphs of:

(i) $y = {}^-3$

(ii) $y = 2x - 1$

(b) Draw the graph of $y = x^2 + 2x - 3$ for values of x from $^-4$ to $^+2$.

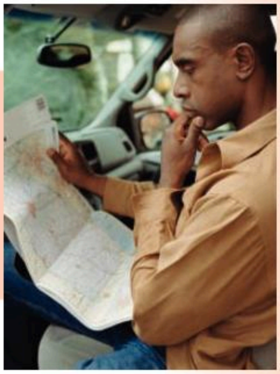
Transformations and vectors 10

Objectives

- ✔ reflect an object in a mirror line
- ✔ find the image of an object after an enlargement
- ✔ solve problems involving similar images
- ✔ describe vectors as 2 × 1 column vectors
- ✔ define and use position vectors to solve problems in geometry
- ✔ find the magnitude of a vector

What's the point?

A suitable enlargement of a photograph of yourself will result in a life-size image. In the same way the transformation of enlargement or diminishment is a critical aspect of map making.

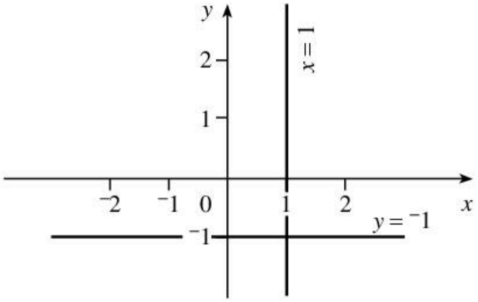


Before you start

You should know ...

1 How to plot points on graphs.

2 How to draw simple lines given their equations.
For example:



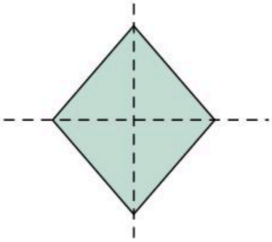
Points on the line $y = {}^{-}1$ always have a y-coordinate of $^{-}1$.

Check in

1 On suitable axes, plot these points.
(a) (5, 7) **(b)** $(^{-}3, 5)$
(c) $(^{-}4, {}^{-}6)$ **(d)** $(8, {}^{-}5)$

2 Draw the graphs of these lines.
(a) $x = 2$ **(b)** $x = {}^{-}3$
(c) $y = 3$ **(d)** $y = {}^{-}2$

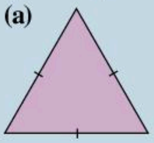
3 How to draw and count lines of symmetry.
For example:



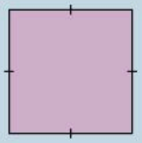
the rhombus has two lines of symmetry.

3 How many lines of symmetry do these shapes have? Copy each shape and draw its lines of symmetry.

(a)

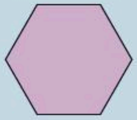


(b)



(c)

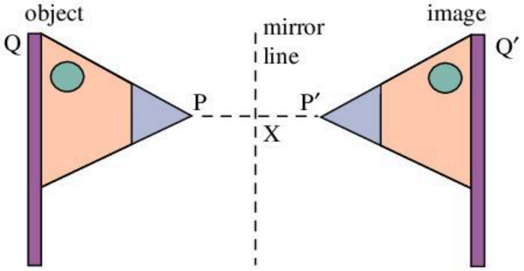
(d)



10.1 Reflection

You will need tracing paper, squared paper and a small mirror.

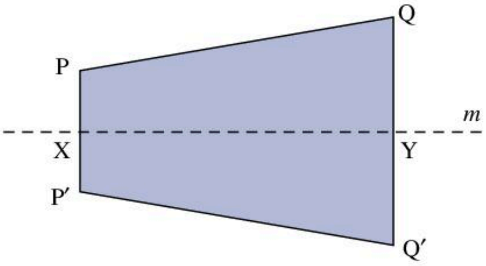
- An object reflected in a mirror line produces an **image**.
  The object and image are identical, but point in opposite directions. The reflection forms an **opposite image**.



The image of P is P′ We say P maps to P′ In the same way Q maps to Q′.
A line from P to P′ intersects the mirror line at X.
The distance PX = P′X.

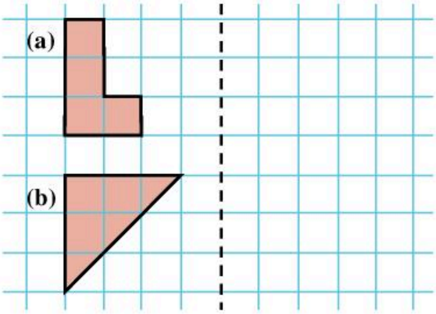
Exercise 10A

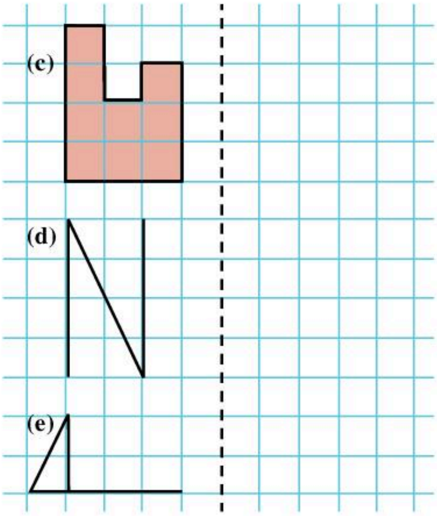
1 In this diagram, PQ is reflected in a mirror line *m* to produce P′Q′.
PP′ meets *m* at X, QQ′ meets *m* at Y.
(a) What is the image of P′Q′?
(b) What is the image of PXYQ?
(c) What is the image of angle PXY?



(d) Without measuring, what can you conclude about the size of angle PXY?
(e) Do you agree that the line joining an object and its image must intersect the mirror line at right angles?

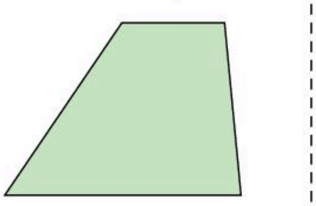
2 Copy these shapes on to squared paper. Draw their reflections in the dashed line.



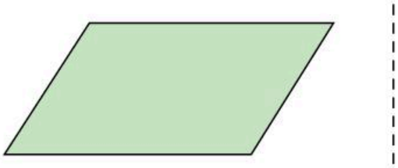


3 Trace each shape and the dashed line. Draw the reflection of the shape in the dashed line.

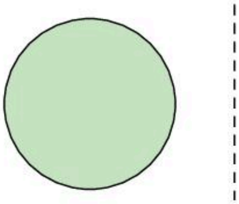
(a)



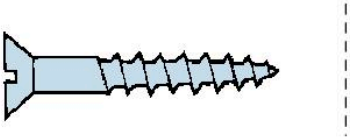
(b)



(c)



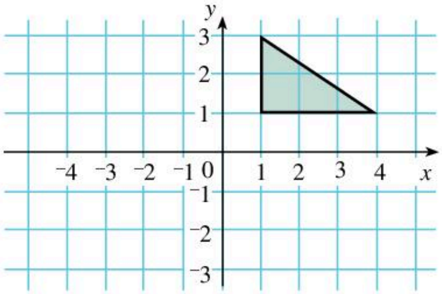
4 **(a)** Copy this drawing of a screw, and draw its reflection in the dashed line.



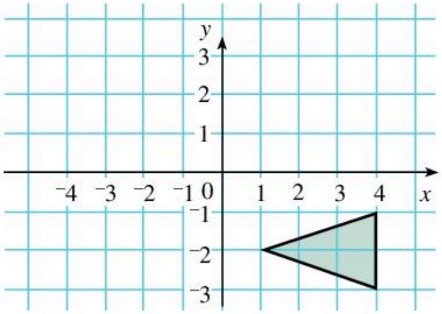
(b) Would a nut that fitted a bolt also fit its reflection?

5 Copy each diagram on to squared paper. Reflect the shape in the y-axis.

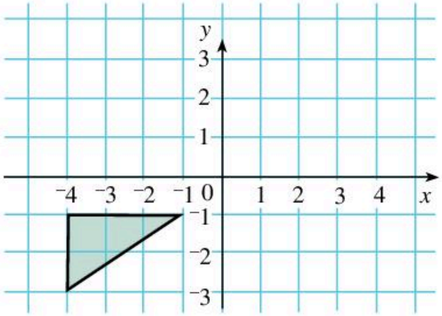
(a)



(b)



(c)



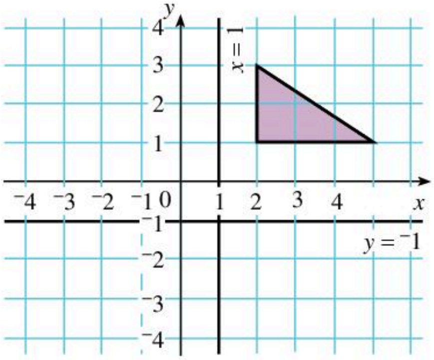
6 On the same diagrams that you drew for Question 5 draw the images formed by reflection in the x-axis.

7 Find the images of the points **(i)** $(3, {}^{-}5)$ **(ii)** $(5, 2)$ and **(iii)** $({}^{-}3, {}^{-}2)$ in the mirror lines

(a) $y = 0$ **(b)** $x = 0$

8 Copy the diagram and draw the image formed by reflection in:

(a) the line $x = 1$ **(b)** the line $y = {}^-1$



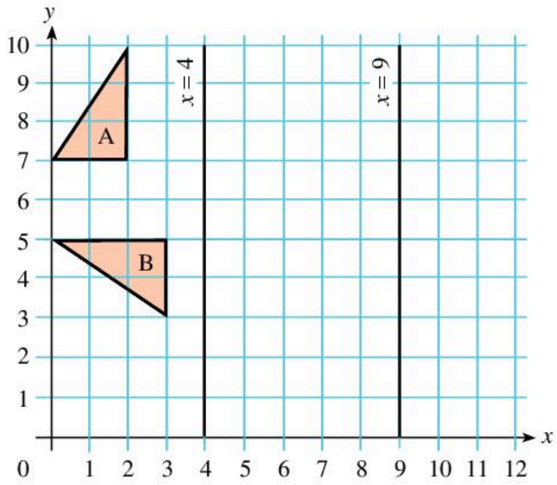
9 Copy the diagram below then draw:

(a) the image A′ of A after reflection in $x = 4$

(b) the image A″ of A′ after reflection in $x = 9$

(c) the image B′ of B after reflection in $x = 4$

(d) the image B″ of B′ after reflection in $x = 9$



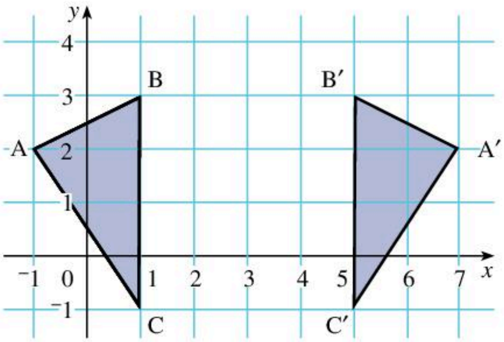
10 On squared paper, plot a triangle with vertices A(0, 3), B(3, 3), C(3, 5). Draw its image after reflection in the line:

(a) $x = 4$ **(b)** $x = {}^-1$

(c) $y = 6$ **(d)** $y = 2$

11 Repeat Question 10 for a triangle with vertices X($^-1$, 3), Y($^-2$, 3), Z($^-2$, 5).

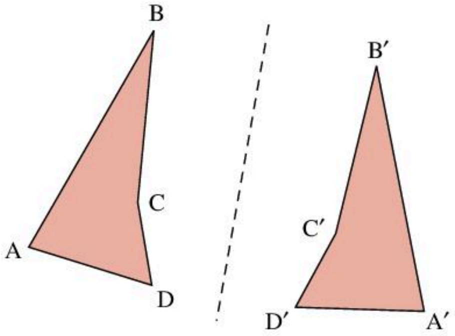
12 A triangle with vertices A($^-1$, 2), B(1, 3), C(1, $^-1$) on reflection gives an image with vertices A′(7, 2), B′(5, 3), C′(5, $^-1$).



(a) Write down the coordinates of three points on the mirror line.

(b) What is the equation of the mirror line?

13 Trace ABCD and its image A′B′C′D′.



(a) Is A′B′C′D′ an opposite image of ABCD?

(b) Draw AA′ and mark its midpoint.

(c) Repeat (b) for the other pairs of vertices.

(d) What do you notice about these midpoints?

(e) Does this prove that the image A′B′C′D′ is a reflection?

14 The triangle ABC in Question 12 is reflected in the line $x = 2$ to give the image XYZ. Triangle XYZ is then reflected in the line $x = 6$ to form the image X′Y′Z′.

(a) Draw a rough sketch to show this.

(b) Is X′Y′Z′ an opposite image of ABC?

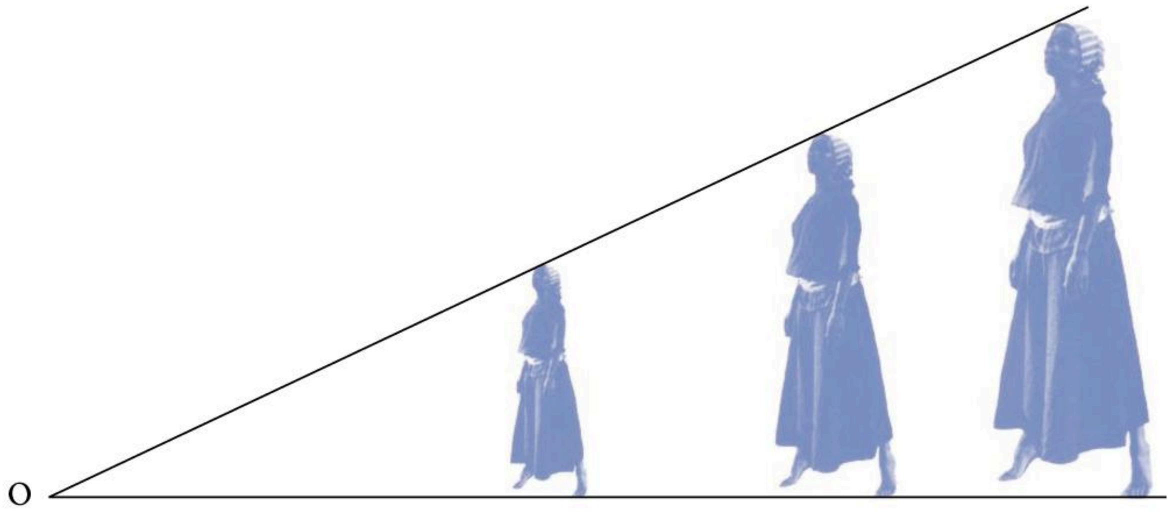
15 Triangle ABC in Question 12 on reflection gives an image with vertices A′($^-1$, 6), B′(1, 5), and C′(1, 9).

(a) Draw a sketch to show this.

(b) What is the equation of the mirror line?

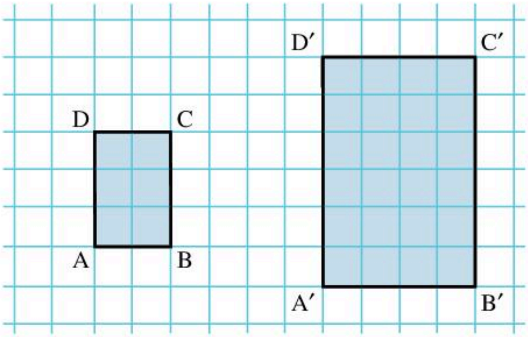
10.2 Enlargement

Here are two enlargements of a photograph.



- An **enlargement** is a transformation that changes the size of an object. The point O is called the **centre of enlargement**.

You will need squared paper.



Look at the two rectangles above.

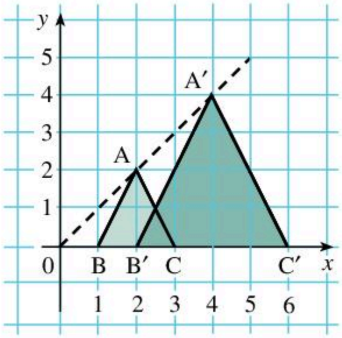
A′B′ = 2 + AB
B′C′ = 2 + BC

We say that rectangle ABCD has been enlarged with scale factor 2.

To define an enlargement you must be given:
- the scale factor of the enlargement
- the centre of the enlargement.

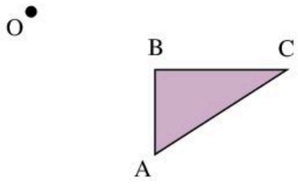
For example, triangle ABC is enlarged by a scale factor of 2 where O is the centre of enlargement:

A(2, 2) maps to A′(4, 4)
B(1, 0) maps to B′(2, 0)
C(3, 0) maps to C′(6, 0)



Example 1

Draw the image of triangle ABC after an enlargement centre O and scale factor 2.

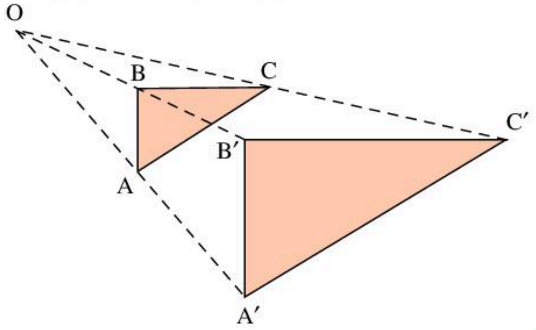


Measure the distances OA, OB and OC with a ruler:

OA = 3 cm, OB = 2 cm, OC = 4 cm.

Since the scale factor is 2

OA′ = 2 × 3 cm = 6 cm
OB′ = 2 × 2 cm = 4 cm
OC′ = 2 × 4 cm = 8 cm

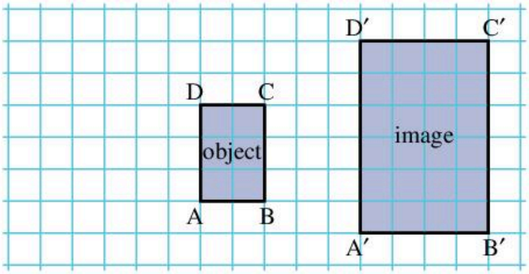


The centre of the enlargement can be found by joining the corners of the image to the corners of the object with straight lines.

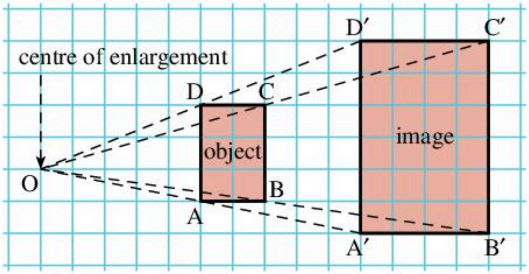
These straight lines meet at the centre of the enlargement.

Example 2

Find the centre of enlargement for this enlargement.



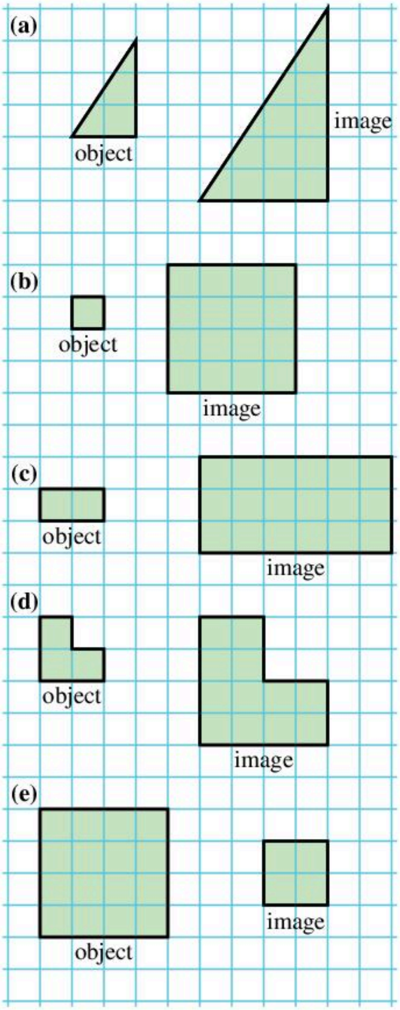
Join the corners A′ to A with a straight line. Do the same for B′B and D′D.



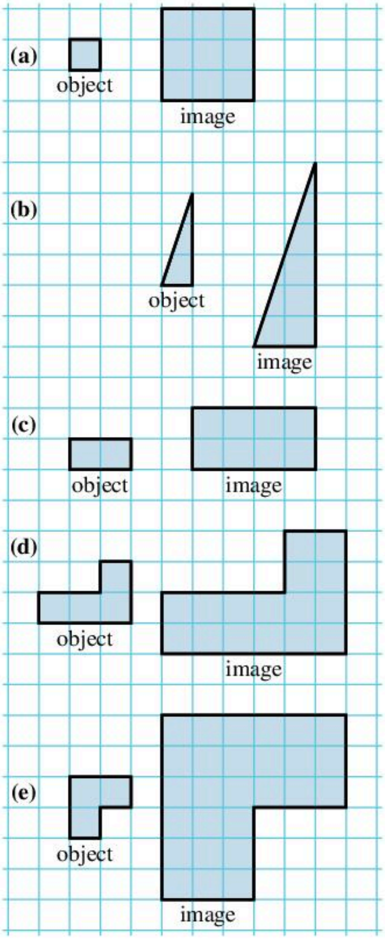
The lines A′A, B′B, C′C and D′D all meet at O. O is the centre of the enlargement.

Exercise 10B

1 Use a ruler to measure the edges of these shapes then write down the scale factor for each enlargement.



2 Copy the shapes on squared paper. By joining the corners of the image to the corners of the object find the centre of enlargement.



3 The triangle P(2, 1), Q(2, 3), R(1, 3) is mapped to P′(4, 1), Q′(4, 5), R′(2, 5) by an enlargement.

(a) Draw PQR and its image on squared paper.

(b) By joining PP′, QQ′ and RR′ find the centre of enlargement. Write down its coordinates.

(c) What do you notice about the sides and angles of triangle PQR and its image?

4 The triangle K(1, 3), L(2, 3), M(2, 4) is mapped to K′(1, 1), L′(3, 1), M′(3, 3) by an enlargement, centre S.

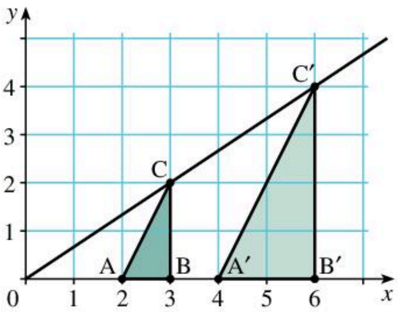
(a) Draw KLM and its image.

(b) Find the coordinates of S.

(c) What do you notice about the lengths of KLM and its image?

(d) Are the angles in K′L′M′ the same size as the angles in KLM?

5 The diagram shows an enlargement of scale factor 2 and centre O (0, 0).



(a) Copy and complete:

$OA' = \square \times OA$

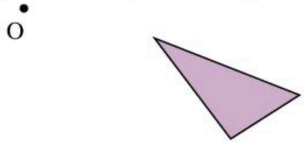
$OB' = \square \times OB$

$OC' = \square \times OC$

(b) Do you agree that $\square$ is equal to the scale factor?

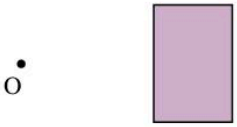
6 Trace each shape and draw the image of the shape after the given enlargement.

(a)



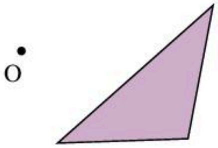
scale factor 2, centre O

(b)



scale factor 2, centre O

(c)



scale factor 3, centre O

7 On squared paper, draw the triangle with vertices (2, 1), (4, 2) and (3, 4). Call it A. With (0, 0) as centre draw:

(a) The enlargement of A with scale factor 3. Call it B.

(b) The enlargement of B with scale factor 2. Call this image C.

What is the scale factor of the enlargement of A to C?

8 Repeat Question 7 using (4, 2) as the centre of enlargement. Do you still find that the two enlargements are equivalent to a single enlargement with scale factor 6?

9 Triangle P with vertices (7, 0), (7, 2), (9, 3) is mapped to triangle Q with vertices (11, 0), (11, 4), (15, 6), by an enlargement.

(a) Draw the two triangles.

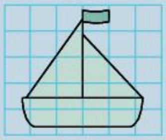
(b) Find the centre of enlargement and the scale factor.

10 **(a)** Draw triangle C with vertices (6, 8) (8, 8) and (6, 2). Draw triangle D with vertices (10, 13), (13, 13) and (10, 4).
Is D an enlargement of C?

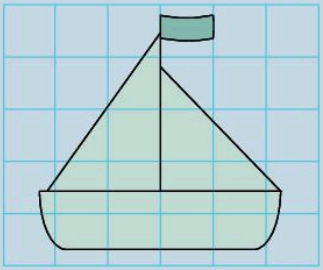
(b) Find the centre of enlargement and the scale factor.

Activity

One method artists use to enlarge drawings is to use a grid.
For example:



Original picture



Enlarged picture

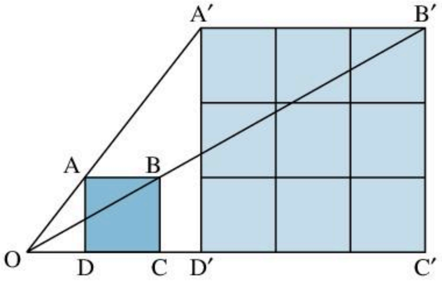
The squares on the enlarged picture are twice as long as those on the original. Hence the image has been enlarged by a scale factor of 2.

- Cut out a photograph or drawing of an object from a magazine.
- Draw grid lines on it.
- Draw an enlarged grid (scale factor 2 or 3) on a sheet of paper.
- Make an enlarged copy of your picture/photograph.
- Repeat with a passport-sized picture of yourself. Enlarge the drawing of your face by a factor of 5.

Areas and enlargements

Exercise 10C

1



A′B′C′D′ above is an enlargement of ABCD.

(a) What is the scale factor?

(b) Copy and complete: The area of A′B′C′D′ is □ times the area of ABCD.

2 **(a)** On squared paper, draw a square ABCD with side 1 cm. Draw its enlargement, centre A, and scale factor 5.

(b) Copy and complete: The area of the image is □ times the area of ABCD.

3 Can you see a connection between the size of the scale factor and the increase in area?

4 In an enlargement of a rectangle ABCD, the image A′B′C′D′ has A′B′ = 4AB. O is the centre of enlargement.

(a) If BC = 3 cm. find B′C′.

(b) If OD = 2 cm. find OD′.

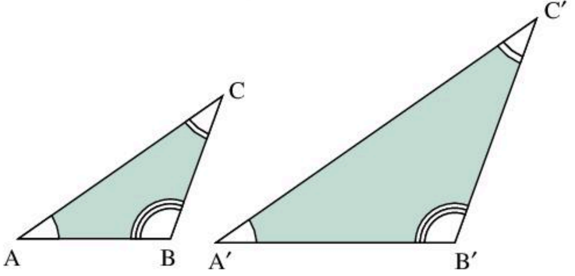
(c) If OC = 1 cm. find CC′.

(d) If area ABCD is 20 cm^2, find area A′B′C′D′.

Similarity

Notice that when a shape is enlarged it does not change its shape. It just becomes bigger or smaller.

Look at these two triangles.



Triangle ABC is an enlargement scale factor 2 of triangle A′B′C′. The shape of the triangles is the same.

With a protractor, measure the angles $C\hat{A}B$, $A\hat{B}C$, $B\hat{C}A$ and $C'\hat{A}'B'$, $A'\hat{B}'C'$, $B'\hat{C}'A'$.

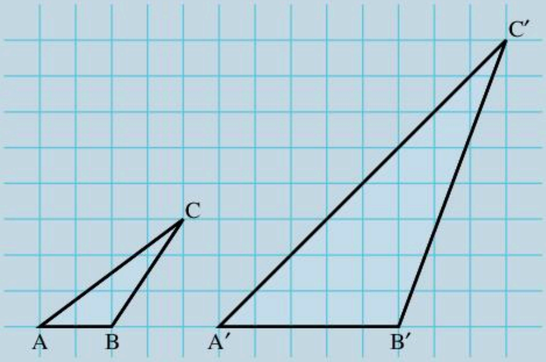
What did you notice?

You should have found that the corresponding angles in each triangle are equal.

The two triangles, ABC and A′B′C′, are called **similar** triangles. The interior angles of each are the same but their sizes are different.

Their **shapes** are the **same** but their **sizes** are **different**.

Activity



Look at the triangles ABC and A′B′C′ in the diagram.

- Measure angles
  - – $C\hat{A}B$ and $C'\hat{A}'B'$
  - – $A\hat{B}C$ and $A'\hat{B}'C'$
  - – $A\hat{C}B$ and $A'\hat{C}'B'$
- What do you notice?
- Measure the lengths
  AB and A′B′
  What is $\frac{A'B'}{AB}$?
- Measure
  AC and A′C′
  What is $\frac{A'C'}{AC}$?
- Measure
  BC and B′C′
  What is $\frac{B'C'}{BC}$?
- What do you notice?

In the Activity you should find that

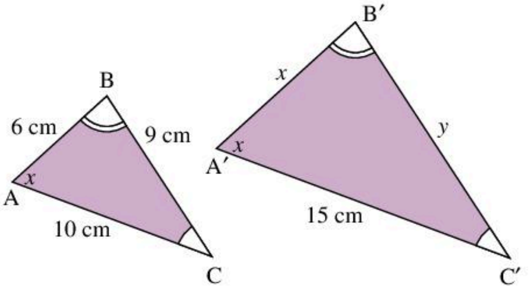
$$\frac{A'B'}{AB} = \frac{A'C'}{AC} = \frac{B'C'}{BC} = 2$$

That is, with similar triangles the ratio of corresponding sides equals the scale factor of enlargement.

You can use this idea of similarity to solve simple problems.

Example 3

Calculate the lengths x and y in these two similar triangles.



Since the triangles are similar:

$$\frac{A'B'}{AB} = \frac{B'C'}{BC} = \frac{A'C'}{AC}$$

So $\quad \frac{x}{6} = \frac{y}{9} = \frac{10}{15}$

That is, $\frac{x}{6} = \frac{10}{15}$

hence $\quad x = 6 \times \frac{10}{15} = 4$ cm

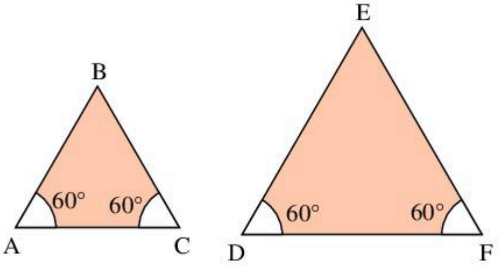
and $\quad \frac{y}{9} = \frac{10}{15}$

so $\quad y = 9 \times \frac{10}{15} = 6$ cm

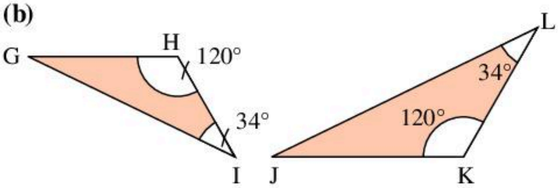
Exercise 10D

1 Which of these pairs of shapes are similar?

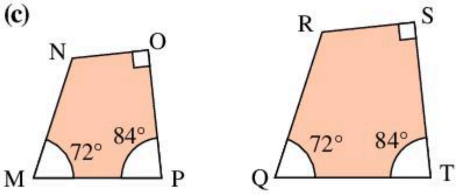
(a)



(b)



(c)



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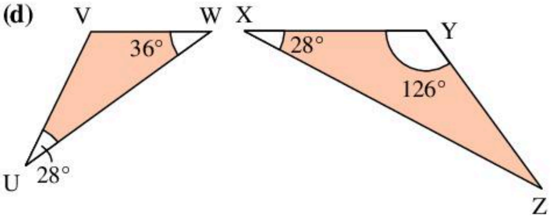


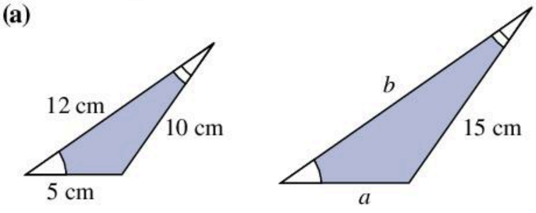
2 **(a)** Draw two different-sized triangles with angles 40°, 50° and 90°.

(b) Measure the length of each side of each triangle.

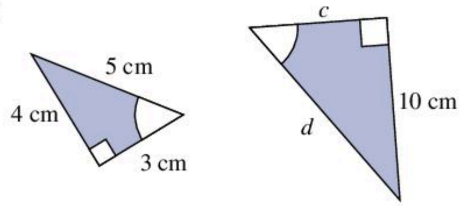
(c) What is the ratio of side lengths?

3 Calculate the lettered lengths in these pairs of similar triangles.

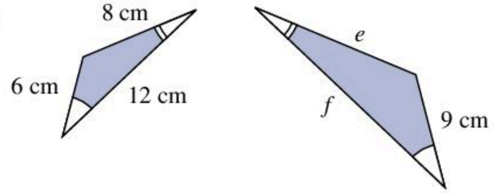
(a)



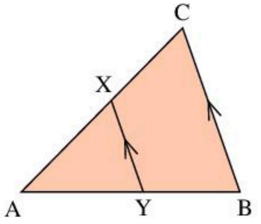
(b)



(c)



4 In the triangle ABC, the line XY is parallel to BC.

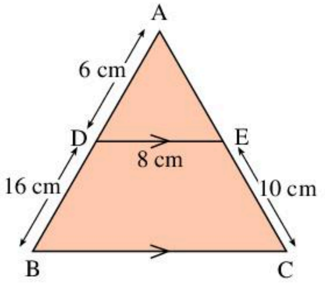


(a) Explain why triangles AXY and ACB are similar.

(b) If AB = 18 cm and AC = 14 cm, CB = 8 cm and XY = 4 cm, find

(i) AX **(ii)** AY

5



In the triangle ABC, DE is parallel to BC. Calculate the lengths

(a) BC **(b)** AE

Technology

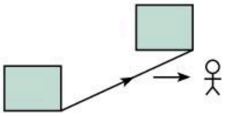
Learn more about similar triangles by visiting the site www.mathsisfun.com/geometry/triangles-similar.html

Make sure you try the questions!

10.3 Vectors in geometry

A **vector** is a quantity that has both **size** (magnitude) and **direction**.

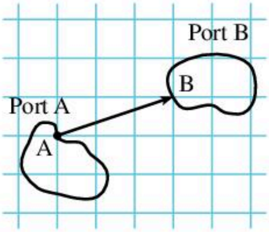
A **translation** is a vector because it represents a movement of a certain distance in a particular direction.



You can represent geometrically a vector by an arrow.

- The length of the arrow represents the size or magnitude of the vector.
- The direction of the arrow gives the direction of the vector.

Vectors are used in navigation by air pilots and ships. For example, a boat sails from Port A to Port B

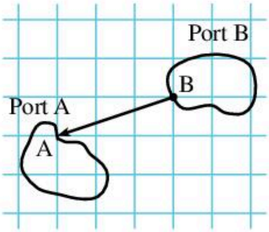


The vector, $\overrightarrow{AB}$, describes the distance and direction the boat travels in from A to B.
You can write a boat's change of position using a column vector.
You can write

$$\overrightarrow{AB} = \begin{pmatrix} 3 \\ 1 \end{pmatrix}$$

that is, the boat has moved
3 squares (across) in the x-direction
and 1 square (up) in the y-direction

On the return journey from Port B to Port A.



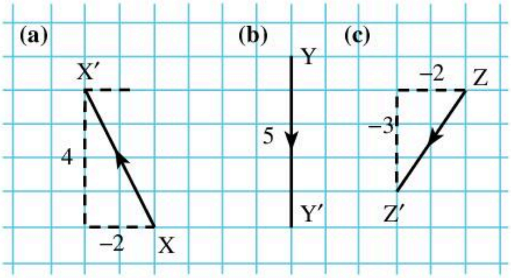
$$\overrightarrow{BA} = \begin{pmatrix} -3 \\ -1 \end{pmatrix}$$

as the boat moves
3 squares (left) in negative x-direction
and 1 square (down) in negative y-direction.

Vectors are sometimes denoted by a lower-case bold letter such as **a**.

Example 4

Write a column vector to describe these translations:



(a) $\overrightarrow{XX'}$ is a movement of 2 units to the left ($^-2$) and 4 units upwards (4),

so $\overrightarrow{XX'} = \begin{pmatrix} ^-2 \\ 4 \end{pmatrix}$

(b) $\overrightarrow{YY'}$ is a movement of 0 units to the right (0) and 5 units downwards ($^-5$),

so $\overrightarrow{YY'} = \begin{pmatrix} 0 \\ ^-5 \end{pmatrix}$

(c) $\overrightarrow{ZZ'}$ is a movement of 2 units to the left ($^-2$) and 3 units downwards ($^-3$),

so $\overrightarrow{ZZ'} = \begin{pmatrix} ^-2 \\ ^-3 \end{pmatrix}$

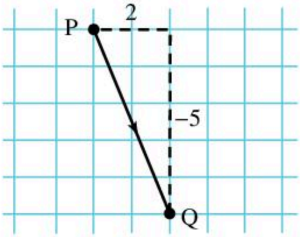
Given the column vector you can easily draw the translation.

Example 5

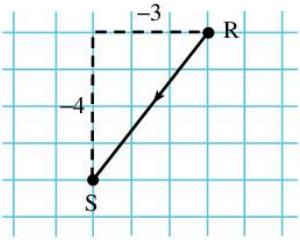
Draw the vectors represented by:

(a) $\overrightarrow{PQ} = \begin{pmatrix} 2 \\ ^-5 \end{pmatrix}$ **(b)** $\overrightarrow{RS} = \begin{pmatrix} ^-3 \\ ^-4 \end{pmatrix}$

(a) $\overrightarrow{PQ} = \begin{pmatrix} 2 \\ ^-5 \end{pmatrix}$ means start at P, move 2 units to the right then go 5 units vertically downwards to Q.

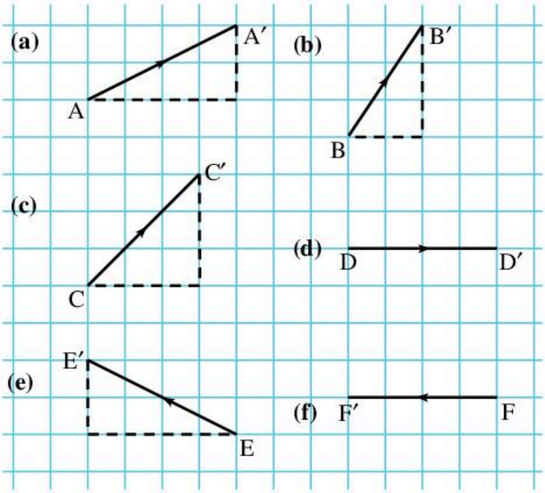


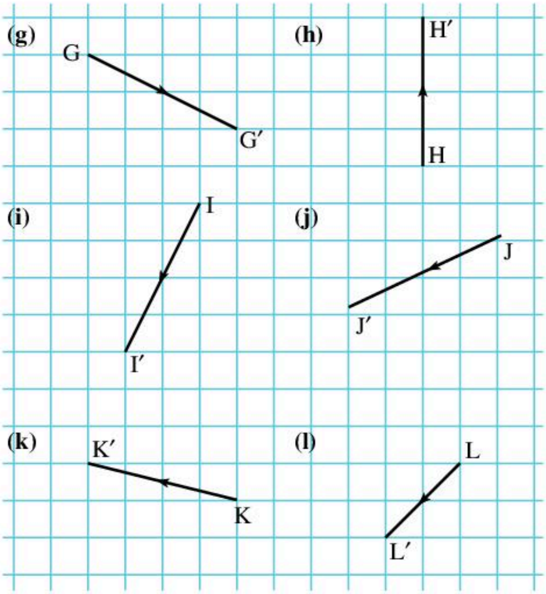
(b) $\overrightarrow{RS} = \begin{pmatrix} ^-3 \\ ^-4 \end{pmatrix}$ means start at R, move 3 units to the left then go 4 units vertically downwards to S.



Exercise 10E

1 Write column vectors to describe each of these translations:





2 On squared paper, draw the translation represented by the column vector:

(a) $\overrightarrow{AB} = \begin{pmatrix} 2 \\ 3 \end{pmatrix}$ **(b)** $\overrightarrow{CD} = \begin{pmatrix} 3 \\ 2 \end{pmatrix}$

(c) $\overrightarrow{EF} = \begin{pmatrix} 4 \\ 0 \end{pmatrix}$ **(d)** $\overrightarrow{GH} = \begin{pmatrix} ^-3 \\ 2 \end{pmatrix}$

(e) $\overrightarrow{IJ} = \begin{pmatrix} ^-4 \\ 3 \end{pmatrix}$ **(f)** $\overrightarrow{KL} = \begin{pmatrix} 2 \\ ^-3 \end{pmatrix}$

(g) $\overrightarrow{MN} = \begin{pmatrix} 5 \\ ^-1 \end{pmatrix}$ **(h)** $\overrightarrow{OP} = \begin{pmatrix} ^-5 \\ ^-2 \end{pmatrix}$

(i) $\overrightarrow{RS} = \begin{pmatrix} ^-3 \\ 0 \end{pmatrix}$ **(j)** $\overrightarrow{TU} = \begin{pmatrix} ^-4 \\ ^-5 \end{pmatrix}$

3 **(a)** On squared paper, draw the vectors:

$$\mathbf{P} = \begin{pmatrix} 2 \\ 1 \end{pmatrix}, \mathbf{q} = \begin{pmatrix} 4 \\ 2 \end{pmatrix}, \mathbf{r} = \begin{pmatrix} 6 \\ 3 \end{pmatrix}$$

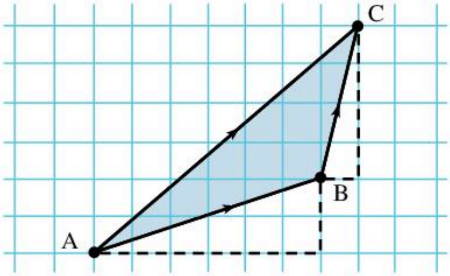
(b) How are **p**, **q** and **r** related?

4 **(a)** On squared paper, draw the vectors:

$$\mathbf{x} = \begin{pmatrix} 3 \\ 4 \end{pmatrix}, \mathbf{y} = \begin{pmatrix} ^-3 \\ ^-4 \end{pmatrix}$$

(b) How are **x** and **y** related?

5



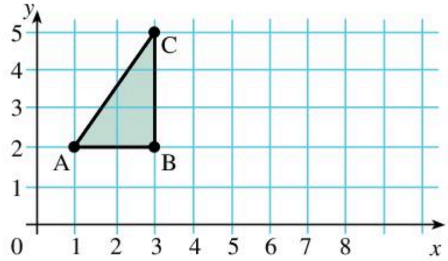
(a) From the diagram write $\overrightarrow{AB}$, $\overrightarrow{BC}$ and $\overrightarrow{AC}$ as column vectors.

(b) Can you see how the column vectors are related?

6 Find the coordinates of the image of the point A(1, 2) after translation:

(a) $\begin{pmatrix} 1 \\ 3 \end{pmatrix}$ **(b)** $\begin{pmatrix} 8 \\ ^-2 \end{pmatrix}$ **(c)** $\begin{pmatrix} ^-5 \\ ^-3 \end{pmatrix}$

7 **(a)** Copy the diagram.



(b) The point A is translated by the vector $\begin{pmatrix} 4 \\ 1 \end{pmatrix}$ to its image A′. Draw the point A′ on your diagram and write down its coordinates.

(c) Translate the points B and C by the vector $\begin{pmatrix} 4 \\ 1 \end{pmatrix}$. Draw their images B′ and C′ on your diagram and write down their coordinates.

(d) Do you agree that the image of the triangle ABC under the translation $\begin{pmatrix} 4 \\ 1 \end{pmatrix}$ is the triangle A′B′C′?

Activity

Scale 1cm = ________ km

Pilots fly across the Caribbean many times each day. They fly their planes on predesignated flight paths. These flight paths can be represented by vectors.

For example, the flight path from Trinidad to Jamaica, $\overrightarrow{TJ} = \begin{pmatrix} \\ \end{pmatrix}$.

(1) Represent as column vectors the flight paths
   (a) Trinidad to Barbados ($\overrightarrow{TB}$)
   (b) Barbados to Jamaica ($\overrightarrow{BJ}$)
   (c) Barbados to Antigua ($\overrightarrow{BA}$)
   (d) Antigua to Jamaica ($\overrightarrow{AJ}$)
   (e) St. Lucia to Antigua ($\overrightarrow{SA}$)
   (f) Dominica to Antigua ($\overrightarrow{DA}$)

(2) A plane flies from Trinidad to Barbados and then to Jamaica. Draw the flight path using arrows (vectors) on squared paper.
   Find $\overrightarrow{TB} + \overrightarrow{BJ}$ Is this the same as $\overrightarrow{TR}$?

(3) Repeat **(2)** for other two stage journeys. What do you notice? How about three stage journeys?

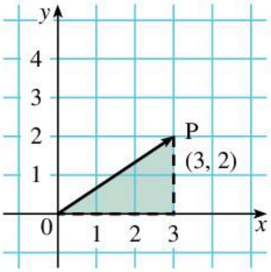
(4) Using the scale, find the actual distance, in km for the flights in **(1)**.

(5) Find out how wind speed may affect the flight paths.

Position vectors

You can use a column vector to describe the position of a point on a coordinate grid.

For example,
The point P (3, 2)



can be represented by the **position vector**, $\overrightarrow{OP} = \begin{pmatrix} 3 \\ 2 \end{pmatrix}$ where O is the origin.

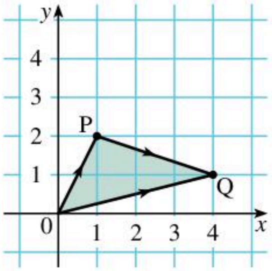
The concept of position vectors can be used to solve simple problems.

Example 6

The points P(1, 2) and Q(4, 1) are joined by a line.

(a) Write down the position vectors
   (i) $\overrightarrow{OP}$ **(ii)** $\overrightarrow{OQ}$

(b) Find the vector $\overrightarrow{PQ}$

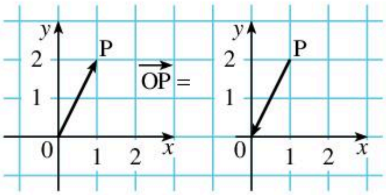


(a) **(i)** $\overrightarrow{OP} = \begin{pmatrix} 1 \\ 2 \end{pmatrix}$ **(ii)** $\overrightarrow{OQ} = \begin{pmatrix} 4 \\ 1 \end{pmatrix}$

(b)
$$\begin{aligned} \overrightarrow{PQ} &= \overrightarrow{PO} + \overrightarrow{OQ} \\ &= -\overrightarrow{OP} + \overrightarrow{OQ} \\ &= -\begin{pmatrix} 1 \\ 2 \end{pmatrix} + \begin{pmatrix} 4 \\ 1 \end{pmatrix} \\ &= \begin{pmatrix} ^{-}1 + 4 \\ ^{-}2 + 1 \end{pmatrix} \\ &= \begin{pmatrix} 3 \\ ^{-}1 \end{pmatrix} \end{aligned}$$

Notice in Example 6 you used two basic ideas:

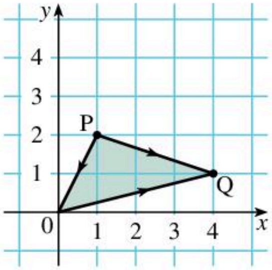
(1) How to find the inverse of a vector $\overrightarrow{OP}$



$\overrightarrow{OP} = \begin{pmatrix}1\\2\end{pmatrix}$ $\qquad$ $\overrightarrow{PO} = \begin{pmatrix}^-1\\^-2\end{pmatrix}$

That is, $\overrightarrow{OP} = -\overrightarrow{PO}$

(2) How to add two vectors



$\overrightarrow{PO} + \overrightarrow{OQ} = \overrightarrow{PQ}$

Exercise 10F

1 A square has coordinates A(1, 1), B(3, 1), C(3, 3), D(1, 3).
Write down the position vectors
(a) $\overrightarrow{OA}$ **(b)** $\overrightarrow{OB}$ **(c)** $\overrightarrow{OC}$ **(d)** $\overrightarrow{OD}$

2 The position vectors of a triangle ABC are

$\overrightarrow{OA} = \begin{pmatrix}0\\2\end{pmatrix}, \overrightarrow{OB} = \begin{pmatrix}2\\2\end{pmatrix}, \overrightarrow{OC} = \begin{pmatrix}2\\4\end{pmatrix}$

Write down the coordinates of the points
(a) A **(b)** B **(c)** C

3 P has coordinates (2, 3) and Q has coordinates (1, 4).
Find

(a) the position vector, $\overrightarrow{OP}$
(b) the position vector, $\overrightarrow{OQ}$
(c) the vector $\overrightarrow{PO}$
(d) the vector $\overrightarrow{PQ}$

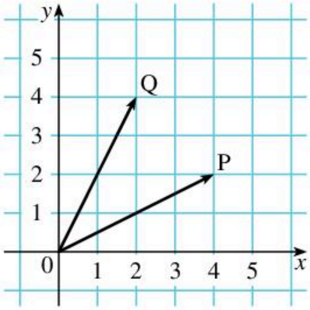
4 A is the point (2, 2) and the vector $\overrightarrow{AB} = \begin{pmatrix}1\\1\end{pmatrix}$.

(a) Draw the point A and the vector $\overrightarrow{AB}$ on a graph.
(b) Write down the position vector $\overrightarrow{OA}$?
(c) Find the position vector, $\overrightarrow{OB}$.
(d) Write down the coordinates of B.

5 The position vectors $\overrightarrow{OX}$ and $\overrightarrow{OY}$ are $\begin{pmatrix}1\\5\end{pmatrix}$ and $\begin{pmatrix}2\\4\end{pmatrix}$ respectively.

(a) Write down the coordinates of the points X and Y.
(b) Find **(i)** $\overrightarrow{XO}$ **(ii)** $\overrightarrow{XY}$

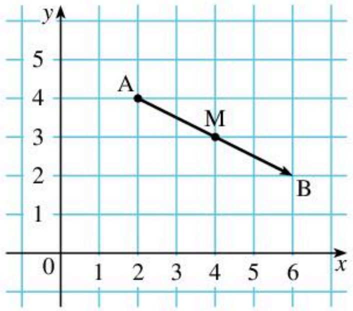
6



(a) In the diagram write down the position vectors
(i) $\overrightarrow{OP}$ **(ii)** $\overrightarrow{OQ}$
(b) Write as a column vector $\overrightarrow{PO}$.
(c) Complete the statement
$\overrightarrow{PQ} = \overrightarrow{PO} +$ ___.
(d) Hence find $\overrightarrow{PQ}$.

7 **(a)** On a coordinate graph show the point, A (2, 1).
(b) Write down the position vector $\overrightarrow{OA}$.
(c) Write down the column vector $\overrightarrow{OA}$ if $\overrightarrow{OA}' = 2\overrightarrow{OA}$.
(d) Show the vector $\overrightarrow{OA}'$ on your graph.
(e) What do you notice about $\overrightarrow{OA}'$ and $\overrightarrow{OA}$?

8



The diagram shows the vector $\overrightarrow{AB}$, with M as the midpoint of AB.

(a) Write down the position vectors

(i) $\overrightarrow{OA}$ **(ii)** $\overrightarrow{OB}$

(b) Write down the column vector $\overrightarrow{AB}$.

(c) Find the column vector $\overrightarrow{AM}$.

(d) Complete the statement $\overrightarrow{OM} = \overrightarrow{OA} +$ ___

Hence, find the position vector, $\overrightarrow{OM}$.

9 The coordinates of a rectangle ABCD are A(1, 0), B(4, 0), C(4, 2), D(1, 2) where O is the origin.

(a) Express as vectors in the form $\begin{pmatrix} x \\ y \end{pmatrix}$

(i) $\overrightarrow{OA}$ **(ii)** $\overrightarrow{OB}$ **(iii)** $\overrightarrow{OC}$

(iv) $\overrightarrow{OD}$ **(v)** $\overrightarrow{AO}$ **(vi)** $\overrightarrow{CO}$

(b) By completing the statements

$\overrightarrow{AB} = \overrightarrow{AO} +$ ___

and $\overrightarrow{CD} = \overrightarrow{CO} +$ ___

write $\overrightarrow{AB}$ and $\overrightarrow{CD}$ as column vectors.

(c) Show that $\overrightarrow{AB} = \overrightarrow{CD}$

(d) Is $\overrightarrow{AC} = \overrightarrow{BD}$? Give reasons.

10 M is the midpoint of the line segment XY and O is the origin.

Find the position vector $\overrightarrow{OM}$, when X and Y are the points.

(a) (2, 2), (4, 4)

(b) (0, 2), (6, 4)

(c) $(3, {}^{-}1), (1, {}^{-}3)$

11 OABC is a parallelogram with O(0, 0), A(2, 3), and B(4, 2).

(a) Write down the position vectors

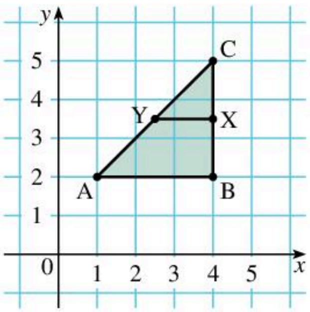
(i) $\overrightarrow{OA}$ **(ii)** $\overrightarrow{OB}$

(b) Write as column vectors

(i) $\overrightarrow{BC}$ **(ii)** $\overrightarrow{AC}$ **(iii)** $\overrightarrow{OC}$

(c) Write down the coordinates of D.

12



In the diagram, ABC is a triangle with coordinates A(1, 2), B(4, 2) and C(4, 5). X and Y are midpoints of BC and AC respectively.

(a) Write down the position vectors

(i) $\overrightarrow{OA}$ **(ii)** $\overrightarrow{OB}$ **(iii)** $\overrightarrow{OC}$

(b) Write as column vectors

(i) $\overrightarrow{AC}$ **(ii)** $\overrightarrow{BC}$ **(iii)** $\overrightarrow{AY}$

(iv) $\overrightarrow{BX}$ **(v)** $\overrightarrow{YX}$

(c) Show that $2\overrightarrow{YX} = \overrightarrow{AB}$.

Technology

Learn more about position vectors. Visit www.onlinemathlearning.com/position-vector.html.

Check the videos!

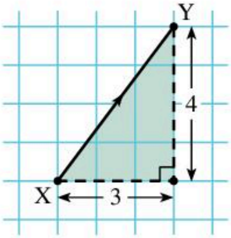
Magnitude of a vector

You can find the magnitude of any vector written in column vector form.

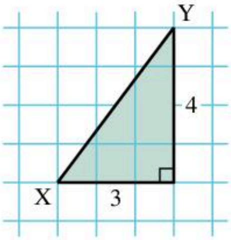
For example

$$\overrightarrow{XY} = \begin{pmatrix} 3 \\ 4 \end{pmatrix}$$

The vector $\overrightarrow{XY}$ can be shown as



It forms a right-angled triangle:



By Pythagoras

$$XY^2 = 3^2 + 4^2$$

That is

$$XY^2 = 9 + 16$$
$$XY^2 = 25$$

So $XY = \sqrt{25} = 5$

The magnitude of the vector $\overrightarrow{XY}$ is 5.

You can write $|\overrightarrow{XY}| = 5$ for short.

Exercise 10G

1 Find the magnitude of these vectors:

(a) $\overrightarrow{AB} = \begin{pmatrix} 3 \\ 4 \end{pmatrix}$ **(b)** $\overrightarrow{CD} = \begin{pmatrix} 5 \\ 12 \end{pmatrix}$

(c) $\overrightarrow{EF} = \begin{pmatrix} 8 \\ 6 \end{pmatrix}$ **(d)** $\overrightarrow{GH} = \begin{pmatrix} 9 \\ 12 \end{pmatrix}$

2 Find the magnitude of these 'unit' vectors:

(a) $\begin{pmatrix} 0 \\ 1 \end{pmatrix}$ **(b)** $\begin{pmatrix} 1 \\ 0 \end{pmatrix}$

(c) $\begin{pmatrix} ^{-}1 \\ 0 \end{pmatrix}$ **(d)** $\begin{pmatrix} 0 \\ ^{-}1 \end{pmatrix}$

3 The line segment AB, is defined by the points A(1, 1), B(4, 5).

(a) Write down the position vector of

(i) $\overrightarrow{OA}$ **(ii)** $\overrightarrow{OB}$

(b) Write $\overrightarrow{AB}$ as a column vector.

(c) Calculate $|\overrightarrow{AB}|$.

4 Find the magnitude when X and Y are the points

(a) X(2, $^{-}3$), Y(5, 1) **(b)** X($^{-}2$, 3), Y(10, 8)

(c) X(–4, $^{-}3$), Y(2, 5) **(d)** X($^{-}5$, 1), Y(4, 13)

Exercise 10H – mixed questions

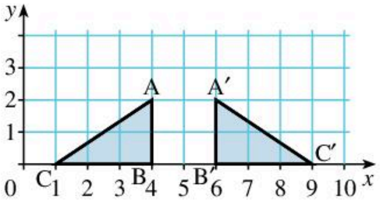
1 A triangle has vertices P(0, 1), Q(8, 4) and R(10, 1). It is reflected in the line $y = 1$ to give an image, P′Q′R′.

(a) Draw a sketch to show this.

(b) Write down the coordinates of the vertices of its image, P′Q′R′.

(c) What shape is formed by the outline PQRQ′.

2 Copy the diagram of triangle ABC and its image A′B′C′ after reflection. Draw in the mirror line. What is the equation of the mirror line?



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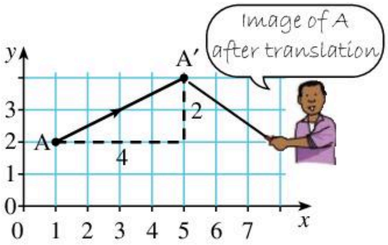


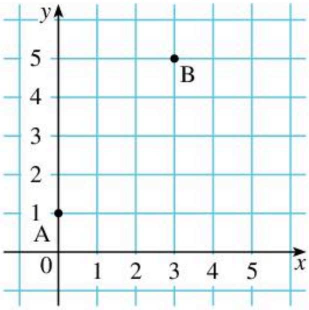
The point A(1, 2) is translated by the vector $\begin{pmatrix} 4 \\ 2 \end{pmatrix}$. What are the coordinates of its image A′?

(a) Draw a triangle ABC and a fixed point O. OA = 2 cm, OB = 3 cm and OC = 1.5 cm.

(b) Draw the enlargement of ABC, centre O, using a scale factor of 2.

4 Pentagon ABCDE is enlarged to A′B′C′D′E′ by a scale factor 5. If A′B′ is 15 cm and the area of ABCDE is 3 cm^2, find the length of AB and the area of A′B′C′D′E′.

5



In the diagram, A is the point (0, 1) and B is the point (3, 5).

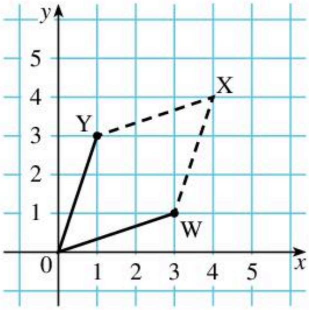
(a) Write down the position vectors

(i) $\overrightarrow{OA}$ **(ii)** $\overrightarrow{OB}$

(b) Express $\overrightarrow{AB}$ as a column vector in the form $\begin{pmatrix} x \\ 4 \end{pmatrix}$.

(c) Calculate $|\overrightarrow{AB}|$.

6



The parallelogram OWXY is defined by the position vectors

$$\overrightarrow{OW} = \begin{pmatrix} 3 \\ 1 \end{pmatrix}, \overrightarrow{OY} = \begin{pmatrix} 1 \\ 3 \end{pmatrix}$$

(a) Find the position vector $\overrightarrow{OX}$.

(b) Write the vector $\overrightarrow{WY}$ as a column vector.

(c) Find $|\overrightarrow{WY}|$.

7 A triangle has vertices at X(3, $^{-}1$) Y(1, 3) and Z(5, 1).
It is reflected to give an image with vertices at X′(9, 5), Y′(5, 7) and Z′(7, 3).

(a) Draw a sketch to show this.

(b) Draw in the mirror line.

(c) Where does the mirror line cut the *x*-axis? Where does it cut the *y*-axis?

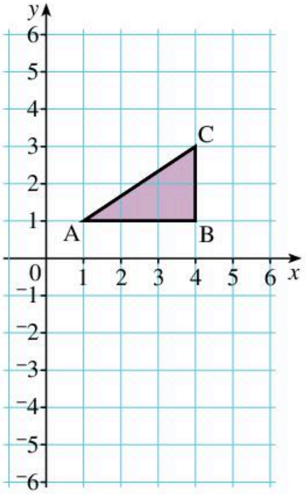
(d) What is the equation of the mirror line?

8 A quadrilateral has vertices A(2, 1), B(1, 4), C(6, 4) and D(5, 1). It is reflected in a line parallel to the *y*-axis, to give the image A′B′C′D′.
The coordinates of C and C′ are the same.

(a) Find the coordinates of A′, B′ and D′.

(b) Write the equation of the mirror line.

9 **(a)** Copy this diagram.



(a) Write down the coordinates of the vertices of the triangle ABC.

(b) Draw the line $y = x$ on your graph.

(c) Draw the image A′B′C′ of ABC after reflection in the line $y = x$.

(d) Write down the coordinates A′, B′, C′,

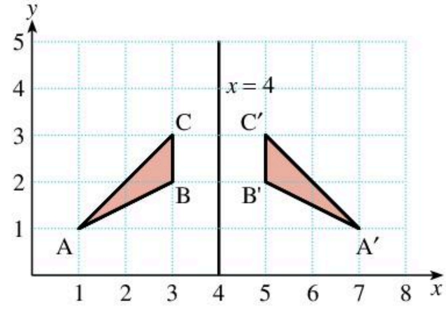
(e) Reflect A′B′C′ in the *x*-axis and then in the line $y = {}^{-}x$. What are the coordinates A″, B″, C″ of the final image?

10 Consolidation

Example 1

Find the image of triangle A(1, 1), B(3, 2), C(3, 3) after reflection in the line $x = 4$.

First draw the triangle on a grid.

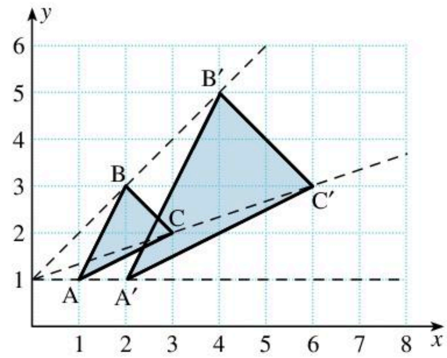


When reflected in $x = 4$,
the coordinates of the image are
A′(7, 1), B′(5, 2), C′(5, 3).

Example 2

Find the image of triangle A(1, 1), B(2, 3), C(3, 2) under an enlargement scale factor 2 and centre (0, 1).

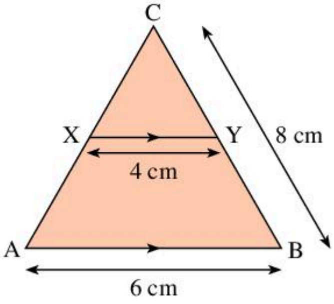
First draw the triangle on a grid.



For an enlargement scale factor 2:
$OA' = 2 \times OA$
$OB' = 2 \times OB$
$OC' = 2 \times OC$
Where O is the centre of enlargement (0, 1).

The image of ABC is A′(2, 1), B′(4, 5), C′(5, 3).

Example 3



In triangle ABC, AB = 6 cm, BC = 8 cm and the points X and Y cut AC and BC respectively such that AB is parallel to XY. Further XY = 4 cm.

Find the length YC.

Triangles ABC and XYC are similar since
$C\hat{A}B = C\hat{X}Y$ and $A\hat{B}C = C\hat{Y}X$ corresponding angles.

Hence $\dfrac{AB}{XY} = \dfrac{AC}{CY}$

that is, $\dfrac{6}{4} = \dfrac{8}{CY}$

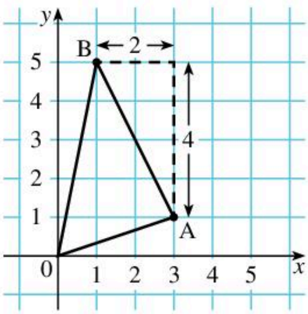
$\Rightarrow 6\,CY = 32$

$\Rightarrow CY = \dfrac{32}{6} = 5.33$ cm

Example 4

A is the point (3, 1) and B the point (1, 5).

Find **(a)** the position vectors $\overrightarrow{OA}$, $\overrightarrow{OB}$
(b) the vector $\overrightarrow{BA}$



(a) From the diagram

$$\overrightarrow{OA} = \begin{pmatrix} 3 \\ 1 \end{pmatrix}, \overrightarrow{OB} = \begin{pmatrix} 1 \\ 5 \end{pmatrix}$$

(b) $\overrightarrow{BA} = \overrightarrow{BO} + \overrightarrow{OA} = -\overrightarrow{OB} + \overrightarrow{OA}$

$$= -\begin{pmatrix} 1 \\ 5 \end{pmatrix} + \begin{pmatrix} 3 \\ 1 \end{pmatrix} = \begin{pmatrix} 2 \\ ^-4 \end{pmatrix}$$

Exercise 10

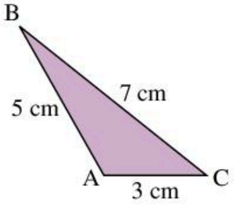
1 Draw these triangles on a grid. In each case, draw the image of each triangle after reflection in the line $x = 1$

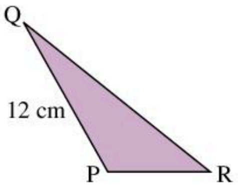
- **(a)** Triangle A($^-1$, 2), B(0, 0), C(1, 3)
- **(b)** Triangle D($^-2$, $^-1$), E($^-1$, 3), F($^-1$, 1)
- **(c)** Triangle G(5, 2), H(3, $^-1$), I(4, 0)

2 Find the image of triangle A(1, 1), B(2, 2), C(2, 4) under an enlargement with:

- **(a)** scale factor 2 and centre (0, 0)
- **(b)** scale factor 2 and centre (1, 0)
- **(c)** scale factor 3 and centre (1, 1).

3





Triangles ABC and PQR are similar.
AB = 5 cm, AC = 3 cm, BC = 7 cm and PQ = 12 cm.
Calculate the lengths

(a) PR **(b)** QR

4 The position vectors of the points A and B are

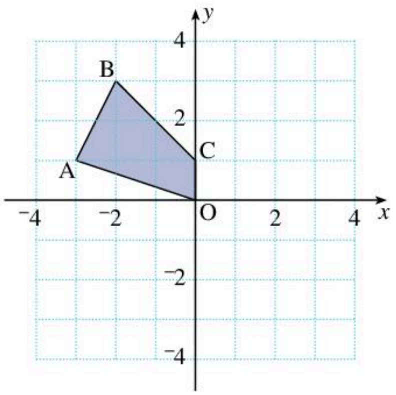
$$\overrightarrow{OA} = \begin{pmatrix} ^-2 \\ 4 \end{pmatrix}, \overrightarrow{OB} = \begin{pmatrix} 2 \\ 1 \end{pmatrix}$$

- **(a)** Show the vectors on a graph.
- **(b)** Express the vector $\overrightarrow{AB}$ as a column vector.
- **(c)** Find the magnitude of $|\overrightarrow{AB}|$.

Application

5 To create a logo, a designer draws the shape OABC on a grid as shown.
To complete the logo he reflects OABC in the line $y = 0$ and then reflects OABC and its image in $x = 0$.

- **(a)** Draw the image, $OA_1B_1C_1$ of OABC after reflection in the line $y = 0$.
- **(b)** Draw the image of OABC and $OA_1B_1C_1$ after reflection in the line $x = 0$.
- **(c)** How many lines of symmetry does the completed logo have?
- **(d)** What is the order of rotational symmetry of the logo?



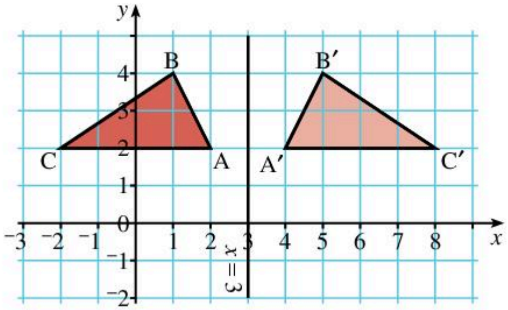
6 A plane flies from city A in an easterly direction for 500 km, then flies directly south for a further 1200 km to city B.

- **(a)** On squared paper show the plane's flight path.
- **(b)** Represent this path, $\overrightarrow{AB}$, as a column vector.
- **(c)** How far is city A from city B?
- **(d)** If the plane flew on a direct path, how long would the flight be if the plane travels at 600 km/h?

Summary

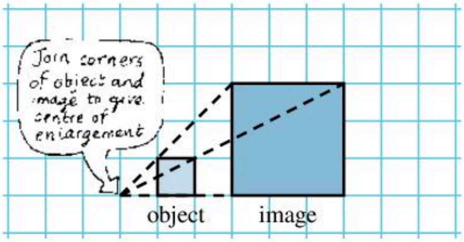
You should know …

1 A reflection produces an opposite image of an object.
For example:



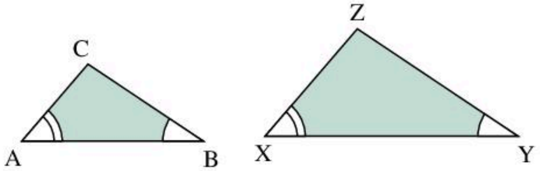
△ABC is reflected in the line $x = 3$ to produce its image △A′B′C′.

2 An enlargement needs a scale factor and a centre of enlargement.
For example: This square has been enlarged by a scale factor of 3.



3 The object and image of an enlargement are similar.

The ratios of corresponding sides in two similar triangles are equal.



$$\frac{AB}{XY} = \frac{AC}{XZ} = \frac{CB}{ZY}$$

Corresponding angles in similar triangles are equal.

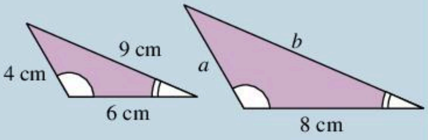
Check out

1 On graph paper, plot the points A(1, 2), B(0, 4) and C(4, 7) and join them to form the triangle ABC. Reflect the triangle ABC in the line $y = x$ and give the coordinates of A′, B′ and C′.

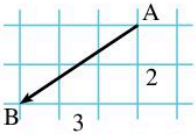
2 **(a)** Draw the triangle with A(2, 3), B(4, 5) and C(5, 7) as vertices on squared paper.

(b) Draw the image of triangle ABC under an enlargement with centre the origin and scale factor 2.

3 Find the length of the lettered sides in this pair of similar triangles.

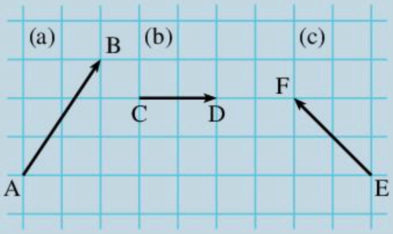


4 A vector is a quantity that has magnitude and direction. It can be represented as a column vector.



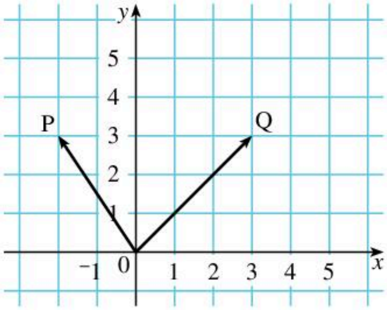
$$\overrightarrow{AB} = \begin{pmatrix} {}^{-}3 \\ {}^{-}2 \end{pmatrix}$$

4 Describe these vectors as column vectors.



5 How to represent a point as a position vector.

For example:



The point P(${}^{-}2$, 3) has position vector $\overrightarrow{OP} = \begin{pmatrix} {}^{-}2 \\ 3 \end{pmatrix}$.

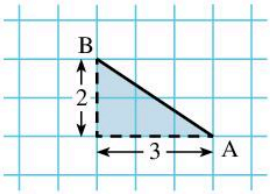
The point Q(3, 3) has position vector $\overrightarrow{OQ} = \begin{pmatrix} 3 \\ 3 \end{pmatrix}$.

5 **(a)** Find the position vectors of the points A(1, 2) and B(3, 6).

(b) Write the vector $\overrightarrow{AB}$ as a column vector.

6 The magnitude of a vector can be found using Pythagoras' theorem.

For example:



$$\overrightarrow{AB} = \begin{pmatrix} {}^{-}3 \\ 2 \end{pmatrix}$$

$$\begin{aligned} \text{So} \quad AB^2 &= 2^2 + 3^2 \text{ by Pythagoras} \\ &= 4 + 9 \\ &= 13 \\ \Rightarrow |\overrightarrow{AB}| &= 13 \end{aligned}$$

6 Calculate the length of the vector $\overrightarrow{XY}$ when

(a) $\overrightarrow{XY} = \begin{pmatrix} 4 \\ 5 \end{pmatrix}$

(b) it is defined by the points X(4, 1) and Y(7, 5).

Statistics and probability 11

Objectives

- ✔ draw and read data from histograms
- ✔ calculate the mean, mode and median from different representations of data
- ✔ understand the concept of probability
- ✔ find the experimental and theoretical probability of simple events

What's the point?

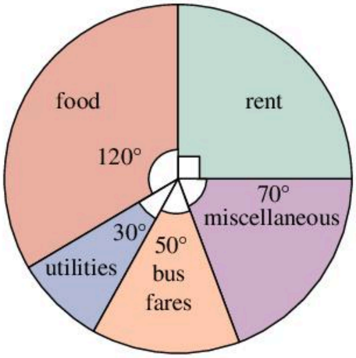
What is the likelihood that your house will catch fire? That you will have a motor accident? That you will be hospitalised? Insurance companies estimate the probability of such events in order to work out the premium you will pay on your insurance policy.

Before you start

You should know ...

1 How to draw and read information from a bar graph or pie chart.

For example:



If Mrs Vernon spends \$2850 per month on housekeeping matters, she spends $\frac{120}{360} \times \$2850 = \$950$ on food.

Check in

1 The favourite sport of 36 students is given:

Cricket	11	Volleyball	5
Football	8	Basketball	7
Tennis	2	Badminton	3

Draw **(a)** a pie chart
(b) a bar graph

to show this information.

2 How to calculate the mean, mode and median of a given set of data.
For example:
Carlene scored a total of 520 marks in eight subjects in her last examination. Her mean score was $\frac{520}{8} = 65$

3 How to draw a frequency table from given data.
For example:
The number of eggs laid each week by Kurt's hens during a 20-week period are:

4, 6, 4, 7, 3, 3, 2, 6, 3, 1
3, 7, 4, 6, 2, 7, 2, 7, 4, 3

This is the frequency table for the data:

No. of eggs	Frequency
1	1
2	3
3	5
4	4
6	4
7	3

2 Find the mode, median and mean of these numbers:

15, 13, 13, 12, 11, 11, 10
10, 10, 10, 9, 9, 8, 8, 7

3 The following are the sizes of shoes worn by 20 people.

7, 9, 6, 10, 8
8, 9, 11, 8, 7
9, 6, 8, 10, 9
8, 7, 7, 8, 9

Draw a frequency table for this information.

11.1 Histograms

Recall there are two basic types of data:

- **discrete** and
- **continuous**.

Discrete data can only take definite values.
For example:

Gender – male, female
Hair colour – black, brown etc
Jeans' size – 6, 7, 8 etc.

Continuous data can take any value.
For example:

Height
Weight
Time

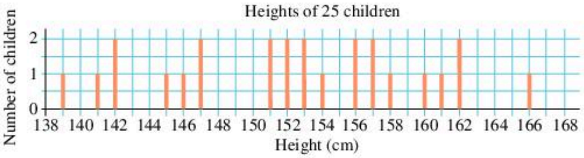
Bar graphs and pie charts are usually used to represent discrete sets of data.

Histograms are very similar to bar charts.
A histogram is usually used to show continuous data. Histograms are also often used for grouped data.

For example, look at the heights of 25 children measured to the nearest centimetre:

139, 141, 142, 142, 145, 146, 147, 147, 151, 151, 152, 152, 153, 153, 154, 156, 156, 157, 157, 158, 160, 161, 162, 162, 166.

Drawing this information on a graph gives.



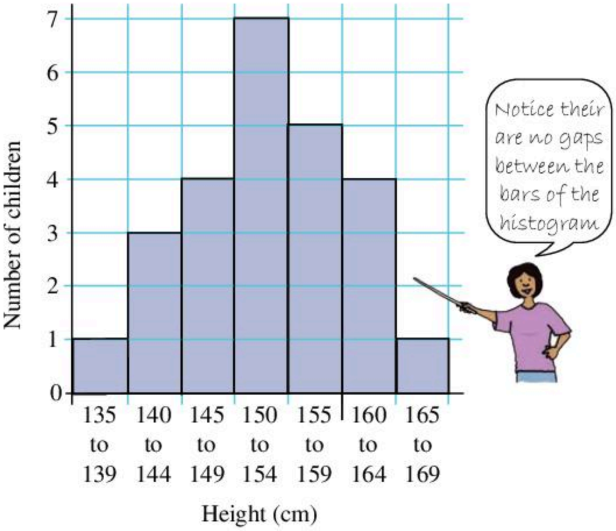
Such a graph is not very useful – explain why.

Instead it is better to construct a grouped frequency table. Using groups 135–139, 140–144 etc. the frequency table is:

Interval	Tally	Frequency
135–139	|	1
140–144	|||	3
145–149	||||	4
150–154	卌 ||	7
155–159	卌	5
160–164	||||	4
165–169	|	1

The histogram of this data is:

Histogram of heights of 25 children



The horizontal axis of the histogram shows the height of the children. This is a continuous variable. It is quite possible to have a height of 138.43 cm etc.

Exercise 11A

1 **(a)** Draw a histogram for the heights of the 25 children using intervals of:
(i) 10 cm **(ii)** 20 cm

(b) Look at all the graphs for this data. Which one shows best how heights vary?

2 Vendra counted the words in the first forty sentences of *High Wind in Jamaica*. She found there were:
8, 17, 7, 3, 8, 13, 11, 79, 22, 4, 10, 16, 40, 6, 2, 16, 7, 11, 14, 3, 12, 9, 4, 11, 11, 23, 7, 51, 61, 43, 21, 5, 34, 14, 56, 48, 10, 12, 15, 20

(a) Draw a frequency table and histogram for this information. Use intervals 0–9, 10–19 etc.

(b) What is the modal interval?

3 The mass in kilograms of 25 children are:

Mass (kg)	10–19	20–29	30–39	40–49	50–59
No. of children	1	4	7	12	1

(a) Draw a histogram to show this information.

(b) What is the modal mass?

4 The heights of 30 plants in centimetres, six weeks after planting were:
13, 6, 10, 7, 13, 18, 16, 14, 12, 8
20, 15, 21, 8, 28, 25, 16, 17, 19, 22,
23, 14, 18, 17, 6, 22, 28, 21, 15, 17

(a) Construct a frequency table using intervals 6–10, 11–15, 16–20, 21–25, 26–30 to show this information.

(b) Draw a histogram to illustrate the data.

5 The waiting time in minutes for 25 patients to see a doctor was:
8, 35, 12, 45, 4, 15, 38, 28, 30,
23, 14, 38, 53, 26, 33, 32, 15, 18,
48, 37, 34, 34, 28, 51, 16

(a) Construct a frequency table using intervals 0–4, 5–9, 10–14 to show the data.

(b) Draw a histogram to illustrate the data.

(c) How many patients waited less than 10 minutes?

(d) What percentage of patients waited longer than half an hour?

Activity

Measure the heights and weights of the students in your class.

(a) Construct suitable frequency tables to show this data.

(b) Draw histograms to illustrate the distribution of heights and weights in your class.

(c) Repeat the process, but this time draw separate tables and histograms for boys and girls.

(d) Comment on your results.

Technology

Review what you have learnt about bar charts and histograms by visiting *www.onlinemathlearning.com*

11.2 Averages

In Book 2 you met three different types of average: the **mode**, the **median** and the **mean**.

The mode

The shoe sizes of 30 children are listed:

3, 5, 2, 4, 7, 6, 6, 5, 3, 2, 6, 5, 2, 8, 3,
4, 4, 5, 4, 3, 4, 5, 5, 4, 5, 6, 3, 8, 5, 3.

Shoe size	Frequency
2	3
3	6
4	6
5	8
6	4
7	1
8	2

The number of times that a shoe size appears is its frequency. The frequency table drawn from the information is shown.

From the table you can see that size 2 appears three times. Three is the frequency of size 2.

- The size with the greatest frequency is the **mode**.

Size 5 is the mode because it occurs eight times.

Exercise 11B

1 The frequency table shows the shoe sizes of a second group of 30 children.

Shoe size	Frequency
1	1
2	5
3	4
4	9
5	3
6	4
7	2
8	2

(a) What is the mode?

(b) Draw a bar graph to show this information.

2 Jo bought 50 g of peanuts. There were 27 pods. The 27 numbers represent the number of peanuts he found in each pod.

3, 3, 3, 2, 4, 3, 3, 4, 3,
2, 3, 3, 4, 4, 2, 3, 3, 3,
4, 2, 4, 2, 3, 3, 3, 4, 3

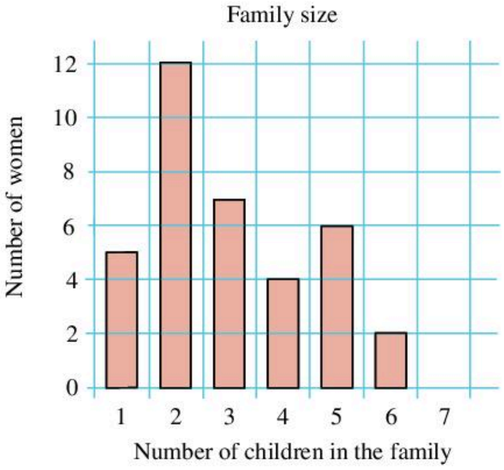
Draw a frequency table and find the mode.

3 Here is a frequency table showing the heights of a sample of people:

Height (cm)	150	151	152	153	154	155
Frequency	1	5	10	16	6	2

What is the modal height?

4 36 women in a women's club in Port of Spain were asked about the number of children they had. The results are shown in this bar graph.



(a) What is the mode?
(b) How many children are there altogether?

5 **(a)** For each student in your class, find out how many children there are in their family.
(b) Draw a frequency table and a bar graph to show the results.
(c) What is the modal family size?

6 In Question 4 of Exercise 11A you showed plant heights in a bar graph. Which interval is the mode?

7 A group of 30 students was asked about the number of cousins each had. Here are their replies:

6, 6, 7, 8, 9, 5, 6, 7, 7, 5,
6, 6, 3, 6, 1, 10, 9, 10, 8, 9,
10, 7, 8, 9, 9, 6, 7, 8, 9, 9

(a) Draw a frequency table and a bar graph of these results.
(b) Which number has the greatest frequency?
(c) Is there more than one mode?

The median

- When a set of values is arranged in order, the middle term is called the **median**.

Bessie Johnson carried out a survey to find the number of people living in the 19 houses in Marville Trace. The results are shown in the table below.

Number of occupants		Total
less than 5	𝍸 ||	7
5	𝍸	5
6	|||	3
7	||	2
more than 7	||	2
	Total	19

To find the median, Bessie arranged the numbers in order:

$$* * * * * * * \underset{\uparrow}{\overline{5\,5\,5\,5\,5\,6\,6\,6\,7\,7}} * *$$

The stars represent *less than 5* and *more than 7*. So you can see that seven of the houses have less than 5 people living in them.

The arrow points to the middle term.
The median number of occupants is 5.

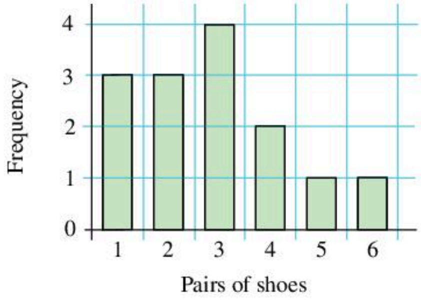
Exercise 11C

1 Look at the bar graph for Question 4 of Exercise 11B.
(a) Write down the numbers of children the women have, in order, as Bessie did.
(b) Pick out the middle number. What is the median?
(c) Is the median the same as the mode, for the number of children per family?

2

Fourteen women were asked how many pairs of shoes they each had.

The bar graph shows the results:



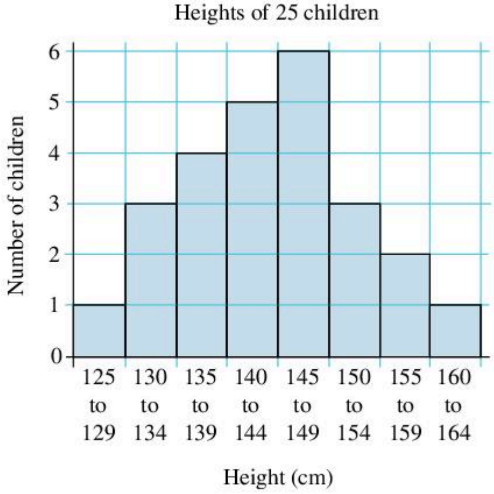
(a) Write down in order the numbers of pairs each woman had.
(b) What is the median?

3 Find the median of each set of numbers.
(a) 7, 19, 15, 18, 24, 17, 7, 21, 17, 14, 21, 16, 12
(b) 6.2, 6.8, 6.7, 6.02, 6.28, 6.82, 6.08, 6.72, 6.27
(c) 5, 3, 2, 5, 4, 5, 2, 3, 5, 3, 4

4 The examination marks of 20 students were: 71, 40, 86, 55, 63, 70, 44, 90, 37, 68, 53, 55, 57, 60, 82, 91, 62, 72, 56, 42.
(a) Arrange the marks in order.
(b) Which mark would you choose for the median? Are there the same number of marks above and below this mark?

5 Here are the heights, in centimetres, of a group of 10 children:
119, 120, 121, 121, 121, 123, 124, 124, 125, 128.
(a) What is the mode?
(b) What is the median?

6 The heights of 25 children, measured to the nearest centimetre, are shown in the histogram.



(a) Which interval is the mode?
(b) In which interval does the median occur?
(c) How many children are there in this interval?
(d) You are not given the height of each child so you cannot state the median. Can you estimate the median height? Write down your estimate.

The mean

In Question 1 of Exercise 11A, Errol Hendry's annual poultry sales, are around 2000.

The total number sold, over eleven years, is 23 100.

The **mean** number sold each year is

$$\frac{23\,100}{11} = 2100.$$

- To find the **mean** you add up all the values, and divide by the number of values.

$$\text{Mean} = \frac{\text{sum of values}}{\text{number of values}}$$

Exercise 11D

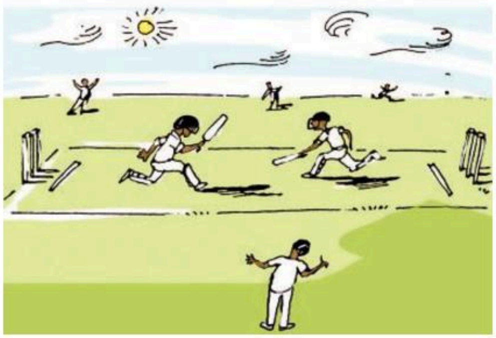
1 On six working days a garage mended 6, 5, 2, 0, 3, 2, punctures. Calculate the mean number of punctures mended per day by the garage.

2 In a test out of 40, the marks of 10 students were 32, 20, 7, 30, 25, 22, 18, 35, 10, 11. Calculate the mean mark for the test.

3 In 10 innings, Michael Dobson's scores were 18, 6, 89, 4, 42, 105, 0, 37, 4, 15.
(a) What was his total score?
(b) What was his mean score?
(c) Write down the mode and median scores.
(d) Which average gives the best idea of how well he batted? Explain why.
(e) In his eleventh innings he scored 43. What does that make his mean score?

4 Work out the mean, to find which is the best batsman:
(a) Jimmy who scored 8, 25, 82, 10, 0, 32, 9, 41, 0 in 9 innings, *or*
(b) Mahendra who scored 75, 83, 104, 0, 2, 68, 0, 0, 1, 7 in 10 innings, *or*

(c) Rikki who scored 14, 56, 102, 37, 76 in 5 innings.



5 **(a)** In a cricket match, Elvin took 3 wickets for 27 runs. What was the mean number of runs per wicket, scored against him?

(b) Ravi took 4 wickets for 38 runs. Who had the better bowling average, Elvin or Ravi?

6 Sham wants to work out his bowling average. In 8 innings, his figures were: 2 for 12, 2 for 27, 3 for 8, 1 for 60, 0 for 32, 6 for 52, 2 for 24, and 0 for 18.

(a) How many runs were made against him altogether?

(b) How many wickets did he take altogether?

(c) Work out the mean number of runs per wicket.

7 Find the missing number.

(a) For the scores 25, 23, 15, 18, 17, and 22, the mean score is □.

(b) For the scores 15, 19, 18, □, 13, 21 and 14, the mean score is 17.

(c) For the scores 26, 31, 54, □ and 49, the mean score is 38.

(d) For the scores 18, 54, 37, 21, □, 41, 14 and 15, the mean score is 33.

8 The scores on a test taken by five students were: 95, 25, 30, 20, 30.

(a) What is the mean score?

(b) What is the median score?

(c) What is the mode?

(d) Which average is the most appropriate representation of the data? What difficulties do you see?

9 Look at this advertisement:

Wanted

Young person to join small team

Average salary $40 000

If you know that the firm has six employees whose salaries are $12 000, $14 000, $16 000, $18 000, $20 000 and $160 000 and that your starting salary would be $12 000, would you say that the advertisement is misleading? Give reasons for your answer.

11.3 Frequency tables and means

The mean can be calculated directly from a frequency table. The following frequency table shows how to find the mean number of peanuts in the 27 pods given in Question 2 of Exercise 11C.

Peanuts per pod	Frequency	Totals
2	5	$5 \times 2 = 10$
3	15	$15 \times 3 = 45$
4	7	$7 \times 4 = 28$
Total	27	83

Mean = $\frac{83}{27}$ = 3.1 (to one decimal place)

Exercise 11E

1 The number of peanuts found in 42 pods is given in this frequency table. Copy and complete it to find the mean.

Peanuts per pod	Frequency	Totals
1	2	
2	9	18
3	22	
4	9	
Total		

2 Rangi counted the number of people in 50 cars that passed her on the road into the town. Here is her record:

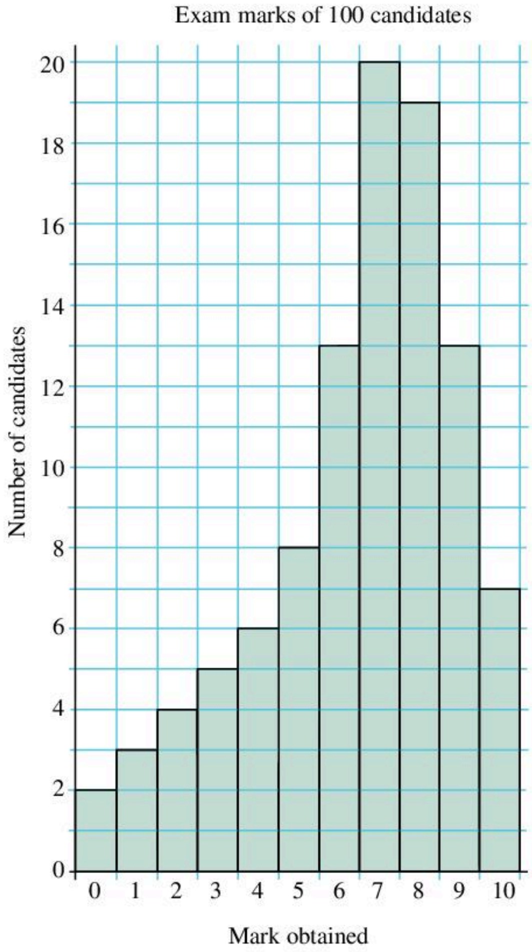
1, 1, 1, 1, 1, 2, 1, 1, 2, 1, 1, 1, 1, 1, 4, 2, 1,
1, 3, 1, 1, 1, 1, 1, 2, 2, 1, 1, 1, 2, 1, 1, 3, 1,
1, 1, 2, 1, 1, 2, 1, 1, 2, 1, 1, 3, 1, 1, 1, 2

Construct a frequency table for these results and use it to find the mean number of people per car.

3 Use a frequency table to calculate the mean of this set of twenty numbers:

6, 2, 1, 5, 3, 1, 2, 4, 4, 3, 4, 5, 6, 2, 1, 1, 3, 2, 2, 5

4 This graph shows the marks obtained by100 candidates in an examination.



(a) Draw a frequency table for the marks.

(b) Use the frequency table to calculate the mean mark.

(c) The mean mark was the pass mark. How many candidates passed the exam?

5 Jasper keeps hens. The frequency table shows the number of eggs he gets per day.

Number of eggs	14	15	16	17	18	19	20	21	22	23
Number of days	1	0	2	4	6	3	4	4	3	1

Calculate the mean egg yield per day.

11.4 What is probability?

Consider these questions:

- Should you take a raincoat to school today?
- Who will win the test match?
- Should 1 buy that newspaper?

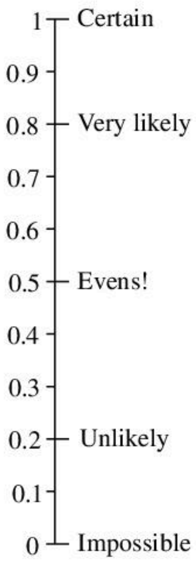
However you answer there is a **chance** that you will be right. There is also a **risk** that you may be wrong.

In mathematics, **probability** is used to describe the likehood of an event occurring.

For example:
There is a high probability that you will put on your school clothes to go to school.

There is a low probability that it will snow tomorrow.

Mathematically, the probability of an event is given as a number between 0 and 1.



An event that is **certain** is assigned a probability of 1. The probability of an **impossible** event is 0.

For example:

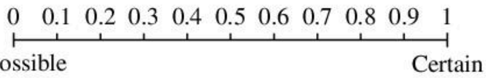
Event	**Likelihood**	**Probability**
I will live to be 200	Impossible	0
I will die someday	Certain	1

Exercise 11F

1 How likely are these events:
- **(a)** I will play cricket for the West Indies
- **(b)** I will pass my next maths test
- **(c)** I will sleep tonight?

2 Write down three events that you think are:
- **(a)** impossible
- **(b)** certain
- **(c)** evens
- **(d)** very likely to happen
- **(e)** very unlikely to happen.

3



On a copy of the probability line above, insert the following events:
- **(a)** I will walk home tonight
- **(b)** I will win the Lotto draw today
- **(c)** It will rain today
- **(d)** School will end at 10.00 am today
- **(e)** At least 50 children will be at school today.

4 Write down two events that you think will have a probability of
- **(a)** 0.5
- **(b)** 0
- **(c)** 0.75
- **(d)** 1
- **(e)** 0.01
- **(f)** 0.92

5 Write down a numerical value for the probability of these events:
- **(a)** I will eat lunch at 3:00 pm today
- **(b)** I will walk to school today
- **(c)** It will snow today
- **(d)** The sun will rise before 8 am tomorrow
- **(e)** I will pass my next maths text.

6 Assign a probability between 0 and 1 to each event.
- **(a)** I will grow 4 cm taller today
- **(b)** The maths lesson will last two hours today
- **(c)** I will be 16 next year
- **(d)** My next class will be art.
- **(e)** In four years time, I will go to college.

11.5 Experimental probability

Probability can be determined through experiments.

In such cases you can define the probability of a successful outcome, P(S), as

$$\mathrm{P(S)} = \frac{\text{number of successful outcomes}}{\text{total number of outcomes}}$$

For example:
You throw a coin 30 times and obtain 16 heads and 14 tails.
The experimental probability of getting a head, P(H) is

$$\mathrm{P(H)} = \frac{\text{number of heads}}{\text{number of throws}} = \frac{16}{30} = \frac{8}{15}$$

Many probabilities are, in fact, found through **surveys** or **statistics**.

Example 2

The number of matches in 30 matchboxes were:

Number of matches	46	47	48	49	50	51
Number of boxes	2	4	10	8	4	2

What is the probability that a box holds
(a) 48 matches **(b)** more than 49 matches?

(a) $\mathrm{P(48\ matches)} = \frac{\text{number of boxes with 48}}{\text{number of boxes}}$

$$= \frac{10}{30} = \frac{1}{3}$$

(b) P(more than 49 matches)

$$= \frac{\text{number of boxes with more than 49 matches}}{\text{number of boxes}}$$

$$= \frac{4+2}{30} = \frac{6}{30} = \frac{1}{5}$$

Exercise 11G

You will need a coin and a dice.

1 **(a)** Toss a coin 20 times. Record the number of times heads comes up.

(b) Complete the table.

	Heads	Tails
Frequency		

(c) Draw a bar chart to show this information.

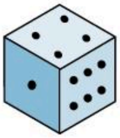
(d) What percentage of the total number of throws were

(i) heads **(ii)** tails?

(e) In this experiment, what is the probability of throwing

(i) a head **(ii)** a tail?

2 Throw a dice 50 times.



(a) Copy and complete the table.

Score	1	2	3	4	5	6
Frequency						

(b) Draw a bar chart to show this information.

(c) What percentage of the throws resulted in a

(i) 6 **(ii)** 1 **(iii)** 3

(d) What fraction of throws are

(i) 2s **(ii)** even numbers

(iii) odd numbers **(iv)** 8s?

(e) In this experiment what is the probability of throwing

(i) 6 **(ii)** a prime number?

3 Petra throws a coin 50 times and gets these results:

Heads	Tails
23	27

Do you think the coin was fair? Explain.

4

(a) Find the shoe size of children in your class.

(b) Copy and complete the table

Shoe size	Frequency

(c) Draw a bar chart to show this data.

(d) What percent of children wear shoes of size

(i) 5 **(ii)** 11?

(e) What is the probability that a child wears shoe size

(i) 2 **(ii)** 5 **(iii)** 9?

5 The scores of 100 students on a test were:

Score	Number of students
0–9	1
10–19	2
20–29	9
30–39	9
40–49	12
50–59	24
60–69	15
70–79	10
80–89	8
90–99	2

What is the probability that a student picked at random scored

(a) 40–49 **(b)** less than 40

(c) 50–59 **(d)** more than 80?

6 **(a)** Measure the height of children in your class. Record your data using the tally chart.

Height (cm)	Tally	Frequency
120–129		
130–139		
140–149		
150–159		
160–169		
170–179		

(b) Draw a suitable chart to show this information.

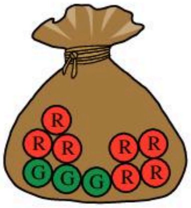
(c) What fraction of the class have heights between

(i) 120–129 cm **(ii)** 150–159 cm?

(d) What is the probability that a child picked at random from the class has height between

(i) 120–129 cm **(ii)** 140–149 cm?

7



(a) Place 7 red and 3 green beads or counters in a bag. Without looking pick one bead or counter. What colour was it?
Return the bead/counter to the bag and repeat 50 times.

(b) Copy and complete the table

Colour	Tally	Frequency
Red		
Green		

(c) Draw a bar chart to show your data.

(d) What percent of the time did you pick

(i) a red bead **(ii)** a green bead?

(e) In this experiment what is the probability of choosing a red bead?

8

(a) Take a story book. Choose a paragraph in the book and mark out the first 200 letters in that paragraph.
Copy and complete the frequency table for the number of different letters in the marked section.

Letter	Tally	Frequency
a		
b		
c		
…		
z		

(b) Draw a bar chart to show this information.

(c) Which letter occurred most frequently?

(d) What percentage of the 200 letters were the letter

(i) E **(ii)** A **(iii)** S **(iv)** Q **(v)** Z?

(e) What is the probability that a letter is a

(i) O **(ii)** N **(iii)** X?

(f) In the game of 'Scrabble' players score more for using letters such as Q or X. Based on your results, do you think this is fair? Give reasons for your answer.

9 A shoe store sells shoes of the following sizes during the course of one week:

Shoe size	2	3	4	5	6	7	8	9	10	11	12
Number sold	4	4	3	6	10	14	33	24	16	10	4

(a) What is the probability that a customer bought shoes with

(i) size 6

(ii) size 5 or less

(iii) size 10 or more?

(b) Why is such data useful for the shoe store's manager?

10 The number of road traffic fatalities on a certain island by age group is shown below.

Age	0–19	20–29	30–39	40–49	50–59	60+
Fatalities	12	23	14	7	10	14

(a) What fraction of fatalities are in the 20–29 age group?
(b) Given a road fatality, what is the probability that the person was 50 years old or older?
(c) Why do you think that automobile insurance premiums are highest for persons in the 20–29 age group?

Investigation

Do people with bigger feet have longer strides?

- Divide the class into two groups, those with shoe size 5 or less and those with shoe size bigger than 5.
- Measure the stride length of everyone.
- Copy and complete the table.

Stride length (cm)	Tally	Frequency
60–69		
70–79		
80–89		
90–99		
100–109		
110–119		

First for those with shoe sizes 5 or less, then for those with shoe size bigger than 5.

- Draw a chart to show your results.
- What percent of the class have
  (a) shoe size bigger than 5
  (b) stride length bigger than 80 cm
  (c) shoe size bigger than 5 and stride bigger than 80 cm?
- What is the probability that a child
  (a) has shoe size 5 or less
  (b) with shoe size 5 or less has a stride length of 900 cm or more
  (c) with a stride length of less than 90 cm has a shoe size of more than 5?
- Do people with bigger feet really have longer strides? Give reasons based on the data collected.

11.6 Theoretical probability

It is usually impractical to carry out experiments to find the probability of events. Instead you can calculate what might be expected to occur. This is called the **theoretical probability**; it is given by the simple formula

$$\text{P(success)} = \frac{\text{number of possible successes}}{\text{number of possible outcomes}}$$

You can use this simple formula to calculate the theoretical probability of events.

Example 3

A bag contains 3 white beads and 2 black beads. What is the probability that a bead picked at random from the bag is black?

$$\text{P(black bead)} = \frac{\text{number of black beads}}{\text{number of beads}}$$

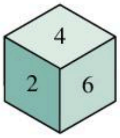
$$= \frac{2}{5}$$

Exercise 11H

1 A bag contains 6 yellow and 4 green marbles. What is the probability that a marble picked at random is:

(a) yellow **(b)** not yellow?

2



A six-faced dice is thrown. What is the probability of throwing:

(a) a 4 **(b)** a 5 **(c)** a 3

(d) 4 or more **(e)** 2 or less **(f)** not a 6?

3 A letter is chosen at random from the word Mississippi. What is the probability that the letter is

(a) M **(b)** I **(c)** S?

4 In a class of 30 children, 12 are boys and 18 are girls. A total of 7 children wear glasses. What is the probability that a child chosen at random is

(a) a girl

(b) wears glasses

(c) does not wear glasses?

5 A bag contains 7 red beads and 3 green beads. What is the probability that a bead picked at random is

(a) red **(b)** green?

6 There are 52 cards in a pack. What is the probability that a card picked at random is:

(a) a 10

(b) a king

(c) red

(d) not red

(e) not a king

(f) a number which is a multiple of 2

(g) a number which is not a multiple of 2?

7 A bag has 6 green, 4 red, 5 yellow and 15 blue marbles.
A marble is picked from the bag. What is the probability that the marble is:

(a) red **(b)** blue

(c) not red **(d)** not blue

(e) red or blue **(f)** neither red nor blue?

8 A coin is tossed twice.

(a) Write down the possible outcomes.

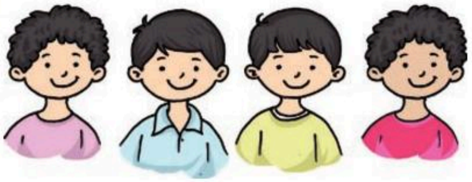
(b) What is the probability of getting 2 heads?

(c) What is the probability of getting 2 tails?

(d) What is the probability of getting one of each?

9 A cricket captain loses the toss five times in a row. What is the probability that he will lose it next time? Explain.

10



(a) Mrs James has four sons. What is the probability that her next child will be a boy?

(b) A mother has three children. What is the probability they are all boys?

Technology

Learn more about probability by visiting the site *www.onlinemathlearning.com/math-probability.html*

Study the examples and watch the videos in preparation for the next section.

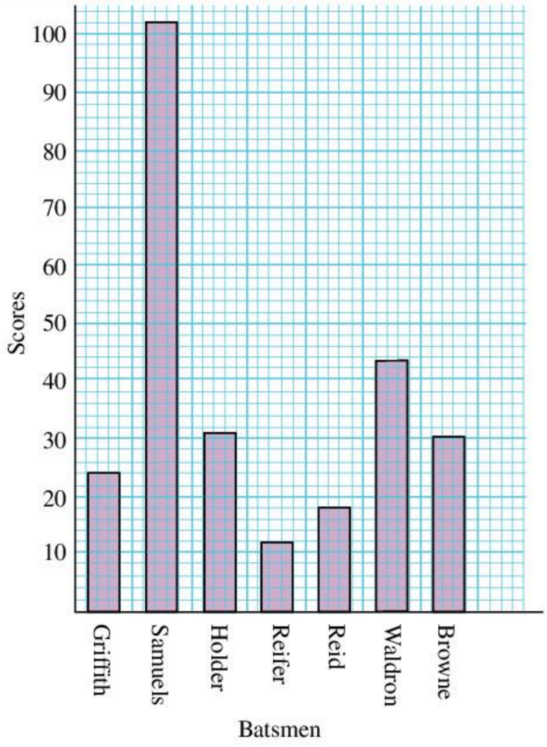
Activity

Research some more on probability.

- Find out which professions make use of probability.
- Give some examples of its use in each profession you listed.
- Share your answers with your class.

Exercise 11I – mixed questions

1



The bar graph shows the scores of seven batsmen for Barbados in a match against Trinidad.

(a) State the number of runs scored by
  (i) Samuels **(ii)** Waldron **(iii)** Reid

(b) Which batsman made the highest score?

(c) What was the total score by Barbados if extras totalled 26?

(d) How many batsmen scored more than 27?

(e) What is the median score? Who scored it?

2 The ages of 36 students are:

15, 15, 16, 15, 16, 15, 14, 14, 16
14, 14, 14, 14, 14, 13, 15, 16, 14
14, 15, 13, 14, 15, 14, 16, 13, 14
14, 15, 16, 14, 15, 14, 13, 15, 13

(a) Arrange the data in a frequency table showing age, tally and frequency.

(b) Draw a bar chart for the data.

(c) What is the probability that a student is aged
  (i) 15 **(ii)** under 15?

3 Eighteen people were asked to name their favourite colour. Their answers are shown in the table.

Colour	green	red	yellow	blue	black
Frequency	5	4	2	5	2

(a) Draw a pie chart to show these results.

(b) What is the probability that a person's favourite colour is blue?

4 **(a)** The ages of the members of a family are 30, 35, 14, 12, 18 and 5 years. Find the mean age.

(b) The heights of 6 people are 157, 149, 158, 160, 152 and 154 cm. Find the median height.

(c) Write down the mode of this set of numbers: 10, 7, 8, 9, 9, 6, 7, 8, 9, 9

5 The frequency table shows the results of a mathematics test for 40 students.

Marks	Number of students
0	2
1	1
2	4
3	3
4	5
5	
6	7
7	4
8	2
9	2
10	1

(a) How many students scored 5?

(b) Calculate the

  (i) mean mark

  (ii) median mark

  (iii) modal mark.

(c) Calculate the fraction of students who scored less than 4.

6 The mean of 5, 8 and y is the same as the mean of 2, 4, 7, 8 and 9. Find the value of y.

7 The average age of a mother and her three children is 10 years. If the ages of the children are 1, 4 and 7 years, how old is the mother?

8 This frequency table shows the heights of a sample of people.

Height (cm)	Frequency
149	1
150	4
151	9
152	10
153	15
154	7
155	1
156	2
157	1

(a) Calculate the mean height.

(b) Determine the median height.

(c) State the modal height.

(d) What is the probability that a person has height

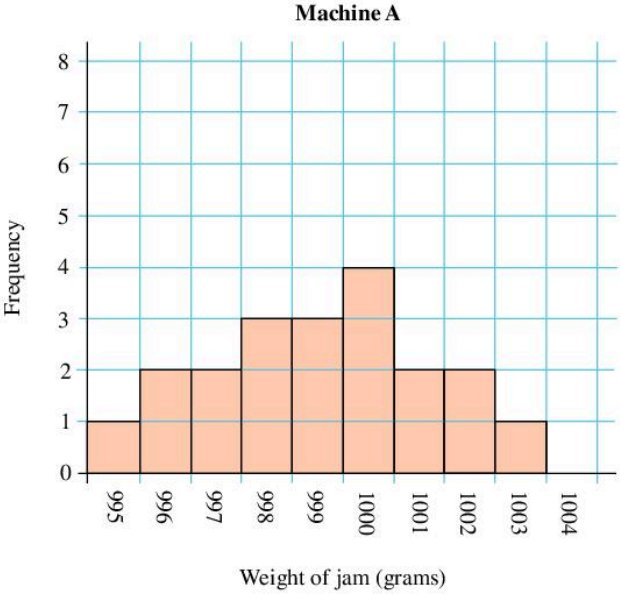
  (i) 153 cm **(ii)** more than 155 cm?

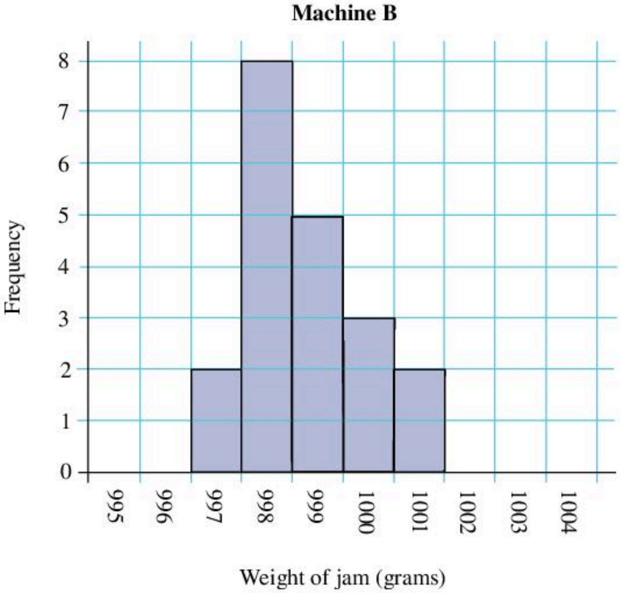
9 Five people each earn \$850 per week, three people each earn \$1120 per week and two people each earn \$1500 per week.
What is the mean wage for the ten people?

10 In the Sweetco factory, glass jars are filled with jam. Each jar should hold 1 kilogram of jam.
There are two machines, A and B. To check that these are working properly, 20 full jars are taken from each machine. The jam in them is weighed carefully to the nearest gram. The graphs show the results.

(a) Draw a frequency table for each machine and use it to calculate the mean.

(b) Which machine gives a mean closest to 1000 g?





(c) Jars have to be refilled if they are more than 2 g different from 1000 g. Which machine produces more jars that need refilling? How did you find your answer?

(d) What is the probability that a jar holds 1000 g or more when chosen at random from
  (i) Machine A **(ii)** Machine B?

(e) Which machine is more useful. Why?

(f) A screw on each machine can be turned, so that the machine cannot fill less than 1000 g. Now which machine is more useful?

(g) Do you think this sort of exercise would be useful in a factory? Why?

11 Consolidation

Example 1

The weights of 20 children in kilograms are:

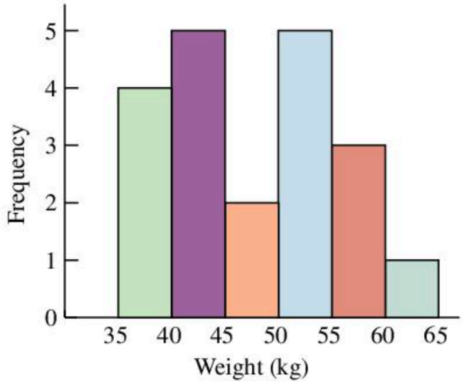
38, 41, 37, 42, 45, 53, 39, 54, 55, 60
45, 48, 52, 54, 39, 44, 47, 58, 61, 59

(a) Construct a frequency table using intervals 36–40 etc. to show the data.
(b) Draw a histogram to represent the data.

(a)

Weight	36–40	41–45	46–50	51–55	56–60	61–65
Tally	||||	𝍸	||	𝍸	|||	|
Frequency	4	5	2	5	3	1

(b)



Example 2

The ages of 20 children are shown in the table:

Age	12	13	14	15	16
Frequency	2	4	7	4	3

Find the **(a)** mean **(b)** median **(c)** mode age.

(a) Mean age $= \dfrac{\text{sum of ages}}{\text{number of children}}$

$$= \frac{\begin{array}{c}12 + 12 + 13 + 13 + 13 + 13 + 14 + 14 + 14 + 14 \\ + 14 + 14 + 14 + 15 + 15 + 15 + 15 + 16 + 16 + 16\end{array}}{20}$$

$= 14.1$

(b) To find median age write ages in order of size:

12, 12, 13, 13, 13, 13, 14, 14, 14, 14, 14, 14, 14,
15, 15, 15, 15, 16, 16, 16

The middle number is 14 so the median age is 14.

(c) Mode = most frequently occurring age = 14.

Example 3

A card is drawn at random from a deck of cards. What is the probability that the card is:

(a) a black card **(b)** a red 4?

(a) $\text{P(Black)} = \dfrac{\text{number of black cards}}{\text{number of cards}} = \dfrac{26}{52} = \dfrac{1}{2}$

(b) $\text{P(Red 4)} = \dfrac{\text{number of red 4s}}{\text{number of cards}} = \dfrac{2}{52} = \dfrac{1}{26}$

Exercise 11

1 The lengths of leaves from a tree measured in millimetres are:

43, 47, 63, 49, 52, 58, 47, 61, 60, 57
39, 42, 57, 56, 54, 63, 62, 58, 55, 37

(a) Construct a frequency table to show the data using intervals 36–40 etc.
(b) Draw a histogram to represent the data.

2 The times in seconds for the 24 children in Form 3 to run 200 m are:

29, 33, 38, 32, 34, 36, 27, 29, 30, 32, 40, 41
33, 28, 31, 33, 35, 29, 34, 34, 31, 35, 30, 31

(a) Construct a frequency table to show this data using intervals 26–30 etc.
(b) Draw a histogram to represent the data.

3 Find the **(a)** mean **(b)** mode values in Questions 1 and 2.

4 A bag contains 6 red, 3 white, 4 yellow and 7 orange marbles. What is the probability that a marble drawn at random is:

(a) red
(b) yellow
(c) not red
(d) white or orange
(e) neither red nor white?

Application

5

The masses of 30 cucumbers grown by a farmer in grams are

421 520 617 384 296 682 462 374 581 214
218 463 389 294 385 262 517 491 382 450
185 284 353 419 384 264 443 368 285 275

(a) Draw a frequency table for the data using intervals 150–199, 200–249 etc.

(b) Construct a histogram to illustrate the data.

(c) What is the probability that a cucumber has a mass of:
(i) less than 200 g **(ii)** more than 400 g?

6 **(a)** Write down the ages of all the students in your class.

(b) Construct a frequency table to show this data.

(c) Calculate the **(i)** mean **(ii)** mode **(iii)** median age of your class.

7 Ask each student in your class the number of minutes they spent on homework last night.

(a) Complete a frequency table to show this data.

(b) Draw a chart to represent the data.

(c) Find the **(i)** mean **(ii)** mode **(iii)** median times.

(d) On the basis of your answer to part (c), what would be your answer to the question 'What is the average time spent on homework each night?'

Support Website

Additional material to support this topic can be found at
www.oxfordsecondary.com/9780198425793

Summary

You should know ...

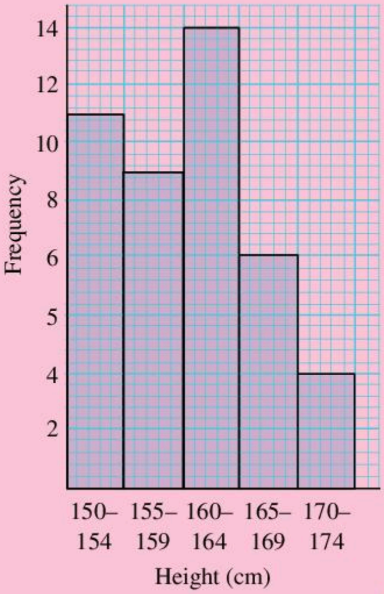
1 How to draw and use frequency tables and histograms with intervals.

For example:
These fifteen test scores:
9, 6, 3, 13, 12, 18, 14, 12, 8, 11, 19, 12, 4, 9, 18
can be shown on a table as:

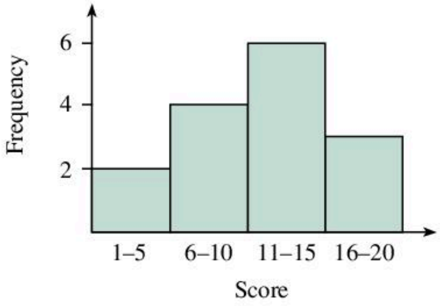
Score	Tally	Frequency
1–5	||	2
6–10	||||	4
11–15	~~||||~~ |	6
16–20	|||	3

Check out

1 The histogram shows the heights (in cm) of some people.



and by a histogram as:



2 How to m = find the mean, mode and median of a data set.
For example:
The heights of 5 children are
130 cm, 142 cm, 130 cm, 148 cm, 135 cm
Mean = $[130 + 142 + 130 + 148 + 135]$ divided by 5 = 137 cm
Mode = most frequent height = 130 cm
To find median, write heights in order of size
130 cm, 130 cm, 135 cm 142 cm, 148 cm
The median height is 135 cm

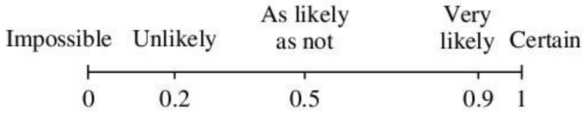
3 How to calculate the mean from a frequency table.
For example:

Number	**Frequency**	No. × Freq.
4	4	16
5	4	20
6	7	42
Total	15	78

The mean for the numbers in this frequency table is $\frac{78}{15} = 5.2$

4 Some events are more likely to happen than others. In mathematics, the likelihood of an event happening is described by a number between 0 and 1.

This number is the probability of the event.



(a) What is the tallest a person could possibly have been in the survey?
(b) How many people are in the range 155–164 cm?

2 Find the:
(a) mean mark
(b) mode
(c) median mark.

3 The savings over three months for a group of children is given in the table.

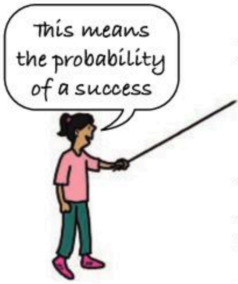
Savings $	**Frequency**
35.00	3
39.50	4
42.00	5
45.50	3
47.00	3
52.00	2

Calculate the mean savings.

4 Write down two events that are:
(a) impossible
(b) certain
(c) very likely to happen
(d) unlikely to happen.

5 How to find the probability of an event by performing an experiment.

The occurrence of a particular event is called a successful outcome.



$$P(\text{success}) = \frac{\text{number of successful outcomes}}{\text{total number of outcomes}}$$

For example:

A coin was thrown 100 times and 52 heads occurred, so

$$P(\text{head}) = \frac{52}{100} = \frac{13}{25}$$

is the experimental probability.

5 The shoe sizes of 20 customers at a shoe store were:

Size	4	5	6	7	8	9	10
Frequency	1	2	3	3	5	4	2

What is the probability that a customer chosen at random

(a) has a size 8 shoe

(b) has a shoe size less than 6?

6 How to find the probability of simple events without performing experiments.

For example:

If a coin is tossed, the probability of getting a head is

$$P(\text{head}) = \frac{\text{number of successful outcomes}}{\text{total number of outcomes}}$$

$$= \frac{1}{2}$$

6 A card is chosen from a pack of cards. What is the probability that the card is:

(a) a black card

(b) a six or a ten

(c) not an ace?

Consumer arithmetic

12

Objectives

- ✔ work out percentages on your calculator
- ✔ calculate simple and compound interest on loans and deposits
- ✔ calculate wages and salaries of employees
- ✔ calculate utility bills

What's the point?

Phone bills, water bills, TV bills, and electricity bills are a fact of life. How they are calculated and how much you have to pay are an important factor in ensuring that you live within your means.

Before you start

You should know ...

1 How to find percentages of amounts.

For example:

$$8\% \text{ of } \$40 = \frac{8}{10\cancel{0}} \times \$4\cancel{0}$$
$$= \$\frac{32}{10} = \$3.20$$

2 Profit = selling price − cost price

For example: If a kettle that cost $96 is sold for $120, profit = $120 − $96 = $24.

Check in

1 Work out:
- **(a)** 5% of $20
- **(b)** 6% of $30
- **(c)** 7% of $50
- **(d)** 11% of $35

2 Find the profit or loss on:
- **(a)** a bicycle bought for $300 and sold for $180
- **(b)** a bed bought for $340 and sold for $385.

3 How to find the cost price or selling price if you know the percentage profit or loss.

For example: If a clock that cost \$75 is sold for a 5% profit,
profit = 5% of \$75 = \$3.75
so selling price = cost price + profit
= \$75 + \$3.75 = \$78.75

3 A salesman buys a watch for \$120. What is his selling price if he makes:
(a) a 20% profit
(b) a 35% profit
(c) a 10% loss?

4 Discount is the amount you subtract from a bill.

For example: A computer game that is usually sold for \$105 is discounted by 20%,
so discount = 20% of \$105 = \$21
and sale price = \$105 − \$21 = \$84

4 A shop is giving a 15% discount on all items. What is the cost of:
(a) a chair marked \$600
(b) a table marked \$1250?

12.1 Fractions and percentages


Frank's Furniture
Invoice

Chair	\$220
15% VAT	\$33
Total	\$253

You may see percentages on bills, notices and in stores.

The easiest way to change a fraction to a percentage is to multiply the fraction by 100%.

Example 1

Write **(a)** $\frac{3}{4}$ **(b)** $\frac{2}{9}$ as percentages.

(a) $\frac{3}{4} = \frac{3}{\cancel{4}_1} \times \frac{\cancel{100}^{25}}{1}\% = 75\%$

(b) $\frac{2}{9} = \frac{2}{9} \times \frac{100}{1}\% = \frac{200}{9} = 22\frac{2}{9}\%$

Alternatively you can use your calculator.

Example 2

Write **(a)** $\frac{3}{4}$ **(b)** $\frac{2}{9}$ as percentages.

(a) On your calculator press

$\boxed{3}\ \boxed{\div}\ \boxed{4}\ \boxed{\times}\ \boxed{1}\ \boxed{0}\ \boxed{0}\ \boxed{=}$

$\boxed{\quad\quad\quad 75.}$

So $\frac{3}{4} = 75\%$

(b) Press

$\boxed{2}\ \boxed{\div}\ \boxed{9}\ \boxed{\times}\ \boxed{1}\ \boxed{0}\ \boxed{0}\ \boxed{=}$

$\boxed{\quad\quad 22.222222}$

So $\frac{2}{9} = 22.2\%$ (to 1 d.p.)

You can also use your calculator to find percentages of amounts.

Example 3

Find $9\frac{1}{2}\%$ of \$15.40

$9\frac{1}{2}\% = \frac{9.5}{100}$, so on your calculator press

$\boxed{9}\ \boxed{\cdot}\ \boxed{5}\ \boxed{\div}\ \boxed{1}\ \boxed{0}\ \boxed{0}$

$\boxed{\times}\ \boxed{1}\ \boxed{5}\ \boxed{\cdot}\ \boxed{4}\ \boxed{=}$

$\boxed{\quad\quad\quad 1.463}$

That is
$9\frac{1}{2}\%$ of \$15.40 = \$1.46

Exercise 12A

1 Change to a percentage:

(a) $\frac{1}{2}$ **(b)** $\frac{1}{4}$ **(c)** $\frac{1}{5}$ **(d)** $\frac{3}{5}$
(e) $\frac{1}{25}$ **(f)** $\frac{3}{20}$ **(g)** $\frac{17}{20}$ **(h)** $\frac{17}{50}$
(i) $\frac{1}{8}$ **(j)** $\frac{3}{8}$ **(k)** $\frac{1}{16}$ **(l)** $\frac{7}{16}$
(m) $\frac{1}{3}$ **(n)** $\frac{5}{6}$ **(o)** $\frac{5}{12}$ **(p)** $\frac{4}{11}$

2 Fill in the gaps in a copy of this table.

$\frac{1}{8}$	$\frac{1}{4}$	$\frac{3}{8}$	$\frac{1}{2}$		$\frac{3}{4}$	1
	25%			$62\frac{1}{2}$%		100%

3 In an exam Beverly scored 88 out of 120. What was her score as a percentage?

4 A coil of rope is 30 m long. 65 cm of the rope is frayed. What percentage of the rope is not frayed?

5 **(a)** What is 20% as a fraction?
(b) Find 20% of:
(i) 5 metres **(ii)** 20 students
(iii) 30 camels **(iv)** 100 bottles
(v) $50 **(vi)** $2500

6 Find:
(a) 10% of $20 **(b)** 6% of $20
(c) 13% of $25 **(d)** 9% of $15
(e) 8% of $6 **(f)** 4% of $7.50
(g) 16% of $9.50 **(h)** 5% of $4.20
(i) $6\frac{1}{2}$% of $26 **(j)** $9\frac{1}{2}$% of $12.50

7 Brenda Matthew buys a dress for $60. She sells it, making a $7\frac{1}{2}$% profit. What is Brenda's selling price?

8 In a school of 700 students, 45% are boys. How many are girls?

9 Mr Miller buys a television for $650 and sells it for $500.
(a) What is his loss?
(b) What is his loss as a fraction of the cost price?
(c) What is his percentage loss?

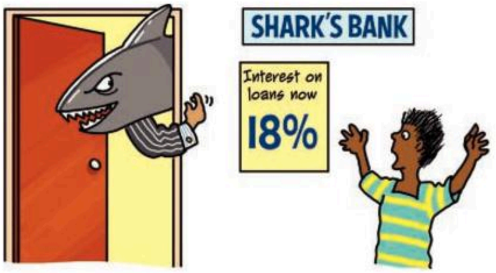
10 Copy and complete the table.

Usual price	Sale price	Discount
$150		35%
$200	$170	
	$ 24	20%
	$ 19.50	25%
$500	$325	

12.2 Loans and interest

If you want to borrow money from a bank, you must repay all the money you borrow plus the fee or interest for that loan.
The interest payable depends on the rate of interest p.a. and the time taken to pay back the loan.

p.a. stands for *per annum* which is Latin for per year. So, 10% p.a. means 10% per year.



Example 4

Casper Lyon borrows $20 000 from Shark's bank. If the interest rate is 18% p.a., how much must Casper repay if he repays after:
(a) 1 year
(b) 5 years?

(a) Interest for 1 year

$$18\% \text{ of } \$20\,000$$
$$= \frac{18}{\cancel{100}} \times \cancel{20\,000}^{200}$$
$$= \$3600$$

He must repay $20 000 + $3600
= $23 600

(b) Simple interest after 5 years

$= 5 \times$ yearly interest

$= 5 \times \$3600$

$= \$18\,000$

Total repayment after 5 years

$= \$20\,000 + \$18\,000$

$= \$38\,000$

The amount of money you borrow or deposit is often called the **principal**.
For Casper, in Example 4, the principal is $20 000.

Example 5

Casper borrows $350 from Jaws Bank on condition he pays it back after 2 months. Work out the total interest charged on the principal if the rate at Jaws is $13\frac{1}{2}\%$ p.a.

Interest for 1 year $= 13\frac{1}{2}\%$ of $350

$= \frac{27}{200} \times \$350$

$= \frac{945}{20}$

$= \$47.25$

Interest for 2 months $= \frac{2}{12} \times \$47.25$

$= \$7.88$

Exercise 12B

1 In Example 4, how much interest will Casper pay if he repays the loan after 4 years?

2 If the bank reduces its interest rate to 11% p.a. work out the total interest Casper has to pay over the 4 years.

3 Another bank offers Casper a loan of $20 000 at $8\frac{1}{2}\%$ p.a. on the condition that he repays it after 3 years. What would be the total interest charged?

4 Find the total amount to be repaid on a loan of:
- **(a)** $10 000 at 6% p.a. for 1 year
- **(b)** $15 000 at 8% p.a. for 3 years
- **(c)** $8000 at 9% p.a. for 5 years.

5 Find the total interest to be paid on these loans. The interest rates are all per annum.
- **(a)** $5000 at 10% for 2 years
- **(b)** $15 000 at 9% for 4 years
- **(c)** $6000 at $8\frac{1}{2}\%$ for $2\frac{1}{2}$ years
- **(d)** $5000 at 10% for 3 months
- **(e)** $12 000 at 9% for 9 months
- **(f)** $25 000 at $13\frac{1}{2}\%$ for 8 months
- **(g)** $15 000 at $33\frac{1}{3}\%$ for $2\frac{1}{2}$ years
- **(h)** $3000 at 10% for 4 months
- **(i)** $5000 at 8% for 6 months
- **(j)** $100 at 16% for $1\frac{1}{4}$ years

6 Find the interest after one year on:
- **(a)** a principal of $100 at a rate of 5%
- **(b)** a principal of $250 at a rate of 4%
- **(c)** a principal of $200 at a rate of 6%
- **(d)** a principal of $*P* at a rate of *R*%

7 For each part of Question 6, write down the interest after:
(i) two years **(ii)** *T* years

Activity

(a) Find the interest rates on loans from banks in your neighbourhood.
(b) Which bank offers the best interest rate on loans? What about deposits?
(c) Which bank would you choose to take a loan from? Why?
(d) Which bank would you choose to deposit your money in? Why?

You could use this activity as a basis for a CSEC SBA project.

The simple interest formula

For Question 7(ii) of Exercise 12B, you should have found that the interest on principal $*P*, at rate *R*%, for *T* years was:

$$\textbf{Interest} = \frac{P \times R \times T}{100}$$

- If I stands for interest, then you can write:

$$I = \frac{P \times R \times T}{100} \text{ or } I = \frac{PRT}{100}$$

Example 6

Find the interest on a loan of \$600 for 2 months at 7% p.a.

$P = \$600$
$R = 7$
$T = 2 \text{ months} = \frac{1}{6} \text{ year}$

$$I = \frac{PRT}{100}$$

$$I = \frac{600 \times 7 \times \frac{1}{6}}{100}$$

$$I = \frac{700}{100}$$

$$I = \$7$$

So the interest is \$7.

If you know the values of I, P and T, you can find the rate of interest R using the simple interest formula.

Example 7

The interest on \$600 deposited in a bank for 4 years is \$120. What is the rate of interest?

$I = \$120$, $P = \$600$, $T = 4$ years

so $$120 = \frac{6\cancel{0}\cancel{0} \times R \times 4}{1\cancel{0}\cancel{0}}$$

$$120 = 24 \times R$$

$(\div 24)$ $$R = \frac{120}{24} = 5$$

Hence the rate of interest = 5%

Exercise 12C

1 Copy and complete this table.

Principal (\$)	Rate (%)	Time (years)	Interest (\$)
200	3	5	
50	5	2	
420	2	5	
1500	4	8	
800	6	0.5	

2 Use the formula to find the interest on:
- **(a)** a deposit of \$300 for 6 months at 8% p.a.
- **(b)** a deposit of \$25 for 2 years at 5% p.a.
- **(c)** a loan of \$550 for 1 month at 12% p.a.
- **(d)** a loan of \$1000 for 3 months at 9% p.a.
- **(e)** a deposit of \$375 for 15 months at 4% p.a.

3 Use the method in Example 7 to find R.
- **(a)** $P = \$100$, $I = \$5$, $T = 1$ year
- **(b)** $P = \$350$, $I = \$7$, $T = 1$ years
- **(c)** $P = \$600$, $I = \$54$, $T = 2$ years
- **(d)** $P = \$750$, $I = \$30$, $T = 6$ months
- **(e)** $P = \$50$, $I = \$10$, $T = 2\frac{1}{2}$ years

4 Copy and complete this table.

Principal	Rate	Time	Interest
\$100	6%		\$3
	5%	2 years	\$32
\$500	5%		\$37.50
	10%	5 years	\$3.50
	9%	2 months	\$4.50
\$50	10%		\$1.25

5 Delbert has \$100 000 to invest. He puts it in a bank that offers an interest rate of 7% per annum. How long should he keep his money there for his principal to increase to \$135 000?

Activity

Find out about compound interest. How does it differ from simple interest? How is it calculated?

What type of interest is charged by banks? Credit card companies? Why?

Write up your findings. Illustrate with some compound interest calculations.

Compound interest

In fact, banks do not use simple interest calculations to compute interest payments. They, instead, use **compound interest**.

In compound interest, the interest after each year is added to the principal.

The interest on the following year is calculated from this new principal.

Example 8

Desmond borrows \$2000 from a bank for two years at 10% rate of interest. How much does he have to repay the bank?

$$\begin{aligned}\text{Interest for first year} &= 10\% \text{ of } \$2000\\ &= \frac{10}{1\cancel{0}\cancel{0}} \times \$20\cancel{0}\cancel{0}\\ &= \$200\end{aligned}$$

After 1 year he has \$2000 + \$200 = \$2200 to repay.

$$\begin{aligned}\text{Interest for second year} &= 10\% \text{ of } \$2200\\ &= \frac{10}{1\cancel{0}\cancel{0}} \times \$22\cancel{0}\cancel{0}\\ &= \$220\end{aligned}$$

After 2 years he has \$2200 + \$220 = \$2420 to repay.

In Example 8, what would be the repayment if the bank just changed simple interest?

$$\begin{aligned}\text{Interest} &= \frac{PRT}{100} = \frac{20\cancel{0}\cancel{0} \times 10 \times 2}{1\cancel{0}\cancel{0}}\\ &= \$400\end{aligned}$$

So repayment would be \$2000 + \$400 = \$2400

Notice that under compound interest his repayment is \$2440.

In compound interest, the interest payment is added successively to the principal, increasing the amount of interest payable or to be paid each year.

Exercise 12D

1 A bank has a compound interest rate of 10% on a loan of \$500.
- **(a)** Find the interest on the loan for one year.
- **(b)** How much must be repaid to the bank after one year?
- **(c)** What would be the interest payment for the second year?
- **(d)** Calculate the total amount to be repaid, if the loan is for a two-year period.

2 Find the compound interest on a loan of:
- **(a)** \$1000 for 2 years at 10%
- **(b)** \$10 000 for 2 years at 8%
- **(c)** \$500 for 2 years at 6%
- **(d)** \$25 000 for 2 years at 12%

3 Find the total amount to be repaid in Question 2.

4 Sophie puts \$5000 in her credit union for a period of two years. The credit union pays interest at 2% per annum.
- **(a)** How much interest does she receive after
  - **(i)** one year **(ii)** two years?
- **(b)** What will be her balance after two years?

5 Find the amount in Victor's bank if he deposits
- **(a)** \$500 for 2 years at 3%
- **(b)** \$10 000 for 2 years at 4%
- **(c)** \$800 for 2 years at 2%
- **(d)** \$25 000 for 2 years at 4%.

6
- **(a)** Find the interest paid on
  - **(i)** \$6000 for 2 years at 5% simple interest
  - **(ii)** \$6000 for 2 years at 4% compound interest.
- **(b)** Which is the better investment?

7 Precious put \$8000 in a deposit account for 2 years at 6% per annum.
- **(a)** Determine the total amount in her account after 2 years under compound interest.
- **(b)** What would she receive after 2 years under simple interest?
- **(c)** How much more profitable is compound interest to her?

8 Delvin borrows \$60 000 from his bank to purchase a car. The bank has an 8% interest rate on vehicle loans.
If he wishes to repay the loan in two years, find
- **(a)** the total compound interest he has to pay
- **(b)** the total he has to repay the bank
- **(c)** his monthly repayment bill.

9 A car valued at \$40 000 depreciates in value by 20% per year.
- **(a)** What would be the value of the car after
  - **(i)** one year
  - **(ii)** two years
  - **(iii)** three years?

10 An insurance agency holds that a vehicle's depreciation rate is 15%. Find the value of the vehicle given its current value is
(a) \$100 000 after 2 years
(b) \$30 000 after 2 years
(c) \$120 000 after 3 years.

Activity

The latest smartphone is on sale for \$3500. You wish to buy it but do not have the money.

- Go to two banks and at least one lending agency and find out the interest rates charged by each.
- Determine the amount you would have to repay if you borrowed the money for
  (i) one year **(ii)** two years
  (iii) three years.
- How much money would you have to repay each month?
- Discuss which lending institutions would you choose.

Note: This activity could be developed into an SBA for CSEC.

12.3 Wages and salaries

Leroy is a security guard. He is paid a **wage** of \$12 per hour.

Celia is a bank teller. She is paid a salary of \$2000 a month.

Workers who are paid by the hour are said to receive **wages**. The more hours they work, the more money they are paid.

Salaried workers are paid a fixed amount of money, usually each month. They do not receive more even if they work longer hours.

In the example above, Celia's **annual salary** is \$24 000 and this is the total amount she is paid for the year. At the end of each month she is paid \$2000. If Leroy works a 40-hour week, he will be paid

$$40 \times \$12 = \$480.$$

Leroy's weekly wage for a 40-hour week is \$480. You can use these ideas to solve simple problems.

Example 9

(a) Jones is paid \$655.20 for 36 hours of work. What is his hourly wage?

$$\begin{aligned}\text{Hourly wage} &= \$655.20 \div 36\\ &= \$18.20\end{aligned}$$

(b) Angela is paid a salary of \$18 000 per annum. What is her monthly income?

$$\begin{aligned}\text{Monthly income} &= \$18\,000 \div 12\\ &= \$1500\end{aligned}$$

The total sum you are paid for your work is known as your **gross** pay.

In practice, the actual amount of money you receive is usually less. This is because taxes and other contributions are **deducted**.

Your **net pay** is given by

net pay = gross pay − deductions

Example 10

Prince has a salary of $3500 per month. Each month he pays $550 in taxes and a $50 union fee. What is his net salary?

Prince's deductions are $550 + $550 = $600

Net pay = $3500 − $600
= $2900

In Example 10, Prince has compulsory tax deductions of $550. These are sometimes called **statutory** deductions because they are enforced by law. His $50 deduction to his Union is probably a voluntary or **non-statutory** deduction.

Exercise 12E

1 What is the monthly salary of a person who earns
- **(a)** $12 000 per annum
- **(b)** $30 000 per annum
- **(c)** $27 000 per annum
- **(d)** $56 000 per annum
- **(e)** $108 000 per annum

2 Jackson is paid a wage of $15 per hour. What is his weekly wage if he works for
- **(a)** 40 hours
- **(b)** 38 hours
- **(c)** 25 hours
- **(d)** $2\frac{1}{2}$ hours
- **(e)** 33 hours?

3 A man is paid $7.50 per hour. How many hours has he worked if he is paid
(a) $300 **(b)** $225 **(c)** $262.50?

4 Gloria is paid a salary of $60 000 per annum. Each year she pays $12 000 in taxes. What is her
- **(a)** gross monthly salary
- **(b)** net annual salary
- **(c)** net monthly salary?

5 Look at Albert Ames' salary slip.

Name: Albert Ames Month-year: 10–18			
Earning		**Deduction**	
Basic	4000	Mortgage Tax	1000 750
Total	4000	Total deduction	1750
		NET PAY	_____

- **(a)** What is Albert's gross annual salary?
- **(b)** Does he have any non-statutory deductions?
- **(c)** What is his net monthly salary?
- **(d)** What is his net yearly income?

6 Look at Edwina Enderby's wage slip.

Name: Edwina Enderby Week: 11th–17th June 2018				
	Hourly rate	**Hours**	**Amount**	**Deduction**
	12.00	38	—	65 (tax)

- **(a)** How much is Edna's gross pay for the week?
- **(b)** What is Edna's net pay?

Activity

- What are the advantages of being a salaried worker?
- What are the disadvantages?
- Talk to some workers.
- Would you prefer to be paid a wage or a salary? Why?

Employers often pay **overtime** to wage earners who work more than the **basic** number of hours per week. Overtime may also be paid to workers who choose to work on official holidays.

Overtime hourly payments are more than basic hourly payments to compensate for the additional work or unfavourable hours.

Example 11

The basic hourly rate at Steve's work is \$8.50. The overtime rate is \$12.50. Steve works 48 hours in one week. Calculate his wage, if the basic rate is paid for 40 hours of work.

$$\begin{aligned}\text{Basic pay} &= 40 \times \$8.50\\ &= \$340\end{aligned}$$

$$\text{Overtime} = 48 - 40 = 8 \text{ hours}$$

$$\begin{aligned}\text{Overtime pay} &= 8 \times \$12.50\\ &= \$100\end{aligned}$$

$$\begin{aligned}\text{Total wage} &= \$340 + \$100\\ &= \$440\end{aligned}$$

Overtime rates are often a fraction of basic rates. For example, if the basic rate is \$12 per hour then an overtime rate of

'time and a quarter' $= 1\frac{1}{4} \times \$12 = \$15$

'time and a half' $= 1\frac{1}{2} \times \$12 = \$18$

'double time' $= 2 \times \$12 = \$24$

Exercise 12F

1 Clive works a 40-hour week for which he is paid \$10 per hour. Overtime is paid at \$12.50 per hour. What is Clive's gross pay if he works 48 hours this week?

2 Leticia is paid a basic rate of \$8.00. Overtime is paid at 'time and a quarter'! What is her pay for a 46-hour week if a normal week is 40 hours?

3 Mr. Stevens works a 38-hour basic week. Any overtime is paid at 'time and a half'! Last week Mr Stevens worked 50 hours. What is his gross pay if the basic rate is \$14 per hour?

4 For a 40-hour week a man is paid \$520.
  (a) What is the basic rate?
  (b) If the man works a further 6 hours overtime at 'double time', what is his total gross wage for the week?

5 At a box-making factory workers are paid a basic rate of \$9.50 per hours for a 40-hour week and an overtime rate of \$12. Find the pay of an employee who works for
  (a) 42 hours
  (b) 45 hours
  (c) 48 hours
  (d) 50 hours.

6 A soft drink company pays its workers \$320 for a 40-hour week. Overtime is paid at 'time and a half'.
Find:
  (a) basic rate of pay
  (b) overtime rate of pay
  (c) the gross wage of a worker who works for 45 hours.

7 The basic rate for a 40-hour week for daily paid workers at the Haddens Company is \$11.20 per hour. Overtime is paid at 'time and a half'. Find the numbers of hours Mrs. Jeffry works if her gross pay is
  (a) \$336 **(b)** \$425.60
  (c) \$464.80 **(d)** \$582.40

8 Rashid is paid \$12.60 per hour for a 40-hour work week. Overtime is paid at 'time and a half'. Calculate his overtime pay if his gross pay for the week is
  (a) \$636.30 **(b)** \$749.70

Commissions

Some salaried workers are sometimes offered **commissions** or **bonuses** when they sell an item to a customer.

The commission is usually a percentage of the sale price of the item.

Example 12

Jenny is given a 5% commission on sales of cosmetics. What is her gross income for the week if her basic wage is $300 and she sells cosmetics to the value of $800?

$$\begin{aligned}\text{Commission on cosmetics} &= 5\% \text{ of } \$800\\ &= \frac{5}{100} \times \$800\\ &= \$40\end{aligned}$$

$$\begin{aligned}\text{Gross wage} &= \text{Basic wage} + \text{commission}\\ &= \$300 + \$40\\ &= \$340\end{aligned}$$

Exercise 12G

1 Find the commission on sales of $2000, if the sales commission is

(a) 3% **(b)** 5% **(c)** 2%

2 Denise receives a monthly salary of $2500. She also receives a 2% commission on all sales over $5000. What is her gross salary if she makes sales of

(a) $4000 **(b)** $6000 **(c)** $60 000?

3 A car salesman is paid 2.5% commission on the value of each car he sells. Calculate

(a) his commission if he sells five cars for $280 000 in the month

(b) his gross salary for that month if his annual salary is $30 000.

4 The gross wage of Brenda Simmons during the first week of June was $650. If her basic wage is $560 and she gets a 3% commission on goods sold find

(a) her commission

(b) the value of goods sold.

5 A computer sales representative has a salary of $35 000 per annum.
In the month of March his commission on sales is $2200. Find his

(a) gross income for March

(b) commission rate if he sold $55 000 worth of computers in March.

(c) value of the goods he sold in April if his gross income for April was $4200.

Technology

Review this work by visiting
www.bbc.co.uk/bitesize/standard/maths_i/numbers/wages/revision/3/

12.4 Utility bills

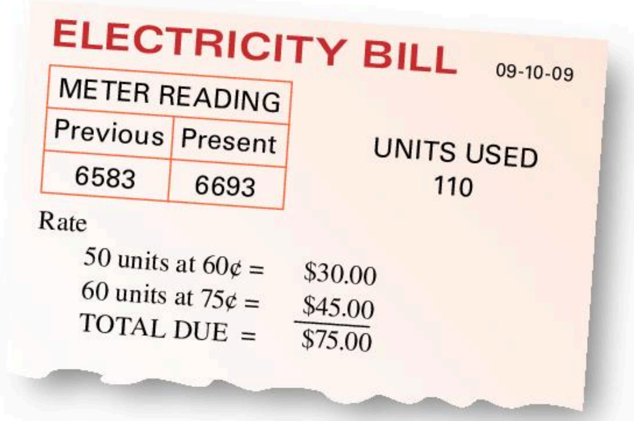
The most important utilities are:

- electricity
- water
- telephone.

These services are provided by utility companies. These companies usually charge consumers for each unit of the utility used.

The cost per unit may vary depending on the number of units used.

For example, an electricity company may charge
60 cents per unit for the first 50 units
and 75 cents per unit for further units.
A typical bill may look like this.



ELECTRICITY BILL 09-10-09

METER READING	
Previous	Present
6583	6693

UNITS USED
110

Rate

50 units at 60¢ = $30.00
60 units at 75¢ = $45.00
TOTAL DUE = $75.00

To find the total utility bill, you need to know the billing rate for units.

Example 13

Water rates on a certain island are:
Fixed charge $25.00
First 1000 gallons $6.50
Next 20 000 gallons $9.25/1000 gal.
What is the water bill of a man who uses 6321 gallons of water?

$$\begin{aligned} \text{Fixed charge} &= \$20.00 \\ \text{First 1000 gallons} &= \$\ 6.50 \\ \text{Next 5321 gallons} &= \$9.25 \times 5.321 \\ &= \$49.22 \\ \text{Total} &= \$75.72 \end{aligned}$$

Exercise 12H

1 The electricity rates at residential homes are:

Fixed charge $25.00

First 100 units 55 cents/unit

Further units 85 cents/unit

What is the electricity bill for someone who uses:

(a) 100 units **(b)** 50 units
(c) 150 units **(d)** 163 units?

2 Using the rates in Question 1, find the bill to be paid if meter readings during last month were:

(a) previous 7134 present 7156
(b) previous 3089 present 3142
(c) previous 9135 present 9241

3 A monthly telephone bill has a $35 rental charge. Local calls cost 28 cents each.

(a) What is the bill if
  (i) 20 units
  (ii) 130 units are used?

(b) How many local calls were made if the monthly bill was
  (i) $36.68
  (ii) $77.84?

4 Find the monthly electricity bills for these people.

(a)

Name	Units used	Fixed charge	Cost per unit
J. Mason	135	$40.00	45¢

(b)

Name	Units used	Fixed charge	Cost per unit
D. Chen	86	$45.00	72¢

(c)

Name	Units used	Fixed charge	Cost per unit
R. Hope	234	$55	66.7¢

5 Calculate the monthly telephone bills for these people.

(a) Janice Sukoo

Call	No. of minutes	Cost/min
Local	84	15¢
Overseas	2	98¢

(b) Michael Chin

Call	No. of minutes	Cost/min
Local	64	20¢
Overseas	28	$1.15

(c) Mona Manly

Call	No. of minutes	Cost/min
Local	153	25¢
Overseas	78	$1.22

6 Look at the electricity bill below.

Meter	Previous Read Date	Current Read Date	Total Days	Previous Reading	Current Reading	Usage	Units	Read Status
006649884	11/24/2008	12/25/2008	31	18219	18408	189	kWh	Regular

Transaction Date	Transaction Description	Amount
Dec 30, 2008	Domestic Block 1 Charge 57.8 cents per kWh	$28.90
Dec 30, 2008	Domestic Block 2 Charge 67.0 cents per kWh	$33.50
Dec 30, 2008	Fuel Surcharge	$45.15
	Total	**$107.55**
Dec 30, 2008	Additional kWh 67.0 cents per kWh	$59.63
Dec 30, 2008	Additional Fuel Surcharge	$40.18
	Sub-Total	**$99.81**
Dec 30, 2008	VAT @ 15%	$14.97
	Summary of New Charges	**$222.33**

(a) What were the current and previous meter readings?
(b) How many units of electricity were used in the billing period?
(c) How many domestic Block 1 units were used?
(d) How many domestic Block 2 units were used?
(e) What was the cost of the first 100 units?
(f) What was the total fuel surcharge?

7 Water rates in Jamaica are given in the table

	Rate per 1000 litres
Litres	**Cost (J$)**
for up to 14 000	97.55
for next 13 000	172.01
for next 14 000	185.73
for next 14 000	237.07
for next 36 000	295.20
over 91 000	380.00

(a) Calculate Amanda Walter's water bill if she uses
(i) 6000 l **(ii)** 15 000 l
(iii) 29 000 l **(iv)** 54 000 l
(b) How many litres of water did Amanda use if her bill was
(i) $390.20 **(ii)** $1881.73
(iii) $5087.67?

8 Electricity rates in Trinidad are

kWh	Rate TT$ per unit
upto 1000	0.32
1000 plus	0.37

with a $6 fixed charge.
(a) Find Phillip Martin's electricity bill if he uses
(i) 820 kWh **(ii)** 1030 kWh
(iii) 8436 kWh
(b) How many kWh of electricity does he use if his bill is
(i) $211.76 **(ii)** $446.25
(iii) $1851.51?

Activity

Find out about the types of electricity meter there are and how to read them.

- Make a handout for consumers on how to read your meter.
- Make your own copy of an electricity bill. Explain each of its parts. Share with your class.

Exercise 12I – mixed questions

1 Value added tax (VAT) is added at 15% to the price of goods. What is the actual selling price of goods marked:
(a) $20.00 + VAT
(b) $3.00 + VAT
(c) $1.50 + VAT

2 A store gives 10% discount on all goods priced at \$50 or more. Calculate the selling price of items marked:
(a) \$1350 **(b)** \$380 **(c)** \$35.95

3 At a recent election in Dominica the number of votes cast was as follows:

Dominica Labour Party (DLP)	15 360
Dominica Freedom Party (DFP)	4862
United Workers Party (UWP)	15 428

(a) Which party won the greatest percentage of the vote?
(b) What percentage of the vote did the DFP win?

4

A tin of fly spray holds 400 ml of spray. An economy tin holds 15% more.
How much spray does the economy tin hold?

5 To buy a television, May borrows \$1500 for one year from her bank at an interest rate of 13% p.a.
(a) How much interest does May pay?
(b) How much does she repay the bank altogether?
(c) How much do you think she pays the bank back each month?

6 The National Bank pays compound interest at a rate of 2% on deposits. Find the amount in a bank on a deposit of
(a) \$5000 for 2 years
(b) \$675 for 2 years.

7 Which gives a better rate of return on \$12 000?
(a) 4% compound interest for 2 years
(b) 3% simple interest for $2\frac{1}{2}$ years

8 Electricity rates on a certain island are

	Cost per unit
First 50 units	\$0.67
Next 100 units	\$0.75
Over 150 units	\$0.84

Work out the electricity bill due on each of the meters

	Meter A	Meter B	Meter C
Present reading	46125	13462	58927
Previous reading	46090	13314	58643

9 A salesman's annual salary is \$48 000. He is given 2% commission on sales more than \$5000 per month. Find
(a) his commission if he makes \$40 000 sales in the month
(b) gross monthly income for the month.

10 Lydia gets a basic wage of \$11.25 per hour for a 40-hour week. Find:
(a) her gross pay for the week
(b) her gross pay for the week if she makes 6 hours overtime at 'time and a half' rates
(c) the number of hours overtime she makes if her gross weekly pay is \$585.

11

A car valued at \$85 000, depreciates in value by 20% a year. Find its value after
(a) one year **(b)** two years **(c)** three years.

12 A shop assistant is paid a basic wage of \$18.50 per hour for a 40-hour week. She is paid at double-time for work on Sundays.
(a) Calculate her basic weekly wage.
(b) Calculate her earnings for the week if she also works 3 hours on Sunday.
(c) How many hours does she work on Sunday if she pay for the week is \$952.75?

13 The electricity rates in a certain island are:

Fixed charge	\$15
First 80 units	\$0.73 per unit
Further units	\$0.95 per unit

What is the electricity bill for a household using:
(a) 45 units **(b)** 86 units **(c)** 214 units?

12 Consolidation

Example 1

Find:

(a) 6% of \$30

$$6\% \text{ of } \$30 = \frac{6}{10\cancel{0}} \times \$3\cancel{0}$$
$$= \frac{\$18}{10} = \$1.80$$

(b) $8\frac{1}{2}$% of \$25

$$8\frac{1}{2}\% \text{ of } \$25 = \frac{17}{\cancel{200}_8} \times \$\cancel{25}$$
$$= \$\frac{17}{8} = \$2.13$$

Example 2

What is the interest to be paid on a loan of \$40 000 for 2 years at

(a) 8% simple interest

(b) 8% compound interest?

(a) Simple interest $= \frac{P \times R \times T}{100}$

$$= \frac{40\,000 \times 8 \times 2}{100}$$
$$= \$6400$$

(b) Interest after 1 year = 8% of \$40 000

$$= \frac{8}{100} \times \$40\,000$$
$$= \$3200$$

Principal after 1 year = \$40 000 + \$3200
= \$43 200

Interest after second year = 8% of \$43 200

$$= \frac{8}{100} \times \$43\,200$$
$$= \$3456$$

Compound interest = \$3200 + \$3456
= \$6656

Example 3

(a) Find Jackson's gross pay for the week if he is paid \$26.25 per hour for a 40-hour week.

(b) If he is paid overtime at 'time and a half', find his gross pay if he works 48 hours.

(a) Gross pay = 40 × \$26.25
= \$1050

(b) He works 48 − 40 = 8 hours overtime

Overtime pay $= 8 \times 1\frac{1}{2} \times \$26.25$
= \$315

Gross pay for 48 hours = \$1050 + \$315
= \$1365

Example 4

The electricity rates for domestic use are:

First 100 units	\$0.78 per unit
Further units	\$1.19 per unit

What is the electricity bill for a person using:

(a) 80 units **(b)** 143 units?

(a) Cost of 80 units = 80 × \$0.78 = \$62.40

(b) Cost of first 100 units = 100 × \$0.78 = \$78.00

Cost of remaining units (143 – 100) = 43 × \$1.19
= \$51.17

Total cost = \$78.00 + \$51.17 = \$129.17

Exercise 12

1 Find:

(a) 5% of \$20 **(b)** 15% of \$30

(c) 7% of \$25 **(d)** 20% of 15 goats

(e) 75% of 72 pigs **(f)** $6\frac{1}{2}$% of \$18

(g) $8\frac{1}{2}$% of \$14.80 **(h)** $13\frac{1}{2}$% of \$24.90

2 Copy and complete the table.

Usual price	Discount	Sale price
\$400	15%	
\$275	8%	
\$1995	$7\frac{1}{2}$%	
\$3085	$5\frac{1}{2}$%	

3 Abby borrowed \$7500 at 10% per annum compound interest.

(a) What was the interest on Abby's loan for the first year?

(b) If she borrowed the money for a two-year period how much did she have to repay?

(c) What would be her monthly payment?

4 Copy and complete the table.

Principal	Rate	Time	Simple interest
\$6000	11%	4 years	
\$3500	10%		\$1050
\$5630		7 months	\$295.58
	$7\frac{1}{2}\%$	6 years	\$4308.75

5 Jeffrey Drigo receives a salary of \$56 000 per annum as an electrical appliance salesman. He also receives a commission of 2% on sales of over \$10 000 he makes each month.

(a) Find his gross monthly salary.

(b) If he makes \$84 000 worth of sales in the month, what would be his gross income for the month?

Application

6 Government workers are given a 3% pay increase. What will be the new salaries of:

(a) a clerk with current salary of \$18 530?

(b) an assistant secretary with a current salary of \$35 875?

7 Davidson invests \$8500 in the Antilles Bank. How long must he wait for his principal to become \$10 000 if the simple interest rate is:

(a) 5% **(b)** 3%?

8 Electricity rates are shown in the table

Units used kWh	Rate (\$)
First 70	0.55
Next 70	0.65
Next 100	0.75
Over 240	0.85

(a) Find Rita's electricity bill if her present and previous meter readings were

4 3 2 6 4 present

4 2 8 4 5 previous

(b) If Rita's bill was \$134.25, how many units did she use?

9 A carpenter is paid \$28 per hour for a basic work week of 40 hours.

(a) What is his gross pay if he works 46 hours for the week with overtime being at 'time and a half'?

(b) What is his net pay for a 48 hour week, if there is an 8% deduction to his pay for taxes?

Summary

You should know ...

1 How to use a calculator to find the percentage of an amount.

For example:

What is 8% of \$13.25?

On your calculator press:

8 ÷ 1 0 0 × 1 3 · 2 5 =

1.06

That is 8% of \$13.25 = \$1.06

2 The simple interest, I, on a principal, P, is given by the formula

$$I = \frac{P \times R \times T}{100}$$

where R is the rate of interest per year and T is the time in years.

Check out

1 Use your calculator to find:

(a) 6% of \$1.25

(b) 7% of \$1344

(c) $17\frac{1}{2}\%$ of \$85.16

(d) $33\frac{1}{3}\%$ of \$167.28

2 **(a)** Find the simple interest on \$600 kept in a bank for 5 years at 4% interest.

(b) How long should you keep \$750 in a bank that offers 5% interest before you receive \$100 in interest?

3 How to find the compound interest on deposit or a loan.

For example:

What is the compound interest on a loan of $2000 at a rate of 8% for two years?

$$\text{Interest in first year} = 8\% \text{ of } \$2000 = \frac{8}{100} \times \$2000 = \$160$$

$$\text{Amount to be repaid after one year} = \$2000 + \$160 = \$2160$$

$$\text{Interest in second year} = 8\% \text{ of } \$2160 = \frac{8}{100} \times \$2160 = \$172.80$$

$$\text{Compound interest} = \$160 + \$172.80 = \$332.80$$

3 Find the compound interest earned on a deposit of

(a) $5000 at 2% for 2 years

(b) $400 at 5% for 2 years

---

4 **(a)** The difference between wages and salaries, overtime and commissions.

(b) How to calculate wages.

For example:

A mechanic is paid $33 per hour for a 36-hour week. Overtime is paid at one and a half times the basic rate. Calculate his wage for a 46-hour week.

$$\text{Basic pay} = 36 \times \$33 = \$1188$$

$$\text{Overtime rate} = 1\tfrac{1}{2} \times \$33 = \$49.50$$

$$\text{Overtime} = 46 - 36 = 10 \text{ hours}$$

$$\text{Overtime pay} = 10 \times \$49.50 = \$495$$

$$\text{Gross pay} = \$1188 + \$495 = \$1683$$

4 **(a)** Explain the terms

(i) wage **(ii)** salary

(iii) commission

(iv) net wage

(b) The pay rates for a mason on a 40-hour week are

Basic	$16.80/hour
Overtime	$22/hour

(i) Find a mason's wage if he works 45 hours.

(ii) How many hours did he work if he receives a pay packet of $826?

---

5 How to read and calculate utility bills.

For example:

Telephone calls in an island are billed per minute. What is Bella's phone bill?

Call	No. of minutes	Cost/min
Local	215	15¢
Overseas	31	85¢

$$\text{Cost of local calls} = 15 \times 215 = \$32.25$$

$$\text{Cost of overseas calls} = 85 \times 31 = \$26.35$$

$$\text{Total bill} = \$32.25 + \$26.35 = \$58.60$$

5 Find the cost of these telephone bills.

(a)

Call	No. of minutes	Cost/ min
Local	68	45¢
Overseas	93	$1.15

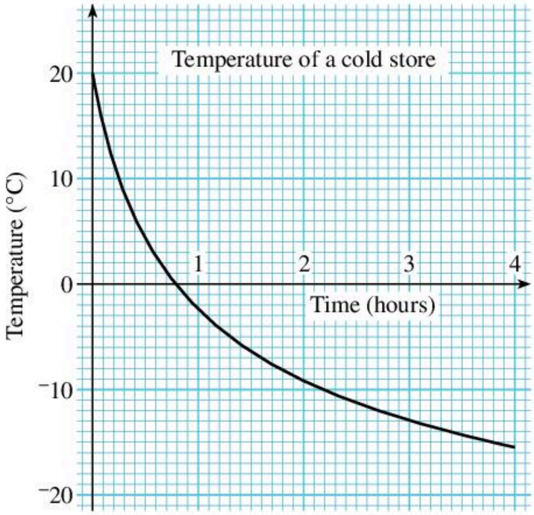
(b)

Call	No. of minutes	Cost/ min
Local	349	27¢
Overseas	187	84¢

Revision exercise 3

Graphs

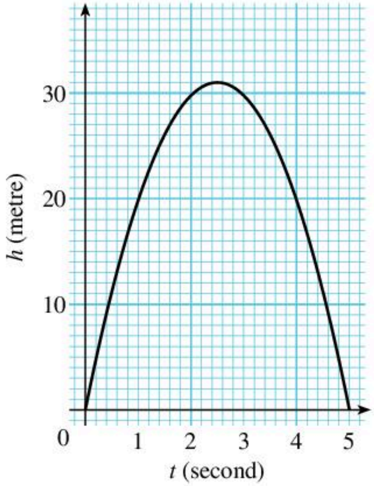
1 The graph below shows the temperature of a cold store after first switching on the cooling unit.



(a) After what time is the temperature
 (i) 0 °C **(ii)** $^{-}10$ °C?

(b) What was the temperature:
 (i) when the unit was switched on
 (ii) after 1 hour
 (iii) after $3\frac{1}{2}$ hours?

2 A cricket ball is thrown up into the air and caught again. The graph shows the height h of the ball at time t.



(a) How long does it take to reach its greatest height?

(b) Estimate the height at $t = 3.5$

(c) At what times is $h = 15$?

3 The height of a plant after w weeks of growth following germination is given by this table:

***w* (weeks)**	2	4	6	8	10	12
Height (cm)	5	10	20	40	60	70

Plot a graph of height against w and use your graph to estimate:

(a) the height after 9 weeks

(b) how long it takes to reach a height of 35 cm.

4 The speed of a model airplane at time t seconds is given by:

Time (s)	0	2	4	6	8	10	12
Speed (m/s)	5	4	4	5	7	6	5

Draw a graph from this information and use it to estimate:

(a) the time when the speed first reaches 6 m/s.

(b) the speed after 9 s.

5 Points on three lines L, M and N are as follows:

L: ($^{-}3$, 0), (0, 3), (2, 5)
M: ($^{-}3$, 1), (0, 1), (4, 1)
N: (0, 6), (2, 2), (4, $^{-}2$)

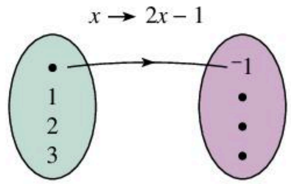
Show these three lines on a graph and find the coordinates of the vertices of the triangle made by the lines.

6 Evaluate:
$f(x) = 4 - x^2$
when

(a) $x = 2$ **(b)** $x = {}^{-}1$

(c) $x = {}^{-}2$ **(d)** $x = 0.5$

7 Copy and complete the arrow graph:



(a) Write down the ordered pairs.
(b) Draw a coordinate graph of $y = 2x - 1$.
(c) What is the gradient of the graph?

8 **(a)** Plot the graphs
$y = 2x - 1$
$x + y = 8$
for $x = {}^{-}2$ to $x = 6$.
(b) Find the point of intersection of the graphs and hence find the solution to the equations.

9 On the same diagram, draw graphs of the lines:
(a) $y = x$ **(b)** $y = x + 2$
(c) $y = 2 - x$ **(d)** $x = {}^{-}2$
Write down the coordinates of the five intersection points.

10 **(a)** Copy and complete this table.

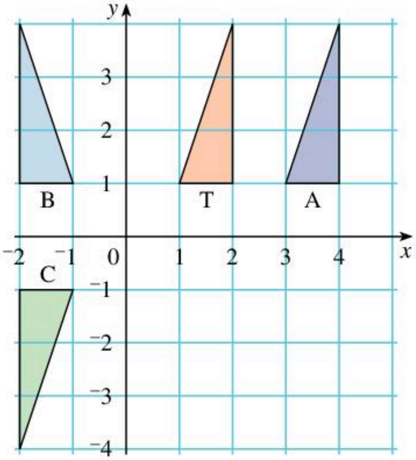
x	0	1	2	3	4	5
x^2	0	1	4			25
$f(x) = x^2 - 4x$	0	$^{-}3$				5

(b) Draw the graph of $f(x) = x^2 - 4x$ for these values of x.
(c) Use your graph to estimate the value of x when $y = 2$.

(d) Where does the graph cut the x-axis? Hence, solve the equation $x^2 - 4x = 0$.

Transformations and vectors

The diagram shows triangle T with coordinates (1, 1) (2, 1) (2, 4). Use it for Questions 11–15.



11 Describe the single transformation
(a) T → A **(b)** T → B **(c)** B → C

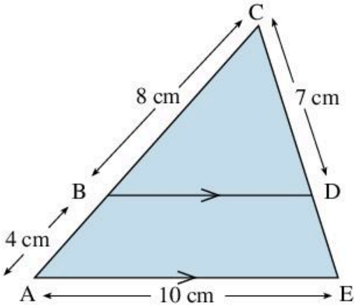
12 On graph paper, draw the triangle T and the triangles D (2, 0) (3, 0) (3, 3) and E ($^{-}1$, $^{-}3$), (0, $^{-}3$) (0, 0).
Use a column vector to describe the translation
(a) T → D **(b)** T → E

13 On graph paper, show the image of triangle T following the translation
(a) $\begin{pmatrix} 2 \\ 1 \end{pmatrix}$ **(b)** $\begin{pmatrix} ^{-}3 \\ 0 \end{pmatrix}$ **(c)** $\begin{pmatrix} ^{-}1 \\ ^{-}4 \end{pmatrix}$

14 Find the coordinates of the image of T following:
(a) reflection in the line $x = 2$
(b) reflection in the line $y = 1$
(c) reflection in the line $y = x$

15 On graph paper, draw triangle T and the triangle F (2, 2) (4, 2) (4, 8).
(a) Describe the transformation T → F.
(b) Find the area of T and F. How much larger is the area of F than the area of T?

16



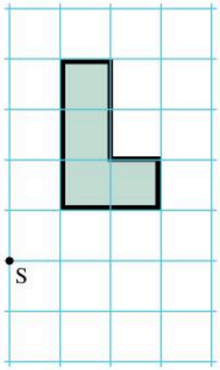
In the diagram above:

(a) State which two triangles are similar.

(b) Calculate the lengths

(i) BD **(ii)** CE.

17 On squared paper, make a copy of this L-shape made from 4 squares.



Draw an enlargement of the shape using S as the centre of enlargement and a scale factor of 2. How many squares are needed to make the enlarged L-shape?

18 X has the coordinates ($^-3$, 2) and Y has coordinates (4, $^-3$).

Write down:

(a) the position vectors $\overrightarrow{OX}$ and $\overrightarrow{OY}$

(b) the vector $\overrightarrow{XO}$

(c) the vector $\overrightarrow{XY}$

(d) the position vector of M if M is the midpoint of $\overrightarrow{XY}$.

19 The coordinates of the square ABCD are (0, 0), (2, 0), (2, 2), (0, 2) respectively.

Determine:

(a) the vector $\overrightarrow{BD}$

(b) the magnitude of $\overrightarrow{BD}$.

20 On squared paper, draw the triangle R ($^-2$, 1) ($^-2$, 3) ($^-1$, 3).

(a) Draw the triangle S, the image of R following a reflection in the *y*-axis.

(b) Draw the triangle T, the image of S following a reflection in the line $x = 2$.

(c) Describe the single transformation R → T.

Statistics and probability

21 These are the lengths (cm) of each specimen of a catch of fifteen fish. Put the numbers in order to find the mode and median length of the batch.

28, 38, 40, 31, 30, 38, 32, 35
29, 30, 32, 37, 38, 38, 39

22 Calculate the mean value of these ten numbers:

7, 3, 2, 3, 5, 4, 6, 2, 1, 8

23 A rowing eight and cox are weighed before a race. Here are the results:

81 73 84 89 79 76 88 75 57 (kg)

Find the median weight and calculate the mean weight of the crew.

24 Mary carries out a letter count of words in a book. The lengths of the first 40 words were:

5 6 2 1 6 2 5 7 3 4
4 3 4 2 5 4 2 3 7 2
7 6 5 2 1 6 2 3 7 5
4 1 6 2 6 5 4 1 3 5

Construct a frequency table and use it to find the mode and the mean word length.

25 These are the heights (cm) of 25 children.

140, 155, 157, 142, 167, 153, 163
154, 154, 152, 145, 152, 153, 143
158, 143, 157, 159, 147, 161, 163
147, 159, 161, 152

(a) Draw up a frequency table of this information using intervals 140–144, 145–149, 150–154, etc.

(b) Use your frequency table to draw a histogram.

(c) What is the mode interval?

26 Five whole numbers have the properties:

range 9 median 14
mode 15 mean 12

What are the five numbers?

27 A traffic survey counted the number of occupants of 40 cars:

Occupants	Frequency
1	23
2	7
3	4
4	5
5	1

Calculate the mean number of occupants per car.

28 The mean age of a group of 10 children is 12 years 6 months. They are joined by two more children aged 11 and 8. Find the mean age of the group of 12 children.

29 A coin is thrown three times.

(a) List all the possible outcomes.

(b) What is the probability that three heads are thrown?

30 A bag contains 6 red, 4 green and 8 yellow marbles. A marble is drawn at random from the bag. What is the probability that the marble is:

(a) red **(b)** yellow

(c) green **(d)** not green?

31 The table shows the number of children who walk to school from a certain class.

	Walk	Do not walk
Boys	11	5
Girls	8	4

(a) How many children are in the class?

(b) What is the probability that a child picked at random

  (i) walks to school

  (ii) is a boy

  (iii) is a girl who does not walk to school?

Consumer arithmetic

32 Copy and complete this table.

Fraction	Decimal	Percentage
$\frac{3}{4}$		
	0.1	
		$66\frac{2}{3}\%$
		$12\frac{1}{2}\%$
$1\frac{1}{5}$		

33 Calculate:

(a) 10% of \$65

(b) 25% of \$170

(c) $12\frac{1}{2}\%$ of \$1600

(d) 125% of \$60

(e) $\frac{1}{2}\%$ of \$5

34 Use the Simple Interest formula $I = \frac{PTR}{100}$ to calculate the total interest on a loan of:

(a) \$600 at 15% p.a. for 2 years

(b) \$1000 at $12\frac{1}{2}\%$ p.a. for 3 years

(c) \$4000 at 13% p.a. for 5 years

35 \$1000 is deposited in a savings account and earns simple interest of 8% p.a. How long does it take for the total to become \$1500?

36 **(a)** Find the compound interest on a loan of \$15 000 when taken for a period of 2 years, given an interest rate of 11%.

(b) What would be a suitable monthly repayment plan?

37 Gretta Cumberbatch's salary slip is shown below.

Gretta Cumberbatch **Position: Executive Officer** **Month: September**			
Earnings		**Deductions**	
Basic	6400	Loan payment	2000
Travel	1200	Tax	900
Total	7600	Total	2900

(a) What is Gretta's net pay for September?

(b) What is her basic gross annual salary?

(c) What non-statutory deductions does she have?

38 Jackson is paid \$15.80 per hour for a 40-hour basic week. Overtime rates are as follows:

Normal overtime – time and a half
Sunday work – double time.

Calculate:
- **(a)** Jackson's weekly pay if he works for 42 hours, no Sunday work.
- **(b)** Jackson's weekly pay if he works 40 hours plus 6 hours on Sunday.

39 A sales executive is paid a 3% commission on sales above \$50 000 in a month.
- **(a)** Find his commission if he has sales of \$150 000 in a month.
- **(b)** What were his sales if he received \$600 commission in a particular month?

40 Water rates are

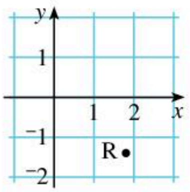
Gallons	**Cost / 1000 gallons**
up to 1000	\$35
1000 – 5000	\$45
5000 +	\$55

- **(a)** Calculate the water bill for a person who uses:
  - **(i)** 648 gallons
  - **(ii)** 3000 gallons
  - **(iii)** 6918 gallons.
- **(b)** How many gallons of water were used if the bill was \$170?

41 A dumper truck valued at \$140 000 depreciates by 20% in value each year. Find its value after
(a) 1 year **(b)** 2 years **(c)** 3 years.

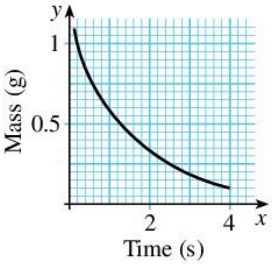
Mixed questions 3

1



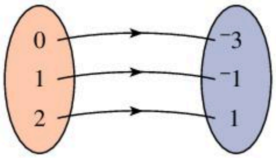
The coordinates of R are
A $(1.8, {}^{-}1.2)$ **B** $({}^{-}1.4, 1.8)$
C $({}^{-}1.2, 1.8)$ **D** $(1.8, {}^{-}1.4)$

2 The graph shows how the mass of certain radioactive material varies with time.
The mass after 2.5 secs is



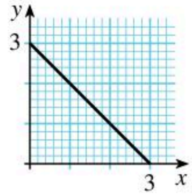
A 0.2 g **B** 0.25 g **C** 0.3 g **D** 0.35 g

3



The arrow graph shows the mapping
A $x \rightarrow x - 3$ **B** $x \rightarrow 2x - 3$
C $x \rightarrow 3x - 3$ **D** $x \rightarrow 3x - 4$

4

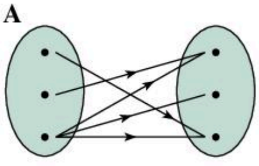


The equation of the line is
A $x = 3$ **B** $y = 3$
C $x + y = 3$ **D** $x - y = 3$

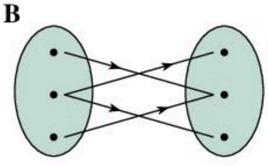
5 Find the value of $f({}^{-}3)$ if $f : x \rightarrow 3 - 4x$
A ${}^{-}9$ **B** ${}^{-}1$ **C** 1 **D** 15

6 Which of the mapping diagrams shows a function?

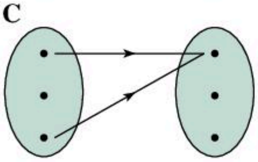
A



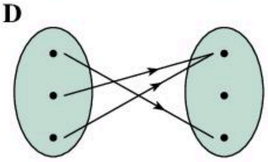
B



C



D

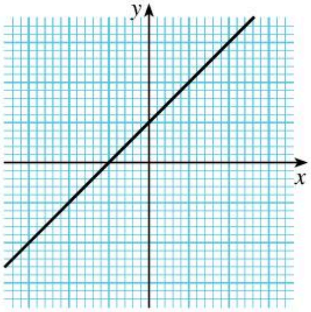


7 The points P(2, 3), Q(10, $^{-}1$) are joined with a line. Find the gradient of the line segment PQ.

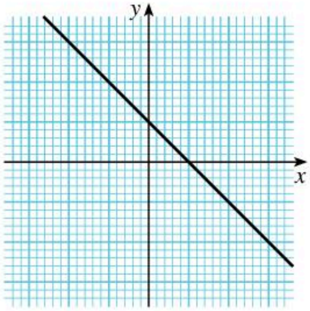
A $^{-}2$ **B** $\frac{-1}{2}$ **C** $\frac{1}{2}$ **D** 2

8 Which graph best illustrates the function $f(x) = ax^2 + b$

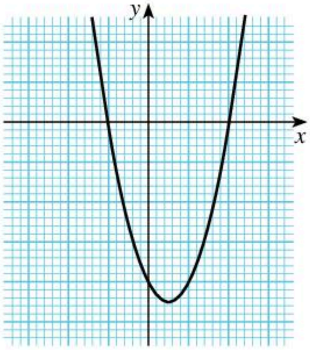
A



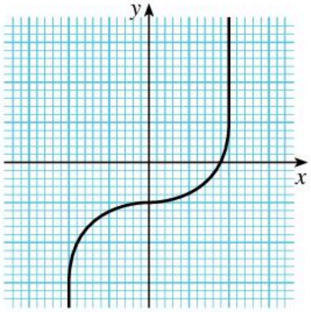
B



C



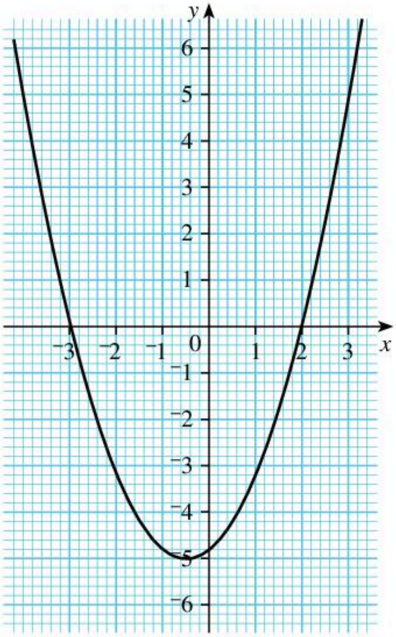
D



9 What is the gradient of the line represented by the equation $2y = 3x - 2$.

A $^{-}2$ **B** $\frac{2}{3}$ **C** $\frac{3}{2}$ **D** 3

10 The diagram shows the graph of $y = x^2 + x - 6$.



What values of x satisfy the equation $x^2 + x - 6 = 0$?

A 3, $^{-}2$ **B** 0, $^{-}6$
C 2, $^{-}5$ **D** $^{-}3$, 2

11 The image of (3, 4) reflected in the x-axis is

A ($^{-}3$, $^{-}4$) **B** (3, $^{-}4$)
C ($^{-}3$, 4) **D** (4, 3)

12 The image of (0, 2) reflected in the line $x = 2$ is

A ($^{-}2$, $^{-}2$) **B** (0, $^{-}2$)
C (2, 2) **D** (4, 2)

13 Under an enlargement from the origin with scale factor $\frac{1}{3}$, the point (6, $^{-}3$) maps to the point

A (18, $^{-}9$) **B** (2, $^{-}1$)
C ($^{-}2$, 1) **D** ($^{-}18$, 9)

14 The property of a triangle which always remains the same under an enlargement is

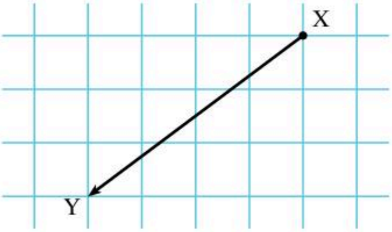
A Area
B The size of an angle
C The length of a side
D Its position on the grid

15 Write down the coordinates of the reflection of the point (1, 2) in the line $y = {}^{-}x$.

A $(^{-}2, 1)$ **B** $(1, {}^{-}2)$

C $(^{-}2, {}^{-}1)$ **D** $(^{-}1, {}^{-}2)$

16

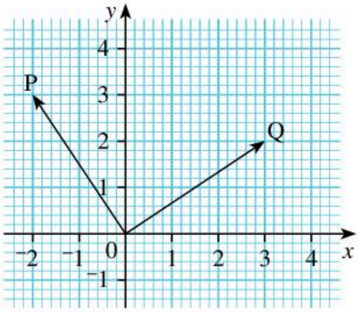


The magnitude of the vector $\overrightarrow{XY}$.

A 3 units **B** 4 units

C 5 units **D** 7 units

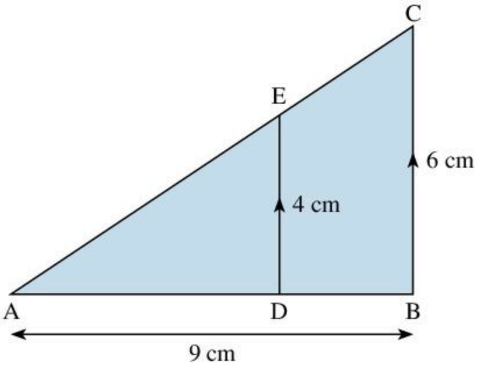
17



The diagram shows the position vectors $\overrightarrow{OP}$ and $\overrightarrow{OQ}$ of the points P and Q. What is the vector $\overrightarrow{PQ}$?

A $\begin{pmatrix} ^{-}2 \\ 3 \end{pmatrix}$ **B** $\begin{pmatrix} ^{-}5 \\ 1 \end{pmatrix}$ **C** $\begin{pmatrix} 5 \\ ^{-}1 \end{pmatrix}$ **D** $\begin{pmatrix} 3 \\ 2 \end{pmatrix}$

18



In triangle ABC, AB = 9 cm and BC = 4 cm. E and D are two points on AC and AB respectively such that ED = 4 cm and ED is parallel to BC. What is the length AD?

A 4 cm **B** 5 cm **C** 6 cm **D** 7 cm

19 A triangle, ABC, with area 2 cm² is enlarged by a scale factor 2 centre (0, 0). What is the area of its image A′B′C′?

A 1 cm² **B** 2 cm² **C** 4 cm² **D** 8 cm²

20 Which of these vectors has magnitude 13?

A $\begin{pmatrix} 6 \\ 7 \end{pmatrix}$ **B** $\begin{pmatrix} 10 \\ ^{-}3 \end{pmatrix}$ **C** $\begin{pmatrix} ^{-}5 \\ 8 \end{pmatrix}$ **D** $\begin{pmatrix} 5 \\ ^{-}12 \end{pmatrix}$

Use the numbers 83, 78, 96, 95, 78 to answer Questions 21–24.

21 The mean of the numbers is

A 18 **B** 78 **C** 83 **D** 86

22 The median of the numbers is

A 18 **B** 78 **C** 83 **D** 86

23 The mode of the numbers is

A 18 **B** 78 **C** 83 **D** 86

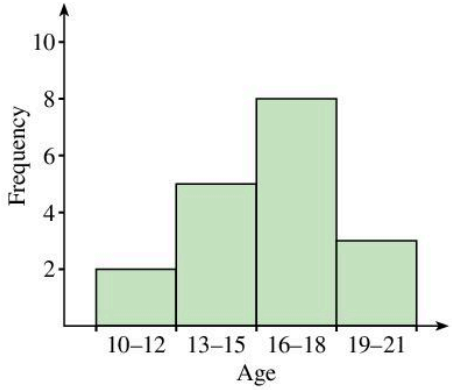
24 The range of the numbers is

A 18 **B** 78 **C** 83 **D** 86

25 The total of 25 numbers is 575, and the total of 5 other numbers is 85. The mean of the 30 numbers is

A 20 **B** 22 **C** 40 **D** 44

Use this histogram to answer Questions 20–22.



26 How many people were in the survey?

A 21 **B** 18 **C** 8 **D** 4

27 What was the modal age?

A 8 **B** 16–18 **C** 10–12 **D** 15

28 What is the probability that a person chosen at random is aged 16–18?

A $\frac{4}{9}$ **B** $\frac{1}{6}$ **C** $\frac{1}{8}$ **D** $\frac{1}{9}$

29 A bag has 5 red, 3 blue and 2 yellow beads. What is the probability that a bead chosen at random is blue?

A 3 **B** $\frac{3}{5}$ **C** $\frac{2}{3}$ **D** $\frac{3}{10}$

30 The outcomes when three coins are thrown together are {HHH, HHT, HTH, THH, TTH, THT, HTT, TTT} where H is a head and T is a tail. What is the probability of throwing two heads?

A $\frac{1}{4}$ **B** $\frac{3}{8}$ **C** $\frac{1}{2}$ **D** $\frac{3}{4}$

31 What is $3\frac{1}{2}\%$ of $60?

A $0.90 **B** $2.10 **C** $9.00 **D** $21.00

32 Myrtle buys a walkman for $240 and sells it for $180. What is her percentage loss?

A 25% **B** $33\frac{1}{3}\%$ **C** 50% **D** 60%

33 What is the simple interest on $250 when invested in a bank at a rate of 5% for 3 years?

A $3.75 **B** $15 **C** $37.50 **D** $150

34 David puts $600 in his bank. What is the bank's interest rate if after 3 years he receives $81 in simple interest?

A 2% **B** $3\frac{1}{2}\%$ **C** $4\frac{1}{2}\%$ **D** 27%

35 Over Christmas a store gives a 5% discount on all items. What would be the cash price of an iron marked $120?

A $6 **B** $114 **C** $115 **D** $126

36 The rate of interest at a lending institution dropped from 4% to $3\frac{1}{2}\%$. What is the difference in the annual interest on a deposit of $4000?

A $20 **B** $40 **C** $140 **D** $160

37 Which is the better investment on $6000?

A 4% simple interest for 3 years
B 2% simple interest for 5 years
C 4% compound interest for 3 years
D 2% compound interest for 5 years

38 Bently is paid $13 per hour for a 40-hour basic week. He is paid $16 for each overtime hour. How many hours does he work if his weekly pay packet is $664?

A 33 **B** 41 **C** 47 **D** 49

39 Electricity charges in a certain country are:

Fixed charges : $10
First 50 units : 15c per unit
More than 50 units : 18c per unit

What is Ann's bill if she uses 160 units?

A $17.50 **B** $26.10 **C** $36.10 **D** $37.30

40 Which is true?

Gross pay is pay

A after statutory deductions
B after all deductions
C before statutory deductions
D before all deductions.

Glossary

1 Computation

Significant figures Significant figures show the relative importance of the digits in a number. The most important digit is the first non-zero number.

For example:

	785 200	0.003 186
3 sig. fig.	785 000	0.003 19
2 sig. fig.	790 000	0.0032
1 sig. fig.	800 000	0.003

Standard form Standard form or scientific notation is a way of displaying a number as a product of a number lying between 1 and 10 and a power of ten.

For example: $785\,200 = 7.852 \times 10^5$

$0.003\,186 = 3.186 \times 10^{-3}$

2 Computation 2

Matrix A rectangular array of numbers.

For example: $\begin{pmatrix} 1 & ^{-}2 \\ 0 & 4 \end{pmatrix}, \begin{pmatrix} 2 & 3 \\ 4 & 2 \\ 0 & 1 \end{pmatrix}$

3 Measurement 1

Arc Any piece of a curve of a circle.

Circumference The distance around a circle.

Perimeter The distance around a closed shape.

Sector A sector of a circle is a shape enclosed by an arc and two radii of the circle.

Trapezium A quadrilateral with one pair of parallel sides.

4 Ratio and proportion

Ratio Ratio is used to compare the size of two quantities.

Binary A number system using base 2.

5 Algebra

Distributive law States that multiplication is distributed over addition as follows $a \times (b + c) = a \times b + a \times c$

Like terms Terms that are identical with respect to their variables.

For example: Pair of like terms: $6x$ and $2x$

$6x^3y$ **and** $2x^3y$

Simultaneous equations Equations involving two or more unknowns that take the same values in each equation.

Quadratic equations Equations involving expressions of the second degree.

For example: $x^2 = 4$

$x^2 + 3x = 2$

6 Geometry

Polygon A closed shape with three or more edges.

Similar triangles Triangles with the same shape but different sizes.

Congruent triangles Triangles that have the same shape and size.

7 Trigonometry

Angle of depression The angle through which someone must look down from the horizontal to see an object.

Angle of elevation The angle through which someone must look up from the horizontal to see an object.

Hypotenuse The longest side of a right-angled triangle.

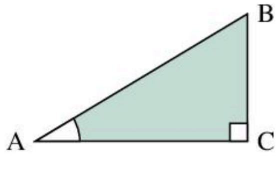
Pythagoras' theorem The relationship between the three sides, a, b and c of a right-angled triangle, stating that $a^2 + b^2 = c^2$ where c is the hypotenuse of the triangle.

Trigonometric ratios The relationships between two sides and an angle in a right-angled triangle.

$\tan \mathrm{A} = \dfrac{\mathrm{BC}}{\mathrm{AC}}$

$\sin \mathrm{A} = \dfrac{\mathrm{BC}}{\mathrm{AB}}$

$\cos \mathrm{A} = \dfrac{\mathrm{AC}}{\mathrm{AB}}$



Trigonometry The study of the relationships between sides and angles of a triangle.

8 Measurement 2

Cylinder A solid with two identical and parallel faces in the form of a circle.

Prism A solid with two identical and parallel faces in the form of a circle.

Surface area The area of the total surface of a solid.

9 Functions and graphs

Arrow diagram A diagram that links some or all elements in one set to the elements of a second set by means of a stated rule.

Function A one-to-one or many-to-one mapping.

Gradient The steepness or slope of a line.

Intercept An intercept of a straight-line graph is the point where it cuts an axis.

Many-to-one map A mapping in which one more than one element of the first set is sent to an element in the second set.

Mapping A mapping is a rule that sends every member of the first set to an element in the second set.

One-to-one map A mapping in which each element of the first set is sent to just one element of the second set.

Ordered pair Two numbers written so as to give the position of a point on a graph.

10 Transformations

Congruent Two shapes are congruent to each other if they have the same size and shape.

Enlargement A transformation that makes a shape larger or smaller but retains the same shape.

Position vector The position vector of a point is the vector formed from the origin to that point.

Similar Two shapes are similar if they have the same shape but are different in size.

Vector A quantity with both magnitude and direction.

For example: A translation, velocity

11 Statistics and probability

Chance Possibility due to favourable circumstances.

Experimental probability The probability that an event will occur as determined by experiment. It is the ratio of the number of times an event occurs to the total number of trials.

Histogram A form of bar chart used for continuous data. It usually takes the form of upright bars, with the area of each bar representing frequency.

Mean The mean value of a set of data is found by adding all the separate values of the data and dividing their sum by the number of data values.

Probability The likelihood of an event occurring.

12 Consumer arithmetic

Commission A percentage paid to a person or institution that provides goods or services.

Compound interest Interest that is calculated on the principal and previously paid interest.

Gross pay Pay before deductions.

Net pay Pay after deductions.

Statutory deductions Deduction of pay by law, for example, income tax.

Interest The extra amount paid or fee charged for the use of someone's money.

Principal The amount borrowed or deposited.

Answers

1 Computation 1

Check in

1 **(a)** 23 **(b)** 2.3 **(c)** 230 **(d)** 23 **(e)** 16.1 **(f)** 161
2 **(a)** 2 **(b)** 0.2 **(c)** 0.24 **(d)** 0.024
(e) 1.31 **(f)** 0.131

Exercise 1A

1 **(a)** $\frac{1}{2}$ **(b)** $\frac{3}{5}$ **(c)** $\frac{1}{4}$ **(d)** 1 **(e)** $\frac{2}{9}$ **(f)** 0
(g) 1 **(h)** $\frac{4}{5}$
2 **(a)** $\frac{3}{4}$ **(b)** $\frac{5}{6}$ **(c)** $\frac{13}{15}$ **(d)** $\frac{37}{56}$ **(e)** $\frac{5}{9}$ **(f)** $\frac{7}{18}$
(g) $\frac{50}{63}$ **(h)** $\frac{61}{72}$
3 **(a)** $\frac{7}{20}$ **(b)** $\frac{3}{8}$ **(c)** $\frac{13}{63}$ **(d)** $\frac{11}{56}$ **(e)** $\frac{1}{4}$
(f) $\frac{1}{4}$ **(g)** $\frac{2}{9}$ **(h)** $\frac{3}{28}$
4 **(a)** $5\frac{7}{12}$ **(b)** $2\frac{1}{4}$ **(c)** $9\frac{5}{12}$ **(d)** $2\frac{11}{35}$ **(e)** $4\frac{5}{6}$
(f) $11\frac{1}{10}$ **(g)** $2\frac{11}{12}$ **(h)** $3\frac{9}{20}$
5 $1\frac{7}{12}$ kg **6** $2\frac{3}{8}$ m **7** **(a)** $\frac{19}{21}$ **(b)** $\frac{2}{21}$
8 **(a)** $3\frac{5}{8}$ litres **(b)** $\frac{7}{36}$

Exercise 1B

1 **(a)** $1\frac{1}{2}$ **(b)** $\frac{1}{4}$ **(c)** $\frac{3}{8}$ **(d)** $\frac{1}{2}$ **(e)** $\frac{7}{20}$ **(f)** $\frac{3}{7}$
(g) $\frac{8}{33}$ **(h)** $\frac{1}{2}$
2 **(a)** $4\frac{1}{2}$ **(b)** $2\frac{1}{4}$ **(c)** $2\frac{13}{16}$ **(d)** $9\frac{1}{6}$ **(e)** $6\frac{1}{4}$ **(f)** $8\frac{29}{32}$
(g) $15\frac{5}{6}$ **(h)** $19\frac{7}{11}$
3 **(a)** 2 **(b)** $1\frac{1}{2}$ **(c)** 3 **(d)** $1\frac{1}{6}$ **(e)** $2\frac{3}{16}$
(f) $\frac{15}{16}$ **(g)** $1\frac{5}{22}$ **(h)** $\frac{3}{8}$
4 **(a)** $\frac{1}{2}$ **(b)** 2 **(c)** $2\frac{5}{8}$ **(d)** $1\frac{11}{21}$ **(e)** $2\frac{16}{25}$ **(f)** $2\frac{5}{13}$
(g) $2\frac{23}{90}$ **(h)** $\frac{9}{16}$
5 **(a)** $1\frac{1}{5}$ **(b)** $\frac{50}{63}$ **(c)** $\frac{13}{54}$ **(d)** $\frac{1}{24}$
6 $\frac{1}{9}$ kg **7** $12\frac{4}{9}$ m **8** **(a)** 219 **(b)** $\frac{433}{658}$

Exercise 1C

1 **(a)** 0.5 **(b)** 9.5 **(c)** 3.6 **(d)** 30.6 **(e)** 13.7 **(f)** 8.1
(g) 7.7 **(h)** 11.65
2 **(a)** 11.85 **(b)** 93.2 **(c)** 20.05 **(d)** 53.03 **(e)** 5.45
(f) 1.983 **(g)** 78.81 **(h)** 42.89
3 **(a)** 18.68 **(b)** 20.35 **(c)** 72.835 **(d)** 10.66 **(e)** 49.84
4 **(a)** 0.62 **(b)** **(i)** 0.81 s **(ii)** 0.02 s
5 **(a)** 10.8 mm **(b)** 1.2 mm
6 **(a)** 2.73 **(b)** 14.9 **(c)** 0.569

Exercise 1D

1 **(a)** 12.88 **(b)** 128.8 **(c)** 1.288 **(d)** 12.88
(e) 0.012 88 **(f)** 1.288 **(g)** 12.88 **(h)** 1.288 m
2 **(a)** 1.8 **(b)** 0.78 **(c)** 7.05 **(d)** 2.66 **(e)** 55.62
(f) 87.6 **(g)** 1.69 **(h)** 23.606
3 **(a)** 0.12 **(b)** 0.64 **(c)** 5.58 **(d)** 0.093 **(e)** 0.574
(f) 2.448 **(g)** 4.991 **(h)** 3.0008
4 **(a)** 15.64 m^2 **(b)** 160.19 m^2 **(c)** 87.9248 m^2
5 **(a)** \$517.80 **(b)** \$263.22 **(c)** \$17.26
6 **(a)** 21 km **(b)** 5.46 km **(c)** 2.52 km
7 **(a)** 0.75 **(b)** 4.96 **(c)** 0.24 **(d)** 0.504 **(e)** 3.12
(f) 2.914 **(g)** 13.803 **(h)** 11.696

Exercise 1E

1 **(a)** 40 **(b)** 20 **(c)** 2.1 **(d)** 3.48 **(e)** 30 **(f)** 300
(g) 30 **(h)** 200
2 **(a)** 17 **(b)** 16 **(c)** 160 **(d)** 1.6 **(e)** 26.8 **(f)** 59.2
(g) 4.56 **(h)** 5.6
3 135 **4** 153
5 **(a)** 7 m **(b)** 9 m **(c)** 16 m **(d)** 288 m
6 \$15.50
7 **(a)** 40 **(b)** 17 **(c)** 5 **(d)** 30 **(e)** 0.4 **(f)** 0.48
(g) 190 **(h)** 60

Exercise 1F

1 **(a)** 0.5 **(b)** 0.6 **(c)** 0.65 **(d)** 0.375 **(e)** 0.24
(f) 0.68 **(g)** 0.875 **(h)** 0.1875 **(i)** 0.8125 **(j)** 0.417
3 **(a)** 0.666 . . . **(b)** 0.166 . . . **(c)** 0.444 . . .
(d) 0.272 . . . **(e)** 0.714 . . . **(f)** 0.315 . . . **(g)** 0.923 . . .
(h) 0.823 . . . **(i)** 0.242 . . . **(j)** 0.860 . . .

Exercise 1G

1 **(a)** $\frac{1}{4}$ **(b)** $\frac{3}{10}$ **(c)** $\frac{1}{20}$ **(d)** $\frac{17}{20}$ **(e)** $\frac{12}{25}$ **(f)** $\frac{3}{100}$
(g) $\frac{91}{100}$ **(h)** $\frac{83}{100}$
2 **(a)** $\frac{1}{8}$ **(b)** $\frac{7}{8}$ **(c)** $\frac{1}{16}$ **(d)** $\frac{27}{400}$ **(e)** $\frac{2}{3}$
(f) $\frac{1}{9}$ **(g)** $\frac{121}{250}$ **(h)** $\frac{38}{175}$ **(i)** $\frac{4}{175}$ **(j)** $\frac{549}{550}$
3 **(a)** 50% **(b)** 75% **(c)** 80% **(d)** 30% **(e)** 68%
(f) $87\frac{1}{2}\%$ **(g)** $66\frac{2}{3}\%$ **(h)** $83\frac{1}{3}\%$ **(i)** $27\frac{3}{11}\%$
(j) $93\frac{1}{3}\%$ **(k)** $76\frac{8}{17}\%$ **(l)** $65\frac{5}{23}\%$ **(m)** $10\frac{10}{29}\%$
(n) $82\frac{34}{63}\%$ **(o)** $4\frac{96}{101}\%$

Exercise 1H

1 **(a)** $\frac{13}{20}$ **(b)** 65% **2** **(a)** $\frac{3}{20}$ **(b)** 0.15
3 **(a)** 0.25 **(b)** 0.3 **(c)** 0.78 **(d)** 0.85
(e) 0.025 **(f)** 0.4275
4 **(a)** 50% **(b)** 65% **(c)** 37% **(d)** 47.5%
(e) 39.5% **(f)** 86.25%

Exercise 1I

1 **(a)** 6.1 **(b)** 6.3 **(c)** 6.7 **(d)** 6.4 **(e)** 6.9 **(f)** 7.0
(g) 6.4 **(h)** 6.8 **(i)** 6.1
2 **(a)** 3.2 **(b)** 4.4 **(c)** 0.8 **(d)** 0.8 **(e)** 0.7 **(f)** 5.0
(g) 6.0 **(h)** 7.0 **(i)** 1.2
3 **(a)** 6.23 **(b)** 0.78 **(c)** 0.97 **(d)** 4.03 **(e)** 4.20
(f) 6.11 **(g)** 4.00 **(h)** 0.03 **(i)** 0.21
4 **(a)** **(i)** 6.418 **(ii)** 6.42 **(iii)** 6.4 **(b)** **(i)** 0.706 **(ii)** 0.71
(iii) 0.7 **(c)** **(i)** 0.022 **(ii)** 0.02 **(iii)** 0.0
(d) **(i)** 9.601 **(ii)** 9.60 **(iii)** 9.6 **(e)** **(i)** 7.181 **(ii)** 7.18
(iii) 7.2 **(f)** **(i)** 0.314 **(ii)** 0.31 **(iii)** 0.3
(g) **(i)** 6.300 **(ii)** 6.30 **(iii)** 6.3 **(h)** **(i)** 4.082
(ii) 4.08 **(iii)** 4.1

5

Number	1 d.p.	2 d.p.	3 d.p.
3.1674	3.2	3.17	3.167
0.0396	0.0	0.04	0.040
7.8074	7.8	7.81	7.807
6.0782	6.1	6.08	6.078
105.1648	105.2	105.16	105.165
94.0718	94.1	94.07	94.072
1.2309	1.2	1.23	1.231
6.7071	6.7	6.71	6.707

6 **(a)** **(i)** 21.78 cm **(ii)** 21.8 cm **(b)** **(i)** 29.19 cm^2 **(ii)** 29.2 cm^2

Exercise 1J

1 **(a)** 400 **(b)** 8000 **(c)** 400 **(d)** 50 **(e)** 5 **(f)** 90 000 **(g)** 10 000 **(h)** 0.003 **(i)** 7
2 **(a)** 960 **(b)** 500 **(c)** 6200 **(d)** 18 000 **(e)** 17 **(f)** 30 **(g)** 0.0039 **(h)** 130 **(i)** 8.0

3

Number	613 752	1.6831	8769.2
3 s.f.	614 000	1.68	8770
2 s.f.	610 000	1.7	8800
1 s.f.	600 000	2	9000

4 **(a)** **(i)**, **(ii)** **(b)** **(iii)** 800 000 **(iv)** 70 **(v)** 950

5 **(a)** 8 135 000 **(b)** 8 130 000 **(c)** 8 100 000 **(d)** 8 000 000

6 **(a)** 5499.99 **(b)** 4500

Exercise 1K

1 **(a)** 0.7 **(b)** 0.9 **(c)** 0.07 **(d)** 0.07 **(e)** 0.1 **(f)** 0.009 **(g)** 0.006 **(h)** 0.0002 **(i)** 0.0009
2 **(a)** 0.67 **(b)** 0.92 **(c)** 0.78 **(d)** 0.094 **(e)** 0.011 **(f)** 0.0037 **(g)** 0.0066 **(h)** 0.0010 **(i)** 0.032
3 **(a)** 0.141 **(b)** 0.389 **(c)** 0.963 **(d)** 0.0398 **(e)** 0.0467 **(f)** 0.0212 **(g)** 0.000726 **(h)** 0.00608 **(i)** 0.0909

4

Numbers	1 s.f.	2 s.f.	3 s.f.
0.42846	0.4	0.43	0.428
0.09626	0.09	0.096	0.0963
0.07207	0.07	0.072	0.0721
0.009966	0.01	0.010	0.00997
0.0007109	0.0007	0.00071	0.000711

5 **(a)** **(ii)** and **(v)**
(b) **(i)** 0.06 **(ii)** 0.062 **(iv)** 0.00089
6 **(a)** 0.694999 **(b)** 0.685
7 0.08470 implies 4 s.f.
8 Suitable answers 8000 (nearest thousand) or 8500 (nearest 500)
9 Students' own research
10 **(a)** 0.03 m **(b)** 0.0152 m **(c)** 0.033 m **(d)** 160 inches **(e)** 100 inches

Exercise 1L

1 **(a)** 6 **(b)** 50 or 60 **(c)** 20 **(d)** 50 **(c)** 42 **(f)** 16
2 **(a)** 64 **(b)** 54 **(c)** 80 **(d)** 14 **(e)** 0.01 **(f)** 0.009 **(g)** 0.003 **(h)** 0.0056

Exercise 1M

1 **(a)** 4 **(b)** 2 **(c)** 4 **(d)** 3 **(e)** 8 **(f)** 8
2 **(a)** 2 **(b)** 0.7 **(c)** 0.5 **(d)** 1 **(e)** 60 **(f)** 0.03
4 **(a)** 51.28 **(b)** 0.86 **(c)** 0.08 **(d)** 1316.54
5 300 people/km^2 **7** **(a)** 9 **(b)** **(i)** 9.31 **(ii)** 9.3

Exercise 1N

1 **(a)** 32 **(b)** 81 **(c)** $4 \times 4 \times 4$ **(d)** 2^4 **(e)** 10^3
2 **(a)** 9 **(b)** 216 **(c)** 125 **(d)** 256 **(e)** 256 **(f)** 1296
3 **(a)** 81 **(b)** 64 **(c)** 625 **(d)** 16807 **(e)** 512 **(f)** 6561
4 **(a)** 10^3 **(b)** 6^2 **(c)** 4 **(d)** 17 **(e)** 6^3 **(f)** 2^{-4}

Exercise 1O

1 **(a)** 3 **(b)** 4^2 **(c)** 6^{-1} **(d)** 2^{-3} **(e)** 4^{-1} **(f)** 7^{-1}
2 **(a)** 2^7 **(b)** 2^6 **(c)** 3^4 **(d)** 3^{-6} **(e)** 6 **(f)** 7^{-1}
3 **(a)** 5^0
4 **(a)** 1 **(b)** $\frac{1}{3}$ **(c)** $\frac{1}{9}$ **(d)** $\frac{1}{343}$
5 **(a)** 5^{-1} **(b)** 5^{-2} **(c)** 2^{-3} **(d)** 10^{-3}

Exercise 1P

1 **(a)** 2 **(b)** 4 **(c)** 9 **(d)** $\frac{1}{2}$ **(e)** $\frac{1}{4}$ **(f)** $\frac{1}{9}$
2 **(a)** 8 **(b)** 125 **(c)** 4 **(d)** $\frac{1}{8}$ **(e)** $\frac{1}{125}$ **(f)** $\frac{1}{4}$
3 **(a)** $\frac{1}{5}$ **(b)** 729 **(c)** $\frac{1}{1000}$ **(d)** $\frac{1}{64}$ **(e)** $\frac{1}{32}$ **(f)** 32

Exercise 1Q

1 **(b)** 1.6×10^4 **(c)** 5.6×10^3 **(d)** 2×10^5 **(e)** 5.31×10^5
2 **(a)** 2×10^3 **(b)** 4×10^2 **(c)** 8×10 **(d)** 9×10^4 **(e)** 4×10^3 **(f)** 7×10^5 **(g)** 3×10^6 **(h)** 4×10^7 **(i)** 1×10^5
3 **(a)** 300 **(b)** 5000 **(c)** 600 000 **(d)** 20 000 000 **(e)** 3000 000 **(f)** 4 000 000 000
4 **(a)** 4.2×10^2 **(b)** 6.3×10^3 **(c)** 1.7×10^5 **(d)** 2.3×10^4 **(e)** 6.13×10^4 **(f)** 9.23×10^3 **(g)** 4.16×10^2 **(h)** 9.8185×10^4 **(i)** $6.310\,04 \times 10^6$
5 **(a)** 1600 **(b)** 280 **(c)** 381 **(d)** 475 000 **(e)** 301 000 000 **(f)** 1 600 000 000

Exercise 1R

1 **(a)** 0.06 **(b)** 0.4 **(c)** 0.008 **(d)** 0.0005 **(e)** 0.000 002 **(f)** 0.000 07

2 **(a)** 4×10^{-2} **(b)** 5×10^{-3} **(c)** 9×10^{-4} **(d)** 3.1×10^{-2} **(e)** 8.35×10^{-3}

3 **(a)** 6.3×10^{-1} **(b)** 7.4×10^{-3} **(c)** 2.8×10^{-2} **(d)** 1.3×10^{-4} **(e)** 2.356×10^{-2} **(f)** 8.2×10^{-4} **(g)** 3.91×10^{-6} **(h)** 1.6×10^{-3} **(i)** 3.83×10^{-3}
4 **(a)** 3.06×10^{-5} **(b)** 4.925×10^{-7} **(c)** 4.2831×10^{-3} **(d)** 9.01×10^{-4} **(e)** 2.5×10^{-10} **(f)** 8.46×10^{-8} **(g)** 8.6×10^{-8} **(h)** 3.675×10^{-12}

5 **(a)** 1.7×10^{-4} **(b)** 1.7×10^{-5} **(c)** 10^{-10} **(d)** 6.9×10^{-8}

Exercise 1T

1 **(a)** 58.42 cm **(b)** 0.11 feet **(c)** 1.25 kg **(d)** 38.17 pints
(e) 9.37 litres
2 **(a)** 3 kg **(b)** 1.38 kg **(c)** 925 g **(d)** 60 ml
3 9 **4** $7.31
5 **(a)** 23.465 m^2 **(b)** $698.08
6 **(a)** 0.362 litres **(b)** $116.56
7 $45.99
8 **(a)** 1 077 386 400 km/h
(b) 9 444 369 182 000 km (using 365.25 days/y)
(c) 160 554 276 100 000 km
9 **(a)** $\frac{13}{15}$ **(b)** $86\frac{2}{3}$ **10** **(a)** 28.4% **(b)** 58.6%
11 **(a)** $\frac{13}{105}$ **(b)** 12.4% **12** **(a)** $\frac{151}{712}$ **(b)** 21.2%
13 **(a)** 149.637 million km **(b)** 940.197 million km
(c) 2.57 million km **(d)** 107254.9 km/h **(e)** 29.8 km/s
14 **(a)** $600 000 **(b)** 92.8% **(c)** $12145 **(d)** $342.72
(e) 9 **(f)** 38
15 **(a)** 18 208 m^2 (answers may vary slightly if students round intermediate answers)
(b) 3793 **(c)** 48 171 kg **(d)** $22 640 **(e)** 24.7%

Consolidation

Exercise 1

1 **(a)** $1\frac{1}{6}$ **(b)** $\frac{53}{56}$ **(c)** $3\frac{31}{35}$ **(d)** $\frac{4}{21}$ **(e)** $3\frac{7}{8}$
2 **(a)** $\frac{2}{5}$ **(b)** $8\frac{1}{3}$ **(c)** $1\frac{2}{7}$ **(d)** $\frac{12}{25}$ **(e)** $1\frac{47}{48}$
3 **(a)** 1.5 **(b)** 3.45 **(c)** 0.858 **(d)** 3.192 **(e)** 0.020 862
4 **(a)** 24 **(b)** 85 **(c)** 19000 **(d)** 231 **(e)** 0.272
5 (answers to 3sf where appropriate)
(a) **(i)** 0.52 **(ii)** 0.725 **(iii)** 0.622 **(iv)** 0.404
(v) 0.452 **(b)** **(i)** 52% **(ii)** 72.5% **(iii)** 62.2%
(iv) 40.4% **(v)** 45.2%

6 **(a)**

	5.613	**0.0138**	**1.0824**
Correct to 2 d.p.	5.61	0.01	1.08
Correct to 1 d.p.	5.6	0.0	1.1

(b)

	83 145	**16.382**	**0.002 59**
Correct to 2 s.f.	83 000	16	0.0026
Correct to 1 s.f.	80 000	20	0.003

7 **(a)** 3.16×10^5 **(b)** 2.51×10^{-2} **(c)** 1.394×10^6
(d) 4.16×10^{-4} **(e)** 8.1×10^9
8 **(a)** $19.34 **(b)** $24.18 **(c)** $110.45
9 **(a)** 37.6 km/hr **(b)** 23.4 miles/hr
(c) **(i)** 112.7 km/hr **(ii)** 70 miles/hr **(d)** 3 times faster

Check out

1 **(a)** $\frac{39}{40}$ **(b)** $\frac{7}{8}$ **(c)** $2\frac{1}{3}$ **(d)** $\frac{4}{9}$
2 **(a)** 0.56 **(b)** 400 **(c)** 3.834 **(d)** 215
3 **(a)** **(i)** 0.75 **(ii)** 0.625 **(iii)** 0.8 **(iv)** 0.4375
(b) **(i)** $\frac{17}{20}$ **(ii)** $\frac{63}{200}$ **(iii)** $\frac{23}{25}$ **(iv)** $\frac{5}{8}$
(c) **(i)** 25% **(ii)** 60% **(iii)** 85% **(iv)** 87.5%

4 **(a)** **(i)** 8710 **(ii)** 0.00486 **(b)** **(i)** 48000 **(ii)** 0.020
(c) **(i)** 60 **(ii)** 0.0008
5 **(a)** $\frac{1}{16}$ **(b)** 8 **(c)** $\frac{1}{6}$ **(d)** 729
6 **(a)** 8×10^3 **(b)** 7.236×10^3 **(c)** 3.8×10^{-2}
(d) 1.82×10^5 **(e)** 6.2×10^{-5} **(f)** 3×10^{0}
7 **(a)** 152.4 cm **(b)** 6.71 m **(c)** 4.02 m **(d)** 77.42 feet
(e) 2.46 feet

2 Computation 2

Check in

1 **(a)** > **(b)** < **(c)** < **(d)** >
2 **(a)** $^{-}5, ^{-}2, 0, 3, 6$ **(b)** $^{-}11, ^{-}7, ^{-}3, ^{-}2, 1, 4$
(c) $^{-}3, ^{-}2, ^{-}1, 1, 2, 4$

Exercise 2A

1 **(a)** $^{-}2+5=3$ **(b)** $^{-}5+3=^{-}2$ **(c)** $1+5=6$ **(d)** $^{-}5+10=5$
2 **(a)** $5-4=1$ **(b)** $3-5=^{-}2$ **(c)** $^{-}2-3=^{-}5$ **(d)** $4-8=^{-}4$
5 **(a)** $^{-}6$ **(b)** $^{-}1$ **(c)** 2 **(d)** $^{-}3$
(e) $^{-}1$ **(f)** $^{-}1$ **(g)** $^{-}2$ **(h)** $^{-}8$
6 $1, 0, ^{-}1, ^{-}2, ^{-}3, ^{-}4, ^{-}5$
7 **(a)** 3 **(b)** 4 **(c)** 1 **(d)** $^{-}2$ **(e)** $^{-}3$
8 **(a)** $^{-}5$ **(b)** $^{-}4$ **(c)** $^{-}1$ **(d)** $^{-}10$ **(e)** $^{-}9$
(f) $^{-}9$ **(g)** $^{-}6$ **(h)** $^{-}20$

9

+	$^{-}4$	$^{-}2$	0	2	4
$^{-}4$	$^{-}8$	$^{-}6$	$^{-}4$	2	0
$^{-}2$	$^{-}6$	$^{-}4$	$^{-}2$	0	2
0	$^{-}4$	$^{-}2$	0	2	4
2	$^{-}2$	0	2	4	6
4	0	2	4	6	8

10 **(a)** $^{-}1$ **(b)** $^{-}4$ **(c)** $^{-}7$ **(d)** $^{-}8$
(e) $^{-}4$ **(f)** $^{-}4$ **(g)** $^{-}3$ **(h)** $^{-}31$

Exercise 2B

1 **(a)** 5 **(b)** 5 **(c)** 7 **(d)** 10 **(e)** 1 **(f)** 1 **(g)** $^{-}1$ **(h)** $^{-}5$
2 **(a)** 1 **(b)** 5 **(c)** $^{-}5$ **(d)** $^{-}1$ **(e)** $^{-}8$ **(f)** 9 **(g)** $^{-}13$ **(h)** $^{-}6$

3

Second number

	−	$^{-}5$	$^{-}3$	$^{-}1$	1	3	5
First number	$^{-}5$	0	$^{-}2$	$^{-}4$	–6	–8	$^{-}10$
	$^{-}3$	2	0	$^{-}2$	$^{-}4$	–6	$^{-}8$
	$^{-}1$	4	2	0	$^{-}2$	$^{-}4$	$^{-}6$
	1	6	4	2	0	$^{-}2$	$^{-}4$
	3	8	6	4	2	0	$^{-}2$
	5	10	8	6	4	2	0

4 **(a)** 9 **(b)** $^{-}3$ **(c)** $^{-}5$ **(d)** $^{-}1$ **(e)** 1 **(f)** $^{-}12$ **(g)** $^{-}4$ **(h)** 0
5 **(a)** 2 **(b)** $^{-}1$ **(c)** 1 **(d)** $^{-}8$ **(e)** $^{-}3$ **(f)** 4 **(g)** $^{-}2$ **(h)** $^{-}3$
6 **(a)** $^{-}1$°C **(b)** 1°C **(c)** $^{-}9$°C **(d)** $^{-}13$°C
7 **(b)** –$50

Exercise 2C

1 **(a)** $^{-}8$ **(b)** $^{-}30$ **(c)** –18 **(d)** 10 **(e)** 27
(f) $^{-}20$ **(g)** $^{-}48$ **(h)** $^{-}48$ **(i)** 33 **(j)** 96
2 **(a)** $^{-}2$ **(b)** –5 **(c)** $^{-}4$ **(d)** 4 **(e)** $^{-}8$
(f) $^{-}4$ **(g)** $^{-}12$ **(h)** $^{-}8$ **(i)** $^{-}24$ **(j)** 6
3 **(a)** $^{-}2$ **(b)** 3 **(c)** $^{-}6$ **(d)** $^{-}2$ **(e)** $^{-}6$ **(f)** $^{-}12$

4 **(a)** $^-6$ **(b)** $^-4$ **(c)** $^-3$ **(d)** $^-45$ **(e)** $^-36$ **(f)** $^-96$

5

×	$^-6$	$^-4$	$^-2$	0	2	4	6
$^-6$	36	24	12	0	$^-12$	$^-24$	$^-36$
$^-4$	24	16	8	0	$^-8$	$^-16$	$^-24$
$^-2$	12	8	4	0	$^-4$	$^-8$	$^-12$
0	0	0	0	0	0	0	0
2	$^-12$	$^-8$	$^-4$	0	4	8	12
4	$^-24$	$^-16$	$^-8$	0	8	16	24
6	$^-36$	$^-24$	$^-12$	0	12	24	36

6 **(a)** 1 **(b)** 9 **(c)** $^-8$ **(d)** $^-64$ **(e)** $^-1$
(f) 1 **(g)** $^-32$ **(h)** 256

7 **(a)**

×	2	$^-3$
2	4	$^-6$
$^-3$	$^-6$	9

(b)

×	$^-3$	4
$^-1$	3	$^-4$
$^-5$	15	$^-20$

8 **(a)**

÷	$^-3$	4
12	$^-4$	3
$^-36$	12	$^-9$

(b)

÷	6	$^-3$	2
30	5	$^-10$	15
12	2	$^-4$	6
$^-6$	$^-1$	2	3

Exercise 2D

1 **(b)** $2 \times 2 \times 2$ **(c)** 3×3 **(d)** $x \times x \times x \times x \times x \times x$
(e) $y \times y \times y \times y$ **(f)** $2 \times a \times a \times a \times a$ **(g)** $3 \times x \times x$
(h) $a \times b \times b \times b$ **(i)** $p \times p \times q \times q \times q$

2 **(b)** 2^7 **(c)** x^6 **(d)** $6p^4$ **(e)** $4p^2q^3$

3 **(b)** $^-1$ **(c)** $^-4$ **(d)** 12 **(e)** 324 **(f)** $^-250$

4 **(a)** **(i)** $^-1$ **(ii)** 1 **(iii)** 1 **(iv)** $^-1$ **(b)** 1, $^-1$ **(c)** $^-1$

5 **(a)** 14 **(b)** $^-28$ **(c)** 31 **(d)** $^-23$ **(e)** $^-35$

6 **(a)** 12 **(b)** 76 **(c)** 54 **(d)** 38 **(e)** $^-18$

7 **(a)** 20, 20 **(b)** 20, 20 **(c)** 40, 40 **(d)** $^-4$, $^-4$

8 **(a)** 56, $^-56$ **(b)** 6, $^-6$ **(c)** $^-20$, 20 **(d)** 26, $^-26$

Exercise 2E

1 **(a)** 20.25, 12.25, 6.25, 2.25, 0.25, 0.25, 2.25, 6.25, 12.25, 20.25 **(c)** yes

2 **(a)** 2.9 **(b)** 10.2 **(c)** 14.4 **(d)** 22.1 **(e)** 22.1
(f) 5.3

3 **(a)** $^-5$, 5 **(b)** yes

4 **(a)** $^-2$, 2 **(b)** $^-1$, 1 **(c)** $^-3$, 3 **(d)** $^-2.5$, 2.5
(e) $^-4.5$, 4.5

5 **(a)** $^-2.2$, 2 2 **(b)** $^-3.9$, 3.9 **(c)** both 0 **(d)** $^-4.8$, 4.8

6 1.44 → $^-1.2$ and 1.2, 1.21 → $^-1.1$ and 1.1, 2.25 → $^-1.5$ and 1.5, 1.69 → $^-1.3$ and 1.3, 1.96 → $^-1.4$ and 1.4

7 **(a)** 0 → 0, 1 → $^-1$ and 1, 4 → $^-2$ and 2
(b) negative numbers cannot be squares and 2 is not a square

Exercise 2F

2 $\{^-1, {}^-2\}, \{^-1, {}^-3\}, \{^-2, {}^-3\}$

3 **(a)** $\{^-3, {}^-2, {}^-1, 0\}$
(b) $\{^-8, {}^-6, {}^-4, {}^-2, 2, 4, 6, 8, 10\}$
(c) $\{1, 2, 3, 5, 6, 10\}$

4 **(a)** $\{^-2\}$ **(b)** $\{3, 5\}$ **(c)** $\{^-3\frac{1}{2}, {}^-1\frac{1}{2}\}$

5 **(a)** $\{^-2\}$ **(b)** $\{^-4, {}^-3, {}^-2, {}^-1, 0, 2\}$
(c) $\{^-5, {}^-3, {}^-2, {}^-1, 1, 3, 5\}$ **(d)** $\{^-2\}$
(e) $\{^-5, {}^-4, {}^-2, 0, 1, 2, 3, 5\}$ **(f)** $\{^-2\}$

6 **(b)** **(i)** $\{^-9, {}^-6, {}^-4, {}^-3, {}^-1\}$ **(ii)** $\{^-9, {}^-6, {}^-4, {}^-2\}$ **(iii)** $\{^-6\}$

7 **(a)** **(i)** $\{^-1, 0, 2, 3, 4\}$ **(ii)** $\{^-3, {}^-2, {}^-1, 0\}$
(iii) $\{^-5, {}^-4, {}^-3, 0, 1, 2\}$
(b) **(i)** $\{^-1, 0\}$ **(ii)** $\{0, 2\}$ **(iii)** $\{^-3, 0\}$
(iv) $\{^-3, {}^-2, {}^-1, 0, 2, 3, 4\}$
(v) $\{^-5, {}^-4, {}^-3, {}^-1, 0, 1, 2, 3, 4\}$
(vi) $\{^-5, {}^-4, {}^-3, {}^-2, {}^-1, 0, 1, 2\}$

Exercise 2G

1 **(a)** A = {1, 2, 3, 4, 6, 12}, B = {1, 2, 4, 8}
(b) **(i)** 6 **(ii)** 4 **(c)** N **(e)** **(i)** {1, 2, 4}
(ii) {1, 2, 3, 4, 6, 8, 12} **(f)** 7

2 **(a)** W, B **(c)** {3, 9} **(d)** 2

3 **(a)** **(i)** 5 **(ii)** 7 **(b)** Z **(d)** **(i)** {4, 8}
(ii) $\{^-8, {}^-4, 0, 2, 4, 8, 10, 12, 14\}$ **(e)** 9

4 **(a)** odd integers
(b) **(i)** 5 **(ii)** 5 **(c)** M **(e)** **(i)** { }
(ii) $\{^-10, {}^-8, {}^-6, {}^-5, {}^-4, {}^-2, {}^-1, 3, 7, 11\}$ **(iii)** 10

5 **(a)** integers that are multiples of 3 **(b)** J, K
(d) **(i)** $\{^-3, 0, 3\}$ **(ii)** {6, 12, 18, . . .} **(e)** 3

6 **(a)** A = {1, 3, 5} B = {2, 4, 6} (other answers possible)
(b) A = {1, 2, 3} B = {2, 4, 6} (other answers possible)
(c) A = {1, 2, 3, 4, 5, 6} B = {2, 4, 6} (other answers possible)

7 **(a)** A = {1, 3, 5, . . .} B = {2, 4, 6, . . .} (other answers possible)
(b) A = {1, 2, 3, . . .} B = {2, 4, 6, . . .} (other answers possible)
(c) A = {1, 2, 3, 4, 5, 6, . . .} B = {2, 4, 6, . . .} (other answers possible)

8 **(a)** **(i)** $\{^-2, 1\}$ **(ii)** $\{^-5, {}^-2, {}^-1, 0, 1, 8, 15, 21\}$
(b) **(i)** 6 **(ii)** 4 **(iii)** 2 **(iv)** 8
(c) yes

9 Students' own answers.
$n(\mathrm{A} \cup \mathrm{B}) = n(\mathrm{A}) + n(\mathrm{B}) - n(\mathrm{A} \cap \mathrm{B})$

Exercise 2H

1 **(a)** 2×2 **(b)** 2×1 **(c)** 1×2 **(d)** 3×1 **(e)** 3×2
(f) 3×3

2 **(a)** $\begin{pmatrix} 5 \\ 3 \end{pmatrix}$ **(b)** $\begin{pmatrix} 5 \\ 3 \end{pmatrix}$ **(c)** $\begin{pmatrix} 7 \\ 4 \end{pmatrix}$ **(d)** $\begin{pmatrix} 7 \\ 4 \end{pmatrix}$ **(e)** $\begin{pmatrix} 6 \\ ^-1 \end{pmatrix}$
(f) $\begin{pmatrix} 6 \\ ^-1 \end{pmatrix}$

3 yes **4** (a) $\begin{pmatrix} 4 & 2 \\ 1 & 5 \end{pmatrix}$ (b) $\begin{pmatrix} 5 & 2 \\ ^-1 & 4 \end{pmatrix}$ (c) $\begin{pmatrix} 0 & 2 \\ 0 & 0 \end{pmatrix}$
(d) $\begin{pmatrix} ^-1 & 2 \\ ^-1 & ^-4 \end{pmatrix}$ **(g)** $\begin{pmatrix} 3 & 2 \\ 0 & 1 \end{pmatrix}$ **(i)** $\begin{pmatrix} ^-2 & 2 \\ 1 & ^-3 \end{pmatrix}$
(e)(f)(h) not possible

5 **(a)** **(i)** $\begin{pmatrix} 6 & 3 \\ 0 & 5 \end{pmatrix}$ **(ii)** $\begin{pmatrix} 6 & 3 \\ 0 & 5 \end{pmatrix}$ **(b)** yes

6 **(a)** $\begin{pmatrix} 6 \\ 2 \\ 5 \end{pmatrix}$ **(b)** $\begin{pmatrix} 8 \\ 5 \\ 10 \end{pmatrix}$ **(c)** $\begin{pmatrix} 2 \\ 3 \\ 5 \end{pmatrix}$ **(d)** Increase in scores

7 **(a)** $\begin{pmatrix} 0 & 2 \\ 2 & 0 \end{pmatrix}$ **(b)** $\begin{pmatrix} 4 & 0 \\ 0 & 4 \end{pmatrix}$ **(c)** $\begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix}$
(d) $\begin{pmatrix} 6 & 0 \\ 0 & 6 \end{pmatrix}$ **(e)** $\begin{pmatrix} ^{-}5 & 1 \\ 1 & ^{-}5 \end{pmatrix}$ **(f)** $\begin{pmatrix} ^{-}6 & 0 \\ 0 & ^{-}6 \end{pmatrix}$

8 **(a)** $\begin{pmatrix} 12 \\ 14 \\ 10 \end{pmatrix}$ **(b)** $\begin{pmatrix} 48 \\ 56 \\ 40 \end{pmatrix}$ **(c)** **(i)** $\begin{pmatrix} 24 \\ 28 \\ 20 \end{pmatrix}$ **(ii)** $\begin{pmatrix} 48 \\ 56 \\ 40 \end{pmatrix}$ **(iii)** $\begin{pmatrix} 60 \\ 70 \\ 50 \end{pmatrix}$
(d) Cost of buying 5 of each item

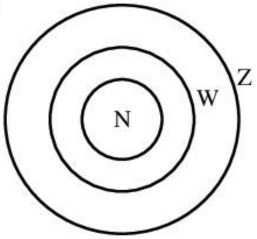
9 **(a)** $\begin{pmatrix} 4 \\ 2 \end{pmatrix}$ **(b)** $\begin{pmatrix} 12 \\ 0 \\ 4 \end{pmatrix}$ **(c)** $\begin{pmatrix} ^{-}2 \\ 0 \end{pmatrix}$ **(d)** $\begin{pmatrix} 10 & 0 \\ 0 & 10 \end{pmatrix}$

10 **(a)** 12 **(b)** 6 **(c)** 4, $^{-}3$ **(d)** 1

Exercise 2I

1 **(a)** $^{-}1$ **(b)** $^{-}1$ **(c)** $^{-}2$ **(d)** 2
(e) $^{-}14$ **(f)** $^{-}4$ **(g)** $^{-}4$ **(h)** $^{-}14$
2 **(a)** –5 **(b)** $^{-}2$ **(c)** 7 **(d)** $^{-}1$
(e) 11 **(f)** $^{-}7$ **(g)** 13 **(h)** $^{-}11$
3 **(a)** $^{-}1$ **(b)** $^{-}3$ **(c)** $^{-}4$ **(d)** $^{-}5$
(e) 5 **(f)** 3 **(g)** $^{-}2$ **(h)** 0
4 **(a)** $^{-}6$ **(b)** $^{-}20$ **(c)** 9 **(d)** $^{-}32$
(e) $^{-}8$ **(f)** $^{-}4$ **(g)** 5 **(h)** $^{-}8$
5 **(a)** 2 **(b)** 4 **(c)** 0 **(d)** 52
6 **(a)** $\mathbb{N}$ = {counting numbers}
$\mathbb{Z}$ = {integers}
$\mathbb{W}$ = {positive whole numbers}

(b)



(c) **(i)** $\mathbb{N}$ **(ii)** $\mathbb{W}$ **(d)** no
7 \$6.50
8 **(a)** 5 **(b)** 4 **(c)** $^{-}5$

9 **(a)** $\begin{pmatrix} 5 & 0 \\ ^{-}1 & 4 \end{pmatrix}$ **(b)** $\begin{pmatrix} 3 & ^{-}1 \\ 3 & ^{-}1 \end{pmatrix}$ **(c)** $\begin{pmatrix} 4 & 1 \\ ^{-}4 & 7 \end{pmatrix}$
(d) $\begin{pmatrix} 9 & 3 \\ ^{-}12 & 18 \end{pmatrix}$ **(e)** $\begin{pmatrix} ^{-}2 & 0 \\ 0 & ^{-}2 \end{pmatrix}$ **(f)** $\begin{pmatrix} 7 & 3 \\ ^{-}12 & 16 \end{pmatrix}$
(g) $\begin{pmatrix} 8 & ^{-}4 \\ 12 & ^{-}8 \end{pmatrix}$ **(h)** $\begin{pmatrix} 5 & ^{-}3 \\ 9 & ^{-}7 \end{pmatrix}$ **(i)** $\begin{pmatrix} 6 & 7 \\ ^{-}25 & 30 \end{pmatrix}$

10 $a = 5$, $b = \frac{^{-}5}{3}$, $c = 4$, $d = 7$

Consolidation

Exercise 2

1 **(a)** $^{-}9$ **(b)** $^{-}4$ **(c)** $^{-}2$ **(d)** 2
2 **(a)** 5 **(b)** $^{-}9$ **(c)** $^{-}9$ **(d)** 3
3 **(a)** 5 **(b)** $^{-}3$ **(c)** $^{-}3$ **(d)** $^{-}1$
4 **(a)** $^{-}3$ **(b)** $^{-}24$ **(c)** 3 **(d)** $^{-}4$
5 **(a)** $^{-}13$ **(b)** $^{-}9$ **(c)** 35 **(d)** $^{-}7$

6 **(a)** **(i)** 27 **(ii)** 9 **(b)** **(i)** 9 **(ii)** $^{-}9$
(c) **(i)** 49 **(ii)** 25 **(d)** **(i)** 92 **(ii)** $^{-}82$
(e) **(i)** 25 **(ii)** $^{-}35$ **(f)** **(i)** 259 **(ii)** 217
7 **(a)** **(i)** $^{-}4$°C **(ii)** $^{-}8$°C **(iii)** $^{-}6$°C
(b) Calgary, Helsinki, Anchorage, Moscow
(c) **(i)** 13°C **(ii)** 2°C

8 **(a)**

x	$^{-}3$	$^{-}2$	$^{-}1$	0	1	2	3
y	$^{-}2$	3	6	7	6	3	$^{-}2$

(c) $x = \pm 2.6$; $\sqrt{7} = 2.6$ (1 d.p.)

Check out

1 **(a)** 1 **(b)** $^{-}9$ **(c)** $^{-}6$ **(d)** $^{-}20$ **(e)** $^{-}15$ **(f)** $^{-}12$
2 **(a)** 9 **(b)** 3 **(c)** 3 **(d)** 1 **(e)** $^{-}9$ **(f)** 44
3 **(a)** $^{-}21$ **(b)** $^{-}7$ **(c)** 8 **(d)** $^{-}91$ **(e)** $^{-}4$ **(f)** $^{-}78$
4 **(a)** 18 **(b)** $^{-}6$ **(c)** 49 **(d)** 30 **(e)** $^{-}3$ **(f)** $^{-}121$
5 **(a)** 9 **(b)** 36 **(c)** $^{-}6$ and 6 **(d)** $^{-}7$ and 7

3 Measurement 1

Check in

2 **(a)** 5 cm^2 **(b)** 2 cm^2 **(c)** roughly 6 cm^2

Exercise 3A

1 **(a)** 20 cm **(b)** 24 cm **(c)** 28 m **(d)** 39 m
2 **(a)** 36 cm **(b)** 34.6 cm **(c)** 42 cm **(d)** 36 cm
(e) 60 cm
3 **(a)** 9 cm **(b)** 9 cm, 5 cm, 4 cm

Exercise 3B

1 **(a)** 37.68 cm **(b)** 47.1 cm
2 **(a)** 44 cm **(b)** 132 cm
3 **(a)** 19.47 cm **(b)** 36.74 cm
4 **(a)** 235.5 cm **(b)** 42.5
5 **(a)** 36 cm **(b)** 25 cm **(c)** 76 cm
6 **(a)** 5 cm **(b)** 1.59 cm
7 12 738 km
8 **(a)** 942 000 000 km **(b)** 2 580 000 km (3sf)
(c) 108 000 km/h (3sf)
9 471 m **10** 0.628 m
11 **(a)** 23.55 mm **(b)** 31.4 mm **(c)** 518.1 mm

Exercise 3C

1 **(a)** 6 **(b)** 6 **(c)** 6 **(d)** $7\frac{1}{2}$ **(e)** 10 **(f)** 9

Exercise 3D

1 **(a)** 21 cm^2 **(b)** 16 cm^2 **(c)** $8\frac{3}{4}$ cm^2 **(d)** 40.89 cm^2
2 **(a)** 6 squares **(b)** $4\frac{1}{2}$ squares **(c)** 10 squares
(d) 6 squares

3

Triangle	Base	Height	$\frac{1}{2} \times$ base $\times$ height
(a)	3	4	6
(b)	3	3	$4\frac{1}{2}$
(c)	5	4	10
(d)	3	4	6

4 **(a)** 14 cm^2 **(b)** 20 cm^2 **(c)** $16\frac{7}{8}$ cm^2 **(d)** 11.39 cm^2

Exercise 3E

1 **(a)** 31 m^2 **(b)** 84 m^2 **(c)** 37 m^2 **(d)** 42 m^2
(e) 58 m^2 **(f)** 70 m^2
2 **(a)** 40 cm^2 **(b)** 56 cm^2 **(c)** 8 m^2 **(d)** 34 m^2
(e) 63 m^2 **(f)** 42 m^2
3 23 m^2 **4** 11.75 m^2 **5** 43.4 m^2 **6** 88 m^2 **7** 106 cm^2
8 **(a)** 42 m^2 **(b)** **(i)** 600 cm^2 **(ii)** 0.06 m^2
(c) 700 **(d)** $910

Activity page 43

(b) yes **(c)** They are exactly the same shape and size.
(e) The area of the parallelogram is twice the area of the shaded triangle. **(f)** area = base length × vertical height

Exercise 3F

1 **(a)** 20 cm^2 **(b)** 24 cm^2 **(c)** 220 cm^2 **(d)** 112 cm^2
2 **(a)** 73.8 cm^2 **(b)** 99.96 cm^2
3 5.78 cm^2
4 **(a)** 28 cm^2 **(b)** 9 cm **(c)** 8 cm **(d)** 7.36 cm^2
(e) 3.7 cm

Exercise 3G

1 **(a)** **(i)** 15 cm^2 **(ii)** 9 cm^2 **(iii)** 24 cm^2
(b) **(i)** 15 cm^2 **(ii)** 9 cm^2 **(iii)** 24 cm^2
2 Yes, they all have an area of 24 cm^2. **3** 28 cm^2

Exercise 3H

1 **(a)** 32.5 cm^2 **(b)** 19 cm^2 **(c)** 42 cm^2 **(d)** 51.1 cm^2
(e) 44.1 cm^2 **(f)** 104 m^2
2 8.4 cm^2
3 **(i)** 90 cm^2 **(ii)** 28 cm^2 **(iii)** 28 cm^2 **(iv)** 95.16 m^2
4 360 cm^2 **5** 3.33 cm
6 **(a)** 84 m^2 **(b)** 123.06 m^2 **(c)** 1200 cm^2 **(d)** 43.2 m^2
(e) 310 cm^2

Exercise 3I

1 **(c)**

Radius, r, of circle (cm)	r^2	Area (cm^2)
2	4	$\cong$12
3	9	$\cong$37
4	16	$\cong$49
5	25	$\cong$77

2 Values are all similar $\cong$3.1
3 **(a)** 113 cm^2 **(b)** 201 cm^2

Exercise 3J

1 **(a)** 50.24 cm^2 **(b)** 78.5 cm^2
3 **(a)** 6 cm **(b)** 113.04 cm^2
4 **(a)** 38.47 cm^2 **(b)** 176.63 cm^2
5 4.37 cm **6** 88.31 cm^2 **7** 1.46 m^2

Exercise 3K

1 **(a)** 23.55 cm, 176.6 cm^2 **(b)** 9.42 cm, 28.3 cm^2
(c) 37.68 cm, 150.7 cm^2
2 **(a)** 2.51 cm^2 **(b)** 53.4 cm^2
3 **(a)** 4.8 cm **(b)** 6.92 cm^2
4 **(a)** 14.13 cm^2 **(b)** 67.0 cm^2
(c) 229.39 cm^2 **(d)** 35.82 cm^2
5 **(a)** 4.71 cm **(b)** 16.75 cm
(c) 45.88 cm **(d)** 9.68 cm
6 1099 cm^2
7 **(a)** 11.4 cm^2 **(b)** 18.24 cm^2 **(c)** 10.75 cm^2 **(d)** 186 cm^2
8 25.6°

Exercise 3L – mixed questions

1 **(a)** 49 m^2 **(b)** 33 m^2 **(c)** 36 m^2 **(d)** 54 m^2
2 **(a)** 12.6 cm^2 **(b)** 113 cm^2 **(c)** 24.6 cm^2
3 **(a)** 184 cm^2 **(b)** 40 cm^2 **(c)** 177 cm^2 **(d)** 103 cm^2
(e) 80 cm^2 **(f)** 48 cm^2 **(g)** $53\frac{1}{3}$ cm^2
4 26 cm^2 **5** 622 cm^2 **6** 330 cm^2
7 **(a)** 630 cm^2 **(b)** $4.91
8 46 cm^2 **9** Dimensions 48, 1; 24, 2; 16, 3; 12, 4; 8, 6
10 174.64 m^2
11 **(a)** 25 cm^2; 100 cm^2; the larger square has an area four times that of the smaller square.
(b) The larger square has an area nine times that of the smaller square.
12 6.928 cm^2 **13** 41.57 cm^2
14 **(a)** 8.84 cm^2 **(b)** 70.7 cm^2
15 Area 9426 m^2, 2357 kg

Consolidation

Exercise 3

1 **(a)** 40 cm^2 **(b)** 40 cm^2 **(c)** 22.4 cm^2 **(d)** 76.93 cm^2
(e) 17.0 cm^2
2 **(a)** 28 cm **(b)** 30 cm **(c)** 21 cm **(d)** 35.98 cm
(e) 16.8 cm
3 **(a)** 1.89 m^2 **(b)** 33.15 m^2
(c) **(i)** 1.38 litres **(ii)** $72.45
4 **(a)** 7200 cm^2 **(b)** **(i)** 225 cm^2 **(ii)** 32
5 1962.5 m^2

Check out

1 **(a)** 28 m **(b)** 40.82 cm **(c)** 17.85 cm
2 28 cm^2 **3** **(a)** 28 cm^2 **(b)** 45.5 cm^2
4 **(a)** 11 cm^2 **(b)** 10.65 cm^2
5 **(a)** 28.3 cm^2 **(b)** 113 cm^2 **(c)** 211 cm^2
6 **(a)** 10.5 cm **(b)** 52.3 cm^2

4 Computation 3

Check in

1 **(a)** $\frac{6}{8}, \frac{9}{12}$ **(b)** $\frac{8}{10}, \frac{12}{15}$
2 **(a)** $\frac{1}{3}$ **(b)** $\frac{4}{5}$ **(c)** $\frac{17}{20}$ **(d)** $\frac{1}{3}$
3 **(a)** 6 : 8 **(b)** 12 : 15 **(c)** 5 : 7 **(d)** 14 : 63

Exercise 4A

1 **(a)** 1 : 2 **(b)** 1 : 3 **(c)** 1 : 2 **(d)** 1 : 1.5 **(e)** 1 : 2.4
(f) 1 : 1.57
2 **(a)** **(i)** 1 : 2 **(ii)** 4 : 9 **(b)** 1 : 2, 1 : 2.25; Garvin
3 4 : 5 shirt

4 (a)

Strike rate
100.05
130.8
92.8
90.2

(b) Russell

5 (a)

Assist / turnover ratio
1:2.06
1:2.14
1:2.09
1:2.14

(b) Hardin

Exercise 4B

1 (a) \$6, \$4 **(b)** \$3, \$7 **(c)** \$2, \$8 **(d)** \$6.50, \$3.50
(e) \$8.50, \$1.50 **(f)** \$8.40, \$1.60
2 (a) 48, 12 **(b)** 6, 34 **(c)** 2, 10 **(d)** 8, 20
3 (a) \$100, \$200, \$600 **(b)** \$200, \$300, \$400
(c) \$200, \$200, \$500 **(d)** \$200, \$250, \$450
4 (a) 12 **(b)** 21 **(c)** 36
5 (a) 3 **(b)** 5 **(c)** $4\frac{1}{2}$
6 (a) 250 **(b)** 375 **(c)** 720 **(d)** 525
7 (a) 345 **(b)** 600 **(c)** 20 **(d)** 90
8 (a) 20 **(b)** $43\frac{1}{2}$ **(c)** 36 **(d)** 7
9 (a) 14 **(b)** 35 **(c)** 126
10(a) 18 m **(b)** 12 m **(c)** 8 cm

Exercise 4C

1 (a) \$70 **(b)** \$60 **(c)** \$210 **(d)** \$140 **(e)** \$455
2 (a) 96 cm **(b)** 144 cm **(c)** 112 cm
3 (a) 21 **(b) (i)** 25 **(ii)** 80
4 (a) \$60 **(b)** 1:2:3 **(c)** $\frac{1}{3}$
5 (a) 8:6:7 **(b) (i)** \$80 **(ii)** \$210

Exercise 4D

7 (a) Each new term is formed by adding the two previous terms.
(b) 21, 34

Exercise 4E

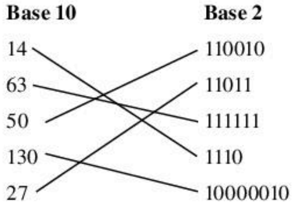
1 (a) 1:100 **(b)** 1:500 **(c)** 1:2000
(d) 1:50000 **(e)** 1:100000 **(f)** 1:500000
2 (a) 200 m **(b)** 500 m **(c)** 100 m
(d) 250 m **(e)** 5 km **(f)** 1 km

Exercise 4F

1 (a) 500 m **(b)** 2.5 km **(c)** 250 m **(d)** 1.6 km
2 (a) 40 cm **(b)** 16 cm **(c)** 8 cm **(d)** 4 cm
3 (a) 1:350 **(b)** 10.5 m **(c)** 1.14 m
4 (a) 1:2000 **(b)** 1:400 000 **(c)** 1:5 000 000
(d) 1:2500
7 (a) 3 cm **(b)** 1.5 km **(c)** 2.2 cm **(d)** 1.1 km
8 (b) 12 cm^2 **(c) (i)** 4800 m^2 **(ii)** 4 800 000 cm^2
(d) 1:4 000 000
9 (a) 4 cm by 3 cm **(b)** 12 cm^2 **(c) (i)** 12 m^2 **(ii)** 120 000 cm^2
(d) 1:10 000
10(a) 1:20 **(b)** 1000 cm^3 **(c) (i)** 8 m^3 **(ii)** 8 000 000 cm^3
(d) 1:64 000

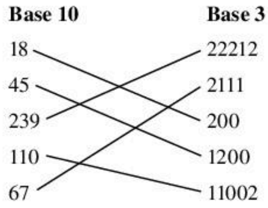
Exercise 4G

1 (a) 3 **(b)** 6 **(c)** 7 **(d)** 9 **(e)** 13
2 1, 10, 11, 100, 101, 110, 111, 1000
3 (a) 8 **(b)** 16 **(c)** 17 **(d)** 33 **(e)** 15 **(f)** 31 **(g)** 29 **(h)** 109
4 (a) 1001 **(b)** 1011 **(c)** 1111 **(d)** 10100
(e) 11111 **(f)** 100000 **(g)** 111011 **(h)** 1000001
5



Exercise 4H

1 (a) 3 **(b)** 4 **(c)** 7 **(d)** 8 **(e)** 23 **(f)** 34
2 1, 2, 10, 11, 12, 20, 21, 22, 100, 101
3 (a) 54 **(b)** 30 **(c)** 75 **(d)** 60 **(e)** 150
(f) 121 **(g)** 242 **(h)** 212
4 (a) 11 **(b)** 110 **(c)** 120 **(d)** 1011 **(e)** 2001
(f) 10001 **(g)** 22222 **(h)** 100001
5



Exercise 4I

1 (a) 101 **(b)** 100 **(c)** 1000 **(d)** 1010
2 (a) 11010 **(b)** 11110
3 (a) 101 **(b)** 111 **(c)** 102 **(d)** 201
4 (a) 11021 **(b)** 10210
5 (a) 10 **(b)** 1 **(c)** 10 **(d)** 1010
6 (a) 10 **(b)** 1100
7 (a) 12 **(b)** 11 **(c)** 22 **(d)** 1102
8 (a) 10120 **(b)** 1121

Exercise 4J

1 (a) \$100, \$20 **(b)** \$70, \$50 **(c)** \$114, \$6 **(d)** \$78, \$42
2 (a) 10 **(b)** 24 **(c)** 18
3 (a) 28 **(b)** 60
4 (a) \$9 **(b)** \$15.75
5 (a) 11100 **(b)** 101010 **(c)** 1011001 **(d)** 1110000
6 (a) 120 **(b)** 1120
7 (a) 79 **(b)** 43 **(c)** 82 **(d)** 205
8 (a) 1:5000 **(b)** 1:10 000 **(c)** 1:40 000 **(d)** 1:25 000 000
9 (a) 5 cm **(b)** 0.5 km **(c)** 4 cm
10 (a) 2 m × 3 m × 4 m **(b)** 192 cm^3
(c) 24 m^3 **(d)** 1:125 000
11 (a) \$117 **(b)** \$90
12 (a) 15:13 **(b)** 20:1 **(c)** 350:17
13 (a) 75 m **(b)** 0.008 cm
14 (a) 75 g **(b)** 1400 g

Consolidation

Exercise 4

1 **(a)** Anton 12 marbles; Dannisha 8 marbles
(b) Anton 16 biscuits; Dannisha 20 biscuits
(c) Anton 3 pencils; Dannisha 6 pencils
(d) Anton \$42; Dannisha \$30
(e) Anton 60 oranges; Dannisha 340 oranges

2 **(a)** **(i)** 60 **(ii)** 100 **(iii)** 248 **(iv)** 400 **(v)** 856
(b) **(i)** 40 **(ii)** 25 **(iii)** 92 **(iv)** 205 **(v)** 197

3 **(a)** Linda \$60; Maryann \$120; Neisha \$120
(b) Linda \$264; Maryann \$264; Neisha \$297
(c) Linda $6\frac{1}{4}$ oranges; Maryann $2\frac{1}{2}$ oranges; Neisha $16\frac{1}{4}$ oranges
(d) Linda 24 cakes; Maryann 36 cakes; Neisha 24 cakes

4 **(a)** 110110 **(b)** 1000100 **(c)** 1100110 **(d)** 11111000

5 **(a)** \$24 **(b)** \$216

6 **(a)** 300 **(b)** 150 : 11

7 **(a)** 4 cm **(b)** 0.8 km **(c)** 10 cm^2 **(d)** 0.4 km^2

8 **(a)** **(i)** 4 1 **(ii)** 6 1 **(iii)** 8 1
(b) **(i)** $7\frac{1}{2}$ 1 **(ii)** 10 1

Check out

1 **(a)** \$40, \$10 **(b)** \$35, \$15 **(c)** \$5, \$45
(d) \$42.50, \$7.50 **(e)** \$12, \$38 **(f)** \$42, \$8

2 **(a)** 40 **(b)** 16

3 **(a)** **(i)** 800 m **(ii)** 2200 m **(b)** **(i)** 5 cm **(ii)** 1 cm

4 **(a)** **(i)** 5 **(ii)** 14 **(iii)** 21 **(iv)** 70
(b) **(i)** 1110 **(ii)** 101011
(c) **(i)** 112 **(ii)** 1121

Revision exercise 1

1 **(a)** 9.2 **(b)** 13.12 **(c)** 68.4 **(d)** 2.34 **(e)** 4.48
(f) 0.45 **(g)** 10.88 **(h)** 1.197 **(i)** 0.0258 **(j)** 2.961

2 **(a)** 219.3 **(b)** 2193 **(c)** 219 300 **(d)** 21.93 **(e)** 2.193
(f) 0.2193 **(g)** 21.93 **(h)** 219.3 **(i)** 0.021 93
(j) 0.002 193

3 **(a)** 1.15 **(b)** 0.9375 **(c)** 355 **(d)** 35
(e) 205.6 **(f)** 60

4 **(a)** 0.23 **(b)** 23 **(c)** 0.0023 **(d)** 0.23
(e) 0.67 **(f)** 6.7

5 14

6 **(a)** 0.75, 75% **(b)** 0.125, 12.5% **(c)** 0.2, 20%
(d) 0.075, 7.5% **(e)** 0.2125, 21.25% **(f)** 0.615, 61.5%

7 **(a)** 0.25, $\frac{1}{4}$ **(b)** 0.65, $\frac{13}{20}$ **(c)** 0.16, $\frac{4}{25}$ **(d)** 0.375, $\frac{3}{8}$
(e) 0.015, $\frac{3}{200}$ **(f)** 0.66 . . . , $\frac{2}{3}$

8 **(a)** 30%, $\frac{4}{13}$, 0.333, 0.35, $\frac{3}{8}$ **(b)** 16%, 0.163, $\frac{9}{55}$, $\frac{7}{40}$, 163%

9

Number	**42819**	**0.038 45**	**1888.3**	**79 000 061**
3 s.f.	42 800	0.0385	1890	79 000 000
2 s.f.	43 000	0.039	1900	79 000 000
1 s.f.	40 000	0.04	2000	80 000 000

10 **(a)** 6.834×10^3 **(b)** 7.943×10^5 **(c)** 2.45×10^8
(d) 9.4356×10^5 **(e)** 4.2×10^{-3} **(f)** 7.8×10^{-7}
(g) 6.76×10 **(h)** 4.1×10^{-1} **(i)** 4.38×10^{11}

11 **(a)** 243 **(b)** 64 **(c)** 256 **(d)** 5
(e) 9 **(f)** 4 **(g)** $\frac{1}{8}$ **(h)** $\frac{1}{3}$

12 \$127.63 **13** \$1.75 **14** $\frac{1}{86400}$, 0.000012 **15** 454.5 g

16 39 526 inches **17** \$1.46 **18** 626.04 km/h

19 30 628 **20** 0.175 kg

21 **(a)** $^{-}4$ **(b)** 4 **(c)** $^{-}10$ **(d)** 5 **(e)** 9 **(f)** $^{-}11$

22 **(a)** 3 **(b)** $^{-}4$ **(c)** $^{-}3$ **(d)** 0 **(e)** 0 **(f)** $^{-}2$

23 **(a)** $7 \rightarrow 2$, $4 \rightarrow {}^{-}1$, ${}^{-}1 \rightarrow {}^{-}6$, $2 \rightarrow {}^{-}3$, ${}^{-}5 \rightarrow {}^{-}10$
(b) $7 \rightarrow 12$, $4 \rightarrow 9$, ${}^{-}1 \rightarrow 4$, $2 \rightarrow 7$, ${}^{-}5 \rightarrow 0$

24

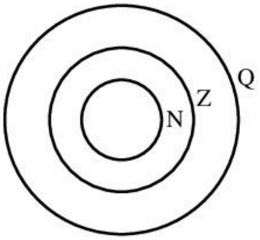
Second number

	Subtract	2	$^{-}3$	$^{-}1$	4
First number	3	1	6	4	$^{-}1$
	$^{-}1$	$^{-}3$	2	0	$^{-}5$
	4	2	7	5	0
	$^{-}2$	$^{-}4$	1	$^{-}1$	$^{-}6$

25 **(a)** $^{-}15$ **(b)** 18 **(c)** 12 **(d)** $^{-}3$ **(e)** $^{-}5$ **(f)** 5

26 **(a)** left 2, right $^{-}6$, bottom $^{-}2$ **(b)** top $^{-}1$, left 3, right 5

27 **(a)**



(b) **(i)** N **(ii)** Z **(iii)** Z
(iv) Q **(v)** N **(vi)** N

28 **(a)** 0 **(b)** $^{-}5$ **(c)** 9 **(d)** $^{-}5$ **(e)** 1 **(f)** 1

29 **(a)** 6 **(b)** 6 **(c)** 42

30 **(a)** $\begin{pmatrix} 5 & 0 \\ 6 & {}^{-}9 \end{pmatrix}$ **(b)** $\begin{pmatrix} {}^{-}1 & 2 \\ 2 & {}^{-}1 \end{pmatrix}$ **(c)** $\begin{pmatrix} 0 & 5 \\ 8 & {}^{-}7 \end{pmatrix}$

31 **(a)** 4 cm^2, 10 cm **(b)** 8.5 cm^2, approx 10 cm
(c) 4 cm^2, approx 13 cm

32 **(a)** 120 cm^2 **(b)** 6 cm

33 **(a)** 18 cm^2 **(b)** 52.5 cm^2 **(c)** 56 cm^2
(d) 28.26 cm^2

34 **(a)** 9 cm^2 **(b)** 32 cm

35 **(a)** 34 cm **(b)** 33.5 cm^2

36 **(a)** 40 cm^2 **(b)** 31 cm^2

37 **(a)** 13.9 cm **(b)** 11.5 cm^2

38 **(a)** 3.7 cm^2 **(b)** 23.3 cm

39 **(a)** 3.44 cm^2 **(b)** 10.6 cm^2 **(c)** 3.56 cm^2 **(d)** 2.28 cm^2

40 **(a)** 22.13 cm^2 **(b)** \$489.35

41 **(a)** 5 : 1 **(b)** 2 : 1 **(c)** 6 : 1 **(d)** 3 : 8
(e) 1 : 30 **(f)** 40 : 1

42 **(a)** 1.33 **(b)** 0.8 **(c)** 0.5 **(d)** 0.25 **(e)** 3.6 **(f)** 14
(g) 6.67 **(h)** 1

43 **(a)** 9 **(b)** 30
(c) **(i)** 20 boys, 12 girls **(ii)** 30 boys, 18 girls

44 **(a)** 252, 108 **(b)** 160, 200 **(c)** 40, 320 **(d)** 210, 150
(e) 175, 185 **(f)** 100, 260

45 **(a)** 2, 10 **(b)** 16, 2 **(c)** 24, 18 **(d)** 20, 25 **(e)** 10, 6
(f) 39, 45 **(g)** 12.6, 4.2 **(h)** 11.2, 14.4 **(i)** 0.35, 0.28
(j) 1536, 1792

46 **(a)** 16 goats, 40 sheep **(b)** 7 cows, 14 goats
(c) 6 cows, 30 sheep **(d)** **(i)** 2 cows, 4 goats, 10 sheep
(ii) 6 cows, 12 goats, 30 sheep
(iii) 9 cows, 18 goats, 45 sheep
(iv) 15 cows, 30 goats, 75 sheep

47 **(a)** \$25, \$175, \$200 **(b)** \$150, \$50, \$50 **(c)** 48, 24, 12
(d) 24, 32, 40 **(e)** 70, 60, 50 **(f)** 78, 52, 39
(g) 40, 40, 48 **(h)** 38, 34, 22 **(i)** 2.9, 14.5, 34.8
(j) 0.56, 0.84, 3.36

48 **(a)** \$144 **(b)** \$240 **(c)** \$784

49 **(a)** 1.48 km **(b)** 10.75 cm

50 **(a)** 24 m **(b)** 200 m^2

51 **(a)** 26 **(b)** 11 **(c)** 151 **(d)** 107

52 **(a)** 111000 **(b)** 11100

Mixed questions 1

1 D **2** D **3** C **4** B **5** D **6** B **7** D **8** D
9 B **10** B **11** B **12** B **13** A **14** D **15** C
16 D **17** B **18** D **19** A **20** A **21** B **22** B
23 A **24** C **25** A **26** A **27** C **28** B **29** C
30 C **31** C **32** C **33** A **34** C **35** D **36** D
37 D **38** B **39** D **40** D **41** B **42** C

5 Algebra

Check in

1 **(a)** $3 > {}^{-}10$ **(b)** ${}^{-}2 < 6$ **(c)** ${}^{-}2 > {}^{-}5$

2 **(a)** 6 cm^2 **(b)** 39 cm^2 **(c)** 58.5 cm^2

Exercise 5A

1 **(a)** $7x$ **(b)** $3y$ **(c)** $3x + 4y$ **(d)** $6y - 2x$ **(e)** $4x - y$
(f) $2ab$ **(g)** $5x - 4y$ **(h)** $2y - x$ **(i)** $5xy - 2x$
(j) $5y - 5x$

2 **(a)** $4x^2$ **(b)** $3y + 6$ **(c)** $5y$ **(d)** 0 **(e)** $7y + 2$
(f) $9 - 9x$ **(g)** $2a^2 - 3a$ **(h)** $2a^2$ **(i)** $5xy$
(j) $4a^3 + a^2$

3 **(a)** 15 **(b)** ${}^{-}9$ **(c)** 7 **(d)** 3 **(e)** 24
(f) 0 **(g)** 19 **(h)** 25 **(i)** ${}^{-}7$ **(j)** 41

Exercise 5B

1 **(a)** x^3y **(b)** x^2y^2 **(c)** x^4y^2 **(d)** a^3b^3

2 **(a)** x^5 **(b)** y^8 **(c)** $2x^7$ **(d)** $3x^9$ **(e)** x^9 **(f)** y^{11}

3 **(a)** x **(b)** 1 **(c)** x^4 **(d)** y^5 **(e)** x^5 **(f)** y^2

4 **(a)** x^8 **(b)** y^6 **(c)** x^{12} **(d)** y^{10} **(e)** a^3b^6 **(f)** $4a^6b^4$

5 **(a)** $2x^4$ **(b)** $4y^3$ **(c)** $\frac{x^3}{y}$ **(d)** $\frac{4x^6}{y^2}$

6 **(a)** x^3y **(b)** $\frac{xy^4}{4}$ **(c)** $9x^6y^6$ **(d)** $8x^7y^2$

Exercise 5C

1 **(a)** $24x^5y^2$ **(b)** $24y^4$ **(c)** $24x^6y^4$ **(d)** $-18x^5y^5$
(e) $-10x^4yz^8$ **(f)** $-12x^7y^7z$ **(g)** $-8x^6y^5z^8$ **(h)** $18x^{10}y^8z^2$
(i) $6x^5y^7z^2$ **(j)** $12x^{11}y^3z^3$

3 **(a)** $\frac{3}{xy}$ **(b)** $\frac{5}{2}xz$ **(c)** $2xyz$ **(d)** $\frac{-2y^3}{xy}$ **(e)** $\frac{-y^2}{2x}$ **(f)** $\frac{-4}{9xz^3}$
(g) $\frac{-7}{y^2}$ **(h)** $\frac{-27x^3}{8y^3z^3}$ **(i)** $16x^{10}y^6z^7$ **(j)** $\frac{-1x^7}{16y^2z^4}$

Exercise 5D

1 **(a)** $3x + 6$ **(b)** $8x - 24$ **(c)** $15x - 15$ **(d)** $24 - 18x$

2 **(a)** $x^2 + 2x$ **(b)** $x^3 + x^4$ **(c)** $3x^2 - 3xy$ **(d)** $8x^2 - 12x^3$
(e) $x^3 + 2x^4 - x^5$ **(f)** $8xy - 4x^2 + 4xy$

3 **(a)** $5x + 8$ **(b)** $6y - 8$ **(c)** $18x - 13$ **(d)** $10 - 17x$
(e) $3x - 19$ **(f)** $18x - 15y$ **(g)** ${}^{-}10y$ **(h)** ${}^{-}3x - 12y$
(i) $4x - 10y$ **(j)** ${}^{-}9x + 27y$

4 **(a)** $7x + 4x^2 + 4x^3$ **(b)** $3y^2 - y$ **(c)** $2x^3 + 3x^2 + 3x$
(d) ${}^{-}5x^2 + 2x$ **(e)** $8x^2 + 10x + 3$ **(f)** ${}^{-}6y^2 + 2y$
(g) ${}^{-}10y^2 + 6y$ **(h)** $12y^2 + 6y$ **(i)** $y^2 + 3y$
(j) ${}^{-}7xy^2 + 5y^2 - 2xy^3$

5 **(a)** $2x - 10y$ **(b)** $4x - xy$ **(c)** ${}^{-}2x^2 - 12x$

Exercise 5E

1 **(a)** $2(x + y)$ **(b)** $3(x - 2y)$ **(c)** $5(x - 3y)$ **(d)** $3(2x - 5y)$
(e) $7(x + 2y)$ **(f)** $5(7x - 3y)$ **(g)** $3(2x + y - 3z)$
(h) $8(2x - 3y + 4z)$

2 **(a)** $b(x + y)$ **(b)** $a(x^2 + y^2)$ **(c)** $a(3x + 4y)$
(d) $3p(2x - y)$ **(e)** $4r(2x + y)$ **(f)** $x(y + 3x)$
(g) $y(5x^2 + 4)$ **(h)** $y(6x + 4 - 3x^2)$

3 **(a)** $x(4x + 3y - yx)$ **(b)** $2(3ax + 2by + 6cz)$ **(c)** $xy(4 - 6x)$
(d) $xy(3x^2 - 2x)$ **(e)** $ab(ab - b + 1)$ **(f)** $ab(3b + 4a + ab)$
(g) $xy(3x - 2y + 4xz)$ **(h)** $xy(4 - 3xy + 5z)$

4 **(a)** $3(x + 3y)$ **(b)** $2(a - 2b)$
(c) $6(x - 2y)$ **(d)** $7(2x - y)$
(e) $3(5x + 6y)$ **(f)** $6(x - 4y)$
(g) $6(x + 12y)$ **(h)** $a(x + y)$

5 **(a)** $a(x + 3y)$ **(b)** $a(n - 3m)$ **(c)** $2r(3x + y)$
(d) $3a(x - 6y)$ **(e)** $5m(n - 3p)$ **(f)** $p(4q - 3r)$
(g) $2r(3 + 2p - q)$ **(h)** $a(3b - 3 + c)$

6 **(a)** $x(3x + 1)$ **(b)** $xy(x + 3y)$ **(c)** $m(4m - 1)$
(d) $ab(a - 3b)$ **(e)** $\pi r^2(1 + h)$ **(f)** $h(\pi r + h)$
(g) $xy(3x - 1 + 3y)$ **(h)** $q(p + 3p^2 - 2q)$

7 **(a)** $ab(4a - 6b + 2)$ **(b)** $xy(xy - 3x + 4y)$
(c) $y^2(4xy - 2x + 3)$ **(d)** $g^2h^2(h - 1 + g)$
(e) $7abc(2 - b + 3c)$ **(f)** $3abc(2b - a + 3c)$

Exercise 5F

1 **(a)** $\frac{4}{5}$ **(b)** $\frac{11}{15}$ **(c)** $\frac{41}{40}$ **(d)** $\frac{85}{112}$

2 **(a)** $\frac{4x}{5}$ **(b)** $\frac{11x}{15}$ **(c)** $\frac{41x}{40}$ **(d)** $\frac{85x}{112}$

3 **(a)** $\frac{25x}{16}$ **(b)** $\frac{19x}{15}$ **(c)** $\frac{21x + 2}{7}$ **(d)** $\frac{3 - 8x}{4}$

4 **(a)** $\frac{9 + 2x}{3x}$ **(b)** $\frac{10 - 3y}{5y}$ **(c)** $\frac{4y + 5x}{xy}$ **(d)** $\frac{3 - 2y}{y}$

5 **(a)** $\frac{4 + x^2}{2x}$ **(b)** $\frac{15 - x^2}{5x}$ **(c)** $\frac{8x + 3xy}{4y}$ **(d)** $\frac{20x^2 - 6y^2}{15xy}$

6 **(a)** $\frac{2a^2 + 15b^2}{5ab}$ **(b)** $\frac{6ab - 10c^2}{15bc}$ **(c)** $\frac{6 - abc}{3ab}$ **(d)** $\frac{2c + 3ab}{abc}$

7 **(a)** $\frac{2ab + 6b + 3a}{3ab}$ **(b)** $\frac{abc + 6c + 9b}{3bc}$ **(c)** $\frac{8ac - 48b - 9c}{12c}$ **(d)** $\frac{6a^2 + 3b^3 + 2ab^2}{3ab^2}$

Exercise 5G

1 **(a)** $\frac{5x + 2}{x(x + 1)}$ **(b)** $\frac{4x - 2}{x(x + 1)}$ **(c)** $\frac{5x + 9}{(x + 1)(x + 3)}$

2 **(a)** $\frac{7x + 10}{x(x + 2)}$ **(b)** $\frac{7x - 9}{x(x - 3)}$ **(c)** $\frac{8x - 28}{(x - 1)(x - 5)}$ **(d)** $\frac{{}^{-}x - 12}{x(x + 4)}$

3 **(a)** $\frac{11x - 1}{12}$ **(b)** $\frac{19x - 29}{6}$ **(c)** $\frac{{}^{-}14x + 14}{15}$ **(d)** $\frac{{}^{-}x - 22}{12}$

4 **(a)** $\frac{{}^{-}x - 2}{(2x - 5)(x - 4)}$ **(b)** $\frac{2x + 2}{(x - 3)(2x - 4)}$ **(c)** $\frac{{}^{-}4x^2 + 11x - 3}{(x - 2)(x - 1)}$ **(d)** $\frac{9x^2 - 25x + 20}{(x - 5)(3x - 7)}$

5 **(a)** $\$\frac{5}{m}$ **(b)** $\$\frac{23}{p}$ **(c)** $\frac{5p + 23m}{mp}$ **(d)** $\frac{5ap + 23bp}{mp}$

Exercise 5H

1 **(a)** $\frac{12c^2}{b}$ **(b)** $\frac{8a^4}{b^3c^2}$ **(c)** $\frac{4b^4c^2}{a^2}$ **(d)** $\frac{4a^5b}{c^5}$

2 **(a)** $\frac{a^3b^2}{c^3}$ **(b)** $\frac{1}{2}$ **(c)** $5b^2$ **(d)** $\frac{2a^2}{3bc}$

3 **(a)** $\frac{a^4b}{4}$ **(b)** $\frac{5ac}{24b^4}$ **(c)** $\frac{12c}{a^4b^2}$ **(d)** $\frac{a^2b}{10c^2}$

4 **(a)** $\frac{2ac^6}{3b^5}$ **(b)** $\frac{b}{2c^2}$ **(c)** $\frac{2}{a^4}$ **(d)** $\frac{-8ac^3}{3b}$

Exercise 5I

1 **(a)** $x^2 + 5x + 6$ **(b)** $x^2 + 6x + 8$
(c) $x^2 + 9x + 18$ **(d)** $a^2 + 2ab + b^2$

2 **(a)** $x^2 + 4x + 3$ **(b)** $x^2 + 7x + 12$ **(c)** $x^2 + 6x + 9$
(d) $x^2 + 10x + 21$ **(e)** $x^2 + 11x + 30$ **(f)** $x^2 + 15x + 56$

3 **(a)** $x^2 - 1$ **(b)** $x^2 - 4$ **(c)** $x^2 - 25$
(d) $x^2 - 36$ **(e)** $x^2 - 64$ **(f)** $x^2 - 81$

4 They are the difference between two squares.

5 **(a)** $x^2 - x - 6$ **(b)** $x^2 + x - 12$ **(c)** $x^2 + 3x - 18$
(d) $x^2 - 2x - 24$ **(e)** $x^2 + x - 30$ **(f)** $x^2 - 2x - 35$

6 **(a)** $x^2 - 5x + 6$ **(b)** $x^2 - 2x - 1$ **(c)** $x^2 - 10x + 25$
(d) $x^2 - 9x + 18$ **(e)** $x^2 - 9x + 20$ **(f)** $x^2 - 15x + 54$

7 **(a)** $ac + bc + ad + bd$ **(b)** $ac + bc - ad - bd$
(c) $ac + ad - bc - bd$ **(d)** $ac - ad - bc + bd$

8 **(a)** $4x^2 + 8x + 3$ **(b)** $2x^2 + 7x + 6$ **(c)** $12x^2 + 13x + 3$
(d) $6x^2 + 19x + 10$ **(e)** $15x^2 + 18x + 3$ **(f)** $8x^2 + 14x + 3$

9 **(a)** $4x^2 - 4x + 1$ **(b)** $9x^2 + 30x + 25$ **(c)** $6x^2 + 7x - 5$
(d) $6x^2 - 7x - 5$ **(e)** $6x^2 - 13x + 5$ **(f)** $12x^2 - 35x + 8$

10 **(a)** $a^2 - 2ab + b^2$ **(b)** $a^2 - b^2$ **(c)** $9a^2 - 4b^2$
(d) $4a^2 - 49b^2$ **(e)** $4a^2 - 12ab + 9b^2$
(f) $4a^2 + 12ab + 9b^2$

Exercise 5J

1 **(a)** 8 **(b)** 2 **(c)** 7 **(d)** 31 **(e)** $^{-}1$ **(f)** 14
2 **(a)** 3 **(b)** 3 **(c)** 3 **(d)** 6 **(e)** 3 **(f)** 3
3 **(a)** 6 **(b)** 20 **(c)** 24 **(d)** $^{-}27$ **(e)** 16 **(f)** $^{-}22$
4 **(a)** 1 **(b)** 1 **(c)** 5 **(d)** 5 **(e)** 9 **(f)** $6\frac{2}{3}$
5 **(a)** 2 **(b)** 3 **(c)** 8 **(d)** 21 **(e)** 20 **(f)** 72

Exercise 5K

1 **(a)** 4 **(b)** 2 **(c)** 4 **(d)** $^{-}2$ **(e)** 9 **(f)** $4\frac{1}{3}$
2 **(a)** 3 **(b)** 2 **(c)** $\frac{1}{2}$ **(d)** $1\frac{2}{7}$ **(e)** 2 **(f)** 12
3 **(a)** 5 **(b)** $^{-}2$ **(c)** 1 **(d)** $^{-}1$ **(e)** 7 **(f)** $1\frac{5}{9}$
4 **(a)** $^{-}10$ **(b)** 1 **(c)** $\frac{5}{7}$ **(d)** $\frac{-1}{3}$ **(e)** $\frac{-1}{2}$ **(f)** $\frac{-3}{5}$
5 **(a)** $4(x + 3) = 32$ **(b)** 5
6 **(a)** $x + (x + 2) + (x + 4) = 6x$ **(b)** 2 cm, 4 cm, 6 cm
7 **(a)** $4x + 4x + (x + 6) + (x + 6) = 48$ **(b)** 14.4 cm, 9.6 cm
8 112, 114

Exercise 5L

1 **(a)** $x + 140 = 180, x = 40$ **(b)** $y + 70 + 60 = 180, y = 50$
2 **(a)** $8w = 48$ **(b)** 6 cm
3 **(a)** $(x + 14)$ cm **(b)** 6
4 **(a)** $2y + 8 = 32$ **(b)** 12 cm
5 **(a)** $x + 20$ **(b)** $2x + 20$ **(c)** 55
6 **(a)** $(2x + 19)$ cm **(b)** 5.5
7 **(a)** $(4w + 10)$ cm **(b)** 15 cm by 10 cm
8 5 cm and 9 cm **9** 9 years old and 12 years old
10 15 and 16
11 23, 24 and 25
12 21, 23 and 25
13 **(a)** $9c = 315$ **(b)** 35 cents and 70 cents
14 $7h + 12$, \$10
15 $2(3n + 7) = 80, n = 11$

Exercise 5M

1 **(a)** $x = 1, y = 1$ **(b)** $x = 2, y = 1$ **(c)** $x = 1, y = 1$
(d) $x = 3, y = 1$

2 **(a)** $x = 1, y = 1$ **(b)** $x = 1, y = 1$ **(c)** $x = 1, y = 3$
(d) $x = 1, y = 1$

3 **(a)** $x = 4, y = 1$ **(b)** $x = 1, y = 1$ **(c)** $x = 3.8, y = 1.6$
(d) $x = 2, y = 2$

4 **(a)** $x = 1, y = 1$ **(b)** $x = 1, y = 1$ **(c)** $x = 5, y = 2$
(d) $x = 1, y = 2$ **(e)** $x = 1, y = {}^{-}1$ **(f)** $x = 2, y = 2$

Exercise 5N

1 **(a)** $x = 4, y = 1$ **(b)** $x = 1, y = 1$ **(c)** $x = 1, y = {}^{-}1$
(d) $x = 1, y = 3$

2 **(a)** $x = 2, y = 1$ **(b)** $x = 2, y = 1$ **(c)** $x = 3, y = 3$
(d) $x = 1, y = 2$

Exercise 5O

1 **(a)** $(x + 4)(x + 1)$ **(b)** $(x + 1)(x + 7)$ **(c)** $(x + 8)(x + 1)$
(d) $(x + 6)(x + 1)$

2 **(a)** $(x + 12)(x + 1)$ **(b)** $(x + 9)(x + 2)$ **(c)** $(x + 5)(x + 3)$
(d) $(x + 10)(x + 2)$

3 **(a)** $(x - 3)(x - 4)$ **(b)** $(x - 5)(x - 3)$ **(c)** $(x - 2)(x - 10)$
(d) $(x - 5)(x - 5)$

4 **(a)** $(x - 4)(x - 2)$ **(b)** $(x - 3)(x - 5)$ **(c)** $(x - 6)(x - 4)$
(d) $(x - 10)(x - 3)$

5 **(a)** $(x + 4)(x - 2)$ **(b)** $(x - 3)(x + 4)$ **(c)** $(x - 2)(x + 9)$
(d) $(x + 3)(x - 12)$

6 **(a)** $(x + 9)(x - 1)$ **(b)** $(x + 2)(x - 8)$ **(c)** $(x - 7)(x + 4)$
(d) $(x + 5)(x - 8)$

7 **(a)** $(x + 1)(x + 17)$ **(b)** $(x - 7)(x + 2)$ **(c)** $(x - 14)(x - 2)$
(d) $(x + 7)(x - 6)$

Exercise 5P

1 **(a)** $^{-}1$, $^{-}3$ **(b)** $^{-}4$, $^{-}3$ **(c)** $^{-}5$, $^{-}3$ **(d)** $^{-}1$, $^{-}22$
2 **(a)** 4, 3 **(b)** 3, 5 **(c)** 2, 5 **(d)** 1, 13
3 **(a)** $^{-}4$, 2 **(b)** $^{-}16$, 1 **(c)** $^{-}23$, 1 **(d)** 2, $^{-}3$
4 **(a)** 7, $^{-}3$ **(b)** 11, $^{-}1$ **(c)** 8, $^{-}4$ **(d)** 7, $^{-}5$

Exercise 5Q – mixed questions

1 **(a)** $5x + 5y + 7$ **(b)** $5x + 11y + 7z$ **(c)** $25x + 2y$
(d) $8x + 4y + 3$

2 **(a)** 7 **(b)** 4 **(c)** $1\frac{7}{9}$ **(d)** 36 **(e)** 48
(f) 81 **(g)** 11

3 **(a)** $6p^7$ **(b)** $8q^7$ **(c)** 3 **(d)** $8y^6$ **(e)** $6x^4$ **(f)** $20x^4y$

4 **(a)** $xy(3x - 2)$ **(b)** $3y(2xy - 1)$ **(c)** $4x^2y^2(2x - 1)$
(d) $4x^2(3x - y)$ **(e)** $y(3x + 4x^2 - y)$ **(f)** $xy(8x - 2y - 1)$

5 **(a)** $\frac{8x+3}{x(x+1)}$ **(b)** $\frac{5y-2x}{xy}$ **(c)** $\frac{4x+3}{x}$ **(d)** $\frac{12y-2}{3y}$
(e) $\frac{8x-2}{(x-1)(x+1)}$ **(f)** $\frac{6x-4}{(x-2)(x-1)}$

6 **(a)** $x^2 + 4x + 1$ **(b)** $x^2 - 4$ **(c)** $x^2 + x - 12$
(d) $x^2 - 6x + 9$ **(e)** $6x^2 + 13x - 33$ **(f)** $6x^2 - 5x + 1$

7 **(a)** $x = 2$ **(b)** $d = \frac{3}{4}$ **(c)** $y = 2$ **(d)** $x = 100$
(e) $x = 39$ **(f)** $x = 2$

8 70

9 **(a)** $x = 1, y = 1$ **(b)** $x = 1, y = \frac{1}{2}$ **(c)** $x = 1, y = 2$

10 18 and 20

11 **(a)** $a + 12$ **(b)** $a - 18$ **(c)** $a + 12 = 3(a - 18)$; 33 years old

12 **(a)** $(x + 7)(x + 6)$ **(b)** $(x - 27)(x + 3)$
(c) $(x - 12)(x + 5)$ **(d)** $(x - 15)(x + 2)$

13 **(a)** $3, {}^{-}4$ **(b)** ± 4 **(c)** $1, {}^{-}4$ **(d)** $9, {}^{-}5$

14 height 16 cm, width 17 cm, length 19 cm

Consolidation

Exercise 5

1 **(a)** $3st^2$ **(b)** p^6 **(c)** $6p^5$ **(d)** $3r^7$ **(e)** p^2 **(f)** $3p$
(g) $9pr$ **(h)** $\frac{1}{4}st^2u^2$

2 **(a)** $6 + 9p$ **(b)** $6x - 12y$ **(c)** $2m^2 - 3mn$ **(d)** $6sr - 15s^2$
(e) $7x - y$ **(f)** $2gh + gf + fh + 2f^2$

3 **(a)** $\frac{21-x^2}{3x}$ **(b)** $\frac{20-x}{4}$ **(c)** $\frac{12y-x^2}{y}$ **(d)** $\frac{5x+11}{(x+1)(x+4)}$
(e) $\frac{13x-11}{(x-2)(2x-1)}$ **(f)** $\frac{-7}{(3x-4)(2x-5)}$

4 **(a)** 2 **(b)** 4 **(c)** 1 **(d)** 15 **(e)** 12 **(f)** 1
(g) ${}^{-}1$ **(h)** $12\frac{1}{3}$

5 **(a)** $3x^2 + 2x - 1$ **(b)** $x^2 - 49$ **(c)** $x^2 + 10x + 25$
(d) $9x^2 - 30x + 25$ **(e)** $4x^2 - 1$ **(f)** $6 - 5x + x^2$

6 **(a)** $x = 1, y = 1$ **(b)** $x = 1, y = \frac{1}{2}$ **(c)** $x = 1, y = {}^{-}1$

7 **(a)** \$480 **(b)** $2\frac{1}{3}$ hrs, i.e. 2 hr 20 mins

8 **(a)** $20(5x) + 5(x) = 945$
(b) 45 twenty dollar bills and 9 five dollar bills

9 **(a)** \$30 **(b)** \$20

10 **(a)** $x(x - 5) = 24$ **(b)** 3 m, 8 m

Check out

1 **(a)** $8y - x$ **(b)** $8xy - 5y$

2 **(a)** $8x - 12$ **(b)** $12x + 42y$ **(c)** $6x - 12xy$
(d) $18x^2 - 24x^3$ **(e)** $3x + 10xy - 2y$ **(f)** ${}^{-}7y^2 + 10y$

3 **(a)** $x(3 - x)$ **(b)** $x(4y - xy)$ **(c)** $6(x + 12)$
(d) $abc(a - 1 + b)$

4 **(a)** $\frac{7+4x}{x+1}$ **(b)** $\frac{8-ab}{4a}$ **(c)** $\frac{3}{(2x-1)(1-x)}$ **(d)** $\frac{a^2-b^2-a+b}{(a-1)(b-1)}$

5 **(a)** $x^2 + 14x + 33$ **(b)** $x^2 - 81$ **(c)** $4x^2 - 28x + 49$
(d) $15 - 11x + 2x^2$

6 **(a)** 1 **(b)** 3 **(c)** 9 **(d)** $1\frac{3}{4}$ **(e)** 1

7 5 cm × 9 cm

8 **(a)** $x = 2, y = 1$ **(b)** $x = 2, y = 2$ **(c)** $x = 1, y = {}^{-}2$

9 **(a)** $x = {}^{-}3$ and ${}^{-}1$ **(b)** $x = 1$ and 5 **(c)** $x = {}^{-}2$ and 4

6 Geometry

Check in

2 **(a)** **(i)** 1.6 cm **(ii)** 2.8 cm

Exercise 6A

1 **(a)** 38° **(b)** 57° **(c)** 77° **(d)** 78°

2 **(a)** $a = 42°, b = 138°, c = 138°$ **(b)** $d = 71.5°$

3 **(a)** $a = 60°$ **(b)** $b = 60°$ **(c)** $c = 45°$
(d) $d = 143°, e = 37°$

Exercise 6B

1 **(a)** $a = 35°, b = 145°$ **(b)** $c = 137°, d = 43°, e = 137°, f = 43°$
(c) $g = 118°, h = 118°, i = 62°$

2 **(a)** $a = 77°, b = 106°$ **(b)** $c = 52°, d = 128°, e = 128°, f = 52°$
(c) $g = 132°, h = 48°, i = 132°$ **(d)** $j = 63°, k = 63°, z = 117°$

3 **(a)** $a = 122°, b = 58°, c = 122°$ **(b)** $d = 78°, e = 102°$
(c) $f = 23°, g = 31°, h = 126°, i = 126°$

4 38°, 142°, 38°

Exercise 6C

1 **(a)** $a = 77°$ **(b)** $b = 21°$ **(c)** $c = 50°, d = 70°$

2 **(a)** $a = 87°$ **(b)** $b = 114°$ **(c)** $c = 43°$

3 **(a)** $a = 217°$ **(b)** $b = 55°, c = 21°$

Exercise 6D

5

Triangle	Number of equal sides	Number of equal angles
Equilateral	3	3
Isosceles	2	2
Scalene	0	0

Exercise 6E

1 **(a)** ABC, MNO; DEF, JKL; GHI, PQR

2 **(a)** yes

3 **(a)** yes **(b)** △s WOX, YOZ; △s WOZ, YOX

4 **(a)** no **(b)** yes

Exercise 6F

4 **(a)**

Quadrilateral	Two pairs of parallel sides	One pair of parallel sides	No parallel sides
A	✓	✗	✗
B	✓	✗	✗
C	✓	✗	✗
D	✗	✓	✗
E	✓	✗	✗
F	✓	✗	✗
G	✓	✗	✗
H	✗	✓	✗
I	✓	✗	✗
J	✗	✗	✓
K	✗	✗	✓

(b) A, B, C, F, G, I **(c)** C, F, I

5

Name of shape	Opposite sides equal	Opposite angles equal	Opposite sides parallel	All sides equal	All angle equal
square	✓	✓	✓	✓	✓
rectangle	✓	✓	✓	✗	✗
parallelogram	✓	✓	✓	✗	✗
rhombos	✓	✓	✓	✓	✗
trapezium	✗	✗	✗	✗	✗
kite	✗	✗	✗	✗	✗

Exercise 6G

3 (a) (ii), (vi) **(b)** (v) **(c)** none **(d)** (iii), (iv)
(e) (i) square **(ii)** kite **(iii)** quadrilateral
(iv) parallelogram **(v)** rectangle **(vi)** trapezium

Exercise 6H

2 (a) trapezium **(b)** parallelogram (or rhombus)

Exercise 6I

1

Name of triangle	Numbers of equal sides
Scalene	0
Isosceles	2
Equilateral	3

2

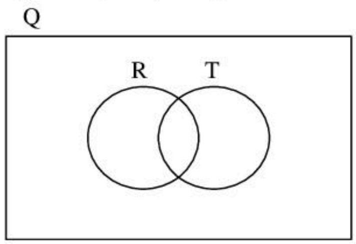
Name of quadrilateral	Number of lines of symmetry
Square	4
Rectangle	2
Kite	1
Parallelogram	0
Rhombus	0

3

Shape	Pairs of equal angles	Pairs of parallel sides
Kite	1	0
Rectangle	2	2
Rhombus	2	2
Square	2	2

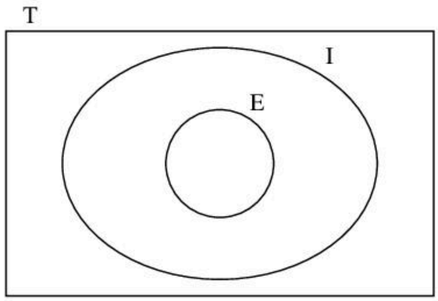
5 Six equal sides, six equal angles

6 (a)



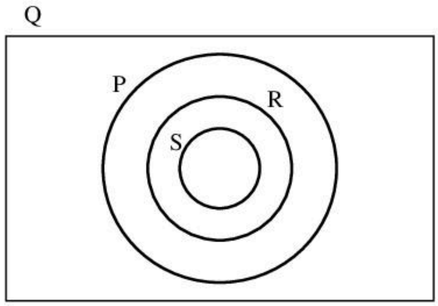
(b) R ∩ T = {squares}

7 (a)



(b) E ∩ T = {equilateral triangles}

8 (a)



(b) yes **(c)** yes

Exercise 6J

1 (b) 4 cm

Exercise 6K

2 (c) The perpendiculars intersect at a single point

Exercise 6N

4 (b) 90°

Exercise 6O

1 (g) a parallelogram

Exercise 6Q – mixed questions

1 **(a)** $a = 46°$ **(b)** $b = 41.5°$ **(c)** $c = 119°$ **(d)** $d = 51°$
2 **(b)** yes
(c) AOB, COD; AOD, COB; ABD, CDA
5 **(a)** $a = 68°, b = 112°, c = 101°$
(b) $d = 143°, e = 37°, f = 143°$

Consolidation

Exercise 6

1 **(a)** $a = 40°$ **(b)** $b = 60°, c = 95°$
(c) $d = 18°, e = 18°, f = 21°$
2 **(a)** $a = 65°, b = 115°$ **(b)** $c = 34°, d = 63°$ **(c)** $f = 92°$
(d) $g = 30°, h = 30°$
3 **(a)**

Triangle	Lines of symmetry	Number of equal sides
Equilateral	3	3
Isosceles	1	2
Right-angled	0	0

(b)

Quadrilateral	Lines of symmetry	Number of right angles	Number of parallel sides
Square	4	4	2
Parallelogram	0	0	2
Kite	1	0 (or possibly 2)	0
Trapezium	0	0 (or possibly 2)	0

5 **(b)** yes
6 **(b)** isosceles **(c)** 72°, 72°, 36°
7 **(b)** 6.34 m

Check out

1 **(a)** $a = 89°$ **(b)** $b = 80°, c = 66°$
2 **(a)** $a = 38°, b = 142°$ **(b)** $c = 64°, d = 116°$
3 **(a)** two equal sides, two equal angles, one line of symmetry
(b) two pairs of adjacent equal sides, two equal angles, one line of symmetry
(c) two pairs of equal sides, two pairs of equal angles, no lines of symmetry
4 **(a)** $\triangle$ABC, ADC **(b)** $\triangle$ABC, ADC

7 Trigonometry

Check in

1 **(a)** 64 cm^2 **(b)** x^2 cm^2
3 **(a)** 80° **(b)** 60°

Exercise 7A

1 **(a)** 9 squares, 16 squares **(b)** 25 squares **(c)** $9 + 16 = 25$
2 **(a)** 1 square, 9 squares **(b)** 10 squares **(c)** $1 + 9 = 10$
3 **(d)** The area of the larger square is equal to the sum of the areas of the two smaller squares.
5 **(a)** 20 mm **(b)** 50 mm **(c)** 13 mm **(d)** 29 mm
(e) 37 mm

6

	a	b	c	a^2	b^2	$a^2 + b^2$	c^2
(a)	12	16	20	144	256	400	400
(b)	30	40	50	900	1600	2500	2500
(c)	5	12	13	25	144	169	169
(d)	20	21	29	400	441	841	841
(e)	12	35	37	144	1225	1369	1369

The numbers in the last two columns are the same.

Exercise 7B

1 $h^2 = 7^2 + 24^2, h^2 = 49 + 576, h^2 = 625, h = \sqrt{625} = 25$
2 **(a)** 13 cm **(b)** 17 cm **(c)** 26 cm **(d)** 20 cm
3 **(a)** 3.61 cm **(b)** 9.43 cm **(c)** 5.66 cm **(d)** 6.08 cm
4 $a^2 + 4^2 = 5^2, a^2 + 16 = 25, a^2 = 9, a = 9$, so $a = 3$
5 **(a)** 8 cm **(b)** 6 cm **(c)** 9 cm **(d)** 16 cm
6 **(a)** 7.81 cm **(b)** 8.94 cm **(c)** 9.54 cm **(d)** 10.3 cm
(e) 9.75 cm **(f)** 5.39 cm **(g)** 10.77 cm **(h)** 6.24 cm
7 **(a)** **(i)** 7.07 cm **(ii)** 4.24 cm
(b) **(i)** 8.06 cm **(ii)** 9.22 cm
8 24.5 cm **9** 12 cm **10** 6 cm and 11.4 cm
11 26 and 16.6 cm **12** 39.4 cm
13 **(b)** and **(c)** 8.5 cm, 10.4 cm, 12 cm **14** **(a)** 2.4 cm
(b) 4.6 cm; angle BCD is not a right angle as $BD^2 \neq BC^2 + CD^2$.

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For example: {6, 8, 10}, {10, 24, 26}

Exercise 7C

1 **(a)** AC, AB, BC **(b)** DF, ED, EF **(c)** GI, HI, GH
(d) JL, JK, KL **(e)** OM, ON, MN **(f)** PR, PQ, QR
2 **(b)** **(i)** 0.25 cm **(ii)** 0.5 cm **(iii)** 1 cm **(iv)** 1.25 cm
(v) 2 cm **(vi)** 2.5 cm **(c)** **(i)** 0.25 **(ii)** 0.25
(iii) 0.25 **(iv)** 0.25 **(v)** 0.25 **(vi)** 0.25

(d)

	Angle	AB (adj)	BC (opp)	BC/AB (= opp/adj)
(i)	14°	1 cm	0.25 cm	$0.25 \div 1 = 0.25$
(ii)	14°	2 cm	0.5 cm	$0.5 \div 2 = 0.25$
(iii)	14°	4 cm	1 cm	$1 \div 4 = 0.25$
(iv)	14°	5 cm	1.25 cm	$1.25 \div 4 = 0.25$
(v)	14°	8 cm	2 cm	$2 \div 8 = 0.25$
(vi)	14°	10 cm	2.5 cm	$2.5 \div 10 = 0.25$

4

	Angle	AB (adj)	BC (opp)	BC/AB (= opp/adj)
(i)	45°	1 cm	1 cm	$1 \div 1 = 1$
(ii)	45°	2 cm	2 cm	$2 \div 2 = 2$
(iii)	45°	4 cm	4 cm	$4 \div 4 = 4$
(iv)	45°	5 cm	5 cm	$5 \div 5 = 5$
(v)	45°	8 cm	8 cm	$8 \div 8 = 8$
(vi)	45°	10 cm	10 cm	$10 \div 10 = 1$

The ratio opp : adj is always 1 : 1 for an angle of 45°.

5 **(c)** and **(d)**

Angle	adj	opp	Ratio (opp/adj)
10°	5 cm	0.9 cm	0.18
20°	5 cm	1.8 cm	0.36
30°	5 cm	2.9 cm	0.58
40°	5 cm	4.2 cm	0.84
50°	5 cm	6 cm	1.19
60°	5 cm	8.7 cm	1.73
70°	5 cm	13.8 cm	2.75

6 **(a)**

Angle	adj	opp	Ratio (opp/adj)
10°	8 cm	1.4 cm	0.18
20°	8 cm	2.9 cm	0.36
30°	8 cm	4.6 cm	0.58
40°	8 cm	6.7 cm	0.84
50°	8 cm	9.5 cm	1.19
60°	8 cm	13.8 cm	1.73
70°	8 cm	22 cm	2.75

(b) yes

7 **(a)** 3.6 cm **(b)** 5.8 cm **(c)** 11.9 cm **(d)** 27.5 cm

Exercise 7D

1 **(a)** 0.176 **(b)** 0.577 **(c)** 1.192 **(d)** 1
(e) 1.600 **(f)** 0.287 **(g)** 2.747 **(h)** 9.514 **(i)** 0.888

2

x°	10	20	30	40	50	60	70	80
tan x°	0.18	0.36	0.58	0.84	1.19	1.73	2.75	5.67

4 **(a)** 0.27 **(b)** 0.47 **(c)** 0.7 **(d)** 1.15
(e) 0.75 **(f)** 2.05 **(g)** 1.88 **(h)** 3.49 **(i)** 4.01

5 **(a)** 22° **(b)** 31° **(c)** 39° **(d)** 45° **(e)** 56°
(f) 63° **(g)** 52° **(h)** 67° **(i)** 60°

6 **(a)** Angles less than 45° have tangents that are less than 1. Angles greater than 45° have tangents greater than 1.
(b) The tangent approaches zero.
(c) The tangent becomes very large.

Exercise 7E

1 **(a)** 33.0° **(b)** 38.0° **(c)** 43.0° **(d)** 48.0°
(e) 54.1° **(f)** 71.0° **(g)** 21.8° **(h)** 41.6° **(i)** 77.7°

2 **(a)** $\frac{8}{15}$ **(b)** $\frac{3}{4}$ **(c)** $\frac{12}{35}$ **(d)** $\frac{8}{15}$ **(e)** $\frac{24}{10}$ **(f)** $\frac{s}{u}$

3 **(a)** 53.1° **(b)** 60.9° **(c)** 35.2° **(d)** 39.8°
(e) 46.6° **(f)** 51.3°

4 **(a)** 1.6, 58.0° **(b)** 0.636, 32.5° **(c)** 0.860, 40.7°
(d) 1.363, 53.7°

5 **(a)** 50.0° **(b)** 70.0° **(c)** 60.0° **(d)** 30.1° **(e)** 30.1°

6 $\tan x = p/q$ **(a)** 1, 45° **(b)** 0.7, 35.0° **(c)** 0.4, 21.8°
(d) 2, 63.4°

Exercise 7F

1 **(a)** 11.0 cm **(b)** 6.71 cm **(c)** 0.36 cm **(d)** 2.91 cm
(e) 16.3 cm **(f)** 21.8 cm

2 **(a)** 5.73 cm **(b)** 9.10 cm

3 4.23 cm **4** 51.2cm **5** 13.7 m

6 **(a)** 16.18 cm **(b)** 1.41 cm **(c)** 9.56 cm

7 7.98 m **8** 6.15 m **9** 39.8

Exercise 7G

1 10.5 m **2** 40.5 m

3 **(a)** 16 m **(b)** 34.3 m **(c)** 90.7 m

4 407 m

Exercise 7H

1 **(b)** **(i)** 0.5 cm **(ii)** 1.0 cm **(iii)** 2.0 cm **(iv)** 2.5 cm
(v) 4.0 cm **(vi)** 5.0 cm **(c)** **(i)** 0.5 cm **(ii)** 0.5
(iii) 0.5 **(iv)** 0.5 **(v)** 0.5 **(vi)** 0.5
(e) All the values are 0.5

2 **(b)** **(i)** 0.9 cm **(ii)** 1.7 cm **(iii)** 3.5 cm **(iv)** 4.3 cm
(v) 6.9 cm **(vi)** 8.7 cm **(c)** **(i)** 0.9 **(ii)** 0.85
(iii) 0.875 **(iv)** 0.86 **(v)** 0.8625 **(vi)** 0.87
(e) All the values are approximately 0.9

3

Angle	Radius	Height	Ratio (height/radius)
10°	5 cm	0.9 cm	0.18
20°	5 cm	1.7 cm	0.34
30°	5 cm	2.5 cm	0.5
40°	5 cm	3.2 cm	0.64
50°	5 cm	3.8 cm	0.76
60°	5 cm	4.3 cm	0.86
70°	5 cm	4.7 cm	0.94
80°	5 cm	4.9 cm	0.98

4 **(b)** The values are the same.

5 **(a)** 5 cm **(b)** 8.7 cm

6 **(a)** **(i)** It is always the same for a given angle.
(ii) It gets bigger as the angle gets closer to 90°.
(b) Yes although it does not get as large as the tangent ratio as the angle gets closer to 90°.

8 **(a)** 0.42 **(b)** 0.57 **(c)** 0.71 **(d)** 0.82

9 **(a)** 17° **(b)** 24° **(c)** 37° **(d)** 44°

Exercise 7I

1 **(a)** 0.530 **(b)** 0.707 **(c)** 0.970 **(d)** 0.906 **(e)** 0.5
(f) 0.866 **(g)** 0.250 **(h)** 0.400 **(i)** 0.910 **(j)** 0.943
(k) 1 **(l)** 0.999

2 **(a)** 34.0° **(b)** 54.0° **(c)** 63.0° **(d)** 64.2° **(e)** 19.3°
(f) 39.7° **(g)** 60.0° **(h)** 53.1° **(i)** 30° **(j)** 45.0°
(k) 26.4° **(l)** 14.7°

3 **(a)** 32.0° **(b)** 30° **(c)** 53.1° **(d)** 36.9° **(e)** 39.5°
(f) 45.1°

4 28.7° **5** 53.1° **6** 41.8° **7** 25.4°

Exercise 7J

1 **(a)** 3.80 cm **(b)** 7.13 cm **(c)** 5.93 cm **(d)** 4.16 cm

2 4.59 m **3** **(a)** 7.5 cm **(b)** 56.4°

4 332 m **5** 92.7 m

Exercise 7K

1 **(a)**, **(b)** and **(c)**

Angle x	Hypotenuse AC	Adjacent AB	$\cos x =$ (adjacent/hypotenuse)
30°	1 cm	0.9 cm	0.9
30°	2 cm	1.7 cm	0.85
30°	4 cm	3.5 cm	0.875
30°	5 cm	4.3 cm	0.86
30°	8 cm	6.9 cm	0.8625
30°	10 cm	8.7 cm	0.87

(c) All the values are approximately 0.9

2 **(a)**, **(b)** and **(c)**

Angle x	Hypotenuse AC	Adjacent AB	$\cos x =$ (adjacent/hypotenuse)
10°	5 cm	4.9 cm	0.98
20°	5 cm	4.7 cm	0.94
30°	5 cm	4.3 cm	0.86
40°	5 cm	3.8 cm	0.76
50°	5 cm	3.2 cm	0.64
60°	5 cm	2.5 cm	0.5
70°	5 cm	1.7 cm	0.34
80°	5 cm	0.9 cm	0.18

(c) It is the same as the last column for Question 3 of Exercise 6H but in reverse order.

3 **(a)** The cosine gets smaller.
(b) The values are the same but in reverse order.

5 **(a)** 0.82 **(b)** 0.71 **(c)** 0.57 **(d)** 0.42 **(e)** 0.5
(f) 0.87 **(g)** 0.17 **(h)** 0.91 **(i)** 0.99 **(j)** 0.97
(k) 0.98 **(l)** 0.02

6 **(a)** 73° **(b)** 66° **(c)** 53° **(d)** 46° **(e)** 76°
(f) 49° **(g)** 57° **(h)** 68°

7 **(a)** 0.743 **(b)** 0.423 **(c)** 0.878 **(d)** 0.992 **(e)** 0.091
(f) 0.983 **(g)** 0.415 **(h)** 0.876

8 **(a)** 53.2° **(b)** 36.0° **(c)** 27.0° **(d)** 70.7° **(e)** 50.3°
(f) 36.9° **(g)** 30.0° **(h)** 45.0°

Exercise 7L

1 **(a)** 48.2° **(b)** 67.4° **(c)** 27.3° **(d)** 45.9°
(e) 36.9° **(f)** 41.4°

2 50.2° **3** 38.2° **4** 70.5° **5** 72.5°

Exercise 7M

1 **(a)** 6.93 cm **(b)** 6 cm **(c)** 9.45 cm **(d)** 7.07 cm
(e) 18.0 cm **(f)** 2.91 m

2 22.6 cm **3** 4.12 cm

4 **(a)** 6 cm **(b)** 60°

5 **(a)** 69.7 m **(b)** 6.1 m

6 140 m

Exercise 7N – mixed questions

1 5.83 m

2 **(a)** 33.7° **(b)** 4.93 cm **(c)** 22.9 cm **(d)** 36.9°
(e) 2.2 cm **(f)** 30° **(g)** 14.9 cm

3 **(a)** 10.4 cm, 60° **(b)** 12 cm, 22.6° **(c)** 7.28 cm, 5.29 cm
(d) 5 cm, 53.1°

4 3.61 m **5** 6.71 m **6** 7.75 m, i.e. 1.04 m higher

7 6.93 m

8 **(b)** 4 km **(c)** 1.44 km

9 17.2 cm

10 **(a)** 3.21 cm **(b)** 44.4° **(c)** 32.2° **(d)** 20.0 cm
(e) 50.8° **(f)** 30.1 cm

11 **(a)** 3.76 m **(b)** 0.12 m

12 **(a)** 16.3° **(b)** 25 cm

13 2.36 m **14** 34.6 cm

15 $r = 18$ cm, $s = 15.6$ cm, $t = 30°$, $u = 60°$

16 1134 m

17 **(a)** 5 **(b)** 3.16

18 20 cm **19** 53.1° **20** 36.9°

21 **(a)** 5 cm **(b)** 13 cm **(c)** 13 cm

22 **(a)** 7.81 cm **(b)** 12.5 cm

23 **(b)** 36.1 cm **(c)** 37.4 cm

24 (b), (c), (d), (e) and (f)

25 **(a)** 8.60 **(b)** 10.5 **(c)** 18.3

26 **(a)** **(i)** both 0.5 **(ii)** both 0.799 **(iii)** both 0.358
(iv) both 0.707
(b) Both values are the same. Both angles come from the same right-angled triangle.

27 **(a)** $5^2 = 3^2 + 4^2$ **(b)** all are 53.1°

28 **(a)** yes **(c)** yes

29 **(a)** 0.28 **(b)** 0.96 **(c)** 0.0784 **(d)** 0.9216 **(e)** 1

30 **(a)** $\sin\theta = b/c$, $\cos\theta = a/c$ **(b)** $\frac{b^2}{c^2}+\frac{a^2}{c^2}=\frac{c^2}{c^2}=1$

31 **(b)** c^2 **(c)** a^2 and b^2

32 **(b)** The area of the larger semicircle equals the sum of the areas of the other two semicircles.
(c) The area of the larger triangle equals the sum of the areas of the other two triangles.

Consolidation

Exercise 7

1 **(a)** 20 cm **(b)** 39 cm **(c)** 8.06 cm (2 d.p.) **(d)** 20 cm
(e) 13.86 cm (2 d.p.) **(f)** 8.66 cm (2 d.p.)

2 **(a)** 60.0° **(b)** 45.0° **(c)** 30.0° **(d)** 53.1°
(e) 87.1° **(f)** 7.1° **(g)** 85.8° **(h)** 52.0°

3 **(a)** 36.9° **(b)** 53.1° **(c)** 36.9° **(d)** 40.0°
(e) 40.9° **(f)** 30.8°

4 **(a)** 2.89 cm **(b)** 11.3 cm **(c)** 11.5 cm **(d)** 6.3 cm
(e) 5.90 cm **(f)** 9.39 cm

5 **(a)** Check students' work **(b)** 50.3 m

6 **(a)** 10 km **(b)** 13.4 km

7 5.05 m

Check out

1 **(a)** 5 **(b)** 11.3 **(c)** 28.9

2 **(a)** 6 cm, 8 cm **(b)** 1.5 m, 2 m **(a)** 1.73 cm **(b)** 39.3°

3 **(a)** 3.54 cm **(b)** 44.0° **(a)** 16 cm **(b)** 56.6°

8 Measurement 2

Check in

2 **(a)** 48 cm^2 **(b)** 18 cm^2 (c) 200.96 cm^2

3 **(a)** 122.5 cm^2 **(b)** 2 cm

Exercise 8A

1 **(a)** m^3 **(b)** cm^3 **(c)** cm^3 **(d)** m^3 **(e)** mm^3 **(f)** cm^3
(g) m^3 **(h)** cm^3 **(i)** cm^3 **(j)** cm^3 or m^3
3 **(a)** 100 **(b)** 10 000 **(c)** 1 000 000
4 **(a)** 4 cm^3 **(b)** 8 cm^3 **(c)** 27 cm^3 **(d)** 21 cm^3

Exercise 8B

1 **(a)** 36 cm^3 **(b)** 45 cm^3 **(c)** 504 cm^3 **(d)** 45 cm^3
2 **(i)** 144 cm^3 **(ii)** 2 cm **(iii)** 20 cm **(iv)** 3 cm
3 **(a)** 4 cm^2, 16 cm^3 **(b)** 8.75 cm^2, 35 cm^3
4 **(i)** 16 cm **(ii)** 41.6 cm^3 **(iii)** 13 cm^3 **(iv)** 8 cm^2
5 3 m **6** 48 **7** 30 **8** 30
9 **(a)** **(i)** 6 **(ii)** 6 000 000 **(b)** 6000
10 1 m

Exercise 8C

1 **(a)** 90 cm^3 **(b)** 30 cm^3 **(c)** 12.5 cm^3 **(d)** 105 cm^3
2 **(a)** 26 cm^2 **(b)** 78 cm^3
3 **(a)** 200 cm^3 **(b)** 202.5 cm^3
4 **(a)** 172.8 cm^3 **(b)** 202.5 cm^3
5 **(a)** 88 cm^2 **(b)** 17 600 cm^3

Exercise 8D

1 **(a)** 1540 cm^3 **(b)** 176 cm^3
2 **(a)** **(i)** 180.9 cm^3 **(ii)** 128.6 cm^3 **(b)** **(i)** 12 cm **(ii)** 6 cm
3 **(a)** 310 860 cm^3 **(b)** 310.86 litres **(c)** 425
4 **(a)** 628 cm^3 **(b)** 72 cm^3 **(c)** 556 cm^3
5 **(a)** 28.3 cm^3 **(b)** 226 cm^3
6 **(a)** 14 cm **(b)** 703 cm^3 **(c)** 1328 g
7 5966 cm^3, 67 416 g

Exercise 8E

1 **(a)** 66.7 cm^3 **(b)** 133 cm^3 **(c)** 100 cm^3
2 **(a)** 117 cm^3 **(b)** 528 cm^3 **(c)** 1410 cm^3
3 24 cm^3 **4** 9 cm **5** 245 m^3
6 707 cm^3 **7** **(a)** 218 cm^3 **(b)** 0.218 litres **(c)** 8.96 cm
8 1.06 m^3

Exercise 8F

1 **(a)** 6 cm^2 **(b)** 54 cm^2
2 144 cm^2
3 **(a)** 37.5 cm^2, 150 cm^2 **(b)** It is multiplied by 4.
4 **(a)** 158 cm^2 **(b)** 72 cm^2 **(c)** 186 cm^2 **(d)** 101 cm^2

7 **(a)** 50 m^2 **(b)** 8.9 litres
8 **(a)** 52 cm^2 **(b)** 60 cm^2 **(c)** 15.4% **(d)** by 31.1%

Exercise 8G

1 **(a)** 408 cm^2 **(b)** 96 m^2
2 **(a)** **(i)** 314 cm^2 **(ii)** 113 cm^2 **(iii)** 565 cm^2
(b) **(i)** 471 cm^2 **(ii)** 170 cm^2 **(iii)** 918 cm^2

4 6.8 m^2 **5** **(a)** 12.28 m^2 **(b)** 1.1 litres

Exercise 8H

1 **(a)** 105 cm^2 **(b)** 224 cm^2 **(c)** 88.5 cm^2
2 32 cm^2 **3** 102.6 cm^2
4 **(a)** 219.8 cm^2 **(b)** 471 cm^2

Exercise 8I – mixed questions

1 **(a)** 2 000 000 cm^3 **(b)** 5000 mm^3 **(c)** 4.3 cm^3
(d) 0.05 m^3
2 225%
3 **(a)** the cube **(b)** 27 **(c)** 750
4 **(a)** 10.8 cm^2 **(b)** 98.5 cm^3
5 **(a)** 15 m^2 **(b)** 10
6 6 cm
7 40
8 **(a)** 16 **(b)** 57.6 litres
9 **(a)** 261.7 cm^3 **(b)** 9.6 cm
10 35.75 m^3
11 16 600 cm^3
12 **(a)** 256 cm^3 **(b)** 870.8 g
13 **(a)** 68.3 cm^2 **b)** 2050 cm^3
14 12 cm
15 **(a)** 7.1 cm **(b)** **(i)** 139 cm^3 **(ii)** 6

Consolidation

Exercise 8

1 Check students' answers for suitability.
2 **(a)** 288 cm^3 **(b)** 176 cm^3 **(c)** 3690 cm^3
(d) 108 cm^3 **(e)** 301 cm^3
3 **(a)** 336 cm^2 **(b)** 286 cm^2 **(c)** 1360 cm^2 **(d)** 150 cm^2
(e) 301 cm^2
4 **(a)** 15.3 cm **(b)** 2.44 cm **(c)** 242 cm^3
5 **(a)** 23.598 m^3 **(b)** 23.6 m^3 **(c)** 1944
6 **(a)** 2.03 m^3 **(b)** 2030 litres **(c)** 53 cm **(d)** 6.78 m^2

Check out

1 **(b)**
2 **(a)** 972 m^3 **(b)** 753.6 cm^3
3 **(a)** 93.3 cm^3 **(b)** 513 cm^3
4 **(a)** 188 m^2 **(b)** 967 cm^2

Revision exercise 2

Give your answers to three significant figures where appropriate

1 **(a)** $9x - 2y$ **(b)** $5a - 4b + 8c$ **(c)** $3x^5 - 7y^5 + z^6$
(d) $6x + 25y$ **(e)** $2xy + 5xz + yz$

2 **(a)** $2(3x + 4y)$ **(b)** $r(a - 2b)$ **(c)** $a(4c - 3b + 6d)$
(d) $abc(4a - 3 + 5b)$
3 **(a)** $\frac{5x+3}{x(x+1)}$ **(b)** $\frac{7x+1}{(x-1)(x+1)}$ **(c)** $\frac{13+12x}{3+2x}$ **(d)** $\frac{x+13}{20}$
4 **(b)** $x^2 + 12x + 35$

5 A = $b(a - b)$ cm^2, B = b^2 cm^2, C = $b(a - b)$ cm^2

6 **(a)** $x^2 + 8x + 15$ **(b)** $x^2 + 2x - 15$ **(c)** $x^2 - 8x + 15$
(d) $x^2 - 25$

7 **(a)** $x = 1, y = 1$ **(b)** $x = 1, y = 3$ **(c)** $x = \frac{1}{2}, y = 1$
(d) $x = 1, y = \frac{1}{2}$

8 **(a)** 7 **(b)** 13 **(c)** 3 **(d)** $x < 3$ **(e)** $x > 6$

9 $2n + 7 = 37$, $n = 15$

10 Rita's age is $c + 5$, Joe's age is $c + 28$;
$c + (c + 5) + (c + 28) = 51$;
Clive is 6, Rita is 11, Joe is 34

11 **(a)** $5c + 3(c + 0.5) = 7.9$
(b) c = \$0.80, adult ticket = \$1.30

12 **(a)** $a = 120°$, $b = 60°$ **(b)** $c = 16°$ **(d)** $d = 66°$

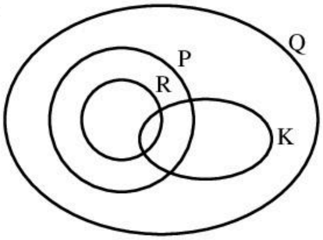
13 e.g. **(a)** kite **(b)** rectangle **(c)** square

14 $x = 30°$

15 **(a)** Two pairs of parallel sides, two pairs of equal angles, four equal sides, diagonals bisect at 90°
(b) Three equal sides, three equal angles, three lines of symmetry, three vertices

18 **(a)** yes **(b)** ADC, ABC and AOD, COD

19 **(a)**



(b) **(i)** R **(ii)** P **(iii)** K

20 **(a)** 5040° **(b)** 168° **21** **(a)** 40° **(b)** 110° **(c)** 6 cm

22 **(a)** 10.6 cm **(b)** 8 cm **(c)** 48 cm **(d)** 10.9 cm

23 **(a)** $x = 5.9$ cm, $y = 10.3$ cm **(b)** $x = 6.9$ cm, $y = 3.5$ cm
(c) $x = 10.05$ cm, $y = 10.10$ cm, $u = 10.15$ cm, $v = 10.20$ cm

24 3.3 m **25** 40.6 km, 11.3 km **26** 8.77 cm

27 **(a)** 0.62 **(b)** 0.75 **(c)** 0.37 **(d)** 0.25 **(e)** 0.88
(f) 0.92

28 **(a)** 57° **(b)** 21.3° **(c)** 31.0° **(d)** 72.4°
(e) 45° **(f)** 73.2°

29 **(a)** 36.9° **(b)** 30° **(c)** 36.9°

30 **(a)** 4.6 cm **(b)** 4.8 cm **(c)** 3.5 cm

31 **(a)** 11.0 cm **(b)** 3.4 cm

32 053° **33** 61°, 49°

34 **(a)** 2.5 cm **(b)** 5.9 cm **(c)** 8.4 cm

35 16.5 m **36** 4.5 m

37 **(a)** 1000000 cm^3 **(b)** 50 000 cm^3 **(c)** 0.2 m^3 **(d)** 0.4 cm^3

38 **(a)** 84 cm^3 **(b)** 96 cm^3 **(c)** 360 cm^3

39 **(a)** 214 cm^2 **(b)** 1190 cm^2

40 **(a)** 4.4 cm **(b)** 166.7 cm^2

41 **(a)** 301 cm^3 **(b)** 267 cm^3

42 166 cm^2

43 **(a)** 2000 litres **(b)** 5000 litres

44 100

45 2.6 cm

Mixed questions 2

1 A **2** C **3** D **4** C **5** B **6** B **7** B
8 D **9** B **10** A **11** A **12** A **13** C **14** C
15 C **16** C **17** B **18** C **19** A **20** A **21** C
22 C **23** A **24** D **25** B **26** C **27** C **28** D
29 C **30** D **31** A **32** A **33** D **34** D **35** D
36 D **37** D **38** C **39** D **40** A **41** B

9 Functions and Graphs

Check in

1 P(1, 4), Q(3, 0), R(5, 1)

2 **(a)** 0.125 **(b)** 0.875 **(c)** 0.818 **(d)** 0.666

3

x	$^-2$	$^-1$	0	1	2	3
y	$^-8$	$^-5$	$^-2$	1	4	7

4 (3, 1), (6, 2), (9, 3), (12, 4)

Exercise 9A

1 A (1, 2), B (1, $^-2$), C ($^-2$, 3), D ($^-4$, 1), E ($^-4$, $^-1$), F ($^-3$, $^-4$), G (0, $^-4$), H (3, $^-2$)

2 **(a)** They all have x-coordinate 4.
(b) They all have y-coordinate 3.

3 **(c)** Cat

4 **(a)** parallelogram, square, rectangle
(b) isosceles, right-angled, scalene
(c) DFG and BCH or FDH and CHD

5 R, V, S, P, W, Y, Q, M, L, N, X, Z, T

6 (2, 2), (2, 3), (8, 3), (8, 4), (9, 3), (9, 1), (8, 1), (8, 2)

Exercise 9B

1 **(a)** **(i)** 0.4 **(ii)** 0.1 **(iii)** 1 **(b)** **(i)** 2 **(ii)** 4 **(iii)** 3

2 A (1, 0.5), B (0.5, $^-0.5$), C (1, 1.4), D (1.5, 0.3), E (2, 0.8), F ($^-0.5$, $^-0.6$), G ($^-0.8$, 0), H (0, $^-0.3$),I ($^-0.8$, $^-0.9$), J ($^-0.5$, 0.4), K ($^-0.4$, 1.2), L (1.4, $^-0.8$), M (1.8, $^-0.4$)

3 **(c)** Rooster

4 **(a)** 5, 0.2
(b) A (1, 2), B (1.4, 1), C (2, 3.8), D (2.6, 3.2), E (5.6, 1.6), F (5.2, 2.8), G (3.4, 0), H (0, 3.6), I (4.5, 1), J (3.8, 2.5), K (4.3, 3.7), L (0.4, 0.3), M (2.2, 1.5), N (5.7, 0.5)

5 **(a)** 1 **(b)** 0.5
(c) A (5, 5), B (5, 15), C (9, 5), D (10, 3.5), E (9, $^-5$), F (3.5, $^-6$), G (1, $^-4$), H ($^-3.5$, 3), I ($^-1.5$, 14), J ($^-2.5$, $^-3$), K ($^-4.5$, $^-8$), L ($^-1.5$, 1), M ($^-2.5$, $^-10$), N (7, 8.5), O (6.5, $^-8.5$), P (11.5, $^-8$)

7 **(a)** 0.4 **(b)** yes
(c) A (7.6, 3.2), B (2, 5), C (2.8, 2), D (5.2, 0.8), E (0.8, $^-2$), F ($^-4$, 2), G ($^-2.4$, 1.2), H ($^-1.2$, 5), I ($^-3.6$, $^-3$), J (3.2, $^-2.2$), K ($^-1.6$, $^-3.6$), L (5.2, $^-3.8$), M (7.8, $^-2.2$)

8 **(b)** Coat hanger

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Figure 1: A (0.4, 0.7), B (1.6, 0.3), C (2.7, 1.4), D (2, $^-0.6$)
Figure 2: A (2, 3.5), B (8, 1.5), C (13.5, 7), D (10, $^-3$)
Figure 3: A (0.8, 1.4), B (3.2, 0.6), C (5.4, 2.8), D (4, $^-1.2$)
Figure 4: A (4, 14), B (16, 6), C (27, 28), D (20, $^-12$)
The coordinates are different in each figure because the scales are different.

Exercise 9C

1 **(a)** 100 people **(b)** 12 minutes **(c)** **(i)** 2300
(ii) 2500 **(iii)** 2950 **(d)** **(i)** 10.36 am **(ii)** 10.48 am
(iii) 11.24 am, 12.30 pm, 2.24 pm

2 **(a)** **(i)** 12 litres **(ii)** 8 litres **(iii)** 7 litres
(b) **(i)** 0 km **(ii)** 15 km **(iii)** 150 km
(c) after 75 km; 9.5 litres

3 **(a)** **(i)** \$7.50 **(ii)** \$10 **(iii)** \$23
(b) **(i)** 10 **(ii)** 26 **(iii)** 96 **(c)** \$2
4 **(a)** **(i)** US\$5.60 **(ii)** US\$7.60 **(iii)** US\$3.00
(b) **(i)** EC\$16 **(ii)** EC\$11.50 **(iii)** EC\$9.50
5 **(a)** **(i)** 40 g **(ii)** 56 g **(iii)** 84 g
(b) **(i)** 25 cm^3 **(ii)** 40 cm^3 **(iii)** 130 cm^3
(c) 0.8 g/cm^3
6 **(a)** **(i)** 28 m/s **(ii)** 40 m/s **(iii)** 68 m/s
(b) **(i)** 7 s **(ii)** 18 s **(iii)** 28 s
7 **(a)** **(i)** 7 hours **(ii)** 3 hours 36 minutes
(iii) 8 hours 48 minutes
(b) **(i)** 48 km/h **(ii)** 32 km/h **(iii)** 35 km/h
8 **(a)** **(i)** 4 kg **(ii)** 7.75 kg **(iii)** 11.25 kg
(b) **(i)** 1 month **(ii)** 9 months **(iii)** 5 months
9 **(a)** **(i)** 3 m **(ii)** 5.3 m **(iii)** 2 m
(b) **(i)** $1\frac{1}{4}$ h, $11\frac{1}{4}$ h **(ii)** 2 h, $10\frac{1}{2}$ h
(iii) 3 hours 23 minutes, 9 hours 8 minutes

Exercise 9E

3 **(a)** add 6 **(b)** square **(c)** is a factor of

Exercise 9F

1 **(a)** **(i)**, **(iii)**
2 **(b)** **(i)**, **(ii)**, **(iii)** are functions

Exercise 9G

3 **(a)** $fx \to x + 2$ **(b)** $f : x \to 3x$ **(c)** $f : x \to 3x + 1$
(d) $f : x \to 3x - 2$
4 **(a)** 12 **(b)** $^{-}1$ **(c)** 2 **(d)** $^{-}5$ **(e)** 4
5 **(a)** **(i)** 11 **(ii)** $^{-}7$ **(iii)** 5
(b) **(i)** 11 **(ii)** 11 **(iii)** 3
(c) **(i)** $\frac{1}{3}$ **(ii)** $^{-}\frac{1}{3}$ **(iii)** 1
(d) **(i)** $^{-}3$ **(ii)** $^{-}3$ **(iii)** 5

Exercise 9H

2 **(a)**, **(c)** and **(d)**
4 **(a)** {3, 5, 7, 9, 11} **(b)** {4, 3, 2, 1, 0}
(c) {7, 4, 3, 4, 7} **(d)** {$^{-}2$, $^{-}1.5$, $^{-}1$, $^{-}0.5$, 0}
5 **(b)**, **(i)**, **(ii)** only
6 **(c)** 11.5 **(d)** 3.3
7 **(b)** **(i)** 1.2 cm **(ii)** 3.6 cm **(c)** 22.5 days **(d)** $h = \frac{d}{2.5}$

Exercise 9I

1 **(b)** (i), (iv)

2 **(a)**

x	$^{-}2$	$^{-}1$	0	1	2	3	4
2*x*	$^{-}4$	$^{-}2$	0	2	4	6	8
+1	1	1	1	1	1	1	1
y	$^{-}3$	$^{-}1$	1	3	5	7	9

3 **(a)** **(i)**

x	$^{-}2$	$^{-}1$	0	1	2	3	4
y	$^{-}6$	$^{-}3$	0	3	6	9	12

(ii)

x	$^{-}2$	$^{-}1$	0	1	2	3	4
y	$^{-}5$	$^{-}2$	1	4	7	10	13

(iii)

x	$^{-}2$	$^{-}1$	0	1	2	3	4
y	$^{-}7$	$^{-}6$	$^{-}5$	$^{-}4$	$^{-}3$	$^{-}2$	$^{-}1$

(iv)

x	$^{-}2$	$^{-}1$	0	1	2	3	4
y	$^{-}9$	$^{-}7$	$^{-}5$	$^{-}3$	$^{-}1$	1	3

(v)

x	$^{-}2$	$^{-}1$	0	1	2	3	4
y	$^{-}1$	$^{-}\frac{1}{2}$	0	$\frac{1}{2}$	1	$1\frac{1}{2}$	2

(vi)

x	$^{-}2$	$^{-}1$	0	1	2	3	4
y	3	2	1	0	$^{-}1$	$^{-}2$	$^{-}3$

5 **(a)**

x	$^{-}3$	$^{-}2$	$^{-}1$	0	1	2	3
y	9	8	7	6	5	4	3

(c) (0, 6)

6 (b), (c), (d)

7 **(a)** **(i)**

x	$^{-}3$	$^{-}2$	$^{-}1$	0	1	2	3
y	$^{-}11$	$^{-}8$	$^{-}5$	$^{-}2$	1	4	7

(ii)

x	$^{-}3$	$^{-}2$	$^{-}1$	0	1	2	3
y	$^{-}10$	$^{-}7$	$^{-}4$	$^{-}1$	2	5	8

(iii)

x	$^{-}3$	$^{-}2$	$^{-}1$	0	1	2	3
y	$^{-}9$	$^{-}6$	$^{-}3$	0	3	6	9

(iv)

x	$^{-}3$	$^{-}2$	$^{-}1$	0	1	2	3
y	$^{-}8$	$^{-}5$	$^{-}2$	1	4	7	10

(v)

x	$^{-}3$	$^{-}2$	$^{-}1$	0	1	2	3
y	$^{-}7$	$^{-}4$	$^{-}1$	2	5	8	11

(vi)

x	$^{-}3$	$^{-}2$	$^{-}1$	0	1	2	3
y	$^{-}6$	$^{-}3$	0	3	6	9	12

(c) They are all parallel. **(d)** **(i)** (0, $^{-}2$) **(ii)** (0, $^{-}1$)
(iii) (0, 0) **(iv)** (0, 1) **(v)** (0, 2) **(vi)** (0, 3)

8 **(a)** **(i)**

x	$^{-}3$	$^{-}2$	$^{-}1$	0	1	2	3
y	$\frac{1}{2}$	1	$1\frac{1}{2}$	2	$2\frac{1}{2}$	3	$3\frac{1}{2}$

(ii)

x	$^{-}3$	$^{-}2$	$^{-}1$	0	1	2	3
y	$^{-}1$	0	1	2	3	4	5

(iii)

x	$^{-}3$	$^{-}2$	$^{-}1$	0	1	2	3
y	$^{-}4$	$^{-}2$	0	2	4	6	8

(iv)

x	$^{-}3$	$^{-}2$	$^{-}1$	0	1	2	3
y	$^{-}7$	$^{-}4$	$^{-}1$	2	5	8	11

(c) They increase in steepness.
(d) They all pass through the same point (0, 2).

Exercise 9J

1 **(b)** **(i)** 4 **(ii)** $\frac{1}{2}$ **(iii)** 3 **(iv)** 2
(c) **(i)** (0, 0) **(ii)** (0, 1) **(iii)** (0, 4) **(iv)** (0, 0)
2 **(b)** $y = 3x + 1$ **(c)** (0, 1) in each case

3

Equation	Gradient	Coordinates of y-intercept
$y = 2x + 1$	2	(0, 1)
$y = 3x - 1$	3	(0, $^-1$)
$y = \frac{1}{2}x + 1$	$\frac{1}{2}$	(0, 1)
$y = {}^-x$	$^-1$	(0, 0)
$y = x$	1	(0, 0)
$y = 4 - x$	$^-1$	(0, 4)
$y = 4 - 2x$	$^-2$	(0, 4)

4 **(a)** $y = 2x + 1$, $y = 3x - 1$, $y = \frac{1}{2}x + 1$, $y = x$
(b) $y = {}^-x$, $y = 4 - x$, $y = 4 - 2x$ **(c)** $y = {}^-x$, $y = 4 - x$

5

Equation	Gradient	Coordinates of y-intercept
$y = 2x + 3$	2	(0, 3)
$y = 3x - 1$	3	(0, $^-1$)
$y = {}^-2x + 5$	$^-2$	(0, 5)
$y = {}^-x - 2$	$^-1$	(0, $^-2$)

Exercise 9K

1 **(a)**

x	$^-1$	0	1	2	3	4	5
y	$^-4$	$^-1$	2	5	8	11	14

x	$^-1$	0	1	2	3	4	5
y	$^-1$	1	3	5	7	9	11

(c) (2, 5) **(d)** $x = 2$, $y = 5$

2 **(a)** $x = 1$, $y = 3$ **(b)** $x = 3$, $y = 0$ **(c)** $x = 4$, $y = 11$
(d) $x = {}^-7$, $y = {}^-17$

3 **(a)** $x = 4$, $y = 2$ **(b)** $x = 3$, $y = 6$ **(c)** $x = 1$, $y = 3$
(d) $x = {}^-4$, $y = {}^-5$

4 **(b)** They are parallel. **(c)** The lines do not meet.

Exercise 9L

1 **(a)**

x	$^-3$	$^-2$	$^-1$	0	1	2	3
x^2	9	4	1	0	1	4	9
$+3$	3	3	3	3	3	3	3
y	12	7	4	3	4	7	12

(d) (0, 3)

2

x	$^-3$	$^-2$	$^-1$	0	1	2	3
y	10	5	2	1	2	5	10

3

x	$^-3$	$^-2$	$^-1$	0	1	2	3
y	12	7	4	3	4	7	12

4

x	$^-3$	$^-2$	$^-1$	0	1	2	3
y	5	0	$^-3$	$^-4$	$^-3$	0	5

5 They have the same shape but are translated up and down the y axis.

6

x	$^-2$	$^-1$	0	1	2
x^2	4	1	0	1	4
$y = 6 - x^2$	2	5	6	5	2

The value of y reaches a maximum rather than a minimum.

7 **(a)**

x	$^-3$	$^-2$	$^-1$	0	1	2	3
x^2	9	4	1	0	1	4	9
$-3x$	9	6	3	0	$^-3$	$^-6$	$^-9$
y	18	10	4	0	$^-2$	$^-2$	0

(b) (0, 0) and (3, 0)

8 **(a)**

x	$^-3$	$^-2$	$^-1$	0	1	2	3
x^2	9	4	1	0	1	4	9
$-2x$	6	4	2	0	$^-2$	$^-4$	$^-6$
3	3	3	3	3	3	3	3
y	18	11	6	3	2	3	6

(c) It does not.

9 **(a)** **(i)**

x	$^-3$	$^-2$	$^-1$	0	1	2	3
x^2	9	4	1	0	1	4	9
$-3x$	9	6	3	0	$^-3$	$^-6$	$^-9$
$+4$	4	4	4	4	4	4	4
$f(x)$	22	18	8	4	2	2	4

(ii)

x	$^-3$	$^-2$	$^-1$	0	1	2	3
x^2	9	4	1	0	1	4	9
$-2x$	6	4	2	0	$^-2$	$^-4$	$^-6$
-1	$^-1$	$^-1$	$^-1$	$^-1$	$^-1$	$^-1$	$^-1$
$f(x)$	14	7	2	$^-1$	$^-2$	$^-1$	2

(iii)

x	$^-3$	$^-2$	$^-1$	0	1	2	3
2	2	2	2	2	2	2	2
$3x$	$^-9$	$^-6$	$^-3$	0	3	6	9
$-x^2$	$^-9$	$^-4$	$^-1$	0	$^-1$	$^-4$	$^-9$
$f(x)$	$^-16$	$^-8$	$^-2$	2	4	4	2

(iv)

x	$^-3$	$^-2$	$^-1$	0	1	2	3
4	4	4	4	4	4	4	4
$-3x$	9	6	3	0	$^-3$	$^-6$	$^-9$
$-x^2$	$^-9$	$^-4$	$^-1$	0	$^-1$	$^-4$	$^-9$
$f(x)$	4	6	6	4	0	$^-6$	$^-14$

(v)

x	$^-3$	$^-2$	$^-1$	0	1	2	3
$3x^2$	27	12	3	0	3	12	27
$-2x$	6	4	2	0	$^-2$	$^-4$	$^-6$
-1	$^-1$	$^-1$	$^-1$	$^-1$	$^-1$	$^-1$	$^-1$
$f(x)$	32	15	4	1	0	7	20

(vi)

x	$^-3$	$^-2$	$^-1$	0	1	2	3
6	6	6	6	6	6	6	6
$-5x$	15	10	5	0	$^-5$	$^-10$	$^-15$
$-2x^2$	$^-18$	$^-8$	$^-2$	0	$^-2$	$^-8$	$^-18$
$f(x)$	3	8	9	6	$^-1$	$^-12$	$^-27$

Exercise 9M

1 (b) $x = 2.4$ or $x = {}^-2.4$

2 (a)

x	$^-3$	$^-2$	$^-1$	0	1	2	3
y	4	$^-1$	$^-4$	$^-5$	$^-4$	$^-1$	4

(b)

x	$^-3$	$^-2$	$^-1$	0	1	2	3
y	12	6	2	0	0	2	6

(c)

x	$^-3$	$^-2$	$^-1$	0	1	2	3
y	16	9	4	1	0	1	4

(d)

x	$^-3$	$^-2$	$^-1$	0	1	2	3
y	16	8	2	$^-2$	$^-4$	$^-4$	$^-2$

(e)

x	$^-3$	$^-2$	$^-1$	0	1	2	3
y	30	20	10	6	2	0	0

3 (b) $^-2.2$, 2.2; 0, 1; 1; 3.5, $^-0.5$; 2, 3

4 (a) $^-2.8$, 2.8 **(b)** –3, 1 **(c)** $^-3$, 2 **(d)** 1, 4

Exercise 9N – mixed questions

1 telephone **2** scissors **3** lamp

4 (2.2, 2.2), (2.2, 1.6), (2.4, 1.3), (2.2, 1.3), (2.2, 0.3), (1.6, 0.3), (1.6, 0.8), (1.3, 0.8), (1.3, 0.3), (0.7, 0.3), (0.7, 1.3), (0.5, 1.3), (1.5, 2.2), (1.8, 1.9), (1.8, 2.2)

5 (a) 7.00 am **(b)** 8 km **(c)** 18 mins **(d)** 8.27 am

6 (a) (i) 3.2 h **(ii)** 6.4 h
(b) (i) 20 km/h **(ii)** 12 km/h **(c)** 100 km

7 (b) (i) 50 insects **(ii)** 420 insects
(c) (i) $3\frac{1}{2}$ weeks **(ii)** 5 weeks 6 days

8 (b) (i) 1–1 **(ii)** m–1 **(iii)** m–m **(iv)** m–m
(c) (i) and **(ii)**

9 (b) (i) $\{4, 3, 2, 1, 0, {}^-1\}$ **(ii)** $\{{}^-5, {}^-2, 1, 4, 7\}$
(iii) $\{3, 0, {}^-1, 0, 3\}$ **(iv)** $\{7, 5, 3, 1, {}^-1\}$

11 (c) $x = 3, y = 0$

12 (b) (i) 26 g **(ii)** 81 g **(c) (i)** 23 cm^3 **(ii)** 57 cm^3

13 (a) 4 **(b)** $^-\frac{1}{2}$ **(c)** $^-1\frac{1}{2}$

14 (b) (i) 80 m **(ii)** 135 m **(c) (i)** 3.2 s **(ii)** 4.7 s

15 (b) (i) 132.7 cm^2 **(ii)** 475.3 cm^2
(c) (i) 3.1 cm **(ii)** 10.2 cm

16 (a) D **(b)** B **(c)** A **(d)** C

17 (a)

x	$^-3$	$^-2$	$^-1$	0	1	2	3
2	2	2	2	2	2	2	2
$-3x$	9	6	3	0	$^-3$	$^-6$	$^-9$
$-2x^2$	$^-18$	$^-8$	$^-2$	0	$^-2$	$^-8$	$^-18$
y	$^-7$	0	3	2	$^-3$	$^-12$	$^-25$

(c) $x = {}^-2, \frac{1}{2}$
(d) $^-2$, 0.5

Consolidation

Exercise 9

1 (a) (i)

x	$^-3$	$^-2$	$^-1$	0	1	2	3	4
y	$^-5$	$^-3$	$^-1$	1	3	5	7	9

(ii)

x	$^-3$	$^-2$	$^-1$	0	1	2	3	4
y	$^-9$	$^-7$	$^-5$	$^-3$	$^-01$	1	3	5

(iii)

x	$^-3$	$^-2$	$^-1$	0	1	2	3	4
y	1.5	2	2.5	3	3.5	4	4.5	5

(iv)

x	$^-3$	$^-2$	$^-1$	0	1	2	3	4
y	12	10	8	6	4	2	0	−2

2 (d) No

3 (a) $x = 6, y = 13$ **(b)** $x = 5, y = {}^-1$ **(c)** $x = 3, y = 0$

5 (b) (i) 30.5 m **(ii)** 5.7 seconds

6 (b) (i) 6.4 cm **(ii)** 1.6 inches **(iii)** 2.4 inches

Check out

1 (a) 5.48 **(b)** 102°F

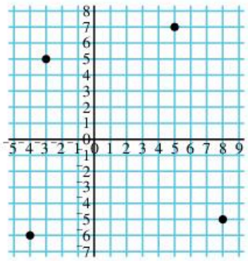
2 (a) and **(b)**

3 (b) (0, $^-7$), (1, $^-5$), (2, $^-3$), (3, $^-1$), (4, 1)

10 Transformations

Check in

1



3 (a) 3 **(b)** 4 **(c)** 2 **(d)** 6

Exercise 10A

1 (a) PQ **(b)** P′XYQ′ **(c)** P′XY
(d) It is a right angle

5 Coordinates are: **(a)** (1, $^-1$), (1, $^-3$), (4, $^-1$)
(b) (1, 2), (4, 3), (4, 1)
(c) ($^-1$, 1), ($^-4$, 1), ($^-4$, 3)

7 (a) (i) (3, 5) **(ii)** (5, $^-2$) **(iii)** ($^-3$, 2)
(b) (i) ($^-3$, $^-5$) **(ii)** ($^-5$, 2) **(iii)** (3, $^-2$)

8 Coordinates are: **(a)** (0, 1), (0, 3), ($^-3$, 1)
(b) (2, $^-3$), (2, $^-5$), (5, $^-3$)

9 Coordinates are: **(a)** (6, 7), (8, 7), (6, 10)
(b) (10, 7), (12, 7), (10, 10)
(c) (5, 3), (5, 5), (8, 5)
(d) (10, 5), (13, 3), (13, 5)

10 Coordinates are: **(a)** (5, 3), (5, 5), (8, 3)
(b) $(^{-}2, 3), (^{-}5, 5), (^{-}5, 3)$
(c) (0, 9), (3, 9), (3, 7)
(d) $(3, 1), (3, ^{-}1), (0, 1)$

11 **(a)** (9, 3)(10, 3)(10, 5) **(b)** $(^{-}1, 3)(0, 3)(0, 5)$
(c) $(^{-}1, 9)(^{-}2, 9)(^{-}2, 7)$ **(d)** $(^{-}1, 1)(^{-}2, 1)(^{-}2, ^{-}1)$

12 **(b)** $x = 3$

13 **(a)** Yes **(d)** They all lie on the mirror line **(e)** Yes

14 **(b)** No **15** **(b)** $y = 4$

Exercise 10B

1 **(a)** 2 **(b)** 4 **(c)** 3 **(d)** 2 **(e)** $\frac{1}{2}$

3 **(b)** (0, 1) **(c)** The angles are the same size, the sides are twice the size.

4 **(b)** (1, 5) **(c)** The lengths are double. **(d)** yes

5 **(a)** $OA' = 2 \times OA$, $OB' = 2 \times OB$, $OC' = 2 \times OC$

7 6 **8** yes

9 **(b)** scale factor 2, centre (3, 0)

10 **(a)** yes **(b)** scale factor $\frac{3}{2}$, centre $(^{-}2, ^{-}2)$

Exercise 10C

1 **(a)** 3 **(b)** 9

2 **(b)** 25

3 Increase in area is scale factor squared

4 **(a)** 12 cm **(b)** 8 cm **(c)** 3 cm **(d)** 320 cm^2

Exercise 10D

1 (a), (b), (c)

3 **(a)** $a = 7.5$ cm, $b = 18$ cm **(b)** $c = 7.5$ cm, $d = 12.5$ cm
(c) $e = 12$ cm, $f = 18$ cm

4 **(b)** **(i)** 7 cm **(ii)** 9 cm

5 **(a)** 29.3 cm **(b)** 3.75 cm

Exercise 10E

1 **(a)** $\begin{pmatrix}4\\2\end{pmatrix}$ **(b)** $\begin{pmatrix}2\\3\end{pmatrix}$ **(c)** $\begin{pmatrix}3\\3\end{pmatrix}$ **(d)** $\begin{pmatrix}4\\0\end{pmatrix}$ **(e)** $\begin{pmatrix}^{-}4\\2\end{pmatrix}$
(f) $\begin{pmatrix}^{-}4\\0\end{pmatrix}$ **(g)** $\begin{pmatrix}4\\^{-}2\end{pmatrix}$ **(h)** $\begin{pmatrix}0\\4\end{pmatrix}$ **(i)** $\begin{pmatrix}^{-}2\\^{-}4\end{pmatrix}$
(j) $\begin{pmatrix}^{-}4\\^{-}2\end{pmatrix}$ **(k)** $\begin{pmatrix}^{-}4\\1\end{pmatrix}$ **(l)** $\begin{pmatrix}^{-}2\\^{-}2\end{pmatrix}$

3 They are parallel.

4 $x = -y$

5 **(b)** AB + BC = AC

6 **(a)** (2, 5) **(b)** (9, 0) **(c)** $(^{-}4, ^{-}1)$

7 **(b)** A′(5, 3) **(c)** B′(7, 3) C′(7, 6) **(d)** yes

Exercise 10F

1 **(a)** $\begin{pmatrix}1\\1\end{pmatrix}$ **(b)** $\begin{pmatrix}3\\1\end{pmatrix}$ **(c)** $\begin{pmatrix}3\\3\end{pmatrix}$ **(d)** $\begin{pmatrix}1\\3\end{pmatrix}$

2 **(a)** (0, 2) **(b)** (2, 2) **(c)** (2, 4)

3 **(a)** $\begin{pmatrix}2\\3\end{pmatrix}$ **(b)** $\begin{pmatrix}1\\4\end{pmatrix}$ **(c)** $\begin{pmatrix}^{-}2\\^{-}3\end{pmatrix}$ **(d)** $\begin{pmatrix}^{-}1\\1\end{pmatrix}$

4 **(b)** $\begin{pmatrix}2\\2\end{pmatrix}$ **(c)** $\begin{pmatrix}3\\3\end{pmatrix}$ **(d)** (3, 3)

5 **(a)** (1, 5), (2, 4) **(b)** **(i)** $\begin{pmatrix}^{-}1\\^{-}5\end{pmatrix}$ **(ii)** $\begin{pmatrix}1\\^{-}1\end{pmatrix}$

6 **(a)** **(i)** $\begin{pmatrix}4\\2\end{pmatrix}$ **(ii)** $\begin{pmatrix}2\\4\end{pmatrix}$ **(b)** $\begin{pmatrix}^{-}4\\^{-}2\end{pmatrix}$ **(c)** $\overrightarrow{OQ}$ **(d)** $\begin{pmatrix}^{-}2\\2\end{pmatrix}$

7 **(b)** $\begin{pmatrix}2\\1\end{pmatrix}$ **(c)** $\begin{pmatrix}4\\2\end{pmatrix}$ **(d)** $\overrightarrow{OA}' = 2\overrightarrow{OA}$

8 **(a)** **(i)** $\begin{pmatrix}2\\4\end{pmatrix}$ **(ii)** $\begin{pmatrix}6\\2\end{pmatrix}$
(b) $\begin{pmatrix}4\\^{-}2\end{pmatrix}$ **(c)** $\begin{pmatrix}2\\^{-}1\end{pmatrix}$ **(d)** $\overrightarrow{AB} = \begin{pmatrix}4\\3\end{pmatrix}$

9 **(a)** **(i)** $\begin{pmatrix}1\\0\end{pmatrix}$ **(ii)** $\begin{pmatrix}4\\0\end{pmatrix}$ **(iii)** $\begin{pmatrix}4\\2\end{pmatrix}$ **(iv)** $\begin{pmatrix}1\\2\end{pmatrix}$
(v) $\begin{pmatrix}^{-}1\\0\end{pmatrix}$ **(vi)** $\begin{pmatrix}^{-}4\\^{-}2\end{pmatrix}$
(b) $\overrightarrow{AB} = \begin{pmatrix}3\\0\end{pmatrix}, \overrightarrow{CD} = \begin{pmatrix}^{-}3\\0\end{pmatrix}$

10 **(a)** $\begin{pmatrix}3\\3\end{pmatrix}$ **(b)** $\begin{pmatrix}3\\3\end{pmatrix}$ **(c)** $\begin{pmatrix}2\\^{-}2\end{pmatrix}$

11 **(a)** **(i)** $\begin{pmatrix}2\\3\end{pmatrix}$ **(ii)** $\begin{pmatrix}4\\2\end{pmatrix}$
(b) **(i)** $\begin{pmatrix}2\\3\end{pmatrix}$ **(ii)** $\begin{pmatrix}4\\2\end{pmatrix}$ **(iii)** $\begin{pmatrix}6\\5\end{pmatrix}$ **(c)** (6, 5)

12 **(a)** **(i)** $\begin{pmatrix}1\\2\end{pmatrix}$ **(ii)** $\begin{pmatrix}4\\2\end{pmatrix}$ **(iii)** $\begin{pmatrix}4\\5\end{pmatrix}$
(b) **(i)** $\begin{pmatrix}3\\3\end{pmatrix}$ **(ii)** $\begin{pmatrix}0\\3\end{pmatrix}$ **(iii)** $\begin{pmatrix}1.5\\1.5\end{pmatrix}$ **(iv)** $\begin{pmatrix}0\\1.5\end{pmatrix}$ **(v)** $\begin{pmatrix}1.5\\0\end{pmatrix}$

Exercise 10G

1 **(a)** 5 **(b)** 13 **(c)** 10 **(d)** 15

2 **(a)** 1 **(b)** 1 **(c)** 1 **(d)** 1

3 **(a)** **(i)** $\begin{pmatrix}1\\1\end{pmatrix}$ **(ii)** $\begin{pmatrix}4\\5\end{pmatrix}$ **(b)** $\begin{pmatrix}3\\4\end{pmatrix}$ **(c)** 5

4 **(a)** **5** **(b)** 13 **(c)** 10 **(d)** 15

Exercise 10H – mixed questions

1 **(b)** $(0, 1), (8, ^{-}2), (10, 1)$ **2** $x = 5$

3 **(a)** (5, 4) **4** AB = 3 cm, 75 cm^2

5 **(a)** **(i)** $\begin{pmatrix}0\\1\end{pmatrix}$ **(ii)** $\begin{pmatrix}3\\5\end{pmatrix}$ **(b)** $\begin{pmatrix}3\\4\end{pmatrix}$ **(c)** 5

6 **(a)** $\begin{pmatrix}4\\4\end{pmatrix}$ **(b)** $\begin{pmatrix}^{-}2\\2\end{pmatrix}$ **(c)** 2.82

7 **(c)** $x = 8$ **(d)** $y = x - 8$

8 **(a)** A′(10, 1), B′(11, 4), D′(7, 1) **(b)** $x = 6$

9 **(a)** A(1, 1), B(4, 1), C(4, 3) **(d)** A′(1, 1), B′(1, 4), C′(3, 4)
(e) $A'(1, ^{-}1), B'(4, ^{-}1), C'(4, ^{-}3)$

Exercise 10

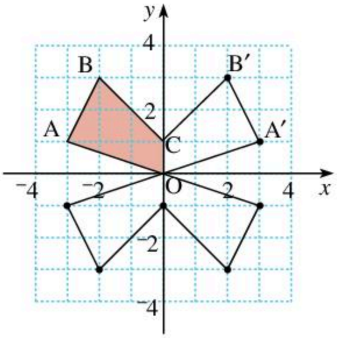
1 **(a)** A′(3, 2), B′(2, 0), C′(1, 3) **(b)** $P'(4, ^{-}1)$, E′(3, 3), F′(3, 1)
(c) $G'(^{-}3, 2), H'(^{-}1, ^{-}1), I'(^{-}2, 0)$

2 After enlargement, corners will be
(a) A′(2, 2), B′(4, 4), C′(4, 8) **(b)** A′(1, 2), B′(3, 4), C′(3, 8)
(c) A′(1, 1), B′(4, 4), C′(4, 10)

3 **(a)** 7.2 cm **(b)** 16.8 cm

4 **(a)** $\begin{pmatrix}4\\^{-}3\end{pmatrix}$ **(c)** 5

5 **(a), (b)**



(c) 2 **(d)** 2

6 **(b)** $\begin{pmatrix} 500 \\ ^{-}1200 \end{pmatrix}$ **(c)** 1300 km **(d)** 2h 10 mn

Check out

1 A(2, 1), B(4, 0), C(7, 4)

3 $a = 5\frac{1}{3}$ cm, $b = 12$ cm

4 **(a)** $\begin{pmatrix} 2 \\ 3 \end{pmatrix}$ **(b)** $\begin{pmatrix} 2 \\ 0 \end{pmatrix}$ **(c)** $\begin{pmatrix} ^{-}2 \\ 2 \end{pmatrix}$

5 **(a)** $\begin{pmatrix} 1 \\ 2 \end{pmatrix}$ and $\begin{pmatrix} 3 \\ 6 \end{pmatrix}$ **(b)** $\begin{pmatrix} 2 \\ 4 \end{pmatrix}$

6 **(a)** $\sqrt{41} = 6.40$ **(b)** 5

11 Statistics and probability

Check in

1 **(a)** sector angles: Cricket 110°, Volleyball 50°, Football 80°, Basketball 70°, Tennis 20°, Badminton 30°

2 mode 10, median 10, mean 10.4

3

Shoe size	Frequency
6	2
7	4
8	6
9	5
10	2
11	1

Exercise 11A

1 **(b)** Intervals of 5 cm, because it is best to use about seven intervals.

2 **(a)**

Number of words	Frequency
0–9	13
10–19	15
20–29	4
30–39	1
40–49	3
50–59	2
60–69	1
70–79	1

(b) 10–19

3 40–49 kg

4 **(a)**

Heights	Tally	Frequency
6–10	卌 I	6
11–15	卌 II	7
16–20	卌 IIII	9
21–25	卌 I	6
26–30	II	2

5 **(a)**

Wait time	Tally	Frequency
0–4	I	1
5–9	I	1
10–14	II	2
15–19	IIII	4
20–24	I	1
24–29	III	3
30–34	卌	5
35–39	IIII	4
40–44		0
45–49	II	2
50–54	II	2

(c) 2 **(d)** 52%

Exercise 11B

1 **(a)** size 4

2

Number of peanuts	Frequency
2	5
3	15
4	7

mode = 3

3 153 cm

4 **(a)** 2 children **(b)** 108

6 16–20 cm

7 **(a)**

Number of cousins	Frequency
1	1
2	0
3	1
4	0
5	2
6	7
7	5
8	4
9	7
10	3

(b) 6 and 9 **(c)** yes, 6 and 9

Exercise 11D

1 **(b)** 3 children **(c)** no

2 **(b)** 3 pairs

3 **(a)** 17 **(b)** 6.28 **(c)** 4

4 **(a)** 37, 40, 42, 44, 53, 55, 55, 56, 57, 60, 62, 63, 68, 70, 71, 72, 82, 86, 90, 91
(b) 61, it is halfway between the middle two numbers.

5 **(a)** 121 cm **(b)** 122 cm

6 **(a)** 145 cm to 149 cm **(b)** 140 cm to 144 cm **(c)** 5 **(d)** 144 cm

Exercise 11E

1 3 **2** 21
3 **(a)** 320 **(b)** 32 **(c)** mode 4, median 16.5
(d) The mean, because the mode and median are too low. **(e)** 33
4 **(a)** 23 **(b)** 34 **(c)** 57; Rikki is the best batsman.
5 **(a)** 9 **(b)** Elvin
6 **(a)** 233 **(b)** 16 **(c)** 14.6
7 **(a)** 20 **(b)** 19 **(c)** 30 **(d)** 64
8 **(a)** 40 **(b)** 30 **(c)** 30 **(d)** median

Exercise 11F

1

Peanuts per pod	Frequency	Totals
1	2	2
2	9	18
3	22	66
4	9	36
Total	42	122

mean = 2.9 peanuts

2

Peanuts per pod	Frequency	Totals
1	36	36
2	10	20
3	3	9
4	1	4
Total	50	69

mean = 1.38 people

3 3.1

4 **(a)**

Mark obtained	Frequency	Totals
0	2	0
1	3	3
2	4	8
3	5	15
4	6	24
5	8	40
6	13	78
7	20	140
8	19	152
9	13	117
10	7	70
Total	100	647

(b) mean = 6.47 **(c)** 59

5 19 eggs per day

Exercise 11G

5 **(a)** $\frac{3}{25}$ **(b)** $\frac{21}{100}$ **(c)** $\frac{6}{25}$ **(d)** $\frac{1}{10}$

9 **(a)** **(i)** $\frac{5}{56}$ **(ii)** $\frac{17}{128}$ **(iii)** $\frac{15}{56}$

10 **(a)** $\frac{23}{80}$ **(b)** $\frac{3}{10}$

Exercise 11H

1 **(a)** $\frac{3}{5}$ **(b)** $\frac{2}{5}$

2 **(a)** $\frac{1}{6}$ **(b)** $\frac{1}{6}$ **(c)** $\frac{1}{6}$ **(d)** $\frac{1}{2}$ **(e)** $\frac{1}{3}$ **(f)** $\frac{5}{6}$

3 **(a)** $\frac{1}{11}$ **(b)** $\frac{4}{11}$ **(c)** $\frac{4}{11}$

4 **(a)** $\frac{3}{5}$ **(b)** $\frac{7}{30}$ **(c)** $\frac{23}{30}$

5 **(a)** $\frac{7}{10}$ **(b)** $\frac{3}{10}$

6 **(a)** $\frac{1}{13}$ **(b)** $\frac{1}{13}$ **(c)** $\frac{1}{2}$ **(d)** $\frac{1}{2}$ **(e)** $\frac{12}{13}$ **(f)** $\frac{5}{13}$ **(g)** $\frac{8}{13}$

7 **(a)** $\frac{2}{15}$ **(b)** $\frac{1}{2}$ **(c)** $\frac{13}{15}$ **(d)** $\frac{1}{2}$ **(e)** $\frac{19}{30}$ **(f)** $\frac{11}{30}$

8 **(a)** HH, HT, TH, TT **(b)** $\frac{1}{4}$ **(c)** $\frac{1}{4}$ **(d)** $\frac{1}{2}$

9 $\frac{1}{2}$ **10** **(a)** $\frac{1}{2}$ **(b)** $\frac{1}{8}$

Exercise 11L – mixed questions

1 **(a)** **(i)** 102 **(ii)** 43 **(iii)** 18 **(b)** Samuels **(c)** 286
(d) 4 **(e)** 30, Browne

2 **(a)**

Age	Frequency
13	5
14	15
15	10
16	6

(c) **(i)** $\frac{5}{18}$ **(ii)** $\frac{5}{9}$

3 **(a)** Sectors: green 100°; red 80°; yellow 40°; blue 100°; black 40° **(b)** $\frac{5}{18}$
4 **(a)** 19 **(b)** 155.5 **(c)** 9
5 **(a)** 9 **(b)** **(i)** 4.925 **(ii)** 5 **(iii)** 5 **(c)** $\frac{1}{4}$
6 5 **7** 28
8 **(a)** 152.5 cm **(b)** 153 cm **(c)** 153 cm
(d) **(i)** $\frac{3}{10}$ **(ii)** $\frac{3}{50}$
9 $1061

10 **(a)**

Machine A Weight (grams)	Frequency
995	1
996	2
997	2
998	3
999	3
1000	4
1001	2
1002	2
1003	1
1004	0

Machine A mean = 999.05

Machine B Weight (grams)	Frequency
995	0
996	0
997	2
998	8
999	5
1000	3
1001	2
1002	0
1003	0
1004	0

Machine B mean = 998.75

(b) Machine A **(c)** Machine A **(d)** **(i)** $\frac{9}{20}$ **(ii)** $\frac{1}{4}$
(d) Machine B – less jars have to be refilled.
(e) Machine B – none of its jars would now have to be refilled.
(f) Yes, it could save time and money.

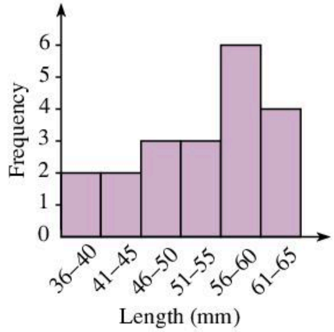
Consolidation

Exercise 11

1 **(a)**

Length (mm)	36–40	41–45	46–50	51–55	56–60	61–65
Frequency	2	2	3	3	6	4

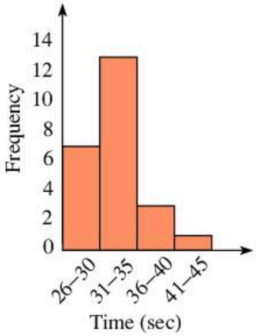
(b)



2 **(a)**

Time (seconds)	26–30	31–35	36–40	41–45
Frequency	7	13	3	1

(b)



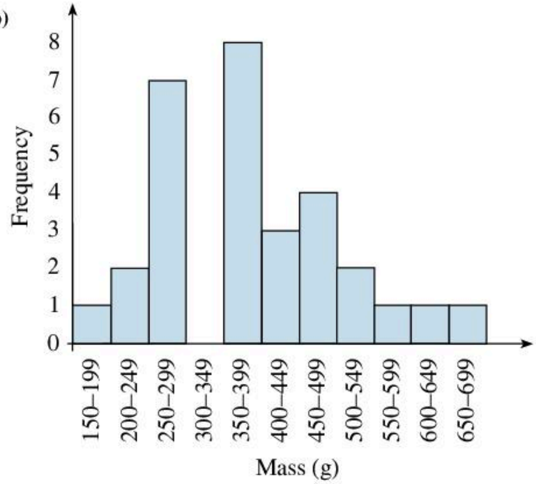
3 **(a)** **(i)** 53 mm **(ii)** 32.7 secs
(b) **(i)** 56–60 **(ii)** 31–35

4 **(a)** $\frac{3}{10}$ **(b)** $\frac{1}{5}$ **(c)** $\frac{7}{10}$ **(d)** $\frac{1}{2}$ **(e)** $\frac{11}{20}$

5 **(a)**

Mass	**Tally**	**Frequency**
150–199	|	1
200–249	||	2
250–299	𝍸 ||	7
300–349		0
350–399	𝍸 |||	8
400–449	|||	3
450–499	||||	4
500–549	||	2
550–599	|	1
600–649	|	1
650–699	|	1

(b)



(c) **(i)** $\frac{1}{30}$ **(ii)** $\frac{2}{5}$

Check out

1 **(a)** 174 cm **(b)** 23
2 **(a)** 6.3 **(b)** 6 or 7 **(c)** 6
3 $42.73
5 **(a)** $\frac{1}{4}$ **(b)** $\frac{3}{20}$
6 **(a)** $\frac{1}{2}$ **(b)** $\frac{2}{13}$ **(c)** $\frac{12}{13}$

12 Consumer arithmetic

Check in

1 **(a)** $1 **(b)** $1.80 **(c)** $3.50 **(d)** $3.85
2 **(a)** $120 loss **(b)** $45 profit
3 **(a)** $144 **(b)** $162 **(c)** £108
4 **(a)** $510 **(b)** $1062.50

Exercise 12A

1 **(a)** 50% **(b)** 25% **(c)** 20% **(d)** 60% **(e)** 4%
(f) 15% **(g)** 85% **(h)** 34% **(i)** 12.5%
(j) 37.5% **(k)** 6.25% **(l)** 43.75% **(m)** 33.33%
(n) 83.33% **(o)** 41.67% **(p)** 36.36%

2

$\frac{1}{8}$	$\frac{1}{4}$	$\frac{3}{8}$	$\frac{1}{2}$	$\frac{5}{8}$	$\frac{3}{4}$	1
$12\frac{1}{2}\%$	25%	$37\frac{1}{2}\%$	50%	$62\frac{1}{2}\%$	75%	100%

3 73.3% **4** 97.8%
5 **(a)** $\frac{1}{5}$ **(b)** **(i)** 1 m **(ii)** 4 students **(iii)** 6 camels
(iv) 20 bottles **(v)** $10 **(vi)** $500
6 **(a)** $2 **(b)** $1.20 **(c)** $3.25 **(d)** $1.35 **(e)** $0.48
(f) $0.30 **(g)** $1.52 **(h)** $0.21 **(i)** $1.69 **(j)** $1.19
7 $64.50 **8** 385 **9** **(a)** $150 **(b)** $\frac{3}{13}$ **(c)** 23.1%

10

Usual price	**Sale price**	**Discount**
$150	$97.50	35%
$200	$170	15%
$30	$24	20%
$26	$19.50	25%
$500	$325	35%

Exercise 12B

1 $14 400 **2** $8800 **3** $5100
4 **(a)** $10 600 **(b)** $18 600 **(c)** $11 600
5 **(a)** $1000 **(b)** $5400 **(c)** $1275 **(d)** $125
(e) $810 **(f)** $2250 **(g)** $12 500 **(h)** $100
(i) $200 **(j)** $20
6 **(a)** $5 **(b)** $10 **(c)** $12 **(d)** $\$\frac{PR}{100}$
7 **(a)** **(i)** $10 **(ii)** 5T$ **(b)** **(i)** $20 **(ii)** 10T$
(c) **(i)** $24 **(ii)** 12T$ **(d)** **(i)** $\$2\frac{PR}{100}$ **(ii)** $\$\frac{PRT}{100}$

Exercise 12C

1

Principal ($)	Rate (%)	Time (years)	Interest ($)
200	3	5	30
50	5	2	5
420	2	5	42
1500	4	8	480
800	6	0.5	24

2 **(a)** $12 **(b)** $2.50 **(c)** $5.50 **(d)** $22.50 **(e)** $18.75
3 **(a)** 5% **(b)** 2% **(c)** 4.5% **(d)** 8% **(e)** 8%
4 6 months; $320; 1.5 years; $7; $300; 3 months

Exercise 12D

1 **(a)** $50 **(b)** $550 **(c)** $55 **(d)** $605
2 **(a)** $210 **(b)** $1664 **(c)** $61.80 **(d)** $6360
3 **(a)** $1210 **(b)** $11664 **(c)** $561.80 **(d)** $31360
4 **(a)** **(i)** $100 **(iii)** $102 **(b)** $5202
5 **(a)** $530.45 **(b)** $10 816 **(c)** $832.32 **(d)** $27 040
6 **(a)** **(i)** $600 **(ii)** $489.60 **(b)** **(i)**
7 **(a)** $8988.80 **(b)** $8960 **(c)** $28.80
8 **(a)** $9984 **(b)** $69984 **(c)** $2916
9 **(a)** $32000 **(b)** $25 600 **(c)** $20 480
10 **(a)** $72250 **(b)** $21 675 **(c)** $73695

Exercise 12E

1 **(a)** $1000 **(b)** $2500 **(c)** $2250 **(d)** $4666.66
(e) $9000
2 **(a)** $600 **(b)** $570 **(c)** $375 **(d)** $37.50 **(e)** $495
3 **(a)** 40 **(b)** 30 **(c)** 35
4 **(a)** $5000 **(b)** $48000 **(c)** $4000
5 **(a)** $48000 **(b)** yes **(c)** $2250 **(d)** $27000
6 **(a)** $456 **(b)** $391

Exercise 12F

1 $500 **2** $380 **3** $784
4 **(a)** $13/hour **(b)** $676
5 **(a)** $404 **(b)** $440 **(c)** $476 **(d)** $500
6 **(a)** $8/hour **(b)** $12/hour **(c)** $380
7 **(a)** 30 **(b)** 38 **(c)** 41 **(d)** 48
8 **(a)** $132.30 **(b)** $245.70

Exercise 12G

1 **(a)** $60 **(b)** $100 **(c)** $40
2 **(a)** $2500 **(b)** $2520 **(c)** $3600
3 **(a)** $7000 **(b)** $9500
4 **(a)** $90 **(b)** $3000
5 **(a)** $5116.67 **(b)** 4% **(c)** $105000

Exercise 12H

1 **(a)** $80 **(b)** $52.50 **(c)** $122.50 **(d)** $133.55
2 **(a)** $37.10 **(b)** $54.15 **(c)** $85.10
3 **(a)** **(i)** $40.60 **(ii)** $71.40
(b) **(i)** 6 **(ii)** 153
4 **(a)** $100.75 **(b)** $106.92 **(c)** $211.08
5 **(a)** $14.56 **(b)** $45 **(c)** $133.41
6 **(a)** 18 408, 18 219 **(b)** 189 **(c)** 50 **(d)** 50
(e) $62.40 **(f)** $85.33
7 **(a)** **(i)** $585.30 **(ii)** $1537.71 **(iii)** $3973.29 **(iv)** $9283.96
(b) **(i)** 4000 **(ii)** 17 000 **(iii)** 35 000
8 **(a)** **(i)** $268.40 **(ii)** $337.10 **(iii)** $3077.32
(b) **(i)** 643 **(ii)** 1325 **(iii)** 5123

Exercise 12I – mixed questions

1 **(a)** $23 **(b)** $3.45 **(c)** $1.73
2 **(a)** $1215 **(b)** $342 **(c)** $35.95
3 **(a)** UWP **(b)** 13.6% **4** 460 ml
5 **(a)** $195 **(b)** $1695 **(c)** $141.25
6 **(a)** $5202 **(b)** $702.27
7 **(a)** is better
8 **(a)** A$23.45; B$107; C$221.06
9 **(a)** $700 **(b)** $4700
10 **(a)** $450 **(b)** $551.25 **(c)** 8
11 **(a)** $68000 **(b)** $54400 **(c)** $43 520
12 **(a)** $740 **(b)** $851 **(c)** $5\frac{3}{4}$ hours
13 **(a)** $47.85 **(b)** $79.10 **(c)** $200.70

Consolidation

Exercise 12

1 **(a)** $1 **(b)** $4.50 **(c)** $1.75 **(d)** 3 goats **(e)** 54 pigs
(f) $1.17 **(g)** $1.26 **(h)** $3.36

2

Usual price	Discount	Sale price
$400	15%	$340
$275	8%	$253
$1995	$7\frac{1}{2}\%$	$1845.38
$3085	$5\frac{1}{2}\%$	$2915.33

3 **(a)** $750 **(b)** $9075 **(c)** $378.13

4

Principal	Rate	Time	Interest
$6000	11%	4 years	$2640
$3500	10%	3 years	$1050
$5630	9%	7 months	$295.58
$9575	$7\frac{1}{2}\%$	6 years	$4308.75

5 **(a)** $4666.67 **(b)** $1480 **(c)** $6146.67
6 **(a)** $19085.90 **(b)** $36 951.25
7 **(a)** Just over $3\frac{1}{2}$ years (3.6 years)
(b) Nearly 6 years (5.9 years)
8 **(a)** $311.15 **(b)** 207 **9** **(a)** $1372 **(b)** $1339.52

Check out

1 **(a)** \$0.08 **(b)** \$94.08 **(c)** \$14.90 **(d)** \$55.76

2 **(a)** \$120 **(b)** 2 years 8 months

3 **(a)** \$202 **(b)** \$41

4 **(b)** **(i)** \$782 **(ii)** 47 hours

5 **(a)** **(i)** \$137.55 **(b)** \$251.31

Revision exercise 3

1 **(a)** **(i)** 0.8 hours **(ii)** 2.2 hours **(c)** **(i)** 20°C
(ii) $^{-}2.5$°C **(iii)** $^{-}14.5$°C

2 **(a)** 2.5 seconds **(b)** 26 m **(c)** 0.7 seconds, 4.3 seconds

3 **(a)** 51 cm **(b)** 7.5 weeks

4 **(a)** 7 seconds **(b)** 6.6 m/s

5 ($^{-}2$, 1), (1, 4), (2.5, 1)

6 **(a)** 0 **(b)** 3 **(c)** 0 **(d)** 3.75

7 **(a)** (0, $^{-}1$), (1, 1), (2, 3), (3, 5) **(c)** 2

8 **(b)** (3, 5), $x = 3$, $y = 5$

9 ($^{-}2$, 0), (0, 2), (1, 1), ($^{-}2$, 4), ($^{-}2$, $^{-}2$)

10 **(a)**

x	0	1	2	3	4	5
x^2	0	1	4	9	16	25
$y = x^2 - 4x$	0	$^{-}3$	$^{-}4$	$^{-}3$	0	5

(c) $x = 3.4$ **(d)** At $x = 0$ and $x = 4$

11 **(a)** translation 2 units right **(b)** reflection in line $x = 0$
(c) reflection in the line $y = 0$

12 **(a)** $\begin{pmatrix} 1 \\ ^{-}1 \end{pmatrix}$ **(b)** $\begin{pmatrix} ^{-}2 \\ ^{-}4 \end{pmatrix}$

14 **(a)** (2, 1), (3, 1), (2, 4) **(b)** (1, 1), (2, 1), (2, $^{-}2$)
(c) (1, 1), (1, 2), (4, 2)

15 **(a)** enlargement with scale factor 2 and centre (0, 0)
(b) 4 times

16 **(a)** ACE and BCD **(b)** **(i)** $6\frac{2}{3}$ cm **(ii)** $10\frac{1}{2}$ cm

17 16 squares

18 **(a)** $\begin{pmatrix} ^{-}3 \\ 2 \end{pmatrix}, \begin{pmatrix} 4 \\ ^{-}3 \end{pmatrix}$ **(b)** $\begin{pmatrix} 3 \\ ^{-}2 \end{pmatrix}$ **(c)** $\begin{pmatrix} 7 \\ ^{-}5 \end{pmatrix}$ **(d)** $\begin{pmatrix} 0.5 \\ ^{-}0.5 \end{pmatrix}$

19 **(a)** $\begin{pmatrix} ^{-}2 \\ 2 \end{pmatrix}$ **(b)** 2.82

20 **(c)** translation 4 units to right

21 38, 35 **22** 4.1 **23** 79, 78

24

Length	Frequency
1	4
2	8
3	5
4	6
5	7
6	6
7	4

mode 2, mean 3.95

25

Height (cm)	Frequency
140–144	4
145–149	3
150–154	7
155–159	6
160–164	4
165–170	1

(c) 150–154 cm

26 6, 10, 14, 15, 15 **27** 1.85 **28** 12 years

29 **(a)** HHH, HHT, HTH, THH, TTH, THT, HTT, TTT
(b) $\frac{1}{8}$

30 **(a)** $\frac{1}{3}$ **(b)** $\frac{4}{9}$ **(c)** $\frac{2}{9}$ **(d)** $\frac{7}{9}$

31 **(a)** **28** **(b)** **(i)** $\frac{19}{28}$ **(ii)** $\frac{4}{7}$ **(iii)** $\frac{1}{7}$

32

Fraction	Decimal	Percentage
$\frac{3}{4}$	0.75	75%
$\frac{1}{10}$	0.1	10%
$\frac{2}{3}$	0.667	$66\frac{2}{3}$%
$\frac{1}{8}$	0.125	$12\frac{1}{2}$%
$1\frac{1}{5}$	1.2	120%

33 **(a)** \$6.50 **(b)** \$42.50 **(c)** \$200 **(d)** \$75 **(e)** \$0.025

34 **(a)** \$180 **(b)** \$375 **(c)** \$2600

35 6.25 years

36 **(a)** \$3481.50 **(b)** \$770.06

37 **(a)** \$4700 **(b)** \$76 800 **(c)** loan, \$2000

38 **(a)** \$679.40 **(b)** \$821.60

39 **(a)** \$3000 **(b)** \$70 000

40 **(a)** **(i)** \$35 **(ii)** \$125 **(iii)** \$320.49
(b) 4000

41 **(a)** \$112 000 **(b)** \$89 600 **(c)** \$71 680

Mixed questions 3

1 D **2** B **3** B **4** C **5** D **6** D **7** B **8** C
9 C **10** D **11** B **12** D **13** B **14** B **15** C
16 C **17** C **18** C **19** D **20** D **21** D **22** C
23 B **24** A **25** B **26** B **27** B **28** A **29** D
30 B **31** B **32** A **33** C **34** C **35** B **36** A
37 C **38** D **39** D **40** D

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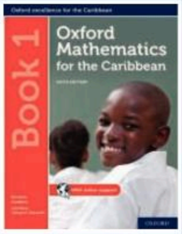

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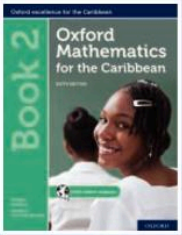

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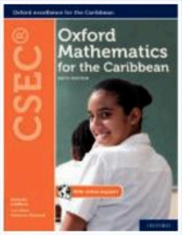

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